Rational Attention Allocation Over the Business Cycle

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Abstract

The literature assessing whether mutual fund managers have skill typically regards skill as an immutable attribute of the manager or the fund. Yet, many measures of skill, such as returns, alphas, and measures of stock-picking and market-timing, appear to vary over the business cycle. Because time-varying ability seems far-fetched, these results call into question the existence of skill itself. This paper offers a rational explanation, arguing that skill is a general cognitive ability that can be applied to different tasks, such as picking stocks or market timing. Using tools from the rational inattention literature, we show that the relative value of these tasks varies cyclically. The model generates indirect predictions for the dispersion and returns of fund portfolios that distinguish this explanation from others and which are supported by the data. In turn, these findings offer useful evidence to support the notion of rational attention allocation.

> "What information consumes is rather obvious: It consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention, and a need to allocate that attention efficiently among the overabundance of information sources that might consume it." Simon (1971)

The literature that evaluates skills of mutual fund managers typically regards skill as an immutable attribute of the manager or the fund.[1] Yet, many skill measures vary over the business cycle, such as returns, alphas (Glode 2011), and measures of stock-picking and market-timing (Kacperczyk, Van Nieuwerburgh, and Veldkamp 2011) (hereafter "KVV"). Because time-varying ability seems far-fetched, these results call into question the existence of skill itself. This paper examines a rational explanation for time-varying skill, where skill is a general cognitive ability that can be applied to different tasks, such as picking stocks or market timing, at different points in time. Each period, skilled managers choose how much of their time or cognitive ability (call that "attention") to allocate to each task. When the economic environment changes, the relative payoffs of paying attention to market timing and stock selection shift. The resulting fluctuations in attention allocation look like time-varying skill. While this story might sound plausible, it leaves open three questions. First, why would a manager want his attention allocation to depend on the state of the business cycle? Second, do the manager's attention choices exhibit the same pattern as the time-varying skill observed in the data? If managers want to allocate more attention to stock-picking in booms, do we see better stock picking in booms? Third, if there are many skilled and unskilled managers in an asset market, would the time-series and cross-sectional portfolio and return patterns resemble those in the data? This paper builds a simple theory of attention allocation and portfolio choice and subjects it to these three tests.

The model uses tools from the rational inattention literature (Sims 2003) to analyze the trade-off between allocating attention to each task. In recessions, the abundance of aggregate risk and its high price both work in the same direction to make market timing more valuable. The model generates indirect predictions for the dispersion and returns of fund portfolios that distinguish this explanation from other potential explanations for time-varying skill. It reveals that when skilled managers devote more time to market timing, portfolio dispersion is higher, both among skilled managers and between skilled and unskilled

[1] For theoretical models, see e.g., Mamaysky and Spiegel (2002), Berk and Green (2004), Kaniel and Kondor (2010), Cuoco and Kaniel (2010), Vayanos and Woolley (2010), Chien, Cole, and Lustig (2010), Chapman, Evans, and Xu (2010), and Pástor and Stambaugh (2010). A number of recent papers in the empirical mutual fund literature also find that some managers have skill, e.g., Kacperczyk, Sialm, and Zheng (2005, 2008), Kacperczyk and Seru (2007), Cremers and Petajisto (2009), Huang, Sialm, and Zhang (2011), Koijen (2010), and Baker, Litov, Wachter, and Wurgler (2010).

managers. It predicts that recessions are times when skilled managers outperform others by a larger margin. Finally, it predicts that volatility and recessions should each have an independent effect on attention, dispersion, and performance. All of these predictions are borne out in the mutual fund data.

These findings offer useful evidence to support a variety of theories that use rational attention allocation to explain phenomena in many economic environments. Recent work has shown that introducing attention constraints into decision problems can help explain observed consumption, price-setting, and investment patterns as well as the timing of government announcements and the propensity for governments to be unprepared for rare events.[2] An obstacle to the progress of this line of work is that information is not directly observable, precluding a direct test of whether decision makers actually allocate their attention in a value-maximizing way. While papers such as Klenow and Willis (2007), Mondria, Wu, and Zhang (2010) and Maćkowiak, Moench, and Wiederholt (2009) have also tested predictions of rational inattention models, none has looked for evidence that attention is reallocated, arguably a more stringent test of the theory.

To surmount the problem that attention is unobservable, our model uses an observable variable – the state of the business cycle – to predict attention allocation. Attention, in turn, predicts aggregate investment patterns. Because the theory begins and ends with observable variables, it becomes testable. To carry out these tests, we use data on actively managed equity mutual funds. A wealth of detailed data on portfolio holdings and returns makes this industry an ideal setting in which to test whether decision makers allocate attention optimally.

To explore whether a rational attention allocation can explain the behavior of mutual fund managers, we build a general equilibrium model in which a fraction of investment managers have skill. These skilled managers can observe a fixed number of signals about asset payoffs and choose what fraction of those signals will contain aggregate versus stock-specific information. We think of aggregate signals as macroeconomic data that affect future cash flows of all firms, and of stock-specific signals as firm-level data that forecast the part of firms' future cash flows that is independent of the aggregate shocks. Based on their signals, skilled managers form portfolios, choosing larger portfolio weights for assets that are more

[2]See, for example, Sims (2003) on consumption, Maćkowiak and Wiederholt (2009a, 2009b), and Matejka (2011) on price setting, and Van Nieuwerburgh and Veldkamp (2009, 2010) and Kondor (2009) on financial investment. Reis (2011) considers the optimal timing of government announcements and Maćkowiak and Wiederholt (2011) use rational inattention constraint to model the allocation of cognitive energy to planning for rare events. A related attention constraint called inattentiveness is explored in Reis (2006). Veldkamp (2011) provides a survey of this literature.

likely to have high returns.

The model produces four main predictions. The first prediction is that attention should be reallocated over the business cycle. In the data, recessions are times when unexpected returns are low, aggregate volatility rises, and the price of risk surges. When we embed these three forces in our model, the first has little effect on attention allocation, but the second and third forces both draw attention to aggregate shocks in recessions. The increased volatility of aggregate shocks makes it optimal to allocate more attention to them, because it is more valuable to pay attention to more uncertain outcomes. The elevated price of risk amplifies this reallocation: Since aggregate shocks affect a large fraction of the portfolio's value, paying attention to aggregate shocks resolves more portfolio risk than learning about stock-specific risks. When the price of risk is high, such risk-minimizing attention choices become more valuable. While the idea that it is more valuable to shift attention to more volatile shocks may not be all that surprising, whether changes in the price of risk would amplify or counteract this effect is not obvious.

The second and third predictions do not come from the reallocation of attention. Rather, they help to distinguish this theory from non-informational alternatives and support the idea that at least some portfolio managers are engaging in value-maximizing behavior. The second prediction is counter-cyclical dispersion in portfolio holdings and profits. In recessions, when aggregate shocks to asset payoffs are larger in magnitude, asset payoffs exhibit more comovement. Thus, any portfolio strategies that put exogenously fixed weights on assets would have returns that also comove more in recessions. In contrast, when investment managers learn about asset payoffs and manage their portfolios according to what they learn, fund returns comove *less* in recessions. The reason is that when aggregate shocks become more volatile, managers who learn about aggregate shocks put less weight on their common prior beliefs, which have less predictive power, and more weight on their heterogeneous signals. This generates more heterogeneous beliefs in recessions and therefore more heterogeneous investment strategies and fund returns.

Third, the model predicts time variation in fund performance. Since the average fund can only outperform the market if there are other, non-fund investors who underperform, the model also includes unskilled non-fund investors. Because asset payoffs are more uncertain, recessions are times when information is more valuable. Therefore, the informational advantage of the skilled over the unskilled increases and generates higher returns for informed managers. The average fund's outperformance rises.

The fourth prediction is perhaps the most specific to our theory. It argues that all three of the above effects of recessions come in part from high aggregate volatility, and in part from the high price of risk. Therefore, periods of high aggregate volatility should be periods in which attention is allocated to aggregate shocks, portfolio dispersion is high, and skilled funds outperform. Then, after controlling for volatility, there should also be an additional positive effect of recessions on all three measures. This additional effect comes from the fact that recessions are also times when the price of risk is high. In other words, both volatility and the price of risk have separate effects on skill, dispersion, and performance.

We test the model's four main predictions on the universe of actively managed U.S. mutual funds. To test the first prediction, a key insight is that managers can only choose portfolios that covary with shocks they pay attention to. Thus, to detect cyclical changes in attention, we should look for changes in covariances. KVV does precisely this. They estimate the covariance of each fund's portfolio holdings with the aggregate payoff shock, proxied by innovations in industrial production growth. This covariance measures a manager's ability to time the market by increasing (decreasing) her portfolio positions in anticipation of good (bad) macroeconomic news. This timing covariance rises in recessions. KVV also calculate the covariance of a fund's portfolio holdings with asset-specific shocks, proxied by innovations in earnings. This covariance measures managers' ability to pick stocks that subsequently experience unexpectedly high earnings. Consistent with the theory, this stock-picking covariance increases in expansions.

Second, we test for cyclical changes in portfolio dispersion. We find that, in recessions, funds hold portfolios that differ more from one another. As a result, their cross-sectional return dispersion increases, consistent with the theory. In the model, much of this dispersion comes from taking different bets on market outcomes, which should show up as dispersion in CAPM betas. We indeed find evidence in the data for higher beta dispersion in recessions as well.

Third, we document fund outperformance in recessions.[3] Risk-adjusted excess fund returns (alphas) are around 1.8 to 2.4% per year higher in recessions, depending on the specification. Gross alphas (before fees) are not statistically different from zero in expansions, but they are positive (2.1%) in recessions.[4] These cyclical differences are statistically and

[3] Empirical work by Moskowitz (2000), Kosowski (2006), Lynch and Wachter (2007), and Glode (2011) also documents such evidence, but their focus is solely on performance, not on managers' attention allocation nor their investment strategies. Furthermore, these studies are silent on the specific mechanism that drives the outperformance result, which is one of the main contributions of our paper.

[4] Net alphas (after fees) are negative in expansions (-0.9%) and positive (1.0%) in recessions. Since funds do not set fees in our model, we have no predictions about after-fee alphas. For a theory about why we should expect net alphas to be zero, see Berk and Green (2004).

economically significant.

Fourth, we document an effect of recessions on covariance, dispersion, and performance, above and beyond that which comes from volatility alone. When we use both a recession indicator and aggregate volatility as explanatory variables, we find that both contribute about equally to our three main results. Showing that these results are truly business-cycle phenomena – as opposed to merely high volatility phenomena – is interesting because it connects these results with the existing macroeconomics literature on rational inattention, e.g., Maćkowiak and Wiederholt (2009a, 2009b).

The rest of the paper is organized as follows. Section 1 lays out our model. After describing the setup, we characterize the optimal information and investment choices of skilled and unskilled investors. We show how equilibrium asset prices are formed. We derive theoretical predictions for funds' attention allocation, portfolio dispersion, and performance. Section 2 tests the model's predictions using the context of actively managed mutual funds. Section 3 discusses alternative explanations. We conclude that while a handful of theories could explain one or two of the facts we document, few, if any, alternatives would explain why covariance, dispersion, and performance all vary both with macroeconomic volatility and with recessions.

1 Model

We develop a stylized model whose purpose is to understand how the optimal attention allocation of investment managers depends on the business cycle and how attention affects asset holdings and asset prices. Most of the complexity of the model comes from the fact that it is an equilibrium model. But in order to study the effects of attention on asset holdings, asset prices and fund performance, having an equilibrium model is a necessity. The equilibrium model makes it clear that, while investors might all pay more attention to a particular asset, they cannot all hold more of that asset, because the market must clear. Similarly, an equilibrium model ensures that for every investor that outperforms, there is someone who underperforms as well.

1.1 Setup

We consider a three-period model. At time 1, skilled investment managers choose how to allocate their attention across aggregate and asset-specific shocks. At time 2, all investors choose their portfolios of risky and riskless assets. At time 3, asset payoffs and utility are realized. Since this is a static model, the investment world is either in the recession (R) or in the expansion state (E).[5]

Assets The model features three assets. Assets 1 and 2 have random payoffs f with respective loadings b_1, b_2 on an aggregate shock a, and face stock-specific shocks s_1, s_2. The third asset, c, is a composite asset. Its payoff has no stock-specific shock and a loading of one on the aggregate shock. We use this composite asset as a stand-in for all other assets to avoid the curse of dimensionality in the optimal attention allocation problem. Formally,

$$\begin{aligned} f_i &= \mu_i + b_i a + s_i,\ i \in \{1, 2\} \\ f_c &= \mu_c + a \end{aligned}$$

where the shocks $a \sim N(0, \sigma_a)$ and $s_i \sim N(0, \sigma_i)$, for $i \in \{1, 2\}$. At time 1, the distribution of payoffs is common knowledge; all investors have common priors about payoffs $f \sim N(\mu, \Sigma)$. Let E_1, V_1 denote expectations and variances conditioned on this information. Specifically, $E_1[f_i] = \mu_i$. The prior covariance matrix of the payoffs, Σ, has the following entries: $\Sigma_{ii} = b_i^2\sigma_a + \sigma_i$ and $\Sigma_{ij} = b_i b_j \sigma_a$. In matrix notation:

$$\Sigma = bb'\sigma_a + \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where the vector b is defined as $b = [b_1\, b_2\, 1]'$. In addition to the three risky assets, there exists a risk-free asset that pays a net return, r.

Investors We consider a continuum of atomless investors. In the model, the only ex-ante difference between investors is that a fraction χ of them have *skill*, meaning that they can choose to observe a set of informative signals about the payoff shocks a or s_i. We call these investors skilled mutual funds and describe their signal choice problem below. The remaining unskilled investors observe no information other than their prior beliefs.

Some of the unskilled investors are mutual fund managers. As in reality, there are also

[5] We do not consider transitions between recessions and expansions, although such an extension would be easy in our setting because assets are short lived and their payoffs are realized and known to all investors at the end of each period. Thus, a dynamic model would amount to a succession of static models that are either in the expansion or in the recession state.

non-fund investors. We assume that they are unskilled.[6] The reason for modeling non-fund investors is that without them, the sum of all funds' holdings would have to equal the market (market clearing) and therefore, the average fund return would have to equal the market return. There could be no excess return in expansions or recessions.

Bayesian Updating At time 2, each skilled investment manager observes signal realizations. Signals are random draws from a distribution that is centered around the true payoff shock, with a variance equal to the inverse of the signal precision that was chosen at time 1. Thus, skilled manager j's signals are $\eta_{aj} = a + e_{aj}$, $\eta_{1j} = s_1 + e_{1j}$, and $\eta_{2j} = s_2 + e_{2j}$, where $e_{aj} \sim N(0, K_{aj}^{-1})$, $e_{1j} \sim N(0, K_{1j}^{-1})$, and $e_{2j} \sim N(0, K_{2j}^{-1})$ are independent of each other and across fund managers. Managers combine signal realizations with priors to update their beliefs, using Bayes' law.

Of course, asset prices contain payoff-relevant information as well. Lemma 2 in Appendix A establishes that managers always prefer to process additional private signals, rather than to use the same amount of capacity to process the information in prices. Therefore, we model managers as if they observed prices, but did not exert the mental effort required to infer the payoff-relevant signals.[7]

Since the resulting posterior beliefs (conditional on time-2 information) are such that payoffs are normally distributed, they can be fully described by posterior means, $(\hat{a}_j, \hat{s}_{ij})$, and variances, $(\hat{\sigma}_{aj}, \hat{\sigma}_{ij})$. More precisely, posterior precisions are the sum of prior and signal precisions: $\hat{\sigma}_{aj}^{-1} = \sigma_a^{-1} + K_{aj}$ and $\hat{\sigma}_{ij}^{-1} = \sigma_i^{-1} + K_{ij}$. The posterior means of the stock-specific shocks, $\hat{s}_{ij}$, are a precision-weighted linear combination of the prior belief that $s_i = 0$ and the signal η_i: $\hat{s}_{ij} = K_{ij}\eta_{ij}/(K_{ij} + \sigma_i^{-1})$. Simplifying yields $\hat{s}_{ij} = (1 - \hat{\sigma}_{ij}\sigma_i^{-1})\eta_{ij}$ and $\hat{a}_j = (1 - \hat{\sigma}_{aj}\sigma_a^{-1})\eta_{aj}$. Next, we convert posterior beliefs about the underlying shocks into posterior beliefs about the asset payoffs. Let $\hat{\Sigma}_j$ be the posterior variance-covariance matrix of payoffs f:

$$\hat{\Sigma}_j = bb'\hat{\sigma}_{aj} + \begin{bmatrix} \hat{\sigma}_{1j} & 0 & 0 \\ 0 & \hat{\sigma}_{2j} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

[6]For our results, it is sufficient to assume that the fraction of non-fund investors that are unskilled is higher than that for the investment managers (funds).

[7]We could allow managers to infer this information and subtract the amount of attention required to infer this information from their total attention endowment. That would not change the basic result that investors prefer to learn more about more volatile risks (see Van Nieuwerburgh and Veldkamp (2009)).

Likewise, let $\hat{\mu}_j$ be the 3×1 vector of posterior expected payoffs:

$$\hat{\mu}_j = [\mu_1 + b_1\hat{a}_j + \hat{s}_{1j},\ \mu_2 + b_2\hat{a}_j + \hat{s}_{2j},\ \mu_c + \hat{a}_j]' \tag{1}$$

For any unskilled manager or investor: $\hat{\mu}_j = \mu$ and $\hat{\Sigma}_j = \Sigma$.

Modeling recessions The asset pricing literature identifies three principal effects of recessions: (1) returns are unexpectedly low, (2) returns are more volatile, and (3) the price of risk is high. Section 2.2 discusses the empirical evidence supporting the latter two effects. To capture the return volatility effect (2) in the model, we assume that the prior variance of the aggregate shock in recessions (R) is higher than the one in expansions (E): $\sigma_a(R) > \sigma_a(E)$. To capture the varying price of risk (3), we vary the parameter that governs the price of risk, which is risk aversion. We assume $\rho(R) > \rho(E)$. We continue to use σ_a and ρ to denote aggregate shock variance and risk aversion in the current business cycle state.

The first effect of recessions, unexpectedly low returns, cannot affect attention allocation because attention must be allocated before returns are observed. Yet, unexpected returns could affect managers' return covariances. The difficulty in analyzing this effect is that since agents in our model always know the current state of the business cycle, they cannot be systematically surprised by low asset payoffs in recessions. When low payoffs are expected, asset prices fall, leaving returns unaffected. Therefore, exploring (1) requires a slightly modified model that relaxes rational expectations. The Supplementary Appendix explores this model numerically and shows that the unexpectedly low returns have little effect on the results.[8] The main body of the paper explores the volatility and price of risk effects.

Portfolio Choice Problem We solve this model by backward induction. We first solve for the optimal portfolio choice at time 2 and substitute in that solution into the time-1 optimal attention allocation problem.

Investors are each endowed with initial wealth, W_0. They have mean-variance preferences over time-3 wealth, with a risk-aversion coefficient, ρ. Let E_2 and V_2 denote expectations and variances conditioned on all information known at time 2. Thus, investor j chooses q_j to maximize time-2 expected utility, U_{2j}:

$$U_{2j} = \rho E_2[W_j] - \frac{\rho^2}{2} V_2[W_j] \tag{2}$$

[8]The supplementary appendix is a separate document, not intended for publication.

subject to the budget constraint:

$$W_j = rW_0 + q_j'(f - pr.) \tag{3}$$

Since there are no wealth effects with exponential utility, we normalize W_0 to zero for the theoretical results. After having received the signals and having observed the prices of the risky assets, p, the investment manager chooses risky asset holdings, q_j, where p and q_j are 3-by-1 vectors.

Asset Prices Equilibrium asset prices are determined by market clearing:

$$\int q_j dj = \bar{x} + x, \tag{4}$$

where the left-hand side of the equation is the vector of aggregate demand and the right-hand side is the vector of aggregate supply. As in the standard noisy rational expectations equilibrium model, the asset supply is random to prevent the price from fully revealing the information of informed investors. We denote the 3×1 noisy asset supply vector by $\bar{x} + x$, with a random component $x \sim N(0, \sigma_x I)$.

Attention Allocation Problem At time 1, a skilled investment manager j chooses the precisions of signals about the payoff-relevant shocks a, s_1, or s_2 that she will receive at time 2. We denote these signal precisions by K_{aj}, K_{1j}, and K_{2j}, respectively. These choices maximize time-1 expected utility, U_{1j}, over the fund's terminal wealth:

$$U_{1j} = E_1\left[\rho E_2[W_j] - \frac{\rho^2}{2} V_2[W_j]\right], \tag{5}$$

subject to two constraints.

The first constraint is the *information capacity constraint*. It states that the sum of the signal precisions must not exceed the information capacity:

$$K_{1j} + K_{2j} + K_{aj} \leq K. \tag{6}$$

Note that our model holds each manager's total attention fixed and studies its allocation in recessions and expansions. In Section 1.9, we allow a manager to choose how much capacity for attention to acquire.

Unskilled investors have no information capacity, $K = 0$. In Bayesian updating with normal variables, observing one signal with precision τ^{-1} or two signals, each with precision $\tau^{-1}/2$, is equivalent. Therefore, one interpretation of the capacity constraint is that it allows the manager to observe N signal draws, each with precision K/N, for large N. The investment manager then chooses how many of those N signals will be about each shock.[9]

The second constraint is the *no-forgetting constraint*, which ensures that the chosen precisions are non-negative:

$$K_{1j} \geq 0 \qquad K_{2j} \geq 0 \qquad K_{aj} \geq 0. \tag{7}$$

It prevents the manager from erasing any prior information, to make room to gather new information about another shock.

1.2 Model Solution

Substituting the budget constraint (3) into the objective function (2) and taking the first-order condition with respect to q_j reveals that optimal holdings are increasing in the investor's risk tolerance, precision of beliefs, and expected return on the assets:

$$q_j = \frac{1}{\rho} \hat{\Sigma}_j^{-1} (\hat{\mu}_j - pr). \tag{8}$$

Since uninformed fund managers and non-fund investors have identical beliefs, $\hat{\mu}_j = \mu$ and $\hat{\Sigma}_j = \Sigma$, they hold identical portfolios $\rho^{-1}\Sigma^{-1}(\mu - pr)$.

Using the market-clearing condition (4), equilibrium asset prices are linear in payoffs and supply shocks. We derive the linear coefficients A, B and C such that:

Lemma 1. $p = \frac{1}{r}(A + Bf + Cx)$

A detailed derivation of expected utility and the proofs of this and all further propositions are in Appendix A.

Substituting optimal risky asset holdings from equation (8) into the first-period objective function (5) yields: $U_{1j} = \frac{1}{2} E_1 \left[(\hat{\mu}_j - pr) \hat{\Sigma}_j^{-1} (\hat{\mu}_j - pr) \right]$. Because asset prices are linear functions of normally distributed payoffs and asset supplies, expected excess returns, $\hat{\mu}_j - pr$,

[9]The results are not sensitive to the additive nature of the information capacity constraint. They also hold, for example, for a product constraint on precisions. The entropy constraints often used in information theory take this multiplicative form. Results available upon request.

are normally distributed as well. Therefore, $(\hat{\mu}_j - pr)\hat{\Sigma}_j^{-1}(\hat{\mu}_j - pr)$ is a non-central χ^2-distributed variable, with mean[10]

$$U_{1j} = \frac{1}{2}trace(\hat{\Sigma}_j^{-1}V_1[\hat{\mu}_j - pr]) + \frac{1}{2}E_1[\hat{\mu}_j - pr]'\hat{\Sigma}_j^{-1}E_1[\hat{\mu}_j - pr]. \tag{9}$$

1.3 Bringing Model to Data

The following sections explain the model's four key predictions: attention allocation, dispersion in investors' portfolios, average performance, and the effect of recessions on these objects beyond that of aggregate volatility. For each prediction, we state a hypothesis and explain how we test it.

Our empirical measures use conventional definitions of asset returns, portfolio returns, and portfolio weights. Risky asset returns are defined as $R^i \equiv \frac{f_i}{p_i} - 1$, for $i \in \{1, 2, c\}$, while the risk-free asset return is $R^0 \equiv \frac{1+r}{1} - 1 = r$. We define the market return as the value-weighted average of the individual asset returns: $R^m \equiv \sum_{i\in\{1,2,c\}} w_i^m R^i$, where $w_i^m \equiv \frac{p_i q_i}{\sum_{k\in\{1,2,c\}} p_k q_k}$ and $q_i \equiv \int_j q_i^j$ is the total demand for asset j. Likewise, a fund j's return is $R^j \equiv \sum_{i\in\{0,1,2,c\}} w_i^j R^i$, where $w_i^j \equiv \frac{p_i q_i^j}{\sum_{k\in\{0,1,2,c\}} p_k q_k^j}$. It follows that end-of-period wealth (assets under management) equals beginning-of-period wealth times the fund return: $W^j = W_0^j(1 + R^j)$.

1.4 Prediction 1: Cyclical Attention Re-allocation

First, we derive from the model the prediction that the optimal attention allocation in expansions differs from that in recessions. Specifically, there should be more attention paid to aggregate shocks in recessions and more attention paid to stock-specific shocks in expansions. Recessions involve changes in the volatility of aggregate shocks and changes in the price of risk. In order to see the effect of each aspect of a recession, we consider each separately, beginning with the rise in volatility.

In the model, each skilled manager ($K > 0$) solves for the choice of signal precisions $K_{aj} \geq 0$ and $K_{1j} \geq 0$ that maximize her time-1 expected utility (9). The choice of signal precision $K_{2j} \geq 0$ is implied by the capacity constraint (6). A first prediction of our model is that it becomes relatively more valuable to learn about the aggregate shock, a, in recessions.

[10]If $z \sim N(E[z], Var[z])$, then $E[z'z] = trace(Var[z]) + E[z]'E[z]$, where $trace$ is the matrix trace (the sum of its diagonal elements). Setting $z = \hat{\Sigma}_j^{-1/2}(\hat{\mu}_j - pr)$ delivers the result.

Proposition 1. *If price noise (σ_x) is sufficiently large (condition (42) holds), then the marginal value of a given skilled investor j reallocating an increment of capacity from stock-specific shock $i \in \{1, 2\}$ to the aggregate shock is increasing in the aggregate shock variance: If $K_{aj} = \tilde{K}$ and $K_{ij} = K - \tilde{K}$, then $\partial^2 U / \partial \tilde{K} \partial \sigma_a > 0$.*

Intuitively, in most learning problems, investors prefer to learn about large shocks that are an important component of the overall asset supply, and volatile shocks that have high prior payoff variance. Aggregate shocks are larger in scale, but are less volatile than stock-specific shocks. Recessions are times when aggregate volatility increases, which makes aggregate shocks more valuable to learn about. The converse is true in expansions. Note that this is a partial derivative result. It holds information choices fixed. In any interior equilibrium, attention will be reallocated until the marginal utility of learning about aggregate and stock-specific shocks is equalized. But it is the initial increase in marginal utility which drives this re-allocation.

It would seem logical that learning about aggregate shocks should always be more valuable in times when those shocks are more volatile. But working through the theory teaches us that this is not true under all circumstances. When the parameter restriction (condition (42)) is violated, more aggregate payoff risk (higher σ_a) creates less risk in expected returns (lower $Var[f - pr]$). This is possible when prices are very good at aggregating information (low σ_x), when many agents acquire lots of information about the aggregate shock (high K_a), and when risk aversion is low. This is a sufficient but not a necessary condition for many of our results to hold. In our numerical work, when we choose parameter values that replicate the observed volatility of aggregate stock market returns and simulate the model, (42) is always easily satisfied.

Next, we consider the effect of an increase in the price of risk. The following result shows that the increase in the price of risk induces managers to allocate even more attention to the aggregate shock in recessions. The additional price of risk effect should show up as an effect of recessions, above and beyond what aggregate volatility alone can explain. The parameter that governs the price of risk in our model is risk aversion. The following result shows that an increase in the price of risk (risk aversion) in recessions is an independent force driving the reallocation of attention from stock-specific to aggregate shocks.

Proposition 2. *If the size of the composite asset $\bar{x}_c$ is sufficiently large, then an increase in risk aversion increases the marginal utility of reallocating a unit of capacity from the firm-specific shock to the aggregate shock: $\partial^2 U / \partial \rho \partial (K_{aj} - K_{1j}) > 0$.*

The intuition for this result is that the aggregate shock affects a large fraction of the value of one's portfolio. Therefore, a marginal reduction in the uncertainty about an aggregate shock reduces total portfolio risk by more than the same-sized reduction in the uncertainty about a stock-specific shock. In other words, learning about the aggregate shock is the most efficient way to reduce portfolio risk. The more risk averse an agent is, the more attractive aggregate attention allocation becomes.

As long as the investor's capacity allocation choice is not a corner solution ($K_{aj} \neq 0$ or $K_{aj} \neq K$), a rise in the marginal utility of aggregate shock information increases the optimal K_{aj}. In these environments, skilled investment managers allocate a relatively larger fraction of their attention to learning about the aggregate shock in recessions. But, that effect can break down when assets become very asymmetric because corner solutions arise. For example, if the average supply of the composite asset, $\bar{x}_c$, is too large relative to the supply of the individual asset supplies, $\bar{x}_1$ and $\bar{x}_2$, the aggregate shock will be so valuable to learn about that all skilled managers will want to learn about it exclusively ($K_{aj} = K$) in expansions and recessions. Similarly, if the aggregate volatility, σ_a, is too low, then nobody ever learns about the aggregate shock ($K_{aj} = 0$ always).

Investors' optimal attention allocation decisions are reflected in their portfolio holdings. In recessions, skilled investors predominantly allocate attention to the aggregate payoff shock, a. They use the information they observe to form a portfolio that covaries with a. In times when they learn that a will be high, they hold more risky assets whose returns are increasing in a. This positive covariance can be seen from equation (8) in which q is increasing in $\hat{\mu}_j$ and from equation (1) in which $\hat{\mu}_j$ is increasing in $\hat{a}_j$, which is further increasing in a. The positive covariances between the aggregate shock and funds' portfolio holdings in recessions, on the one hand, and between stock-specific shocks and the portfolio holdings in expansions, on the other hand, directly follow from optimal attention allocation decisions switching over the business cycle. As such, these covariances are the key moments that enable us to test the attention allocation predictions of the model.

Following KVV, we define a fund's *fundamentals-based timing ability*, *Ftiming*, as the covariance between its portfolio weights in deviation from the market portfolio weights, $w_i^j - w_i^m$, and the aggregate payoff shock, a, over a T-period horizon, averaged across assets:

$$Ftiming_t^j = \frac{1}{TN^j} \sum_{i=1}^{N^j} \sum_{\tau=0}^{T-1} (w_{it+\tau}^j - w_{it+\tau}^m)(a_{t+\tau+1}), \tag{10}$$

where N^j is the number of individual assets held by fund j. The subscript t on the portfolio weights and the subscript $t + 1$ on the aggregate shock signify that the aggregate shock is unknown at the time of portfolio formation. Relative to the market, a fund with a high *Ftiming* overweights assets that have high (low) sensitivity to the aggregate shock in anticipation of a positive (negative) aggregate shock realization and underweights assets with a low (high) sensitivity.

When skilled investment managers allocate attention to stock-specific payoff shocks, s_i, information about s_i allows them to choose portfolios that covary with s_i. *Fundamentals-based stock picking ability*, *Fpicking*, hich measures the covariance of a fund's portfolio weights of each stock, relative to the market, with the stock-specific shock, s_i:

$$Fpicking_t^j = \frac{1}{N^j} \sum_{i=1}^{N^j} (w_{it}^j - w_{it}^m)(s_{it+1}). \tag{11}$$

How well the manager can choose portfolio weights in anticipation of future asset-specific payoff shocks is closely linked to her stock-picking ability.

Ftiming and *Fpicking* are closely related to commonly-used measures of market-timing and stock-picking ability. Typical measures of market-timing ability estimate how a fund's holdings of each asset, relative to the market, covary with the systematic component of the stock return, over the next T periods. Before the market return rises, market timers overweight assets that have high betas. Likewise, they underweight assets with high betas in anticipation of a market decline. Similarly, stock picking typically measures how a fund's holdings of each stock, relative to the market, covary with the idiosyncratic component of the stock return. A fund that successfully picks stocks overweights assets that have subsequently high idiosyncratic returns and underweights assets with low subsequent idiosyncratic returns.

The key difference between our measures and the conventional ones is that picking and timing measure how a portfolio covaries with returns, while *Fpicking* and *Ftiming* measure how a portfolio covaries with aggregate and firm-specific fundamentals. KVV examine the cyclical behavior of funds' picking and timing ability, as measured in this more conventional way and show that picking also rises in recessions and timing also rises in expansions, just as *Fpicking* and *Ftiming* do. To test our theory as directly as possible, we use the fundamentals-based measures because they correspond more closely to the idea in the model that funds are learning about fundamentals and using signals about those fundamentals to time the market and pick stocks.

1.5 Prediction 2: Dispersion

Since many studies detect no skill, perhaps the most controversial implication of the previous finding is that investment managers are processing information at all. Our second and third predictions speak directly to that implication.

In recessions, as aggregate shocks become more volatile, the firm-specific shocks to assets' payoffs account for less of the variation, and the comovement in stock payoffs rises. Since asset payoffs comove more, the payoffs to all investment strategies that put fixed weights on assets should also comove more. But when investment managers are processing information, this prediction is reversed. To see why, consider the Bayesian updating formula for the posterior mean of asset payoffs. It is a weighted average of the prior mean μ and the fund j's signal $\eta_j|f \sim N(f, \Sigma_\eta)$, where each is weighted by their relative precision:

$$E[f|\eta_j] = \left(\Sigma^{-1} + \Sigma_\eta^{-1}\right)^{-1} \left(\Sigma^{-1}\mu + \Sigma_\eta^{-1}\eta_j\right) \tag{12}$$

In recessions, when the variance of the aggregate shock, σ_a, rises, the prior beliefs about asset payoffs become more uncertain: Σ rises and Σ^{-1} falls. This makes the weight on prior beliefs μ decrease and the weight on the signal η_j increase. The prior μ is common across agents, while the signal η_j is heterogeneous. When informed managers weigh their heterogeneous signals more, their resulting posterior beliefs become more different from each other and more different from the beliefs of uninformed managers or investors. More disagreement about asset payoffs results in more heterogeneous portfolios and portfolio returns.

Thus, the model's second prediction is that in recessions, the cross-sectional dispersion in funds' investment strategies and returns should rise. The following Proposition shows that funds' portfolio holdings and returns, $q_j'(f - pr)$, display higher cross-sectional dispersion when aggregate risk is higher, in recessions.

Proposition 3. *If condition (42) holds, $\chi K < \sigma_a^{-1}$, then for given K_{aj} and K_{ij}, an increase in aggregate risk, σ_a, increases the dispersion of funds' portfolios $E[\sum_{i\epsilon\{1,2,c\}}(q_{ij} - \bar{q}_i)^2]$, and their portfolio returns $E[((q_j - \bar{q})'(f - pr))^2]$, where $\bar{q} \equiv \int q_j dj$.*

As before, the parameter restriction is sufficient, but not necessary, and is not very tight when calibrated to the data.

Next, we consider the effect of an increase in the price of risk. The following result shows that an increase in the price of risk increases the dispersion of portfolio returns.

Proposition 4. *If the variance of asset supply shocks (σ_x) is sufficiently high (conditions (56) and (57) hold), then for given $K_{aj}, K_{ij}\,\forall j$, an increase in risk aversion ρ increases the dispersion of funds' portfolio returns $E[((q_j-\bar{q})'(f-pr))^2]$.*

The primary reason return dispersion increases is that a higher ρ increases the price of risk and thus the average level of returns. Since the dispersion in returns is increasing in the level of returns, return dispersion increases as well. But this effect has to offset a counter-acting force. Recall that the optimal portfolio for investor j takes the form $q=(1/\rho\hat{\sigma}_j)(\hat{\mu}_j-pr)$. If ρ increases, the scale of q falls. The increase in returns needs to increase dispersion enough to offset the decrease in dispersion coming from the effect of $1/\rho$ reducing q.

To connect this Proposition to the data, we measure portfolio dispersion as the sum of squared deviations of fund j's portfolio weight in asset i at time t, w_{it}^j, from the average fund's portfolio weight in asset i at time t, w_{it}^m, summed over all assets held by fund j, N^j:

$$Portfolio\ Dispersion_t^j=\sum_{i=1}^{N^j}\left(w_{it}^j-w_{it}^m\right)^2 \tag{13}$$

This measure is similar to the portfolio concentration measure in Kacperczyk, Sialm, and Zheng (2005) and the active share measure in Cremers and Petajisto (2009). It is the same quantity as in Proposition 3, except that the number of shares q is replaced with portfolio weights w. Our numerical example shows that the model's fund $Portfolio\ Dispersion$, defined over portfolio weights w, is higher in recessions as well. In recessions, the portfolios of the informed managers differ more from each other and more from those of the uninformed investors. Part of this difference comes from a change in the composition of the risky asset portfolio and part comes from differences in the fraction of assets held in riskless securities. Fund j's portfolio weight w_{it}^j is a fraction of the fund's assets, including both risky and riskless, held in asset i. Thus, when one informed fund gets a bearish signal about the market, its w_{it}^j for all risky assets i falls. Dispersion can rise when funds take different positions in the risk-free asset, even if the fractional allocation among the risky assets remains identical.

The higher dispersion across funds' portfolio strategies translates into a higher cross-sectional dispersion in fund abnormal returns (R^j-R^m). To facilitate comparison with the data, we define the dispersion of variable X as $|X^j-\bar{X}|$. The notation $\bar{X}$ denotes the equally weighted cross-sectional average across all investment managers (excluding non-fund investors).

When funds get signals about the aggregate state a that are heterogenous, they take different directional bets on the market. Some funds tilt their portfolios to high-beta assets and other funds to low-beta assets, thus creating dispersion in fund betas. To look for evidence of this mechanism, we form a CAPM regression for fund j

$$R_t^j = \alpha^j + \beta^j R_t^m + \sigma_\varepsilon^j \varepsilon_t^j \tag{14}$$

and test for an increase in the beta dispersion in recessions as well.

1.6 Prediction 3: Performance

The third prediction of the model is that the average performance of investment managers is higher in recessions than it is in expansions. To measure performance, we want to measure the portfolio return, adjusted for risk. One risk adjustment that is both analytically tractable in our model and often used in empirical work is the certainty equivalent return, which is also an investor's objective (5). The following Proposition shows that the average certainty equivalent of skilled funds' returns exceeds that of unskilled funds or investors by more when aggregate risk is higher, that is, in recessions.

Proposition 5. *If (42) holds, then an increase in aggregate shock variance increases the difference between an informed investor expected certainty equivalent return and the expected certainty equivalent return of an uninformed investor:* $\partial(U_j - U^U)/\partial\sigma_a > 0$.

Corollary 1 in Appendix A.9 shows that a similar result holds for (risk unadjusted) abnormal portfolio returns, defined as the fund's portfolio return, $q_j'(f - pr)$, minus the market return, $\bar{q}'(f - pr)$.

Because asset payoffs are more uncertain, recessions are times when information is more valuable. Therefore, the advantage of the skilled over the unskilled increases in recessions. This informational advantage generates higher returns for informed managers. In equilibrium, market clearing dictates that alphas average to zero across all investors. However, because the data only include mutual funds, our model calculations must similarly exclude non-fund investors. Since investment managers are skilled or unskilled, while other investors are only unskilled, an increase in the skill premium implies that an average manager's risk-adjusted return rises in recessions.

Next, we consider the effect of an increase in the price of risk on performance. The following result shows that the average certainty equivalent of skilled funds' returns exceeds that of unskilled funds by more when the price of risk is higher, that is, in recessions.

Proposition 6. *For given K_{aj}, K_{1j}, K_{2j} strictly positive, an increase in risk aversion ρ for all investors increases the difference in expected certainty equivalent returns between an informed and an uninformed investor: $\partial(U_j - U^U)/\partial\rho > 0$.*

The reason that a higher price of risk leads to higher performance is that information can resolve risk. Therefore, informed managers are compensated for risk that they do not bear because the information has resolved some of their uncertainty about that random outcome. When the price of risk rises, the value of being able to resolve this risk rises as well. Put differently, informed funds take larger positions in risky assets because they are less uncertain about their returns. These larger positions pay off more on average when the price of risk is high.

We measure outperformance by looking at risk-adjusted returns. One way to do that risk adjustment is to estimate (14) for each fund and look at the α of that equation. We also compute αs for similar models with multiple risk factors.

1.7 Do the Theoretical Measures and Empirical Measures Have the Same Properties?

The theoretical propositions refer to payoffs and quantities that have analytical expressions in a model with CARA preferences and normally distributed asset payoffs. But they do not correspond neatly to the returns and portfolio weights that are commonly used in the empirical literature. The commonly used empirical measures, however, are not tractable analytically. This raises the concern that, if we constructed $Ftiming$ and $Fpicking$ measures in the model, allocating attention to aggregate shocks might not manifest itself as high $Ftiming$ and allocating attention to stock-specific risks might not be captured by high $Fpicking$. To allay this concern, we choose parameters and simulate our model in which each fund manager allocates attention and chooses his portfolio optimally. Then, we compute equilibrium prices and portfolio weights and estimate the same regressions on the model-generated data as we do in the real data. This exercise verifies that the empirical and theoretical measures have the same comparative statics.

The supplementary appendix explains how parameters are chosen to match moments of the aggregate and individual stock returns in expansions and recessions, and it documents a complete set of results. For brevity, we only discuss the key comparative statics here.

For our benchmark parameter values, all skilled managers exclusively allocate attention to stock-specific shocks in expansions. In contrast, the bulk of skilled managers learn about the aggregate shock in recessions (87%, with the remaining 13% equally split between shocks 1 and 2). Thus, managers reallocate their attention over the business cycle. Such large swings in attention allocation occur for a wide range of parameters.

This shift in attention allocation is clearly reflected in the fluctuations in $Ftiming$ and $Fpicking$. The simulation results show that skilled investors' $Ftiming$ in recessions is orders of magnitude higher than in expansions. Similarly, we find that skilled funds have positive $Fpicking$ ability in expansions, when they allocate their attention to stock-specific information. Our numerical results also confirm that there is a higher dispersion in the funds' betas, and in their abnormal returns, in recessions. Lastly, the simulations confirm that abnormal returns and alphas, defined as in the empirical literature, and averaged over all funds, are higher in recessions than in expansions. Skilled investment managers have positive excess returns, while the uninformed ones have negative excess returns. Aggregating returns across skilled and unskilled funds results in higher average alphas in recessions.

1.8 Who Underperforms?

The requirement that markets clear implies that not all investors can be successful stock-pickers or market-timers. In each period, someone must make poor stock-picking or market-timing decisions. We explain now why rational, unskilled investors underperform in equilibrium.

Unskilled, passive investors have negative timing ability in recessions. When the aggregate state a is low, most skilled investors sell, pushing down asset prices, p, and making prior expected returns, $(\mu - pr)$, high. Equation (8) shows that uninformed investors' asset holdings increase in $(\mu - pr)$. Thus, their holdings covary negatively with aggregate payoffs, making their $Ftiming$ measure negative. Since no investors learn about the aggregate shock in expansions, prices do not fall when unexpected aggregate shocks are negative. Since the price mechanism is shut down, $Ftiming$ is close to zero for both skilled and unskilled in expansions. Taken together, the average fund exhibits some ability to time the market and exploits that ability at the expense of the uninformed investors, in recessions.

Likewise, unskilled investors will show negative stock-picking ability in expansions. When the stock-specific shock s_i is low, and some investors know that it will be low, they will sell and depress the price of asset i. A low price raises the expected return on the asset $(\mu_i - p_i r)$ for uninformed investors. The high expected return induces them to buy more of the asset. Since they buy more of assets that subsequently have negative asset-specific payoff shocks, these uninformed investors display negative stock-picking ability.

1.9 Endogenous Capacity Choice

So far, we have assumed that skilled investment managers choose how to allocate a fixed information-processing capacity, K. We now extend the model to allow for skilled managers to add capacity at a cost $\mathcal{C}(K)$.[11] We draw three main conclusions. First, the proofs of Propositions 1 and 2 hold for any chosen level of capacity K, below an upper bound, no matter the functional form of $\mathcal{C}$. Endogenous capacity only has quantitative, not qualitative implications. Second, because the marginal utility of learning about the aggregate shock is increasing in its prior variance (Proposition 1), skilled managers choose to acquire higher capacity in recessions. This extensive-margin effect amplifies our benchmark, intensive-margin result. Third, the degree of amplification depends on the convexity of the cost function, $\mathcal{C}(K)$. The convexity determines how elastic equilibrium capacity choice is to the cyclical changes in the marginal benefit of learning. The supplementary appendix discusses numerical simulation results from the endogenous-K model; they are similar to our benchmark results.

2 Evidence from Equity Mutual Funds

Our model studies attention allocation over the business cycle, and its consequences for investors' strategies. We now turn to a specific set of investors, active U.S. mutual fund managers, to test the predictions of the model. The richness of the data makes the mutual fund industry a great laboratory for these tests. In principle, similar tests could be conducted for hedge funds, other professional investment managers, or even individual investors.

2.1 Data

Our sample builds upon several data sets. We begin with the Center for Research on Security Prices (CRSP) survivorship bias-free mutual fund database. The CRSP database provides comprehensive information about fund returns and a host of other fund characteristics, such as size (total net assets), age, expense ratio, turnover, and load. Given the nature of our tests and data availability, we focus on actively managed open-end U.S. equity mutual funds. We further merge the CRSP data with fund holdings data from Thomson Financial. The total number of funds in our merged sample is 3,477. In addition, for some of our exercises, we map

[11] We model this cost as a utility penalty, akin to the disutility from labor in business cycle models. Since there are no wealth effects in our setting, it would be equivalent to modeling a cost of capacity through the budget constraint. For a richer treatment of information production modeling, see Veldkamp (2006).

funds to the names of their managers using information from CRSP, Morningstar, Nelson's Directory of Investment Managers, Zoominfo, and Zabasearch. This mapping results in a sample with 4,267 managers. We also use the CRSP/Compustat stock-level database, which is a source of information on individual stocks' returns, market capitalizations, book-to-market ratios, momentum, liquidity, and standardized unexpected earnings (SUE). The aggregate stock market return is the value-weighted average return of all stocks in the CRSP universe.

Following KVV, we use changes in monthly industrial production, obtained from the Federal Reserve Statistical Release, as a proxy for aggregate shocks. Industrial production is seasonally adjusted. We measure recessions using the definition of the National Bureau of Economic Research (NBER) business cycle dating committee. The start of the recession is the peak of economic activity and its end is the trough. Our aggregate sample spans 312 months of data from January 1980 until December 2005, among which 38 are NBER recession months (12%). We consider several alternative recession indicators and find our results to be robust.[12]

2.2 Motivating Fact: Aggregate Risk and Prices of Risk Rise in Recessions

At the outset, we present empirical evidence for the main assumption in our model: Recessions are periods in which individual stocks contain more aggregate risk and when prices of risk are higher.

Table 1 shows that an average stock's aggregate risk increases substantially in recessions whereas the change in idiosyncratic risk is not statistically different from zero. The table uses monthly returns for all stocks in the CRSP universe. For each stock and each month, we estimate a CAPM equation based on a twelve-month rolling-window regression, delivering the stock's beta, β_t^i, and its residual standard deviation, $\sigma_{\varepsilon t}^i$. We define the aggregate risk of stock i in month t as $|\beta_t^i \sigma_t^m|$ and its idiosyncratic risk as $\sigma_{\varepsilon t}^i$, where σ_t^m is formed monthly as the realized volatility from daily return observations. Panel A reports the results from a time-

[12]We have confirmed our results using an indicator variable for negative real consumption growth, the Chicago Fed National Activity Index (CFNAI), and an indicator variable for the 25% lowest stock market returns as alternative recession indicators. While its salience makes the NBER indicator a natural benchmark, the other measures may be available in a more timely manner. Also, the CFNAI has the advantage that it is a continuous variable, measuring the strength of economic activity. The results on performance are, if anything, stronger using the CFNAI measure than they are with the NBER indicator. Results are omitted for brevity but are available from the authors upon request.

series regression of the aggregate (Columns 1 and 2) and the idiosyncratic risk (Columns 3 and 4), both averaged across stocks, on the NBER recession indicator variable.[13] The aggregate risk is twenty percent higher in recessions than it is in expansions (6.69% versus 8.04% per month), an economically and statistically significant difference. In contrast, the stock's idiosyncratic risk is essentially identical in expansions and in recessions. The results are similar whether one controls for other aggregate risk factors (Columns 2 and 4) or not (Columns 1 and 3). Panel B reports estimates from pooled (panel) regressions of a stock's aggregate risk (Columns 1 and 2) or idiosyncratic risk (Columns 3 and 4) on the recession indicator variable, $Recession$, and additional stock-specific control variables including size, book-to-market ratio, and leverage. The panel results confirm the time-series findings.

A large literature in economics and finance presents evidence supporting the results in Table 1. First, Ang and Chen (2002), Ribeiro and Veronesi (2002), and Forbes and Rigobon (2002) document that stocks exhibit more comovement in recessions, consistent with stocks carrying higher systematic risk in recessions. Second, Schwert (1989, 2011), Hamilton and Lin (1996), Campbell, Lettau, Malkiel, and Xu (2001), and Engle and Rangel (2008) show that aggregate stock market return volatility is much higher during periods of low economic activity. Diebold and Yilmaz (2008) find a robust cross-country link between volatile stock markets and volatile fundamentals. Third, Bloom, Floetotto, and Jaimovich (2009) find that the volatilities of GDP and industrial production growth, obtained from GARCH estimation, and the volatility implied by stock options are much higher during recessions. The same result holds for the uncertainty in several establishment-, firm- and industry-level payoff measures they consider.[14]

The idea that the price of risk rises in recessions is supported by an empirical literature that documents the counter-cyclical nature of risk premia on equity, bonds, options, and currencies.[15] The counter-cyclicality of the variance risk premium suggests that agents are willing to pay a higher price for assets whose payoffs are high when return volatility is high (Drechsler and Yaron 2010). A large theoretical literature has developed that generates such counter-cyclical risk premia, e.g., the external habit model of Campbell and Cochrane (1999)

[13]The reported results are for equally weighted averages. Unreported results confirm that value-weighted averaging across stocks delivers the same conclusion.

[14]Several other pieces of evidence also corroborate the link between volatility and recessions. First, labor earnings volatility is substantially counter-cyclical (Storesletten, Telmer, and Yaron (2004)). Second, small firms face more risk in recessions (Perez-Quiros and Timmermann (2000)). Finally, the notion of Shumpeterian creative destruction is also consistent with such link.

[15]Among many others, Cochrane (2006), Ludvigson and Ng (2009), Lustig, Roussanov, and Verdelhan (2010), and the references therein.

or the variable rare disasters model of Gabaix (2009).

2.3 Testing Prediction 1: Time-Varying Skill

Turning to our main model predictions, we first test whether skilled investment managers reallocate their attention over the business cycle in a way that is consistent with measures of time-varying skill. Learning about the aggregate payoff shock in recessions makes managers choose portfolio holdings that covary more with the aggregate shock. Conversely, in expansions, their holdings covary more with stock-specific information. In a purely empirical paper, KVV show that the extent of market-timing and stock-picking skill varies over the business cycle. We add to their findings, by disentangling the risk price and volatility components of the effect, show that both are independently significant, and that the direction of both effects supports the model's predictions.

To estimate time-varying skill, we need measures of $Ftiming$ and $Fpicking$ for each fund j in each month t. KVV proxy for the aggregate payoff shock with the innovation in log industrial production growth. A time series of $Ftiming_t^j$ is obtained by computing the covariance of the innovations and each fund j's portfolio weights (as in equation (10)), using twelve-month rolling windows. Following equation (11), $Fpicking$ is computed in each month t as a cross-sectional covariance across the assets between the fund's portfolio weights and firm-specific earnings shocks (SUE). KVV then estimate the following two equations using pooled (panel) regression model and calculating standard errors by clustering at the fund and time dimensions.

$$Picking_t^j = a_0 + a_1 Recession_t + \mathbf{a_2 X_t^j} + \epsilon_t^j, \tag{15}$$
$$Timing_t^j = a_3 + a_4 Recession_t + \mathbf{a_5 X_t^j} + \varepsilon_t^j, \tag{16}$$

$Recession_t$ is an indicator variable equal to one if the economy in month t is in recession, as defined by the NBER, and zero otherwise. X is a vector of fund-specific control variables, including the fund age, the fund size, the average fund expense ratio, the turnover rate, the percentage flow of new funds, the fund load, and the fund style characteristics along the size, value, and momentum dimensions.

The KVV parameter estimates appear in columns 1, 2, 4 and 5 of Table 2. Column 1 shows the results for a univariate regression model. In expansions, $Ftiming$ is not different from zero, implying that funds' portfolios do not comove with future macroeconomic information in those periods. In recessions, *Ftiming* increases. The increase amounts to ten percent of a standard deviation of *Ftiming*. It is measured precisely, with a t-statistic of 3. To remedy the possibility of a bias in the coefficient due to omitted fund characteristics correlated with recession times, we turn to a multivariate regression. Our findings, in Column 2, remain largely unaffected by the inclusion of the control variables. Columns 4 and 5 of Table 2 show that the average *Fpicking* across funds is positive in expansions and substantially lower in recessions. The effect is statistically significant at the 1% level. It is also economically significant: Fpicking decreases by approximately ten percent of one standard deviation. KVV show that these results are robust to alternative measures of picking and timing and alternative recession indicator variables, and they investigate in more detail the strategies funds use to time the market.

Our model predicts that *Ftiming* should be higher in recessions, which means that the coefficient on *Recession*, a_4, should be positive. Conversely, the fund's portfolio holdings and its returns covary more with subsequent firm-specific shocks in expansions. Therefore, our hypothesis is that *Fpicking* should fall in recessions, or that a_1 should be negative. The data support both predictions. Portfolio holdings are more sensitive to aggregate shocks in recessions and more sensitive to firm-specific shocks in expansions.

Testing for Separate Effects of Volatility and Recessions. To identify a more nuanced prediction of the model, we can split the recession effect into that which comes from aggregate volatility and that which comes from an increased price of risk. Proposition 1 predicts that an increase in aggregate volatility alone should cause managers to reallocate attention to aggregate shocks. Furthermore, there should be an additional effect of recessions, after controlling for aggregate volatility, that comes from the increase in the price of risk (Proposition 2).

To test for these two separate effects, we re-estimate the previous results with both an indicator for recessions and an indicator for high aggregate payoff volatility. The high-volatility indicator variable equals one in months with the highest volatility of aggregate earnings growth, where aggregate volatility is estimated from Shiller's S&P 500 earnings growth data.[16] We include both NBER recession and high aggregate payoff volatility indicators as explanatory variables in an empirical horse race.

Columns 3 and 6 of Table 2 show that both recession and volatility contribute to a lower

[16]We calculate the twelve-month rolling-window standard deviation of aggregate earnings growth. We choose the volatility cutoff such that 12% of months are selected, the same fraction as NBER recession months.

Fpicking in expansions and a higher Ftiming in recessions. For some of the results the recession effect is slightly stronger, while for others the volatility effect is slightly stronger. Clearly, there is an effect of recessions beyond the one coming through volatility. This is consistent with the predictions of our model, where recessions are characterized both by an increase in aggregate payoff volatility and an increase in the price of risk.

2.4 Testing Prediction 2: Dispersion

The second main prediction of the model states that heterogeneity in fund investment strategies and portfolio returns rises in recessions. To test this hypothesis, we estimate the following regression specification, using various return and investment heterogeneity measures, generically denoted as $Dispersion_t^j$, the dispersion of fund j at month t.

$$Dispersion_t^j = b_0 + b_1 Recession_t + \mathbf{b_2 X_t^j} + \epsilon_t^j, \tag{17}$$

The definitions of *Recession* and other controls mirror those in regression (15). Our coefficient of interest is b_1.

The first dispersion measure we examine is $Portfolio\ Dispersion$, defined in equation (13). It measures a deviation of a fund's investment strategy from a passive market strategy, and hence, in equilibrium, from the strategies of other investors. The results in Columns 1 and 2 of Table 3 indicate an increase in average $Portfolio\ Dispersion$ across funds in recessions. The increase is statistically significant at the 1% level. It is also economically significant: The value of portfolio dispersion in recessions goes up by about 15% of a standard deviation.

Since dispersion in fund strategies should generate dispersion in fund returns, we next look for evidence of higher return dispersion in recessions. To measure dispersion, we use the absolute deviation between fund j's return and the equally weighted cross-sectional average, $|return_t^j - \overline{return}_t|$, as the dependent variable in (17). Columns 5 and 6 of Table 3 show that return dispersion increases by 80% in recessions. Finally, portfolio and return dispersion in recessions should come from different directional bets on the market. This should show up as an increase in the dispersion of portfolio betas. Columns 3 and 4 show that the CAPM-beta dispersion also increases by about 30% in recessions, all consistent with the predictions of our model.

These findings are robust. Counter-cyclical dispersion in funds' portfolio strategies is also found in measures of fund style shifting and sectoral asset allocation. The dispersion in returns is also found for abnormal returns and fund alphas. Results available on request.

Testing for Separate Effects of Volatility and Recessions. Propositions 3 and 4 tell us that return dispersion increases in recessions for two reasons. One is that the volatility of aggregate shocks increases and the other reason is that the price of risk increases. We can disentangle these two effects by regressing return dispersion on volatility and recession simultaneously. The model would predict that volatility should be a significant determinant of dispersion and that after controlling for volatility, there should be some additional explanatory power of recessions that comes from the price of risk effect.

Column 7 of Table 3 shows that both the return and the volatility effects are present in the data. Both are associated with a significant increase in the dispersion of returns. After including the volatility variable, the magnitude of the coefficient on recessions falls by 47%, suggesting that volatility and price of risk fluctuations have roughly an equal effect on portfolio dispersion.

2.5 Testing Prediction 3: Performance

The third prediction of our model is that recessions are times when information allows funds to earn higher average risk-adjusted returns. We evaluate this hypothesis using the following regression specification:

$$Performance_t^j = c_0 + c_1 Recession_t + \mathbf{c_2 X_t^j} + \epsilon_t^j \tag{18}$$

where $Performance_t^j$ denotes fund j's performance in month t, measured as fund abnormal returns, or CAPM, three-factor, or four-factor alphas. All returns are net of management fees. The coefficient of interest is c_1.

Column 1 of Table 4 shows that the average fund's net return is 3bp per month lower than the market return in expansions, but it is 34bp per month higher in recessions. This difference is highly statistically significant and becomes even larger (42bp), after we control for fund characteristics (Column 2). Similar results (Columns 3 and 4) obtain when we use the CAPM alpha as a measure of fund performance, except that the alpha in expansions becomes negative. When we use alphas based on the three- and four-factor models, the recession return premiums diminish (Columns 5-8). But in recessions, the four-factor alpha still represents a non-trivial 1% per year risk-adjusted excess return, 1.6% higher than the -0.6% recorded in expansions (significant at the 1% level). The advantage of this cross-sectional regression model is that it allows us to include a host of fund-specific control variables. The disadvantage is that performance is measured using past twelve-month rolling-window regressions. Thus, a given observation can be classified as a recession when some or even all of the remaining eleven months of the window are expansions.

To verify the robustness of our cross-sectional results, we also employ a time-series approach. In each month, we form the equally weighted portfolio of funds and calculate its net return, in excess of the risk-free rate. We then regress this time series of fund portfolio returns on *Recession* and common risk factors, adjusting standard errors for heteroscedasticity and autocorrelation. We find similar outperformance in recessions. Our results are also robust to alternative performance measures, such as *gross* fund returns, gross alphas, or the information ratio (the ratio of the CAPM alpha to the CAPM residual volatility). All increase sharply in recessions. Finally, we find similar results when we lead alpha on the left-hand side by one month instead of using a contemporaneous alpha. All results point in the same direction: Outperformance increases in recessions.

Testing for Separate Effects of Volatility and Recessions. As before, two forces increase the performance of funds relative to non-funds in recessions: the increase in volatility and the increase in the price of risk (propositions 5 and 6). Column 9 of Table 4 shows that the data are consistent with each force having a distinct effect on fund outperformance. We use the 4-factor alpha as the dependent variable for this exercise because we want to avoid conflating more risk taking in recessions with greater fund outperformance in recessions. When we regress each fund's 4-factor alpha on a recession indicator and a volatility measure, both have positive, significant coefficients. Adding the volatility variable reduces the size of the recession effect by 28%. This suggests that fund outperformance in recessions is due mostly to the increased price of risk and is due to a lesser extent to the higher volatility of aggregate shocks. But the fact that both variables have a significant relationship with fund outperformance, dispersion, and attention, in the direction predicted by the theory offers solid support for the model.

3 Alternative Explanations

The existing literature has not yet advanced any alternative explanations for time-varying skill, as far as we know. We briefly review categories of mutual fund theories and detail the facts with which they are compatible and incompatible. Ultimately, we conclude that while various explanations can account for some of the facts, they are unlikely to account for all facts *jointly*.

Fund skill changes because fund managers change. While our model is silent about the distinction between funds and fund managers, in practice, skill could be embodied in the manager or be produced by the organizational setup of the fund. Table 5 re-estimates our main results using the cross section of data at the manager level. The effects of recession on $Ftiming/Fpicking$, dispersion, and performance are essentially unchanged, suggesting that the distinction is not important for our results.

Sample selection Suppose that managers have heterogeneous skill, but they do not display the cyclical variation in attention allocation we envision. Furthermore, suppose that the best managers leave the sample in good times, maybe because they go to a hedge fund. Then the composition effect would deliver lower alphas and less dispersion in expansions. If for some reason skill is associated with high $Ftiming$ and low $Fpicking$, it could also explain the attention allocation results.

This story is incompatible with at least four facts. First, Table 5 shows that the results all survive when managers are the unit of observation and when we include manager fixed effects. Including fixed effects in a regression model is a standard response to sample selection concerns. We also add fund fixed effects to our original specification. The results do not change in either case. Second, KVV show that the same managers who have high $Fpicking$ in expansions also have high $Ftiming$ in recessions. This is inconsistent with a composition effect. Third, even though there is a higher chance of being promoted or picked off by a hedge fund in expansions, there is also a higher likelihood of being fired or demoted in recessions. The former effect could explain low return dispersion in expansions, but the latter effect would also depress dispersion in recessions, leaving little or no cyclical fluctuation. Fourth, KVV find no systematic differences in age, educational background, or experience of fund managers in recessions versus expansions.

Convex flow-performance relationship Kaniel and Kondor (2010) show that the convex relationship between mutual fund performance and fund inflows – typically a determinant of the manager's compensation – can explain outperformance and higher portfolio dispersion in recessions. Section 1 already explained that any theories where managers learn, even if they do not actively reallocate their attention, can also explain these facts. While Kaniel and Kondor explain counter-cyclical dispersion and other salient long-run features of asset markets that our theory does not address, they do not offer a competing answer to our main question, which is why skill measures are cyclical. Similarly, existing theories do not explain why dispersion and performance have a component that is correlated with macroeconomic volatility, even after controlling for recessions. These are the facts most specific to our theory.

Career concerns Chevalier and Ellison (1999) show that young managers with career concerns may have an incentive to herd. It would seem logical that the concern for being fired would be greatest in recessions. But if that were the case, herding should be most prevalent in recessions and it should make the dispersion in portfolios decline. Instead, the results in Table 3 show that portfolio dispersion rises in recessions.

Since portfolio dispersion rises in recessions, one might be tempted to instead construct a story whereby career concerns are actually stronger in expansions. But if that is true, then there should be an interaction effect: Younger managers should be more likely to hold portfolios with low dispersion in expansions, meaning that in recessions, their portfolio dispersion should increase by more. Conversely, older managers' portfolio dispersion should change less over the cycle. This suggests that when we regress portfolio dispersion on recession, age and the interaction of recession and age, the interaction term should have a negative sign (dispersion for older managers decreases less in recessions). Instead, we find a significantly positive interaction effect. While labor market considerations may be important for understanding many aspects of the behavior of mutual fund managers, the above argument suggests that they cannot account for the empirical patterns we document.

Time-varying marginal utility Glode (2011) argues that funds outperform in recessions because their investors' marginal utility is highest in such periods. While complementary to our explanation, his work remains silent on what strategies investment managers pursue to achieve this differential performance. Therefore, it does not explain why the nature of skill changes over time.

Mechanical effects The final alternative explanation we consider is that our effects arise mechanically from the properties of asset returns. To rule this out, KVV verify that several mechanical mutual fund strategies cannot reproduce the observed features of fund returns. The mechanical strategies include 1) an equally-weighted portfolio of 75 (or 50 or 100) randomly chosen stocks by all funds; 2) half the funds choose 75 random stocks from the top half of the alpha distribution and the other half choose 75 stocks from the bottom half of the alpha distribution; and 3) half the funds pick from the top half of the beta distribution with the other half of funds choosing from the bottom half. None of the strategies explored generate anything that resembles time-varying skill, counter-cyclical dispersion, or counter-cyclical performance.

4 Conclusion

Do investment managers add value for their clients? The answer to this question matters for problems ranging from the discussion of market efficiency to a practical portfolio advice for households. The large amount of randomness in financial asset returns makes it a difficult question to answer. The multi-billion investment management business is first and foremost an information-processing business. We model investment managers not only as agents making optimal portfolio decisions, but also as human beings with finite mental capacity, who optimally allocate that capacity to process information (their attention) at each point in time. Since the optimal attention allocation varies with the state of the economy, so do investment strategies and fund returns. As long as a subset of skilled investment managers can process information about future asset payoffs, the model predicts a higher covariance of portfolio holdings with aggregate asset payoff shocks, more cross-sectional dispersion in portfolio investment strategies and returns across funds, and a higher average outperformance in recessions. We observe these patterns in investments and returns of actively managed U.S. mutual funds. Hence, the data are consistent with a world in which some investment managers have skill.

Beyond the mutual fund industry, a sizeable fraction of GDP currently comes from industries that produce and process information (consulting, business management, product design, marketing analysis, accounting, rating agencies, equity analysts, etc.). Ever increasing access to information has made the problem of how to best allocate a limited amount of information-processing capacity even more relevant. While information choices have consequences for real outcomes, they are often poorly understood because they are difficult to measure. By predicting how information choices are linked to observable variables (such as the state of the economy) and by tying information choices to real outcomes (such as portfolio investment), we show how models of information choices can be brought to the data. This information-choice-based approach could be useful in examining other information-processing sectors of the economy.

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Table 1: **Individual Stocks Have More Aggregate Risk in Recessions**

For each stock i and month t, we estimate a CAPM equation based on twelve months of data (a twelve-month rolling-window regression). This estimation delivers the stock's beta, β_t^i, and its residual standard deviation, $\sigma_{\varepsilon t}^i$. We define stock i's aggregate risk in month t as $\left|\beta_t^i \sigma_t^m\right|$ and its idiosyncratic risk as $\sigma_{\varepsilon t}^i$, where σ_t^m is the realized volatility from daily market return observations. Panel A reports results from a time-series regression of the average stock's aggregate risk, $\frac{1}{N}\sum_{i=1}^N \left|\beta_t^i \sigma_t^m\right|$, in Columns 1 and 2, and of the average idiosyncratic risk, $\frac{1}{N}\sum_{i=1}^N \sigma_{\varepsilon t}^i$, in Columns 3 and 4 on *Recession*. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. In Columns 2 and 4 we include several aggregate control variables: the market excess return (MKTPREM), the return on the small-minus-big portfolio (SMB), the return on the high-minus-low book-to-market portfolio (HML), the return on the up-minus-down momentum portfolio (UMD). The data are monthly from 1980-2005 (309 months). Standard errors (in parentheses) are corrected for autocorrelation and heteroscedasticity. Panel B reports results of panel regressions of each stock's aggregate risk, $\left|\beta_t^i \sigma_t^m\right|$, in Columns 1 and 2 and of its idiosyncratic risk, $\sigma_{\varepsilon t}^i$, in Columns 3 and 4 on *Recession*. In Columns 2 and 4 we include several firm-specific control variables: the log market capitalization of the stock, $log(Size)$, the ratio of book equity to market equity, $B-M$, the average return over the past year, *Momentum*, the stock's ratio of book debt to book debt plus book equity, *Leverage*, and an indicator variable, *NASDAQ*, equal to one if the stock is traded on NASDAQ. All control variables are lagged one month. The data are monthly and cover all stocks in the CRSP universe for 1980-2005. Standard errors (in parentheses) are clustered at the stock and time dimensions.

	(1)	(2)	(3)	(4)
	Aggregate Risk		Idiosyncratic Risk	
Panel A: Time-Series Regression				
Recession	1.348	1.308	0.058	0.016
	(0.693)	(0.678)	(1.018)	(1.016)
MKTPREM		-4.034		-1.865
		(3.055)		(3.043)
SMB		8.110		12.045
		(3.780)		(4.293)
HML		0.292		9.664
		(5.458)		(8.150)
UMD		-4.279		-1.112
		(2.349)		(3.888)
Constant	6.694	6.748	13.229	13.196
	(0.204)	(0.212)	(0.286)	(0.276)
Observations	309	309	309	309
Panel B: Pooled Regression				
Recession	1.203	1.419	0.064	0.510
	(0.242)	(0.238)	(0.493)	(0.580)
Log(Size)		-0.145		-1.544
		(0.021)		(0.037)
B-M Ratio		-0.934		-2.691
		(0.056)		(0.086)
Momentum		0.097		2.059
		(0.101)		(0.177)
Leverage		-0.600		-1.006
		(0.074)		(0.119)
NASDAQ		0.600		1.937
		(0.075)		(0.105)
Constant	4.924	4.902	12.641	12.592
	(0.092)	(0.095)	(0.122)	(0.144)
Observations	1,312,216	1,312,216	1,312,216	1,312,216

Table 2: **Attention Allocation is Cyclical**

Dependent variables: Fund j's $Ftiming_t^j$ is defined in equation (10), where the rolling window T is 12 months and the aggregate shock a_{t+1} is the change in industrial production growth between t and $t+1$. A fund j's $Fpicking_t^j$ is defined as in equation (11), where s_{it+1} is the change in asset i's earnings growth between t and $t+1$. All are multiplied by 10,000 for readability.
Independent variables: *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. $Log(Age)$ is the natural logarithm of fund age in years. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Turnover* is the fund turnover ratio. *Flow* is the percentage growth in a fund's new money. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. *Volatility* is an indicator variable for periods of volatile earnings. We calculate the twelve-month rolling-window standard deviation of the year-to-year log change in the earnings of S&P 500 index constituents; the earnings data are from Robert Shiller for 1926-2008. Volatility equals one if this standard deviation is in the highest 10% of months in the 1926-2008 sample. During 1985-2005, 12% of months are such high volatility months. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered by fund and time.

	(1)	(2)	(3)	(4)	(5)	(6)
		Ftiming			Fpicking	
Recession	0.011	0.011	0.011	-0.682	-0.696	-0.503
	(0.004)	(0.004)	(0.004)	(0.159)	(0.150)	(0.166)
Volatility			0.000			-0.477
			(0.003)			(0.106)
Log(Age)		-0.002	-0.002		0.423	0.416
		(0.001)	(0.001)		(0.060)	(0.060)
Log(TNA)		-0.001	-0.001		-0.173	-0.167
		(0.000)	(0.000)		(0.029)	(0.029)
Expenses		-0.330	-0.331		88.756	90.911
		(0.244)	(0.247)		(11.459)	(11.599)
Turnover		-0.004	-0.004		-0.204	-0.199
		(0.001)	(0.001)		(0.053)	(0.063)
Flow		-0.008	-0.008		1.692	1.668
		(0.010)	(0.010)		(0.639)	(0.632)
Load		0.017	0.017		-9.644	-10.009
		(0.023)	(0.023)		(1.972)	(1.984)
Constant	-0.001	-0.001	-0.001	3.084	3.086	3.124
	(0.001)	(0.001)	(0.001)	(0.069)	(0.070)	(0.074)
Observations	224,257	224,257	224,257	166,328	166,328	166,328

Table 3: **Portfolio and Return Dispersion Rises in Recessions**

Dependent variables: Portfolio dispersion is the Herfindahl index of portfolio weights in stocks $i \in \{1, \cdots, N\}$ in deviation from the market portfolio weights $\sum_{i=1}^{N}(w_{it}^{j} - w_{it}^{m})^2 \times 100$. Return dispersion is $|return_t^j - \overline{return_t}|$, where $\overline{return}$ denotes the (equally weighted) cross-sectional average. The CAPM beta comes from twelve-month rolling-window regressions of fund-level excess returns on excess market returns (and returns on SMB, HML, and MOM). Beta dispersion is constructed analogously to return dispersion. The right-hand side variables, the sample period, and the standard error calculation are the same as in Table 2.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Portfolio Dispersion		Beta Dispersion		Return Dispersion		
Recession	0.205	0.147	0.082	0.083	0.530	0.561	0.298
	(0.027)	(0.026)	(0.015)	(0.014)	(0.108)	(0.101)	(0.148)
Volatility							0.581
							(0.223)
Log(Age)		0.203		-0.009		-0.064	-0.118
		(0.028)		(0.002)		(0.007)	(0.018)
Log(TNA)		-0.179		0.003		0.029	0.025
		(0.014)		(0.001)		(0.004)	(0.011)
Expenses		28.835		5.460		13.816	23.264
		(4.860)		(0.235)		(1.152)	(2.655)
Turnover		-0.092		0.020		0.074	0.092
		(0.025)		(0.001)		(0.006)	(0.014)
Flow		0.122		0.022		0.479	0.017
		(0.104)		(0.017)		(0.088)	(0.221)
Load		-1.631		-0.444		-1.738	-3.501
		(0.907)		(0.042)		(0.182)	(0.532)
Constant	1.525	1.524	0.229	0.229	0.586	0.659	1.845
	(0.024)	(0.022)	(0.006)	(0.006)	(0.018)	(0.027)	(0.079)
Observations	230,185	230,185	227,159	227,159	226,745	226,745	226,745

Table 4: **Fund Performance Improves in Recessions**

Dependent variables: *Abnormal Return* is the fund return minus the market return. The alphas come from twelve-month rolling-window regressions of fund-level excess returns on excess market returns for the CAPM alpha, additionally on the SMB and the HML factors for the three-factor alpha, and additionally on the UMD factor for the four-factor alpha. The independent variables, the sample period, and the standard error calculations are the same as in Table 2.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Abnormal Return		CAPM Alpha		3-Factor Alpha		4-Factor Alpha		
Recession	0.342	0.425	0.337	0.404	0.043	0.073	0.107	0.139	0.100
	(0.056)	(0.058)	(0.048)	(0.047)	(0.034)	(0.028)	(0.041)	(0.032)	(0.035)
Volatility									0.125
									(0.064)
Log(Age)		-0.031		-0.036		-0.028		-0.039	-0.036
		(0.009)		(0.008)		(0.006)		(0.006)	(0.006)
Log(TNA)		0.046		0.033		0.009		0.012	0.010
		(0.005)		(0.004)		(0.003)		(0.003)	(0.003)
Expenses		-1.811		-2.372		-7.729		-7.547	-8.187
		(1.046)		(0.945)		(0.782)		(0.745)	(0.787)
Turnover		-0.023		-0.044		-0.074		-0.065	-0.067
		(0.016)		(0.010)		(0.010)		(0.008)	(0.008)
Flow		2.978		2.429		1.691		1.536	1.536
		(0.244)		(0.172)		(0.097)		(0.096)	(0.096)
Load		-0.809		-0.757		-0.099		-0.335	-0.223
		(0.226)		(0.178)		(0.131)		(0.141)	(0.141)
Constant	-0.027	-0.033	-0.059	-0.063	-0.059	-0.060	-0.050	-0.052	-0.065
	(0.027)	(0.026)	(0.025)	(0.024)	(0.020)	(0.018)	(0.023)	(0.021)	(0.021)
Observations	226,745	226,745	226,745	226,745	226,745	226,745	226,745	226,745	226,745

Table 5: **Robustness: Managers as the Unit of Observation**

The dependent variables are fundamentals-based market-timing ability (Ftiming), fundamentals-based stock-picking ability (Fpicking), portfolio dispersion (*Dispersion*), and the four-factor alpha (4-Factor Alpha), all of which are tracked at the manager level. Columns with a 'Y' include manager fixed effects. The independent variables, the sample period, and the standard error calculations are the same as in Table 2.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Ftiming		Fpicking		Dispersion		4-Factor Alpha	
Recession	0.008	0.007	-0.701	-0.824	0.105	0.142	0.167	0.141
	(0.003)	(0.002)	(0.130)	(0.132)	(0.031)	(0.024)	(0.035)	(0.035)
Log(Age)	-0.002	-0.004	0.460	0.268	0.154	0.017	-0.032	-0.069
	(0.001)	(0.001)	(0.066)	(0.065)	(0.032)	(0.021)	(0.006)	(0.008)
Log(TNA)	-0.000	-0.000	-0.126	-0.139	-0.131	-0.093	0.007	0.003
	(0.000)	(0.000)	(0.032)	(0.033)	(0.018)	(0.011)	(0.004)	(0.005)
Expenses	0.130	0.060	127.222	45.894	43.920	13.241	-8.225	-10.590
	(0.105)	(0.146)	(13.972)	(13.867)	(6.012)	(4.504)	(0.794)	(1.138)
Turnover	-0.005	-0.004	-0.287	0.210	-0.127	-0.014	-0.081	-0.035
	(0.002)	(0.002)	(0.077)	(0.090)	(0.030)	(0.020)	(0.010)	(0.009)
Flow	-0.009	-0.016	1.037	1.186	0.154	-0.374	1.832	1.483
	(0.009)	(0.009)	(0.613)	(0.554)	(0.121)	(0.101)	(0.097)	(0.089)
Load	-0.009	-0.069	-16.064	-5.832	-4.674	-0.231	-0.426	0.097
	(0.017)	(0.023)	(2.393)	(2.307)	(1.284)	(0.809)	(0.151)	(0.172)
Fixed Effect	N	Y	N	Y	N	Y	N	Y
Constant	-0.002	-0.002	2.966	2.977	1.438	1.447	-0.045	-0.043
	(0.001)	(0.001)	(0.072)	(0.068)	(0.026)	(0.008)	(0.024)	(0.021)
Observations	332,676	332,676	249,942	249,942	332,776	332,776	332,776	332,776

A Appendix: Proofs of Propositions

A.1 Proof of Lemma 1

Proof. Following Admati (1985), we conjecture that the price vector p is linear in the payoff vector f and the supply vector x: $pr = A + Bf + Cx$. We now verify that conjecture by imposing market clearing

$$\int q_j dj = \bar{x} + x \tag{19}$$

Using (35) to substitute out the left hand side, and rearranging,

$$pr = -\rho\bar{\Sigma}(\bar{x} + x) + f + \bar{\Sigma}\Sigma^{-1}(\mu - f)$$

Thus, the coefficients A, B, and C are given by

$$A = -\rho\bar{\Sigma}\bar{x} + \bar{\Sigma}\Sigma^{-1}\mu \tag{20}$$

$$B = I - \bar{\Sigma}\Sigma^{-1} \tag{21}$$

$$C = -\rho\bar{\Sigma} \tag{22}$$

which verifies our conjecture. □

A.2 Mathematical Preliminaries

Matrices in terms of fundamental variances To determine the effect of changes in aggregate shock variance on dispersion and profits, we need to express some of the matrices in terms of σ_a^{-1}. If we can decompose the matrices into components that depend on σ_a and those that do not, we can differentiate the expressions more easily.

First, we decompose the payoff precision matrices. To do this decomposition, we need to invert Σ. Doing it by hand yields

$$\Sigma^{-1} = \begin{bmatrix} \sigma_1^{-1} & 0 & -b_1\sigma_1^{-1} \\ 0 & \sigma_2^{-1} & -b_2\sigma_2^{-1} \\ -b_1\sigma_1^{-1} & -b_2\sigma_2^{-1} & \sigma_a^{-1} + b_1^2\sigma_1^{-1} + b_2^2\sigma_2^{-1} \end{bmatrix} \tag{23}$$

Similarly, the posterior precision matrix for investor j is

$$\hat{\Sigma}_j^{-1} = \begin{bmatrix} \hat{\sigma}_1^{-1} & 0 & -b_1\hat{\sigma}_1^{-1} \\ 0 & \hat{\sigma}_2^{-1} & -b_2\hat{\sigma}_2^{-1} \\ -b_1\hat{\sigma}_1^{-1} & -b_2\hat{\sigma}_2^{-1} & \hat{\sigma_a}^{-1} + b_1^2\hat{\sigma}_1^{-1} + b_2^2\hat{\sigma}_2^{-1} \end{bmatrix} \tag{24}$$

It is useful to separate out the terms that depend on σ_a from those that do not. Define

$$S \equiv \begin{bmatrix} \sigma_1^{-1} & 0 & -b_1\sigma_1^{-1} \\ 0 & \sigma_2^{-1} & -b_2\sigma_2^{-1} \\ -b_1\sigma_1^{-1} & -b_2\sigma_2^{-1} & b_1^2\sigma_1^{-1} + b_2^2\sigma_2^{-1} \end{bmatrix} \tag{25}$$

and let $\hat{S}_j$ be the posterior S, meaning that each σ_1 is replaced with the posterior variance $\hat{\sigma}_{1j}$ and each σ_2 is replaced with the posterior variance $\hat{\sigma}_{2j}$.

$$\Upsilon_a \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \Upsilon_1 \equiv \begin{bmatrix} 1 & 0 & -b_1 \\ 0 & 0 & 0 \\ -b_1 & 0 & b_1^2 \end{bmatrix} \tag{26}$$

so that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma_a}^{-1} = \Upsilon_a$ and $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma_1}^{-1} = \Upsilon_1$.

Then,

$$\Sigma^{-1} = S + \sigma_a^{-1}\Upsilon_a \tag{27}$$

$$\hat{\Sigma}_j^{-1} = \hat{S}_j + \hat{\sigma_{a}}_j^{-1}\Upsilon_a \tag{28}$$

Define the average posterior precision matrix when a fraction χ of investment managers have capacity to be $(\bar{\Sigma})^{-1} \equiv \int_j \hat{\Sigma}_j^{-1} dj$. Similarly, let $S^a \equiv \int_j \hat{S}_j dj$ and let $\bar{K}_a$ be the average amount of capacity that an agent devotes to processing aggregate information. For example, if a fraction χ of investors are skilled, and all skilled investors devote all their capacity K to processing aggregate information, $\bar{K}_a = \chi K$. Recall that $\hat{\sigma_{a}}_j^{-1} = \sigma_a^{-1} + K_{aj}$, then

$$(\bar{\Sigma})^{-1} = S^a + (\sigma_a^{-1} + \bar{K}_a)\Upsilon_a \tag{29}$$

Useful matrices and expressions We derive several matrices and expressions that recur frequently in the proofs below.

1) $\Delta \equiv \hat{\Sigma}_j^{-1} - (\bar{\Sigma})^{-1} = \Sigma_{\eta j}^{-1} - (\bar{\Sigma}_\eta)^{-1}$: The difference between the precision of an informed manager's posterior beliefs and the precision of the average manager's posterior beliefs is computed as:

$$\hat{\Sigma}_j^{-1} - (\bar{\Sigma})^{-1} = \hat{S}_j - S^a + (K_{aj} - \bar{K}_a)\Upsilon_a \tag{30}$$

Now we show the equality. Bayes' rule for variances of normal variables is $\hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_{\eta j}^{-1}$. Integrating the left and right sides of this expression over managers j yields $(\bar{\Sigma})^{-1} = \Sigma^{-1} + (\bar{\Sigma}_\eta)^{-1}$. Subtracting one expression from the other yields $\hat{\Sigma}_j^{-1} - (\bar{\Sigma})^{-1} = \Sigma_{\eta j}^{-1} - (\bar{\Sigma}_\eta)^{-1}$. Therefore, $\Delta \equiv \hat{\Sigma}_j^{-1} - (\bar{\Sigma})^{-1} = \Sigma_{\eta j}^{-1} - (\bar{\Sigma}_\eta)^{-1}$. Observe that Δ is positive definite as long as the investor is more informed than the average: $K_{aj} > \bar{K}_a$ and $K_{ij} > \bar{K}_i$ for $i \in \{1, 2\}$.

2) $\bar{\Sigma}$: To compute the average variance we need to invert $(\bar{\Sigma})^{-1}$. Replacing σ_a with $(\sigma_a^{-1} + \bar{K}_a)^{-1}$ and

σ_i with $(\sigma_i^{-1}+\bar{K}_i)^{-1}$, $i \in \{1,2\}$ in Eq. (24), and following the same inversion steps backwards, we get

$$\bar{\Sigma} = (\sigma_a^{-1}+\bar{K}_a)^{-1}bb' + \Phi, \tag{31}$$

where b is the 3×1 vector of loadings of each asset on aggregate risk, and if $\bar{K}_1$ and $\bar{K}_2$ represent the average amount of capacity devoted to processing information about assets 1 and 2,

$$\Phi \equiv \begin{bmatrix} (\sigma_1^{-1}+\bar{K}_1)^{-1} & 0 & 0 \\ 0 & (\sigma_2^{-1}+\bar{K}_2)^{-1} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

3) $\bar{\Sigma}\Sigma^{-1}$: Let $\bar{\sigma}_a \equiv (\sigma_a^{-1}+\bar{K}_a)^{-1}$, $\bar{\sigma}_1 \equiv (\sigma_1^{-1}+\bar{K}_1)^{-1}$ and $\bar{\sigma}_2 \equiv (\sigma_2^{-1}+\bar{K}_2)^{-1}$. Then we have the following results:

$$\begin{aligned} \bar{\Sigma}\Sigma^{-1} &= (\bar{\sigma}_a bb' + \Phi)(S + \sigma_a^{-1}\Upsilon_a) \\ &= \bar{\sigma}_a bb'S + \sigma_a^{-1}\Phi\Upsilon_a + \bar{\sigma}_a\sigma_a^{-1}bb'\Upsilon_a + \Phi S \\ &= \begin{bmatrix} \bar{\sigma}_1\sigma_1^{-1} & 0 & b_1(\bar{\sigma}_a\sigma_a^{-1} - \bar{\sigma}_1\sigma_1^{-1}) \\ 0 & \bar{\sigma}_2\sigma_2^{-1} & b_2(\bar{\sigma}_a\sigma_a^{-1} - \bar{\sigma}_2\sigma_2^{-1}) \\ 0 & 0 & \bar{\sigma}_a\sigma_a^{-1} \end{bmatrix} \end{aligned} \tag{32}$$

where we have used that $bb'S = 0$, $\Phi\Upsilon_a = 0$, $bb'\Upsilon_a$ is a matrix of zeros except for the last column which is equal to b and ΦS is equal to the first rows of S multiplied by $\bar{\sigma}_1$ and $\bar{\sigma}_2$ respectively. We have that $trace(\bar{\Sigma}\Sigma^{-1}) = \bar{\sigma}_1\sigma_1^{-1} + \bar{\sigma}_2\sigma_2^{-1} + \bar{\sigma}_a\sigma_a^{-1}$.

4) $\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}$:

$$\begin{aligned} \bar{\Sigma}\Sigma^{-1}\bar{\Sigma} &= (\bar{\sigma}_a\sigma_a^{-1}bb'\Upsilon_a + \Phi S)(\bar{\sigma}_a bb' + \Phi) \\ &= \bar{\sigma}_a^2\sigma_a^{-1}bb'\Upsilon_a bb' + \bar{\sigma}_a\sigma_a^{-1}bb'\Upsilon_a\Phi + \bar{\sigma}_a\Phi S bb' + \Phi S\Phi \\ &= \bar{\sigma}_a^2\sigma_a^{-1}bb' + diag(\tilde{\sigma}_1^2\sigma_1^{-1}, \tilde{\sigma}_2^2\sigma_2^{-1}, 0) \\ &= \begin{bmatrix} \bar{\sigma}_a^2\sigma_a^{-1}b_1^2 + \tilde{\sigma}_1^2\sigma_1^{-1} & \bar{\sigma}_a^2\sigma_a^{-1}b_1b_2 & \bar{\sigma}_a^2\sigma_a^{-1}b_1 \\ \bar{\sigma}_a^2\sigma_a^{-1}b_1b_2 & \bar{\sigma}_a^2\sigma_a^{-1}b_2^2 + \tilde{\sigma}_2^2\sigma_2^{-1} & \bar{\sigma}_a^2\sigma_a^{-1}b_2 \\ \bar{\sigma}_a^2\sigma_a^{-1}b_1 & \bar{\sigma}_a^2\sigma_a^{-1}b_2 & \bar{\sigma}_a^2\sigma_a^{-1} \end{bmatrix} \end{aligned} \tag{33}$$

where we have used that $bb'S = 0$, $\Phi\Upsilon_a = 0$, $bb'\Upsilon_a bb' = bb'$ and $\Phi S\Phi$ is equal to the matrix described as a diagonal above. We have that the $trace(\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}) = \bar{\sigma}_a^2\sigma_a^{-1}[1 + b_1^2 + b_2^2] + \tilde{\sigma}_1^2\sigma_1^{-1} + \tilde{\sigma}_2^2\sigma_2^{-1}$.

Signal about asset payoffs f It is convenient for the proofs below to express the signal as the true asset payoff plus noise as follows: $\eta_j = f + \epsilon_j$, where $\epsilon_j \sim \mathcal{N}(0, G\Sigma_{ej}G')$, $\Sigma_{ej} = diag\{K_a^{-1}, K_1^{-1}, K_2^{-1}\}$, and $G = \begin{bmatrix} b_1 & 1 & 0 \\ b_2 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Then the posterior precision is given by $\hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_{\eta j}^{-1}$, where $\Sigma_{\eta j}^{-1} = \Sigma_{\epsilon j}^{-1} = G\Sigma_{ej}G'$. Note that $\Sigma_{\eta j}^{-1}$ and its inverse are positive definite as long as K_a, K_1, K_2 are strictly positive. We will assume this throughout the proofs.

Average portfolio holdings The optimal portfolio for investor j is

$$q_j = \frac{1}{\rho}\hat{\Sigma}_j^{-1}(\hat{\mu}_j - pr) \tag{34}$$

This comes from the first order condition and is a standard expression in any portfolio problem with CARA or mean-variance utility. Next, compute the portfolio of the average investor. Let the average of all investors' posterior precision be $(\bar{\Sigma})^{-1} \equiv \int \hat{\Sigma}_j^{-1} dj$. Use the fact that $\hat{\mu}_j = \hat{\Sigma}_j \Sigma^{-1}\mu + (I - \hat{\Sigma}_j\Sigma^{-1})\tilde{\eta}_j$ and the fact that the signal noise is mean-zero to get that $\int \hat{\Sigma}_j^{-1}\hat{\mu}_j dj = \Sigma^{-1}\mu + ((\bar{\Sigma})^{-1} - \Sigma^{-1})f$. This is true because the mean of all investors' signals are the true payoffs f and because the signal errors are uncorrelated with (but of course, not independent of) signal precision.

$$\bar{q} \equiv \int q_j dj = \frac{1}{\rho}\left(\Sigma^{-1}\mu + ((\bar{\Sigma})^{-1} - \Sigma^{-1})f - (\bar{\Sigma})^{-1}pr\right) \tag{35}$$

Using Bayes' rule for the posterior variance of normal variables, we can rewrite this as

$$\bar{q} \equiv \int q_j dj = \frac{1}{\rho}\left(\Sigma^{-1}\mu + \bar{\Sigma}_{\tilde{\eta}}^{-1} f - (\bar{\Sigma})^{-1}pr\right) \tag{36}$$

where $\bar{\Sigma}_{\tilde{\eta}}^{-1} \equiv \bar{\Sigma}^{-1} - \Sigma^{-1}$ is the average investor's signal precision.

Mean and variance of asset returns The vector of asset returns is $(f - pr)$. Now replace pr with $A + Bf + Cx$, where A, B, and C are given by Appendix A.1,

$$f - pr = (I - B)f - A - Cx \tag{37}$$

Substituting in the coefficients in the pricing equation reveals that $(I - B)\mu - A = \rho\bar{\Sigma}\bar{x}$, that $I - B = \bar{\Sigma}\Sigma^{-1}$, and that $C = -\rho\bar{\Sigma}$, and therefore $f - pr = \bar{\Sigma}(\Sigma^{-1}(f - \mu) + \rho x + \rho\bar{x})$. Since a linear combination of two normal variables is also a normal variable, we can write

$$f - pr = w + V^{1/2}z \quad \text{where } z \sim N(0, I), \tag{38}$$

$$V \equiv Var[f - pr] = \bar{\Sigma}(\Sigma^{-1} + \rho^2\sigma_x I)\bar{\Sigma} \tag{39}$$

$$w \equiv E[f - pr] = \rho\bar{\Sigma}\bar{x}. \tag{40}$$

A key partial derivative for evaluating comparative statics will be $\frac{\partial V}{\partial \sigma_a}$. So, it is useful to evaluate that here. Use (31) and (33) to write V as

$$V = \bar{\sigma}_a^2\sigma_a^{-1}bb' + diag(\tilde{\sigma}_1^2\sigma_1^{-1}, \tilde{\sigma}_2^2\sigma_2^{-1}, 0) + \rho^2\sigma_x[\bar{\sigma}_a bb' + \Phi]^2$$

When taking the partial derivative with respect to σ_a, we hold K_a fixed. Note that $\bar{\sigma}_a^{-1} - \sigma_a^{-1} + K_a$. Thus,

$$\partial V/\partial\sigma_a = \bar{\sigma}_a^2\sigma_a^{-2}[2\bar{\sigma}_a\sigma_a^{-1} - 1]bb' + 2\rho^2\sigma_x\bar{\sigma}_a^2\sigma_a^{-2}\bar{\Sigma}bb' \tag{41}$$

Note that this is multiplied by $\bar{\sigma}_a^2\sigma_a^{-2} > 0$ throughout and post-multiplied by bb', which is positive definite.

Since a product of positive definite matrices is positive definite, a sufficient condition for $\partial V/\partial\sigma_a$ positive definite is

$$[2\bar{\sigma}_a\sigma_a^{-1}-1]I+2\rho^2\sigma_x\bar{\Sigma} \quad \text{is positive definite.} \tag{42}$$

This is the condition under which the variance of expected returns increases when aggregate shock variance increases. If $2\bar{\sigma}_a\sigma_a^{-1}>1$, which implies that $\bar{K}_a<\sigma_a^{-1}$, this derivative will be positive. Since the maximum possible average capacity devoted to the aggregate shock is the total capacity of informed investors K times the fraction of informed investors, a sufficient condition for $\partial V/\partial\sigma_a$ to be positive definite is that $\chi K<\sigma_a^{-1}$. Note also that the second term is a positive definite matrix. Suppose we do an eigen-decomposition: Let $\bar{\Sigma}=\bar{\Gamma}\bar{\Lambda}\bar{\Gamma}'$. Note that $I=\bar{\Gamma}I\bar{\Gamma}'$. Thus, the eigenvalue matrix of the sum in (42) is $[2\bar{\sigma}_a\sigma_a^{-1}-1]I+2\rho^2\sigma_x\bar{\Lambda}$. Since $\bar{\Sigma}$ is positive definite, all the entries of $\bar{\Lambda}$ are positive. Thus, if σ_x is sufficiently large, the whole eigenvalue matrix will be positive and therefore, $\partial V/\partial\sigma_a$ will be positive definite. Thus a second sufficient condition for (42) to hold is σ_x sufficiently large.

Portfolio dispersion Using the optimal portfolio expressions, (34) and (36), and Bayes' rule ($\hat{\mu}_j=\hat{\Sigma}_j(\Sigma^{-1}\mu+\Sigma_{\eta j}^{-1}\eta_j)$), the difference in portfolios $(q_j-\bar{q})$ is

$$(q_j-\bar{q})=\frac{1}{\rho}\left[\Sigma_{\eta j}^{-1}\eta_j-(\bar{\Sigma}_\eta)^{-1}f+((\bar{\Sigma})^{-1}-\hat{\Sigma}_j^{-1})pr\right]$$

where $(\bar{\Sigma}_\eta)^{-1}\equiv\int\Sigma_{\eta j}^{-1}dj$ is the average manager's signal precision.

Next, we need to take into account that signals and payoffs are correlated. To do this, replace the signal η_j with the true payoff, plus signal noise: $\eta_j=f+\epsilon_j$,

$$(q_j-\bar{q})=\frac{1}{\rho}\left[\Sigma_{\eta j}^{-1}\epsilon_j+(\Sigma_{\eta j}^{-1}-(\bar{\Sigma}_\eta)^{-1})f-(\hat{\Sigma}_j^{-1}-(\bar{\Sigma})^{-1})pr\right]$$

Recall that $\Delta\equiv\hat{\Sigma}_j^{-1}-(\bar{\Sigma})^{-1}=\Sigma_{\eta j}^{-1}-(\bar{\Sigma}_\eta)^{-1}$. Substituting this in and combining terms yields

$$(q_j-\bar{q})=\frac{1}{\rho}\left[\Sigma_{\eta j}^{-1}e_j+\Delta(f-pr)\right]$$

Substituting (38) yields

$$(q_j-\bar{q})=\frac{1}{\rho}\left[\Sigma_\eta^{-1}\epsilon_j+\Delta(V^{1/2}z+w)\right] \tag{43}$$

To work out the expectation of this quantity squared, recognize that this is the square of a sum of one constant and two, independent, mean-zero, normal variables. Since ϵ_j and x are independent, all the cross terms drop out, leaving

$$E[(q_j-\bar{q})'(q_j-\bar{q})]=\frac{1}{\rho^2}[Tr(\Sigma_{\eta j}^{-1})+Tr(\Delta V\Delta)+w'\Delta\Delta w] \tag{44}$$

Expected portfolio return Let $z\sim N(0,I)$ and $V\equiv\bar{\Sigma}(\Sigma^{-1}+\rho^2\sigma_x I)\bar{\Sigma}$ as in the previous section, then $f-pr$ can be expressed as:

$$f-pr=((I-B)f-A-Cx)=\bar{\Sigma}(\rho\bar{x}+\Sigma^{-1}(f-\mu)+\rho x)=w+V^{1/2}z$$

Recalling (43), the expected profits are given by:

$$\begin{aligned}
E[(q_j-\bar{q})'(f-pr)] &= \frac{1}{\rho}E\Big[[\Sigma_{\eta j}^{-1}\epsilon_j+\Delta(V^{1/2}z+w)]'(V^{1/2}z+w)\Big] \\
&= \frac{1}{\rho}E\Big[\epsilon_j'\Sigma_{\eta j}^{-1}w+\epsilon_j'\Sigma_{\eta j}^{-1}V^{1/2}z+(w'+z'V^{1/2})\Delta(V^{1/2}z+w)\Big] \\
&= \frac{1}{\rho}E\Big[\epsilon_j'\Sigma_{\eta j}^{-1}w+\epsilon_j'\Sigma_{\eta j}^{-1}V^{1/2}z+2w'\Delta V^{1/2}z+w'\Delta w+z'V^{1/2}\Delta V^{1/2}z\Big] \\
&= \frac{1}{\rho}E\Big[w'\Delta w+z'V^{1/2}\Delta V^{1/2}z\Big] \\
&= \frac{1}{\rho}\Big[\rho^2\bar{x}'\bar{\Sigma}\Delta\bar{\Sigma}\bar{x}+Tr\Big(V^{1/2}\Delta V^{1/2}E(zz')\Big)\Big] \\
&= \frac{1}{\rho}\Big[\rho^2\bar{x}'\bar{\Sigma}\Delta\bar{\Sigma}\bar{x}+Tr(\Delta V)\Big] \\
&= \rho Tr(\bar{x}'\bar{\Sigma}\Delta\bar{\Sigma}\bar{x})+\frac{1}{\rho}Tr(\Delta V) \qquad (45)
\end{aligned}$$

where the fourth equality comes from the fact that ϵ_j and z are mean zero and uncorrelated.

Dispersion of portfolio returns Using results from the previous part, the dispersion in fund profits is given by:

$$E[((q_j-\bar{q})'(f-pr))^2]=E\left[\left(\frac{1}{\rho}[\Sigma_\eta^{-1}e_j+\Delta V^{1/2}z+\Delta w]'(w+V^{1/2}z)\right)^2\right]$$

Using the fact that for any random variable x we have that $V(x)=E(x^2)-E^2(x)$, the dispersion of funds' portfolio returns is equal to:

$$\begin{aligned}
E[((q_j-\bar{q})'(f-pr))^2] &= \frac{1}{\rho^2}E\Big[\Big([\Sigma_{\eta j}^{-1}e_j+\Delta V^{1/2}z+\Delta w]'(V^{1/2}z+w)\Big)^2\Big] \\
&= \frac{1}{\rho^2}Var\Big([\Sigma_{\eta j}^{-1}e_j+\Delta V^{1/2}z+\Delta w]'(V^{1/2}z+w)\Big) \\
&+ \frac{1}{\rho^2}\Big(E[\Sigma_{\eta j}^{-1}e_j+\Delta V^{1/2}z+\Delta w]'(V^{1/2}z+w)\Big)^2
\end{aligned}$$

We compute each term separately.

$$\begin{aligned}
Var(\cdot) &= Var\Big[e_j'\Sigma_{\eta j}^{-1}w+e_j'\Sigma_{\eta j}^{-1}V^{1/2}z+2w'\Delta V^{1/2}z+w'\Delta w+z'V^{1/2}\Delta V^{1/2}z\Big] \\
&= Var\Big[e_j'\Sigma_{\eta j}^{-1}w+e_j'\Sigma_{\eta j}^{-1}V^{1/2}z+2w'\Delta V^{1/2}z+w'\Delta w+z'V^{1/2}\Delta V^{1/2}z\Big] \\
&= w'\Sigma_{\eta j}^{-1}w+0+4w'\Delta V\Delta w+0+2Tr(\Delta V\Delta V) \\
&= \rho^2Tr(\bar{x}'\bar{\Sigma}'\Sigma_{\eta j}^{-1}\bar{\Sigma}\bar{x})+4\rho^2Tr(\bar{x}'\bar{\Sigma}'\Delta V\Delta\bar{\Sigma}\bar{x})+2Tr(\Delta V\Delta V)
\end{aligned}$$

and

$$\begin{aligned}
E(\cdot)^2 &= (w'\Delta w)^2+Tr^2(\Delta V)+2w'\Delta wTr(\Delta V) \\
&= \rho^4Tr^2(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x})+Tr^2(\Delta V)+2\rho^2Tr(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x})Tr(\Delta V)
\end{aligned}$$

Substituting back we have:

$$\begin{aligned} E[((q_j-\bar{q})'(f-pr))^2] &= Tr(\bar{x}'\bar{\Sigma}'\Sigma_{\eta j}^{-1}\bar{\Sigma}\bar{x}) + 4Tr(\bar{x}'\bar{\Sigma}'\Delta V\Delta\bar{\Sigma}\bar{x}) + \frac{2}{\rho^2}Tr(\Delta V\Delta V) \\ &+ \rho^2 Tr^2(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x}) + \frac{1}{\rho^2}Tr^2(\Delta V) + 2Tr(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x})Tr(\Delta V) \end{aligned} \tag{46}$$

Expected Utility From (9) in the main text, we know that expected utility is

$$U_{1j} = \frac{1}{2}trace(\hat{\Sigma}_j^{-1}Var[\hat{\mu}_j - pr]) + \frac{1}{2}E_1[\hat{\mu}_j - pr]'\hat{\Sigma}_j^{-1}E_1[\hat{\mu}_j - pr]$$

where we have normalized initial wealth $W_0 = 0$. The excess return is given by:

$$\hat{\mu}_j - pr = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_{\eta j}^{-1}\eta_j) - A - Bf - Cx$$

The signal η can be expressed as the true asset payoff f, plus orthogonal signal noise ϵ_j.

$$\hat{\mu}_j - pr = \hat{\Sigma}_j\Sigma^{-1}\mu - A + (\hat{\Sigma}_j\Sigma_{\eta j}^{-1} - B)f + \hat{\Sigma}_j\Sigma_{\eta j}^{-1}\epsilon_j - Cx$$

Since μ and A are known constants and f, ϵ_j, and x are independent, with variances Σ, $\Sigma_{\eta j}$, and $\sigma_x I$ respectively, the variance term is given by:

$$Var[\hat{\mu}_j - pr] = (\hat{\Sigma}_j\Sigma_{\eta j}^{-1} - B)\Sigma(\hat{\Sigma}_j\Sigma_{\eta j}^{-1} - B)' + \hat{\Sigma}_j\Sigma_{\eta j}^{-1}\hat{\Sigma}_j + CC'\sigma_x$$

Substituting in for the price coefficients using (20), (21), and (22) yields

$$Var[\hat{\mu}_j - pr] = (\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)' + \hat{\Sigma}_j\Sigma_{\eta}^{-1}\hat{\Sigma}_j + \rho^2\sigma_x\bar{\Sigma}\bar{\Sigma}$$

Next, work out the second term by using the expression above for $\hat{\mu}_j - pr$ and taking the expectation: $E[\hat{\mu}_j - pr] = \hat{\Sigma}_j\Sigma^{-1}\mu - A + (\hat{\Sigma}_j\Sigma_{\eta j}^{-1} - B)\mu$. Substituting in the coefficients A and B, and simplifying reveals that $E[\hat{\mu}_j - pr] = \rho\bar{\Sigma}\bar{x}$. Thus,

$$E_1[\hat{\mu}_j - pr]'\hat{\Sigma}_j^{-1}E_1[\hat{\mu}_j - pr] = \rho^2\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$$

Thus, expected utility is

$$U_{1j} = \frac{1}{2}trace\left(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)' + \Sigma_{\eta j}^{-1}\hat{\Sigma}_j + \rho^2\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{\Sigma}\sigma_x\right) + \frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$$

We can simplify the terms inside the trace to get the final expression which is convenient for the following proofs:

$$U_{1j} = \frac{1}{2}trace\left(\hat{\Sigma}_j^{-1}[\bar{\Sigma}(\rho^2\sigma_x I + \Sigma^{-1})\bar{\Sigma}] + I - 2\Sigma^{-1}\bar{\Sigma}\right) + \frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x} \tag{47}$$

or equivalently

$$U_{1j} = \frac{1}{2}trace\left(\hat{\Sigma}_j^{-1}V + I - 2\Sigma^{-1}\bar{\Sigma}\right) + \frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x} \tag{48}$$

A.3 Lemma 2: Investors Prefer Not To Learn Price Information

The idea behind this result is that an investor who learns from price information, will infer that the asset is valuable when its price is high and infer that the asset is less valuable when its price is low. Buying high and selling low is generally not a way to earn high profits. This effect shows up as a positive correlation between $\hat{\mu}$ and pr, which reduces the variance $Var[\hat{\mu}_j - pr]$.

Mathematical Preliminaries: Note that $B^{-1}(pr - A) = f + B^{-1}Cx$. Since x is a mean-zero shock, this is an unbiased signal about the true asset payoff f. The precision of this signal is $\Sigma_p^{-1} \equiv \sigma_x^{-1}B'(CC')^{-1}B$.

Lemma 2. *A manager who could choose either learning from prices and observing a signal $\tilde{\eta}|f \sim N(f, \tilde{\Sigma}_\eta)$ or not learning from prices and instead getting a higher-precision signal $\eta|f \sim N(f, \Sigma_\eta)$, where the signals are conditionally independent across agents, and where $\Sigma_\eta^{-1} = \Sigma_p^{-1} + \tilde{\Sigma}_\eta^{-1}$, would prefer not to learn from prices.*

Proof. From (9) in the main text, we know that expected utility is

$$U_{1j} = \frac{1}{2}trace(\hat{\Sigma}_j^{-1}Var[\hat{\mu}_j - pr]) + \frac{1}{2}E_1[\hat{\mu}_j - pr]'\hat{\Sigma}_j^{-1}E_1[\hat{\mu}_j - pr]$$

By Bayes' rule, the two options yield equally informative posterior beliefs: $\hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_\eta^{-1} = \Sigma^{-1} + \Sigma_p^{-1} + \tilde{\Sigma}_\eta^{-1}$. Likewise, since both possibilities give the manager unbiased signals, beliefs are a martingale, meaning that $E_1[\hat{\mu}_j - pr]$, is identical under the two options.

Thus, the only term in expected utility that is affected by the decision to learn information from prices is $Var[\hat{\mu}_j - pr]$. Let $\hat{\mu}_j = E[f|\eta]$ be the posterior expected value of payoffs for the manager who learns from the conditionally independent signal and $\tilde{\mu} = E[f|p, \tilde{\eta}]$ the posterior expected value for the manager that chooses to learn information in prices.

From properties of the variance we have that:

$$Var[\hat{\mu}_j - pr] - Var[\tilde{\mu} - pr] = Var[\hat{\mu}_j] - Var[\tilde{\mu}] + 2(Cov[\tilde{\mu}, pr] - Cov[\hat{\mu}_j, pr])$$

Step 1: show that the first two terms are equal and thus cancel out. By Bayes' law: $\hat{\mu}_j = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_{\eta j}^{-1}\eta)$. Writing the signal as $\eta = f + \epsilon$ (see Preliminaries) we have:

$$\hat{\mu}_j = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_{\eta j}^{-1}f + \Sigma_{\eta j}^{-1}\epsilon)$$

Then $Var[\hat{\mu}_j] = \hat{\Sigma}_j\Sigma_{\eta j}^{-1}(\Sigma\Sigma_{\eta j}^{-1} + I)\hat{\Sigma}_j$.

Again, by Bayes' law, $\tilde{\mu} = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_p^{-1}B^{-1}(pr - A) + \tilde{\Sigma}_\eta^{-1}\tilde{\eta})$. Write the signal as $\tilde{\eta} = f + \tilde{\epsilon}$ and use the price equation $pr = A + Bf + Cx$ we get:

$$\begin{aligned}
\tilde{\mu} &= \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_p^{-1}B^{-1}(Bf + Cx) + \tilde{\Sigma}_\eta^{-1}(f + \tilde{\epsilon})) \\
&= \hat{\Sigma}_j(\Sigma^{-1}\mu + (\Sigma_p^{-1} + \tilde{\Sigma}_\eta^{-1})f + \Sigma_p^{-1}B^{-1}Cx + \tilde{\Sigma}_\eta^{-1}\tilde{\epsilon}) \\
&= \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_\eta^{-1}f + \Sigma_p^{-1}B^{-1}Cx + \tilde{\Sigma}_\eta^{-1}\tilde{\epsilon})
\end{aligned}$$

where the last equality comes from the assumption that $\tilde{\Sigma}_\eta^{-1} + \Sigma_p^{-1} = \Sigma_\eta^{-1}$.

Then taking variance:

$$\begin{aligned}
Var[\tilde{\mu}] &= \hat{\Sigma}_j(\Sigma_\eta^{-1}\Sigma\Sigma_\eta^{-1} + \sigma_x\Sigma_p^{-1}B^{-1}CC'B^{-1}\Sigma_p^{-1} + \tilde{\Sigma}_\eta^{-1}])\hat{\Sigma}_j \\
&= \hat{\Sigma}_j\Sigma_\eta^{-1}(\Sigma\Sigma_\eta^{-1} + I)\hat{\Sigma}_j + \sigma_x\hat{\Sigma}_j(\Sigma_p^{-1}B^{-1}CC'B^{-1}\Sigma_p^{-1} - \Sigma_p^{-1})\hat{\Sigma}_j \\
&= \hat{\Sigma}_j\Sigma_\eta^{-1}(\Sigma\Sigma_\eta^{-1} + I)\hat{\Sigma}_j + \hat{\Sigma}_j\Sigma_p^{-1}(\sigma_x B^{-1}CC'B^{-1}\Sigma_p^{-1} - I)\hat{\Sigma}_j \\
&= \hat{\Sigma}_j\Sigma_\eta^{-1}(\Sigma\Sigma_\eta^{-1} + I)\hat{\Sigma}_j
\end{aligned}$$

where the last term cancels because $\Sigma_p^{-1} \equiv \sigma_x^{-1}B'(CC')^{-1}B$. Therefore, $Var[\hat{\mu}_j] = Var[\tilde{\mu}]$.

Step 2: Show that $Cov[\tilde{\mu}, pr] > Cov[\hat{\mu}_j, pr]$. Now we compute the covariance terms, using the fact that adding constant terms do not change change the value of the covariance.

$$\begin{aligned}
Cov[\hat{\mu}_j, pr] &= Cov[\hat{\mu}_j - \hat{\Sigma}_j(\Sigma_{\eta j}^{-1} + \Sigma^{-1})\mu, pr - A] \\
&= Cov[\hat{\Sigma}_j\Sigma_{\eta j}^{-1}(f-\mu) + \hat{\Sigma}_j\Sigma_{\eta j}^{-1}\epsilon, B(f-\mu) + Cx] \\
&= Cov[\hat{\Sigma}_j\Sigma_{\eta j}^{-1}(f-\mu), B(f-\mu)] \\
&= E[\hat{\Sigma}_j\Sigma_{\eta j}^{-1}(f-\mu)(f-\mu)'B'] \\
&= \hat{\Sigma}_j\Sigma_{\eta j}^{-1}\Sigma B'
\end{aligned}$$

$$\begin{aligned}
Cov[\tilde{\mu}, pr] &= Cov[\tilde{\mu} - \hat{\Sigma}_j(\Sigma_\eta^{-1} + \Sigma^{-1})\mu, pr - A] \\
&= Cov[\hat{\Sigma}_j\Sigma_{\eta j}^{-1}(f-\mu) + \hat{\Sigma}_j\Sigma_p^{-1}B^{-1}Cx + \hat{\Sigma}_j\tilde{\Sigma}_\eta^{-1}\tilde{\epsilon}, B(f-\mu) + Cx] \\
&= Cov[\hat{\Sigma}_j\Sigma_{\eta j}^{-1}(f-\mu), B(f-\mu)] + Cov[\hat{\Sigma}_j\Sigma_p^{-1}B^{-1}Cx, Cx] \\
&= \hat{\Sigma}_j\Sigma_{\eta j}^{-1}\Sigma B' + E[\hat{\Sigma}_j\Sigma_p^{-1}B^{-1}Cxx'C'] \\
&= \hat{\Sigma}_j\Sigma_{\eta j}^{-1}\Sigma B' + \hat{\Sigma}_j\Sigma_p^{-1}(\sigma_x^{-1}B'(CC')^{-1}B)^{-1}B' \\
&= Cov[\hat{\mu}_j, pr] + \hat{\Sigma}_j B'
\end{aligned}$$

In summary, we have shown that $Var[\hat{\mu}_j - pr] - Var[\tilde{\mu} - pr] > 0$. The difference in utility from learning conditionally independent information and learning price information is given by

$$\frac{1}{2}trace(\hat{\Sigma}_j^{-1}(Var[\hat{\mu}_j - pr] - [Var[\tilde{\mu} - pr]))$$

Since the expression inside the trace is a product of positive definite matrices, the difference in expected utilities is positive. □

A.4 Proof of Proposition 1

If (42) holds, then the marginal value of a given investor j reallocating an increment of capacity from stock-specific shock $i \in \{1,2\}$ *to the aggregate shock is increasing in the aggregate shock variance* σ_a*: If* $K_{aj} = \tilde{K}$ *and* $K_{ij} = K - \tilde{K}$*, then* $\partial^2 U/\partial \tilde{K} \partial \sigma_a > 0$.

Proof. We want to take a cross-partial derivative of utility with respect to σ_a and $\tilde{K}$. To do this, we will substitute out the $\hat{\Sigma}_j^{-1}$ terms using (27), (28), and (29). Then, we will use the fact that, by the chain rule, $\partial U/\partial \tilde{K} = \partial U/\partial K_{aj} - \partial U/\partial K_{ij}$. Therefore,

$$\partial^2 U/\partial \tilde{K} \partial \sigma_a = \partial^2 U/\partial K_{aj} \partial \sigma_a - \partial^2 U/\partial K_{ij} \partial \sigma_a$$

We consider each of these two cross-partial derivatives separately in cases a and b.

Part a: *The marginal value of a given investor j having additional capacity* K_{aj} *devoted to learning about the aggregate shock a is increasing in the aggregate shock variance:* $\partial^2 U/\partial K_{aj} \partial \sigma_a > 0$.

Recall the expression for utility in (47):

$$U_{1j} = \frac{1}{2} trace\left(\hat{\Sigma}_j^{-1}[\bar{\Sigma}(\rho^2 \sigma_x I + \Sigma^{-1})\bar{\Sigma}] + I - 2\Sigma^{-1}\bar{\Sigma}\right) + \frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$$

Sign last term: $\frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$

Note that K_{aj} appears only in $\hat{\Sigma}_j^{-1}$. Recall that $\partial \hat{\Sigma}_j^{-1}/\partial \hat{\sigma_a}^{-1} = \Upsilon_a$. Since $\hat{\sigma_a}^{-1} = \sigma_a{}^{-1} + K_{aj}$, the chain rule implies that $\partial \hat{\Sigma}_j^{-1}/\partial K_{aj} = \Upsilon_a$. Thus, the last term has K_{aj} derivative $(\rho^2/2)\bar{x}'\bar{\Sigma}\Upsilon_a\bar{\Sigma}\bar{x}$. The only term in this expression that varies in σ_a is $\bar{\Sigma}$. Since $\bar{\Sigma}$ has every entry increasing in σ_a (equation 31), and $\bar{\Sigma}$ and Υ_a are positive semi-definite matrices, this term has a positive cross-partial derivative $\partial^2/\partial K_j \partial \sigma_a > 0$. One can also compute it analytically and obtain: $\frac{\rho^2}{\sigma_a^2(\sigma_a^{-1}+\bar{K}_a)^2}\Upsilon_a\bar{\Sigma}\bar{x}\bar{x}' > 0$. Thus, a sufficient condition for $\partial^2 U/\partial K_j \partial \sigma_a > 0$ is for the trace term to have a positive cross partial derivative.

Sign trace: $trace\left(\hat{\Sigma}_j^{-1}[\bar{\Sigma}(\rho^2 \sigma_x I + \Sigma^{-1})\bar{\Sigma}] + I - 2\Sigma^{-1}\bar{\Sigma}\right)$

Observe that only the first element of the trace depends on K_{aj} and exclusively through $\hat{\Sigma}_j^{-1}$. We take derivative of the trace with respect to K_{aj} to get:

$$trace(\Upsilon_a \bar{\Sigma}(\rho^2 \sigma_x I + \Sigma^{-1})\bar{\Sigma})$$

Note from equation (39) that $\bar{\Sigma}(\rho^2 \sigma_x I + \Sigma^{-1})\bar{\Sigma} = V$. We know that the derivative of V with respect to σ_a is positive if condition (42) holds. Since *upa* is invariant in σ_a and is positive semidefinite, the partial derivative of the product is positive semidefinite, and therefore the trace is ≥ 0 if (42) holds. Thus, (42) is a sufficient condition for the cross-partial to be positive.

□

Part b: *The marginal value of a given investor j having additional capacity* K_{ij} *devoted to learning about stock-specific shock i is constant in the aggregate shock variance:* $\partial^2 U/\partial K_{ij} \partial \sigma_a = 0$.

Proof. Without loss of generality, we consider reallocating capacity from the asset 1 shock to the aggregate shock ($i = 1$). The same proof follows if it were asset 2 instead.

Sign last term: $\frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$.

Note that K_{1j} appears only in $\hat{\Sigma}_j^{-1}$. Recall that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma_1}^{-1} = \Upsilon_1$. Since $\hat{\sigma_1}^{-1} = \sigma_1{}^{-1} + K_{1j}$, using the chain rule, we get $\partial\hat{\Sigma}_j^{-1}/\partial K_{1j} = \Upsilon_1$. Therefore, $\partial/\partial K_{1j}(\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}) = \bar{x}'\bar{\Sigma}\Upsilon_1\bar{\Sigma}\bar{x}$.

Because of the structure of the Υ_1 matrix, it turns out that using (26) and (31) to multiply out the three matrices $\bar{\Sigma}\Upsilon_1\bar{\Sigma}$ delivers

$$\bar{\Sigma}\Upsilon_1\bar{\Sigma} = \begin{bmatrix} (\sigma_1^{-1} + \bar{K}_1)^{-2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{49}$$

Since this has no σ_a term in it and both $\bar{x}$ and ρ are exogenous, the cross-partial derivative $\partial^2/\partial K_{1j}\partial\sigma_a$ of the last terms is zero.

Sign trace: $trace\left(\hat{\Sigma}_j^{-1}[\bar{\Sigma}(\rho^2\sigma_x I + \Sigma^{-1})\bar{\Sigma}] + I - 2\Sigma^{-1}\bar{\Sigma}\right)$

Observe that only the first element of the trace depends on K_{1j} and exclusively through $\hat{\Sigma}_j^{-1}$. We take derivative of the trace with respect to K_{1j} to get:

$$trace(\Upsilon_1\bar{\Sigma}(\rho^2\sigma_x I + \Sigma^{-1})\bar{\Sigma})$$

From the multiplication above, we have that $trace(\bar{\Sigma}\Upsilon_1\bar{\Sigma}) = (\sigma_1^{-1} + \bar{K}_1)^{-2}$. Also, the sparse form of Υ_1 causes the matrix multiplication of $\Upsilon_1\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}'$ to turn out neatly. Using (23), (26) and (31) to multiply out the four matrices delivers

$$\Upsilon_1\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}' = \begin{bmatrix} \sigma_1^{-1}(\sigma_1^{-1} + \bar{K}_1)^{-2} & 0 & 0 \\ 0 & 0 & 0 \\ -b_1\sigma_1^{-1}(\sigma_1^{-1} + \bar{K}_1)^{-2} & 0 & 0 \end{bmatrix}. \tag{50}$$

The trace of this matrix is $\sigma_1^{-1}(\sigma_1^{-1}+\bar{K}_1)^{-2}$. With these facts, the expression for $\partial/\partial K_{ij}$ can be rewritten as: $(\sigma_1^{-1} + \bar{K}_1)^2(\sigma_1^{-1} + \rho\sigma_x)$. Since this expression has no σ_a term in it, the cross-partial derivative $\partial^2/\partial K_{1j}\partial\sigma_a$ is zero.

In conclusion, if $\bar{K}_a \leq \sigma_a^{-1}$ then $\partial^2 U/\partial K_{aj}\partial\sigma_a > 0$ and $\partial^2 U/\partial K_{ij}\partial\sigma_a = 0$ and the difference of the two terms is positive. Thus, if the average attention allocated to the aggregate shock is not too high, the marginal value of a given investor j reallocating an increment of capacity from shock 1 to the aggregate shock is increasing in the aggregate shock variance: $\partial^2 U/\partial\tilde{K}\partial\sigma_a = \partial^2 U/\partial K_{aj}\partial\sigma_a - \partial^2 U/\partial K_{ij}\partial\sigma_a > 0$. □

A.5 Proof of Proposition 2

If the size of the composite asset $\bar{x}_3$ is sufficiently large, then an increase in risk aversion increases the marginal utility of reallocating a unit of capacity from the idiosyncratic shock to the aggregate shock: $\partial/\partial\rho(\partial U/\partial(K_{aj} - K_{1j}) > 0$.

Proof. We can rewrite $\partial/\partial\rho(\partial U/\partial(\hat{\sigma}_{aj}^{-1} - \hat{\sigma}_{1j}^{-1}))$ as $\partial^2 U/\partial\rho\partial\hat{\sigma}_{aj}^{-1} - \partial^2 U/\partial\rho\partial\hat{\sigma}_{1j}^{-1} > 0$.

We will work out each of these two terms separately. But first, both depend on the partial derivative of utility with respect to risk aversion. Taking the partial derivative of utility in (47) with respect to ρ yields

$$\frac{\partial U}{\partial \rho} = \rho\sigma_x Tr[\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{\Sigma}] + \bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}. \tag{51}$$

The next step is to differentiate (51) with respect to $\hat{\sigma}_{aj}^{-1}$. Since $\hat{\sigma}_{aj}^{-1}$ is the precision of agent j's information, it does not affect aggregate variables such as $\bar{\Sigma}$. Recalling that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma}_{aj}^{-1} = \Upsilon_a$,

$$\frac{\partial^2 U}{\partial\rho\partial\hat{\sigma}_{aj}^{-1}} = \rho\sigma_x Tr[\Upsilon_a\bar{\Sigma}\bar{\Sigma}] + \bar{x}'\bar{\Sigma}\Upsilon_a\bar{\Sigma}\bar{x}. \tag{52}$$

Next, we follow the same steps to differentiate (51) with respect to $\hat{\sigma}_{1j}^{-1}$, which also affects only $\hat{\Sigma}_j$. Recalling that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma}_{1j}^{-1} = \Upsilon_1$, and using the fact that the trace is invariant to matrix ordering,

$$\frac{\partial^2 U}{\partial\rho\partial\hat{\sigma}_{1j}^{-1}} = \rho\sigma_x Tr[\bar{\Sigma}\Upsilon_1\bar{\Sigma}] + \bar{x}'\bar{\Sigma}\Upsilon_1\bar{\Sigma}\bar{x}. \tag{53}$$

The extent to which risk aversion affects the utility of reallocating precision from risk 1 to risk a is the difference of (52) and (53):

$$\partial/\partial\rho(\partial U/\partial(\hat{\sigma}_{aj}^{-1} - \hat{\sigma}_{1j}^{-1})) = \rho\sigma_x Tr[\bar{\Sigma}(\Upsilon_a - \Upsilon_1)\bar{\Sigma}] + \bar{x}'\bar{\Sigma}(\Upsilon_a - \Upsilon_1)\bar{\Sigma}\bar{x}. \tag{54}$$

If $b_1 < 1$, then $\Upsilon_a - \Upsilon_1 = \begin{bmatrix} -1 & 0 & b_1 \\ 0 & 0 & 0 \\ b_1 & 0 & 1 - b_1^2 \end{bmatrix}$ is positive definite. Since $\bar{\Sigma}$ is positive definite, both the trade and the second term will be positive and we have the result.

We can derive a different sufficient condition as follows. Multiplying out term-by-term $\bar{\Sigma}\Upsilon_a\bar{\Sigma}$ reveals that it equals $\bar{\sigma}_a^2 bb'$. Multiplying out $\bar{\Sigma}\Upsilon_1\bar{\Sigma}$ reveals that it equals

$$\bar{\Sigma}\Upsilon_1\bar{\Sigma} = \begin{bmatrix} \bar{\sigma}_1^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus,

$$Tr[\bar{\Sigma}(\Upsilon_a - \Upsilon_1)\bar{\Sigma}] = \bar{\sigma}_a^2(b_1^2 + b_2^2 + 1) - \bar{\sigma}_1^2$$

and

$$\bar{x}'\bar{\Sigma}(\Upsilon_a - \Upsilon_1)\bar{\Sigma}\bar{x} = \bar{\sigma}_a^2(b_1\bar{x}_1 + b_2\bar{x}_2 + \bar{x}_3)^2 - \bar{x}_1^2\bar{\sigma}_1^2$$

If these two terms are positive, then $\partial/\partial\rho(\partial U/\partial(\hat{\sigma}_{aj}^{-1} - \hat{\sigma}_{1j}^{-1})) > 0$.

Note that the whole second partial derivative is increasing in x_3, the supply of the composite asset:

$$\partial/\partial\rho(\partial U/\partial(\hat{\sigma}_{aj}^{-1} - \hat{\sigma}_{1j}^{-1})) = \rho\sigma_x(\bar{\sigma}_a^2(b_1^2 + b_2^2 + 1) - \bar{\sigma}_1^2) + \bar{\sigma}_a^2(b_1\bar{x}_1 + b_2\bar{x}_2 + \bar{x}_c)^2 - \bar{x}_1^2\bar{\sigma}_1^2 \tag{55}$$

Thus, as long as the composite asset, meant to represent the entire market capitalization, aside from the two assets 1 and 2, is large enough relative to assets 1 and 2, the cross-partial derivative will be positive. □

A.6 Proof of Proposition 3

If (42) holds, then for given precisions of an investor j who is more informed than the average (i.e. for fixed $K_{aj} > \bar{K}_a$ and $K_{ij} > \bar{K}_i$ for $i \in \{1, 2\}$), an increase in aggregate risk σ_a: a) increases the dispersion of fund portfolios $E[(q_j - \bar{q})'(q_j - \bar{q})]$ where $\bar{q} \equiv \int q_j dj$ and b) increases the dispersion of funds' portfolio returns $E[((q_j - \bar{q})'(f - pr))^2]$.

Proof of Part a):

Proof. From equation (44) in the preliminaries, we know that portfolio dispersion is given by:

$$E[(q_j - \bar{q})'(q_j - \bar{q})] = \frac{1}{\rho^2} Tr(\Delta V \Delta) + Tr(\bar{x}' \Delta \bar{\Sigma} \bar{\Sigma} \Delta \bar{x})$$

Now we take partial derivatives with respect to σ_a.

Term 1: The first term depends only on information choice variables. So, holding choices fixed, the partial derivative with respect to σ_a is zero.

Term 2: Recall that $\Delta \equiv \hat{\Sigma}_j^{-1} - (\bar{\Sigma})^{-1}$ depends only on information choices, which we hold fixed, and thus $\partial Tr(\Delta V \Delta)/\partial \sigma_a = Tr(\Delta \left[\partial V/\partial \sigma_a\right] \Delta)$. Furthermore, if the posterior precision of the investor is higher than the average, Δ is positive definite too. From the preliminaries we know that $\partial V/\partial \sigma_a$ is positive semi-definite if condition (42) holds. Therefore, (42) is a sufficient condition for the second term to be positive.

Term 3: The derivative of term 3 with respect to σ_a is: $Tr(\bar{x}' \Delta \left[\partial(\bar{\Sigma}\bar{\Sigma})/\partial \sigma_a\right] \Delta \bar{x})$. This will be positive if $\left[\partial(\bar{\Sigma}\bar{\Sigma})/\partial \sigma_a\right]$ is positive semi-definite. We have that $\bar{\Sigma}\bar{\Sigma} = (\sigma_a^{-1} + \bar{K}_a)^{-2} bb'bb' + 2(\sigma_a^{-1} + \bar{K}_a)^{-1} bb'\Phi + \Phi^2$ and thus the derivative with respect to σ_a is: $2\sigma_a^{-2}(\sigma_a^{-1} + \bar{K}_a)^{-3} bb'bb' + 2\sigma_a^{-2}(\sigma_a^{-1} + \bar{K}_a)^{-2} bb'\Phi > 0$.

In sum, all four terms in the expression for dispersion are increasing in σ_a. □

Proof of Part b):

Proof. From equation (46) in the preliminaries, we know that dispersion of portfolio returns is given by:

$$\begin{aligned} E[((q_j - \bar{q})'(f - pr))^2] &= Tr(\bar{x}'\bar{\Sigma}'\Sigma_{\eta j}^{-1}\bar{\Sigma}\bar{x}) + \rho^2 Tr^2(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x}) + 4Tr(\bar{x}'\bar{\Sigma}'\Delta V \Delta \bar{\Sigma}\bar{x}) \\ &+ \frac{2}{\rho^2} Tr(\Delta V \Delta V) + \frac{1}{\rho^2} Tr^2(\Delta V) + 2Tr(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x}) Tr(\Delta V) \end{aligned}$$

Observe that $\Sigma_{\eta j}^{-1}$ and Δ depend only on choice variables, which we hold fixed. Also, every entry of $\bar{\Sigma}$ is increasing in σ_a. If condition (42) holds, $\partial V/\partial \sigma_a$ is a positive semi-definite matrix, which guarantees that the whole expression is increasing in σ_a.

□

A.7 Proposition 4

If σ_x is sufficiently high, then for given $K_{aj}, K_{ij}\,\forall j$, an increase in risk aversion ρ increases the dispersion of funds' portfolio returns $E[((q_j-\bar{q})'(f-pr))^2]$.

Proof. From equation (46) in the preliminaries, we have that the dispersion of portfolio returns is given by:

$$\begin{aligned} E[((q_j-\bar{q})'(f-pr))^2] &= Tr(\bar{x}'\bar{\Sigma}'\Sigma_{\eta j}^{-1}\bar{\Sigma}\bar{x}) + 4Tr(\bar{x}'\bar{\Sigma}'\Delta V\Delta\bar{\Sigma}\bar{x}) + \frac{2}{\rho^2}Tr(\Delta V\Delta V) \\ &+ \rho^2 Tr^2(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x}) + \frac{1}{\rho^2}Tr^2(\Delta V) + 2Tr(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x})Tr(\Delta V) \end{aligned}$$

Taking derivatives with respect to ρ we get:

$$\begin{aligned} \frac{\partial E[((q_j-\bar{q})'(f-pr))^2]}{\partial\rho} &= 4Tr(\bar{x}'\bar{\Sigma}'\Delta\frac{\partial V}{\partial\rho}\Delta\bar{\Sigma}\bar{x}) + \frac{2}{\rho^2}Tr\Big(\frac{\partial(\Delta V\Delta V)}{\partial\rho}\Big) - \frac{4}{\rho^3}Tr(\Delta V\Delta V) \\ &+ 2\rho Tr^2(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x}) + \frac{2}{\rho^2}Tr(\Delta V)Tr\Big(\Delta\frac{\partial V}{\partial\rho}\Big) - \frac{2}{\rho^3}Tr^2(\Delta V) \\ &+ 2Tr(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x})Tr\Big(\Delta\frac{\partial V}{\partial\rho}\Big) \end{aligned}$$

Substitute (39) for V, $\frac{\partial V}{\partial\rho} = 2\rho\sigma_x\bar{\Sigma}\bar{\Sigma}$ and $\frac{\partial(\Delta V\Delta V)}{\partial\rho} = 4\rho\sigma_x\Delta\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}\Delta\bar{\Sigma}\bar{\Sigma} + 4\rho^3\sigma_x^2\Delta\bar{\Sigma}\bar{\Sigma}\Delta\bar{\Sigma}\bar{\Sigma}$:

$$\begin{aligned} &= \underbrace{8\rho\sigma_x Tr(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{\Sigma}\Delta\bar{\Sigma}\bar{x})}_{>0} + \underbrace{2\rho Tr^2(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x})}_{>0} + \underbrace{4\rho\sigma_x Tr(\bar{x}'\bar{\Sigma}'\Delta\bar{\Sigma}\bar{x})Tr\Big(\Delta\bar{\Sigma}\bar{\Sigma}\Big)}_{>0} \\ &+ \underbrace{Tr\Big(\Delta\bar{\Sigma}\bar{\Sigma}\Big[4\rho\sigma_x^2\Delta\bar{\Sigma} - \frac{4}{\rho^3}\Sigma^{-1}\Delta\Sigma^{-1}\Big]\bar{\Sigma}\bar{\Sigma}\Big)}_{I} \\ &+ \underbrace{2\rho\sigma_x^2 Tr^2(\Delta\bar{\Sigma}\bar{\Sigma}) - \frac{2}{\rho^3}Tr^2(\Delta\bar{\Sigma}\Sigma^{-1}\bar{\Sigma})}_{II} \end{aligned}$$

We need a condition such that I and II are positive and thus the derivative will be positive. For (I), we need $4\rho\sigma_x^2\Delta\bar{\Sigma} \geq \frac{4}{\rho^3}\Sigma^{-1}\Delta\Sigma^{-1}$, or

$$Tr(\Delta\bar{\Sigma}(\rho^4\sigma_x^2 I - \Delta^{-1}\bar{\Sigma}^{-1}\Sigma^{-1}\Delta\Sigma^{-1})) \geq 0. \tag{56}$$

For (II), we need $2\rho\sigma_x^2 Tr^2(\Delta\bar{\Sigma}\bar{\Sigma}) \geq \frac{2}{\rho^3}Tr^2(\Delta\bar{\Sigma}\Sigma^{-1}\bar{\Sigma})$, or

$$Tr(\Delta\bar{\Sigma}(\rho^2\sigma_x I - \Sigma^{-1})\bar{\Sigma}) \geq 0. \tag{57}$$

In both cases, if σ_x is sufficiently high, the conditions are met. □

A.8 Proof of Proposition 5

If (42) holds, then an increase in aggregate shock variance increases the difference between an informed investor expected certainty equivalent return and the expected certainty equivalent return of an uninformed *investor:* $\partial(U_j - U^U)/\partial\sigma_a > 0$.

Proof. From (48) we know that time-1 expected utility for an agent who has posterior belief precision $\hat{\Sigma}_j^{-1}$ is given by

$$U_{1j} = \frac{1}{2}trace\left(\hat{\Sigma}_j^{-1}V + I - 2\Sigma^{-1}\bar{\Sigma}\right) + \frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$$

Since the result is about the difference in utility between an informed and an uninformed agent, who has posterior belief precision $\hat{\Sigma}_j = \Sigma$, we take the difference of these two expected utilities:

$$U_j - U^U = \frac{1}{2}trace\left([\hat{\Sigma}_j^{-1} - \Sigma^{-1}]V\right) + \frac{\rho^2}{2}\bar{x}'\bar{\Sigma}[\hat{\Sigma}_j^{-1} - \Sigma^{-1}]\bar{\Sigma}\bar{x}$$

Recall that $\hat{\Sigma}_j^{-1} - \Sigma^{-1} = \Sigma_{\eta j}^{-1}$, then:

$$U_j - U^U = \frac{1}{2}trace\left(\Sigma_{\eta j}^{-1}V\right) + \frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\Sigma_{\eta j}^{-1}\bar{\Sigma}\bar{x}$$

Since $\Sigma_{\eta j}^{-1}$ is a choice variable it does not change in σ_a. Therefore we have that the partial derivative of the utility difference with respect to σ_a is:

$$\frac{\partial(U_j - U^U)}{\partial\sigma_a} = \frac{1}{2}trace\left(\Sigma_{\eta j}^{-1}\left[\frac{\partial V}{\partial\sigma_a}\right]\right) + \rho^2 trace\left(\bar{x}'\Sigma_{\eta j}^{-1}\left[\frac{\partial\bar{\Sigma}}{\partial\sigma_a}\right]\bar{\Sigma}\bar{x}\right)$$

Differentiating (31) tells us that $\partial\bar{\Sigma}/\partial\sigma_a$ is a positive scalar times bb', which is positive semi-definite. If (42) holds, then $\partial V/\partial\sigma_a$ is also positive semi-definite. Therefore, both trace terms are positive and $\partial(U_j - U^U)/\partial\sigma_a > 0$.

□

A.9 Proof of Corollary 1

If (42) holds, then for given precisions of an investor j who is more informed than the average (i.e. for fixed $K_{aj} > \bar{K}_a$ and $K_{ij} > \bar{K}_i$ for $i \in \{1,2\}$ strictly positive), an increase in aggregate risk σ_a increases the expected profit of an informed fund, $E[(q_j - \bar{q})'(f - pr)]$, where $\bar{q} \equiv \int q_j dj$.

Proof. From (45), the expected profits are given by:

$$E[(q_j - \bar{q})'(f - pr)] = \rho Tr(\bar{x}'\bar{\Sigma}\Delta\bar{\Sigma}\bar{x}) + \frac{1}{\rho}Tr(\Delta V)$$

Then

$$\frac{\partial E[(q_j - \bar{q})'(f - pr)]}{\partial\sigma_a} = 2\rho Tr\left(\bar{x}'\Delta\left[\frac{\partial\bar{\Sigma}}{\partial\sigma_a}\right]\bar{\Sigma}\bar{x}\right) + \frac{1}{\rho}Tr\left(\Delta\left[\frac{\partial V}{\partial\sigma_a}\right]\right) > 0$$

since $\Delta = \Sigma_{\eta j}^{-1} - (\bar{\Sigma}_\eta)^{-1}$ does not depend on σ_a and both $\bar{\Sigma}$ and V are increasing in σ_a as long as condition (42) holds. The requirement that the investor is more informed than the average ensures that Δ is positive semi-definite.

□

A.10 Proposition 6

For given K_{aj}, K_{1j}, K_{2j} strictly positive, an increase in risk aversion ρ for all investors increases the difference in expected certainty equivalent returns between an informed and an uninformed investor: $\partial(U_j - U^U)/\partial\rho > 0$.

Proof. From the proof of proposition (5) we have that the difference in expected utilities between an informed investor and an uninformed investors is:

$$U_j - U^U = \frac{1}{2}Tr(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)') - \frac{1}{2}Tr\left(\Sigma^{-1}(\bar{\Sigma} - \Sigma)\Sigma^{-1}(\bar{\Sigma} - \Sigma)'\right)$$

$$+\frac{1}{2}Tr(\Sigma_{\eta j}^{-1}\hat{\Sigma}_j) + \frac{\rho^2}{2}\Big\{\sigma_x Tr(\bar{\Sigma}\Sigma_{\eta j}^{-1}\bar{\Sigma}) + \bar{x}'\bar{\Sigma}\Sigma_{\eta j}^{-1}\bar{\Sigma}\bar{x}\Big\}$$

where $\Sigma_{\eta j}^{-1} = \hat{\Sigma}_j^{-1} - \Sigma^{-1}$. Differentiating with respect to ρ yields:

$$\frac{\partial(U_j - U^U)}{\partial\rho} = \rho\Big\{\sigma_x Tr(\bar{\Sigma}\Sigma_{\eta j}^{-1}\bar{\Sigma}) + \bar{x}'\bar{\Sigma}\Sigma_{\eta j}^{-1}\bar{\Sigma}\bar{x}\Big\}$$

As long as K_{aj}, K_{ij} are strictly positive, then $\Sigma_{\eta j}$ and its inverse will be a positive definite matrices. Since $\bar{\Sigma}$ is clearly positive definite, we obtain the desired result. □