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Supply chain structure and demand risk☆

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Abstract

V. Agrawal and S. Seshadri (2000) [Risk intermediation in supply chains. *IIE Transactions*, *32*, 819–831] considered a problem in which a single risk neutral distributor supplies a short-lifecycle, long-leadtime product to several retailers that are identical except in their attitudes towards risk. They proved that the distributor should not offer the same terms to every retailer but instead offer less risky (from the demand risk perspective) contracts to more risk averse retailers. They did not prove the optimality of their menu.

In this paper we reconstruct their results when the number of retailers is infinite and their coefficient of risk aversion is drawn from a continuous distribution. We use optimal control theory to solve this problem. We show that this distribution uniquely determines the channel structure. Moreover, the optimal contract menu not only has the same structure as in Agrawal and Seshadri but is also optimal among nearly all contracts. The implications of these findings for channel design are discussed.
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Keywords: Supply chain structure; Demand risk; Hidden information; Newsvendor model

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1. Introduction

In the standard newsvendor model, a vendor offers a contract to price-taking retailers that face uncertain demand. The retailers choose an order quantity knowing that if the realized demand is larger than her quantity, excess demand will be met through an emergency purchase order at a higher price; otherwise, the unsold product will be re-sold at the salvage price. This contract will be called the "original newsvendor contract" (ONC). The ONC is common in many supply chains. Standard analysis shows that the optimal order quantity under the ONC is given by the "fractile rule" which depends on both the demand distribution as well as the retailer's utility function.

Agrawal and Seshadri (2000) showed that, if retailers have different risk preferences, the single contract offered by the vendor may not achieve the optimal risk reduction. Thus, in practice risk intermediation is often employed. A risk-neutral intermediary (the distributor) can take the vendor's ONC and instead offer a menu of contracts to the retailers. Since the distributor can absorb the risk at a lower cost, she gets benefits from offering risk-reducing contracts to retailers.

In Agrawal and Seshadri's menu, less risky (from the demand risk perspective) contracts are given to more risk averse retailers. Such a menu of contracts increases the distributor's expected profit because the distributor can trade-off the expected value obtained by risk averse retailers against the gain in utility from risk reduction. They left unaddressed the question whether the menu of contracts designed by them is optimal.

In this paper, we reconstruct their results when the number of retailers is infinite and their coefficient of risk aversion is drawn from a continuous distribution. We apply optimal control theory to solve the contract design problem. Surprisingly, the optimal menu not only has the same structure as that given by Agrawal and Seshadri but is also optimal among nearly all contracts. We also show that the distribution of the risk aversion coefficient uniquely determines the channel structure. Thus, distribution systems for products with long supply leadtimes and short lifecycles should bear marked similarities reflecting the attitude towards risk of channel participants.

The rest of the paper is organized as follows. In the next section, we briefly discuss demand risk and its impact on ordering

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decisions, and then present the model in Section 3. Section 4 provides the optimal contract menu when all retailers are offered a contract, and in Section 5 we show that the proposed contract is optimal among all contract menus. Section 6 concludes with suggestions for future work.

2. Demand risk

The impact of risk aversion on the order quantity has been examined in the framework of the "risk averse newsvendor problem." In this problem, the retailer is offered the ordinary newsvendor contract (denoted as ONC) in which items that are ordered before the realization of demand are supplied at the cost of c per unit, items ordered after demand has been realized at e per unit, and unsold goods are taken back at s per unit. For this problem, under the ONC, it is well known that the risk averse retailer's order quantity (i.e., the one that maximizes his expected utility) will be smaller than the order quantity that maximizes his expected profit (see Baron, 1973; Eeckhoudt, Gollier, & Schlesinger, 1995; Horowitz, 1970). The reduction in the order quantity of the retailer leads to lower expected profit (for the retailer). Eeckhoudt et al. give examples in which risk averse retailers will order nothing due to high demand uncertainty. Therefore, risk aversion of the retailers has been portrayed in the literature as leading to the loss of efficiency in supply chains.

Agrawal and Seshadri (2000) showed not only that this loss of efficiency can be eliminated through risk reducing pricing contracts, but also that any risk neutral intermediary will find it beneficial to offer such risk reducing contracts to the retailers. In their model, the intermediary is referred to as the distributor[1] who purchases the goods as per the terms of the ONC from the vendor, and in turn offers the goods to the retailers on contract terms that are less risky from the retailers' viewpoint.

They proposed that, as opposed to the ONC, under the risk reducing contracts offered by the distributor to the retailers, the emergency purchase and the salvage prices should be set equal to the regular purchase price, and in addition a fixed payment should be made by the distributor to the retailer. Therefore, a retailer's payoff consists of a fixed component (independent of the demand) and a variable component that increases linearly with the realized demand. Consequently, as the retailer's payoff depends only upon the demand, the retailer is indifferent to the order quantity decision and is content to relegate the responsibility of determining an order quantity to the distributor. The distributor makes the order quantity decision fully aware that he has to satisfy all the demand faced by the retailer. The distributor bears the cost if necessary of buying the product at the emergency purchase cost and also the cost of disposing any unsold product at the salvage price.

[1] The distributor can be an independent firm, or the vendor, or one of the retailers. For the sake of clarity we will refer to the intermediary as the distributor, and the risk averse players facing uncertain demand as the retailers. The analysis, though, is valid for any two levels in a vertical marketing channel, where the lower level facing uncertain demand is risk averse and the upper level is risk neutral (or less risk averse).

By performing this function of "(demand) risk intermediation", the distributor raises the retailers' order quantities such that the maximum efficiency is obtained. The key contribution in their paper is to establish that the contracts offered to the retailer not only maximize the efficiency in the supply chain but are also optimal from the distributor's viewpoint within the class of contracts that have a fixed payment and a linear price schedule. We show that such contracts are actually optimal for the distributor amongst a much broader class of contracts, thus making the menu designed by Agrawal and Seshadri much more attractive!

Contracts similar to the ones proposed by them are being adopted within the context of vendor managed inventory (VMI) programs. In many VMI programs the vendors make the inventory decisions on behalf of the retailers and also bear the risks and costs associated with these decisions (Andel, 1996). In addition to the contracts found in VMI programs, we have observed several supply contracts that transfer the demand risk from the buyer to the vendor, for example in the publishing (Carvajal, 1998), cosmetics (Moses & Seshadri, 2000), computers (Kirkpatrick, 1997), apparel (Bird & Bounds, 1997), and grocery industries (Lucas, 1996). Other work on risk reducing contracts includes that of Chen, Sim, Simchi-Levi, and Sun (2004), Donohue (2000), Eppen and Iyer (1997), Feng and Sethi (2004), Fisher, Hammond, Obermeyer, and Raman (1994), Fisher and Raman (1996), and Gan, Sethi, and Yan (2004). In contrast to the majority of this work which deals with a single retailer, we focus on the optimal contract for multiple risk averse retailers.

In the last two decades the concept of risk intermediation has been used to create not only novel investment and insurance products but also a global marketplace for such products and services. A large number of firms now offer a menu of products with different risk-return choices to customers worldwide. The existence of a similar market for hedging risky payoffs resulting from uncertain demand should not be entirely surprising. The contracts observed in some of the industries studied by us further confirm the insight provided by our analysis. It is also logical that such contracts are seen for products that have short life cycle or are perishable such as grocery, personal computers and apparel, as these are the industries that are the most vulnerable to demand uncertainty. The single period inventory model as the decision making framework embodied in the newsperson problem is appropriate for such products as well.

3. Model

We consider a single period model in which multiple risk averse retailers purchase a single product from a common vendor. We assume that the retailers operate in identical and independent markets. The retailers face uncertain customer demand with a fixed selling price p, and they accordingly make their purchase order quantity decisions to maximize their expected utility. The distribution of demand faced by a retailer is $F_{\mathrm{D}}(\cdot)$, which is independent of the contracts offered either by the vendor or by the distributor. The vendor has to offer

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the same supply contract(s) to each retailer. The terms of the contract offered to the retailers are to be determined.

Retailers are assumed to be risk averse, but have different degrees of risk aversion. We adopt a mean–variance utility approach, which can be regarded as the order-2 approximation of the original utility function via a Taylor series expansion. That is, let ρ_i denote the Arrow–Pratt risk aversion measure ($\rho_i \equiv -u_i''/u_i'$ where $u_i(\cdot)$ is the retailer i's original utility), and Z be a gamble. Then, retailer i's expected utility is given by $E[Z] - \rho_i(Var[Z]/2)$ (Pratt, 1964). This approach is valid in the small gambles framework since higher-order terms vanish in the Taylor series expansion.

The decision problem of a retailer is to either select a contract from the menu offered by the distributor, or to accept the vendor's ONC. An ONC is characterized by three per-unit parameters c, s, e: c is the purchase price, s is the salvage value, and e is the emergency purchase price. We assume $p \geqslant e$, thus all demand is met. In Agrawal and Seshadri's (2000) model, the distributor offers a menu of contracts, each of which specifies only two terms: the fixed payment $F(\rho)$, and the purchase/salvage/emergency price $c(\rho)$. Later we will show that this restricted class of contracts is broad enough for constructing optimal contract menus.

Retailers are expected utility rather than expected profit maximizers. We define the reservation utility of retailer i, denoted by r_i, as the expected utility she will get upon accepting the vendor's contract ONC. We assume that retailer i will choose a contract from the distributor's menu if it provides at least an expected utility of r_i. We show in Section 4 that r_i's are ordered according to the coefficient of risk aversion.

In the main departure from Agrawal and Seshadri's (2000) model, we assume that the coefficient of risk aversion ρ can take values in the interval [0, 1]. We assume that it has the density function $f_r(\rho)$. In this representation, the fraction of retailers in the population whose coefficient of risk aversion lies in the interval $[\rho, \rho + \mathrm{d}\rho]$ is given by $f_r(\rho)\,\mathrm{d}\rho$. The distribution function of risk aversion and its complement are denoted by F_r and F_r^c. We also assume that the reservation utility is a differentiable and convex function of ρ.

4. Main results

In this section, we first review some results in Agrawal and Seshadri (2000) where they consider a fixed number of retailers (instead of a continuum). The rest of the section focuses on the optimal contract menu with a continuum of retailers.

4.1. Review of the model with discrete types

4.1.1. Single contract

Agrawal and Seshadri find that the distributor has an incentive to cover fewer retailers if the distributor is allowed to offer only a single contract. The number of retailers covered decreases as the demand becomes more volatile (i.e., σ/μ increases), as the emergency cost e increases, and, when the retailers' margin $p - c$ increases. These results are a consequence of the fact that higher s, e, or $p - c$, allow the distributor to make greater profit per retailer. Thus with increasing s, e, or $p - c$ it becomes more attractive for the distributor to "skim" the market and serve only the more profitable retailers.

4.1.2. Menu of contracts

Let us recall the setting of Agrawal and Seshadri (2000). The set of all retailers is denoted by N, where $N = \{1, \ldots, n\}$. Retailer i has a reservation utility r_i and coefficient of risk aversion ρ_i where $\rho_i \leqslant \rho_j,\ \forall i \leqslant j$.[2] Assume that the distributor offers a family of contracts $C = \{F_i, c_i\}$, where the distributor makes payment F_i to retailer i, supplies both regular and emergency orders at price c_i and also accepts returns at the same price c_i. Denote the set of retailers that accept the contract as $S(C)$. Note that the contract (F, p, p, p) is a risk-free contract under which the retailer gets a side payment F and passes the demand to the distributor at unit price of p. The following theorem summarizes their main results:

Theorem 1 (*Agrawal and Seshadri (2000)*). *In the optimal contract menu, there exists a fixed number k such that retailers $k, k+1, \ldots, N$ accept the risk free contract, $F_k = r_k, c_k = p$, and the distributor's profit is maximized by offering the contract $(F_i, c_i),\ i \leqslant k-1$ given by*

$$c_i = p - \left[\frac{2(r_{i+1} - r_i)}{(\rho_{i+1} - \rho_i)\sigma^2}\right]^{0.5},$$

$$F_i + (p - c_i)\mu - \rho_i \frac{(p - c_i)^2 \sigma^2}{2} = r_i. \tag{1}$$

The distributor will offer a menu of contracts $C^* = ((F_1, c_1), \ldots, (F_{k-1}, c_{k-1}), (r_k, p))$, and every retailer will choose a contract, i.e., $S(C^*) = N$. The expected value of contracts is ordered by $\{r_i\}$'s, which is decreasing in ρ_i, and the distributor makes a profit on all contracts.

Remark. Agrawal and Seshadri (2000) prove that the choice of $((F_1, c_1), \ldots, (F_{k-1}, c_{k-1}), (r_k, p))$ eliminates the incentive of any retailer to select a contract that is not designed for her. In particular, as retailer k prefers the risk-free contract (r_k, p) to any other contracts, Property 5 in Agrawal and Seshadri (2000) implies that all retailers $j \geqslant k$ strictly prefer (r_k, p) to all others.

4.2. Optimal contract menu in the continuous case

We now discuss the model with a continuum of $\rho \sim F_r(\cdot)$. We work with the probability triple $([0, 1], \mathscr{B}, F_r(\cdot))$ with $\mathscr{B}$ being the Borel sets on [0, 1]. We make the following assumption on the distribution of ρ in the sequel.

[2] In their paper, the ordering is reversed. We modify it here to make the discrete and continuous cases consistent.

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Assumption 2. $xF_r^c(x)$ is unimodal and has a unique maximum at an interior point $k \in (0, 1)$.

Remark. Unimodality of $xF_r^c(x)$ is commonly assumed in many papers on revenue management, e.g., Lariviere and Porteus (2001) and Yoshida (2002).[3] Note that this assumption is scale invariant, i.e., if x is scaled to bx then $xF_r^c(x/b)$ remains unimodal. Moreover, the "point" that achieves the maximum is also scale invariant. See the proof of Lemma 5 for details.

First, assume that *all* retailers are offered a contract and focus on the design of the optimal menu of contracts. Motivated by Agrawal and Seshadri (2000), we formulate the optimal contract design problem in two stages. In the first stage, we assume that there exists a constant $\tau \in [0, 1]$ such that retailers with $\rho \in [\tau, 1]$ will choose the risk-free contract $(r(\tau), p, p, p)$ from the menu, where $r(\rho)$ is the reservation utility of retailer with risk aversion coefficient ρ. Note that $(r(\tau), p, p, p)$ is the cheapest risk-free contract that satisfies the participation constraints for all retailers $\rho \in [\tau, 1]$ for two reasons: it is risk-free, so has the lowest expected value of all contracts that provide a utility of $r(\tau)$. We show below that it provides utility greater than or equal to $r(\rho)$ for all retailers with ρ in $[\tau, 1]$. We first develop optimal incentive compatible contracts to every retailer $\rho \in [0, \tau)$ under such assumptions. In the second stage, we optimize over the choice of τ. Since we do not exclude the possibility of $\tau = 1$, i.e., only the most risk-averse retailer is offered the risk-free contract, and hence this is without loss of generality.

Consider any menu such that $g(x)$ and $h(x)$ are, respectively, the mean and variance of the payoff to a retailer if menu item x is chosen, where x can take values in the interval $[0, \tau]$ and $\tau \in [0, 1]$. We therefore do not restrict ourselves to the family of contracts considered in Agrawal and Seshadri (2000), and instead consider the most general form of the contracts. This is the most general form because retailers are concerned only about the mean and the variance of the payoff. With some abuse of notation, let a retailer with coefficient of risk aversion equal to $x \leqslant \tau$ choose menu item x. Given a fixed τ, the distributor's problem is to choose $\{(g(\rho), h(\rho)), \rho \in [0, \tau)\}$ that solves the following maximization problem:

$$\begin{aligned}
&\max \left\{ [EV(S_{\text{opt}}^{EV}, c, s, e) - r(\tau)]F_r^c(\tau) + \int_0^\tau (EV(S_{\text{opt}}^{EV}, c, s, e) - g(\rho)) f_r(\rho)\, \mathrm{d}\rho \right\},\\
&\text{s.t. (IC-1) } \rho \in \operatorname*{argmax}_{z\in[0,\tau)} g(z) - \rho h(z), \quad \forall \rho \in [0, \tau),\\
&\quad\text{(IC-2) } r(\tau) \geqslant \max_{z\in[0,\tau)} g(z) - \rho h(z), \quad \forall \rho \in [\tau, 1],\\
&\quad\text{(IR-1) } g(\rho) - \rho h(\rho) - r(\rho) \geqslant 0, \quad \forall \rho \in [0, \tau),\\
&\quad\text{(IR-2) } r(\tau) \geqslant r(\rho), \quad \forall \rho \in [\tau, 1], \qquad (2)
\end{aligned}$$

where $S_{\text{opt}}^{EV} \equiv F_{\text{D}}((e-c)/(e-s))$ ($F_{\text{D}}(\cdot)$ denotes the demand distribution) is the expected value maximizing order quantity as defined in Eq. (2) of Agrawal and Seshadri (2000). $EV(S_{\text{opt}}^{EV}, c, s, e)$ is the expected cost that the distributor has to pay for buying the vendors' ONC.

In Eq. (2), the first two inequalities are incentive compatibility (IC) conditions for, respectively, the retailers that receive a specific contract designed for her and the retailers that accept the risk-free contract. In (IC-1), the contract menu is incentive compatible since the utility of retailer ρ is maximized if she chooses the contract with mean $g(\rho)$ and variance $h(\rho)$. On the other hand, $r(\tau)$ is the utility of retailer $\rho \geqslant \tau$ when she receives the risk-free contract, and (IC-2) guarantees that she prefers this to any other contracts $g(z), h(z)$ with $z \in [0, \tau)$.

The last two inequalities in Eq. (2) represent individual rationality (IR) conditions, i.e., each retailer shall get at least her reservation utility. Note that the reservation utility $r(\rho)$ can be explicitly expressed as $r(\rho) = \max_S \{E[\Pi(S, 0, c, s, e)] - \rho(Var[\Pi(S, 0, c, s, e)]/2)\}$, where $\Pi(S, 0, c, s, e)$ is the profit if the ONC is accepted and the order quantity is S. Lemma 3.1 in Agrawal and Seshadri (2000) shows that with this expression $r(\rho)$ is strictly decreasing in ρ, and hence the last inequality (IR-2) is automatically satisfied. To see this let $\rho_1 < \rho_2$. If retailer ρ_1 uses retailer ρ_2's order quantity, she gets the same mean and variance but a higher expected utility because ρ_1 is less than ρ_2. If she optimizes the order quantity, then her expected utility can only be higher. Thus, $r(\rho_1) \geqslant r(\rho_2)$.

Our strategy is to first ignore the IC conditions for retailers $\rho \in (\tau, 1]$ (IC-2), and then verify that they are satisfied by our proposed menu. We assume that for retailer $\rho \in [0, \tau)$ the first-order condition for interior optimality (or local optimality—LO) hold:

$$\text{(LO)} \quad \left[\frac{\mathrm{d}g(z)}{\mathrm{d}z} - \rho \frac{\mathrm{d}h(z)}{\mathrm{d}z}\right]_{z=\rho} = 0, \quad \forall \rho \in [0, \tau]. \qquad (3)$$

We shall replace constraints (IC-1) and (IC-2) in Eq. (2) by (LO), and obtain the necessary conditions for optimality for the modified version of the problem. A candidate menu will then be proposed based on this relaxed optimization problem, and later we prove that (LO) for that menu ensures that each retailer $\rho \in [0, \tau)$ is choosing her contract optimally.

Denote the expected value of the utility obtained by retailer ρ as $\hat{r}(\rho) = g(\rho) - \rho h(\rho)$. Using Eq. (3) gives

$$\frac{\mathrm{d}\hat{r}(\rho)}{\mathrm{d}\rho} = \left[\frac{\mathrm{d}g(\rho)}{\mathrm{d}\rho} - \rho \frac{\mathrm{d}h(\rho)}{\mathrm{d}\rho} - h(\rho)\right] = -h(\rho). \qquad (4)$$

This implies that

$$g(\rho) = \hat{r}(\rho) + \rho h(\rho) = \hat{r}(\rho) - \rho \frac{\mathrm{d}\hat{r}(\rho)}{\mathrm{d}\rho}. \qquad (5)$$

Now we come back to the distributor's optimization problem Eq. (2). Observing that the term $[EV(S_{\text{opt}}^{EV}, c, s, e) - r(\tau)]F_r^c(\tau)$ is independent of the choice of $\{(g(\rho), h(\rho)),\ \rho \in [0, \tau)\}$, the distributor's problem becomes

$$\max \int_0^\tau \left[-\hat{r}(\rho) + \rho \frac{\mathrm{d}\hat{r}(\rho)}{\mathrm{d}\rho}\right] f_r(\rho)\, \mathrm{d}\rho$$

[3] The support need not be [0, 1] for distributions we discuss here. We restrict to [0, 1] in this paper for merely notational ease. Another popular assumption for demand unimodality in revenue management is that the revenue is concave in demand, see Gallego and van Ryzin (1994).

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subject to $\hat{r}(\rho) \geqslant r(\rho)$, $\hat{r}(\tau) = r(\tau)$, $\hat{r}(0) = r(0)$. (Why pay the risk neutral retailer more than the expected value? Therefore $\hat{r}(0) = r(0)$.) We can rewrite the problem as

$$\max \int_0^\tau \left[-\hat{r}(\rho) + r(\rho) + \rho \frac{\mathrm{d}\hat{r}(\rho)}{\mathrm{d}\rho} - \rho \frac{\mathrm{d}r(\rho)}{\mathrm{d}\rho} \right] f_r(\rho)\,\mathrm{d}\rho, \quad (6)$$

subject to the constraints since the added terms are independent of the policy $g(\rho)$. Let $x(\rho) \equiv \hat{r}(\rho) - r(\rho)$ be the state variable, and $u(\rho) = \mathrm{d}(\hat{r}(\rho) - r(\rho))/\mathrm{d}\rho$ be the control. Through this transformation, the design of the optimal menu of contracts can be recast as an optimal control problem and can be solved by use of calculus of variation. The Hamiltonian is given by

$$H(\rho) = (-x(\rho) + \rho u(\rho)) f_r(\rho) + \lambda(\rho) u(\rho). \quad (7)$$

The adjoint equation is given by $\mathrm{d}\lambda(\rho)/\mathrm{d}\rho = -\partial H/\partial x = f_r(\rho)$, and the transversality condition gives no information. Let $\lambda(\tau) = c$ where c is some constant, we obtain

$$\lambda(\rho) = c - F_r^c(\rho). \quad (8)$$

The necessary condition for optimality is that the Hamiltonian is maximized by the choice of u. From Eqs. (7) and (8), H is linear in u, and the coefficient of u in H is

$$c + \rho f_r(\rho) - F_r^c(\rho). \quad (9)$$

If the expression in Eq. (9) were positive, the solution would be unbounded. Due to our assumption about the uniqueness of the maximum, it is not hard to see that $c = 0$. Note that $\rho f_r(\rho) - F_r^c(\rho)$ is the derivative of $-\rho F_r^c(\rho)$, and hence from Assumption 2, $\rho f_r(\rho) - F_r^c(\rho) > 0$ if $\rho > k$, and $\rho f_r(\rho) - F_r^c(\rho) < 0$ if $\rho < k$. The case $\rho = k$ has measure zero and hence it will not contribute to the integral Eq. (6). If $\rho f_r(\rho) - F_r^c(\rho) > 0$, there is no maximum since we can take $u \to \infty$. On the other hand, when $\rho f_r(\rho) - F_r^c(\rho) < 0$ we should make u as negative as possible. But, the boundary conditions $\hat{r}(\rho) \geqslant r(\rho)$ on $[0, \tau]$ on the other hand they require that $u(\rho)$ be greater than or equal to zero whenever $\hat{r}(\rho) = r(\rho)$. It therefore follows that $\hat{r}(\rho) = r(\rho)$ for all ρ in $[0, \tau]$.

The following theorem summarizes our results thus far.

Theorem 3. *Suppose Assumption* 2 *holds*, *retailers with* $\rho \in [\tau, 1]$ *choose contract* $(r(\tau), p, p, p)$, *where constant* $\tau \in [0, 1]$, *and the distributor has to serve all retailers. Then the necessary conditions for the optimal contract menu are* (i) (*LO*) *in Eq.* (3) *and* (ii) *retailers* $\rho \in [0, \tau]$ *receive their reservation utilities*.

4.2.1. Candidate menu and verifying the necessary and sufficient conditions

Now we will propose a candidate menu of contracts. The inspiration is due to the optimal menu in the discrete version, i.e., the one proposed in Theorem 1. We will focus on the class of contracts with a fixed franchise fee and common cost $\{F(\rho), c(\rho)\}$, and prove that this class is broad enough to achieve the optimality. Passing to the limit in Eq. (1), the cost $c(\rho)$ charged to the retailer with a coefficient of risk aversion equal to ρ and the corresponding fixed side payment $F(\rho)$ are given by the solution to

$$c(\rho) = p - \left(-\frac{2\mathrm{d}r(\rho)}{\mathrm{d}\rho} \frac{1}{\sigma^2} \right)^{0.5},$$

$$F(\rho) + (p - c(\rho))\mu - \rho \frac{(p - c(\rho))^2 \sigma^2}{2} = r(\rho). \quad (10)$$

The corresponding $g(\rho)$ and $h(\rho)$ are $F(\rho) + (p - c(\rho))\mu$ and $(p - c(\rho))^2 \sigma^2/2$, $\forall \rho \in [0, \tau)$.

We will verify now that the proposed contract menu satisfies the necessary and sufficient conditions.

Checking condition (ii) *in Theorem* 3. With the menu shown in Eq. (10), retailers with $\rho \in [0, \tau]$ receive their reservation utilities.

Checking (IC-1). We now verify that (LO) implies global optimality for $\rho \in [0, \tau)$. Suppose a retailer $\rho \in [0, \tau]$ chooses the contract designed for retailer $z \in [0, \tau]$. The fixed side payment is $F(z) = r(z) - (p - c(z))\mu + z((p - c(z))^2 \sigma^2/2)$, and hence her payoff by doing so will be $r(z) - (\rho - z)((p - c(z))^2 \sigma^2/2)$. From Eq. (10), we have $p - c(z) = (-2\mathrm{d}r(y)/\mathrm{d}y|_{y=z}(1/\sigma^2))^{0.5}$, and hence retailer ρ's payoff becomes $r(z) - (\rho - z)\mathrm{d}r(y)/\mathrm{d}y|_{y=z}$. Recall that if she chooses her own contract, she receives her reservation utility $r(\rho)$. Thus, (IC-1) boils down to

$$r(\rho) \geqslant r(z) + (\rho - z) \left. \frac{\mathrm{d}r(y)}{\mathrm{d}y} \right|_{y=z}, \quad \forall z \in [0, \tau), \quad (11)$$

which is equivalent to saying that $r(\rho)$ is convex.

The convexity of $r(\rho)$ is established in Lemma 3.2 of Agrawal and Seshadri (2000), and we briefly present the proof here. Suppose that $\rho_1 < \rho_2 < \rho_3$ and S_2 is the optimal ordering quantity under the ONC for retailer with ρ_2. If retailers with ρ_1 and ρ_2 use S_2 as the ordering quantity, we have $r(\rho_2) = g(S_2) - \rho_2 h(S_2)$ and $r(\rho_1) \geqslant g(S_2) - \rho_1 h(S_2)$. Thus $r(\rho_1) - r(\rho_2) \geqslant (\rho_2 - \rho_1) h(S_2)$, which yields $h(S_2) \leqslant (r(\rho_1) - r(\rho_2))/(\rho_2 - \rho_1)$. On the other hand, if using S_2 as the ordering quantity for both retailers with ρ_2 and ρ_3, we obtain

$$r(\rho_2) - r(\rho_3) \leqslant (\rho_3 - \rho_2) h(S_2) \quad \Rightarrow h(S_2) \geqslant \frac{r(\rho_2) - r(\rho_3)}{\rho_3 - \rho_2}.$$

Combining both cases, we have $\forall \rho_1 < \rho_2 < \rho_3$,

$$\frac{r(\rho_1) - r(\rho_2)}{\rho_2 - \rho_1} \geqslant \frac{r(\rho_2) - r(\rho_3)}{\rho_3 - \rho_2},$$

i.e.,

$$\frac{r(\rho_2) - r(\rho_1)}{\rho_2 - \rho_1} \leqslant \frac{r(\rho_3) - r(\rho_2)}{\rho_3 - \rho_2}.$$

Therefore $r(\rho)$ is convex.

Since $r(\rho)$ is convex, Eq. (11) is valid, and retailer ρ's payoff attains its maximum when contract $(F(\rho), c(\rho))$ is selected, $\forall \rho \in [0, \tau)$. Thus, (IC-1) is true.

Checking (IC-2). The IC condition for retailer τ yields $r(\tau) \geqslant F(z) - (p - c(z))\mu - \tau\sigma^2 (p - c(z))^2/2$, $\forall z \in [0, \tau)$, and hence for retailer $\rho \in (\tau, 1]$, choosing the risk-free contract

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(which gives $r(\tau)$) is strictly preferred since

$$r(\tau) \geqslant F(z) - (p - c(z))\mu - \tau\frac{\sigma^2(p-c(z))^2}{2}$$
$$> F(z) - (p - c(z))\mu - \rho\frac{\sigma^2(p-c(z))^2}{2}, \quad \forall z \in [0, \tau).$$

The above discussions establish the necessity of optimality given a fixed τ.

Checking the sufficiency. As the Hamiltonian Eq. (7) is linear in the state variable, the derived Hamiltonian is concave in the state variable and satisfies the sufficient condition for optimality (see Theorem 2.2 of Sethi & Thompson, 1981).

Remark. The standard approach to prove optimality (e.g., see Salanie, 1998) is to first assume the single-crossing (sorting) condition, or so-called Spence–Mirrlees condition, whose definition is given below. Suppose that a retailer's payoff is $F - u(q, \rho)$, where F is the monetary transfer, q is the quality level (contract terms), and ρ is the retailer's unobservable "type" (the coefficient of risk aversion). The single-crossing condition, labelled as (SC), requires that $\partial^2 u(q,\rho)/\partial\rho\partial q < 0,\ \forall q$. In words, this condition ensures that types can be ranked according to their marginal utilities, and it implies that utilities of two distinct retailers intersect at most once.

With the single-crossing condition, it can be shown that the necessary and sufficient conditions for (IC-1) and (IR-1) conditions are (LO) and the monotonicity (M) of $h(\rho)$, see Salanie (1998) for details. We now verify that for our proposed menu, both (SC) and (M) hold.

Checking (*M*). The term corresponding to the variance $h(\rho)$ is $\sigma^2(p-c(\rho))^2/2 = -\mathrm{d}r(y)/\mathrm{d}y|_{y=\rho}$, which is indeed monotonic in ρ from the convexity of $r(\rho)$.

Checking (*SC*). Define $q(\rho) = \sigma^2(p - c(\rho))^2/2$. Recall that the retailers possess mean–variance utility $g(z) - \rho h(z)$, and hence the mean $(g(z))$ does not contribute to $\partial^2 u(q,\rho)/\partial\rho\partial q$. Thus $\partial^2 u(q,\rho)/\partial\rho\partial q = -1 < 0,\ \forall q \geqslant 0$, i.e., (SC) is satisfied.

4.2.2. Optimal choice of τ

Now we turn to the second stage: optimizing over the choice of τ. Let $\Xi(\tau)$ denote the profit function of the distributor when retailers that have a coefficient of risk aversion greater than τ are offered the risk-free contract. Using Eq. (10), $\Xi(\tau)$ can be restated as

$$\Xi(\tau) = (EV(S_{\mathrm{opt}}^{EV}, c, s, e) - r(\tau))F_r^c(\tau)$$
$$+ \int_0^\tau (EV(S_{\mathrm{opt}}^{EV}, c, s, e) - r(\rho) + r\,\frac{\mathrm{d}r(\rho)}{\mathrm{d}\rho}) f_r(\rho)\,\mathrm{d}\rho.$$

Using the rule for differentiating under the integral we obtain

$$\frac{\mathrm{d}\Xi(\tau)}{\mathrm{d}\tau} = -\frac{\mathrm{d}r(\tau)}{\mathrm{d}\tau}(F_r^c(\tau) - \tau f_r(\tau)). \qquad (12)$$

From Eq. (12) and the fact that $-\mathrm{d}r(\tau)/\mathrm{d}\tau > 0$, the maxima of the profit function are independent of the reservation utility. Moreover, note that the expression in parentheses in Eq. (12) is the derivative of $\tau F_r^c(\tau)$. Therefore, if the function $\tau F_r^c(\tau)$ has a unique maximum in the interior of [0, 1], then the optimal value of τ is independent of the reservation utility. In other words, the fraction of retailers who select the risk free contract is independent of *product characteristics* if the distribution is unimodal.

Recall that $k \in (0, 1)$ is the value of τ at which the function $\tau F_r^c(\tau)$ attains its maximum. Thus,

$$F_r^c(\tau) - \tau f_r(\tau) \geqslant 0, \quad \tau \in [0, k], \qquad (13)$$

and the necessary condition for optimality of $\Xi(\tau)$ is $\tau = k$. We use $C^* = \{F^*(\rho), c^*(\rho)\}$ to denote the contract menu where $(F^*(\rho), c^*(\rho))$ are as given in Eq. (10) and the corresponding payment when $\rho \in [0, k)$ and $(F^*(\rho), c^*(\rho)) = (r(k), p),\ \forall \rho \in [k, 1]$. Notice that in the continuous case the menu C^* we propose again gives a risk-free contract to all retailers with coefficient higher than k, which is chosen in Eq. (12). This completes the characterization of the optimal menu of contracts, and therefore we have

Theorem 4. *Suppose Assumption 2 holds. Let $k = \mathrm{argmax}_{\tau\in[0,1]}\, \tau F_r^c(\tau)$ and $C^* = \{(F^*(\rho), c^*(\rho)), \rho \in [0, k]\}$. Then the proposed C^* is optimal among the class of menus that serve all retailers. Moreover, under the optimal menu of contracts, all retailers $\rho \in [0, k]$ receive their reservation utilities, and retailers $\rho \in (k, 1]$ are offered the same risk-free contract.*

Note that the class of menus we consider include all menus since retailers' utility functions are of the mean–variance format. Hence, if all retailers ought to be served, C^* is indeed the optimal menu.

Remark. Since $1 \times F^c(1) = 0$, $\tau = 1$ can never occur in optimality.

Remark. If we assume instead the distributor can offer a contract to only one retailer, it becomes an adverse selection problem. This may be of interest to study in future work.

5. Verification of optimality

In Theorem 4, we have shown that if all retailers are served, our proposed contract menu C^* yields the highest expected payoff to the distributor. The purpose of this section is to show that our proposed menu of contracts is indeed optimal even when we allow the distributor to exclude some retailers (for example, offer contracts only to those whose coefficient of risk aversion falls in $[0, 0.25) \cup [0.7, 0.993)$). We do this through three lemmas and a theorem as stated below. The proofs are given in the appendix.

Let $S(\mathscr{C})$ be the set of retailers that receive and accept contracts from the menu $\mathscr{C}$. For each $x \in S(\mathscr{C})$, the menu $\mathscr{C}$ specifies a bundle $(F(x), c(x))$. Needless to say, the sets $S(\mathscr{C})$ of interest should be measurable with respect to the probability space $([0, 1], \mathscr{B}, F_r(\cdot))$. Due to the special structure of our proposed contract, we show that if the distributor wants to serve merely the retailers on an interval $I \subset [0, 1]$ and ignore all other retailers, the optimal one-segment contract menu

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coincides with the proposed contract C^* restricted to the interval I (denoted as $C^*|_I$):

Lemma 5 (*Decomposition*). *Suppose* $C^* = (F^*(x), c^*(x))$ *is the optimal contract menu for* $S(C^*) = [0, 1]$. *Then for any interval* $I \subset [0, 1]$, $C^*|_I$ *is also optimal.*

This lemma says for any given contract C with arbitrary number of segments, the distributor will be better off if she replaces C by menu C^* in every segment. Next we will study two properties of the proposed contract menu C^*, namely the no-skip property and push-to-the-end property.

Lemma 6 (*No-skip property*). *Suppose the distributor adopts menu* C^* *and* $S(C^*)$ *is composed of two disjoint intervals* I_1 *and* I_2, *then the distributor will be better off by offering contracts to all retailers in* I_1, I_2, *and also those between* I_1 *and* I_2.

Applying this lemma inductively, we obtain that if the distributor offers the menu C^*, then the optimal $S(C^*)$ will be an interval. Next we show that while offering family of contracts C^*, the distributor should not leave any uncovered intervals of retailers from both ends.

Lemma 7 (*Push-to-the-end property*). *Suppose the distributor adopts menu* C^* *and* $S(C^*)$ *is nonempty. Let* $\bar{s} \equiv \sup\{x : x \in S(C^*)\}$ *Then it is in the distributor's interest to set* $\bar{s} = 1$. *On the other hand, if* $\underline{s} \equiv \inf\{x : x \in S(C^*)\}$, *the distributor will set* $\underline{s} = 0$.

Combining Lemmas 5–7, if the distributor offers $C^* = \{F^*(x), c^*(x)\}$, she will offer contracts to the entire interval $[0, 1]$ to maximize her profit. Bearing in mind the structure of C^*, we are ready to prove its optimality among all feasible contract menus:

Theorem 8. *The proposed contracts* $(F^*(x), c^*(x))$ *are optimal among all contracts that offer a menu to a measurable set of retailers.*

In our model, the reservation utility of a retailer comes from her alternative "accepting the ONC." Therefore, the reservation utility varies from type to type in nature, and is *decreasing* in ρ. The optimal contract menu C^* enables the distributor to extract all the information rent of retailers who are *less* risk averse, while leaving the retailers with higher risk aversion the full information rent. This result is in strict contrast to the case when reservation utilities are the same for all players, but they differ in their aversion to risk. In that case, it is optimal to give no rent to the most risk-averse player, and give information rent to the rest. In our case, because the players differ in their reservation utilities, we are able to capture some of the difference in reservation utility. Moreover, as less risk-averse retailers have higher utility, we capture their rent and not that of the most risk-averse retailers. This corresponds to case 2 of Section 3.3.1 in Laffont and Martimort (2002) where the discrete case is discussed.

The fact that a continuum of retailers receive a risk-free contract is also worth noting. It is known as the "bunching" phenomenon (Laffont & Martimort, 2002), which may occur in the standard case when the monotone hazard rate property of types fails. Here the bunching occurs in retailers with high risk aversion and the contract offers them the efficient level, i.e., it fully covers the demand risk for those risk averse retailers. More discussion on type-dependent participation constraints can be found in Jullien (2000).

Finally, we show in the following corollary that the proposed menu C^* is unique up to a measure-zero modification, which means all menus properly different from C^* are suboptimal.

Corollary 9. *The menu* C^* *is the cheapest menu that achieves the optimal profit uniquely up to a measure-zero set.*

6. Conclusion and extension

In this paper we show that the contract menu proposed by Agrawal and Seshadri (2000) is indeed optimal among all possible menus, provided that the distribution of risk aversion is continuous and satisfies some mild condition commonly adopted in the revenue management literature. The channel structure is uniquely determined by the distribution, independent of the underlying ONC contract and the demand distribution.

The same results hold for other cases if there exists a parameter y and two functions $g(\cdot), h(\cdot)$ such that the utility of a type-y retailer receiving the contract C is $g(C) + yh(C)$, and the reservation utility is differentiable and decreasingly convex in y (required in Eqs. (10) and (12)). If the payoff of retailers is normally distributed, such an utility structure may show up since the first and second moments are sufficient statistics for all of its moments.

The reason why it does not work for a general utility function (e.g., $g(C) + yh(C) + \delta(y, C)$ where $\delta(y, C)$ is the higher-order term) can be seen by examining the proof of Theorem 4. With this extra term $\delta(y, C)$, Eqs. (4) and (5) both fail, and therefore the optimal control problem cannot be solved simply by use of calculus of variation. Further investigation on general utility functions is needed, especially when the retailers cannot be ordered by a single parameter.

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Appendix A. Proofs in Section 5

Proof of Lemma 5. Let k be the cutoff point such that under contracts C^* retailers in $[0, k]$ get the risk-free contract. First we consider the case when $I \subset [k, 1]$. In this case, if the distributor provides $C^*|_I$ to the retailers, their IC conditions are satisfied and their rents are fully extracted. Thus, no other contract will yield a higher expected revenue for the distributor. If $I \cap [0, k]$ is nonempty, we can derive the optimal one-segment contract C' on I. For ease of explanation we define $I = [\alpha, \beta]$. Recall that the

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retailers are located on I with a rescaled distribution F_r' where $F_r'(x)=0$ if $x<\alpha$, $F_r'(x)=(F_r(x)-F_r(\alpha))/(F_r(\beta)-F_r(\alpha))$, if $\alpha \leqslant x \leqslant \beta$, and $F_r'(x)=1$ if $x \geqslant \beta$.

Now we revisit the second equation of the first-order condition, we find that $(F_r')^c(x) - xf_r'(x) = 1/(F_r(\beta) - F_r(\alpha))[F_r^c(x) - xf_r(x)]$ has again k as the cutoff point since $1/(F_r(\beta)-F_r(\alpha))$ is an irrelevant constant. Thus, the new contract C' on I offers the risk-free contract to retailers in $[\alpha, k]$ and extracts retailers' rent in $[k, \beta]$, which exactly coincides with $C^*|_I$. □

Proof of Lemma 6. It suffices to consider the case with closed intervals since F_r is atomless. Suppose that these two intervals are $[\alpha_1, \beta_1]$ and $[\alpha_2, \beta_2]$ with $\beta_1 < \alpha_2$. Since these two intervals are disjoint, we can find two points a, b such that $\beta_1 < a < b < \alpha_2$. Let $C' = C'' \cup C^*$ where $S(C'') = (a, b)$ and $C'' = \{(F^*(x), c^*(x)),\ \forall x \in S(C'')\}$, i.e., we propose contracts $(F^*(x), c^*(x))$ to those retailers on (a, b). Since the choice of $(F^*(x), c^*(x))$ directly ensures their IC conditions, these retailers on $[\alpha_1, \beta_1]$ and $[\alpha_2, \beta_2]$ will not deviate to choose any contract of C''. The IC conditions for a retailer x on (a, b) again follow from the construction of $(F^*(\cdot), c^*(\cdot))$. By Theorem 4, retailer x in $[a, b]$ receives her reservation utility $r(x)$.

We now state and use Lemma 2.2 in Agrawal and Seshadri (2000). Suppose F_x and F_y are distribution functions of, respectively, random variables X and Y, and $F^{-1}(a) = \inf\{b \in R : F(b) \geqslant a\}$ denotes the "inverse" of distribution F. X is said to be less than Y in the dispersive order if and only if

$$F_x^{-1}(\omega_2) - F_x^{-1}(\omega_1) \leqslant F_y^{-1}(\omega_2) - F_y^{-1}(\omega_1), \quad \forall 0 < \omega_1 \leqslant \omega_2 < 1.$$

Lemma 2.2 of Agrawal and Seshadri (2000) shows that $\Pi(S_{\text{opt}}^{EV}, F^*(x), c^*(x), c^*(x), c^*(x)) \equiv F^*(x) + (p - c^*(x))D$, where D denotes the realized demand, is smaller than the profit under the ONC in the dispersive order. Thus, the distributor benefits from offering the contract $(F^*(x), c^*(x))$ to retailer x. □

Proof of Lemma 7. Suppose $\bar{s} < 1$, then there exists an interval (a, b) such that $\bar{s} < a < b < 1$. Let $C'' = C \cup C'$, where $S(C') = (a, b)$ and $C' = \{(F^*(x), c^*(x),\ \forall x \in (a, b)\}$. Since the IC conditions of any $x \in S(C'')$ are satisfied, no deviation can occur. Hence the distributor receives a higher payoff under C'' than under C^*, which contradicts the assumption that C^* is optimal. Therefore setting $\bar{s} = 1$ is in the distributor's interest.

On the other hand, let us suppose that $\underline{s} > 0$. Lemma 3.5 in Agrawal and Seshadri (2000) implies that when ρ is discrete, if $\underline{s} > 0$, then the distributor will find it profitable by offering a contract to a retailer with $\rho < \underline{s}$. A similar argument shows that in the continuous case, the distributor will be better off by offering contracts to retailers in (c, d), where $0 < c < d < \underline{s}$. Thus $\underline{s} = 0$ is optimal. □

Proof of Theorem 8. Let $\tilde{C}$ be the family of contracts that is optimal and $S(\tilde{C})$ be its associated set. Since $S(\tilde{C})$ is measurable and the probability measure of ρ is equivalent to the Lebesgue measure, for an arbitrarily small constant ε, we can find a closed set $G \subset S(\tilde{C})$ such that the measure of $\{\rho \in G\}$ is greater than or equal to the measure of $\{\rho \in S(\tilde{C})\} - \varepsilon$. Moreover, G is compact by its closedness and the fact $G \subset [0, 1]$, and hence there exists a finite open covering O that covers set G (see, e.g., Royden, 1988). Grouping all the connected sets, we can decompose O into a finite number of disjoint open components, i.e., $O = \sum_{j=1}^{J} O_j$ where $\{O_j\}$'s are mutually nonoverlapping open intervals and J is the number of these intervals.

Define $\pi(C)$ as the expected payoff if the distributor adopts menu C. Now we will propose a new menu of contracts C' whose set $S(C')$ is an interval and $\pi(C') \geqslant \pi(\tilde{C}) - M\varepsilon$, where M is the maximal payoff that the distributor can gain by offering a contract to a retailer. M should be bounded (otherwise the distributor extracts infinite profit from a single retailer!) and can be found by solving the single-retailer problem:

$$M \leqslant \sup_{\rho \in [0,1]} \max_{F, c'} \left\{ E[\Pi(S_{\text{opt}}^{EV}(0, c, s, e)] - (F + (p - c')\mu) : F + (p - c')\mu - \rho(p - c')^2 \frac{\sigma^2}{2} \geqslant r(\rho) \right\},$$

which is bounded by the continuity of the objective and the compactness of $[0, 1]$. Note also that M is a fixed constant independent of the choice of $\tilde{C}$.

First, we replace the contracts offered to those retailers on O by $C'' \equiv (F^*(\rho), c^*(\rho))$ and $S(C'') = O$. On each open interval O_j, Lemma 5 shows that it is optimal within this interval O_j and hence the payoff that the distributor gets from retailers in O_j under C'' is higher than that under $\tilde{C}$. Moreover, since the IC conditions under C'' are a subset of the IC conditions under $(F^*(\rho), c^*(\rho))$ on $[0, 1]$, a retailer ρ will weakly prefer to choose her own $(F^*(\rho), c^*(\rho))$ over all other contracts $(F^*(\rho'), c^*(\rho')), \forall \rho' \in O$. Note that there are some retailers in O that are not considered in $\tilde{C}$ before. Property 8 in Agrawal and Seshadri (2000) says that if a retailer receives the same utility while accepting a contract as that under the ONC, then the distributor will earn positive profit from this retailer. Thus, offering contracts to these retailers while keeping others' contracts fixed still satisfies all IC conditions and can only benefit the distributor. Thus from the retailers on O, the distributor gets at least as much payoff under menu C'' as what she gets on G under $\tilde{C}$.

If $J = 1$, i.e., the open subcovering O is itself an interval, we can define $C' = C''$, and the distributor's payoff under C' is $\pi(C'') \geqslant \pi(\tilde{C}|G) \geqslant \pi(\tilde{C}) - M\varepsilon$, where $\pi(C|G)$ denotes the expected payoff that the distributor gets from retailers in $G \in S(C)$ by offering contract menu C. We define $\pi(C) \equiv \pi(C|S(C))$.

Next we focus on the case $J > 1$. Let α_i, β_i be, respectively, the left-hand and right-hand endpoints of O_j, and we assume without loss of generality that $\beta_i < \alpha_{i+1}, \forall i \in \{1, \ldots, J-1\}$. Since O_j and O_{j+1} are disjoint where $j \leqslant J - 1$, we can find an open interval (a, b) such that $\beta_j < a < b < \alpha_{j+1}$. Note that (a, b) has strictly positive measure. Let $C' = C'' \cup C^*$ where $C^* = (F^*(\rho), c^*(\rho))$ and $S(C^*) = (a, b)$, i.e., we propose contracts $(F^*(\rho), c^*(\rho))$ to those retailers on (a, b). Applying

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Lemma 6, all these retailers in O will not deviate to choose any contract of C^*. The IC conditions on (a, b) are satisfied from the construction of C^*.

Consequently, as long as O is not connected, we can always offer contracts to some retailers in between two intervals O_j and O_{j+1} and yield a (weakly) higher payoff. We then obtain by induction that the optimal contract C' with $O \subset S(C')$ has the "no-skip" property (Lemma 6), i.e., $S(C')$ is an interval. The distributor's payoff under C' is $\pi(C') \geqslant \pi(C'|O) \geqslant \pi(\tilde{C}|G) \geqslant \pi(\tilde{C}) - M\varepsilon$, where the second inequality follows from Lemma 5.

So far we have established that for a given ε, there exists a contract menu C' such that $\pi(C') > \pi(\tilde{C}) - M\varepsilon$ and $S(C')$ is an interval. Finally, since ε can be arbitrarily small and M is fixed, the distributor's payoff under our proposed contracts can be made arbitrarily close to the optimal level, which completes the proof. □

Proof of Corollary 9. An argument similar to the proof of Lemma 5 shows that in every interval, our contract menu C^* is the cheapest menu that extracts the reservation utility $r(\rho)$ from retailers with $\rho \in [k, 1]$. If there exists another contract $\hat{C}$ such that $\hat{C}$ extracts retailers' reservation and is cheaper than C^*, then it can be cheaper in at most a set of countable points (otherwise we would have found an interval over which $\hat{C}$ outperforms C^*). Since the distribution F_r is equivalent to Lebesque measure (it has a density and no singularities), every single point has measure zero, and a countable union of measure zero points also has measure zero (Ash & Doleans-Dade, 1999). Thus the corollary is true. □

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