Risk-Sharing and the Creation of Systemic Risk

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Abstract

We address the paradox that financial innovations aimed at risk-sharing appear to have made the world riskier. Financial innovations facilitate hedging idiosyncratic risks among agents; however, aggregate risks can be hedged only with liquid assets. When risk-sharing is primitive, agents self-hedge and hold more liquid assets; this buffers aggregate risks, resulting in few correlated failures. As risk-sharing improves, hedging low-probability aggregate risks with liquid assets becomes costly to agents; aggregate liquidity falls; and, the system becomes more exposed to aggregate risks, resulting in more correlated failures. This insight provides a rationale for minimum margin requirements for derivatives and clearing houses.
1 Introduction

Over much of the last several decades, instruments and contracts aimed at facilitating risk-sharing between financial agents have seen explosive growth. By and large (although not uniformly) over this period, regulators viewed this as a benign and even beneficial development, a point of view perhaps best exemplified by Federal Reserve Board Chairman Alan Greenspan’s remarks at the American Bankers Association Annual Convention in October 2004.\(^1\) Referring to the unbundling and transfers of risks facilitated by derivatives, Greenspan suggested that, as a consequence

\[\text{...not only have individual financial institutions become less vulnerable to shocks from underlying risk factors, but also the financial system as a whole has become more resilient.}\]

The advent of the financial crisis in 2007-08 led to a sharp reappraisal of these views. Today, it is widely acknowledged that the very markets and contracts designed with the ostensible purpose of facilitating risk-sharing among financial intermediaries—including derivatives such as credit default swaps and securitization instruments such as collateralized debt obligations—played a central role in fanning the crisis, nearly bringing about a collapse of the entire financial system.

How could the facilitation of risk-sharing increase systemic risk and systemic fragility? In this paper, we suggest an explanation of this apparently paradoxical situation. The intuition behind our formal analysis is rather simply expressed. We begin with the observation that while risk-sharing arrangements are effective at hedging against idiosyncratic shocks (such shocks are by definition uncorrelated across agents), they have, at best, limited potential to provide a hedge against aggregate shocks. When an aggregate shock hits the economy, everyone is affected by the common shock and any existing risk-sharing arrangements face increased likelihood of counterparty default. The only effective ex-ante defense against systemic shocks is to, so to speak, save for the rainy day, viz., to increase holdings of safe, liquid assets (“cash”) and reduce investment in risky and illiquid assets.

Now consider an agent who faces shocks that could be idiosyncratic or aggregate. Aggregate shocks are those that hit all agents and firms; they may be far less likely (they may be “tail risks”) than the idiosyncratic risk exposures, and as such, the agent may find it privately costly to hold liquidity purely for withstanding such aggregate shocks. But when risk-sharing opportunities are absent or scarce, the more-likely idiosyncratic shocks also need to be hedged, providing the agent with an incentive to hold liquid assets to withstand the shocks and making it privately optimal to save to hedge the risk exposures. As a consequence, in the (perhaps low-likelihood) event that an aggregate risk materializes, the system has adequate liquidity to deal with it, so systemic crises are averted or unlikely.

In contrast, when financial innovations allow agents and firms to share idiosyncratic risks with each other, the hedging motive for holding liquidity to withstand these shocks is reduced. Risk-sharing allows the financial system to economize on its pool of liquidity. Now, the principal benefit of holding liquidity arises in case of aggregate shocks, but if these are low-likelihood events, it will not generally be in the interest of individual agents or firms to hedge against these states. As a consequence, while idiosyncratic risks are shared and risk-sharing promises are honored when idiosyncratic shocks arise, there is inadequate liquidity and insufficient savings for the rainy day when an aggregate shock hits, making it difficult or impossible to honor risk-sharing promises. The result is financial fragility or systemic risk as an equilibrium outcome.

In short, financial innovations facilitating risk-sharing may increase systemic vulnerability by inducing agents to invest resources more fully in high-return risky projects by reducing liquidity for dealing with low-risk aggregate shocks. To be sure, the increased systemic fragility could be very costly for the economy when an aggregate shock arises since a meltdown of the financial sector could produce an elongated period of financial disruption affecting payment and settlement systems, transactional services, and savings schemes. This suggests that while prohibiting all instruments of risk-sharing may (evidently) be undesirable, unbridled financial innovation without coincident investments in ensuring resilience of the system to aggregate shocks may also be undesirable from a societal standpoint.
We deliver this insight in a series of models. Section 3 introduces the foundation on which we build, the problem of an individual firm seeking to hedge against total risk exposure in the absence of risk-sharing possibilities. (We call this no-risk-sharing setting one of “autarky.”) We show that if the risk exposure is sufficiently large, the autarkic firm finds it privately optimal to fully hedge against the exposure by holding adequate liquidity. In Sections 4-6, we embed this one-firm setting in different multi-firm contexts (with each firm’s risk exposure consequently broken into idiosyncratic and systematic parts), and study the consequences of permitting risk-sharing among the firms.

Section 4 looks at the simplest and most transparent of these extensions, that of a two-firm setting in which the firms can write a risk-sharing contract to make transfers to each other in idiosyncratic risk states. When risk-sharing is disallowed, each firm is, of course, in the autarky setting of Section 3, and for the reasons explained in the previous paragraph, there is adequate liquidity in the system to handle the aggregate risk state. But when risk-sharing is allowed, we show that the firms optimally choose to economize on liquidity and increase their investment in risky assets; and, as a consequence, when the (possibly low-likelihood) aggregate risk state does materialize, the system-wide liquidity proves inadequate to meet the shock, and both firms fail.

Sections 5 and 6 then generalize the setting to a setting with a large number of firms and a richer description of uncertainty, and analyze two risk-sharing arrangements of interest. Section 5 looks at the setting where risk-sharing takes the form of insurance against shocks that may be purchased from a third-party insurance firm (think of AIG Financial Products) operating in a competitive insurance sector. Section 6 looks at risk-sharing via a co-insurance arrangement akin to a clearinghouse. Under insurance, the insurance firm collects premia up-front to build its reserves that are then used to honor insurance claims from individual firms that are hit by shocks; the insurance firm is insolvent if claims exceed the reserves so accumulated. In the clearinghouse-style setting too, the clearinghouse collects premia up-front to build its reserves but it can further also make ex-post “capital calls” on liquidity-surplus banks to aid (co-insure) the liquidity-short firms. Clearinghouse insolvency and systemic failure occur if the reserves and capital calls are collectively insufficient to meet the needs of liquidity-short firms.
We show that in either case our general insight prevails: risk-sharing arrangements can incentivize firms to take on greater risk at the individual level, reduce overall liquidity, and increase systemic fragility. The level of correlation in firm risks plays a key role. When the correlation between underlying risks of firms is low, risk-sharing arrangements are enabled that do not require high upfront premia, and both the insurance and clearinghouse settings provide risk-sharing of good quality in most states of the world, offering a clear benefit over a world without risk-sharing. But when a large aggregate shock materializes, these arrangements fail, creating greater systemic fragility than in a world without risk-sharing. Conversely, when underlying risks of firms are highly correlated, upfront premia required by insurance and clearinghouse rise correspondingly, reducing the attractiveness of risk-sharing. At sufficiently high correlations, premia become so high that autarky is preferable and aggregate outcomes mirror the no risk-sharing setting. In either case, when collective failures result in negative externalities and large societal costs (accompanying the breakdown of a payments system), regulation in the form of minimum reserve requirements or minimum margin requirements may improve matters.

These issues have come to the fore as the Dodd-Frank Act in the United States and similar reforms of the financial sector world-wide are moving standardized (or “standardizable”) derivative products to centrally cleared arrangements and mandating a minimum level of risk-management standards at clearinghouses. For products that remain over the counter, minimum margin requirements are being designed. To provide normative guidance for considerations that should drive the minimum margin requirements on derivative contracts and clearinghouse arrangements, we analyze a regulatory design problem for setting these margin requirements when the regulator has imperfect information about how correlated the risks are (we consider risk-sharing via a clearinghouse setting; the results are similar for the case of third-party insurance). The optimal clearinghouse margin level trades off the benefit of limiting systemic risk when the underlying correlation turns out to be high against the cost of limiting investments excessively when it turns out to be low; we illustrate the precise nature of this trade off via a detailed numerical example. Importantly, regulation in the form of minimum margin requirements—even under incomplete information—can lower the probability of systemic failure compared to the private setting.
These models help illustrate our main point that aggregate risk translates into systemic risk or collective failures depending on the liquidity choices of agents which are endogenous to the available risk-sharing opportunities.

The ideas in this paper may have general applicability beyond the immediate concerns analyzed here. Consider any setting in which much of the activity is “routine” but there is an occasional need to face a complex unexpected scenario. (Most organizations can, in fact, be described in this fashion as largely requiring a set of routine activities which in principle may be implemented following protocol or “rules,” but occasionally an unexpected—and consequential—query arises that cannot be addressed within the rules. In our model, the routine is the management of idiosyncratic risk, the dramatic is that of systemic risk.) With primitive technology, the same resources are used to manage both sets of activities, the routine and the rare, so, as a consequence, there are generally adequate resources available to address crises when they arise. But as technology improves facilitating specialized handling of activities, the management of the routine can be separated from the management of the unexpected; the consequent specialization reduces resources available to address the rare hit (it may no longer be economically viable to maintain resources just to address the rare complex event), and so makes the organization more vulnerable to derailment from tail events.

Section 2 discusses the related literature. Sections 3-6 contain our analysis. Section 7 examines social costs of bank failures and the case for regulatory action. Proofs not contained in the main body of the paper may be found in the appendices.

2 Related Literature

As noted in the Introduction, prior to the financial crisis, the view that risk-sharing and derivatives enhanced systemic stability was widely held, but there were important notes of dissent. Speaking in Jackson Hole, Wyoming, in August 2005, Raghuram Rajan, then Chief Economist of the IMF, commented that
While the system now exploits the risk-bearing capacity of the economy better by allocating risks more widely, it also takes on more risks than before. ... [T]he linkages between markets, and between markets and institutions, are more pronounced. While this helps diversify the system against small shocks, it also exposes the system to large systemic shocks.\(^2\)

Rajan’s (qualitative) case for why financial innovations may have made the world riskier focuses on tail-risk seeking that is driven by short-termism of financial sector pay and incentives. Our paper offers a formalization that is complementary and leads to similar conclusions that the growth of markets for risk-sharing could result in greater risk-taking and make the system more vulnerable to “large systemic” shocks.

Acharya, Cooley, Richardson and Walter (2010) document in detail the process of “manufacturing tail risks” that took hold during 2003-07 and the reasons behind it, in particular, excessive seeking of aggregate risk due to the presence of government guarantees, imperfect regulation and its arbitrage, and inadequate internalization by the financial sector of externalities from collective failures. Our paper offers a viewpoint that is complementary to these. It suggests that improvements in financial risk-sharing technology and innovations, and the resulting moral-hazard effect on private liquidity choices, may be the subtle underlying force behind the growing inadequacy of private reserves to withstand aggregate shocks.\(^3\)

Several papers consider the effect of risk-sharing on risk-taking. In early work, Bhattacharyya and Gale (1987) consider how inter-bank contracts to share liquidity can lead to free-riding on the common pool of liquidity as individual banks, privately informed about own liquidity shocks, aim to transfer risks to counterparties. Acemoglu and Zilibotti (1997) provide a model in which risk-averse agents when offered diversification opportunities are willing to take on risks that were privately too risky to undertake otherwise. Allen and Carletti (2006) model how ability to transfer risks from banking to the insurance sector can create risk of contagion from one sector to the other.

\(^2\)See Cassidy (2009), Chapter 1, p.21.

\(^3\)This, in turn, may also explain the explosive growth of the financial sector over the past two decades (Philippon and Reshef, 2012) relative to other sectors of the economy; the growth occurred at the cost of grave risk of future crises whose costs were partly borne by sectors other than the financial sector.
Acharya and Bisin (2009) examine how managerial incentives to take on aggregate or idiosyncratic risks are altered by the ability to privately hedge these risks by trading in capital markets. Zawadowski (2009) considers a market for insurance in an “entangled” financial network and shows how the presence of network externalities can lead in equilibrium to inefficiently little insurance purchase against low-risk events. Yorulmazer (2013) studies how access to insurance against default risk can lead the insured and the insurer to “herd” as they collectively prefer correlated underlying risks to transfer risks outside of the financial sector, e.g., on to the taxpayer.

Like these papers, our paper too links risk-sharing to risk-taking, but our primary insight is somewhat different—that the ability to share risks alters the cost-benefit tradeoff in holding liquidity to hedge against aggregate risk states and this alteration can endogenously transform such states into collective failure or systemic risk states.

Also related to our paper is the recent literature on clearinghouses. Duffie and Zhu (2009) study whether risks will be better pooled by single versus multiple clearinghouses. Leitner (2013) and Acharya and Bisin (2014) examine the extent to which clearinghouses can resolve incomplete information about agents’ positions. Pirrong (2009) argues that costs relating to information asymmetry are higher with a clearinghouse and may potentially outweigh the benefits associated with the mutualization of risks. Menkveld (2015) identifies the systemic risk arising from crowded trades by clearinghouse members and proposes a margin methodology to account for it. Tucker (2014) is a discussion of the moral hazard consequences of clearinghouse arrangements that resonates with both our positive analysis of these arrangements (the creation of systemic risk relative to autarky) as well as the normative analysis (the need for minimum margin requirements).

Koeppl, Monnet and Temzelides (2012) consider the tradeoff faced by a clearinghouse in providing liquidity to members, while ensuring incentives are in place for carrying out and settling transactions. Biais, Heider and Hoerova (2011) examine if centralized clearing makes trading parties better off and whether it eliminates counterparty risk of its members. In their model, risk-averse buyers of protection trade with risk-neutral sellers who may default. To reduce this default risk, buyers have an incentive to expend effort and trade with a “good” protection seller. They show that if risks are idiosyncratic, the clearinghouse is able to fully insure agents against possible
future losses; but if there is aggregate risk, agents expend effort to search for good counterparties and are fully insured only in the case when effort is observable and contractible. These findings are broadly in line with our own, but our model is evidently different in that focuses on the assumption of risk undertaken by every bank (and hence the aggregate) when risk sharing is and is not possible. We show that risk sharing endogenously incentivizes the agents in our model to take on more risk leading to an increased likelihood of systemic failure.

Finally, Rochet and Roger (2014) build a general contracting theory of “risky utilities” - firms that provide public benefits while still at the risk of default, such as clearinghouses, large banks and infrastructure companies. They show that the optimal regulatory contract is implemented with a capital requirement, which, if breached, results in restructuring and expropriation. In contrast, we show that a simple margin requirement can serve to increase welfare when clearinghouse members do not internalize the cost of systemic risk, and that this regulation is at its most effective when the correlation between the shocks to member banks is high.

3 The One-Bank Setting

In this section, we introduce the foundational one-bank model that we embed in multi-bank settings in Sections 4–6. We consider a model with three dates \( t \in \{0, 1, 2\} \). At date 0, a bank has initial investable capital of $1. Banks have access to two investment opportunities. The first is a safe asset (“cash”). The safe asset has a net return of zero; an investment of \( \ell \) at time \( t \) returns \( \ell \) at time \( t + 1 \).

The second investment opportunity is in a risky asset or risky project. Figure 1 summarizes the consequences of investing in this opportunity. Investment in this risky project at time 0 yields returns at time 2. The risk in the project comes from the potential need to refinance the investment at time 1. There is a probability \( \alpha \) that such refinancing will be required. Refinancing is a zero-one decision; partial refinancing of the project is not possible. The refinancing amount required is \( \phi \) per $1 that was invested in the project. For specificity, we take \( \phi = 1 \); i.e., if refinancing is required, the refinancing need is 100% of the initial invested amount in the risky project.
Figure 1: The Risky Investment Opportunity

This figure summarizes the salient features of the risky investment opportunity described in Section 3.

If the project does not require refinancing at time 1, or if it requires refinancing and is refinanced, then the expected return at time 2 is $R > 1$ per unit investment in the project at time 0. If refinancing is needed and the project is not refinanced, the investment returns zero with certainty at time 2.

Since $R > 1$, it is always worth refinancing the project at time 1 should such refinancing be required (and should it be feasible). We make two assumptions concerning the refinancing. First, we assume that the refinancing need is observable and verifiable. This assumption plays no role in the current section, but is used in a later section when we consider allowing the bank to use a contingent claim to guard against this state. Second, we assume that the cash flows from the risky project are not pledgeable, so the bank cannot raise outside financing against these cash flows. For the time being (until we introduce risk-sharing possibilities), this means the project may only be refinanced using the bank’s own internal resources, which in turn means it can only be refinanced using the amount the bank has invested in the riskless asset.
The Bank’s Optimal Investment Decision

The tradeoff driving the bank’s investment decision is a simple one: the riskless asset’s return is lower than that from the risky project, but investing (a suitable amount) in the riskless asset provides the bank with the ability to refinance the project should refinancing be required—which happens with probability \( \alpha \). We show that if \( \alpha \) is small, then it is not worthwhile for the bank to divert investment away from the risky project; there is no investment in the riskless asset and the risky project fails with probability \( \alpha \). As \( \alpha \) increases past a critical level, then refinancing becomes optimal; the bank fully self-insures and invests enough in the riskless asset to meet the refinancing need (should it arise), and the risky project never fails. Finally, at very high levels of \( \alpha \), investment in the risky project becomes unprofitable and all investment is made in the riskless asset.

Specifically, let \((\ell, 1 - \ell)\) denote the investment strategy in which the bank chooses to invest \( \ell \) in the riskless asset and \( 1 - \ell \) in the risky technology. We prove the following result:

**Proposition 3.1** Let \( \lambda(R) \) and \( \nu(R) \) be defined by

\[
\lambda(R) = \frac{R - 1}{2R - 1}; \quad \nu(R) = R - 1.
\]

Then:

1. If \( \alpha \in [0, \lambda(R)) \), then the optimal action is \( \ell^* = 0 \). Since there is no investment in the riskless asset, the risky project is never refinanced so fails with probability \( \alpha \). The bank’s expected payoff is \( (1 - \alpha)R \).

2. If \( \alpha \in [\lambda(R), \nu(R)] \), the optimal action is \( \ell^* = 1/2 \). There is always exactly enough to refinance the risky project if this is required. The risky project is always refinanced, so never fails. The bank’s expected payoff is

\[
\frac{1}{2} [R + 1 - \alpha]. \tag{1}
\]
3. If $\alpha > \nu(R)$, the optimal action is $\ell^* = 1$. There is no investment in the risky project and the bank’s expected payoff is 1.

Remark Note that Case 3 is irrelevant when $R \geq 2$ since $\alpha \leq \nu(R)$ is always satisfied in this case.

Proof See Appendix A.

In the sections following we extend this model in several directions. Section 4 considers a basic extension to a two-bank setting and looks at the consequences of permitting risk sharing in this set-up. Sections 5 and 6 develop a many-firms model and look at the consequences of permitting risk-sharing via a third-party insurance-style arrangement and a clearinghouse structure, respectively.

4 Two-Banks: Risk-Sharing and Increasing Systemic Risk

Consider a two-bank setting in which each bank faces exactly the same investment opportunities as in the previous section with exactly the same payoffs, but in which the refinancing needs may be correlated. Thus, there are four states of the world that are possible:

<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
<th>Risky Projects Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_1$</td>
<td>Neither project requires refinancing</td>
</tr>
<tr>
<td>2</td>
<td>$p_2$</td>
<td>Only Bank 2 needs refinancing</td>
</tr>
<tr>
<td>3</td>
<td>$p_3$</td>
<td>Only Bank 1 needs refinancing</td>
</tr>
<tr>
<td>4</td>
<td>$p_4$</td>
<td>Both banks need refinancing</td>
</tr>
</tbody>
</table>

We refer to States 2 and 3 as the “idiosyncratic” risk states and to State 4 as the one of “aggregate” or “systemic” risk. The total probability of Bank 1 needing refinancing ($p_3 + p_4$) and the total probability of Bank 2 needing refinancing ($p_2 + p_4$) are each taken to equal $\alpha$, where $\alpha$ is the probability
in the one-bank setting of a refinancing need. For an arbitrary correlation $\rho$ in refinancing needs, the probabilities $p_i$ take on the form

\begin{align*}
    p_1 &= \rho(1 - \alpha) + (1 - \rho)(1 - \alpha)^2 \\
    p_2 &= (1 - \rho)\alpha(1 - \alpha) \\
    p_3 &= (1 - \rho)\alpha(1 - \alpha) \\
    p_4 &= \rho\alpha(1 - \alpha) + \alpha^2
\end{align*}

Note that the banks are ex-ante identical. We focus on the case $\alpha \in [\lambda(R), \nu(R)]$. As noted in Proposition 3.1, the bank under autarky then invests in the risky project but fully (self-)insures this investment so there is no failure.

We allow the banks to “risk-share” by trading in contingent claims that pay off in specified states of the world. When contingent claim trading is disallowed, each bank faces precisely the one-bank model of Section 3 the solution to which is identified in Proposition 3.1; we refer to this as the model of “autarky.” Of course, even if contingent claim trading is allowed, each bank has the option to not participate in the contingent claim market; it will elect to participate if and only if participation raises its expected payoff beyond that in the autarkic solution.

Risk-sharing affords each bank the possibility of lowering its investment in the safe asset while increasing its investment in the risky asset and relying instead on additional funds from the contingent claims should refinancing of its risky project be required. There is a chance, of course, that the refinancing shock may be “systemic,” i.e., both banks may require refinancing and there may not be enough cash to meet the combined refinancing needs. Thus risk-sharing involves a trade-off between the larger returns obtained from investing in the risky project and its failure in the systemic state.

Observe that risk-sharing benefits accrue in the idiosyncratic-risk States 2 and 3 but not in the systematic-risk State 4. The likelihood of the idiosyncratic-risk states decreases, and that of

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Note that if there was enough cash to meet combined refinancing need, each bank must have invested enough in the riskless asset to meet at least its own refinancing need, so we are effectively in the autarky solution.
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the systematic-risk state increases, as the correlation $\rho$ in refinancing needs increases. Intuitively, therefore, one would expect that there is a critical cut-off level of $\rho$ below which risk-sharing may dominate the autarkic outcome but above which autarky should dominate. Proposition 4.1 establishes precisely this to be the case:

**Proposition 4.1** Fix any $R > 1$. There is a critical level of correlation $\rho^* > 0$ given by

$$\rho^* = \frac{1}{2R}$$

such that

1. For $\rho < \rho^*$:
   
   (a) There is an interval of values $[\lambda(R), \alpha^*)$ of $\alpha$ under any of which there is a risk-sharing equilibrium in which each bank invests an amount $2/3$ in the risky project and an amount $1/3$ in the riskless asset.

   (b) Outcomes in this risk-sharing equilibrium strictly dominate outcomes under autarky.

   (c) There is a positive probability given by $\rho \alpha (1 - \alpha) + \alpha^2$ in this risk-sharing equilibrium of systemic failure (i.e., of both projects needing refinancing and neither obtaining it).

2. For $\rho > \rho^*$, the outcomes under autarky dominate those under risk-sharing.

**Proof** See Appendix B.

The risk-sharing equilibrium described in Proposition 4.1 has the following properties:

1. There is greater individual risk-taking in the form of investment in the risky project by each bank under risk-sharing than under autarky ($2/3$ compared to $1/2$), and correspondingly lower total liquidity/cash reserves held in the system under risk-sharing than under autarky ($2/3$ compared to $1$).
2. A systemic failure (State 4 with no accompanying refinancing) happens with positive probability in the risk-sharing equilibrium whereas, for the same parameters, there is no systemic failure under autarky; indeed, there are no individual failures under autarky because each bank maintains adequate reserves to insure itself.

3. Thus, expected systemic shortfall is zero under autarky but, under risk-sharing, each bank falls 2/3 short of its refinancing need in the systemic failure state, so expected systemic shortfall is equal to

\[ 2 \times \frac{2}{3} \times (\rho \alpha(1 - \alpha) + \alpha^2) = \frac{4}{3} (\rho \alpha(1 - \alpha) + \alpha^2). \]

The results of this section have been derived in a setting that is simple and transparent but also obviously restrictive (only two firms sharing risks, etc.). In the sections that follow, we describe a richer framework and examine the consequences of two methods of risk-sharing: insurance purchased from a third-party (Section 5) and a clearinghouse-style co-insurance arrangement (Section 6).

A final comment is relevant. The results of this section do not, in themselves, carry any welfare implications, in particular, that risk-sharing is “bad” in the sense of being welfare-decreasing. Indeed, if there are no externalities from systemic failures, then welfare is (trivially) increased by allowing for risk-sharing. However, if there are social and macroeconomic costs of bank bankruptcy and systemic failure (resulting, for example, from disruptions in the payment system), risk-sharing may well be welfare-reducing also. We discuss this subject separately in Section 7.

5 Many Banks: A Centralized Insurance Model

In this section, we move to a setting with a large number of banks and a richer description of uncertainty. Refinancing needs are driven by a common factor and also by idiosyncratic factors. When viewed in isolation, each individual bank in this model continues to face the same decision
problem as in the one-bank setting of Section 3; thus, if the bank decides to remain in autarky and not participate in risk-sharing, its optimal actions are those specified by Proposition 3.1.

Risk-sharing in this section is achieved by paying a premium and buying insurance as described below. To describe the full model, we first develop an \(n\)-bank model for finite \(n\) and then let \(n \to \infty\).

The \(n\)-Bank Model: Description of Uncertainty

Suppose there are \(n\) banks. If Bank \(i\) invests an amount \(\ell > 0\) in the risky project, it receives an interim signal \(A_i \sim N(0, 1)\) concerning the returns from the project. If \(A_i \leq c_i\) (where \(c_i\) is a given critical cut-off), then the project needs to be refinanced, i.e., an additional amount \(\ell\) needs to be invested in the project. If the project is not refinanced, it returns zero with certainty. If it is refinanced, it returns \(R > 1\) with certainty.

The banks are ex-ante identical, and in particular \(c_1 = \cdots = c_n (= c, \text{ say})\). Denoting the cumulative standard normal distribution by \(N(\cdot)\), we let \(\alpha = \text{Prob}(A_i \leq c) = N(c)\) be the ex-ante probability that a bank will need refinancing. We assume that the signals \(A_i\) satisfy\(^5\)

\[
A_i = \sqrt{\rho}A + \sqrt{1 - \rho} \epsilon_i
\]

where \(A, \epsilon_i \sim N(0, 1)\), and \(A\) and the \(\epsilon_i\)'s are independent. Under this condition, banks’ refinancing needs have a pairwise correlation of \(\rho \in (0, 1)\). We refer to \(A\) as the common or systematic factor in determining refinancing needs and to the \(\epsilon_i\)'s as bank-specific or idiosyncratic factors. Conditional on \(A = a\), note that the probability \(p(a)\) that a generic bank needs refinancing is

\[
p(a) = \text{Prob}\{\sqrt{\rho}a + \sqrt{1 - \rho} \epsilon_i \leq c\}
\]

\[
= \text{Prob}\left\{\epsilon_i \leq \frac{c - \sqrt{\rho}a}{\sqrt{1 - \rho}}\right\} = N\left(\frac{c - \sqrt{\rho}a}{\sqrt{1 - \rho}}\right).
\]

\(^5\)The uncertainty structure is similar to that introduced in Vasicek (2002) in a different context.
Insurance and Solvency

There is a competitive insurance sector where banks may buy insurance against the refinancing need by paying a premium of $\pi \in [0, 1]$ per unit refinancing need in the risky project. That is, if Bank $i$ purchases insurance for a refinancing of $b$, it pays a premium $\pi b$; in exchange, the insurance company undertakes to provide the bank with up to an amount $b$ to refinance the project should refinancing be required. We show in Appendix C.1 that if a bank purchases insurance, it is inoptimal for the bank to also self-insure by investing in the riskless asset, i.e., that without loss of generality the bank can be restricted to two choices: self-insure (autarky) or buy insurance and invest in the risky asset. In the latter case, denoting the amount of investment in the risky asset by $b$, the maximum size of $b$ for the bank is determined by its resource constraint $b\pi + b = 1$, so $b = 1/(1 + \pi)$. Since there is a one-to-one relationship between $b$ and $\pi$, we will, for expositional simplicity, refer in the sequel to $k = b\pi$ as the premium charged by the insurance company in exchange for insuring a refinancing need of $1 - k$. Refinancing needs are assumed to be observable and verifiable; a bank cannot file a false refinancing-need claim.

The insurance company’s only source of funds is the premiums it collects; it has no reserves. We assume that the insurance company holds the premia it collects in the most liquid form (the riskless asset—cash—in our model). The insurance company may become insolvent if the claims exceed the total premium collection. Insolvency of the insurance company represents systemic risk in this setting since it leads to failure of those banks unable to refinance their risky projects; note however, that the number of such banks is random. Assuming a symmetric equilibrium in which all $n$ banks buy insurance, the insurance company is solvent as long as the number of banks $m$ filing for refinancing satisfies

$$nk \geq m(1 - k),$$
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or, equivalently, as long as the fraction $m/n$ of banks requiring refinancing satisfies

$$\frac{m}{n} \leq \frac{k}{1-k}.$$  

For a given premium $k$, this identifies the maximum number of claimants $M(k)$ that will still leave the insurance company solvent: $M(k) = \text{Int}[nk/(1-k)]$, where $\text{Int}[x]$ is the largest integer that is less than or equal to $x$. If the insurance company remains solvent, it keeps any profits (excess of premia collected over payouts) from its operations; if it becomes insolvent, we assume that all premia are returned to the banks.\(^6\)

**The Limiting Model as $n \to \infty$**

For a given realization $a$ of the common factor $A$, let $X_i = 1$ denote the event that Bank $i$ needs refinancing, and $X_i = 0$ the event that it does not. We have, for any $i$,

$$\text{Prob}(X_i = 1) = 1 - \text{Prob}(X_i = 0) = p(a).$$

Moreover, conditional on $a$, the $X_i$’s are independent. So the probability that Bank $i$ needs refinancing and will be able to get it (which is the probability that $i$ and at most $(M(k) - 1)$ of the remaining $n-1$ banks need refinancing) is

$$p(a) \times \text{Prob}\left\{ \sum_{j \neq i} X_j \leq \frac{nk}{1-k} - 1 \right\} = p(a) \times \text{Prob}\left\{ \frac{1}{n-1} \sum_{j \neq i} X_j \leq \frac{1}{n-1} \left( \frac{nk}{1-k} - 1 \right) \right\}$$

By the Strong Law of Large Numbers, the term $[1/(n-1)]\sum_{j \neq i} X_j$ on the left-hand side inside the braces converges to $p(a)$ almost surely as $n \to \infty$, while the term on the right-hand side is non-stochastic and converges to $k/(1-k)$ as $n \to \infty$. It follows that as $n$ becomes unboundedly large, the probability of the term inside the braces is either 1 or 0, depending on whether $k/(1-k)$

\(^6\)Since the banks are ex-ante identical, the analysis is essentially unchanged if we instead assume that in the event of insolvency, the total premia collected are shared amongst only those banks filing refinancing claims.
is greater than \( p(a) \) or not. Therefore, as the number of banks becomes very large, the probability that Bank \( i \) needs refinancing \textit{and} will receive it from the insurance company is

\[
\begin{align*}
\left\{ \begin{array}{ll}
p(a), & \text{if } \frac{k}{1-k} > p(a); \\
0, & \text{otherwise.}
\end{array} \right.
\]

Let \( a^*(k) \) denote the critical value of \( a \) below which \( p(a) < k/(1-k) \); of course, \( a^*(k) \) also depends on \( \rho \), but in the interests of notational economy, we suppress this dependence. For a given value of \( k \), a generic bank obtains:

- a payoff of \( R(1-k) \) if the insurance company remains solvent \((a > a^*(k))\), regardless of whether or not the bank needed refinancing;

- a payoff of \( R(1-k) \) if the insurance company becomes insolvent \((a < a^*(k))\) \textit{and} the bank did not require refinancing (which happens with probability \( 1 - p(a) \)); and,

- a refund of its insurance premium \( k \) if the insurance company becomes insolvent.

Therefore, the bank’s expected payoff \( E\Pi(k) \) from purchasing insurance is given by

\[
E\Pi(k) = \int_{a^*(k)}^{\infty} R(1-k)\phi(a) \, da + \int_{-\infty}^{a^*(k)} [R(1-k)(1-p(a)) + k]\phi(a) \, da
\]

where \( \phi(a) \) denotes the density of the standard normal distribution. Simplifying this expression, we obtain

\[
E\Pi(k) = R(1-k) + kN(a^*(k)) - R(1-k) \int_{-\infty}^{a^*(k)} p(a)\phi(a) \, da. \tag{3}
\]

Since the insurance sector is assumed to be competitive, equilibrium outcomes under insurance are determined by the value of \( k \) that maximizes (3). The bank chooses insurance if this maximized payoff dominates the payoff under autarky; else, it elects to remain autarkic and self-insure.

Our analysis in this section focuses on the situation where \( \alpha \in [\lambda(R), \nu(R)] \); under this condition, recall that it is optimal under autarky to carry enough liquidity to fully self-insure investment.
in the risky project; so there are no individual (hence, no aggregate) bank failures. The autarky payoff is given by

\[ \frac{1}{2} [1 + R - \alpha]. \]

**Insurance versus Autarky**

The principal questions in which we are interested are (i) when, if at all, the equilibrium payoff under insurance can dominate the payoff under autarky (i.e., when will banks prefer buying insurance to self-insuring), and, (ii) when risk-sharing is preferable, whether it leads to greater aggregate risk than under autarky (in particular, whether it leads to a greater probability of systemic failure). And in either case, what is the role played by the correlation \( \rho \)?

Under autarky, every bank is optimally fully self-insured with an investment of 1/2 in the riskless asset and 1/2 in the risky asset. A bank will never buy insurance if \( k \geq 1/2 \).\(^7\) If a bank buys insurance for \( k < 1/2 \), the principal advantage gained over autarky is that it can invest \( 1 - k > 1/2 \) in the risky asset to generate greater returns. The main downside to purchasing insurance is that if the insurance company becomes insolvent—which will result from a suitably bad aggregate shock \( a \)—the risky project cannot be refinanced if required. (Note too that under insurance, the premium is lost if the individual bank does not need refinancing and the insurance company remains solvent, a further downside.) Intuitively, then, we would expect that the benefits of insurance are maximal at low correlation levels—other things being equal, a lower level of correlation implies a lower probability of insolvency of the insurance company—so insurance should dominate autarky for low values of correlation. At high levels of correlation, we would expect the opposite: given that banks’ refinancing needs are occurring all together, the insurance company is likely to face insolvency unless premia are very high, and either factor will make autarky more attractive than purchasing insurance. And, indeed, we establish the following result:

\(^7\) A bank can always fully self-insure by investing 1/2 in the risky asset, so buying insurance for \( k \geq 1/2 \) is suboptimal. Indeed, it is strictly suboptimal even at \( k = 1/2 \) because the premium is lost under insurance if the bank does not need refinancing and the insurance company is solvent, whereas under insurance, the bank retains control of the investment in the riskless asset.
Proposition 5.1 For $\alpha \in [\lambda(R), \nu(R)]$:

1. For low correlation levels, risk-sharing via insurance dominates autarky. The situation is reversed at high correlation levels.

2. When $\rho > 0$ and equilibrium involves risk-sharing, there is a strictly positive probability of insolvency of the insurance company and systemic failures.

Proof See Appendix C.2.

Proposition 5.1 reinforces the two key ideas developed in the simpler model of Section 4, that risk-sharing could lead to (a) greater risk-taking by banks, and (b) a greater likelihood of systemic failure. We now move to a characterization of the firm- and market-level implications of equilibrium, focusing in particular on how outcomes change as correlation changes.

Properties of Equilibrium

Unfortunately, closed-form solutions to the problem are not possible when $\rho \neq \{0, 1\}$, so we use numerical analysis to examine equilibrium outcomes. The figures that follow describe the result of numerical simulations as $\rho$ varies for $R = 2.50$ and two values of $\alpha$, $\alpha = 0.40$ and $\alpha = 0.45$. (Similar payoff patterns hold for other values of the parameters; for reasons of space, we do not report them here.) The figures reinforce the following key points:

1. For low values of correlation, insurance dominates autarky, but the distance between the two narrows as correlation rises, and for high enough values of $\rho$, autarky dominates (upper left panel of Figure 2).

2. Aggregate systemic liquidity is strictly lower under insurance than under autarky (so aggregate risk-taking—investment in the risky asset—is higher under insurance than autarky) (upper right panel of Figure 2).
3. Insurance leads to a strictly positive probability of systemic failure, and this probability increases as correlation increases (lower left panel of Figure 2).

The upper left panel of Figure 2 describes payoffs under insurance and autarky, and the resulting equilibrium payoffs. Equilibrium payoffs are strictly decreasing in $\rho$ for low values of $\rho$ and become flat at higher values. At low values of $\rho$, insurance dominates autarky (as noted in Proposition 5.1) and equilibrium payoffs are those resulting from insurance. Insurance becomes less attractive as correlation increases. But as $\rho$ increases beyond a point, insurance payoffs dip below autarky payoffs; equilibrium payoffs are now given by the autarky payoffs which are independent of correlation.

The upper right panel of Figure 2 plots aggregate systemic liquidity as correlation varies. At low levels of correlation, equilibrium outcomes are those resulting from insurance; aggregate systemic liquidity is equal to the total premia collected by the insurance sector. At high correlation levels, autarky dominates; aggregate systemic liquidity is the total savings by individual banks (i.e., their investment in the risk-free asset which, by Proposition 3.1, is equal to $1/2$). Aggregate systemic liquidity is increasing under insurance as correlation increases but is always strictly lower under insurance than under autarky; therefore, aggregate risk-taking (investment in the risky asset) is always strictly higher when equilibrium involves risk-sharing. Intuitively, as correlation increases, banks are more likely to need financing together and this leads to an increase in the equilibrium premium. However, as noted in footnote 7, equilibrium premia under insurance can never exceed $1/2$ because at such levels, autarky would strictly dominate insurance for the individual bank. Thus, equilibrium liquidity typically involves a discontinuous jump up when autarky begins to dominate insurance.

The lower left panel of Figure 2 presents the probability of failure of insurance as correlation changes. The figure considers the same parameter configurations as the upper panel. As intuition would suggest, the probability of insurance insolvency and systemic failure increases with $\rho$ over the relevant range where insurance also dominates autarky. When autarky begins to dominate, the probability of failure drops to zero since there is always full self-insurance under autarky for the
Figure 2: Equilibrium Under Insurance and Autarky

The four panels of this figure plot various properties of equilibrium in the model of Section 5. In all the figures, the return from the risky project is set to $R = 2.50$. In Panels 1 and 4 (top-left and bottom-right, respectively), the value of $\alpha$ (the probability that refinancing of the risky project will be required) is set to 0.45; the remaining two panels consider two values each of $\alpha$ as noted in the figures. Panel 1 (top left) plots the payoff to a generic bank as correlation changes. Panel 2 (top right) plots aggregate systemic liquidity in equilibrium as correlation changes. Panel 3 (bottom left) plots the probability of systemic failure (i.e., insolvency of the insurance company) as correlation changes. Lastly, Panel 4 (bottom right) describes the distribution function of the fraction of banks that fail in equilibrium for three levels of the correlation $\rho = 0.05, 0.15$, and 0.25.
Finally, what is the distribution of the fraction of banks which could fail under insurance? That is, how severe could “systemic” risk be? The bottom right panel of Figure 2 provides an answer. The figure takes the refinancing-need probability to be $\alpha = 0.45$ and describes the cumulative distribution function for the number of failing banks for each of three values of correlation $\rho$. (For all three values, insurance dominates autarky.) These distribution functions are consistent with the lower-left panel which showed that the probability of default and systemic failure increases with $\rho$ (the distribution functions get “pushed down” as correlation rises). As also implied in that panel, for each value of $\rho$, there is a substantial probability of zero failures (i.e., of insurance remaining solvent), but in all cases when insurance does fail, it involves a significant fraction of failing banks.

Once again, it must be emphasized that the possibility of lower liquidity and systemic failure under risk-sharing has no immediate normative implications. Whether it is welfare enhancing—and, if not, whether it can be made welfare-enhancing through regulation—depends on the social and macroeconomic costs of bank failures. In Section 7, we use the distribution of failed banks resulting from any choice of premium $k$ to identify the first-best solution under insurance when the costs of bank failures is taken into account. But first we undertake an investigation of an alternative interesting form of risk-sharing.

6 Clearinghouse: A Co-Insurance model

In this section, we move from a third-party insurance provider model to one in which insurance takes the form of co-insurance. Before we describe the specific structure of risk-sharing, we make one small change to the underlying one-bank model; in this section, that model will be taken to evolve according to the description in Figure 3. The only change from the earlier setting of Figure 1 is that, when refinancing is not required, the risky project payoff is realized at time 1 itself. Observe that this makes no difference at all to either the autarkic outcome (which continues to be
as described in Proposition 3.1) or to the equilibrium and outcomes in the model of Section 5. But having successful banks’ payoffs realized at time 1 enables us to consider a co-insurance setting where these payoffs are used to fund those banks that experience refinancing needs. We describe this setting below.

We assume that there is a “clearinghouse” wholly owned by the banks that provides insurance to member banks from counterparty credit risk. The clearinghouse is funded with up-front premium or “margin” payments of size \( k \) from each of the participating banks; in addition, the clearinghouse also has the right to make ex-post capital calls on its members. Specifically, in period 1, when banks’ refinancing needs are known, these are first funded out of the margins collected by the clearinghouse. Should this prove insufficient, the clearinghouse can then tap the payoffs realized by the investments of those banks that did not experience a refinancing need. However, if the initial margins and the capital calls prove collectively inadequate to meet the total refinancing needs, then the clearinghouse becomes insolvent and no bank’s refinancing insurance contract is honored. In this event, we assume that the initial margins are returned to the banks.

Banks take into account this possibility of ex-post transfers in their decision to participate in the clearinghouse. Conditional on insurance not becoming insolvent, the size of the transfers from the successful banks to the failed banks is a function of the fraction of banks requiring refinancing. This fraction (or more accurately, the distribution of this fraction) depends on the size of the aggregate shock. We look to characterize the equilibria that result in this setting.

As mentioned, \( k \) denotes the up-front margin paid by each bank to the clearinghouse. Notation is otherwise unchanged from previous sections. Per dollar invested in the risky project, the bank obtains a return of \( R \) if it is successful on the investment. As in Section 5, each bank is subject to an aggregate shock \( a \) as well as an idiosyncratic shock \( \epsilon_i \) and needs to refinance if \( \sqrt{\rho} a + \sqrt{1 - \rho} \epsilon_i < c \). Banks are ex-ante identical. Conditional on the aggregate shock \( a \), let \( p(a) \) be the fraction of banks requiring refinancing. For expositional simplicity, we write \( p \) for \( p(a) \) in the steps that follow.

Conditional on the aggregate shock \( a \), the aggregate revenue of the banks not requiring refinancing is \( R(1 - p)(1 - k) \), while the total refinancing need is \( p(1 - k) \). Given the initial margin
collection of $k$, this leaves a deficit of $p(1 - k) - k$ to be shared between those banks that were successful, i.e., did not experience a refinancing need. Thus, the share $\eta(p)$ of its realized revenues that each successful bank must give up to the clearinghouse is

$$\eta(p) = \frac{p(1 - k) - k}{R(1 - k)(1 - p)}$$

If $\eta(p) > 1$, then the clearinghouse cannot meet the refinancing needs even with the ex-post capital calls, so becomes insolvent. Note that we may also have $\eta(p) < 0$; this simply means that the clearinghouse has a surplus left over after meeting refinancing needs and returns this amount to the successful banks.

Therefore, we now have two cases:

- $p > \frac{[R(1 - k) + k]}{[(1 - k)(1 + R)]}$, i.e. $\eta(p) > 1$: The clearinghouse fails and a generic
bank gets $R(1 - k)$ if it succeeds and 0 otherwise. The initial margin is rebated back to member banks.

- $p \leq \frac{[R(1 - k) + k]}/[(1 - k)(1 + R)]$: Failed banks have their refinancing needs honored, realizing (after the bailout) a payoff of $R(1 - k)$. Successful banks also lose a fraction $\eta(p)$ of their payoff to fund the bailout.

Recall that (using the same notation as in Section 5)

$$p = p(a) = N \left( \frac{c - \sqrt{\rho a}}{\sqrt{1 - \rho}} \right).$$

Let $a^*(k)$ be such that $p(a^*(k)) = \frac{[R(1 - k) + k]}/[(1 - k)(1 + R)]$. Then the clearinghouse fails if $a < a^*(k)$, and is solvent for $a \geq a^*(k)$. Repeating exactly the analogous steps in Section 5, we obtain the following expressions for the likelihood of a generic bank’s requiring refinancing and the clearinghouse being solvent, conditional on $a$:

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability if $a &lt; a^*(k)$</th>
<th>Probability if $a \geq a^*(k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank needs refinancing, clearinghouse is solvent</td>
<td>0</td>
<td>$p(a)$</td>
</tr>
<tr>
<td>Bank needs refinancing, clearinghouse is insolvent</td>
<td>$p(a)$</td>
<td>0</td>
</tr>
<tr>
<td>Bank does not need refinancing, clearinghouse is solvent</td>
<td>0</td>
<td>$1 - p(a)$</td>
</tr>
<tr>
<td>Bank does not need refinancing, clearinghouse is insolvent</td>
<td>$1 - p(a)$</td>
<td>0</td>
</tr>
</tbody>
</table>

while the payoffs of the bank in the four cases are:
Risk-Sharing and the Creation of Systemic Risk

<table>
<thead>
<tr>
<th>Event</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank needs refinancing, clearinghouse is solvent</td>
<td>( R(1 - k) )</td>
</tr>
<tr>
<td>Bank needs refinancing, clearinghouse is insolvent</td>
<td>( k )</td>
</tr>
<tr>
<td>Bank does not need refinancing, clearinghouse is solvent</td>
<td>( R(1 - k)[1 - \eta(p(a))] )</td>
</tr>
<tr>
<td>Bank does not need refinancing, clearinghouse is insolvent</td>
<td>( R(1 - k) + k )</td>
</tr>
</tbody>
</table>

Integrating over \( a \), the ex-ante expected profit of a representative bank as a function of \( k \) is, therefore:

\[
E\Pi^c(k) = \int_{-\infty}^{a^*(k)} [R(1 - k)(1 - p(a)) + k] \varphi(a) \, da \\
+ \int_{a^*(k)}^{\infty} \{ R(1 - k)[1 - \eta(p(a))](1 - p(a)) + R(1 - k)p(a) \} \varphi(a) \, da
\]

where, as usual, \( \varphi(\cdot) \) is the density of the standard normal distribution. Substituting the expression for \( \eta(\cdot) \) and simplifying, we obtain

\[
E\Pi^c(k) = R(1 - k) + k - R(1 - k) \int_{-\infty}^{a^*(k)} p(a) \varphi(a) \, da - (1 - k) \int_{a^*(k)}^{\infty} p(a) \varphi(a) \, da
\]

The initial margin \( k \) is chosen by the mutually-owned clearinghouse to maximize this profit. The choice of \( k \) will evidently depend on the correlation \( \rho \) and other factors. We look to characterize the optimal outcome, focusing in particular on the questions of whether risk-sharing can dominate autarky; whether risk-sharing leads to greater individual risk-taking than autarky; and whether risk-sharing leads to greater systemic failure (here insolvency of the clearinghouse) than autarky.
Properties of Equilibrium

Our main result is that co-insurance is never worse than autarky, and at low correlation levels, it is strictly better. Moreover, whenever coinsurance dominates autarky, it involves greater investment in the risky asset than under autarky and a non-zero probability of clearinghouse failure:

Proposition 6.1

1. For any correlation $\rho$, the payoff for a generic bank under co-insurance is at least as large as that under autarky.

2. If $\rho < 1/2$, then co-insurance leads to strictly higher payoffs than autarky; involves a greater investment in the risky asset than autarky; and has a non-zero probability of systemic (i.e., clearinghouse) failure.

3. There is $\rho^* \in (1/2, 1)$ such that for $\rho \geq \rho^*$, the equilibrium value of $k$ is either $k = 0$ or $k = 1/2$.

Proof: See Appendix D.2.

The four panels of Figure 4 illustrate the coinsurance-autarky comparison. The upper-left panel presents a typical plot of payoffs in the clearinghouse model as correlation varies. The figure considers $R = 1.6$, and $\alpha = 0.45$. The payoff with the clearing house is at least as large as that under autarky. As correlation increases, the benefits of risk-sharing decrease, so co-insurance payoffs fall towards the autarky payoff.

The upper-right panel describes the size of the initial margin (the only cash reserves under coinsurance since there is no other investment in the riskless asset) for two values of $\alpha$. In either case, when correlation is very low, the optimal initial margin is zero; the system relies entirely on post-facto successful investments to bail out failing firms. But as correlation increases, more firms fail together, so reliance on successful investments alone is insufficient and the initial margin level becomes positive, eventually rising towards the autarky level of 0.50. The investment in the risky
asset under coinsurance is therefore equal to 1, the maximum possible amount, at low levels of correlation but decreases towards the autarky level of 1/2 as correlation increases.

The lower-left panel plots the probability of failure of the clearinghouse against correlation. For the initial low levels of correlation where the optimal initial margin is zero, this probability of failure increases with correlation. Beyond this point, the collection of initial margin causes the probability of failure to fall back down, eventually reaching the zero probability level of autarky. Observe the interesting feature that the peak probability of clearinghouse failure is higher for lower $\alpha$, since a lower alpha increases the zone over which zero initial margins are optimal.

Finally, the lower-right panel describes the cumulative distribution functions for the fraction of bank failures for three different levels of correlation.

**Insurance vs Co-insurance**

One might expect that, given the possibility of ex-post transfers, co-insurance does at least as well as the centralized insurance setting of Section 5 for any given level of correlation, and indeed, this is not difficult to establish:

**Proposition 6.2** Banks have a higher expected payoff under co-insurance than under centralized insurance. $E\Pi^c > E\Pi^{ins}$ where $E\Pi^{ins}$ is the expected profit under centralized insurance.

**Proof:** See Appendix D.2.

We can also compare the systemic risk generated under each of the two models by comparing the distribution of failures under each framework. We omit figures here, but it can be shown that the number of failures under the centralized insurance model first-order stochastically dominates outcomes under co-insurance, i.e., for any given threshold number of failures, the likelihood of failures exceeding this threshold is always higher under the centralized insurance model than under co-insurance. Thus the introduction of a clearinghouse lowers systemic risk, without eliminating it entirely.
Figure 4: Equilibrium Under Co-Insurance and Autarky

The four panels of this figure plot various properties of equilibrium in the model of Section 6. In all the figures, the return from the risky project is set to $R = 1.60$. In Panels 1 and 4 (top-left and bottom-right, respectively), the value of $\alpha$ (the probability that refinancing of the risky project will be required) is set to 0.45; the remaining two panels consider two values each of $\alpha$ as noted in the figures. Panel 1 (top left) plots the payoff to a generic bank as correlation changes. Panel 2 (top right) plots aggregate systemic liquidity in equilibrium as correlation changes. Panel 3 (bottom left) plots the probability of systemic failure (i.e., insolvency of the insurance company) as correlation changes. Lastly, Panel 4 (bottom right) describes the distribution function of the fraction of banks that fail in equilibrium for three levels of the correlation $\rho = 0.05, 0.15, \text{ and } 0.25$. 

![Figure 4: Equilibrium Under Co-Insurance and Autarky](image-url)
7 Costs of Systemic Failures and the Case for Regulation

The paper has thus far focussed on the positive aspects of equilibrium and not its normative implications. In particular, we have not yet modeled the potential social/macroeconomic costs bank failure imposes. We now examine this issue. We assume that the social cost of bank failures is convex (specifically, quadratic) in the number of failures. This is a natural condition to impose: the marginal cost of a bank’s failing is likely low if the failure is an isolated event but could be reasonably expected to increase as the number of other failed banks increases, i.e., as the failure becomes “more” systemic.

We measure aggregate welfare by the total profits of the banks less the social costs of bank failures. Since the latter quantity is not internalized by banks, equilibrium and first-best outcomes will in general differ. The question we are interested in is whether and to what extent regulatory requirements can improve welfare and lower systemic risk and how equilibrium outcomes under regulation compare both to equilibrium without regulation and to the first-best outcomes.

We focus on the co-insurance model of Section 6 and take the policy tool available to regulators in this setting to be the minimum level of initial margins the clearinghouse must require of its members. We assume that the regulator acts in an environment of incomplete information (otherwise, the regulator could trivially just impose the first-best margin level); we operationalize the incomplete information by assuming that the regulator knows the range within which the correlation $\rho$ in bank refinancing needs lies but not its exact level. The regulator maximizes expected welfare to solve for the minimum margin level the clearinghouse should charge. Equilibrium outcomes are then determined as in Section 6, but with this minimum margin level as a constraint.

We are interested in four questions concerning this “constrained” equilibrium: (i) How does the equilibrium margin level compare to the unconstrained equilibrium and to the first best?, (ii) How do the levels of welfare compare across the three settings?, (iii) How does the level of systemic risk—i.e., failure of the clearinghouse—compare across the three settings?, and (iv) What are the tradeoffs faced by the regulator in setting these margin levels and how are they reflected in the

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8Qualitatively very similar results obtain in the insurance model.
welfare and systemic risk outcomes?

**Failure Costs and the First-Best Solution**

Consider the coinsurance setting of Section 6. Let \( \Pi(k) \) denote the ex-ante expected profit to a generic bank when the clearinghouse charges an up-front initial margin of \( k \). In a first-best solution, a social planner chooses premium levels \( k^* \) to maximize welfare taking into account the costs of insurance failure:

\[
\max_{k \in [0,1]} \left[ E\Pi(k) - cE(f(k)^2) \right]
\]

Here, \( f(k) \) is the fraction of failures given the insurance premium \( k \), and \( c \) is the parameter governing the social cost of bank failures; a higher value of \( c \) implies bank failures are more costly. Expanding the objective function, the social planner picks \( k^* \) to solve

\[
\max_k \left[ R(1 - k) + k - R(1 - k) \int_{-\infty}^{a^*(k)} p(a)\phi(a) \, da - (1 - k) \int_{a^*(k)}^{\infty} p(a)\phi(a) \, da \right.

\[
- c \int_{-\infty}^{a^*(k)} p(a)^2\phi(a) \, da \left. \right]
\]

**Constrained “Second-Best” Equilibrium: The Regulator’s Objective**

A regulator in possession of perfect information concerning the model parameters could simply mandate the level of initial margin that comes out of the first-best solution. Regulators in the real world, however, typically operate under conditions of (very) incomplete information; bank regulators may not know the exact composition of bank balance sheets and assets, and utility regulators may be unaware of the utilities’ true cost structures. To operationalize this uncertainty in our stylized setting, we assume that the regulator does not know the exact value of the correlation
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\[ \rho \] in banks’ refinancing needs. Rather, the regulator knows only its ex-ante distribution \( F \) with support \([\rho_{\min}, \rho_{\max}]\). Let \( W(\rho, k) \) denote the equilibrium welfare level (taking into account the cost of bank failures) when the true correlation is \( \rho \) and the regulator imposes a minimum margin level of \( k \) on the clearinghouse. Then, the regulator picks \( k^* \) to solve

\[
\max_k \int_{\rho_{\min}}^{\rho_{\max}} W(\rho, k) \, dF(\rho), \tag{4}
\]

Our objective is to compare the outcome of this constrained equilibrium to those under the first best and the unconstrained equilibrium.

**Regulatory Outcomes and their Properties**

Since it is not possible to solve for these equilibrium quantities in closed form, we undertake numerical analysis. To this end, we assume that the regulator’s beliefs \( F \) concerning \( \rho \) are uniformly distributed on \([\rho_{\min}, \rho_{\max}]\) with \( \rho_{\min} = 0.10 \) and \( \rho_{\max} = 0.20 \).

Figure 5 compares the outcomes. The figure has three panels. The upper panel compares equilibrium and the first-best margin levels, the lower left panel compares welfare levels, and the lower right panel compares the likelihood of systemic risk. Outcomes are shown for a range of values of the uncertain parameter, \( \rho \), fixing \( R \) and \( \alpha \) (at 1.8 and 0.4 respectively).

From the upper panel, we observe that in the unconstrained case, the equilibrium margin level is zero for all \( \rho \) in the relevant range (0.10 to 0.20). The first-best level of initial margin is also zero when \( \rho \) is sufficiently small (for reasons explained in Section 6), but then becomes positive and rises with \( \rho \), reaching a level of over 0.30 by the time \( \rho \) is at 0.20. The constrained equilibrium has a margin level that is throughout equal to the regulator-imposed minimum margin level of 0.16.

The welfare levels shown in the lower left panel reflect these margin levels. For very low values of \( \rho \), the first-best and unconstrained equilibrium levels of \( k \) are very similar, so they lead to similar welfare levels. However, the constrained equilibrium faces a much higher minimum margin
Figure 5: Constrained Equilibrium, Unconstrained Equilibrium, and the First-Best

This figure compares outcomes under the constrained second-best equilibrium of Section 7 to the unconstrained equilibrium and to the first-best solution. The figure has three panels. The upper panel compares the equilibrium margin level across the three settings. The lower left panel compares the welfare levels that result while the lower right panel compares the level of systemic risk. The value of $\tilde{R}$ in all cases is taken to be 1.8 and that of $\alpha$ to be 0.4.
Figure 6: Second-Best Margins

This figure plots the optimal margin levels set by the regulator for different values of $R$ for three different values of $\alpha$ (0.35, 0.40 and 0.45). As described in the text, the regulator’s belief concerning $\rho$ is a uniform distribution on $[0.10, 0.20]$.

requirement, so leads to a lower welfare level. But as $\rho$ rises, and the first-best and unconstrained levels of $k$ begin to diverge so do their welfare levels, and the constrained equilibrium now does better than the unconstrained one, achieving very favorable welfare levels compared to the first best.

Similarly concerning systemic risk, shown in the lower right panel. At very low levels of $\rho$, the first-best and unconstrained equilibrium offer similar levels of systemic risk, but the constrained equilibrium (which has “too high” a level of $k$ for low values of $\rho$) leads to an even lower probability of systemic risk than either. As $\rho$ increases, the probability of systemic risk under both the constrained and unconstrained equilibria increases, whereas that under the first-best drops off sharply, and falls below that of the constrained equilibrium. In all cases, however, the constrained outcome has a lower probability of systemic risk than the unconstrained outcome.

While Figure 5 analyzes the constrained equilibrium outcome (relative to the unconstrained and first-best equilibria) for a given value of $R$ and $\alpha$, we now look to understand how the second-best
margin (given by the solution to equation 4) varies across $R$ and $\alpha$. Figure 6 plots the second-best margin levels as a function of $R$ when $\alpha$ equals 0.35, 0.40 and 0.45. The regulator continues to face uncertainty over the correlation parameter which ranges between 0.10 and 0.20 as before. We observe that the second-best margin is higher when the riskiness of the project ($\alpha$) increases and is lower as $R$ increases. This behavior is intuitive. As $\alpha$ increases, there is greater risk of failures and output losses in the unconstrained equilibrium, and as $R$ increases, there is greater ex-post payoff to meet refinancing needs so that the risk of failures and output losses is diminished.

Figure 7 illustrates the tradeoffs involved in the design of the second-best margins. The four panels of the figure show the potential gains from regulation in the form of changes in welfare (upper two panels) and changes in systemic risk (lower two panels) as $R$ and $\alpha$ vary. The upper two panels display three statistics - the average, maximal and minimal welfare change - as $R$ changes, and when $\alpha$ equals 0.35 and 0.45 respectively. The three statistics are over different $\rho$, i.e. for a given realization of $R$ and $\alpha$, we compute the unconditional expected gain (or loss) in welfare, where the uncertainty is over $\rho$, and similarly we compute the realized maximum and minimum welfare change over different realizations of $\rho$. The lower two panels display the average, maximal and minimal reductions in systemic risk (again, over realizations of $\rho$) as $R$ changes for the same two values of $\alpha$.

From the upper and lower left panels, we observe that when $\alpha = 0.35$, regulation always lowers systemic risk, but it may also lower welfare. In particular, the minimal welfare change can be negative, which occurs for realizations of low correlation. It is intuitive that regulation is at its most effective when correlation between banks’ refinancing needs is high, and so is the expected output loss from systemic failures. Similarly, the regulation is least effective when this correlation is low. Note that when $R > 1.7$, the optimal second-best margin requirement set by the regulator is exactly the same as the unconstrained equilibrium margin that banks choose.

The upper and lower right panels present the same data for $\alpha = 0.40$, and we observe the same qualitative effects. While regulation always lowers systemic risk and increases welfare on average, there are states of the world where regulation may be counterproductive, lowering welfare. It is the efficiency loss when realization of correlation is low that drives the tradeoff in the determination
Figure 7: Welfare and Systemic Risk Effects of Regulation

This figure presents the effects of regulation on welfare levels (upper panels) and on the likelihood of systemic failure (lower panels). The panels on the left consider a value of $\alpha = 0.35$ and those on the right take $\alpha = 0.40$. The results are presented for a range of values of $R$ from 1.60 to 1.90.
of second-best margins between reducing systemic risk and reducing economic output.

In summary, regulation even under incomplete information can lower the probability of systemic failure compared to the unconstrained setting. It leads to increased overall welfare in those settings where the probability of systemic failure is high (because of high $\rho$) magnifying the social costs of bank failure. However, when the correlation between banks’ refinancing needs is low, regulation may be welfare-reducing. The policy implication of this welfare analysis is that the lack of precise knowledge of all parameters in the economic environment gives rise to a tradeoff that clearinghouses face between economic efficiency and systemic risk reduction while designing margin requirements. The optimal margins that take account of this tradeoff should be higher when the failure risk of clearing members ($\alpha$) is high and their profitability ($R$) low.

8 Conclusion

This paper makes the fundamental point that whether aggregate shocks lead to systemic risk or collective failure of financial firms depends on the liquidity choices of firms but that these liquidity choices are intimately shaped by the risk-sharing opportunities in the economy. When risk-sharing opportunities are abundant, holding liquidity to hedge against low-risk aggregate shocks is privately costly resulting in excessive investment and too little reserves in the system for dealing with such shocks. This novel insight helps resolve the paradox that risk-sharing appears to have led to greater systemic risk and incidence of financial crises. In turn, it provides a sound rationale for minimum reserve requirements for insurance and co-insurance arrangements and minimum margin requirements for derivative contracts. The need and effectiveness of such regulatory requirements is the greatest when the correlation between banks’ refinancing needs is high. Important future work that remains is to understand how such regulation would affect the price of liquidity and the return on investments, both of which we took as given in this paper.
References


A Proof of Proposition 3.1

Pick a strategy \((\ell, 1 - \ell)\). Since refinancing costs are a dollar per dollar of investment in the risky asset, the total refinancing need (should a refinancing need arise) will be \((1 - \ell)\). Since only internal resources may be used to refinance, this means the bank can refinance the risky investment if and only if \(\ell \geq (1 - \ell)\), i.e., if and only if \(\ell \geq 1/2\).

If the bank chooses \(\ell < 1/2\), then refinancing is impossible, and investment in the risky project yields a return of \(R\) (per dollar investment) with probability \(\alpha\) and 0 with probability \(1 - \alpha\). Since the riskless project has a return of 1 (per dollar investment) with certainty, the expected payoff from the strategy \((\ell, 1 - \ell)\) is

\[
\ell + (1 - \alpha)(1 - \ell)R = \ell[1 - R + \alpha R] + (1 - \alpha)R. \tag{5}
\]

On the other hand, if the bank chooses \(\ell \geq 1/2\), then refinancing is always feasible. Since refinancing is optimal when it is feasible, and since refinancing costs of a dollar (per dollar investment) are incurred with probability \(\alpha\), the expected payoff per dollar of investment in the risky strategy is \(R - \alpha\). So the payoff from the strategy \((\ell, 1 - \ell)\), conditional on choosing \(\ell \geq 1/2\), is

\[
\ell + (1 - \ell)R - \alpha(1 - \ell) = \ell[1 - R + \alpha] + R - \alpha. \tag{6}
\]

To identify the optimal strategy for the bank, we compare the maximized values of the payoffs (5) and (6), i.e., (i) the maximal payoff conditional on \(\ell < 1/2\), and (ii) the maximal payoff conditional on \(\ell \geq 1/2\). Consider expression (5) first. It is apparent from visual inspection that the optimal action is to choose \(\ell^* = 0\) if \(1 - R + \alpha R \leq 0\), and to choose \(\ell^*\) as high as possible (subject to \(\ell < 1/2\)) if \(1 - R + \alpha R > 0\). Thus, the maximum value of the payoff (or the supremum since
there is no maximum in the latter case) is

\[
\left\{
\begin{array}{ll}
(1 - \alpha)R, & \text{if } 1 - R + \alpha R \leq 0 \\
\frac{1}{2} [1 + R - \alpha], & \text{if } 1 - R + \alpha R > 0
\end{array}
\right.
\]

Now consider the choice of \(\ell\) that maximizes expression (6) subject to \(\ell \geq 1/2\). Again, simple visual inspection shows that the optimal action is the smallest possible value of \(\ell\) (which is \(\ell^* = 1/2\)) if \(1 - R + \alpha \leq 0\) and the largest possible value of \(\ell\) (which is \(\ell^* = 1\)) if \(1 - R + \alpha \geq 0\). Thus, the maximized value of the payoff is

\[
\left\{
\begin{array}{ll}
\frac{1}{2} [1 + R - \alpha], & \text{if } 1 - R + \alpha \leq 0 \\
1, & \text{if } 1 - R + \alpha > 0
\end{array}
\right.
\]

Since \(R > 1\), this gives this three cases to consider:

- **Case 1.** \(0 \geq 1 - R + \alpha R > 1 - R + \alpha\).

  In this case, the maximized payoff under \(\ell < 1/2\) is \((1 - \alpha)R\), which corresponds to a choice of \(\ell^* = 0\), while the maximized payoff under \(\ell \geq 1/2\) is

\[
\frac{1}{2} [1 + R - \alpha]
\]

(7)
The former is greater if and only if $2(1 - \alpha)R > 1 + R - \alpha$, i.e., if and only if

$$\alpha < \frac{R - 1}{2R - 1} = \lambda(R)$$

Thus, for $\alpha < \lambda(R)$, the optimal action is $\ell^* = 0$ leading to a payoff of $1 - \alpha)R$ if $\alpha < \lambda(R)$, while the optimal action is $\ell^* = 1/2$ leading to a payoff of (7) if $\alpha \in [\lambda(R), (R - 1)/R]$. This establishes Part 1 of the proposition.

• **Case 2.** $1 - R + \alpha R > 0 > 1 - R + \alpha$.

Note that $1 - R + \alpha < 0 \iff \alpha < \nu(R)$ as defined in the statement of Proposition 3.1.

The supremum of the payoffs (there is no maximum) subject to $\ell < 1/2$ here is

$$\frac{1}{2} [1 + R - \alpha R], \quad (8)$$

while the maximized value of the payoffs subject to $\ell \geq 1/2$ (corresponding to the action $\ell^* = 1/2$) is the same as given by (7). Since $R > 1$, the latter is always greater, so it remains optimal to choose $\ell^* = 1/2$ and thence to obtain the expected payoff (7). This establishes Part 2 of the proposition.

• **Case 3.** $1 - R + \alpha R > 1 - R + \alpha \geq 0$.

In this case, the supremum of the payoffs (there is no maximum) subject to $\ell < 1/2$ continues to be given by (8), while the maximized payoff subject to $\ell \geq 1/2$, corresponding to the optimal action $\ell^* = 1$, is 1. The latter is greater as long as

$$1 + R - \alpha R < 2 \iff R(1 - \alpha) < 1$$

But this always holds since we are considering the case $1 - R + \alpha R > 0$. Thus, the optimal action in this case is to set $\ell^* = 1$ and to receive an expected payoff of 1. This establishes Part 3 and completes the proof of the proposition.
B Proof of Proposition 4.1

Recall that when $\alpha \in [\lambda(R), \nu(R)]$ (the case we are considering), under autarky each bank invests an amount of $1/2$ in the riskless asset and $1/2$ in the risky project, and receives an ex-ante expected payoff of

$$\frac{1}{2} (1 + R - \alpha).$$

Consider the following strategies for the two banks that corresponds to a “mutualization” of risk (equivalently, to each bank buying a specific contingent claim from the other bank):

1. Each bank contributes an amount $\ell^* = 1/3$ to a common insurance “pool.” The pool is invested in the riskless asset.

2. Each bank invests its remaining funds $1 - \ell^* = 2/3$ in its risky project.

3. Either bank can claim the amount in the pool if its risky project requires refinancing. (Recall that the refinancing need is observable and verifiable, so false claims cannot be made.)

4. If more than one bank makes a claim at the same time, then the banks share the available money equally.

5. If neither bank experiences a refinancing need, the available money in the pool is equally divided between the two banks at time 2.

Since each bank invests $2/3$ in the risky project, and there is a total of $2/3$ invested in the riskless asset, there is exactly enough to refinance one project but not both. So if only one bank needs refinancing (which happens in States 2 and 3), the project can be refinanced, but if both banks need refinancing (State 4), refinancing is impossible and there is “systemic” failure. Note too that if either bank deviates unilaterally to a lower contribution to the pool, there will not be
enough cash to refinance even one project, so the only possible deviation that could dominate the chosen allocation is autarky, a point we will return to shortly.

The state-wise payoff from the specified strategies for each bank is:

<table>
<thead>
<tr>
<th>State</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1/3 + 2R/3$</td>
</tr>
<tr>
<td>2</td>
<td>$2R/3$</td>
</tr>
<tr>
<td>3</td>
<td>$2R/3$</td>
</tr>
<tr>
<td>4</td>
<td>$1/3$</td>
</tr>
</tbody>
</table>

So the expected payoff to each bank from the described strategies is

$$\frac{1}{3} (p_1 + p_4) + \frac{2}{3} (1 - p_4)R,$$

or, substituting for the probabilities $p_i$,

$$\frac{1}{3} \left[ 1 - 2(1 - \rho)\alpha(1 - \alpha) \right] + \frac{2}{3} (1 - \alpha)(1 + \alpha - \alpha \rho)R \quad (11)$$

This risk-sharing payoff (11) dominates the autarky payoff (9) as long as

$$\left\{ \frac{1}{3} [1 - 2(1 - \rho)\alpha(1 - \alpha)] + \frac{2}{3} (1 - \alpha)(1 + \alpha - \alpha \rho)R \right\} \geq \frac{1}{2} (1 + R - \alpha)$$

A little bit of algebra shows that this condition is equivalent to the condition that

$$(R - 1) [1 - 4\alpha(\alpha + \rho - \alpha \rho)] \geq \alpha \quad (12)$$

When does (12) hold? As $\rho \downarrow 0$, this reduces to the condition that

$$(R - 1)(1 - 4\alpha^2) \geq \alpha \quad (13)$$
Now, as $\alpha$ approaches its lower bound $\lambda(R) = (R - 1)/(2R - 1)$, the left-hand side of (13) tends to
\[
\left( \frac{R - 1}{2R - 1} \right) \left( \frac{4R - 3}{2R - 1} \right)
\]
while the right-hand side approaches $(R - 1)/(2R - 1)$. Therefore, in the limit, the left-hand side is strictly larger as long as
\[
\frac{4R - 3}{2R - 1} > 1,
\]
which always holds since $R > 1$. Since the inequality holds strictly, it follows that there exists an interval $[\lambda(R), \alpha^*)$ such that for $\alpha$ in this interval, risk-sharing strictly dominates the autarky payoff.

It is easy to see from this that for any such $\alpha$ the proposed strategies constitute an equilibrium. From the dominance, neither party will prefer to revert to an autarkic equilibrium. Neither party will also wish to deviate to a lower (but positive) contribution to the pool because there will then not be enough resources in the pool to refinance even one project. And neither party will wish to deviate to a zero contribution since payoff from a zero contribution leads, at best, to the autarkic payoff which is dominated by the proposed risk-sharing strategies.

By continuity in $\rho$, it follows that for all suitably small $\rho$, there is an interval $[\lambda(R), \alpha^*)$ of possible values of $\alpha$ such that the risk-sharing equilibrium dominates autarky for all $\alpha$ in this interval. The upper-bound $\alpha^*$ depends on $\rho$ (and indeed, declines in $\rho$, since it is easily checked that the payoff (11) from risk-sharing is declining in $\rho$).\(^9\)

At high correlations, however, the situation reverses itself, i.e., autarky dominates the payoff under risk-sharing:
\[
\frac{1}{3} [1 - 2(1 - \rho)\alpha(1 - \alpha)] + \frac{2}{3} (1 - \alpha)(1 + \alpha - \alpha\rho)R < \frac{1}{2} (1 + R - \alpha). \tag{14}
\]
\(^9\)How large can $\alpha^*$ be? It must clearly be strictly less than 1/2, since (13) clearly fails for $\alpha \geq 1/2$. 


To see (14), note that as $\rho \uparrow 1$, the left-hand side tends to the limit

$$\frac{1}{3} + \frac{2}{3} (1 - \alpha) R,$$

and this is strictly less than the right-hand side of (14) if

$$2 + 4(1 - \alpha) R < 3(1 + R - \alpha)$$

i.e., if $\alpha > (R - 1)/(4R - 3)$. But $\alpha > (R - 1)/(2R - 1)$ by hypothesis, and $(2R - 1) < (4R - 3)$, so certainly $\alpha > (R - 1)/(4R - 3)$, so the strict inequality (14) holds in the limit. Once again, by continuity in $\rho$, it follows that for sufficiently high $\rho$, autarky dominates risk-sharing.

Finally, as observed earlier, the expected payoff from risk-sharing (11) is strictly decreasing in $\rho$: differentiating this payoff with respect to $\rho$, we obtain

$$\frac{2}{3} \alpha(1 - \alpha) - \frac{2}{3} \alpha(1 - \alpha) R = \frac{2}{3} \alpha(1 - \alpha)(1 - R) < 0$$

(15)

It follows that there is a unique $\rho^*$ such that (a) for $\rho > \rho^*$, autarky dominates risk-sharing for all $\alpha \geq \lambda(R)$, while (b) for $\rho < \rho^*$, there is an interval $[\lambda(R), \alpha^*)$ of possible values of $\alpha$ such that risk-sharing dominates autarky for all $\alpha$ in this interval. Since $\alpha^*$ declines in $\rho$, we can solve explicitly for $\rho^*$ by taking $\alpha$ equal to $\lambda(R)$ and setting the two sides of (12) equal:

$$(R - 1) \left[ 1 - 4 \frac{(R - 1)^2}{(2R - 1)^2} - 4\rho^* \frac{R - 1}{2R - 1} \left( 1 - \frac{R - 1}{2R - 1} \right) \right] = \frac{R - 1}{2R - 1}$$

or, rearranging and canceling common terms,

$$\left[ 1 - 4 \frac{(R - 1)^2}{(2R - 1)^2} - 4\rho^* \frac{R(R - 1)}{(2R - 1)^2} \right] = \frac{1}{2R - 1}$$

Multiplying through by $2R - 1$,

$$2R - 1 - 4 \frac{(R - 1)^2}{2R - 1} - 4\rho^* \frac{R(R - 1)}{2R - 1} = 1$$
So
\[ 2R - 2 = 4 \left( \frac{(R - 1)^2}{2R - 1} + 4\rho^* \frac{R(R - 1)}{2R - 1} \right) \]

Multiplying through again by \((2R - 1)\) and re-arranging, we have
\[ 2(R - 1)(2R - 1) - 4(R - 1)^2 = 4\rho^* R(R - 1) \]

The left-hand side of this expression is \(2(R - 1)[(2R - 1) - 2(R - 1)] = 2(R - 1)\), so we obtain
\[ 2(R - 1) = 4\rho^* R(R - 1) \]

which finally gives us
\[ \rho^* = \frac{1}{2R} \quad (16) \]

This completes the proof of Proposition 4.1.

C Proofs for Section 5

C.1 Strategies in the Insurance Setting

In Section 5, we restricted attention to two choices of strategy for a generic bank: either stay in autarky, or buy insurance (say, total premium paid = \(k\)) and invest the entire remaining balance \(1 - k\) in the risky asset. We show here that this restriction is without loss of generality, i.e., that even if we allow for the bank to purchase insurance, and invest in both the risky and riskless assets, it will not be an equilibrium action for the bank to do so.

To this end, note that the only reason to invest additionally in the riskless asset (after purchasing insurance) is to have enough funds for refinancing, so without loss, the only two possibilities we need to consider when purchasing insurance are: The firm buys insurance for \(k\) and either

1. invests \(1 - k\) in the risky asset; or
2. invests $1/2$ in the risky asset and $(1 - 2k)/2$ in the riskless asset.

Under the second strategy, the firm invests less in the risky asset than under the first strategy (so there is less reward if that project pays off) but compensating for this, the firm always has enough to bail out the risky project if the insurance company becomes insolvent.

We show that the payoff of the second strategy is always dominated by autarky. This means that it is never an equilibrium action, so in identifying equilibria, it suffices to consider only the choice between autarky and the first strategy, which is precisely what we do in Section 5. The payoff under the second strategy is

\[
\begin{cases}
  \frac{1}{2} R + \left(\frac{1}{2} - k\right), & \text{if the insurance company remains solvent} \\
  \frac{1}{2} R + \frac{1}{2}, & \text{if no refinancing needed + insolvency} \\
  \frac{1}{2} R, & \text{if refinancing needed + insolvency}
\end{cases}
\]

So its expected payoff is

\[
\frac{1}{2} R + \left(\frac{1}{2} - k\right) \int_{\alpha^*(k)}^{\infty} \phi(a) \, da + \frac{1}{2} \int_{-\infty}^{\alpha^*(k)} (1 - p(a)) \phi(a) \, da
\]

or, simplifying

\[
\frac{1}{2} R + \frac{1}{2} - k \int_{\alpha^*(k)}^{\infty} \phi(a) \, da - \frac{1}{2} \int_{-\infty}^{\alpha^*(k)} p(a) \phi(a) \, da
\]

The payoff from autarky is
\begin{align*}
\frac{1}{2} R + \frac{1}{2} - \frac{1}{2} \alpha &= \frac{1}{2} R + \frac{1}{2} - \frac{1}{2} \int_{-\infty}^{\infty} p(a) \phi(a) \, da \\
\text{so autarky dominates if and only if} \\
&\quad k \int_{a^*(k)}^{\infty} \phi(a) \, da + \frac{1}{2} \int_{-\infty}^{a^*(k)} p(a) \phi(a) \, da \geq \frac{1}{2} \int_{-\infty}^{\infty} p(a) \phi(a) \, da \\
i.e., if and only if \\
&\quad k \int_{a^*(k)}^{\infty} \phi(a) \, da \geq \frac{1}{2} \int_{a^*(k)}^{\infty} p(a) \phi(a) \, da \\
\text{Since } p(a) < k/(1 - k) \text{ for } a > a^*(k), \text{ we observe that} \\
&\quad \frac{1}{2} \int_{a^*(k)}^{\infty} p(a) \phi(a) \, da < \frac{1}{2} p(a^*(k)) \int_{a^*(k)}^{\infty} \phi(a) \, da = \frac{k}{2(1 - k)} \int_{a^*(k)}^{\infty} \phi(a) \, da \\
\text{Therefore, to prove that} \\
&\quad k \int_{a^*(k)}^{\infty} \phi(a) \, da \geq \frac{1}{2} \int_{a^*(k)}^{\infty} p(a) \phi(a) \, da \\
it suffices to show that \\
&\quad k \int_{a^*(k)}^{\infty} \phi(a) \, da \geq \frac{k}{2(1 - k)} \int_{a^*(k)}^{\infty} \phi(a) \, da \\
\text{But this is always true since } k \leq 1/2.
\end{align*}
C.2 Proof of Proposition 5.1

We first prove the following result for the two extreme values of $\rho$:

**Proposition C.1** For $\alpha \in [\lambda(R), \nu(R)]$:

1. When $\rho = 0$, insurance strictly dominates autarky. There are no systemic failures under autarky or insurance. However, insurance leads to higher aggregate risk-taking: the equilibrium under insurance involves a strictly larger investment in the risky project by each bank than under autarky.

2. When $\rho = 1$, autarky strictly dominates insurance.

**Proof** When correlation is zero, the systematic factor $a$ is irrelevant (effectively, $a^*(k) = -\infty$). From the law of large numbers, the fraction of banks requiring refinancing is, with probability one, equal to $\alpha$, the failure probability of any one bank. Thus, for the insurance company to remain solvent with certainty, the premium $k$ collected from each bank must be at least equal to the expected payout of $\alpha(1 - k)$ to that bank; any lower level of premium will lead to insolvency with probability one. Solving for $k$, we obtain

$$k = \frac{\alpha}{1 + \alpha}$$

Given this value of $k$, the expected payoff to each bank is the certainty quantity

$$R(1 - k) = \frac{R}{1 + \alpha}$$

This is greater than the payoff $(1 + R - \alpha)/2$ under autarky if and only if

$$2R \geq (1 + \alpha)(1 - \alpha) + (1 + \alpha)R,$$

which is precisely the condition that $R \geq 1 + \alpha$ or $\alpha \leq R - 1 = \nu(R)$. Moreover, since the insurance company remains solvent with probability one, there are no failures of the risky project.
in the aggregate. Finally, note that under insurance, each bank invests an amount $1/(1 + \alpha)$ in the risky project whereas under autarky, the amount invested in the risky project is $1/2$. The former amount is always larger than the latter since $\alpha < 1$. This establishes the first part of the proposition.

The second part is trivial. When correlation is perfect, the banks all need refinancing together, so the only way the insurance company will remain solvent is if each bank’s insurance premium is enough to refinance the bank. Thus, the premium is the same as the investment in the riskless asset under autarky, but unlike autarky, this amount is lost under insurance if the bank does not require refinancing.

\[ \square \]

The proof of Proposition 5.1 now follows easily:

**Proof of Proposition 5.1** Part 1 of the Proposition just follows from Proposition C.1 by continuity. For Part 2, note that $a^*(k) > -\infty$ for any $k$ and, in particular, for the equilibrium value of $k$. So the probability that the insurance company becomes insolvent is given by the strictly positive quantity $1 - N(a^*(k))$. \[ \square \]

### D Proofs for Section 6

**D.1 Strategies in the Co-Insurance Setting**

If a bank in the clearinghouse setting buys insurance from the clearinghouse, the only motivation for further investing in the risk-free asset is to ensure that it can refinance its risky project even if the clearinghouse is insolvent. That is, if the bank buys insurance from the clearinghouse for (say) $k$, we may without loss restrict attention to two investment strategies for the bank:

1. Invest the remaining $1 - k$ in the risky asset.

2. Invest $(1/2 - k)$ in the riskless asset and $1/2$ in the risky asset.

In this section, we show that the second of these strategies is always suboptimal, meaning that we may, in our analysis of coinsurance, restrict attention to the first strategy and autarky. Consider
a bank that invests \( k \) as margin in the clearing house and \( 1/2 - k \) in the riskless asset. If the clearing house fails, the bank’s payoff equals

\[
\begin{cases} 
\frac{1}{2} (1 + R), & \text{if bank does not require refinancing} \\
\frac{1}{2} R, & \text{otherwise}.
\end{cases}
\]

Recall that the clearing house fails if the aggregate shock \( a \) is less than \( a^*(k) \). If each bank invests exactly \( 1/2 \) in the risky asset, the transfer function (conditioning on a fraction of \( p \) failures) satisfies

\[
k + \frac{R(1-p)}{2} \eta(p) = f/2,
\]

or

\[
\eta(p) = \frac{p/2 - k}{R(1-p)/2}.
\]

If the insurance company remains solvent, it automatically refinances all projects. Further, banks also obtain the payoff from their riskless investment. Therefore, the banks’ expected profit becomes

\[
\begin{align*}
E \Pi^c(k) &= \int_{-\infty}^{a^*(k)} [(\frac{R}{2} + \frac{1}{2})(1 - p(a)) + \frac{R}{2} p(a)] \phi(a) \ da \\
&\quad + \int_{a^*(k)}^{\infty} [(\frac{R}{2} (1 - p(a))(1 - \eta(p(a))) + \frac{R}{2} p(a) + (\frac{1}{2} - k)] \phi(a) \ da
\end{align*}
\]

Doing some algebra, we get that this equals \((1 + R - \alpha)/2\), independent of \( k \) and equaling autarky payoff. This proves the result.
D.2 Proof of Proposition 6.1

The expected profit to a bank under the clearinghouse model

\[ E\Pi^{ins} = R(1 - k) + k - R(1 - k) \int_{a^*(k)}^{a_\ast} p(a)\phi(a)da - (1 - k) \int_{a^*(k)}^{\infty} p(a)\phi(a)da \]

Here, \( a^*(k) \) is the threshold value of \( a \) below which insurance fails;

\[ p(a^*(k)) = \frac{R(1 - k) + k}{(R + 1)(1 - k)} \]

where \( p(a) = N\left(\frac{c - \sqrt{\rho a}}{\sqrt{1 - \rho}}\right) \)

This implies

\[ a^*(k) = \frac{c - \sqrt{1 - \rho}N^{-1}(R/(1 + R) + k/((1 + R)(1 - k)))}{\sqrt{\rho}} \]

Now, observe that if \( k = 1/2 \), \( a^*(k) = -\infty \), so \( E\Pi^c = (1 + R - \alpha)/2 \), which equals the payoff under autarky. The optimal \( k^* \) under the clearinghouse arrangement must do at least as well as \( k = 1/2 \), so this establishes Part 1 of the result that payoffs under the clearinghouse are never inferior to those under autarky.

To see Part 2, note first that since each bank can fully self-insure at \( k = 1/2 \), we may restrict attention without loss of generality to the clearinghouse choosing \( k \in [0, 1/2] \). Now, the probability of systemic risk (i.e., of clearinghouse failure) is given by \( N(a^*(k)) \). Whenever \( a^*(k) > -\infty \), this probability is always positive; moreover, as is easily checked, \( a^*(k) > -\infty \) whenever \( k < 1/2 \) (and more generally that as \( k \uparrow 1/2 \), \( a^*(k) \downarrow -\infty \)).

We will show that for \( \rho < 1/2 \), we have \( k^* < 1/2 \). This will establish Part 2 since (i) the investment in the risky asset under the clearinghouse is then \( 1 - k^* > 1/2 \), and (ii) for \( k^* < 1/2 \), the probability of systemic failure is \( N(a^*(k)) > 0 \). Since \( k^* \in [0, 1/2] \), we can prove this by
showing that the payoff function is decreasing in $k$ at $k = 1/2$, i.e., that $\partial E\Pi^c(k)/\partial k < 0$ at $k = 1/2$.

Now,

$$\frac{\partial E\Pi^c(k)}{\partial k} = -R + 1 + \alpha - (R - 1)(1 - k)p(a^*(k))\phi(a^*(k))\frac{\partial a^*(k)}{\partial k} + (R - 1)\int_{-\infty}^{a^*(k)} p(a)\phi(a)da$$

Notice that

$$\lim_{k \uparrow 1/2} p((a^*(k)) = 1; \lim_{k \uparrow 1/2} \int_{-\infty}^{a^*(k)} p(a)\phi(a)da = 0$$

Also,

$$\frac{\partial a^*(k)}{\partial k} = -\frac{\sqrt{1 - \rho}}{\sqrt{\rho}(1 + R)(1 - k)^2} \phi(N^{-1}\frac{R}{1+R} + \frac{k}{(1+R)(1-k)})$$

Doing some algebra,

$$\phi(a^*(k))\frac{\partial a^*(k)}{\partial k} = C \exp \left\{ \frac{1}{2}[\frac{c - \sqrt{\rho}a^*(k)}{\sqrt{1 - \rho}}]^2 - a^*(k)^2 \right\}$$

where

$$C = -\frac{\sqrt{1 - \rho}}{\sqrt{\rho}} \frac{1}{(1 + R)(1 - k)^2}$$

We can see that, if $\rho < 1/2$, then $\lim_{k \uparrow 1/2}[\phi(a^*(k)) \cdot \partial a^*(k)/\partial k] = 0$. Therefore,
\[
\lim_{k \uparrow 1/2} \frac{\partial E\Pi^c(k)}{\partial k} = -R + 1 + \alpha < 0,
\]
where the final inequality follows from the maintained assumption that \( R > 1 + \alpha \). This establishes the second part of Proposition 6.1.

To see Part 3 of the Proposition, we consider the behavior of the second derivative of the payoff function with respect to \( k \):

\[
\frac{\partial^2 E\Pi^c(k)}{\partial k^2} = \frac{\partial}{\partial k} \left[ -(R - 1)(1 - k)p(a^*(k))\phi(a^*(k))\frac{\partial a^*(k)}{\partial k} + (R - 1) \int_{-\infty}^{a^*(k)} p(a)\phi(a)da \right]
= 2(R - 1)p(a^*(k))\phi(a^*(k))\frac{\partial a^*(k)}{\partial k} - (R - 1)(1 - k) \frac{\partial p(a^*(k))\phi(a^*(k))}{\partial k} \frac{\partial a^*(k)}{\partial k}
\]

Now,

\[
p(a^*(k))\phi(a^*(k))\frac{\partial a^*(k)}{\partial k} = \left[ \frac{R}{1 + R} + \frac{k}{(R + 1)(1 - k)} \right] \frac{1}{(1 - k)^2} \exp \left( \frac{1}{2} \eta(k) \right) C
\]

where \( C = C(k)(1 - k)^2 = \frac{-\sqrt{-\rho}}{\sqrt{\rho}} \frac{1}{(1 + R)} \). Also,

\[
\frac{\partial}{\partial k} \left[ \frac{R}{(1 + R)(1 - k)^2} + \frac{k}{(R + 1)(1 - k)^3} \right] = \frac{2R}{(1 + R)(1 - k)^3} + \frac{(1 + 2k)}{(R + 1)(1 - k)^4} \equiv f(k)
\]

Therefore, the derivative of (17) is given by

\[
\frac{\partial(17)}{\partial k} = C \exp \left( \frac{1}{2} \eta(k) \right) \left[ f(k) + \frac{1}{2} \eta'(k) \times \frac{p(a^*(k))}{(1 - k)^2} \right]
\]
Now, we obtain the expression for $\eta'(k)$. Recall that

$$\eta(k) = \left[ \frac{c^2 + (2\rho - 1)a^*(k)^2 - 2c\sqrt{\rho}a^*(k)}{1 - \rho} \right]$$

Therefore,

$$\eta'(k) = \frac{1}{1 - \rho} \left[ (2\rho - 1) \times 2\frac{\partial a^*(k)}{\partial k}a^*(k) - 2c\sqrt{\rho} \frac{\partial a^*(k)}{\partial k} \right]$$

$$= 2\frac{\partial a^*(k)}{\partial k} \left[ \frac{1}{1 - \rho}((2\rho - 1)a^*(k) - c\sqrt{\rho}) \right]$$

Substituting the expression for $\frac{\partial a^*(k)}{\partial k}$, the above reduces to

$$\eta'(k) = -2\sqrt{\frac{1 - \rho}{\rho}} \left[ (2\rho - 1) \left( \frac{c - \sqrt{(1 - \rho)N^{-1}(p(a^*(k)))}}{\sqrt{\rho}} \right) - \frac{c\sqrt{\rho}}{(1 - \rho)} \right]$$

$$\times \frac{1}{(1 + R)(1 - k)^2} \phi(N^{-1}[\frac{R}{1+R} + \frac{k}{(1+R)(1-k)}])$$

After some algebra, this becomes

$$\eta'(k) = 2 \left[ \frac{c\sqrt{1 - \rho}}{\rho} + \frac{(2\rho - 1)}{\rho} N^{-1}(p(a^*(k))) \right] \times \frac{1}{(1 + R)(1 - k)^2} \phi(N^{-1}[\frac{R}{1+R} + \frac{k}{(1+R)(1-k)}])$$

(19)

Now, we go back to the second derivative. Using the fact that $C \exp(\frac{1}{2}\eta(k)) = (1 - k)^2\phi(a^*(k)) \frac{\partial a^*(k)}{\partial k}$, we have that $\partial^2 \Pi^c(k)/\partial k^2$ equals

$$2(R - 1)p(a^*(k))\phi(a^*(k))\frac{\partial a^*(k)}{\partial k} - (R - 1)\phi(a^*(k))\frac{\partial a^*(k)}{\partial k} (1 - k)^3 \left[ f(k) + \frac{1}{2}\eta'(k) \times \frac{p(a^*(k))}{(1 - k)^2} \right]$$

Observing that $f(k)(1 - k)^3 = 2p(a^*(k)) + \frac{1}{(1+R)(1-k)}$, the above simplifies to
\[
\frac{\partial^2 E\Pi^c(k)}{\partial k^2} = -(R - 1)\phi(a^*(k)) \frac{\partial a^*(k)}{\partial k} \left[ \frac{1}{(1 + R)(1 - k)} + \frac{1}{2} \eta'(k) \times (1 - k)p(a^*(k)) \right]
\]

Now, noticing that \(\partial a^*(k)/\partial k < 0\), we see that

\[
\frac{\partial^2 E\Pi^c(k)}{\partial k^2} > (\leq) 0 \; i f f \; \left[ \frac{1}{(1 + R)(1 - k)} + \frac{1}{2} \eta'(k) \times (1 - k)p(a^*(k)) \right] > (\leq) 0
\]

For \(\rho\) close to 1, we observe that \(\eta'(. > 0\), and so the above second derivative is positive. In general, for \(c < 0\), there will be a cutoff value of \(\rho\) above which \(\eta' > 0\) for all \(k\). The payoff function is then convex and so the maximand of the function is is either \(k = 0\) or \(k = 1/2\); \(k = 0\) will be chosen if \(E\Pi^c(0) = R - \alpha - (R - 1) \int_{-\infty}^{a^*_0(k)} p(a)\phi(a)da > (1 + R - \alpha)/2 = E\Pi^c(1/2)\). This completes the proof of Part 3 of Proposition 6.1.

\[\Box\]

D.3 Proof of Proposition 6.2

Recall that

\[E\Pi^{ins} = R(1 - k) + kN(a^*_0(k)) - R(1 - k) \int_{-\infty}^{a^*_0(k)} p(a)\phi(a)da\]

where \(a^*_0(k) = \frac{c - \sqrt{1 - \rho N^{-1}(\frac{k}{1 - k})}}{\sqrt{\rho}}\) is the threshold value of \(a\) below which insurance defaults in the centralized insurance model. Note that \(\Delta\Pi^{ins} = E\Pi^c - E\Pi^{ins}\) is given by
\[ \Delta \Pi^{\text{ins}} = k(1 - N(a_0^*(k))) - (1 - k) \int_{a^*_0(k)}^{\infty} p(a) \phi(a) da + R(1 - k) \int_{a^*_0(k)}^{a^*(k)} p(a) \phi(a) da \]

\[ = k(1 - N(a_0^*(k))) - (1 - k) \int_{a^*_0(k)}^{\infty} p(a) \phi(a) da + (R - 1)(1 - k) \int_{a^*_0(k)}^{a^*(k)} p(a) \phi(a) da \]

\[ > k(1 - N(a_0^*(k))) - (1 - k)p(a^*_0(k)) \int_{a^*_0(k)}^{\infty} \phi(a) da + (R - 1)(1 - k) \int_{a^*_0(k)}^{a^*(k)} p(a) \phi(a) da \]

\[ = (R - 1)(1 - k) \int_{a^*_0(k)}^{a^*(k)} p(a) \phi(a) da \]

\[ > 0 \]

This completes the proof of Proposition 6.2