

# A Theory of Systemic Risk and Design of Prudential Bank Regulation<sup>1</sup>

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## **A Theory of Systemic Risk and Design of Prudential Bank Regulation**

### **Abstract**

Systemic risk is modeled as the endogenously chosen correlation of returns on assets held by banks. The limited liability of banks and the presence of a negative externality of one bank's failure on the health of other banks give rise to a *systemic risk-shifting* incentive where all banks undertake correlated investments, thereby increasing economy-wide aggregate risk. Regulatory mechanisms such as bank closure policy and capital adequacy requirements that are commonly based only on a bank's *own* risk fail to mitigate aggregate risk-shifting incentives, and can, in fact, accentuate systemic risk. Prudential regulation is shown to operate at a collective level, regulating each bank as a function of both its *joint* (correlated) risk with other banks as well as its individual (bank-specific) risk.

# 1 Introduction

## 1.1 General Overview

A financial crisis is “systemic” in nature if many banks fail together, or if one bank’s failure propagates as a contagion causing the failure of many banks. At the heart of bank regulation is a deep-seated concern that social and economic costs of such systemic crises are large. It is thus broadly understood that the goal of prudential regulation should be to ensure the financial stability of the system as a whole, i.e., of an institution not only individually but also as a part of the overall financial system.<sup>1</sup> Different reform proposals such as the ones by the Bank of International Settlements (1999) have been made with the objective of improving bank regulation, and in the aftermath of the global financial crisis of 2007-2009, many more proposals will come to the fore. A central issue is to examine these proposals under a common theoretical framework that formalizes the (often implicit) objective of ensuring efficient levels of systemic failure risk. This paper seeks to fill this important gap in the literature.

The standard theoretical approach to the design of bank regulation considers a “representative” bank and its response to particular regulatory mechanisms such as taxes, closure policy, capital requirements, etc. Such partial equilibrium approach has a serious shortcoming from the standpoint of understanding sources of, and addressing, inefficient systemic risk. In particular, it ignores that in general equilibrium, each bank’s investment choice has an externality on the payoffs of other banks and thus on their investment choices. Consequently, banks can be viewed as playing a strategic Nash game in responding to financial externalities and regulatory mechanisms. Recognizing this shortcoming of representative bank models, this paper develops a unified framework with multiple banks to study the essential properties of prudential bank regulation that takes into account both individual and systemic bank failure risk.

Our analysis has two features: one positive and one normative. The positive feature of the analysis provides a precise definition and an equilibrium characterization of systemic risk. Unlike most of the extant literature on systemic risk (see Section 2) that has focused on bank liability structures, we define systemic risk as the joint failure risk arising from the correlation of returns on asset-side of bank balance-sheets. Moreover, we give a characterization of conditions under which in equilibrium, banks prefer an inefficiently high correlation of asset returns (“herd”), giving rise to systemic or aggregate risk.

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<sup>1</sup>For example, Stephen G. Cecchetti, former Director of Research at the Federal Reserve Bank of New York, mentioned in his remarks at a symposium on the future of financial systems, “The need to protect consumers gives rise to prudential regulation whose main focus is on the failure of the individual firm.... The second basic justification for regulation is to reduce systemic risk. In this capacity, the regulator really functions as the risk manager for the financial system as a whole.” (Cecchetti, 1999)

The normative feature of the analysis involves the design of optimal regulation to mitigate inefficient systemic risk. To this end, we first demonstrate that the design of regulatory mechanisms, such as bank closure policy and capital adequacy requirements, based *only* on individual bank risk could be suboptimal in a multiple bank context, and may well have the unintended effect of accentuating systemic risk. Next, we show that optimal regulation should be “collective” in nature and should involve the *joint* failure risk of banks as well as their individual failure risk. In particular, (i) bank closure policy should exhibit little forbearance upon joint bank failures and conduct bank sales upon individual bank failures, and (ii) capital adequacy requirements should be increasing in the correlation of risks across banks as well as in individual risks.

## 1.2 Model Overview

In our model, banks have access to deposits that take the form of a simple debt contract. Upon borrowing, banks invest in risky and safe assets. In addition, they choose the “industry” in which they undertake risky investments. The choice of industry by different banks determines the correlation of their portfolio returns. Systemic risk arises as an *endogenous* consequence when in equilibrium, banks prefer to lend to similar industries.<sup>2</sup>

Since deposit contract is not explicitly contingent on bank characteristics, the depositor losses resulting from bank failures are not internalized by the bankowners. This externality generates a role for regulation. The regulator in our model is a *central bank* whose objective is to maximize the sum of the welfare of the bankowners and the depositors net of any social costs of financial distress.

In this setting with multiple banks, when one bank fails, there are two conflicting effects on other banks. First, there is a *reduction* in the aggregate supply of funds (deposits) in the economy, and hence, in aggregate investment. This results in a *recessionary spillover* (a negative externality) to the surviving banks through an increase in the market-clearing rate for deposits, that reduces the profitability of banks.<sup>3</sup> Second, surviving banks have a *strategic benefit* (a positive externality) from the failure of other banks due to an increase in scale or an expansion, resulting from the migration of depositors from the failed banks to the surviving banks, or, due to strategic gains from acquisition of failed banks’ assets and business.

Over a robust set of parameters, the negative externality effect exceeds the positive ex-

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<sup>2</sup>In practice, joint failure risk may be determined by a more complex pattern of inter-bank loans, derivatives, and other transactions.

<sup>3</sup>Diamond and Rajan (2005) have a somewhat similar general equilibrium effect, wherein a bank that receives liquidity shock is forced to sell assets due to hardness of deposit contracts, but in the process reduces aggregate liquidity available to other banks, causing a rise in their costs of borrowing and reduction in value. This can possibly lead to a contagion.

ternality effect, in which case banks find it optimal to increase the probability of surviving together, and thus failing together, by choosing asset portfolios with greater correlation of returns. This would arise, e.g., if (i) the reduction in aggregate investment is substantial upon a bank's failure, i.e., banks are 'large'; or (ii) the depositors of the failed bank do not migrate to the surviving banks, i.e., banks are 'essential'; or (iii) other banks cannot benefit from acquiring the business facilities of the failed bank, i.e., banks are 'unique' or anti-trust regulations prevent such acquisitions. The preference for high correlation arises as a joint consequence of the limited liability of the banks' equityholders and the nature of the externalities described above. This equilibrium characterization of systemic risk is the first contribution of the paper. We call such behavior as *systemic risk-shifting* since it can be viewed as a multi-agent counterpart of the risk-shifting phenomenon studied in corporate finance by Jensen and Meckling (1976), and in credit rationing by Stiglitz and Weiss (1981).

In the first-best allocation, however, different banks undertake investments in assets with lower correlation of returns. This is because losses to depositors, and to the economy, from a joint failure exceed those from individual failures. In individual bank failures, depositors of the failed bank migrate to surviving banks and intermediation role played by the failed bank is not fully impaired. However, such a possibility does not exist in a joint failure and there is a greater reduction in aggregate investment compared to states of individual bank failures.

The central bank attempts to mitigate systemic and individual risk-shifting incentives of bankowners through its design of bank closure policy and capital requirements. Our second contribution is to illustrate the design of bank closure policies that takes into account the collective investment policies of banks. We model the closure policy as a bail out of the failed bank with a dilution of bankowners' equity claim, greater dilution implying a less forbearing closure policy. A bank bail out eliminates the financial externalities discussed above but also induces moral hazard depending upon the extent of forbearance exercised. The optimal *ex-ante* closure policy is shown to be "collective" in nature: it exhibits lower forbearance towards bankowners upon joint failure than upon individual failure. The costs of nationalizing a large number of banks however may render such a policy suboptimal from an *ex-post* standpoint, i.e., time-inconsistent and hence, lacking in commitment. The resulting (implicit) "too-many-to-fail" guarantee, where bankowners anticipate greater forbearance upon joint failure than in individual failure, accentuates systemic risk by inducing banks to make correlated investments so as to extract greater regulatory subsidies.

Further, a "myopic" closure policy that does not take into account the collective response of banks and hence, does not distinguish between forbearance in individual and joint failures, also fails to mitigate systemic risk-shifting behavior. It is strictly dominated by collective regulation that counteracts any residual systemic moral hazard induced through "too-many-to-fail" guarantee, by conducting bank sales (possibly subsidized) upon failure of individual banks. This increases the charter value of banks, in a relative sense, in the states where they

survive but other banks fail, in turn, inducing a preference for lower correlation.<sup>4</sup>

Our third important contribution concerns the design of capital adequacy regulation. The current BIS capital requirement is a function *only* of a bank’s individual risk and does not penalize banks for holding asset portfolios with high correlation of returns. We show that under such a structure, each bank may optimally reduce its individual failure risk, but systemic risk arising from high correlation remains unaffected. To remedy this, we propose a “correlation-based” capital adequacy requirement that is increasing, not only in the individual risk of a bank, but is also increasing in the correlation of a bank’s asset portfolio returns with that of other banks in the economy. We propose an intuitively appealing implementation by considering a portfolio theory interpretation. The risks undertaken by banks can be decomposed into exposures to “general” risk factors and “idiosyncratic” components. For any given level of individual bank risk, correlation-based regulation would encourage banks to take idiosyncratic risks by charging a higher capital requirement against exposure to general risk factors.

Many financial institutions already employ a collective approach to capital budgeting (see Section 2) and regulators have also acknowledged the role of *intra-bank* correlations by proposing a long-term shift towards portfolio models for credit risk measurement (BIS, 1999). The proposed reforms of the BIS regulation appears however to have focused too much on the portfolio risk of each bank and ignored the *inter-bank* correlation effects for diversification of the economy-wide banking portfolio. Given the attention being devoted to possible reforms of the capital adequacy regulation and lender-of-last-resort policies, we believe our advocacy of collective regulation of systemic risk is particularly germane.

The remainder of the paper is structured as follows. Section 2 discusses the related literature. Section 3 describes the model set-up for the multiple-bank economy. Section 4 characterizes the systemic risk-shifting phenomenon in the intermediated economy. Section 5 considers the design of bank closure policies and Section 6 looks at the design of capital adequacy requirements. Finally, Section 7 concludes with a brief discussion of possible avenues for related research and the relevance of our results for other economic phenomena. Appendix A contains certain regularity assumptions and Appendix B contains proofs. The Addendum contains appendices from the unabridged version, Acharya (2001), referred to in the text.

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<sup>4</sup>It is also possible that a myopic closure policy that provides too-many-to-fail guarantee is the only sub-game perfect outcome unless the regulator can commit to a time-inconsistent closure policy, as argued by Acharya and Yorulmazer (2007, 2008b).

## 2 Related literature

A discussion of the seminal papers in banking regulation can be found in Dewatripont and Tirole (1993), and Freixas and Rochet (1997).

There is a burgeoning literature on models of contagion among banks: Rochet and Tirole (1996), Kiyotaki and Moore (1997), Freixas and Parigi (1998), Freixas, Parigi, and Rochet (1999), and Allen and Gale (2000c), to cite a few. The primary focus of these studies is on characterizing the sources of contagion and financial fragility. These studies examine the liability structure of banks, whereas in our model systemic risk arises from a high correlation of returns on the asset side of their balance-sheets.

In an incomplete markets model based on private information about agents' idiosyncratic endowments, Rampini (1999) defines systemic risk as default correlation. In his model, a substantial correlation of default arises to enable risk-sharing when an aggregate shock is low. This approach is different from ours since its focus is on optimal systemic risk from a risk-sharing standpoint and not on systemic risk that is suboptimal and that is an outcome of the collective risk-shifting incentives inherent in a multi-bank financial system.

Our general approach of considering the interaction of investment choices across banks has the flavor of the approach adopted by Maksimovic and Zechner (1990), Shleifer and Vishny (1992), and Rajan (1994). Maksimovic and Zechner study the endogenous choice of riskiness of cashflows and debt levels in an "industry equilibrium". Shleifer and Vishny focus on a "market equilibrium" where the liquidation value of a firm depends on the health of its peers. Rajan's paper is about bank lending policies and is somewhat more related.

Rajan models the information externality across two banks where reputational concerns and short-termism induce banks to continue to lend to negative NPV projects. He derives a theory of expansionary (or liberal) and contractionary (or tight) bank credit policies which influence, and are influenced by other banks and conditions of borrowers. However, his model does not examine the issue of whether banks lend to different industries or to similar industries. Further, the source of agency problem in his model is the short-term nature of managerial decisions, whereas in our model it is the bankowners' limited liability.

The importance of taking into account "covariances" of agents, in addition to their "variances", has been underlined by Froot and Stein (1998). They propose the need for centralized capital allocation within a financial institution. They criticize the RAROC (risk-adjusted return on capital) based approach which attends only to the individual risk of each line of business or lending activity. They suggest that bank-wide risk considerations must enter into the setting of hurdle rates in capital budgeting. Thus, in their setup too, the optimal design consists of a "central planner" who pools information across the different activities within a financial institution. Empirical evidence supporting such capital budgeting is provided by

James (1996) in the case study of Bank of America.

Our analysis and proposal of the collective regulation of banks is closest in spirit to that of Froot and Stein. However, there are important differences. Froot and Stein focus on undertaking of multiple projects or activities, but do not model agency issues that may drive the preference for correlation of returns across projects. Instead, we model these agency issues explicitly as arising from the limited liability of bank’s equityholders. Second, Froot and Stein study the problem of a single institution, i.e., they are concerned with *intra-bank* correlations. Our motivation instead is based on *inter-bank* correlations.

### 3 Model

We build a multi-period general equilibrium model with many agents, viz. banks and depositors, and many markets, viz. markets for safe and risky assets, and market for deposits. In order to study systemic risk and its prudential regulation, the model incorporates (i) the likelihood of default by banks on deposits; (ii) financial externalities from failure of one bank on other banks; (iii) regulatory incentives; and (iv) the interaction of these features. The model builds upon the Allen and Gale (2000a) model of bubbles and crises which is a one-period, single-investor model of risk-shifting. A schematic of the model is presented in Figure 1.

**Banks and depositors:** There are two periods and three dates  $t = 0, 1, 2$  with a single consumption good at each date. The economy consists of two banking “sectors,” possibly heterogeneous. The two banking sectors, which can be interpreted as being geographically separated, are denoted as ‘ $A$ ’ and ‘ $B$ ’. All variables in sector  $A$  are indexed by  $A$ . First, we describe a single banking sector. In sector  $i$ ,  $i \in \{A, B\}$ , there is

- (i) a single bank, owned by risk-neutral intermediaries (referred to as bankowners or equityholders), who have no wealth of their own; and
- (ii) a continuum of risk-neutral depositors, with  $D_{it} > 0$  units of good to invest at  $t = 0, 1$ .

Depositors have no investment opportunities, and hence lend their goods to banks. For simplicity, the bankowners and the depositors are assumed to have no time-preference. The deposits are assumed inelastic with no secondary trading, i.e., we rule out any revelation of information about the bank’s risk through deposit prices. The only deposit contract allowed is the simple debt contract with no conditioning of the deposit rate on the size of the deposit or on asset returns.<sup>5</sup> Since deposits cannot be conditioned on their size, banks can borrow

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<sup>5</sup>There are conditions such as costly state verification as in Townsend (1979) or Gale and Hellwig (1985) which justify such a simple debt contract. Alternately, the costs to the depositors of enforcing contracts where returns are explicitly contingent are too high.



as much as they like at the going rate of (gross) interest, denoted as  $r_{Dt}$ ,  $t = 0, 1$ . The banks are assumed to be price-takers when they borrow deposits since our model is an abstraction of an economy with a large number of banks accessing the deposit market even though, for ease of exposition, we have chosen to focus on the interaction of just two banks.<sup>6</sup>

In each period, the banks can invest in a “safe” asset and a “risky” asset, and also determine the “industry” in which they make the risky investments.

**Safe asset:** The safe asset is common to both banking sectors, and is available for investment only to the banks, not to depositors.<sup>7</sup> It pays a fixed gross return  $r_{St}$  at  $t + 1$  on a unit of investment at  $t = 0, 1$ . We interpret the safe asset as capital goods leased to the corporate sector (or riskless corporate debt). Competition in market for capital goods ensures that the rate of return on the safe asset is the marginal product of capital. We assume a neo-classical, diminishing returns-to-scale production technology  $f(x)$ ,  $f'(x) > 0$ ,  $f''(x) < 0$ ,  $f'(0) = \infty$ ,  $f'(\infty) = 0$ ,  $\forall x > 0$ . The equilibrium rate of interest is given by  $r_{St} = f'(x_t)$ ,  $x_t$  being the total investment in the safe asset at date  $t$  from both banking sectors.

**Risky asset:** The risky asset is to be interpreted as loans to entrepreneurs. Entrepreneurs holding the risky asset (a claim to their business profits) supply it to the bank in exchange for goods. Unlike the safe asset, these loans are information sensitive and each bank has a monopoly over the entrepreneurs in its sector to whom it lends. Relationships based on informational monopoly as in Rajan (1992) justify such an assumption. The supply of the risky asset in a sector is thus determined by the amount of risky investment by the corresponding bank. A particular risky asset is a portfolio of constituent loans that produce a given level of risk and return as described below.

For bank  $i$ , the risky asset gives a random gross return  $R_{it}$  at  $t + 1$  on one unit of investment at  $t = 0, 1$  that is distributed over the support  $[0, R_{max}]$ . The bank in sector  $i$  picks a risky asset (a portfolio) that gives a return  $R_{it} \sim h_i(\cdot; \sigma)$ , from a family of distributions  $\mathcal{H}_i$ , indexed by the risk parameter  $\sigma$ ,  $\sigma \in [\sigma_{min}, \sigma_{max}]$ . The bank selects  $\sigma_{it}$ , the riskiness of the asset, and in addition, it chooses the scale of investment in this portfolio. The risky asset technology,  $\mathcal{H}_i$ , is assumed to be identical in both the periods and short sales are not allowed.<sup>8</sup>

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<sup>6</sup>Appendix B in the unabridged version, Acharya (2000), analyzes a version of the model with a continuum of banks and develops qualitatively similar results.

<sup>7</sup>The assumption that depositors have no access to the safe asset is made purely for expositional ease. In extension (1) in Appendix D of the unabridged version, Acharya (2001), we relax this assumption and show that our results remain unchanged. Footnote 13 discusses this point in some detail.

<sup>8</sup>Note that such a risk choice arises naturally from a portfolio allocation between imperfectly correlated loan returns that are otherwise identical. The scale of investment determines the ‘size’ of the portfolio, and the riskiness of investment determines the ‘relative weights’ of the loans in the portfolio.

There are no direct linkages between the two sectors, i.e., no inter-bank contracts. However, the risky asset technologies for the two banking sectors may be correlated. The likelihood of joint default of the banks is determined by the individual risk of each bank’s investment as well as by the “correlation” of their investments, the latter being denoted as  $\rho_t$ .

**Choice of industry:** In each period, the risky investments in sector  $i$  can be made to two types of industries, say, “manufacturing” and “farming”. Information costs of investing in both industries are assumed to be exorbitantly high so that each bank invests in only one industry.<sup>9</sup> The correlation of returns on the risky investments when both banks invest in manufacturing or when both banks invest in farming, denoted as  $\rho_h$ , is higher than the correlation of returns when the banks invest in different industries, denoted as  $\rho_l$ , i.e.,  $\rho_h > \rho_l$ .

Within an industry, the banks can pick the scale and the risk level of the investment as described above. This is tantamount to assuming that the choice of risk and industry is over a space rich enough such that any combination of risk and correlation is feasible. Finally, we make certain regularity assumptions on the risky technologies and their correlations.

**Regularity assumptions on the risky technology:** In words, we want the portfolios of the two banks to satisfy the following: *ceteris paribus*,

- (i) increasing a bank’s risk increases its likelihood of failure and expected losses in failure;
- (ii) increasing a bank’s risk increases the likelihood of joint failure and expected losses in joint failure; and
- (iii) increasing the correlation across banks increases the likelihood of joint failure and expected losses in joint failure.

For simplicity, we restrict attention to a family of mean-preserving spreads.<sup>10</sup>

These assumptions are formally stated in Appendix A.

**Costs of investing in the risky asset:** We assume that there is a non-pecuniary cost of investing in the risky asset. We want to introduce costs in a way that restricts the size of individual portfolios (diminishing returns-to-scale) and at the same time ensures that banks make positive expected profits. These could be thought of as costs of loan initiation, monitoring, administration, etc. There are ways of dealing with pecuniary costs with some difficulty, but we choose to model these as non-pecuniary. This leads to a simple analysis and

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<sup>9</sup>Allowing banks to invest in many industries adds an interesting dimension to our results. It gives rise to a hitherto ignored tradeoff between “focus” and “diversification” in the banking industry, from a systemic risk standpoint, and is discussed in Section 6.3.

<sup>10</sup>The analysis can be carried out without any qualitative difference if the expected return is assumed to be non-decreasing and concave in risk ( $\sigma$ ).

lets us illustrate our results in a succinct manner. The cost function,  $c(x)$ , when the amount of risky investment is  $x$ , satisfies the neo-classical assumptions:  $c(0) = 0$ ,  $c'(0) = 0$ ,  $c'(x) > 0$ , and  $c''(x) > 0$ ,  $\forall x > 0$ . This cost technology is identical for both the sectors.

**Possible states at  $t = 1$ :** To keep the analysis simple, we assume that all profits of the bankowners from first period, if any, are paid out as dividends or consumed, and all returns on the first period deposits are consumed by the depositors.<sup>11</sup> Since the banks make risky investments at  $t = 0$ , they may fail to pay the depositors their promised rate of return in some states at  $t = 1$ . Upon such failure, the banks are closed. The possibility of bail outs and bank sales will be admitted to the model at a later stage. Depending on the survival or the failure of the two banks, there are four states possible at  $t = 1$  (see Figure 2):

(1) **Both banks survive (state “ $ss$ ”):** In this case, the depositors in sector  $i$  lend their goods,  $D_{i1}$ , to bank  $i$ ; and the costs of risky investments for each bank are given by  $c(x)$ .

(2) **Bank  $A$  survives but bank  $B$  fails (state “ $sf$ ”):** In this case,

– a fraction  $s$ ,  $s \in [0, 1)$ , of the depositors from sector  $B$  migrates to bank  $A$  so that its deposit pool is enlarged to  $D_{A1} + s \cdot D_{B1}$ . The remaining fraction,  $1 - s$ , has no investment opportunities and simply holds its deposits in the form of the consumption good till  $t = 2$ . The parameter  $s < 1$  implies that not all depositors from the failed sector are able to access the surviving bank. Hence, the aggregate level of deposits with banks is lower in state “ $sf$ ” than in state “ $ss$ .” It will be shown that higher  $s$  will imply a smaller *recessionary spillover* from the failed sector to the surviving sector.

– the costs to the surviving bank  $A$  of investing in the risky asset are reduced to  $\alpha \cdot c(x)$ , where  $\alpha \in [\alpha_{min}, 1]$ ,  $\alpha_{min} > 0$ . This captures the possibility that the surviving bank may have a *strategic benefit* from acquisition of loan facilities of the failed bank which make its lending operations more efficient, and as a result, may “expand”.<sup>12</sup> A higher value of  $\alpha$  implies a lower strategic benefit.

(3) **Bank  $A$  fails but bank  $B$  survives (state “ $fs$ ”):** This is symmetric to state “ $sf$ .”

(4) **Both banks fail (state “ $ff$ ”):** There is no investment in the economy in any assets and the depositors in both sectors simply hold their consumption good till  $t = 2$ .

Thus, the second period in our model can be treated as a repetition of the first period, with the two important differences outlined in state “ $sf$ ” (and “ $fs$ ”) above. These differences

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<sup>11</sup>The reason as to why relaxing this assumption does not affect the qualitative nature of our results is discussed in extension (3) of Appendix D of the unabridged version, Acharya (2000).

<sup>12</sup>Such a benefit may arise due to various reasons in practice and could have also been modeled as an improvement in the return on the risky asset (or “asset quality”) for the surviving bank. Alternately, in some states of the world, the surviving bank may be capacity-constrained and may “expand” upon receipt of additional deposits (as would be the case if there were economies of scale up to some capacity).

enable us to model the negative externality (through the recessionary spillover parameter  $s$ ) or the positive externality (through the strategic benefit parameter  $\alpha$ ) of one bank's failure on the welfare of the other bank.

These, in turn, will be shown to determine the incentives of the bankowners to undertake correlated or uncorrelated investments at  $t = 0$ . At that point, more general interpretations of these externalities will be provided.

**Payoffs to bankowners and depositors:** At time  $t = 0$  and at  $t = 1$ , bank  $i$  (if it has survived) makes the following investment choices:

- (i) the amount of safe investment,  $X_{Sit}$ ;
- (ii) the amount of risky investment,  $X_{Rit}$ ;
- (iii) the level of risk of the risky asset,  $\sigma_{it}$ ; and
- (iv) the industry in which risky investments are made,  $I_{it}$ .

The banks' choice of industries determines the correlation of their investments,  $\rho_t$ .

In each period, all depositors are treated symmetrically. When the return on the risky asset is low, the bank cannot pay the promised return  $r_{Dt}$  to the depositors and, as a result, it defaults or fails. Let the critical return on the risky asset below which bank  $i$  defaults at  $t + 1$  be denoted as  $R_{it}^c$ . In what follows, we drop the time subscript  $t$  and specialize it later.

$R_i^c$  is given by the condition  $r_S X_{Si} + R_i^c X_{Ri} = r_D (X_{Si} + X_{Ri})$ , so that

$$R_i^c = r_D + (r_D - r_S) \cdot \frac{X_{Si}}{X_{Ri}}. \tag{3.1}$$

When the realized return on the risky asset,  $R_i$ , exceeds  $R_i^c$ , the dividends to bank  $i$  are  $r_S X_{Si} + R_i X_{Ri} - r_D (X_{Si} + X_{Ri})$ . For  $R_i < R_i^c$ , the bank gets nothing. Let the cost technology be  $c(\cdot)$ . Then, the current period expected payoff of the bankowners, denoted as  $v_i(r_D, r_S, \sigma_i, X_{Si}, X_{Ri})$ , and that of all the depositors that lend to bank  $i$ , denoted as  $u_i(r_D, r_S, \sigma_i, X_{Si}, X_{Ri})$ , are respectively:

$$v_i(\cdot) = \int_{R_i^c}^{R_{max}} [r_S X_{Si} + R X_{Ri} - r_D (X_{Si} + X_{Ri})] h_i(R; \sigma_i) dR - c(X_{Ri}), \tag{3.2}$$

$$u_i(\cdot) = \int_{R_i^c}^{R_{max}} r_D (X_{Si} + X_{Ri}) h_i(R; \sigma_i) dR + \int_0^{R_i^c} (r_S X_{Si} + R X_{Ri}) h_i(R; \sigma_i) dR. \tag{3.3}$$

Note that  $(r_D, r_S, \sigma_i, X_{Si}, X_{Ri})$  are determined in equilibrium and in general vary across times  $t = 0, 1$  and across the different states at  $t = 1$ . The total amount of deposits borrowed by bank  $i$  is denoted as  $\hat{D}_{it}$  and it takes on different values depending upon  $t$  and the state (at  $t = 1$ ).

We assume that  $\hat{D}_{it}$  is high enough (or  $c(\cdot)$  is convex enough) to ensure that the choice of  $X_{Rit}$  is smaller than  $\hat{D}_{it}$ .

**First-period payoffs:** The critical return,  $R_{i0}^c$ , the expected payoff of bank  $i$ ,  $v_{i0}^{ss}(\cdot)$ , and the expected payoff of the depositors who lend to bank  $i$  (depositors of sector  $i$ ),  $u_{i0}^{ss}(\cdot)$ , are given by equations (3.1)–(3.3). Under the symmetric equilibrium,  $\hat{D}_i \equiv D_{i0}$ .

**Second-period payoffs in state “ $ss$ ”:** This state occurs when  $R_{i0} > R_{i0}^c, \forall i \in \{A, B\}$ . In this state,  $R_{i1}^c, v_{i1}^{ss}(\cdot)$ , and  $u_{i1}^{ss}(\cdot)$ , are given by (3.1)–(3.3) respectively, and again,  $\hat{D}_i \equiv D_{i1}$ .

**Second-period payoffs in state “ $sf$ ”:** This state occurs when  $R_{A0} > R_{A0}^c$  and  $R_{B0} < R_{B0}^c$ , i.e., bank  $A$  survives but bank  $B$  fails. For bank  $A$ , the investment choice is identical to that of the first period except that its deposit pool is  $D_{A1} + sD_{B1}$ , and its cost technology is lowered to  $\alpha \cdot c(\cdot)$ . Thus,  $R_{A1}^c, v_{A1}^{sf}(\cdot)$ , and  $u_{A1}^{sf}(\cdot)$ , are given by (3.1)–(3.3) respectively,  $c(X_{RA})$  being replaced by  $\alpha \cdot c(X_{RA})$ , and  $\hat{D}_A \equiv D_{A1} + sD_{B1}$ . Note that  $u_{A1}^{sf}(\cdot)$  is the expected payoff of depositors who lend to bank  $A$ , viz. depositors of sector  $A$  plus the fraction  $s$  of the depositors of sector  $B$  that migrate.  $u_{B1}^{sf}$ , the expected payoff of the remaining fraction  $(1 - s)$  of the depositors of sector  $B$  who do not migrate, is  $(1 - s)D_{B1}$ , since they simply store their consumption goods. Finally,  $v_{B1}^{sf} \equiv 0$ .

**Second-period payoffs in state “ $fs$ ”:** This state is symmetric to the state “ $sf$ .”

**Second-period payoffs in state “ $ff$ ”:** This occurs when  $R_{A0} < R_{A0}^c$  and  $R_{B0} < R_{B0}^c$ , i.e., both banks fail. In this state, there is no investment and  $v_{i1}^{ff} \equiv 0, u_{i1}^{ff} \equiv D_{i1}, \forall i \in \{A, B\}$ .

Note that the likelihood of survival of bank  $A$  (union of states “ $ss$ ” and “ $sf$ ”) depends upon the realization of  $R_{A0}$  only, whereas the likelihood of survival of both banks (state “ $ss$ ”) depends upon the joint realization of  $R_{A0}$  and  $R_{B0}$ .

## 4 Systemic risk-shifting in the intermediated economy

We demonstrate a *systemic risk-shifting* phenomenon where both banks undertake correlated investments by investing in similar industries at  $t = 0$ . In the presence of standard debt contract, there is risk-shifting at *collective* level in addition to the risk-shifting behavior at individual bank level. This collective behavior aggravates joint failure risk in the economy.

### 4.1 Equilibrium in the intermediated economy

We solve the equilibrium by backwards induction, i.e., by first solving the bank’s investment problem in different states at  $t = 1$ , and then solving the bank’s investment problem at  $t = 0$ .

**Equilibrium in the state “ $sf$ ” at  $t = 1$ :** The only relevant investment problem in this state arises for bank  $A$ . The investment problem of the bank is

$$\max_{\sigma_A, X_{SA}, X_{RA}, I_A} \quad v_{A1}^{sf}(r_D, r_S, \sigma_A, X_{SA}, X_{RA}) \quad \text{where} \tag{4.1}$$

$$v_{A1}^{sf}(\cdot) = \int_{R_A^c}^{R_{max}} [(r_S - r_D)X_{SA} + (R - r_D)X_{RA}] h_A(R; \sigma_A) dR - \alpha \cdot c(X_{RA}), \quad \text{and} \tag{4.2}$$

$$R_A^c = r_D + (r_D - r_S) \cdot \frac{X_{SA}}{X_{RA}}. \tag{4.3}$$

In equilibrium, market-clearing for the safe asset requires  $r_S = f'(X_{SA})$ , and the budget constraint requires  $X_{SA} + X_{RA} = \hat{D}_A = D_A + sD_B$ . We have dropped the time subscript ( $t = 1$ ) from the variables to reduce notational burden.

Since the choice of  $I_A$  does not affect the choice of  $\sigma_A$ ,  $X_{SA}$ , and  $X_{RA}$ ,  $v_{A1}^{sf}$  is independent of  $I_A$ , and thus, the choice of industry is irrelevant.

We show first that in equilibrium,  $r_D$  must equal  $r_S$ . Consider for given  $\sigma_A$ , the choice of  $X_{SA}$ . For  $r_D > r_S$ ,  $R_A^c$  is increasing in  $X_{SA}$  and  $v_{A1}^{sf}$  is decreasing in  $X_{SA}$ . It follows that for  $r_D > r_S$ ,  $X_{SA} = 0$  so that the bank has no demand for the safe asset. But in equilibrium,  $r_S = f'(X_{SA}) = f'(0) = \infty$ , a contradiction. On the other hand, for  $r_D < r_S$ ,  $R_A^c$  is decreasing in  $X_{SA}$  and  $v_{A1}^{sf}$  is increasing in  $X_{SA}$  so that  $X_{SA} = \infty$ , i.e., the bank has an infinite demand for the safe asset, and either the budget constraint or the short-sales constraint ( $X_{RA} \geq 0$ ) is violated. Thus,  $r_D = r_S$  in equilibrium and we will denote it simply as  $r$ .

Incorporating this, we get  $R_A^c = r$ , and using the budget constraint,  $X_{SA} = \hat{D}_A - X_{RA}$ ,

$$v_{A1}^{sf}(r, \sigma_A, X_{RA}) = \int_r^{R_{max}} (R - r)X_{RA} h_A(R; \sigma_A) dR - \alpha \cdot c(X_{RA}). \tag{4.4}$$

Given  $r$  and  $\sigma_A$ , the optimal risky investment,  $X_{RA}(r, \sigma_A)$ , is given by the first order condition:

$$\int_r^{R_{max}} (R - r) h_A(R; \sigma_A) dR = \alpha \cdot c'(X_{RA}), \tag{4.5}$$

where LHS represents the expected marginal gain to the bank and RHS represents the marginal cost of an additional unit of risky investment. This can be rewritten as

$$\bar{R} - \alpha \cdot c'(X_{RA}) = r - \int_0^r (r - R) h_A(R; \sigma_A) dR. \tag{4.6}$$

Assuming  $\bar{R} > c'(\hat{D}_A)$  guarantees an interior solution,  $X_{RA}(r, \sigma_A) \in (0, \hat{D}_A)$ . Next, let  $\hat{\sigma}_A(r) = \arg \max_{\sigma_A} v_{A1}^{sf}(r, \sigma_A, X_{RA}(r, \sigma_A))$ , and  $\hat{X}_{RA}(r) = X_{RA}(r, \hat{\sigma}_A(r))$ . Then, equilibrium at  $t = 1$  in the state “ $sf$ ,” denoted as  $(r_1^{sf}, \sigma_A^{sf}, X_{RA}^{sf})$ , is determined by the fixed-point (market-clearing condition for the safe asset):

$$r_1^{sf} = f'[D_A + sD_B - \hat{X}_{RA}(r_1^{sf})]. \tag{4.7}$$

It is easy to show (Lemma A.1 in the appendix) that the equilibrium,  $(r_1^{sf}, \sigma_A^{sf}, X_{RA}^{sf})$ , exists and  $\sigma_A^{sf} \equiv \sigma_{max}$ . Since the bank does not bear the cost of a low return on its investments, its payoff is truncated. This convexity of its payoff leads to a preference for risk. This is the classic problem of “risk-shifting” or “asset-substitution” by borrowers, studied in corporate finance by Jensen and Meckling (1976) and in credit-rationing by Stiglitz and Weiss (1981).

Note that with a positive level of risky investment, there is default whenever the realized return on the risky asset is smaller than  $r$ , and hence, the expected rate of return on deposits is smaller than  $r$ , the promised rate of return.<sup>13</sup>

Finally, denote the maximized objective function as  $V_{A1}^{sf} \equiv v_{A1}^{sf}(r_1^{sf}, \sigma_A^{sf}, X_{RA}^{sf})$ . This represents the continuation value of the equityholders at  $t = 1$  in state “ $sf$ ” and will also be called the bank’s *charter-value*. The state “ $fs$ ” is symmetric to the state “ $sf$ ” and its equilibrium  $(r_1^{fs}, \sigma_B^{fs}, X_{RB}^{fs})$  satisfies the counterpart of Lemma A.1.

**Equilibrium in the state “ $ss$ ” at  $t = 1$ :** In this case, the investment problem of both the banks needs to be solved. A little thought reveals that as in the case of state “ $sf$ ,” the

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<sup>13</sup> If the depositors could invest directly in the safe asset and hence charged a rate  $r_D > r_S$  that takes into account the likelihood of default, our results remain unaffected. This is because the safe investments currently made by the banks would be made by the depositors instead and the banks would intermediate only the risky investments. Importantly, the deposit rate  $r_D$  would move in tandem with  $r_S$ . To see this, note that in equilibrium,  $r_D$  (for given  $r_S, X_R$ , and  $\sigma$ ) would solve the fixed-point equation:

$$r_S = r_D \cdot \int_{r_D}^{R_{max}} h(R; \sigma) dR + \int_0^{r_D} R h(R; \sigma) dR.$$

The first term is the return on deposits in case of no default, and the second term is the return on deposits upon default. This generalization is considered in Appendix D of the unabridged version, Acharya (2001). What drives our model is risk-shifting which arises (ex-post) in any model with a standard debt contract, so long as the rate charged on the contract (ex-ante) is not contingent.

choice of industry in which the banks make their investments is irrelevant. This can be seen formally by examining the expression for  $v_{i1}^{ss}$  in the model section. It follows that each bank's problem is similar to the investment problem of bank  $A$  in state “ $sf$ ” studied above. The equilibrium is denoted as  $(r_1^{ss}, \sigma_A^{ss}, X_{RA}^{ss}, \sigma_B^{ss}, X_{RB}^{ss})$ . The only differences between determining the equilibrium in state “ $ss$ ” and in state “ $sf$ ” are the following:

- (i)  $v_{i1}^{ss}$  replaces  $v_{i1}^{sf}$  in equation (4.1);
- (ii) there is no migration of deposits, i.e.,  $\hat{D}_i = D_i$ , so  $s = 0$  in the budget-constraint;
- (iii) there is no reduction in the costs of investing in the risky asset, so  $\alpha = 1$  in the first order condition (4.6) to determine  $X_{Ri}(r, \sigma_i)$ ; and finally,
- (iv) the equilibrium safe asset return is the fixed-point:  $r_1^{ss} = f'[\sum_i D_i - \sum_i \hat{X}_{Ri}(r_1^{ss})]$ .

With these modifications, we can show that the equilibrium,  $(r_1^{ss}, \sigma_A^{ss}, X_{RA}^{ss}, \sigma_B^{ss}, X_{RB}^{ss})$ , exists and  $\sigma_i^{ss} \equiv \sigma_{max}$ ,  $\forall i \in \{A, B\}$  (as in Lemma A.1).

We denote the maximized objective function of bank  $i$ , its charter-value in state “ $ss$ ,” as  $V_{i1}^{ss} = v_{i1}^{ss}(r_1^{ss}, \sigma_i^{ss}, X_{Ri}^{ss})$ .

**The nature of externality at  $t = 1$ :** Whether bank  $i$  benefits or is hurt by the failure of bank  $j$  at  $t = 1$  depends on the difference in charter-values,  $V_{i1}^{sf} - V_{i1}^{ss}$ . When this difference is less than zero, there is a *negative externality* of bank  $j$ 's failure on bank  $i$ , whereas when this difference is greater than zero, there is a *positive externality*.<sup>14</sup> This depends crucially on the two parameters that affect the charter-value in state “ $sf$ ”:

- (i)  $s \in [0, 1)$ , the *recessionary spillover* parameter; and
- (ii)  $\alpha \in [\alpha_{min}, 1]$ ,  $\alpha_{min} > 0$ , the *strategic benefit* parameter.

On the one hand, when bank  $j$  fails, since only a part of the depositors migrate ( $s < 1$ ), there is a reduction in the overall investment in the economy. This raises the equilibrium return on the safe asset which is also the promised return on the deposits, increasing the cost of borrowing for the surviving bank  $i$ . This constitutes a *recessionary spillover* to bank  $i$  when bank  $j$  fails.<sup>15</sup>

On the other hand, when bank  $j$  fails, bank  $i$  “expands”. This is because, bank  $i$  is able

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<sup>14</sup>Note that we have used the state “ $sf$ ” to mean ‘when  $i$  survives and  $j$  fails’. Strictly speaking, the relevant difference is  $V_{A1}^{sf} - V_{A1}^{ss}$  for bank  $A$ , and  $V_{B1}^{sf} - V_{B1}^{ss}$ . However, switching from “ $sf$ ” for bank  $A$  to “ $fs$ ” for bank  $B$ , introduces unnecessary notational burden. Instead, we will simply use  $V_{i1}^{sf}$ ,  $\forall i \in \{A, B\}$ .

<sup>15</sup>This effect is much akin to the “liquidity” effect of a monetary shock, empirically documented in business-cycle literature. This effect will arise also due to the fact that failure of a bank leads to reduction in aggregate depositor wealth, even if there were a perfect migration of depositors. In our model, real rates of interest rise in “recession,” i.e., upon a reduction in total depositor wealth. In the business-cycle evidence, nominal rates of interest rise upon a reduction in growth of  $M_1$  (see, Cooley and Hansen, “Money and the Business Cycle,” in Cooley, 1995).



to acquire some of the “human capital” of bank  $j$ , such as lending desks, loan administration facilities, etc. which reduce its costs of loan initiation from  $c(\cdot)$  to  $\alpha \cdot c(\cdot)$ ,  $\alpha < 1$ . This, in turn, implies that bank  $i$  invests more in the risky technology, which makes it more profitable. This constitutes a *strategic benefit* to bank  $i$  when bank  $j$  fails.

The recessionary spillover is decreasing in  $s$ , whereas the strategic benefit is decreasing in  $\alpha$ . These parameters,  $s$  and  $\alpha$ , are to be treated as exogenous parameters of the economy or its current state. When the recessionary spillover dominates, i.e., when  $s$  is ‘small’ and  $\alpha$  is ‘large’, the overall externality is negative. On the other hand, when the strategic benefit dominates, i.e., when  $s$  is ‘large’ and  $\alpha$  is ‘small’, the externality is positive. These intuitions are formalized below and lead to a key result in our theory of systemic risk.

**Lemma 1** *The charter-value at  $t = 1$  in the state “sf”,  $V_{i1}^{sf}$ , is (i) increasing in  $s$  for a given  $\alpha$ ; and (ii) decreasing in  $\alpha$  for a given  $s$ .*

**Proposition 1 (Nature of Externality)** *The sign of the externality,  $V_{i1}^{sf} - V_{i1}^{ss}$ , is characterized by the following:*

(i) *For a given  $\alpha$ ,  $\exists$  a critical level,  $s^c(\alpha)$ , such that  $V_{i1}^{sf} - V_{i1}^{ss} < 0$ ,  $\forall s < s^c(\alpha)$  (negative externality), and  $V_{i1}^{sf} - V_{i1}^{ss} > 0$ ,  $\forall s > s^c(\alpha)$  (positive externality). Further,  $s^c(\alpha)$  is increasing in  $\alpha$ .*

(ii) *For a given  $s$ ,  $\exists$  a critical level,  $\alpha^c(s)$ , such that  $V_{i1}^{sf} - V_{i1}^{ss} < 0$ ,  $\forall \alpha > \alpha^c(s)$  (negative externality), and  $V_{i1}^{sf} - V_{i1}^{ss} > 0$ ,  $\forall \alpha < \alpha^c(s)$  (positive externality). Further,  $\alpha^c(s)$  is increasing in  $s$ .<sup>16</sup>*

This proposition implies that the two-dimensional space  $[0, 1] \times [\alpha_{min}, 1]$  is divided by a curve  $C$  into two regions, such that the region to the north-west of  $C$  supports a negative externality of a bank’s failure on the surviving bank, and the region to the south-east of  $C$  supports a positive externality (see Figure 3). The equilibrium at  $t = 0$  is characterized next.

**Equilibrium at  $t = 0$ :** The investment choice of each bank at  $t = 0$  anticipates the states at  $t = 1$  and the charter-values in those states. In particular, unlike the investment choice at  $t = 1$ , the choice of industry by each bank is relevant. This choice determines the likelihood of the states at  $t = 1$  (“ss”, “sf”, “fs”, “ff”), and hence, the magnitude of the externality of a bank’s failure on the other bank. Further, this choice affects only the correlation of the asset returns of the two banks,  $\rho$ . Thus, we can translate the choice of industries into a preference of the banks for low correlation ( $\rho_l$ ) or high correlation ( $\rho_h$ ).<sup>17</sup>

<sup>16</sup>The critical levels may coincide with the boundaries of the parameter space (see Appendix B).

<sup>17</sup>Indeed, we have chosen two industries only for simplicity. The translation of choice of industry into the preference for correlation is robust in a richer model with greater number of industries.

Thus, in equilibrium, when the banks prefer a low correlation,  $\rho_l$ , one of them invests in “manufacturing” and the other in “farming”, whereas when they prefer a high correlation,  $\rho_h$ , either both of them invest in “manufacturing” or both of them invest in “farming”.

Since the externality at  $t = 1$  induces a dependence of one bank’s investment choice on the investment choice of the other, we need to enrich the notion of equilibrium by a *Nash equilibrium* of the two banks’ investment choices. In what follows, all time subscripts for  $t = 0$  are omitted and the time subscripts for  $t = 1$  are explicitly employed.

The *strategy* of bank  $i$  is denoted as  $\Omega_i = (\sigma_i, X_{Ri}, \rho_i) \in [\sigma_{min}, \sigma_{max}] \times [0, D_i] \times \{\rho_l, \rho_h\}$ . The *best-response* of bank  $i$  to bank  $j$ ’s strategy is denoted as  $\Omega_i(\Omega_j)$ . The equilibrium of the economy at  $t = 0$ ,  $(r_D^*, r_S^*, \Omega_A^*, \Omega_B^*)$ , satisfies the following:

- (i)  $\Omega_A^*$  and  $\Omega_B^*$  constitute a Nash equilibrium:  $\Omega_A(\Omega_B^*) = \Omega_A^*$ , and  $\Omega_B(\Omega_A^*) = \Omega_B^*$ ;
- (ii) the banks prefer the same correlation:  $\rho_A^* = \rho_B^*$  (to be denoted as  $\rho^*$ ); and
- (iii) the market for the safe asset clears:  $r_S^* = f'(\sum_i D_i - \sum_i X_{Ri}^*)$ .

For simplicity, since it makes no difference to the analysis, we incorporate below the equilibrium budget constraint,  $X_{Si} + X_{Ri} = D_i$ , directly into bank  $i$ ’s maximization problem. Then, for given  $r_D$ ,  $r_S$ , and  $\Omega_j$ , bank  $i$ ’s best-response,  $\Omega_i(\Omega_j)$ , is determined by the solution to the following maximization problem:

$$\max_{\sigma_i, X_{Ri}, \rho_i} \quad v_{i0}(r_D, r_S, \sigma_i, X_{Ri}) + \tag{4.8}$$

$$V_{i1}^{ss} \cdot Pr[R_i > R_i^c, R_j > R_j^c] + V_{i1}^{sf} \cdot Pr[R_i > R_i^c, R_j < R_j^c] \quad \text{where}$$

$$v_{i0}(r_D, r_S, \sigma_i, X_{Ri}) = \int_{R_i^c}^{R_{max}} [(r_S - r_D)D_i + (R - r_S)X_{Ri}] h_i(R; \sigma_i) dR - c(X_{Ri}), \tag{4.9}$$

$$R_i^c = r_D + (r_D - r_S) \cdot \left( \frac{D_i}{X_{Ri}} - 1 \right). \tag{4.10}$$

As before  $r_D = r_S$  in equilibrium, denoted as  $r$ . This implies that  $R_i^c = r, \forall i \in \{A, B\}$ . Incorporating these equilibrium requirements and the identity  $Pr[R_i < r, R_j > r] = Pr[R_i < r] - Pr[R_i < r, R_j < r]$ , the maximization problem above can be rewritten as:

$$\max_{\sigma_i, X_{Ri}, \rho_i} \quad \int_r^{R_{max}} (R - r)X_{Ri} h_i(R; \sigma_i) dR - c(X_{Ri}) + \tag{4.11}$$

$$V_{i1}^{ss} \cdot Pr[R_i > r] + (V_{i1}^{sf} - V_{i1}^{ss}) \cdot Pr[R_i > r, R_j < r].$$

Consider the best-response of bank  $i$ . For given  $r$  and  $\sigma_i$ , the first-order condition w.r.t.  $X_{Ri}$ , the amount of risky investment, can be expressed as:

$$\bar{R} - c'(X_{Ri}) = r - \int_0^r (r - R) h_i(R; \sigma_i) dR. \tag{4.12}$$

Assuming  $\bar{R} > c'(D_i)$  guarantees an interior solution,  $X_{Ri}(r, \sigma_i) \in (0, D_i)$ . Given  $X_{Ri}(r, \sigma_i)$  and a correlation  $\rho$ , inspection of the maximand in (4.11) reveals that the best-response for the level of risk can be denoted as  $\hat{\sigma}_i(r, \sigma_j)$ . Given these best-responses, we examine the choice of  $\rho$ , the preference of the banks for correlation of returns on their risky investments, that is central to our theory of systemic risk. The convexity of the bankowners' payoff interacts with the nature of the externality (positive or negative) and endogenously determines whether the banks choose the same industry at  $t = 0$ , i.e., "standardize" or "syndicate", or choose different industries, i.e., "specialize" or "differentiate".

The objective function in equation (4.11) reveals that correlation affects only the externality term,  $(V_{i1}^{sf} - V_{i1}^{ss}) \cdot Pr[R_i > r, R_i < r]$ . In particular, it affects only  $Pr[R_i > r, R_j < r]$ , the probability of state "sf," which is decreasing in  $\rho$  (Assumption 5, Appendix A). Thus, when the externality of bank  $j$ 's failure is negative ( $V_{i1}^{sf} < V_{i1}^{ss}$ ), bank  $i$  has a preference for as high a correlation as possible. To see this in a diagram, notice that in Figure 4, if  $(V_{i1}^{sf} - V_{i1}^{ss}) < 0$  then bank  $i$  prefers state "ss" over state "sf" (and does not care for states "fs" and "ff" due to limited liability). On the other hand, when the externality is positive ( $V_{i1}^{sf} > V_{i1}^{ss}$ ), bank  $i$  has a preference for as low a correlation as possible. The following lemma is a consequence of Proposition 1 and formalizes this discussion.

**Lemma 2** *The choice of correlation by bank  $i$ ,  $\rho_i$ , is (i)  $\rho_h$ , for  $s < s^c(\alpha)$  or  $\alpha > \alpha^c(s)$  (negative externality); and (ii)  $\rho_l$ , for  $s > s^c(\alpha)$  or  $\alpha < \alpha^c(s)$  (positive externality).*

Thus, in the case of negative externality, i.e., when the recessionary spillover dominates, for any levels of risk,  $\sigma_i$  and  $\sigma_j$ , bank  $i$  has a preference to survive when bank  $j$  survives (and thus, fail when bank  $j$  fails). If the banks are symmetric in all respects (which we assume henceforth for simplicity), then bank  $j$ 's preference for correlation,  $\rho_j$ , satisfies the same property. Thus, each bank prefers more correlation with the other bank to less. In the case of positive externality, the strategic benefit dominates and each bank prefers a low correlation with the other bank.<sup>18</sup>

**Proposition 2 (Intermediated Equilibrium)** *The equilibrium of the intermediated economy at  $t = 0$ ,  $[r^*, \Omega_A^*, \Omega_B^*]$ , exists and is characterized by the following properties:*

(i) *at high charter-values  $(V_{i1}^{ss}, V_{i1}^{sf})$ , an interior solution  $\sigma_i^* \in [\sigma_{min}, \sigma_{max})$  exists,  $\forall i$ . At low charter-values,  $\sigma_i^* \equiv \sigma_{max} \forall i$ ;*

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<sup>18</sup>In practice, strategic benefit may be small if banks are 'large' so that anti-trust restrictions prevent the acquisition of other bank's facilities ( $\alpha \approx 1$ ) and/or opening of new branches and ATMs in the failed bank's "sector" of operation ( $s \approx 0$ ). In addition, 'uniqueness' of bank assets (for example, due to information asymmetry and resulting bank-client relationships), would increase the bite of the negative externality.

(ii) for  $s < s^c(\alpha)$  or  $\alpha > \alpha^c(s)$  (negative externality), both banks choose to be as highly correlated as possible, i.e.,  $\rho_i^* = \rho_j^* = \rho_h$ , whereas for  $s > s^c(\alpha)$  or  $\alpha < \alpha^c(s)$  (positive externality), both banks choose to be as little correlated as possible, i.e.,  $\rho_i^* = \rho_j^* = \rho_l$ .

A crucial factor that drives the preference for high correlation in the case of negative externality is the limited liability of banks. Traditional corporate finance has focused on risk-shifting at the level of a single firm in the presence of standard debt contract and limited liability. Our result shows that in the case of multiple firms (in our model, banks), in addition to limited liability there is a negative externality of default of one firm on the profitability of others, then the firms *collectively* increase the aggregate risk by undertaking highly correlated investments. We thus call this phenomenon *systemic risk-shifting*.

In systemic risk-shifting, the banks can afford to increase the value of their equity by being highly correlated precisely because the cost of doing so (which is an increase in the likelihood of joint failure) is not borne by the banks. Given their limited liability, bankowners have no preference between failing individually or failing together, however they have a preference for surviving together in the case of negative externality.

An outcome of systemic risk-shifting is that the aggregate banking portfolio looks highly concentrated or poorly diversified. To demonstrate that this is indeed a risk-shifting phenomenon and is suboptimal for social welfare, we need to show that the first-best investment choices in the centralized economy imply a lower level of correlation in equilibrium. We do this next.

## 4.2 The first-best: equilibrium in the aligned economy

The “central bank” in our model is effectively the central planner of the economy. Its objective is to maximize the sum of the welfare of bankowners and depositors, net of any costs of financial distress. This objective is taken to capture “the safety and soundness of the financial sector,” as regulation often claims.<sup>19</sup> Thus, unlike what would be appropriate in single-bank models, the central bank is potentially concerned not only about *individual* bank failures but also about *joint* failures.

We assume that the deadweight costs of bank failures are proportional to the extent of risky investment,  $X_R$ , the proportionality factor being  $\delta(r, R) > 0$ , where  $r$  is the promised return on deposits and  $R$  is the realization of risky return in failure ( $R < r$ ). Such costs arise

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<sup>19</sup>For example, see Cecchetti (1999). The question as to why do we need a central bank is fascinating and remains open, see Goodhart (1987) for a discussion. We focus on the central bank’s role in the maintenance of financial stability, as suggested by these studies, and provide a rationale for bank regulation based on systemic and individual risk-shifting incentives of bankowners.

from legal and administrative fees in work-outs, delayed recovery on defaulted assets, disruption of the payments system, allocation inefficiencies following crises, financing constraints for viable firms, etc.<sup>20</sup>

Finally, note that in general, the central bank may put greater weight on the welfare of the depositors and on the deadweight costs than on the welfare of bank’s equityholders. This affects both the extent of deviation between the first-best and the intermediated case, and the exact level of optimal regulation, but not their qualitative nature. Hence, we assume that it weighs all three equally. For simplicity, we also assume that the central bank puts equal weight on the two sectors in its objective function.

**Equilibrium at  $t = 1$ :** In the first-best, the banks aligned with the central bank coordinate and jointly pick their investment choices,  $[\boldsymbol{\sigma} \equiv (\sigma_A, \sigma_B), \mathbf{X}_R \equiv (X_{RA}, X_{RB}), \rho]$  to solve:

$$\max_{\boldsymbol{\sigma}, \mathbf{X}_R, \rho} \sum_i \left[ v_{i1}(r, \sigma_i, X_{Ri}) + u_{i1}(r, \sigma_i, X_{Ri}) - \int_0^r \delta(r, R) X_{Ri} h_i(R; \sigma_i) dR \right] \quad (4.13)$$

The budget constraint  $X_{Si} + X_{Ri} = \hat{D}_i, \forall i$ , is already incorporated in the specification above. Also, we have made use of the fact  $r_D = r_S$  (as in the intermediated economy), denoted simply as  $r$ . Further, as before, the choice of correlation  $\rho$  is irrelevant at  $t = 1$ .

Let us consider the state “ss.”  $v_{i1}^{ss}$  and  $u_{i1}^{ss}$  are as defined in Section 3. Since  $v_{i1} + u_{i1} = r\hat{D}_i + (\bar{R} - r)X_{Ri} - c(X_{Ri})$ , the first order condition w.r.t.  $X_{Ri}$  can be expressed as:

$$\bar{R} - c'(X_{Ri}) = r + \int_0^r \delta(r, R) h_i(R; \sigma_i) dR. \quad (4.14)$$

Once again, our assumptions guarantee a unique solution,  $X_{Ri}(r, \sigma_i)$ . Comparison with response of the bank in the intermediated economy (equation 4.6) reveals that the aligned bank takes into account both the welfare of the depositors as well as the costs of bank failure.

With mean-preserving spreads, additional risk only leads to a greater likelihood of failure, implying lower welfare of depositors and greater expected costs of failure. Since these are internalized by the aligned bank, it always prefers the lowest level of risk,  $\hat{\sigma}_i(r) = \sigma_{min}, \forall r$ . Let  $\hat{X}_{Ri}(r) = X_{Ri}(r, \sigma_{min})$ . Then, equilibrium is obtained by the market-clearing condition for the safe asset which yields the fixed-point:  $r_1^{ss} = f'[\sum_i D_i - \sum_i \hat{X}_{Ri}(r_1^{ss})]$ .

As in Lemma A.1, the equilibrium  $[r_1^{ss}, \boldsymbol{\sigma}^{ss} = (\sigma_{min}, \sigma_{min}), \mathbf{X}_R^{ss} = (\hat{X}_{RA}(r_1^{ss}), \hat{X}_{RB}(r_1^{ss}))]$  exists. We denote the expected payoffs in equilibrium of bank  $i$  and the depositors that lend to bank  $i$  as  $V_{i1}^{ss} = v_{i1}^{ss}(r_1^{ss}, \sigma_{min}, X_{Ri}^{ss})$  and  $U_{i1}^{ss} = u_{i1}^{ss}(r_1^{ss}, \sigma_{min}, X_{Ri}^{ss})$ , respectively.

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<sup>20</sup>Sprague (1986), James (1991), and Saunders (2000) document that such direct bankruptcy costs range from 4–10% of book liabilities (or assets) and are much greater than those in corporate bankruptcies. There may also be real costs associated with financial distress.

The equilibrium in state “ $sf$ ” (and “ $fs$ ”) is derived similarly: in the budget constraint,  $X_{SA} + X_{RA} = \hat{D}_A = D_A + sD_B$ ; and the cost function for bank  $A$  is reduced to  $\alpha \cdot c(\cdot)$ . This gives the equilibrium  $[r_1^{sf}, \sigma_A^{sf} = \sigma_{min}, \hat{X}_{RA}(r_1^{sf})]$ , where  $r_1^{sf} = f'[D_A + sD_B - \hat{X}_{RA}(r_1^{sf})]$ . We denote the expected payoffs in equilibrium of bank  $A$  and the depositors that lend to bank  $A$  as  $V_{A1}^{sf} = v_{A1}^{sf}(r_1^{sf}, \sigma_{min}, X_{RA}^{sf})$  and  $U_{A1}^{sf} = u_{A1}^{sf}(r_1^{sf}, \sigma_{min}, X_{RA}^{sf})$ , respectively. Note that  $V_{B1}^{sf} = 0$  and  $U_{B1}^{sf} = (1 - s)D_B$ . Let

$$W_1^j = \sum_i \left[ V_{i1}^j + U_{i1}^j - \int_0^r \delta(r, R) X_{Ri}^j h_i(R; \sigma_{min}) dR \right], \tag{4.15}$$

where  $i \in \{A, B\}$ ,  $j \in \{ss, sf, fs, ff\}$ . Then, the following lemma captures the intuitive result that the welfare losses from bank failures are “systemic” in nature, i.e., the failure of both sectors leads to greater losses than the failures of only one of the sectors.<sup>21</sup>

**Lemma 3 (Systemic Costs of Failure)**  $\forall s \in [0, 1]$  and  $\forall \alpha \in (\alpha_{min}, 1]$ ,  $(W_1^{ss} - W_1^{sf}) + (W_1^{ss} - W_1^{fs}) < (W_1^{ss} - W_1^{ff})$ , i.e.,  $W_1^{ss} + W_1^{ff} < W_1^{sf} + W_1^{fs}$ .

Taking the state “ $ss$ ” as the benchmark, the total welfare loss in single bank failure states, “ $sf$ ” and “ $fs$ ,” is given by  $(W_1^{ss} - W_1^{sf}) + (W_1^{ss} - W_1^{fs})$ . This is smaller than the loss incurred in the joint failure state “ $ff$ ” which is  $(W_1^{ss} - W_1^{ff})$ . The intuition is as follows. If only one of the banks fails, the deposits from the distressed sector migrate to the surviving sector. The ability to migrate increases the welfare of the depositors. Further, if  $\alpha < 1$ , the charter-value of the surviving bank may increase as well. On the other hand, in the case of a joint failure, there is no investment and all depositors in the economy simply store their goods.

**Equilibrium at  $t = 0$ :** The aligned banks coordinate their investment choices to solve:

$$\begin{aligned} \max_{\sigma, \mathbf{X}_R, \rho} \quad & \sum_i \left[ v_{i0}(r, \sigma_i, X_{Ri}) + u_{i0}(r, \sigma_i, X_{Ri}) - \int_0^r \delta(r, R) X_{Ri} h_i(R; \sigma_i) dR \right] \\ & + W_1^{ss} \cdot Pr[R_A > r, R_B > r] + W_1^{sf} \cdot Pr[R_A > r, R_B < r] \\ & + W_1^{fs} \cdot Pr[R_A < r, R_B > r] + W_1^{ff} \cdot Pr[R_A < r, R_B < r]. \end{aligned} \tag{4.16}$$

In equilibrium,  $r = f'(\sum_i D_i - \sum_i X_{Ri})$ .

As before, with mean-preserving spreads, it is suboptimal to undertake any risk  $\hat{\sigma}_i(r) = \sigma_{min}$ ,  $\forall r$ , and  $\hat{X}_{Ri}(r) = X_{Ri}(r, \sigma_{min})$  (given by equation 4.14). The equilibrium in the aligned economy at  $t = 0$ ,  $[r^o, \sigma^o \equiv (\sigma_{min}, \sigma_{min}), \mathbf{X}_R^o = (\hat{X}_{RA}(r^o), \hat{X}_{RB}(r^o)), \rho^o]$ , is given by the fixed-point:  $r^o = f'[\sum_i D_i - \sum_i \hat{X}_{Ri}(r^o)]$ .

<sup>21</sup>The lemma is always true for  $s < s^c(\alpha)$  or  $\alpha > \alpha^c(s)$ , the case of negative externality, as required in our results. It holds more generally  $\forall s, \alpha$  whenever the level of deposits,  $D_i$ , is sufficiently high (Appendix B).

The only investment choice to be determined is that of the correlation,  $\rho^o$ . Intuitively, increasing the correlation across banks is harmful if the costs of doing so are systemic in nature. Lemma 3 implies that the aligned banks choose to be as little correlated as possible, i.e.,  $\rho^o = \rho_l$ , since increasing the correlation increases the likelihood of joint failure (Assumption 4) and in turn, increases the social costs. In other words, a “diversified” aggregate banking portfolio where the constituent bank investments are as little correlated as possible minimizes the likelihood of joint failure and is preferred to a “concentrated” one.

**Proposition 3 (First-best: Aligned Equilibrium)** *The equilibrium of the aligned economy,  $[r^o, \sigma^o, X_R^o, \rho^o]$ , exists and is characterized by the following properties:*

- (i) *The aligned banks pick the lowest level of risk, i.e.,  $\sigma_i^o = \sigma_{min}, \forall i$ .*
- (ii) *The aligned banks pick the lowest level of correlation, i.e.,  $\rho^o = \rho_l$ .*

### 4.3 Individual and systemic risk-shifting

The analysis so far leads to the important result that in the intermediated economy with multiple-banks, there is risk-shifting at both the individual level (through a higher  $\sigma$  and  $X_R$ ) and *also* at the collective level (through a higher  $\rho$ ).

From Propositions 2–3, we obtain

**Corollary 1 (Individual Risk-shifting)** *Comparing the intermediated and the first-best equilibria,  $\forall t, r^* > r^o, \sigma_i^* > \sigma_i^o$ , and  $X_{Ri}^* > X_{Ri}^o, \forall i$ .*

The intermediated economy is characterized by a greater choice of riskiness of the risky asset, a greater investment in the risky asset, and a higher rate of interest on deposits, compared to their first-best counterparts. It follows from Assumption 2 that the likelihood of default and expected losses upon failure are greater in the intermediated economy due to two effects: a direct effect from a greater risk choice, and an indirect, endogenous effect of greater investment in the risky asset giving rise to a higher borrowing rate in equilibrium. Further, the endogenous effect is especially perverse: an increase in the risky investment by each bank increases borrowing rate for all banks. Next,

**Corollary 2 (Systemic Risk-shifting)** *There is systemic risk-shifting through a preference for higher correlation in the intermediated equilibrium, i.e.,  $\rho^* = \rho_h > \rho^o = \rho_l$  for  $s < s^c(\alpha)$  or  $\alpha > \alpha^c(s)$  (negative externality).*

From Assumptions 3–4, the likelihood of joint failure and expected costs from joint failures are higher in the intermediated economy due to individual risk-shifting, *as well as* due

to systemic risk-shifting. In particular, if we consider the intermediated and the aligned economies at the same levels of risk,  $\sigma_i$  and  $\sigma_j$ , and at the same rate of interest,  $r$ , these measures would be higher in the intermediated economy purely due to a higher correlation of assets (whenever there is a *negative externality* of one bank's failure on the other bank). The effect is only exacerbated by the fact that  $\sigma_i^* > \sigma_i^o$ ,  $\forall i \in \{A, B\}$ , and  $r^* > r^o$ , as well.

**Discussion on the negative externality:** A word about our choice of model for the negative externality or the recessionary spillover is in order here. In reality, systemic risk in the financial sector arises due to a variety of factors: (i) In extension (2) in Appendix D in Acharya (2001), the reduction in aggregate depositor wealth upon bank failures accentuates the spillover even if it is assumed that depositor migration to surviving banks is perfect. (ii) There are *network externalities* from bank services such as the payments and settlements system. These may be disrupted upon bank failures.<sup>22</sup> (iii) The failure of a few big banks could hamper the orderly functioning of the markets for inter-bank loans, over-the-counter derivative contracts, etc. that connect banks and financial institutions.<sup>23</sup> (iv) Asymmetric information about the positions that different banks hold may give rise to information externalities (as in Rajan, 1994). For example, the cost of capital for a bank could rise if it is deemed less healthy upon a failure of its peers (as in Acharya and Yorulmazer, 2008a).

In the situations described above, the financial sector is “healthier” when most banks survive since the failure of one bank translates into a reduced profitability of the other banks. This effect is endogenous in our model. The common safe asset across the two sectors, the imperfect migration of deposits upon one bank's failure, and their effect on the cost of borrowing deposits for the surviving bank, generate the negative externality in general equilibrium in a parsimonious way. While the imperfect migration can be justified on the basis of “distance” across sectors either based on geographical location or based on a richer segmentation between banks and depositors, it could also be considered as a “metaphor” for a variety of *ex-post* spillovers that would also generate endogenous systemic risk *ex-ante*.

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<sup>22</sup>Cecchetti (1999) observes, “(financial systems are characterized by) what are now termed *network externalities*: the overall value that arises from an individual's participation in a particular network is greater than the individual's private value because an additional party in the network raises everyone else's utility. Many of the products provided by financial intermediaries share these characteristics. Payments and settlement services are a clear example: they display both scale economies and network externalities.”

<sup>23</sup>In recent crises, markets for securities trading (e.g., mortgage-backed securities in 1994, 1998, 2007-2009) shut down following the insolvency of large institutions, adversely affecting those with a “franchise” in those markets.



## 5 Design of Bank Closure Policies

In the previous section, we showed that risk-shifting arises at individual and collective levels because deposit contracts that are conditional upon observable bank characteristics cannot be written. This creates a “missing market.” (as discussed in Gorton and Mullineaux, 1987). The central bank can design mechanisms to overcome the inefficiencies arising due to this “missing market”. However, we show below that lack of judicious regulation could also induce such risk-shifting behavior. We will focus on the *bank closure policy* adopted by the central bank (the regulator), which is the ex-post mechanism employed to manage financial crisis. We model the closure policy as a bail out of the bank with a possible dilution of equityholders’ claim upon bail out. The rationale for this modeling choice is the following.

In a bail out, the central bank covers the shortfall to the depositors, and the bank is not closed. The closure of a bank entails ex-post costs in the form of a loss of its charter-value and the welfare losses to its depositors. If the opportunity cost to the central bank of the funds needed to cover the shortfall is smaller than these costs of bank closures, it is *ex-post optimal* to bail out the failed bank always. We assume this to be the case. This insurance however has a negative feedback effect on the ex-ante risk choice of the bankowners. The resulting moral hazard in the form of excessive risk-taking implies that the ex-post optimal policy of bailing out always will, in general, fail to be *ex-ante optimal*.

To counteract this moral hazard, the central bank upon bail out subjects the bankowners to a “dilution” by acquiring warrants or by nationalizing the bank.<sup>24</sup> Thus, we assume that upon bail out, the bankowners retain only a fraction  $\beta$  of their equity claim, where  $0 \leq \beta \leq 1$ . The remaining fraction,  $1 - \beta$ , is taken over by the central bank. Thus,  $\beta = 1$  is interpreted as complete forbearance towards the bank;  $0 < \beta < 1$  as the central bank holding a partial equity stake in the bank; and  $\beta = 0$  as a complete nationalization of the bank.<sup>25</sup>

The moral hazard in the form of excessive risk-taking by a single bank is well-understood. Hence, we focus only on the moral hazard arising in the form of collective risk-taking. We assume that there is no equity capital in the bank (the bankowners are wealth-constrained), the bankowners do not (or cannot) issue outside equity, and no capital adequacy regulation is

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<sup>24</sup>Alternately, the central bank could employ a mixed strategy where a bank is bailed out with some probability  $p$ ,  $0 < p < 1$ . This is often referred to as “constructive ambiguity” (e.g., Freixas, 1999). Such policies are however time-inconsistent, as Mailath and Mester (1994) note. Further, there is vast evidence that upon a bank’s survival, equityholders lose a significant stake in the bank either to an acquiring bank or to the assisting government, as documented by Sprague (1986) and by Dewatripont and Tirole (1993).

<sup>25</sup>Note that, the central bank may impose penalties as well through firing of managers, CEO’s, and even the Board. Limited liability will however bind and such penalties are limited to the losses resulting to the incumbent managers upon severance of their contract. An earlier draft allowed for penalties that could be contingent on extent of bank failure and found that allowing for them is not very crucial to our results.

in place.<sup>26</sup> Note that since a bank is never closed upon failure, all externality effects considered in Section 4.1 arising either due to a spillover or due to strategic benefit are eliminated. The charter-values are thus identical in all states at  $t = 1$  and in particular, equal to those in the state “ $ss$ ”. Hence, we refer to it simply as  $V_1$ . This enables us to focus exclusively on the effect of the bank closure policy on the correlation preference of banks. Finally, we make the natural assumption that bail outs are more costly in joint failures than in individual failures, for example, due to a convex cost of funds faced by the central bank. It is straightforward to show (as in the proof of Proposition 3) that this induces a preference for low correlation across bank asset returns in the social first-best, i.e.,  $\rho^o = \rho_l$ .

### 5.1 Systemic Moral Hazard

Consider a bank closure policy consisting of a bail out of a bank upon failure with a dilution function given as  $(\beta_i^{fs}, \beta_i^{ff})$ . Thus upon bail out, bank  $i$  retains  $\beta_i^{fs}$  fraction of its equity when it fails but bank  $j$  survives (state “ $fs$ ”), and it retains  $\beta_i^{ff}$  fraction of its equity both banks fail (state “ $ff$ ”). We call such a closure policy as “collective” in nature if  $\beta_i^{fs} \neq \beta_i^{ff}$ , since in this case bank  $i$ ’s payoff is affected by whether bank  $j$  survives or fails. On the other hand, we say that the closure policy is “myopic” in nature, i.e., if  $\beta_i^{fs} = \beta_i^{ff}$ . We examine the correlation preference of the banks under such closure policies.

The objective function of bank  $i$  facing the closure policy is given as:

$$\begin{aligned} \max_{\sigma_i, X_{Ri}, \rho_i} & \int_r^{R_{max}} (R - r)X_{Ri} h_i(R; \sigma_i)dR - c(X_{Ri}) + V_{i1} \cdot Pr[R_i > r] \\ & + \beta_i^{fs}V_{i1} \cdot Pr [R_i < r, R_j > r] + \beta_i^{ff}V_{i1} \cdot Pr [R_i < r, R_j < r] \end{aligned} \quad (5.1)$$

Substituting  $Pr[R_i < r, R_j > r] = Pr[R_i < r] - Pr[R_i < r, R_j < r]$ , we can rewrite this as

$$\begin{aligned} \max_{\sigma_i, X_{Ri}, \rho_i} & \int_r^{R_{max}} (R - r)X_{Ri} h_i(R; \sigma_i)dR - c(X_{Ri}) + V_{i1} \cdot Pr[R_i > r] \\ & + \beta_i^{fs}V_{i1} \cdot Pr [R_i < r] + (\beta_i^{ff} - \beta_i^{fs}) V_{i1} \cdot Pr [R_i < r, R_j < r]. \end{aligned} \quad (5.2)$$

Denote the equilibrium correlation induced by the closure policy as  $\rho^\beta$ . Then, the following important result is a consequence of Equation (5.2) and Assumption 4.

**Proposition 4 (Systemic Moral Hazard)** *A collective closure policy that exhibits greater forbearance in the joint failure state compared to the individual welfare state induces systemic*

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<sup>26</sup>A detailed study of the joint design of bank closure policies and capital adequacy requirements is undertaken in Acharya (2003) which also contains relevant references on the topic.

risk-shifting, i.e., if  $\beta_i^{ff} > \beta_i^{fs}$ , then  $\rho^\beta = \rho_h > \rho^o = \rho_l$ . On the other hand, a collective closure policy that exhibits less forbearance in the joint failure state compared to the individual welfare state eliminates systemic risk-shifting, i.e., if  $\beta_i^{ff} < \beta_i^{fs}$ , then  $\rho^\beta = \rho^o = \rho_l$ .

Intuitively, since the costs of bank bail outs are systemic in nature, prudential regulation should reward banks less in such states and cause them to internalize these costs. Importantly, the proposition shows that implicit or explicit government guarantees can have a perverse feedback effect on systemic risk. The insurance to a group of banks or financial institutions in the form of greater forbearance in joint failure provides them an incentive to be highly correlated. This increases the ex-ante likelihood of such a joint failure and generates *systemic moral hazard*. A subtle but a fundamental point revealed by this result is the following: while the *absolute* level of forbearance affects the moral hazard that manifests as individual risk-shifting, it is the *relative* levels of forbearance in the individual and the joint failure states that affect the moral hazard that manifests as systemic risk-shifting.

**“Too-Many-To-Fail” and systemic risk:** In practice, systemic failures can thus be quite problematic. It may be difficult to implement the *ex-ante optimal* policy: it may be impossible to nationalize a large number of banks to produce a greater dilution in joint failure states. Similarly, the replacement of top-level personnel and boards that would impose severe penalties in the joint failure states may be infeasible due to labor market constraints if such penalties are required for many institutions simultaneously. Further, since welfare losses to the depositors (and thus to the economy at large) are greater in the joint failure state, the group of failed banks may have greater bargaining power with the central bank and may be able to renegotiate a bail out with weaker terms.<sup>27</sup> This implicit “too-many-to-fail” guarantee renders the ex-ante optimal policy time-inconsistent and hence, lacking in commitment. It may thus play a significant role in sustaining systemic risk across banks as their equilibrium response to extract greater regulatory subsidies.<sup>28</sup>

Another immediate implication of equation (5.2) is the following. Under a myopic closure policy ( $\beta_i^{fs} = \beta_i^{ff}$ ), there is no dependence of each bank’s welfare on the joint characteristics of banks’ portfolios, and the choice of correlation is indeterminate. Thus, such a policy has a shortcoming in the multiple-bank economy: it fails to cause a bank to internalize the costs of systemic distress. As a result, if banks choose to be correlated due to other reasons, e.g.,

<sup>27</sup>Bongini, Claessens, and Ferri (1999) study the political economy of the distress of East Asian banks during the crisis of late 1990’s and document the difficulty faced by the regulators in managing multiple bank failures.

<sup>28</sup>The author thanks Enrico Perotti for suggesting the nice, intuitive term: “too-many-to-fail” guarantee. Acharya and Yorulmazer (2008b) develop a more exhaustive analysis of too-many-to-fail and the associated time-inconsistency in bank closure regulation.

synergies from sharing information as is used to motivate loan syndications, then the myopic rescue policy does not penalize them for such correlation.

**Proposition 5 (Suboptimality of Myopic Closure Policy)** *Under a myopic closure policy, i.e., if  $\beta_i^{fs} = \beta_i^{ff}$ , the equilibrium correlation of banks’ asset returns,  $\rho^\beta$ , is indeterminate. Under the worst case,  $\rho^\beta = \rho_h$  and the systemic risk-shifting remains completely unmitigated.*

This justifies the use of the term *myopic* for such designs. While the term “myopic” usually gives the connotation of short-termism or single-period focus in inter-temporal problems, we employ this term in its broader meaning of being “short-sighted”. Alternately, such designs could be called “narrow” in scope or “micro” as different from “broad” or “macro”.<sup>29</sup> To see that the optimal collective design is not myopic (in general), consider a myopic policy  $(\beta^f, \beta^f)$ . Let the choice of bank  $i$ ’s risk under this policy be  $\sigma$ . Consider a collective policy  $(\beta^{fs}, \beta^{ff})$ , where  $\beta^{fs} = \beta^f + \delta$ , and  $\beta^{ff} = \beta^f - \epsilon$ , with  $\delta, \epsilon > 0$ . It follows that the correlation choice induced by this policy is  $\rho_l$  (Proposition 4). From equation (5.1), given any  $\epsilon > 0$ , we can find a  $\delta > 0$  such that under the policy  $(\beta^{fs}, \beta^{ff})$ , bank  $i$ ’s choice of risk is again  $\sigma$ . The dilution factors are only transfers between the banks and the central bank. Thus, for any myopic policy, there exists a collective policy with  $\beta^{ff} < \beta^{fs}$  that dominates it.

To end this section, we consider a regulatory policy that creates “value” for banks when they survive and others fail. We show that such a policy can be effectively used to reduce the bite of implicit “too-many-to-fail” guarantee and in turn, to mitigate systemic risk.

## 5.2 The Incentive Role of Bank Sales

Let us augment the collective closure policy to  $(\beta_i^{fs}, \beta_i^{sf}, \beta_i^{ff})$ ,  $\beta_i^{sf}$  being the proportion of bank  $i$ ’s charter-value that is awarded to it in the state where it survives but the bank  $j$  fails (state “ $sf$ ”). Under this policy, Bank  $i$  picks  $(\sigma_i, X_{Ri}, \rho_i)$  to maximize

$$\int_r^{R_{max}} (R - r) X_{Ri} h_i(R; \sigma_i) dR - c(X_{Ri}) + V_{i1} \cdot Pr[R_i > r] \tag{5.3}$$

$$+ \beta_i^{fs} V_{i1} \cdot Pr [R_i < r, R_j > r] + \beta_i^{sf} V_{i1} \cdot Pr [R_i > r, R_j < r] + \beta_i^{ff} V_{i1} \cdot Pr [R_i < r, R_j < r].$$

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<sup>29</sup>There is another sense in which the myopic closure policy is suboptimal. In the intermediated equilibrium, systemic risk-shifting arises only if  $s < s^c(\alpha)$ , the case of negative externality (Proposition 1). In the case of positive externality, i.e., when  $s > s^c(\alpha)$ , banks in fact prefer a low correlation of asset returns even in the absence of any regulation. Thus unconditional bail outs with a myopic closure policy are dominated by a policy of conditional bail outs, i.e., bail out only if the bank’s failure imposes a negative externality on the rest of the system, else simply close the bank. This gives a justification for “too-big-to-fail” kind of guarantee from an ex-ante standpoint, in addition to the conventional ex-post argument.

Substituting  $Pr[R_i < r, R_j > r] = Pr[R_i < r] - Pr[R_i < r, R_j < r]$ , we can rewrite this as

$$\int_r^{R_{max}} (R - r) X_{Ri} h_i(R; \sigma_i) dR - c(X_{Ri}) + V_{i1} \cdot Pr[R_i > r] + \beta_i^{fs} V_{i1} \cdot Pr[R_i < r] + \beta_i^{sf} V_{i1} \cdot Pr[R_i > r] + (\beta_i^{ff} - \beta_i^{fs} - \beta_i^{sf}) V_{i1} \cdot Pr[R_i < r, R_j < r]. \tag{5.4}$$

Thus, a necessary and a sufficient condition to induce low correlation is that  $\beta_i^{ff} < \beta_i^{fs} + \beta_i^{sf}$ . It is *not* necessary for the closure policy to have  $\beta_i^{fs} > \beta_i^{ff}$ , if  $\beta_i^{sf}$  is sufficiently high. In words, the perverse feedback effect of a “too-many-to-fail” bail out policy can be mitigated if the banks gain a large *strategic benefit*, measured by  $\beta_i^{sf}$ , when other banks fail and they survive.<sup>30</sup> Such strategic benefit may accrue to banks by acquiring, partly or fully, the failed banks. Bank sales, known in the U.S. as “purchase and assumption,” are commonly employed by the receiver of the failed banks in many countries, including the U.S. and Norway (see, James, 1991 and Dewatripont and Tirole, 1993). Since many countries do not have the market or the tradition of conducting bank sales, it may be a good policy directive for their regulators to encourage the development of such a market.

While the simple structure of our model prevents us from undertaking a thorough study of other implications of conducting bank sales, such as the creation of monopolies and a change in the organizational structure of the banking industry, it points to a hitherto neglected incentive effect of including bank sales in the closure policy of a central bank.<sup>31</sup> Bank sales increase the charter-value of banks precisely in those states where they survive and other banks fail. This reduces the preference amongst banks to be highly correlated in order to extract rents from closure policies that are not sufficiently stringent upon joint failures. In fact, the central banks may find it optimal to transfer value to the surviving banks via subsidized sales, i.e., sales that occur at lower than the fair value of the failed banks.

## 6 Design of Capital Adequacy Regulation

As discussed in the previous section, optimal closure policy designs may be difficult to implement as they are time-inconsistent. Hence, we examine the *ex-ante* mechanism, viz. the

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<sup>30</sup> Assuming symmetric banks, the restriction is that  $\beta_i^{fs} + \beta_i^{sf} \leq 1$ , the remaining equity stake,  $1 - \beta_i^{fs} - \beta_i^{sf}$ , being taken up by the central bank in a partial nationalization.

<sup>31</sup> It is often argued that from a policy standpoint, there is no difference between a bail out and a sale. Sprague (1986) claims, “In practice, the effect of a sale or a bail out is virtually the same.... In either instance, the management is out. Then what is the problem? It simply is... Bail out is a bad word.” This argument is flawed since it ignores that a sale creates value for the surviving banks whereas a bail out does not and this in turn, gives incentive to banks to be uncorrelated. Ignoring this mitigating effect of bank sales on systemic risk-shifting incentive leads to the “myopic” conclusion that their net effect is identical to that of bail outs.

capital requirements. We first discuss the suboptimality of myopic capital adequacy that is based only a bank’s *own* risk, and next show that the optimal capital adequacy also takes into account the *joint* risk of banks, in particular, their correlation.

Consider the economy as described in Section 3. We continue to assume that there is no closure policy in place. Each bank’s equityholders are wealth-constrained and have no capital of their own. As a result, any bank capital must be raised in the form of outside equity, which corresponds to Tier 1 capital required by the current regulation.<sup>32</sup> For simplicity, we assume that depositors and capital providers are different agents in the economy, consistent with a “segmented markets” explanation.<sup>33</sup>

Raising such equity is privately costly since it dilutes the claim of existing equityholders if they are required to pay a higher than fair, expected rate of return on equity. These costs (transfers) arise due to (i) informed trading in capital markets, (ii) asymmetric information of the equityholders, and/or (iii) manager-shareholder conflicts.<sup>34</sup> We can extend the model of Section 3 to endogenize such costs. For simplicity however, we take the net dilution cost of outside equity for each bank to be simply  $\theta(K)$ , where  $K$  is the amount of outside equity issued,  $\theta'(K) > 0$  and  $\theta''(K) > 0$ .

The budget-constraint for bank  $i$  is now given by  $X_{Si} + X_{Ri} = D_i + K_i$ . There is default whenever  $rX_{Si} + R_i X_{Ri} < rD_i$ , i.e., whenever  $R_i$  is below the threshold  $R_i^c = r \cdot \left(1 - \frac{K_i}{X_{Ri}}\right) \leq r$ . The “buffer” role of capital is thus to act as an ex-ante liability of the bankowners and lower the threshold return below which the bank defaults. Incorporating this lower threshold, the value of old equityholders of bank  $i$  given the investment choices of both banks and their respective levels of outside capital, is the following where  $R_j^c = r \cdot \left(1 - \frac{K_j}{X_{Rj}}\right)$ :

$$\begin{aligned}
 V_i^{old}(\cdot) &= \int_{R_i^c}^{R_{max}} (R - R_i^c) X_{Ri} h_i(R; \sigma_i) dR - c(X_{Ri}) - \theta(K_i) \\
 &+ V_{i1}^{ss} \cdot Pr[R_i > R_i^c] + \left( V_{i1}^{sf} - V_{i1}^{ss} \right) \cdot Pr[R_i > R_i^c, R_j < R_j^c],
 \end{aligned}
 \tag{6.1}$$

## 6.1 Suboptimality of myopic capital adequacy

Consider a capital adequacy regulation of the form,  $K_i(\cdot)$ , which is independent of the correlation across the banks’ assets. We call such a scheme as “myopic” since it is not based on

<sup>32</sup>See BIS (1988, 1996) for details on the regulatory specification of what qualifies as bank capital.

<sup>33</sup>Gorton and Winton (1999), and Diamond and Rajan (2000), consider the effect of requiring bank capital on the extent of deposits that the banks can raise.

<sup>34</sup>Empirical evidence on underpricing costs of outside equity can be found in Lee et al. (1996). Theoretical justifications for dilution cost of outside equity have been provided by Leland and Pyle (1977), Myers and Majluf (1984), and Rock (1986). Alternative explanations based on manager-shareholder agency costs are employed in Dewatripont and Tirole (1993), Froot, Scharfstein and Stein (1993), and Froot and Stein (1998).

the joint characteristics of banks' investments. Note that, in general,  $K_i(\cdot)$  may depend on  $\sigma_i$  and  $X_{Ri}$  (depending upon the contracting possibilities). Crucially,  $K_i(\cdot)$  does not depend on  $\rho$  in a myopic design. When such a scheme is employed in the multiple bank context, the collective incentives of the banks remain unaffected.

To see this, note that when banks face a myopic capital requirement, their objective function in equation (6.1) is qualitatively similar to that in equation (4.11), with the threshold point of failure driven down from  $r$  to  $R_i^c$ . The case of interest is one where  $R_i^c > 0$ , i.e.,  $K_i < X_{Ri}$ , since the participation constraint of the bankowners will bind in general. In other words, the case where  $K_i = X_{Ri}$  would require 100% bank capital which is unrealistic because the accompanying dilution cost would drive the bankowners out of banking activity. With  $R_i^c > 0$ , the form of the externality term  $(V_{i1}^{sf} - V_{i1}^{ss}) \cdot Pr[R_i > R_i^c, R_j < R_j^c]$ , which affects the preference for correlation amongst banks, is essentially unchanged.

It was shown (Proposition 1) that the recessionary spillover from the failure of one bank on the health of the surviving bank dominates the strategic benefit of the surviving bank ( $V_{i1}^{sf} < V_{i1}^{ss}$ ), whenever  $s < s^c(\alpha)$  or alternately,  $\alpha > \alpha^c(s)$ . In these cases, there is a systemic risk-shifting in the intermediated economy, i.e.,  $\rho^* = \rho_h$ . It follows that myopic capital adequacy regulation does a poor job of mitigating the systemic risk-shifting incentive, *even if* it succeeds in mitigating the individual risk-shifting incentive.<sup>35</sup>

**Proposition 6 (Suboptimality of Myopic Capital Adequacy)** *Under any myopic capital adequacy regulation, systemic risk-shifting is left unmitigated, i.e., banks choose to be highly correlated whenever  $s < s^c(\alpha)$  or  $\alpha > \alpha^c(s)$  (negative externality).*

A subtle point is in order. A myopic capital adequacy can be interpreted as a value-at-risk constraint since the level of bank capital essentially determines the likelihood of bank failure. A feasible option for the regulators, one that is currently employed, is to increase the confidence level employed in calculating value-at-risk, increase capital charge, lower the threshold point of bank failure, and in turn, reduce the magnitude of systemic risk-shifting effect. However, this also has a perverse side effect: the bankowners who suffer huge dilution costs of additional capital may respond by underinvesting, transforming “capital crunch” into a “credit crunch”. The optimal capital adequacy regulation is correlation-based and *strictly* dominates any myopic scheme.

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<sup>35</sup>Note that strictly speaking, the charter-values,  $V_{i1}^{sf}$  and  $V_{i1}^{ss}$ , will also be affected by capital requirements. In fact, reduction of future values may lead banks to increase risk today. This perverse feedback effect must be taken into account in the optimal design, but is not the focus of our argument.

## 6.2 Correlation-based capital adequacy

A “correlation-based” (or “collective”) capital adequacy scheme is one where the capital requirement is explicitly contingent on *inter-bank correlations*. In other words, it is of the form  $K_i(\cdot, \rho)$ . Before we proceed to show how such a scheme can mitigate systemic risk-shifting, two things deserve mention. First, in general, design of such a scheme should be undertaken in conjunction with all other observables that the scheme is contingent on. For example, the current capital requirements depend on size ( $X_R$ ) and a measure of riskiness ( $\sigma$ ) of the assets. The complete design  $K_i(\sigma, X_R, \rho)$  was undertaken in an earlier draft.

Second, a fundamental question is whether such contingent contracts are feasible in the first place. Such contracts are infeasible from the point of view of depositors due to multiplicity of monitoring costs and/or lack of coordination. The regulator, a “delegated monitor” of sorts who represents the depositors, can however write and enforce more sophisticated contracts, e.g., via capital adequacy schemes. Our goal is to prescribe as a normative rule what is the optimal contract, with the understanding that inter-bank correlations are at least partially observable and contractible by the regulators. The issue of robustness of the optimal contract to imprecise measurement and asymmetric information between banks and regulators is an interesting issue for future research.

We show next that  $K_i(\cdot, \rho)$  can be structured so that the banks facing such a scheme respond by undertaking investments in assets with a low correlation of returns.

**Proposition 7 (Correlation-based Capital Adequacy)** *Under a capital adequacy scheme,  $K_i(\cdot, \rho)$ , that is increasing sufficiently steeply in  $\rho$ , banks choose to be as little correlated as possible. Thus, systemic risk-shifting is completely mitigated. A necessary and a sufficient condition is that for given investment choices  $(\sigma_i, X_{Ri})$ ,  $i \in \{A, B\}$ ,*

$$\frac{dK_i}{d\rho} \cdot \left\{ \theta'_i(K_i) - \Delta V \cdot \frac{dR_i^c}{dK_i} \cdot \frac{d}{dR_i^c} Pr[R_i > R_i^c, R_j < R_j^c] \right\} > \Delta V \cdot \frac{d}{d\rho} Pr[R_i > R_i^c, R_j < R_j^c],$$

where  $\Delta V = V_{i1}^{sf} - V_{i1}^{ss}$ , and  $R_i^c = r \cdot (1 - \frac{K_i}{X_{Ri}})$ .

Note that the LHS above is the marginal cost to bank  $i$  from increasing its correlation with bank  $j$ , and the RHS is the marginal benefit. In the case of negative externality (when systemic risk-shifting occurs),  $\Delta V < 0$ . By Assumption 5,  $Pr[R_i > R_i^c, R_j < R_j^c]$  is decreasing in  $\rho$ . Thus, under our maintained assumption that the issuance of capital is privately costly to the bank (so that the term inside  $\{\cdot\}$  in LHS above is positive), it is necessary that  $\frac{dK_i}{d\rho} > 0$ . The intuition is clear: the negative externality in state “ $sf$ ” which induces in banks a preference for the state “ $ss$ ” is counteracted by a higher cost of capital that banks must incur if they increase the probability of state “ $ss$ ” by being highly correlated. The



magnitude of the capital charge or “penalty” for increasing correlation depends upon the negative externality endogenous to the general equilibrium. The resulting collective design mitigates systemic risk-shifting and strictly improves upon the myopic design.

This illustrates a fundamental point: in one principal, many agent problems with externalities across agents, the optimal mechanism in general will offer payoffs to agents that are dependent on the heterogeneity of agents *and* on the actions of other agents. The correlation-based scheme,  $K_i(\cdot, \rho)$ , has both these features. It depends on the joint action of the banks ( $\rho$ ). In addition, the extent of dependence is related to the precise nature of investment opportunities available to the banks ( $\mathcal{H}_i$ ).

### 6.3 Discussion: Implications for current regulation

Attempts at collective regulation so far have mainly been *ex-post*, and have manifested either as temporary position limits on investments in the industry (or counterparty) that caused the systemic event or as an increase in capital charge for such investments. No systematic attempt has been made however, to anticipate and contain the extent of systemic risk *ex-ante*.<sup>36</sup> The following discussion provides an intuitively appealing implementation of our proposal.

**Portfolio theory interpretation:** Regulation has encouraged banks to consider the portfolio effects of their activities (trading, lending, etc.) while measuring their enterprise-wide risk. The recommendation is based on the concern that returns on different activities may be correlated and simply adding up their risks may understate the true risk of the portfolio. This recognition of *intra-bank correlations* for market risk (BIS, 1996) and for credit risk (BIS, 1999) has however not been extended from within the banks to the economy at large, where a similar consideration arises due to *inter-bank correlations*. Regulating the risk of each bank affects the variance terms in the inter-bank covariances but leaves the contribution from the correlation terms unaffected. Optimal regulation takes account of both contributions.

This portfolio interpretation suggests that we can decompose the risk of each bank into two components: (i) exposures to “general” factors such as interest rate, foreign exchange rate, industry, etc., and (ii) exposures to idiosyncratic or a “specific” factors, e.g., as suggested by Arbitrage-Pricing Theory. Prudential regulation should require that banks hold greater capital against general risks than against specific risks for the same level of risk. This would

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<sup>36</sup>It seems that financial regulators had recognized the importance of inter-bank correlations. On 21<sup>st</sup> September 2000, Andrew Crockett, General Manager and Chairman of the Financial Stability Forum at BIS, made the following suggestions for ‘marrying the micro- and the macro- dimensions of financial stability’: “More often than not, episodes of financial distress arise from the exposure of groups of institutions to common risk factors. Unless the authorities take into account the impact of the collective behavior of institutions on economic outcomes, they may fail to monitor risks and take remedial action appropriately.”

give incentives to the banks to be less correlated and thus reduce systemic risk.

**Portfolio compositions vs. summary statistics:** Over the last decade, bank-wide risk management has moved towards the value-at-risk approach where each bank aggregates its risk and reports a single summary statistic. While such a number may be sufficient to regulate the individual risk of a bank, it is clearly insufficient to compute the value-at-risk of the aggregate banking portfolio. It is paramount for the operation of correlation-based capital adequacy that the banks report not just their value-at-risk numbers but also their portfolio compositions. The exposures of each bank’s portfolio to the general risks can be supplied to the regulator who can consolidate these exposures across banks and determine the collective risk capital charge for each bank, in addition to the individual risk contributions. While detailed consolidation may be costly due to current lack of standardization in reporting, a simpler approach via the general and specific risk decomposition may suffice. Consolidation can also provide “macro-prudential indicators” that can be employed in setting aggregate position limits, i.e., constraints of the form that the consolidated exposure across all banks to a general factor (e.g., aggregate industry concentration) not exceed a limit.<sup>37</sup>

**Centralized capital budgeting:** The collective “pricing” of risks is in fact operational within the financial sector. Each bank allocates capital to an activity taking into account its contribution to the overall risk of the bank’s portfolio. James (1996) while discussing the implementation of RAROC at Bank of America, states that “the amount of capital allocated varies with the contribution of the project to the overall volatility of earnings.” Similarly, sophisticated banks calculate the counterparty credit charge on their derivative transactions based on a portfolio pricing approach. A trading desk doing transactions sends relevant information to a central group that aggregates the counterparty risks of different transactions and in return provides the desk with a credit charge number to be applied to the derivative’s price. Our visualization of the central bank calculating the capital charge for the individual banks is not much different than its micro incarnations listed above.

**Focused vs. diversified banks:** Consider two types of economies with two industries and two banks in each. In economy A, banks are “focused” and invest in different industries. The industries are imperfectly correlated and hence, neither of the banks achieves possible

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<sup>37</sup>Such limits could have helped prevent some recent crises, such as (i) the near collapse of Long Term Capital Management, where almost all large investment houses had significant exposures to a single counterparty – LTCM (exceeding a billion dollars in some cases); a consolidation of real-estate lending from the call reports of banks could have also forewarned of the New England banking crisis in the early 1990s, and (ii) the crisis of 2007-2009 where ex post it turned out that banks had in fact not transferred enough risks of the mortgage-backed assets, and indirectly thus of housing sector, but instead retained big chunks on their balance sheets (Acharya and Schnabl, 2009).

diversification. In economy B, each bank invests in both industries and is thus “diversified”. In economy A, the individual risk of each bank is higher due to lack of diversification, however the joint failure risk is lower. On the other hand, in economy B, the individual risk of each bank is lower, but both banks are perfectly correlated and always fail together. This illustrates the tradeoff between focus and diversification. Diversified banks are attractive since the risk of each bank is reduced, but this is achieved at the cost of an increase in systemic risk. This tradeoff determines an optimal level of focus for the banking industry.<sup>38</sup>

**Effect of competition on systemic risk:** A related point is with regards to encouraging competition in the banking sector. Many authors (e.g., Allen and Gale, 2000b, and the references therein) have argued that increase in competition amongst banks may lead to greater risk-shifting incentives for the banks. Our analysis shows that this effect may be especially perverse since encouraging competition may also increase the correlation of banks’ portfolio returns as they all invest in similar sectors. The costs of resulting financial instability must be weighed against the efficiency gains arising from greater competition.<sup>39</sup>

## 7 Conclusion

We have developed a positive theory of systemic risk and a normative theory of its prudential regulation in a multi-period general equilibrium model with many banks and depositors that incorporates (i) the likelihood of default by banks on deposits; (ii) financial externalities from failure of one bank on other banks; (iii) regulatory incentives; and (iv) the interaction of these features. Several applications of our analysis and results are immediate.<sup>40</sup>

A result such as ours where the optimal mechanism for an agent depends on the characteristics and actions of other agents is to be expected in general in any model with heterogeneous agents and some externality. It applies to many economic phenomena where agents undertake similar strategies and modes of behavior. Our analysis suggests that examining the comple-

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<sup>38</sup>It is a little appreciated statistical fact that pooling of risks increases the likelihood of joint survival and joint failure, as Shaffer (1994) notes. In fact, there is a strong sense in which a certain level of “focus” is *always* optimal. If all banks have low but positive risk of failure and are perfectly correlated, then the probability of joint failure equals the probability of individual failure *independent* of the number of banks. On the other hand, if banks are imperfectly correlated, the probability of joint failure converges to zero for sufficiently large number of banks. This is particularly relevant since the regulators have recognized that most investment houses today hold virtually identical balance-sheets and the failure of one would in most cases be the same event as the failure of most of them. The author is pursuing an extension of this model where this tradeoff is formalized by explicitly modeling general and specific risk factors.

<sup>39</sup>The author thanks Qiang Dai for bringing this point to his attention.

<sup>40</sup>In particular, additional details about how to empirically implement a capital requirement that is based on joint failure risk of banks is presented in Acharya, Pedersen, Philippon and Richardson (2009).

mentarities of agents' actions and the underlying agency problems may be a fruitful direction towards explaining the collective behavior of agents as their equilibrium response.

The most relevant application seems to be in delegated portfolio management. The bonus schemes of traders in banks are often implicitly based on group performance. Losses to a single desk could generate lower compensation for all other traders. This is a negative externality of the failure of one trader on the profitability of others. Given their limited liability, the traders have an incentive to undertake trading strategies such that they survive together and fail together rather than see their profits subsidize the failure of others. Enterprise-wide risk-management and capital budgeting that is based on correlations across desks should be designed *jointly* with the incentive schemes of different desks to mitigate such behavior.

We have modeled systemic risk through the choice of correlation across assets of different banks. Systemic risk can also arise due to inter-bank contracts. Our analysis implies that regulating each bank's risk cannot capture fully the risks that could propagate through a nexus of contracts. This propagation is of particular concern in banking given the opaqueness of banks' assets and investments. The effect of regulation on the endogenous choice of inter-bank contracts deserves careful scrutiny. In addition, we believe that characterizing systemic risk as an equilibrium response of the financial intermediaries is a crucial step towards building a model of systemic risk in the traditional general equilibrium set-ups.

## A Regularity Assumptions on Risky Technology

Bank  $i$  picks a risky portfolio that gives a return  $R \sim h_i(\cdot; \sigma)$ , from a family of distributions  $\mathcal{H}_i$ , indexed by the “risk” parameter  $\sigma$ .

**Assumption 1 (Mean-preserving Spreads)** *The expected return,  $\int_0^{R_{max}} R h_i(R; \sigma) dR$ , is constant  $\forall \sigma$  and is denoted as  $\bar{R}$ .*

Assumptions 2–5 are assumed to hold over the relevant range of  $r$  for the analysis.

**Assumption 2 (Increasing Risk of Default)** *The family of risky portfolios  $\mathcal{H}_i$  is ordered by risk parameter  $\sigma$ ,  $\sigma \in [\sigma_{min}, \sigma_{max}]$ , in the sense of ‘increasing risk of default’:*

- (i) *likelihood of failure,  $\int_0^r h_i(R; \sigma) dR$ , is increasing and convex in  $\sigma$ ; and*
- (ii) *expected losses in failure,  $\int_0^r (r - R) h_i(R; \sigma) dR$ , are increasing and convex in  $\sigma$ .*

We will assume continuity of these functions and their derivatives. A family of “mean-preserving spreads” in the sense of Rothschild and Stiglitz (1970) satisfies Assumptions 1–2.<sup>41</sup>

The “correlation” of the risky portfolios is denoted as  $\rho \in \{\rho_l, \rho_h\}$ .

**Assumption 3 (Increasing Risk of Joint Default)** *Given  $\rho$  and  $\sigma_j$ , the distributions  $R_i \sim h_i(\cdot; \sigma_i)$  and  $R_j \sim h_j(\cdot; \sigma_j)$  satisfy the following properties:*

- (i) *likelihood of joint failure,  $Pr[R_i < r, R_j < r]$ , is increasing and convex in  $\sigma_i$ ; and*
- (ii) *expected losses upon joint failure,  $E[(r - R_i) \cdot 1_{\{R_i < r, R_j < r\}}]$ , are increasing and convex in  $\sigma_i$ .*

An analogous assumption is made with respect to  $\sigma_j$  for given  $\rho$  and  $\sigma_i$ . Note that  $1_{\{\mathcal{F}\}}$  is the indicator function, equal to 1 when the event  $\mathcal{F}$  occurs, and 0 otherwise.

**Assumption 4 (Correlation Increases Risk of Joint Default)** *Given  $\sigma_i$  and  $\sigma_j$ , the distributions  $R_i \sim h_i(\cdot; \sigma_i)$  and  $R_j \sim h_j(\cdot; \sigma_j)$  satisfy the following properties:*

- (i) *likelihood of joint failure,  $Pr[R_i < r, R_j < r]$ , is increasing in  $\rho$ ; and*
- (ii) *expected losses upon joint failure,  $E[(r - R_i) \cdot 1_{\{R_i < r, R_j < r\}}]$ , are increasing in  $\rho$ .*

Assumption 4 and the assumption that the choice of industries (in effect,  $\rho$ ) does not affect the choice of risk ( $\sigma$ ) imply the following which is stated as an assumption:

**Assumption 5**  *$Pr[R_i < r, R_j > r] = Pr[R_i < r] - Pr[R_i < r, R_j < r]$ , is decreasing in  $\rho$ .*

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<sup>41</sup>It is possible for mean-preserving spreads not to increase the likelihood of default and expected losses in default, but they cannot decrease these measures.

## B Proofs

**Lemma A.1** *The equilibrium,  $(r_1^{sf}, \sigma_A^{sf}, X_{RA}^{sf})$ , exists with  $\sigma_A^{sf} \equiv \sigma_{max}$ .*

Since  $X_{RA}(r, \sigma_A)$ , given by equation (4.6), is optimal for given  $r$  and  $\sigma_A$ , it follows from equation (4.4) that  $\frac{d}{d\sigma_A} v_{A1}^{sf}(r, \sigma_A, X_{RA}(r, \sigma_A)) = X_{RA}(r, \sigma_A) \cdot \frac{d}{d\sigma_A} \int_r^{R_{max}} (R - r) h_A(R; \sigma_A) dR = X_{RA}(r, \sigma_A) \cdot \frac{d}{d\sigma_A} \int_0^r (r - R) h_A(R; \sigma_A) dR > 0$  where the last equality follows the identity  $\int_r^{R_{max}} (R - r) h_A(R; \sigma_A) dR = \bar{R} - r - \int_0^r (R - r) h_A(R; \sigma_A) dR$ , and the last inequality follows Assumption 2. This holds for any  $\sigma_A$ , hence  $\hat{\sigma}_A(r) = \sigma_{max}, \forall r$ . The equilibrium is given by the fixed-point:  $r_1^{sf} = f'[D_A + sD_B - \hat{X}_{RA}(r_1^{sf})]$  where  $\hat{X}_{RA}(r) = X_{RA}(r, \sigma_{max})$ . Assuming  $X_{RA}(r, \sigma) \in (0, D_A + sD_B), \forall r, \sigma$ , this fixed point exists by Brouwer's Fixed Point Theorem (Sundaram, 1996).  $\square$

**Lemma 1:** Without loss of generality, let  $i = A$ . We prove first that  $V_{A1}^{sf}$  increases in  $s$  for given  $\alpha$ . Denote  $\hat{X}_{RA}(r) = X_{RA}(r, \sigma_{max}), V_{A1}^{sf}(r) = \int_r^{R_{max}} (R - r) \hat{X}_{RA}(r) h_A(R; \sigma_{max}) dR - \alpha \cdot c(\hat{X}_{RA}(r)), r_1^{sf} = f'[D_A + sD_B - \hat{X}_{RA}(r_1^{sf})]$  (the fixed-point), and  $V_{A1}^{sf} = V_{A1}^{sf}(r_1^{sf})$ .

- (i)  $\frac{d\hat{X}_R(r)}{dr} < 0$ : Differentiating equation (4.6) w.r.t.  $r$  yields  $-\alpha \cdot c''(\hat{X}_{RA}(r)) \cdot \frac{d\hat{X}_R(r)}{dr} = 1 - \frac{d}{dr} \int_0^r (r - R) h_A(R; \sigma_{max}) dR > 0$  by chain-rule. The convexity of  $c(\cdot)$  implies that  $\frac{d\hat{X}_R(r)}{dr} < 0$ .
- (ii)  $\frac{dV_{A1}^{sf}(r)}{dr} < 0$ : Differentiating  $V_{A1}^{sf}(r)$  w.r.t.  $r$  yields  $\frac{dV_{A1}^{sf}(r)}{dr} = \hat{X}_{RA}(r) \cdot \frac{d}{dr} \int_r^{R_{max}} (R - r) h_A(R; \sigma_{max}) dR = \hat{X}_{RA}(r) \cdot \frac{d}{dr} [\bar{R} - r - \int_0^r (r - R) h_A(R; \sigma_{max}) dR] < 0$  by chain-rule.
- (iii)  $\frac{dr_1^{sf}}{ds} < 0$ : This follows the fixed-point equation for  $r_1^{sf}$  and the concavity of  $f(\cdot)$ .
- (iv) Thus,  $\frac{dV_{A1}^{sf}}{ds} = \frac{dV_{A1}^{sf}(r)}{dr} \cdot \frac{dr_1^{sf}}{ds} > 0$ , for given  $\alpha$ .

The second part that  $V_{A1}^{sf}$  decreases in  $\alpha$  for a given  $s$  can be proved similarly along these steps:

- (i)  $\frac{d\hat{X}_R(r)}{d\alpha} < 0$ . (ii)  $\frac{dr_1^{sf}}{d\alpha} > 0$ . (iii) Thus,  $\frac{dV_{A1}^{sf}}{d\alpha} = \frac{dV_{A1}^{sf}(r)}{dr} \cdot \frac{dr_1^{sf}}{d\alpha} < 0$ , for given  $s$ .  $\square$

**Proposition 1:** Consider first the cut-off  $s^c(\alpha)$ . We will denote the charter-value in the state “ $sf$ ” by the parameters,  $s$  and  $\alpha$ , as  $V_{i1}^{sf}(s, \alpha)$ . Consider  $\alpha = 1$  for illustration.

- (i)  $V_{i1}^{sf}(1, 1) > V_{i1}^{ss}$ . With  $\alpha = 1$ , the objective function of bank  $i$  is the same in state “ $sf$ ” and “ $ss$ ,” the only difference being in the return on safe asset. With  $s = 1$ , we must have  $r_1^{ss} > r_1^{sf}$ . Otherwise,  $r_1^{sf} < f'[\sum_i D_i - \sum_i \hat{X}_{Ri}(r_1^{sf})] < f'[\sum_i D_i - \sum_i \hat{X}_{Ri}(r_1^{ss})] = r_1^{ss}$ , a contradiction, where the first inequality follows the concavity of  $f(\cdot)$ , and the second inequality follows from  $\frac{d\hat{X}_{Ri}(r)}{dr} < 0$  (proof of Lemma 1). Since,  $\frac{dV_{i1}^{sf}}{dr} < 0$  (proof of Lemma 1), it follows that for  $\alpha = 1, V_{i1}^{sf}(1, 1) > V_{i1}^{ss}$ .

- (ii)  $V_{i1}^{sf}(0, 1) < V_{i1}^{ss}$ . This is because with  $s = 0, r_1^{sf} > r_1^{ss}$ . Else,  $r_1^{sf} = f'[D_A - \hat{X}_{Ri}(r_1^{sf})] < f'[\sum_i D_i - \sum_i \hat{X}_{Ri}(r_1^{ss})] = r_1^{ss}$ , a contradiction. The claim now follows the result  $\frac{dV_{i1}^{sf}}{dr} < 0$ .

Since  $V_{i1}^{sf}(s, 1)$  is increasing in  $s$  (Lemma 1), it follows that  $\exists s^c(1) \in (0, 1)$  such that  $\forall s < s^c(1), V_{i1}^{sf} < V_{i1}^{ss}$  (negative externality), and  $\forall s > s^c(1), V_{i1}^{sf} > V_{i1}^{ss}$  (positive externality).

- (iii) The existence of  $s^c(\alpha)$  for any  $\alpha$  follows similarly. If  $V_{i1}^{sf}(0, \alpha_{min}) < V_{i1}^{ss}$  as well, then  $\forall \alpha, s^c(\alpha) \in (0, 1)$ . Else, there is a threshold  $\bar{\alpha} > \alpha_{min}$ , such that  $\forall \alpha > \bar{\alpha}, s^c(\alpha) \in (0, 1)$ , else

$s^c(\alpha) \equiv 0$ , i.e., the externality is always positive for  $\alpha < \bar{\alpha}$  (sufficiently high strategic benefit).

(iv)  $V_{i1}^{sf}(s, \alpha)$  is decreasing in  $\alpha$  (Lemma 1). Consider  $\alpha' > \alpha$ . Then,  $V_{i1}^{ss} \leq V_{i1}^{sf}(s^c(\alpha), \alpha) < V_{i1}^{sf}(s^c(\alpha), \alpha')$ . It follows now that  $s^c(\alpha') \geq s^c(\alpha)$ , so that  $s^c(\alpha)$  is increasing in  $\alpha$ .

To prove the analogous result for cut-off  $\alpha^c(s)$ , we proceed exactly as above. Under the assumption that  $V_{i1}^{sf}(0, \alpha_{min}) < V_{i1}^{ss}$ , it can be shown that  $\exists \underline{s}, \bar{s}, 0 < \underline{s} < \bar{s} < 1$ , such that (i)  $\forall s < \underline{s}, \alpha^c(s) = \alpha_{min}$  (the externality is always negative for sufficiently high recessionary spillover), (ii)  $\forall s > \bar{s}, \alpha^c(s) = 1$  (the externality is always positive for sufficiently low recessionary spillover), and (iii)  $\forall s, \underline{s} < s < \bar{s}, \alpha^c(s) \in (\alpha_{min}, 1)$  so that  $V_{i1}^{sf} < V_{i1}^{ss}$  for  $\alpha > \alpha^c(s)$  (negative externality), and  $V_{i1}^{sf} > V_{i1}^{ss}$  for  $\alpha < \alpha^c(s)$  (positive externality). The details of the proof are omitted here.  $\square$

Next, we prove two lemmas for the intermediated economy, to be used in latter proofs.

**Lemma A.2** *The amount of risky investment,  $X_{Ri}(r, \sigma_i)$ , is increasing in risk,  $\sigma_i$ .*

Differentiating equation (4.12) w.r.t.  $\sigma_i$  yields  $c''(X_{Ri}) \cdot \frac{dX_{Ri}}{d\sigma_i} = \frac{d}{d\sigma_i} \int_0^r (r - R) h_i(R; \sigma_i) dR$ . From Assumption 2 and the convexity of  $c(\cdot)$ , it follows that  $\frac{dX_{Ri}(r, \sigma_i)}{d\sigma_i} > 0$ .  $\square$

**Lemma A.3** *The choice of risk by bank  $i$ ,  $\hat{\sigma}_i(r, \sigma_j)$ , is (i) increasing in  $\sigma_j$  for  $s < s^c(\alpha)$  or  $\alpha > \alpha^c(s)$  (negative externality); and (ii) decreasing in  $\sigma_j$  for  $s > s^c(\alpha)$  or  $\alpha < \alpha^c(s)$  (positive externality). Further,  $\hat{\sigma}_i(r, \sigma_j)$  is decreasing in  $s$  (for a given  $\alpha$ ) and increasing in  $\alpha$  (for a given  $s$ ).*

Differentiating the maximand in equation (4.11) w.r.t.  $\sigma_i$  at  $X_{Ri}(r, \sigma_i)$  (given by equation 4.12), we get

$$\begin{aligned} & X_{Ri}(r, \sigma_i) \cdot \frac{d}{d\sigma_i} \int_r^{R_{max}} (R - r) h_i(R; \sigma_i) dR \\ & + V_{i1}^{ss} \cdot \frac{d}{d\sigma_i} Pr[R_i > r] \\ & + \left( V_{i1}^{sf} - V_{i1}^{ss} \right) \cdot \frac{d}{d\sigma_i} Pr[R_i > r, R_j < r]. \end{aligned}$$

The effect of  $\sigma_j$  on  $\hat{\sigma}_i(r, \sigma_j)$  depends upon the last term, whose sign depends upon the nature of the externality. From Assumption 3,  $Pr[R_i > r, R_j < r]$  is increasing in  $\sigma_j \forall \sigma_i$ , and decreasing in  $\sigma_i \forall \sigma_j$ . From Proposition 1, when  $s < s^c(\alpha)$  or  $\alpha > \alpha^c(s)$  (negative externality), we obtain  $V_{i1}^{sf} < V_{i1}^{ss}$ . It follows that in this case,  $\hat{\sigma}_i(r, \sigma_j)$  is increasing in  $\sigma_j$ . The result for the positive externality case follows analogously. Finally,  $\hat{\sigma}_i(r, \sigma_j)$  is decreasing in  $s$  (for given  $\alpha$ ) and increasing in  $\alpha$  (for given  $s$ ) since (i)  $V_{i1}^{sf}$  is the only term in the derivative above that depends on  $s$  and  $\alpha$ , and (ii) from Lemma 1,  $V_{i1}^{sf}$  is increasing in  $s$  (for given  $\alpha$ ) and decreasing in  $\alpha$  (for given  $s$ ).  $\square$

**Proposition 2:** We prove both parts of the proposition for the best-responses, next demonstrate the existence of the equilibrium, and then the results carry over to the equilibrium best-responses.

For the best-responses: (i) Part 1. The derivative w.r.t.  $\sigma_i$  in Lemma A.3 can be rewritten as:

$$\begin{aligned} & X_{Ri}(r, \sigma_i) \cdot \frac{d}{d\sigma_i} \int_0^r (r - R) h_i(R; \sigma_i) dR \\ & + V_{i1}^{ss} \cdot \frac{d}{d\sigma_i} Pr[R_i > r, R_j > r] \end{aligned}$$

$$+ V_{i1}^{sf} \cdot \frac{d}{d\sigma_i} Pr[R_i > r, R_j < r].$$

The first term, the preference for higher risk due to truncated first-period payoff, is positive by Assumption 2 whereas the next two terms, the risk-reducing effect of the second period charter-values, are negative by Assumption 3. Thus, at low charter-values  $V_{i1}^{ss}$  and  $V_{i1}^{sf}$ , the first term dominates implying  $\hat{\sigma}_i(r, \sigma_j) \equiv \sigma_{max}$ , whereas at high charter-values, we obtain an interior solution  $\hat{\sigma}_i(r, \sigma_j) \in [0, \sigma_{max})$ . (ii) Part 2 is a direct consequence of Lemma 2.

Next, we demonstrate the existence of the equilibrium. Consider the Nash equilibrium of the two banks for a given level of  $r$ . Both banks are price-takers. Thus, their best-responses are obtained as  $\Omega_i(r, \Omega_j) = [\hat{\sigma}_i(r, \sigma_j), \hat{X}_{Ri}(r, \sigma_j), \rho_i]$ . From Lemma 2, we have  $\rho_i = \rho_h$  for  $s < s^c(\alpha)$  and  $\rho_i = \rho_l$  for  $s > s^c(\alpha)$ . Further, Lemmas A.2–A.3 imply that  $\forall s, \alpha$ ,  $\hat{X}_{Ri}(r, \sigma_j)$  and  $\hat{\sigma}_i(r, \sigma_j)$  are monotone and continuous in  $\sigma_j$ .

It follows from Brouwer’s Fixed Point Theorem that a Nash equilibrium given by the investment choices,  $\Omega_i^*(r)$  and  $\Omega_j^*(r)$ , exists  $\forall r$ , with  $\rho_i^* = \rho_j^*$ . In equilibrium, market-clearing for the safe asset determines its return  $r = f'[\sum_i D_{i0} - \sum_i X_{Ri}^*(r)]$ . Thus, we define a map  $\hat{r}(r) = f'[\sum_i D_{i0} - \sum_i X_{Ri}^*(r)]$ . Under an additional technical condition that guarantees interior solution for  $X_{Ri}^*$ , the fixed-point  $r^*$  exists by Brouwer’s Fixed Point Theorem.  $\square$

**Lemma 3:** The proof requires several steps: Let

(i)  $\bar{W}_1(r, s, \alpha) \equiv 2 \cdot [ (\bar{R} - r)\hat{X}_R(r) - \alpha \cdot c(\hat{X}_R(r)) + (1 + r)D_1 - \int_0^r \delta(r, R)\hat{X}_R(r) h(R; \sigma_{min})dR + (r - 1)sD_1 ]$ , where  $\hat{X}_R(r) = X_R(r, \sigma_{min})$ , given by equation (4.14) for a given value of  $\alpha$ . Then, some algebra reveals that  $W_1^{ss} + W_1^{ff} = \bar{W}_1(r_1^{ss}, 0, 1)$ , and  $W_1^{sf} + W_1^{fs} = \bar{W}_1(r_1^{sf}, s, \alpha)$ ,  $s \in [0, 1)$ ,  $\alpha \in [\alpha_{min}, 1]$ . Note that we have assumed symmetry of the states “sf” and “fs.”

(ii) For  $D_1$  sufficiently high,  $\frac{d}{dr}\bar{W}_1(r, s, \alpha) > 0$ . Differentiating (i) w.r.t.  $r$  yields

$$\frac{d}{dr}\bar{W}_1(r, s, \alpha) = 2 \cdot [ (1 + s)D_1 - \hat{X}_R(r)(1 + \frac{d}{dr} \int_0^r \delta(r, R) h(R; \sigma_{min})dR) ],$$

which is greater than zero  $\forall s$  if  $\hat{X}_R(r) \in (0, D_1)$ , as is assumed throughout the paper.

(iii) Let  $s = 0$ . In this case,  $r_1^{sf} = f'[D_1 - \hat{X}_R(r_1^{sf})]$  (for given  $\alpha$ ) and  $r_1^{ss} = f'[2(D_1 - \hat{X}_R(r_1^{ss}))]$  (with  $\alpha = 1$ ). We have  $r_1^{ss} < r_1^{sf}$  (as in the proof of Proposition 1). It follows that  $\bar{W}_1(r_1^{ss}, 0, 1) < \bar{W}_1(r_1^{sf}, 0, \alpha)$  from (ii) and the fact that  $\bar{W}_1(r, s, \alpha)$  is decreasing in  $\alpha$  (can be verified easily).

(iv) In the extreme, let  $s = 1$ . In this case,  $r_1^{sf} = f'[2D_1 - \hat{X}_R(r_1^{sf})]$  (for given  $\alpha$ ) so that  $r_1^{sf} < r_1^{ss}$ . Thus, it is important now to consider the overall effect of  $s$  on  $\bar{W}_1(\cdot)$ . We can write

$$\bar{W}_1(r_1^{ss}, 0, 1) - \bar{W}_1(r_1^{sf}, s, \alpha) = [ \bar{W}_1(r_1^{ss}, 0, 1) - \bar{W}_1(r_1^{sf}, 0, \alpha) ] + [ \bar{W}_1(r_1^{sf}, 0, \alpha) - \bar{W}_1(r_1^{sf}, s, \alpha) ].$$

The second term above equals  $-2 \cdot (1 + r_1^{sf}) \cdot sD_1 < 0$ . This captures the loss to the depositors in the joint failure state. The first term is negative whenever  $r_1^{sf} > r_1^{ss}$ , the case of negative externality. However, it need not be negative when  $s$  is large as it implies  $r_1^{sf} < r_1^{ss}$ . However, as  $D_1$  increases, the difference between  $r_1^{sf}$  and  $r_1^{ss}$  becomes smaller (since  $f(\cdot)$  is concave) so that the first term becomes less positive and the second term becomes more negative. Thus, for the case of negative externality or more generally when  $D_1$  is sufficiently high, we can assure that  $\forall s$ ,  $\bar{W}_1(r_1^{ss}, 0, 1) < \bar{W}_1(r_1^{sf}, s, \alpha)$ , i.e.,  $W_1^{ss} + W_1^{ff} < W_1^{sf} + W_1^{fs}$ , as required. The requirement that  $D_1$  be sufficiently high becomes



weaker as  $\alpha$  decreases since the first term above decreases in  $\alpha$ .  $\square$

**Proposition 3:** First, we rewrite the expected continuation value of the economy as:

$$W_1^{ss} + (W_1^{sf} - W_1^{ss}) \cdot Pr[R_i > r, R_j < r] + (W_1^{fs} - W_1^{ss}) \cdot Pr[R_i < r, R_j > r] + (W_1^{ff} - W_1^{ss}) \cdot Pr[R_i < r, R_j < r],$$

in turn rewritten as:

$$W_1^{ss} + (W_1^{sf} - W_1^{ss}) \cdot Pr[R_j < r] + (W_1^{fs} - W_1^{ss}) \cdot Pr[R_i < r] + (W_1^{ff} + W_1^{ss} - W_1^{sf} - W_1^{fs}) \cdot Pr[R_i < r, R_j < r].$$

From Lemma 3, we have  $W_1^{sf} - W_1^{ss} < 0$ ,  $W_1^{fs} - W_1^{ss} < 0$ , and also  $W_1^{ff} + W_1^{ss} - W_1^{sf} - W_1^{fs} < 0$ .

Consider the maximization problem of the aligned banks (equation 4.16). Increasing  $\sigma_i$  and  $\sigma_j$  given the best-response  $X_{Ri}(r, \sigma_i)$  (equation 4.14) increases (i) the costs of distress through  $\delta(r, R)$  term (Assumption 2), and (ii) the losses in continuation welfare of the economy through  $Pr[R_i < r]$ ,  $Pr[R_j < r]$ , and  $Pr[R_i < r, R_j < r]$  terms (Assumptions 2–3). Thus, the best-response of aligned bank is  $\hat{\sigma}_i(r, \sigma_j) = \sigma_{min}$ ,  $\forall r, \sigma_j, \forall i$ . Further, the correlation  $\rho$  affects only the joint failure term in the representation above which is negative. It follows now from Assumption 4 that  $\rho = \rho_l$ .

Next, we show that the equilibrium exists. Denote  $\hat{X}_{Ri}(r) = X_{Ri}(r, \hat{\sigma}_i(r, \sigma_j)) = X_{Ri}(r, \sigma_{min})$ , with  $X_{Ri}(r, \sigma_i)$  as in equation (4.14). Market clearing for the safe asset in equilibrium requires  $r = f'[\sum_i D_{i0} - \sum_i X_{Ri}(r, \sigma_{min})]$ . The fixed point,  $r^o$ , exists as in the proof of Proposition 2, yielding the equilibrium  $[r^o, \sigma^o = (\sigma_{min}, \sigma_{min}), \mathbf{X}_R^o = (\hat{X}_{RA}(r^o), \hat{X}_{RB}(r^o)), \rho^o = \rho_l]$ .  $\square$

**Corollary 1:** Compare the first order conditions in the intermediated and the aligned case, equations (4.12) and (4.14), respectively. Since  $c'(\cdot) > 0$ , we have  $X_{Ri}^*(r, \sigma_i) > X_{Ri}^o(r, \sigma_i)$ , where we have used the superscripts  $*$  and  $o$  for the intermediated and the aligned case, respectively. Since  $\sigma_i^o = \sigma_{min}$ , and  $\sigma_i^* > \sigma_{min}$ , it follows that  $\hat{X}_{Ri}^o(r) = X_{Ri}^o(r, \sigma_i^o) < X_{Ri}^*(r, \sigma_i^o) < X_{Ri}^*(r, \sigma_i^*) = \hat{X}_{Ri}^*(r)$ , where the last inequality follows from Lemma A.2. At equilibrium,  $r^* = f'[\sum_i D_{i0} - \sum_i \hat{X}_{Ri}^*(r^*)]$ , and  $r^o = f'[\sum_i D_{i0} - \sum_i \hat{X}_{Ri}^o(r^o)]$ .

Suppose that  $r^o > r^*$ . Then,  $r^o < f'[\sum_i D_{i0} - \sum_i \hat{X}_{Ri}^o(r^o)] < f'[\sum_i D_{i0} - \sum_i \hat{X}_{Ri}^o(r^*)] < f'[\sum_i D_{i0} - \sum_i \hat{X}_{Ri}^*(r^*)] = r^*$ , a contradiction. The last inequality follows from the observation that  $\hat{X}_{Ri}^*(r)$  (and also  $\hat{X}_{Ri}^o(r)$ ) is decreasing in  $r$ , as shown in the proof of Lemma 1. Hence, we must have  $r^o < r^*$ , which in turn implies  $X_{Ri}^o < X_{Ri}^*$  in equilibrium.  $\square$

**Proposition 7:** The banks prefer low correlation of asset returns to high *iff* their value net of the dilution cost of capital,  $V_i^{old}(\cdot)$ , is decreasing in  $\rho$ . In other words, we need  $\frac{d}{d\rho} V_i^{old}(\cdot) < 0$ . Differentiating equation (6.1) w.r.t.  $\rho$  and using the chain-rule, this is equivalent to requiring

$$- \theta'_i(K_i) \cdot \frac{dK_i}{d\rho} + (V_{i1}^{sf} - V_{i1}^{ss}) \cdot \left\{ \frac{d}{d\rho} Pr[R_i > R_i^c, R_j < R_j^c] + \frac{d}{dR_i^c} Pr[R_i > R_i^c, R_j < R_j^c] \cdot \frac{dR_i^c}{dK_i} \cdot \frac{dK_i}{d\rho} \right\}$$

be  $< 0$ . Note that  $R_i^c = r \cdot (1 - \frac{K_i}{X_{Ri}})$ , so that  $\frac{dR_i^c}{dK_i} = -\frac{1}{X_{Ri}} \cdot r$ . The equation above can be rearranged to obtain the necessary and the sufficient condition as stated in Proposition 7.  $\square$

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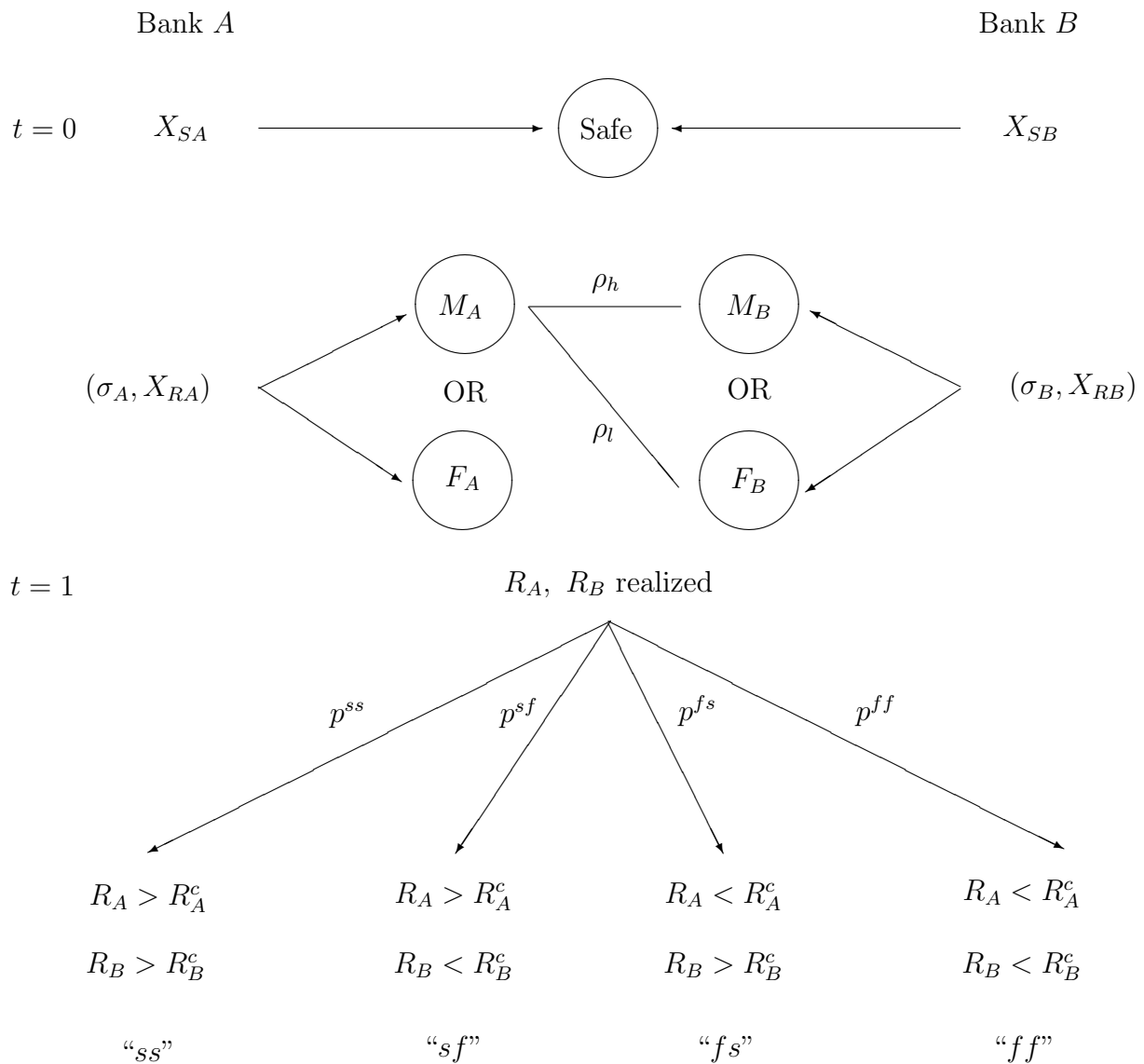
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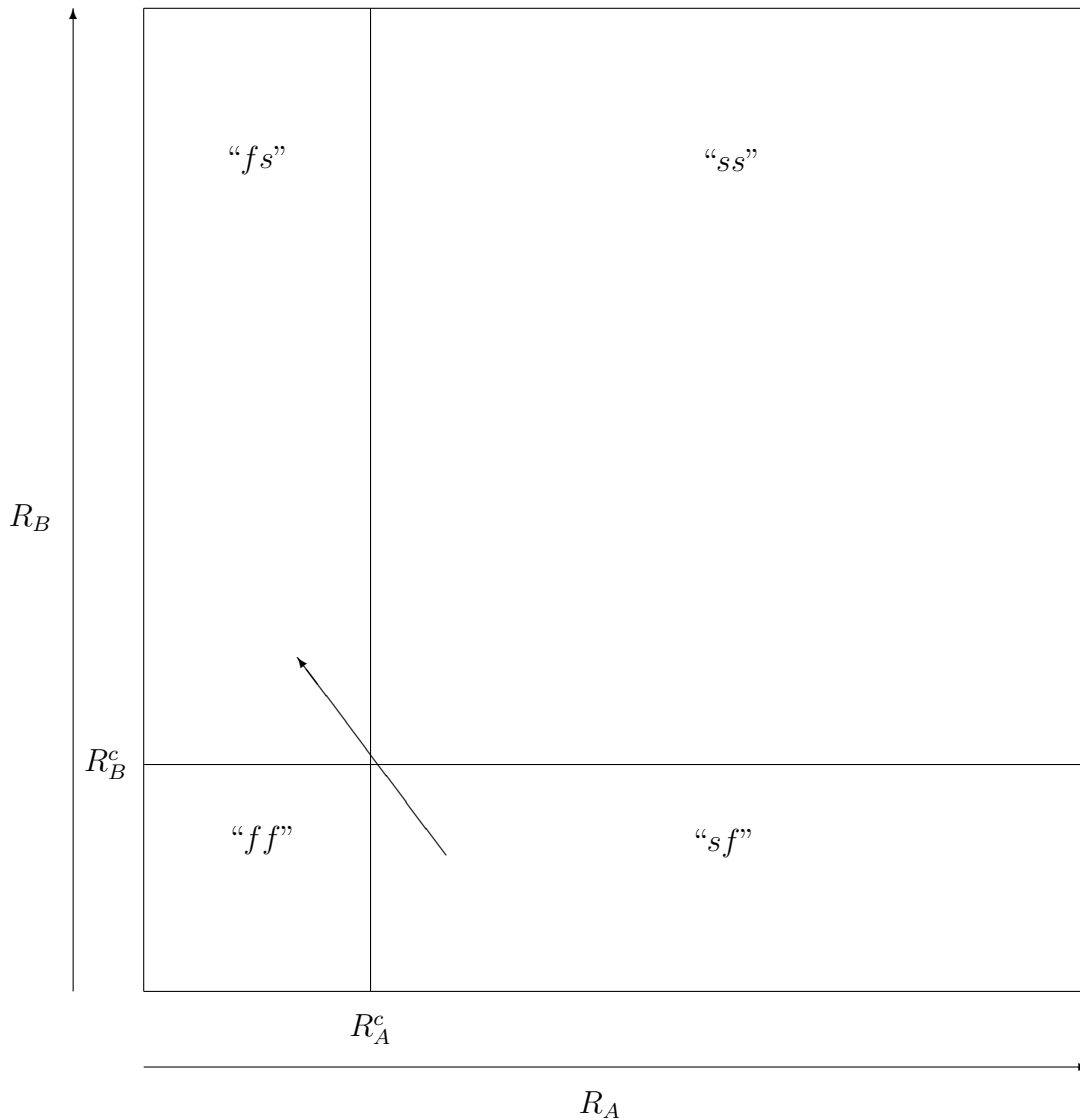
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Figure 1: Time-line and Investment Choices of Banks



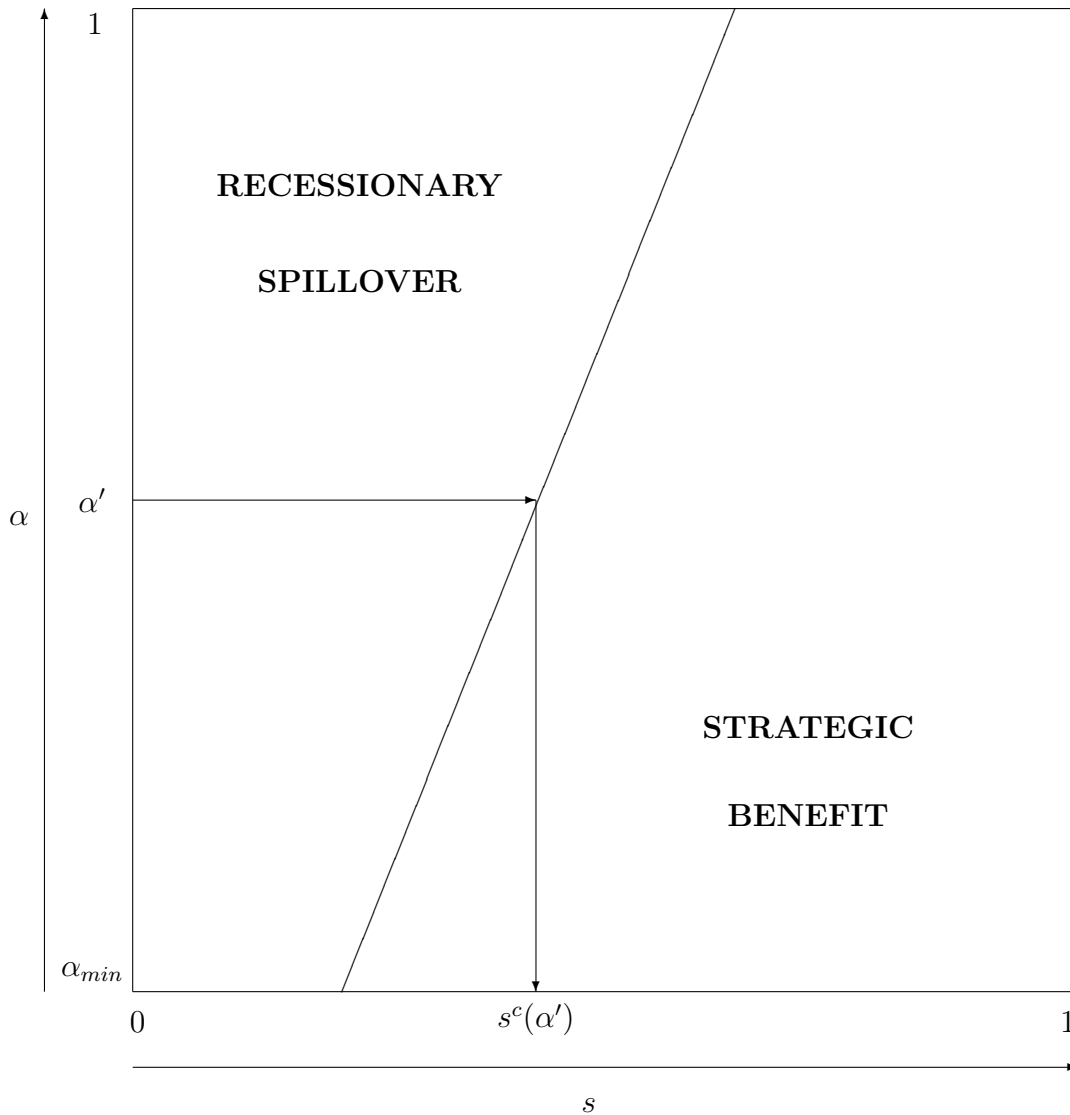
Note that the deposit market is not included in the diagram. Each bank borrows deposits from its sector in a competitive deposit market. The size of the deposit pool in the two sectors is  $D_A$  and  $D_B$ , respectively.

Figure 2: States of the Economy at  $t = 1$



This figure illustrates the four kinds of states possible at  $t = 1$ , viz., "ss", "sf", "fs", and "ff".  $R_i$  denotes the realized return on risky investments of bank  $i$ ,  $i \in \{A, B\}$ .  $R_i^c$  is the threshold level of return on risky investments of bank  $i$  below which it 'defaults'. An increase in correlation of risky assets across banks increases the likelihood of the off-diagonal states, "ss" and "ff", and decreases the likelihood of the diagonal states, "sf" and "fs".

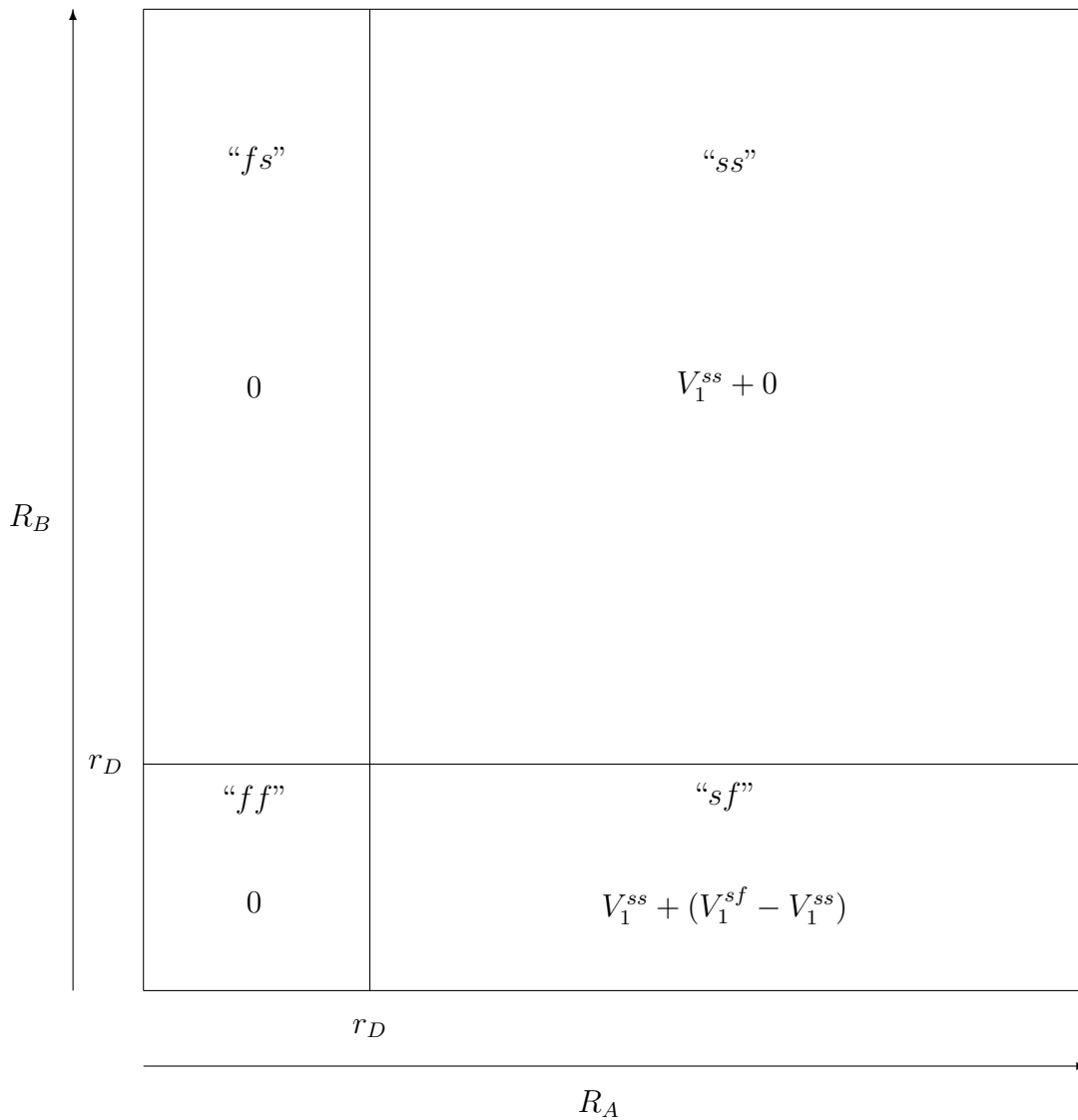
Figure 3: Recessional Spillover, Strategic Benefit, and the Nature of Externality



This figure illustrates the nature of the externality of one bank’s failure on the health of the other bank in various regions of  $(s, \alpha)$  space. Note that  $s$  is the fraction of depositors that migrate from the failed bank to the surviving bank; and  $\alpha$  is the proportional factor by which the costs of the surviving bank decrease upon other bank’s failure. When  $s$  is small and  $\alpha$  is large, the recessionary spillover dominates the strategic benefit. This happens in the north-west region of the space which is thus characterized by negative externality. On the other hand, when  $s$  is large and  $\alpha$  is small, the strategic benefit dominates the recessionary spillover. This happens in the south-east region of the space which is thus characterized by positive externality. For a given  $\alpha'$ , the critical level  $s^c(\alpha')$  is such that there is negative externality for all  $s < s^c(\alpha')$ .



Figure 4: Nature of Externality and Choice of Correlation



This figure illustrates the interaction of (i) the limited liability of banks, and (ii) the nature of externality of failure of one bank on the health of the other bank, and how this interaction determines their preference to undertake risky investments with a high correlation of returns. For bank *A*, the externality from the failure of bank *B* is given by the term  $V_{A1}^{sf} - V_{A1}^{ss}$ . When  $V_{A1}^{sf} < V_{A1}^{ss}$ , the externality is negative, and bank *A* prefers state “*ss*” over state “*sf*”, in turn implying that it prefers a high correlation of returns with bank *B*. Similarly, when  $V_{A1}^{sf} > V_{A1}^{ss}$ , the externality is positive, and bank *A* prefers state “*sf*” over state “*ss*”, in turn implying that it prefers a low correlation of returns with bank *B*.