Liquidity, liquidity everywhere, not a drop to use

Why flooding banks with central bank reserves may not expand liquidity[1]

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Abstract

Central bank balance sheet expansion is financed by commercial banks. Apart from substituting liquid central bank reserves for other assets held by commercial banks, which enhances the availability of liquidity, it entails an increase in commercial bank liabilities, such as demandable deposits, issued to finance reserves. Banks typically also write other claims on reserve holdings. During future episodes of stress when many claims on liquidity are exercised and surplus banks hoard reserves, the demand for liquidity can exceed available reserves, exacerbating liquidity stress. This may attenuate any positive monetary effects of reserve expansion on economic activity.

Despite a significant expansion in central bank balance sheets, some markets like the US money market have experienced increasing interest rate volatility, including alarming spikes in the repo rate, notably in December 2018 and September 2019 (see D'Avernas and Vanderweyer (2021)). The disruption in markets that depend intimately on the availability of liquidity seems puzzling when Fed officials set reserve levels not just at a multiple of levels before the global financial crisis, but also above their estimate of private sector demands, with adequate buffers for unexpected variations such as in the Treasury's Fed account (see Logan (2019)).[2] Greater private sector reserves holdings do not seem to have made markets for liquidity fully immune to liquidity shocks. Indeed, even after the Fed injected more reserves in September 2019, markets were disrupted yet again in March 2020 at the onset of the COVID-19 pandemic, and the banking system was found short in its ability to accommodate the demand for liquidity.[3] In response, the Federal Reserve expanded its balance sheet yet more (see, for example, Kovner and Martin (2020)), buying financial assets from the private sector and placing large quantities of liquid reserves with it (or promising to do so). Where had all the prior liquidity gone?

A number of explanations to this question have been suggested, including changes in liquidity or capital regulations and supervision, and changes in the public demand for liquidity.[4] While these explanations have merit, our intent is to augment them by asking a fundamental question: what theoretical effect should the expansion of central bank balance sheets have on systemic liquidity, taking into account the endogenous response of the banking system? We argue that commercial banks offset their increased holdings of liquid central bank reserves by issuing a variety of claims on liquidity, both on- and off-balance sheet. On net then, the free liquidity generated by central bank balance sheet expansion can be much smaller than one might think if bank behavior were disregarded. Indeed, central bank balance sheet expansion can leave the financial system more dependent on liquidity, and hence more vulnerable to adverse liquidity shocks such as those the existing explanations analyze. Particularly consequential could be liquidity shocks emanating from adverse shocks in the real sector, such as the onset of the pandemic.

To elaborate, intuition would suggest that when the central bank issues more liquid central bank reserves to finance its balance sheet expansion, the supply of liquidity to the financial system increases,

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[2] Copeland, Duffie and Yang (2021) estimate that banks made do with about $50 billion in reserve balances before the global financial crisis, and had $1.3 trillion in balances in September 2019.

[3] Corporate debt, but also segments of the US Treasuries market experienced significant illiquidity, see Duffie (2020), Fleming and Ruela (2020), He, Nagel and Song (2020), Liang and Parkinson (2020), Schrimp, Shin and Sushko (2020)), and Vissing-Jorgensen (2020). Corporates drew down bank credit lines, see Kashyap (2020) and Acharya, Engle and Steffen (2021); and, dealer banks found it difficult to make markets, see Boyarchenko, Kovner and Shachar (2020), Breckenfelder and Ivashina (2021), Kargar *et al.* (2021), and Vissing-Jorgensen (2020).

[4] See, for example, Copeland, Duffie and Yang (2021), Correa, Du and Liao (2021), D'Avernas and Vanderweyer (2021, 2022), IAWG Treasury Market Surveillance Report (2021), Lopez-Salido and Vissing-Jorgensen (2022), Poszar (2019), and Yang (2021).

bringing down illiquidity premia in markets, and reducing private financing costs. This intuition neglects three key private sector responses. First, central banks effectively issue these reserves to commercial banks (henceforth "banks"), which typically finance them with short-term liabilities such as deposits, an offsetting claim on liquidity. Second, available reserves could shrink, or get further encumbered in future times of stress through additional bank-issued liquidity claims (such as lines of credit) and regulations preventing free use. Third, banks may exacerbate shortages by hoarding spare liquidity. In short, reserves expansion works through commercial bank balance sheets, which could raise the future demand for liquidity and constrain its supply, limiting its benefits and potentially creating costs.

Let us elaborate. We assume the central bank wants to expand its balance sheet (during Quantitative Easing or QE), buying financial assets from the private sector with newly issued reserves. We take any direct effect of the asset purchases on economic activity initially as given, so as to focus on what happens to liquidity after that. We assume the reserves eventually find their way back to commercial bank balance sheets (so non banks cannot hold reserves and their cash holdings do not go up). Key in the analysis that follows is the mix of how banks finance these reserves. A number of authors (Calomiris and Kahn (1991), Dang, Gorton, and Holmstrom (2010), Flannery (1986), and Gorton and Pennacchi (1990), among others) have argued that banks have a comparative advantage in issuing short-term or demandable debt. Others (see, for example, Diamond and Dybvig (1984) or Stein (2012)) have attributed an implicit liquidity/money premium to demandable bank liabilities that makes them relatively attractive for investors, and Diamond and Rajan (2001) argue that one leads to the other. We are agnostic as to why longer-term financing (that is, capital) is costlier for banks, but assume functional forms that make it so. Naturally then, a first effect of QE is that banks finance a large portion of the reserve expansion with demandable deposits, an effect the literature has underemphasized.[5]

We assume that after commercial banks get reserves, make loans, and set their capital structure accordingly, there is a probability that the economy will become *liquidity stressed*, and the demand for liquidity in the real economy will increase significantly. Demand will be concentrated on some banks. Call these the *stressed* banks. We assume that their wholesale depositors, fearful of any loss, withdraw their cash in such states, increasing the stressed banks' need for funds.

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[5] If the central bank simply buys long-term assets (e.g., Treasuries) from the commercial banks, the commercial banks will be swapping Treasuries for central bank reserves. This requires no financing. However, a significant expansion of the central bank balance sheet will require it to buy financial assets from non-banks. In this case, commercial bank balance sheet expansion occurs automatically; the non-banks deposit the proceeds in their banks, giving the commercial banks both reserves and offsetting deposits. Of course, banks and non-banks might eventually want to rebalance (which we assume in our model), but banks in aggregate have to hold the reserves.

In times of liquidity stress healthy banks may see a valuable *convenience yield* to liquid reserves – for instance, because it is dry powder in case conditions worsen.[6] Consequently, a fraction of healthy banks may hoard liquidity and seek to maintain unimpeachable balance sheets, in order to be perceived as safe and attract more deposit flows, rather than lending reserves out to stressed banks.

Banks may also find some of the initially issued bank-held reserves are not available to pay out in stressed states. They may have shrunk because of Quantitative Tightening (QT), where the central bank sold commercial banks bonds that cannot be used for payments. Reserves may also be encumbered because banks have sold liquidity guarantees; a bank holding highly liquid reserves, with the reserves being required only in situations of liquidity stress, will want to try and "sell" liquidity in all the states it does not need it, for instance, by offering contingent lines of credit. Unfortunately, such commitments typically are also called upon in states where the economy is liquidity-stressed.[7] Regulators may also add to the encumbrances on reserves (perhaps to offset these very private sector responses), for example by mandating a liquidity coverage ratio. They could, of course, suspend an ex-ante-imposed liquidity requirement in the face of ex-post stress so that more reserves are available to alleviate market illiquidity. Diamond and Kashyap (2016) explain the regulator may not want to do this for fear that localized stress morphs into a full-blown panic.

These three effects – the financing of eventually shrunken or partially encumbered reserves with short-term deposits, coupled with reserve hoarding by some of the healthy banks that are recipients of flight-to-safety deposits – sets up an interesting dynamic in episodes of liquidity stress: loan rates in the interbank market can shoot up as stressed banks try and attract liquidity from healthy banks (see, Acharya and Mora (2015) for empirical documentation of such a dynamic during 2007-08).

Importantly, central bank quantitative easing may contribute far less to current activity than might be suggested if bank actions are ignored. Under some circumstances, every additional dollar of reserves the central bank issues up front can increase the net demand for liquidity in situations of liquidity stress,

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[6] For earlier work on liquidity hoarding, see for example, Acharya, Shin and Yorulmazer (2011), Acharya, Gromb and Yorulmazer (2012), Alfonso, Kovner, and Schoar (2011), Bech (2008), Copeland, Duffie, and Yang (2021), and Diamond and Rajan (2011).

[7] Yankov (2020) examines the changes in the liquidity management at banks and nonbank financial firms in the United States that occurred following the initial proposal of the liquidity coverage ratio (LCR) requirement in 2010 and its finalization in 2014. He concludes that "*While banks increased their liquid assets to meet the new regulatory liquidity requirements, nonbank financial institutions–such as insurance companies, finance companies, real estate investment trusts, pension funds, asset managers, mutual funds, and others–decreased their liquid assets and increased their reliance on bank credit lines to manage their liquidity risks.*" Thus banks effectively sold claims that would be collectively called upon in states where liquidity would likely be scarce.

and can increase the interbank borrowing premium. [8] A higher anticipated bank borrowing rate in the future then cascades up front into a higher rate for term loans made by banks (as in Diamond and Rajan (2011), Shleifer and Vishny (2010), or Stein (2012)), lower firm investment, and lower aggregate activity.

In sum, the demand for reserves is not static, it ratchets up with supply, leaving fewer "free" reserves than one might otherwise think. That *the ex-ante supply of reserves affects the ex-post demand for them* is because QE operates through commercial bank balance sheets; if central bank reserves were instead placed directly with households, or with financial intermediaries that did not issue claims on liquidity after swapping their assets for reserves, the effects we hypothesize would be mitigated.

Evidence for the underlying assumptions of our paper and our results has accumulated since our early drafts. Exhibit 1A-1D are from Acharya, Chauhan, Rajan, and Steffen (2022). Exhibit 1A suggests that as the Federal Reserve expanded reserves through various rounds of quantitative easing between late 2008 and 2014, commercial banks increased their issuance, not just of deposits, but eventually also of lines of credit. Within deposits, Exhibit 1B suggests that as reserves increased, time deposits came down while demand deposits increased sharply, suggesting that even the financing of bank balance sheets became more demandable. Indeed, Acharya et al. (2022) find that overall deposits went up approximately one for one with reserves, but demandable deposits went up more than reserve issuance. Moreover, Exhibit 1C suggests it was uninsured demandable deposits, the most volatile form of borrowing, which increased the most with reserve expansion. Finally, Exhibit 1D shows that the ratio of claims on liquidity (outstanding credit lines plus demandable deposits) to reserves and eligible securities (securities that could be used to borrow reserves) reached a peak in September 2019, when the Fed was forced to resume quantitative easing to alleviate liquidity stress. Bank-issued claims on liquidity seem important to understand liquidity stress.

Turning to pricing, Lopez-Salido and Vissing-Jorgensen (2022) argue that the opportunity value of liquidity, measured as the effective federal funds rate minus the central-bank-paid interest on excess reserves, is affected not just by the quantity of outstanding reserves but also by the outstanding stock of commercial banking deposits. In the cross-section, Acharya et al. (2022) show that banks with substantial exogenous quantities of reserves do issue more demandable deposits to finance them. They also reduce the spread they are willing to pay for time deposits, originate more credit lines to firms, and lower the fee they charge for drawdowns on these lines of credit. More generally, as our model suggests, the marginal

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[8] We show that if an implicit or explicit reserve requirement leads demand deposits to grow faster than reserves, a pattern suggested by the evidence, this possibility can arise even without any shrinkage of, or encumbrance on reserves.

"liquidity" value of additional reserves is negatively related to the supply of reserves, *but only after correcting for claims on these reserves*, which grow endogenously with reserves.

Closely related to our paper is Diamond, Jiang, and Ma (2021), who ask how the reserve build-up by the Federal Reserve could affect bank lending. While they too emphasize the need to finance reserves, their focus is on the crowding-out effects of such reserve holdings on corporate loans. Our focus instead is on the effects of reserves on ex-post liquidity, and how that would consequently impact corporate lending.

Studies of the recent rate spikes in usually liquid money markets have attributed them to regulatory and supervisory action (see, for example, Correa, Du and Liao (2021), D'Avernas and Vanderweyer (2021, 2022), IAWG Treasury Market Surveillance Report (2021)), to sudden increases in demand for, and decreases in supply of, reserves due to Treasury actions (see, for example, Copeland, Duffie and Yang (2021)), or to an exogenous growth in household financial assets and thus, mechanically, their demand for deposits (Lopez-Salido and Vissing-Jorgensen (2022)). Our theory, with banks affecting both demand for, and supply of reserves once the central bank issues them, and making the system more prone to disruption, complements these explanations. For instance, a binding capital or Liquidity Coverage Ratio requirement or rising household demand for deposits would not explain why banks deliberately reduced their share of financing from time deposits during QE, increased their share of financing from uninsured demand deposits (typically not households), and increased their issuance of lines of credit (typically to firms). The endogeneity of bank responses to reserve issuance also suggests that while an understandable response to liquidity stress is to permanently expand the quantum of reserves, if the claims on liquidity eventually grow with higher levels of reserves, as we argue and Acharya et al. (2022) document, there will be a ratcheting up in needed reserves.

Are there are any costs then to the central bank supplying reserves ex post as needed. Such intervention crowds out lending by healthy banks in our model, increasing the scale of the needed intervention. Market participants would be even more inclined ex ante to write future claims on liquidity and become ever more dependent on the central bank backstop. The central bank then starts dominating the markets for liquidity, crowding out price discovery (see, for example, Filardo (2020)) and asset prices as well as asset allocations could potentially get distorted. The central bank moves from temporary emergency infusions of liquidity to ever increasing permanent infusions. The size of the central bank balance sheet may then eventually run up against political costs and limits (Plosser (2018)) as well as any aggregate constraints on the stock of pledgeable safe assets. Of course, long before that limit is reached, monetary concerns such as the mixed message sent when simultaneously raising policy rates and expanding the central bank balance sheet, or the dangers of monetary financing of the fiscal deficit at a time of high inflation, may limit central bank balance sheet expansion, forcing a dangerous end game.

There are many moving parts in a model with firms, banks, investors, and the central bank. We introduce the model in section I, and analyze it in section II, first assuming a fixed exogenous fraction of hoarding by healthy banks and next endogenizing the fraction. In section III we compare the privately optimal choices to the socially optimal (central bank/planner's) choices. In section IV we provide various interpretations of, and ways to endogenize, the extent of reserve shrinkage or encumbrance. In section V, we examine robustness and extensions, concluding in section VI.

I. The Model

Consider an economy with three dates, 0, 1, and 2. Subscripts denote the date in what follows and Greek letters are parameters. There are four sets of agents in the economy: firms, banks, risk-averse savers, and risk-neutral savers (with the central bank playing a cameo role in determining reserves). The state of the economy $\tilde{y}$ is revealed at date 1. It can be healthy ($y = 0$) or liquidity stressed ($y = 1$). Firms and banks maximize expected profits.

1.1. Firms

Each firm could be thought of as representing an entire sector of the real economy. The firm has access to an investment opportunity at date 0. The state of the firm $\tilde{z}$ is revealed at date 1. It is always healthy ($z = 0$) when the economy is healthy. However, the firm can be hit by an independent and identically distributed shock that makes it stressed ($z = 1$) with probability θ when the economy is liquidity stressed, which occurs with probability $\frac{q}{\theta}$. So the date-0 probability of a firm getting stressed at date 1 is q. The time line for the state space of economic outcomes is in Figure 1 (we will shortly explain the bank-level outcomes illustrated there).

An investment of I_0 at date 0 produces $g_0(I_0)$ at date 2 if the firm is healthy. If stressed, the firm produces nothing at date 2 from its original investment. However, it has the possibility of "rescuing" some of its earlier investment at date 1 by investing an additional amount I_1. The expected output from such investment is $g_1(I_1)$. This output of the rescue investment is high enough in expectation to allow the firm to repay the expected value of its loans, both for the initial investment and the rescue investment. There is, however, a non-zero probability that nothing is produced from the rescue investment also and the entire sequence of investments is a write-off. Both g_0 and g_1 are increasing and concave, and obey Inada conditions.

Liquidity stress in our model stems from real needs for spending at date 1, which in turn precipitate larger financial demands for liquidity. A model where losses on financial investment precipitate margin calls, which necessitate new funding to avoid fire sales, would have similar effects.
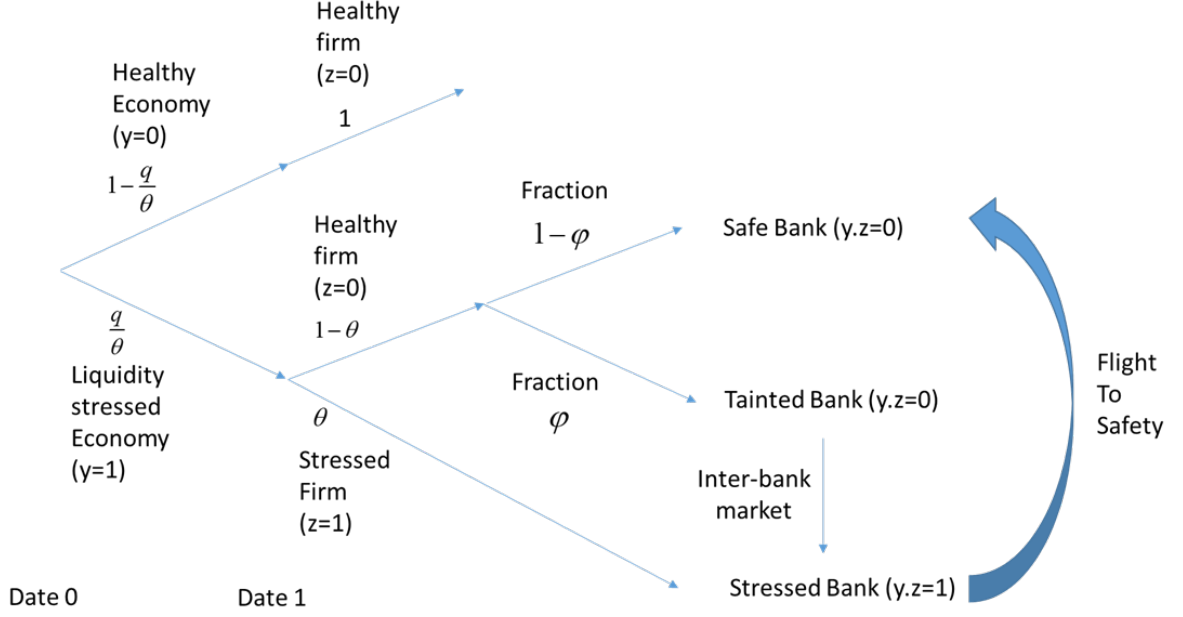

Figure 1: The state space of economic and bank-level outcomes

The firm starts out with own funds of W_0^F, and will supplement it with L_0^F of long-term borrowing from the bank. Apart from the real investment at date 0, it can also place deposits of D_0^F in the bank. We can think of this as the firm's precautionary liquidity holdings, and is isomorphic (up to the fees charged) to pre-contracted credit lines from the bank. At date 1, the stressed firm can withdraw its deposit, as well as borrow from the bank, in order to make its rescue investment, I_1.

1.2. Banks

Each bank lends to a firm (or in the alternative interpretation, an entire sector). So a bank and a firm constitute a pair. The analysis will be conducted on a per bank-firm pair basis. At date 0, the bank can make a two-period loan of amount L_0 at a cumulative gross interest rate of R_0^L. The bank incurs a cost of $\frac{1}{2}\lambda\left(L_0\right)^2$ in making the loan – the cost is increasing and convex because the bank has to manage, and lay off, an increasing amount of risk. At date 0, each bank also has to hold S_0 of reserves that the central bank has issued. For now, we assume it has no choice about the size of reserves it holds,

these arise automatically from its (symmetric) share of financial activity, which is given. At date 1, we will refer to a bank that has lent to a firm that has become stressed also as "stressed".

1.3. Bank Financing

To finance its asset holdings, a bank can raise deposits at date 0 from the risk-averse saver, whose rate of time preference is 1. So if D_0 is the quantum of overall deposits it raises, then $(D_0 - D_0^F)$ is what it raises from the public, receiving the rest from the firm. Implicit here is the assumption that there are only a limited number of risk-neutral savers in the economy so deposits cannot be financed by them.

The risk-averse saver has log utility over consumption at date 2. We assume that if the low probability event that the stressed firm repays nothing on the rescue loan materializes at date 2, the bank will have to default on deposits at date 2. Anticipating their deposits to be haircut, risk-averse depositors will certainly run on the bank at date 2 to avoid being the one at the back of the line that gets nothing. In turn, anticipating a run at date 2 and thus possible zero consumption even with small probability, risk-averse depositors will ask for their money back from a stressed bank at date 1. Put differently, even though the bank is solvent at date 1, as in Stein (2012) it will have to repay its risk-averse depositors immediately if stressed.[9] The stressed firm also withdraws some or all of its bank deposits to make the date-1 rescue investment.

The bank can raise long-term bank capital (consisting of long-term bonds or equity) from the risk-neutral investor (a Warren Buffet or a sovereign wealth fund), at both date 0 and date 1. The bank faces a repayment cost of $e_t + \frac{\alpha_t}{2} e_t^2$ at date 2 when it raises amount e_t at date t. The quadratic term represents costs associated with long-term capital relative to short-term deposits, including higher illiquidity premia, higher term premia, higher borrower moral hazard, and due diligence costs. These costs could be higher at date 1, when the economy is liquidity stressed, than at date 0.

1.4. Firm Financing in the Stressed State at Date 1

To make its date-1 rescue investment, the stressed firm can borrow l_1^F from its bank at date 1 to supplement the deposits it withdraws. The bank will have to do due diligence and monitor the loan, given

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[9] Such behavior mirrors that of institutional wholesale depositors such as corporations or financial institutions where their CFOs lose their job if they have inadvertently left low-yielding transaction deposits in a bank that is risky or fails – this induces extreme risk-aversion about wholesale (uninsured) transaction deposit accounts. Alternatively, depositors could believe that only perfectly safe assets have the requisite "moneyness" (Stein (2012)) or they have no ability to monitor a bank's risky claims (Dang, Gorton, Holmstrom (2010)).

the stressed state of the firm, so the interest rate charged will be $(1 + r_1 + \gamma)$ where γ is the bank's deadweight screening and monitoring costs which are passed on to the firm. For simplicity, we assume that all interest rates reflect expected values (so that face values are set to deliver that rate after accounting for any default risk). This reduces notation and lets us focus on liquidity.

1.5. Reserves Shrinkage or Encumbrance at Date 1

We assume that a bank can use only $(1 - \tau)$ fraction of its initial reserves to meet depositor or lending needs at date 1. The shrinkage of bank reserves can arise due to quantitative tightening in which a portion of the reserves are swapped for financial assets by the central bank. A portion of reserves may be encumbered and not free, either for regulatory reasons (such as liquidity or capital requirements) or because they have been pledged elsewhere (such as for posting margins against speculative positions as part of prime brokerage service for clients). We will elaborate further and present ways to endogenize this fraction in Section IV. Importantly, bank assets and liabilities are optimally set assuming this shrinkage or encumbrance of reserves.

1.6. Interbank Market

A stressed bank can borrow in the interbank market, where healthy banks with surplus reserves can lend. The gross interest rate over the second period in the inter-bank market is 1 if there is an excess of loanable funds relative to demand. If not, the gross interest rate will rise to $(1 + r_1)$ to equalize the demand and supply for funds. When this is the case, stressed banks and healthy banks that lend in the interbank market will find it attractive to issue some capital at date 1.

1.7. Flow of Reserves due to Deposit Flight

When there are ample reserves, the location of reserves is unimportant. In stressed times, when everyone dashes for cash for final settlement, location is paramount. Where do deposits that flee the distressed banks (as well as the incremental deposits that are created by the payments resulting from the date-1 rescue investment) go? This is a critical issue and will influence important results in the paper. We assume these deposits get parked in *safe* banks. But what is safe? Any healthy bank that lends in the interbank market bears some risk of not being repaid, raising concerns among risk-averse institutional depositors about how much risk the bank is taking; in particular, these institutional depositors may learn from interbank markets that the bank has run down its reserve balances. We therefore assume that to be seen as *safe*, a healthy bank should maintain an unimpeachable balance sheet and, in particular, not lend to distressed banks in the date-1 interbank market. It will then attract a proportional share (along with other safe banks) of the flight-to-safety deposits that flee the distressed banks (see Figure 1).

Of course, given the high rate prevailing in the interbank market at date 1, some healthy banks may lend. These banks become *tainted*. Tainted banks will not attract any flight-to-safety deposits.[10]

1.8. Convenience Yield on Reserves in Stressed State of the Economy

For a healthy bank's choice to remain safe to be interesting, there should be some value to attracting flight-to-safety deposits and passing up the opportunity to earn a premium in lending to the interbank market. To this end, we assume that when the economy is liquidity stressed, each dollar of reserves has a *convenience yield* $\delta \geq 0$ to the final holder. This could be thought of as the precautionary value of reserves in case there is further un-modeled stress (or in case assets are sold at a fire sale discount in the future), their value in signaling a "fortress balance sheet" to investors looking for safety, or the franchise value of deposits associated with those reserves. Since the convenience yield is enjoyed by the final holder, any reserve transfer effects a private wealth transfer that washes out in the aggregate. However, the convenience yield significantly affects banks' responses to a liquidity shock.[11]

1.9. Payments and Reserve Transfers

Any purchase and payment leads to a transfer of reserves. We assume that any date-1 bank capital issued is bought by risk-neutral investors who first acquire deposits in safe banks (for instance, by selling risk-averse depositors their treasury bills), and then transfer the safe bank's reserves to the capital-issuing bank by writing the latter a check (alternative assumptions would worsen the date-1 illiquidity problem). Conversely, any payment by firms while purchasing equipment for the rescue investment is deposited in the safe banks. Note that the banking system does not gain or lose reserves as a result of date-1 capital issuance or payments; the former moves reserves from safe banks to capital-issuing banks, whereas the latter moves reserves from lending banks to safe banks. Reserves are necessary for final settlement – for instance, to pay depositors who withdraw money from the bank. Finally,

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[10] We assume depositors of tainted banks do not run, though alternative assumptions are easily handled.

[11] Why do safe banks do not compete for flight-to-safety deposits by raising deposit rates? Acharya and Mora (2015) show that safe banks did not raise deposit rates during the GFC, while distressed banks did. One explanation is that safe banks were trying to signal that they did not need funds in order to avoid the stigma associated with risky banks. Relatedly, the inflow from risk-averse flight-to-safety institutional depositors may be driven by convenience and a desire for principal protection rather than to exploit small differences in rates. For instance, depositors may flee their stressed bank to the most proximate safe bank. Finally, a bank will have to pay any higher rate to all its depositors. If the flight-to-safety deposits are only a small fraction of a receiving bank's overall deposits, safe banks may be reluctant to compete for them. This is a similar effect to Drechsler, Savov and Schnabl (2017), who document that banks in concentrated banking areas are reluctant to pay flighty depositors higher rates when the Fed raises rates, since they have to also pay captive depositors that rate. Given these considerations, we assume safe banks cannot (or will not) raise rates to attract more flight-to-safety deposits.

i) Banks hold reserves symmetrically. We do not initially allow for central bank reserves to be held directly by the non-bank sector. We study private incentives to hold reserves later.

ii) We net out the volume of deposit creation engendered by the issuance of high-powered reserves, looking only at final "reduced-form" balance sheets. The pyramiding of deposits via the money multiplier typically introduces additional complications as to how claims are run upon, netted and settled (see, for example, Kashyap (2020)) that would magnify the problems we examine.

II. Analysis

We first solve the model assuming that the fraction of healthy banks that choose to become tainted by lending in the interbank market at date 1 is exogenously set at φ, which is known at date 0, and the convenience yield, δ, from holding reserves is zero. We will then allow $\delta > 0$ and endogenize the fraction of tainted healthy banks φ at date 1.

2.1. Exogenous Share φ of Healthy Banks that are Active in the Inter-Bank Market

2.1.1. The Firm's Problem

To ease understanding of the calculations that follow, we present firm and bank balance sheets at date 0 and date 1 in Exhibit 2. With probability q the firm will be stressed at date 1, and it will be healthy with probability $(1-q)$. So its maximization problems at date 0 and date 1 are as follows:

Date 0: $\displaystyle \max_{L_0^F, D_0^F} (1-q)\left[g_0(I_0) + D_0^F\right] + q\left[g_1(I_1) - l_1^F\left(1+\gamma+r_1\right)\right] - R_0^L L_0$

Date 1: $\displaystyle \max_{l_1^F} g_1(I_1) - l_1^F\left(1+\gamma+r_1\right)$

s.t. $I_0 = L_0 + W_0^F - D_0^F$ and $I_1 = l_1^F + D_0^F$

The constraints are just budget constraints at each date. The firm's first order conditions (FOCs) then are

w.r.t. firm's long-term borrowing from bank L_0: $\quad (1-q)g_0{}' - R_0^L = 0 \qquad\qquad (1.1)$

w.r.t. firm's deposit in the bank D_0^F: $\quad (1-q)\left(-g_0{}'+1\right) + q\left(g_1{}'\right) = 0 \qquad (1.2)$

w.r.t. date-1 firm borrowing from bank l_1^F: $\quad g_1{}' - \left(1+\gamma+r_1\right) = 0 \qquad\qquad (1.3)$

Substituting the value of g_1' from (1.3) into (1.2), we get $(1-q)g_0' = (1+q\gamma + qr_1)$. Term the right-hand side of this expression R_0^{DF}: it is the expected opportunity return for the firm of holding an additional dollar of deposit, and thus avoiding borrowing from the bank at date 1 if stressed. Comparing with (1.1) where the firm's marginal expected return on date 0 investment is equal to the cost of long-term borrowing from the bank, we get $R_0^L = R_0^{DF}$. In words, the cost of long-term borrowing is equal to the opportunity return on holding an additional dollar of deposit. Let us now turn to the bank's problem.

2.1.2. The Bank's Problem

The bank maximizes profits given constraints, that is,

$$\underset{L_0^B, e_0, e_1}{Max} \quad R_0^L L_0 + S_0 - e_0 - \frac{\alpha_0}{2} e_0^2 - D_0$$

$$+ \frac{q}{\theta}\theta\left[-\frac{\alpha_1}{2}e_1^2 - r_1\left(b_1(y=1, z=1) - l_1^B\right)\right]$$

$$+ \frac{q}{\theta}(1-\theta)\varphi\left[-\frac{\alpha_1}{2}e_1^2 - r_1 b_1(y=1, z=0)\right]$$

s.t. $D_0 + e_0 = L_0 + \frac{1}{2}\lambda(L_0)^2 + S_0$ (1.4)

$b_1(y=1, z=1) = l_1^B + D_0 - S_0(1-\tau) - e_1$ (1.5)

$b_1(y=1, z=0) = -S_0(1-\tau) - e_1$ (1.6)

$l_1^B = l_1^F = I_1 - D_0^F$ (1.7)

The first line of the maximization is the bank's expected profits at date 2 from date-0 lending and financing activities. The second line is the expected loss for a stressed bank. The stressed bank's loan to the stressed firm (in equilibrium, $l_1^B (= l_1^F)$) and its deposit repayments are funded by unencumbered reserves, date-1 capital raised, and interbank borrowing (of $b_1(y=1, z=1)$, see (1.5)), and its expected loss is the financing cost net of the return on the loan to the firm. The third line of the maximization is the expected profits to a healthy bank from becoming tainted and lending $-b_1(y=1, z=0)$ into the interbank market when the economy is stressed at date 1, net of financing costs. It makes the loan out of its

unencumbered reserves and the capital it raises (see (1.6)). Note that both the stressed bank and the tainted bank will raise the same amount of capital in equilibrium because they will both see the same marginal return of using that capital in the interbank market (for the stressed bank it reduces borrowing, and for the tainted bank it increases loans). The constraint (1.4) simply reflects the sources and uses of funds at date 0 (the bank raises money from deposits and long-term capital, and invests in long-term loans, the cost of making loans, as well as forced reserve holdings). Then, the first order conditions are

w.r.t. long-term loans to the firm L_0: $R_0^L - (1 + \lambda L_0)(1 + qr_1) = 0$

From the bank's perspective, the date-0 return from making another dollar of loan should equal the cost of funding that dollar (and the associated upfront marginal cost of managing the risk of the additional loan, λL_0) via flighty deposits, which cost a net rate of r_1 per dollar if the bank gets stressed. Let us term as R_0^{DB} the expected cost of funding via deposits, which equals $(1 + qr_1)$. Hence, $R_0^L = (1 + \lambda L_0)R_0^{DB}$.

FOC w.r.t. date-0 capital issuance e_0: $-(1 + \alpha_0 e_0) + (1 + qr_1) = 0$

This implies the marginal cost of raising an additional dollar of long-term funding or capital at date 0 should equal the saving on funding via deposits. So $e_0 = \dfrac{(R_0^{DB} - 1)}{\alpha_0} = \dfrac{qr_1}{\alpha_0}$. In words, the bank raises more capital at date 0 the higher the expected premium it will pay in the interbank market in the stressed state. Finally, FOC w.r.t. date-1 capital issuance e_1: $-(1 + \alpha_1 e_1) + (1 + r_1) = 0$

So at the margin, the bank's cost of raising an additional dollar of capital at date 1 equals the cost of borrowing in the interbank market. Simplifying,

$$r_1 = \alpha_1 e_1 \qquad\qquad (1.8)$$

Hence the premium r_1 in the interbank market drives capital-raising at date 1 by stressed and healthy tainted banks and vice versa. Firm and bank maximizations also link the various interest rates to it. So

$$R_0^L = R_0^{DF} = (1 + q\gamma + qr_1) = (1 + \lambda L_0)R_0^{DB} \qquad (1.9)$$

2.1.3. Market-clearing at Date 1

We know that the inter-bank premium is necessary in order to equalize the date-1 demand and supply of funds when the economy is liquidity stressed – essentially the premium draws forth more date-1 issuance

in the capital market by stressed and tainted banks even while reducing rescue investment by stressed firms, and the associated spillover demand for funds. The net date-1 shortfall in the interbank market in the liquidity stressed economy is $\theta\left[I_1 + (D_0 - D_0^F)\right] - \left[\varphi(1-\theta) + \theta\right]S_0(1-\tau)$. The first term in the first square bracket is the "rescue" investment by the stressed firms and the second term is the expected withdrawal by the risk-averse depositors from stressed banks (which is redeposited in safe banks). The sum is the call on liquidity by the system, which is reduced by the last term, the available shrunken reserves with stressed and tainted healthy banks. This overall shortfall exactly equals $\left[\varphi(1-\theta) + \theta\right]e_1$, the date-1 capital raised by the tainted and stressed banks (note that safe banks do not raise date-1 capital because they have no profitable way to deploy it). So when r_1 is positive, we have from (1.8),

$$\left[\varphi(1-\theta) + \theta\right]\alpha_1^{-1}r_1 = \theta\left[I_1 + (D_0 - D_0^F)\right] - \left[\varphi(1-\theta) + \theta\right]S_0(1-\tau)$$

or $r_1 \equiv \alpha_1 f(r_1, S_0)$ where $f(r_1, S_0) \equiv \dfrac{\theta}{\left[\varphi(1-\theta) + \theta\right]}\left[I_1 + (D_0 - D_0^F)\right] - S_0(1-\tau)$. \hfill (1.10)

Recall that in equilibrium $D_0^F = L_0 + W_0^F - I_0$ and $D_0 = L_0 + \frac{1}{2}\lambda(L_0)^2 + S_0 - e_0$. We denote the equilibrium interbank rate premium as $\overline{r_1}$. If there are sufficient unencumbered reserves in the banking system to meet funding needs without date-1 capital issuance, then $\overline{r_1} = 0$.

2.1.4. Date-1 Interbank Rate

Let us now analyze how the equilibrium interbank rate varies with reserves issued, S_0. It is straightforward to check that since $\dfrac{\partial f}{\partial r_1}$ is negative (see proof of Lemma 1 in Appendix I), so

$Sign\left(\dfrac{d\overline{r_1}}{dS_0}\right) = Sign\left(\dfrac{\theta}{\left[\varphi(1-\theta) + \theta\right]} - (1-\tau)\right)$. Rearranging the right hand side, we get

Lemma 1: The date-1 equilibrium interest rate in the inter-bank market is unique. If

(i) $\theta > \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}, \dfrac{d\overline{r_1}}{dS_0} > 0$; (ii) $\theta < \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}, \dfrac{d\overline{r_1}}{dS_0} < 0$; (iii) $\theta = \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}, \dfrac{d\overline{r_1}}{dS_0} = 0$.

Proof: See Appendix I.

Lemma 1 (ii) is the more traditional view that more reserves injected at date 0 will reduce the date-1 interbank premium – essentially each dollar of reserves is not eaten up entirely by encumbrances and bank-issued claims on liquidity that come due if the economy is stressed. Lemma 1 (i) is more novel, suggesting the stress in the interbank market, and the associated equilibrium rate for funds, can *increase* with the reserves that the central bank injects into the system at date 0. Importantly, this will also reduce date-0 and date-1 real investments. At first pass, the result seems counterintuitive. How can more liquidity supply at date 0 increase liquidity stress at date 1? However, this result is counterintuitive only from a partial-equilibrium perspective – not taking into account the effect of running reserves through bank balance sheets. Recognize first that the marginal source of funding of the reserves is demand deposits, which potentially create their own demand for liquidity in the stressed state (in proportion to the fraction of stressed banks, θ). Moreover, only a fraction φ of the $(1-\theta)$ healthy banks use their unencumbered reserves to meet the liquidity demands of stressed banks, and only $(1-\tau)$ of each dollar of their reserves is available at date 1. Put differently, $\dfrac{d\bar{r_1}}{dS_0} > 0$ whenever the marginal liquidity provided by each dollar of reserves, $(1-\tau)\left[\varphi(1-\theta)+\theta\right]$, is lower than θ, the marginal call on liquidity when demand deposits are withdrawn from stressed banks. This leads to the condition $\theta > \dfrac{\varphi(1-\tau)}{\tau+\varphi(1-\tau)}$ in Lemma 1.

Note that a fraction $(1-\varphi)$ of healthy banks not only hoard *all* of their reserves, but also any flight-to-safety reserves they obtain. With all healthy banks hoarding ($\varphi = 0$), the condition for Lemma 1 (i) is always met as long as $\tau > 0$; conversely, when $\varphi = 1$ so that all healthy banks lend in the interbank market, then Lemma 1 (i) holds whenever $\theta > (1-\tau)$.

2.1.5. When Do Reserves Raise or Lower the Date-1 Interbank Rate

Differentiating (1.10) totally with respect to S_0 and recognizing that $f(r_1, S_0)$ is increasing in S_0, whether more date-0 reserves raise or lower the date-1 interbank rate depends on whether

$$S_0\left(\frac{\theta}{\left[\varphi(1-\theta)+\theta\right]}-(1-\tau)\right) > \frac{\theta}{\left[\varphi(1-\theta)+\theta\right]}\left(W_0^F - I_0 - I_1 - \frac{1}{2}\lambda(L_0)^2\Big|_{r_1=0}\right) \equiv NLS \qquad (1.11)$$

The left hand side is the net liquidity demand created by reserves. The right hand side is the net liquidity supplied (NLS) by the corporate sector anticipating a date-1 interbank premium of zero (and adjusting for any cost to the bank of long term lending). NLS is high when the corporate sector has a high level of

starting internal funds W_0^F and a relatively low demand for funds for investment and loans. The equilibrium interbank rate (and date-1 capital market rate) is positive if the net liquidity demand exceeds supply at a rate of zero. Since $f(r_1, S_0)$ decreases in r_1, and changes in S_0 and τ only shift the term containing S_0 and not the slope of $f(r_1, S_0)$ with respect to r_1, using Lemma 1 we can describe the level of date-0 central bank reserves $\hat{S}_0$, at which the interbank rate turns positive. We can also describe how the interbank rate moves with reserves around that threshold. We have

Theorem 1:

(i)   If $\theta > \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$, then $\bar{r}_1 > 0$ is the unique equilibrium for $S_0 > \hat{S}_0$ with $\bar{r}_1$ increasing in

   S_0; and, $\bar{r}_1 = 0$ for $S_0 \leq \hat{S}_0$. Note that $\hat{S}_0 \leq 0$ if $NLS \leq 0$.

(ii)   If $\theta \leq \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$, then $\bar{r}_1 > 0$ is the unique equilibrium for $S_0 < \hat{S}_0$ with $\bar{r}_1$ decreasing in

   S_0; and, $\bar{r}_1 = 0$ for $S_0 \geq \hat{S}_0$. Note that $\hat{S}_0 \leq 0$ if $NLS \geq 0$.

Proof: Follows from the discussion above.

Theorem 1 (ii) is the traditional view of reserves. An increase in reserves should alleviate future illiquidity, reduce the interbank rate, and increase current (and future) real investment. A preponderance of reserves, $S_0 \geq \hat{S}_0$, ensures that the date-1 interbank interest rate premium will be zero.

Let us plot in Figure 2A the threshold value of reserves at which the date 1 interbank rate is zero, $\hat{S}_0$, for different values of θ (the fraction of the banking sector that becomes liquidity stressed). We do this for the more plausible case that the corporate sector absorbs liquidity so $NLS < 0$ (in Figure 2B in Appendix I we analyze $NLS > 0$). The size of reserves is on the vertical axis and θ is on the horizontal axis. When $\theta < \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ (to the left of the vertical axis), $\hat{S}_0$ is positive, and rises in θ. Because higher ex-ante reserves loosen liquidity conditions, $\bar{r}_1$ falls in S_0, and the unhatched region below the $\hat{S}_0$ curve is where $\bar{r}_1$ is positive.

Theorem 1 (i) is the alternative view our model also offers. When $\theta > \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ (to the right

of the vertical axis), $\hat{S}_0$ is negative, and increases in θ. Because higher ex-ante reserves tighten liquidity

in the stressed state, $\overline{r}_1$ increases in S_0, and the unhatched region above the $\hat{S}_0$ curve is where $\overline{r}_1$ is

positive. In this unconventional case, the central bank cannot provide the demanded date-1 liquidity via

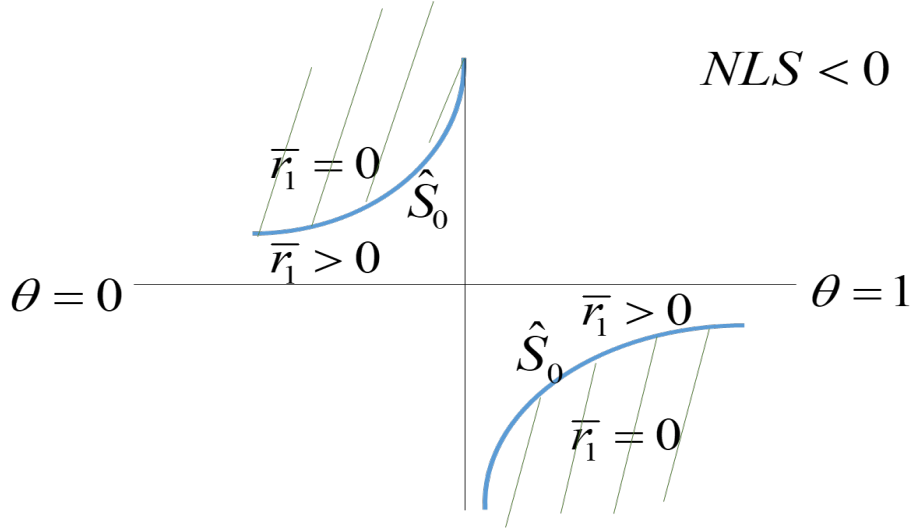banks through asset purchases at date 0 – reserve issuance also tends to absorb liquidity on net.

Figure 2A: Reserves are on the y axis, θ on x axis, with the axes intersecting at $\theta = \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$

In sum, the inter-bank premium is always zero when the economy is healthy. Any idiosyncratic liquidity

demands are likely to be diversified across a large set of liquidity suppliers (see Kashyap, Rajan, and

Stein (2002), for example). Central bank supplied liquidity is likely to be ample for such needs and the

location of reserves is unimportant. However, in stressed situations, liquidity demands become much

more strongly positively correlated (for example, as witnessed by banks in the United States during 2007-

08, see Acharya and Mora (2015), or as witnessed by firms at the onset of the pandemic in March 2020,

see Kashyap (2020), and Acharya, Engle and Steffen (2021)), and liquidity hoarding can further restrict

supply. There is then a threshold central bank reserve issuance level at date 0 beyond which further

issuance increases the net interbank date-1 rate r_1.

Whether increased ex-ante reserves reduce ex-post liquidity is an empirical question. The broader

point, however, is that the "free" reserves available to deal with liquidity stress at date 1 will be

significantly lower than the aggregate reserves issued at date 0 because of the endogenous issuance of

claims on reserves by the banking sector and liquidity hoarding. Reserve demand is not static, indeed supply will create some of its own demand, a phenomenon the literature has underemphasized.

2.2 Endogenous φ : Liquidity Hoarding

We assumed so far that in stressed situations an *exogenous* fraction φ of healthy banks lend in interbank markets, and the remaining fraction receives the flighty deposits of stressed banks. To endogenize the fraction φ, we will allow healthy banks to choose between lending in the interbank markets and consequently becoming *tainted*, or staying clear of profitable albeit risky interbank lending and instead attracting flight-to-safety deposits on which recipient banks earn a convenience yield $\delta > 0$. At the equilibrium value of φ and the equilibrium interbank rate, healthy banks must be indifferent between choices. Importantly, we will see the interbank market may remain shut, that is, $\varphi = 0$.

2.2.1. Equilibrium as S_0 changes

In the benchmark model of Section 2.1, the interbank market was always open so that r_1 was both the interbank rate as well as the lending rate to stressed firms net of bank monitoring cost γ (that is, firms borrow at $r_1 + \gamma$ but the bank earns the net rate r_1). Now, the interbank market may be shut. However, stressed firms will still borrow from their banks, and r_1 is the bank lending rate to the firm net of the monitoring cost. As before, we will use the notation $\overline{r_1}$ for the equilibrium interbank lending rate when the interbank market is open (in which case it will equal r_1). However, when the interbank market is closed in a stressed situation, we will denote the equilibrium r_1 as the *autarky* rate r_1^A; this is simply the rate that equilibrates liquidity demand and supply (via capital issuances) when the interbank market is closed. Finally, the presence of a convenience yield for reserves implies that the interbank market will be open only if the interbank rate exceeds a "breakeven rate" which we will denote as r_1^φ.

Now, in the liquidity stressed state, three cases may arise (details are in Appendix II):

<u>Case 1:</u> *Stressed banks have enough liquidity (while raising date-1 capital commensurate with the convenience yield) to meet the needs of deposit outflows and to fund rescue investment without accessing the inter-bank market.*

This first case arises when the level of reserves, and in turn of demandable deposits, is adequately low.

Case 2: *The liquidity needs of each stressed bank can be entirely met by its raising date-1 capital (beyond that warranted by the convenience yield).*

In this second case, the level of reserves is moderately high and the interbank market remains shut ($\varphi = 0$) even though stressed banks are liquidity-deficient; formally, this occurs because the autarky rate r_1^A is below the breakeven rate r_1^φ that healthy banks require to enter the interbank market.

Case 3: *The liquidity needs of the stressed banks are high enough that at the equilibrium autarkic interest rate, some of the healthy banks are willing to lend in the interbank market and become tainted. The equilibrium rate then is lower than the (now counterfactual) autarkic rate.*

The third case arises when the level of reserves is high enough that the autarky rate r_1^A rises above the breakeven rate r_1^φ and the inter-bank market opens up ($\varphi > 0$). Some surplus banks are induced by the high inter-bank premium to provide liquidity to deficient banks.[12]

2.2.2 Determining φ

Let us characterize this third case to understand when in equilibrium the interbank market opens up or remains shut. The φ healthy banks that choose to lend in the date-1 interbank market will lend all their unencumbered reserves as well as the capital raised at date 1 at rate r_1. The date-1 profits from doing so are $V_1^\varphi = \left[(r_1 - \delta) S_0 (1 - \tau) + r_1^2 \Big/ 2\alpha_1 \right]$, where the first term is the incremental value from lending out own unencumbered reserves, and the second term is the profit from raising capital ($e_1 = r_1 \big/ \alpha_1$) and lending the proceeds. The net reserve outflows from the stressed and now tainted banks amount to $S_0(1 - \tau)\left[\theta + (1 - \theta)\varphi \right]$ and these go to the $(1 - \theta)(1 - \varphi)$ banks that choose to be safe. So the profit from being seen as safe and attracting the flight-to-safety deposits is $V_1^{1-\varphi} = \dfrac{\delta S_0 (1 - \tau) \left[\theta + (1 - \theta)\varphi \right]}{(1 - \theta)(1 - \varphi)} =$

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[12] We assume that δ is sufficiently large for these cases to arise. The condition is formally stated in Appendix II, in the Proof of Theorems 2-3. When the convenience yield δ is small, it is possible that only Cases 1 and 3 arise since the breakeven rate r_1^φ may be lower than δ at the level of reserves that requires a switch out of Case 1.

$\delta S_0 (1-\tau) \left(\dfrac{1}{(1-\theta)(1-\varphi)} - 1 \right)$. In equilibrium, healthy banks should be indifferent between choosing to

become tainted or stay safe. So $V_1^{\varphi} = V_1^{1-\varphi}$, and rearranging terms

$$(1-\varphi) = \frac{\delta S_0 (1-\tau)}{(1-\theta)\left(r_1 S_0 (1-\tau) + {r_1^2}\big/{2\alpha_1} \right)} \qquad (1.12)$$

Inspecting (1.12), it is clear that $\dfrac{\partial \varphi}{\partial S_0} < 0, \dfrac{\partial \varphi}{\partial \delta} < 0, \dfrac{\partial \varphi}{\partial r_1} > 0$. In words, the share of healthy banks lending

in the interbank market falls in the ex-ante level of reserves (because, as S_0 increases, the relative profits

from raising and lending capital fall relative to attracting the flight-to-safety deposits) as well as in the

convenience yield, and increases in the available interbank rate.[13]

Then, requiring that $\varphi > 0$ yields the "breakeven interbank rate" r_1^{φ} , that induces some banks to

lend in the interbank market. When $\varphi > 0$, the equilibrium interest rate $\overline{r_1}$ and φ are now jointly

determined by equations (1.10) and (1.12). Finally, comparing the breakeven interbank rate r_1^{φ} and the

autarky rate r_1^{A} determines when Case 2 versus Case 3 arise. These details are worked out in Appendix II

where we show formally that

Theorem 2: For $\delta > 0$ and $\tau > 0$, there exist critical thresholds for the level of reserves, S_0^{*} and S_0^{**},

where $S_0^{**} > S_0^{*} > 0$, such that the inter-bank market is open, that is, $\varphi > 0$, only for $S_0 > S_0^{**}$, and

(i) for $S_0 \leq S_0^{*}$, stressed banks are not liquidity-deficient (taking into account their capital raise dictated

by the convenience yield), and r_1 charged to firms equals δ ;

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[13] As an aside, as $\delta \to 0$, we have $\varphi \to 1$. That is, as the convenience yield of reserves falls to zero, virtually all healthy banks choose to lend in the interbank market. Only a sliver of the healthy banks prefers being seen as safe, and these attract all the flight-to-safety reserves, which carry an infinitesimal convenience yield δ .

(ii) for $S_0 \in \left(S_0^*, S_0^{**}\right]$, stressed banks are liquidity-deficient and raise more capital at date1 than dictated by the convenience yield, but the inter-bank market remains shut (autarky). Furthermore, the autarkic lending rate to firms r_1^A satisfies $r_1^A > \delta$, $\dfrac{dr_1^A}{dS_0} > 0$, and $r_1^A\left(S_0^{**}\right) = r_1^{\varphi}\left(S_0^{**}\right) > 0$; and,

(iii) for $S_0 > S_0^{**}$, stressed banks are liquidity-deficient and raise capital as well as borrow in the inter-bank market at date 1; the inter-bank rate satisfies $\overline{r}_1\left(S_0\right) \geq r_1^{\varphi}\left(S_0\right) > 0$, with $r_1^{\varphi}\left(S_0\right)$ increasing in S_0.

We also show in Appendix II that a sufficient condition for the equilibrium r_1 to be increasing in S_0 in Case (iii) is $\theta > \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ as in section 2.1. It is only a sufficient condition, however, since with endogenous φ, the incentive to hoard and attract deposits also increases in S_0, further increasing the equilibrium interest rate. Furthermore, since φ rises from zero at the breakeven interest rate r_1^{φ} at which the interbank market opens, there is always a region in Case (iii) in which r_1 increases in S_0.

Finally, as the convenience yield δ associated with the possession of reserves increases, the inter-bank market remains shut over a wider range of the level of reserves, and the level of the inter-bank rate increases with δ whenever the inter-bank market is open. Formally,

Theorem 3: (i) S_0^* and S_0^{**} are increasing in δ; and, (ii) for $S_0 > S_0^{**}$, $\dfrac{d\overline{r}_1}{d\delta} > 0$.

Interestingly, because safe banks do not make any decisions after date 0 (other than deciding whether to lend), and because banks take φ and $\overline{r}_1$ as given, the ex-ante bank maximization problem is the same as what we analyzed earlier, though φ, $\overline{r}_1$, and bank expected profits will be altered by δ.

2.2.3. Examples and details

Figures 3A and 3B illustrate model outcomes for a specific parameterization where $\delta = 0.2, \tau = 0.2$. In Figure 3A, $\theta = 0.8 = (1-\tau)$ and in Figure 3B, $\theta = 0.6 < (1-\tau)$. Other parameters are $\lambda = 1, \gamma = 0.4, q = 0.1, \alpha_0 = \alpha_1 = 1, W_0^F = 2, g_0' = g_1' = 1/I$. The breakeven interbank rate $r_1^{\varphi}\left(S_0\right)$ is in green, the autarkic bank lending rate $r_1^A\left(S_0\right)$ is in yellow (with its hypothetical value extrapolated if the inter-bank market were to remain shut even for $S_0 > S_0^{**}$), and the equilibrium interbank rate $\overline{r}_1\left(S_0\right)$ when some healthy banks choose to enter the inter-bank market is in blue. While the entry of some

healthy banks pulls the inter-bank rate down (blue line relative to the yellow line), it nevertheless remains above $r_1^\varphi(S_0)$ and is increasing in S_0 for both sets of parameters.

We also illustrate the effects of varying the convenience yield δ in Figures 4A and 4B, where θ is set at 0.6, less than $(1-\tau)$. In Figure 4C, δ takes values close to zero, whereas in Fig 4B, it takes significantly higher values. In both cases, as δ increases, the threshold reserve level above which the inter-bank market opens shifts higher to the right, though this shift is relatively modest at low values of δ; also, as δ increases, the inter-bank rate is higher whenever the inter-bank market is open. Finally, Figure 4C shows that as δ increases, the proportion φ of surplus banks that enter the inter-bank market decreases.

2.2.4. Endogenous δ

Thus far, we have assumed the date-1 convenience yield is exogenous. Of course, it likely depends on the net liquidity mismatch (so it does not necessarily fall with gross ex-ante reserves). A good proxy for the date-1 mismatch is r_1. The higher is r_1, the more worried market participants may be about hidden prospective problems (and opportunities to buy in fire sales) emanating from the current stress, and the more they may value the convenience yield in holding reserves. If $\delta(r_1) = \delta^A + \delta^B r_1^2$ where $\delta^A \geq 0$, $\delta^B \geq 0$, we show in Appendix II that depending on parameters, it is possible the interbank market never opens (in particular when δ^B is sufficiently high, because the convenience yield grows fast with the inter-bank premium, increasing the returns to hoarding more than lending), or that it opens only if the date-1 rate lies in a range and is closed otherwise, or that it opens above a certain rate as with a fixed δ. The point, however, is that date-1 stress could be magnified significantly with endogenous δ, i.e., a convenience yield on reserves that rises in liquidity-stressed states.

III. The Central Bank/Planner 's Problem and Optimal Reserves

In taking the ex-ante level of reserves as given, we have in mind the central bank setting reserves for un-modeled monetary purposes, for instance to effect quantitative easing.[14] Let us now examine where the central bank/planner would set reserves with the view of maximizing welfare in our framework, ignoring

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[14] Central banks have sought to expand reserves in order to implement quantitative easing, where the effects range from signaling monetary policy stance (see Krishnamurthy and Vissing-Jorgensen (2011)), recapitalizing banks through the back door, or repairing markets (see, for example, Acharya, Eisert, Eufinger, and Hirsch (2019)).

the un-modeled monetary effects of setting S_0. We then consider whether the planner can counteract some of the financial stability implications of reserves by altering the choice of bank capital structures.

3.1. The Central Bank/Planner 's Problem and Optimal Reserves

The central bank/planner wants to maximize output net of real costs, that is, maximize w.r.t. S_0

$$U \equiv \left((1-q)g_0(I_0) - I_0\right) + q\left(g_1(I_1) - I_1 - (I_1 - D_0^F)\gamma\right) - \tfrac{1}{2}\alpha_0 e_0^2 - \frac{q}{\theta}\left[(1-\theta)\varphi + \theta\right]\left(\tfrac{1}{2}\alpha_1 e_1^2\right) - \tfrac{1}{2}\lambda\left(L_0\right)^2$$

(1.13)

where $(I_1 - D_0^F)$ is the firm's date-1 borrowing from the bank that is associated with a per unit

deadweight monitoring cost γ. It follows that $\dfrac{dU}{dS_0} = \dfrac{\partial U}{\partial \overline{r_1}} \cdot \dfrac{d\overline{r_1}}{dS_0}$ since $\dfrac{\partial U}{\partial S_0} = 0$ (the central bank has no

direct cost of supplying reserves as suggested by Friedman (1969)). It is easily shown (see Appendix III)

that $\dfrac{\partial U}{\partial \overline{r_1}} < 0$. Consequently, the central bank wants to raise S_0 only if it brings down the date-1

interbank market rate premium, that is, $\dfrac{d\overline{r_1}}{dS_0} < 0$. Conversely, if $\dfrac{d\overline{r_1}}{dS_0} > 0$, the central bank wants to

reduce ex-ante reserve issuance. In sum, the central bank will set the reserves at any level such that the

anticipated interbank rate premium $\overline{r_1}$ is zero.[15]

3.2. Discussion

The most important takeaway thus far is that the claims banks issue on liquidity in response to central bank balance sheet expansion may qualify, even alter, conclusions about appropriate policies. For

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[15] What if the threshold level of reserves, $\hat{S}_0$, is negative? When $\theta < \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$, the corporate sector is in

liquidity surplus and any positive level of reserves will set the interbank premium to zero since every dollar of

reserves adds to date-1 liquidity. The problem arises when $\theta > \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$; additional reserves exacerbate the

liquidity shortage since every dollar of date-0 reserves subtracts from date-1 net liquidity. While "negative" reserves may be required in theory, it is unclear how this can be implemented. Perhaps it is best to recognize that when reserves do not add to net future liquidity, the central bank should find other instruments – for instance make long-term loans to the banking sector to encourage the purchase of long-term corporate financial assets/loans, and then be prepared to lend against those assets in case the economy becomes liquidity stressed. Parenthetically, this may resemble the European Central Bank's Long-Term Refinancing Operation (LTRO) interventions.

instance, some commentators argue that central bank balance sheet expansion might reduce systemic risks (see Stein (2012) and Greenwood, Hanson, and Stein (2016)). Essentially, the argument is that central bank reserves will compete with short-term bank deposits for place on private investor portfolios. Being more money-like, the former will displace the latter, and make the financial system safer (by avoiding deposit-induced run risk).

Our analysis qualifies this conclusion by noting that because reserve expansion typically runs through commercial bank balance sheets, the demand for liquidity is affected by the issuance of reserves. Far from crowding out bank deposits, central bank reserve issuance may enhance them (as also pointed out by Nagel (2016)), and may encourage bank issuance of other claims on liquidity.

Greenwood, Hanson, and Stein (2016) argue that the optimal way for the central bank to crowd out the money premium in deposits is to do reverse repo transactions directly with a broader set of non-bank investors – as the Fed started doing in 2021 with money market funds. However, so long as much of central bank reserve expansion runs through bank balance sheets, our qualifications hold.

We must also note an important difference in policy implications between papers that attribute market stress to regulatory and supervisory action (see, for example, Correa, Du and Liao (2021), D'Avernas and Vanderweyer (2021, 2022), IAWG Treasury Market Surveillance Report (2021)) or to an exogenous growth in household financial assets and thus, based on constant portfolio allocations, household demand for deposits (Lopez-Salido and Vissing-Jorgensen (2022)). One policy solution in those papers is to permanently increase reserves and the size of the central bank balance sheet (see Copeland, Duffie, and Yang (2021), for example). Our theory, with banks affecting both the demand for, and the supply of, reserves once the central bank issues them, would qualify this implication because the subsequent endogenous bank response to permanent injections may make matters worse over the medium term (see later).

3.3. The Central Bank/Planner's Problem in Setting the Bank Capital Structure

Could bank capital requirements be an added instrument for the central bank – if for example, it needs to set reserves for monetary reasons at a level that induces a positive r_1? The answer is no when we take as given the fraction φ of banks that lend in the interbank market. While there is a pecuniary externality when a bank makes its financing choice between deposits and capital (the bank takes r_1 as given and does not internalize the fact that its choice of higher deposit financing will increase the net demand for liquidity at date 1 and raise r_1), it is well known that pecuniary externalities need not cause a divergence between the private optimal and social optimal choices. Indeed, we show in Appendix III that

this is the case in our model, unlike Lorenzoni (2008) or Stein (2012) where banks finance excessively with deposits. So the planner will not want to alter bank capital choices.

However, with endogenous φ, there is a divergence between the private and social optimal, but interestingly in a different direction from Lorenzoni (2008) and Stein (2012). The social planner would finance with more deposits and less capital at date 0 than the bank privately would!

Theorem 4: If $\overline{r_1} > 0$, $\left. \dfrac{dU}{de_0} \right|_{e_0 = \frac{qr_1}{\alpha_0}} < 0$, so at the private bank's optimal financing choice, the central

bank/planner wants the bank to finance reserve holdings with less capital.

Proof: See Appendix III.

The reason is interesting. Higher capital issuance up front (by each bank) reduces the ex-post inter-bank rate, thereby increasing the degree of liquidity hoarding, and thus increasing the degree of liquidity shortfall that has to be met by date-1 dissipative capital issues (by all stressed and tainted banks). This externality, it turns out, swamps the direct effect of higher date-0 capital issues reducing the need for date-1 capital issues.

Our result differs from Stein (2012) because the nature of the spillover differs. In Stein (2012), higher bank capital reduces the fire-sale discount, causing lending by non-banks at date 0 to increase (the source of spillover). In a sense, non-banks hoard less ex ante anticipating possible fire sales and that increases activity. In our framework, as we saw in the previous section, the bank makes all lending decisions. So the pecuniary externality embedded in r_1 (our measure of fire-sale discounts) does not distort private financing choices away from the social optimal – there are no non-banks to be influenced. However, when φ is endogenous, something the bank ignores in its maximization, the planner wants to achieve the lowest dissipative capital issues associated with a desired r_1. A higher date-0 capital issue directly reduces liquidity stress and hence r_1, but the higher induced hoarding by others indirectly increases the liquidity shortfall and increases dissipative date-1 equity issuance. The indirect effect swamps the direct effect, implying the social planner wants lower date-0 capital than the private optimal.

Diamond, Jiang, and Ma (2021) and Liang and Parkinson (2020) suggest the supplementary leverage ratio (requiring capital to be held against *all* assets including the relatively safe ones) should not apply to reserves. However, their reason for weaker capital requirements is to eliminate a regulatory encumbrance on the use of reserves (see section IV). Our point is that lower capital may reduce bank incentives to hoard liquidity ex post. Importantly, our focus is on liquidity. Incorporating costly bank insolvency in our model could certainly alter these implications.

3.4. Ex-post Central Bank Intervention

Arguably, the convenience yield on reserves δ is higher in environments where bank assets other than reserves are very illiquid, and where the incidence of systemic stresses are positively serially correlated. This then leads to the possible desirability of ex-post central bank intervention. The central bank could try to bring down $\overline{r_1}$ by lending to banks at date 1 if the economy becomes liquidity stressed. However, emergency borrowing from the central bank, whether from the discount window or the newly created Statutory Repo Facility, could be problematic if it prompts rumors of potential stress at the bank (see Hu and Zhang (2020) or Nelson (2022) for example), which cause other lending to freeze up.

Assuming banks overcome such stigma in extreme situations, there are still important caveats here. First, the most effective way for the central bank to intervene ex post is for it to lend unsecured into the interbank market. However, this entails significant risk of central bank loan losses. If it does lend against high-quality securities, though, the financial sector will have to hold those high-quality securities ex ante. If they are financed with deposit issuance, they add no extra liquidity to a stressed bank, even allowing for central bank lending against them. Of course, the central bank can broaden the range of assets it will lend against (for example, lend against corporate securities) even while increasing the size of the haircut it levies on collateral value. The larger the quantum of intervention, the more the central bank is likely to depart from alleviating just liquidity risk, instead taking on other risks such as credit risk.

All this suggests that there may be a maximum aggregate amount the central bank will feel comfortable injecting in reserves over date 0 and date 1 without taking credit- or political risk (for instance, an amount limited by the aggregate quantum of safe or eligible assets in the economy). If so, it is easy to show that ex-ante central bank balance sheet expansion not only enhances the claims banks write on liquidity at date 0, but also reduces the extent of ex-post central bank intervention, increasing the cum-intervention equilibrium date-1 interest rate.

Central bank reserve injection at date 1 will reduce the interbank rate. But then fewer healthy banks will want to lend into that interbank market. So the act of intervention ex post will crowd out private lending and associated price discovery (see Filardo (2020)), potentially keep interbank markets shut over a wider range of ex-ante reserves, and increase the ex-post quantum of needed central bank intervention. Another subtle effect is also at play. Note that in our model, the incentive to hoard reserves is diminished by the capital issuance of stressed and tainted banks, which reduces the reserves hoarded by safe banks. If, however, capital issuances fall because the central bank adds new reserves to the system (which eventually find their way to the safe banks), it further increases the incentive to hoard over and above any effect on the interbank rate because a greater net stock of reserves flows to the safe banks.

Of course, central bank intervention may also reduce the convenience yield on reserves (assuming the private sector, in response, does not increase the demands for liquidity again – see below), especially if the central bank can commit to lending freely in the foreseeable future without much concern for security or return. This can reduce the incentive to hoard, but if liquidity still does not flow freely through the system under stress, as is likely, it disproportionately raises profits and distorts ex-ante actions of market participants that have privileged access to central bank loans. [16]

Consider this last issue. If leveraged illiquid banks expect to receive central bank funds ex post, they may not reduce their illiquid assets in a timely manner, taking on further liquidity risk in the process (Acharya and Tuckman, 2014). Similarly, the more the financial sector expects central bank intervention, the more it will increase the ex-ante issuance of claims on liquidity, effectively reducing liquidity holdings net of liquidity promises (see Acharya, Shin and Yorulmazer (2011), Diamond and Rajan (2012) or Farhi and Tirole (2012)), and necessitating intervention of yet greater magnitude.[17]

In sum, in the context of the model, illiquidity premia can be brought down ex post through central bank intervention – indeed, some see this as the fundamental purpose of a central bank. Yet, because of the private sector responses we have modeled, it will not be emergency "once and done" intervention, and will tend to be more permanent and increasing, as it has over the past three decades (in 2020, the Fed rolled out many of the programs it had created during the Great Recession plus some more).

Outside the model, the need for further quantitative easing when the central bank is trying to curb inflation may send confusing messages on its monetary stance, and indeed hamper the fight against inflation. Furthermore, balance sheet expansion typically eases government financing, and progressively larger central bank balance sheets induced by commercial bank illiquidity may be tantamount to monetary financing of fiscal deficits, a large cost when an economy is close to fiscal dominance. If there are eventual limits to central bank intervention (for instance tied to the aggregate availability of safe assets, the political willingness to intervene, or the inflation situation), the private sector's greater liquidity dependence at that time could eventually result in significant losses.

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[16] The central bank could offer secured lending to a wider set of market participants against high quality assets than just to the banking sector (see Liang and Parkinson (2020), and the recent move to Standing Repurchase Facilities). This will prevent the banking sector from becoming an impediment to the transfer of liquidity to stressed firms in the financial sector. It will not necessarily ensure that financial firms with surplus liquidity will recirculate it to stressed firms in the real economy or in the banking sector.

[17] Indeed, some central banks recognize that their provision of liquidity on demand creates dependence for more. Nelson (2019) cites a Norges Bank statement in 2010 justifying its move to a deficit reserves position in the system thus: *"When Norges Bank keeps reserves relatively high for a period, it appears that banks gradually adjust to this level…With ever increasing reserves in the banking system, there is a risk that Norges Bank assumes functions that should be left to the market. It is not Norges Bank's role to provide funding for banks…If a bank has a deficit of reserves towards the end of the day, banks must be able to deal with this by trading in the interbank market."*

IV. Shrinkage and Encumbrance of Reserves

We have argued that share τ of reserves are not available at date 1 for any inter-bank transfers, either because of shrinkage or encumbrances. We now explain in greater detail the role of quantitative tightening or QT in shrinking reserves. Then we show that speculation and regulation are two channels through which reserves encumbrance can be endogenized. Finally, we argue our results are robust to assuming τ to be a fixed *level* of reserves (instead of a fixed *share* of reserves).

4.1. Reserves Shrinkage Factor τ due to Quantitative Tightening

Exhibits 1A-1D show that while reserves injection into the banking system during quantitative easing (QE) leads to an expansion of bank balance-sheets via a corresponding increase in demandable bank claims (such as demand deposits and lines of credit), reserves withdrawal during quantitative tightening (QT) is *not* associated with a corresponding reduction in these claims.

To generalize our model in this direction, suppose QT (where the central bank sells government bonds for reserves) just before date 1 shrinks each bank's reserves by τS_0. It also shrinks bank deposits in the process (symmetrically across banks for simplicity) by a factor $\tau^D S_0$ where $\tau^D \leq \tau$. Thus, the bank now holds $(\tau - \tau^D) S_0$ bonds.[18] The date-1 market-clearing condition (1.10) takes the modified form

$$\left[\varphi(1-\theta)+\theta\right]\alpha_1^{-1}r_1 = \theta\left[I_1 + (D_0 - D_0^F) - \tau^D S_0\right] - \left[\varphi(1-\theta)+\theta\right]S_0(1-\tau).$$

It is then straightforward to show that a *necessary* condition for higher S_0 followed by QT to raise r_1 is that $\tau^D < \tau$. Conversely, if QT is exactly a reversal of QE, i.e., $\tau^D = \tau$, then we have to appeal to encumbrances (see below) rather than QT to explain liquidity stress. Further, the sufficient condition for reserves to be destabilizing now takes the form $\theta > \dfrac{\varphi(1-\tau)}{\left(\tau-\tau^D\right)+\varphi(1-\tau)}$.

Empirically, Acharya et al. (2022) find $\tau^D \approx 0$ in the post-QE period before the pandemic, validating our starting assumption in the model that $\tau^D = 0$. This could be, for example, because QT is in large part an asset-swap with banks. Furthermore, Acharya et al. (2002) document that the liquidity distribution across banks deteriorates during QT relative to QE. In our model, this would correspond to τ

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[18] None of our earlier analysis changes since stressed banks cannot use bonds to pay running depositors or to make loans. Of course, bonds can be used as collateral to borrow reserves, but we have assumed no bank borrowing constraints, so the analysis is unaffected when aggregate reserves are in short supply.

being higher for liquidity-stressed banks and/or τ^D being lower for these banks, which would expand the parameter space over which an expansion of ex-ante reserves is likely to be destabilizing.

4.2. Endogenizing τ as Encumbrance Share due to Speculation

Reserves, as we argued in the introduction, have an optionality embedded in them. Ideally, banks would like to sell that option for states they do not need it in (when the economy is healthy), and retain it when the economy is liquidity stressed. Unfortunately, such selective sales of liquidity may be difficult.

In Appendix IV, we sketch a model of how bank prime-brokerage services can expand in line with bank reserves – the model is isomorphic to one of banks issuing contingent lines of credit. Briefly, the model features three ingredients: (i) Speculators, who engage in state-contingent bets with each other because of their private beliefs, incur costs in seeking "prime-brokerage" liquidity support from banks – costs that are declining in the level of bank reserves; (ii) Liquidity support requires banks to fund significant margin calls on the speculator positions in times of stress at date 1;[19](iii) Banks and clients agree on fees (to be paid at date 2 when speculation is "successful") to compensate banks for the opportunity cost of encumbered reserves in the form of margins posted at date 1. Since this opportunity cost arises in the liquidity stress state at date 1 when interbank rate premium $\overline{r_1}$ can exceed zero, prime-brokerage fees rise in $\overline{r_1}$. In turn, the size of speculative positions increases in reserves but at a declining rate in $\overline{r_1}$, and so does the margin-linked encumbrance, which takes the form $\tau(\overline{r_1})\, S_0$, where $\tau'(\overline{r_1}) \leq 0$.

Effectively, this implies then that at low levels of the expected rate $\overline{r_1}$, there is greater speculation, and if liquidity needs in the stressed state rise, then speculative activity is tempered by the expectation of a rising interbank rate, creating an additional equilibrating force that clears the market for reserves. Using $\tau'(\overline{r_1}) \leq 0$, and logic analogous to the proof of Theorem 1, it can be shown that

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[19] The banks may offer liquidity support in all states, but claims may be drawn only in states of liquidity stress, when asset price volatility increases and there is higher counterparty risk (see Aramonte, Schrimpf, and Shin (2021)). Barth and Kahn (2021) provide evidence, for instance, that speculative hedge fund positions in relative value trades in Treasury markets grew from under $200 billion in 2013 to $800 billion in January 2020, rising sharply during 2018 and 2019, with required margins on their futures position rising sharply with the rise of VIX in March 2020.

Theorem 5:

(i) If $\theta > \dfrac{\varphi(1-\tau(0))}{\tau(0)+\varphi(1-\tau(0))}$, then $\overline{r_1} > 0$ is the unique equilibrium for $S_0 > \hat{S}_0$; $\overline{r_1}$ increases with S_0

over a range $[\hat{S}_0, \hat{S}_0^{\,*}]$ till it reaches r_1^{*} where $\dfrac{\varphi(1-\tau(r_1^{*}))}{\tau(r_1^{*})+\varphi(1-\tau(r_1^{*}))} = \theta$, after which $\overline{r_1}$ does not increase

with further increases in S_0. Also $\overline{r_1} = 0$ for $S_0 \leq \hat{S}_0$.

(ii) If $\theta \leq \dfrac{\varphi(1-\tau(0))}{\tau(0)+\varphi(1-\tau(0))}$, then $\overline{r_1} > 0$ is the unique equilibrium for $S_0 < \hat{S}_0$; $\overline{r_1}$ increases as S_0 falls

till it reaches r_1^{**} at $S_0 = \hat{S}_0^{\,**}$ where $\dfrac{\varphi(1-\tau(r_1^{**}))}{\tau(r_1^{**})+\varphi(1-\tau(r_1^{**}))} = \theta$, after which $\overline{r_1}$ does not increase with

further decreases in S_0. Also $\overline{r_1} = 0$ for $S_0 \geq \hat{S}_0$.

In essence, case (i) which formalizes our novel insight continues to hold with the endogenous modeling for reserves encumbrance. As long as additional reserves create a net demand for liquidity when the interbank rate is zero, that is, $\theta > \dfrac{\varphi(1-\tau(0))}{\tau(0)+\varphi(1-\tau(0))}$, increasing reserves eventually leads to an interbank rate that is greater than zero. It rises with reserves until the speculative encumbrance τ falls to the point that an incremental increase in reserves does not change the net demand for liquidity (or $\overline{r_1}$ or $\tau(\overline{r_1})$). Importantly, reserves encumbrance can be readily endogenized to mirror the reality that banks and speculators have mutual interest in "locking up" reserves for future risk management.

4.3. Endogenizing τ as Encumbrance Share due to Regulation

To offset speculation, regulators may place their own encumbrances on reserves (see Farhi, Golosov and Tsyvinski (2009) or Calomiris, Heider, and Hoerova (2014)). The most obvious such regulation is a requirement that a certain fraction of assets have to be held at all times in the most liquid form (see, for example, Diamond and Kashyap (2016)) or a capital requirement that binds precisely when a bank ought to lend out its excess reserves (see, for example, Vanderweyer (2019)). Such regulations are likely to be insufficiently contingent.

Why cannot such requirements be dropped in times of stress? As Goodhart (2008) emphasizes, a policy of having at least one taxi at the station is of little benefit to the late-arriving traveler if it cannot be used. Diamond and Kashyap (2016) argue, however, that it may make sense for the regulator to prevent a bank from using up liquid reserves in stressed times if the anticipation of use causes the stress to spread –

if savers believes healthy banks have no mandated liquid assets and might lend them all to stressed banks, they may run on all banks. We incorporate such regulatory requirements in Appendix IV.

Cautious bank behavior in response to uncertain regulation could also amplify encumbrances. D'Avernas and Vanderweyer (2021) attribute enhanced volatility and fragility in repo markets to implicit supervisory mandates on intra-day bank liquidity holdings. They cite Jamie Dimon, CEO of JP Morgan *"[...] we have $120 billion in our checking account at the Fed, and it goes down to $60 billion and then back to $120 billion during the average day. But we believe the requirement under CLAR and resolution and recovery is that we need enough in that account, so if there's extreme stress during the course of the day, it doesn't go below zero."* In other words, uncertainty about enforcement appears to have forced JP Morgan to hold a portion of reserves back for really extreme market events – since no one really knows what these might be, some portion of the reserves might be permanently encumbered.

Relatedly, Nelson (2019) documents that in a Bank Policy Institute (BPI) survey conducted in January 2019, bank examiner expectations about liquidity holdings were mentioned overwhelmingly as "important" or "very important" reasons for reserve demand by banks. Indeed, Nelson points out that in times of abundant reserves, bank supervisors scrutinize any drawdowns carefully, creating a ratchet effect (higher the held reserves, higher the reserves supervisor expect) limiting the ability of healthy banks to redeploy reserves when needed. Indeed, a recent discussion paper by the Bank of England (2022) flags such behavior during stress episodes in the pandemic, and seeks (as part of its prudential liquidity framework) to induce banks to use their surplus liquidity even when the system is stressed.

4.4. Fixed Encumbrance on Reserves ($\tau S_0 \equiv \overline{E}$)

Suppose that instead of a constant fraction, the regulatory encumbrance is a fixed amount $\overline{E}$ of required reserves, independent of total reserves, S_0. Our analysis of Section 2.2 carries over to this case even though with a fixed encumbrance, an increase in reserves cannot shrink ex-post liquidity simply because it is financed with deposits. However, the novelty here is that as long as there is a convenience yield on reserves in the stressed state, an increase in reserves increases the returns to hoarding and staying safe to attract flight-to-quality deposits; this reduces in turn the fraction of surplus banks in the interbank market (which may remain shut altogether). For reasons of space, we leave the formal statement of results and analysis to Appendix IV (Theorem 6).

V. Robustness

We now elaborate on some of the assumptions we have made and discuss their robustness.

5.1. Reserve Expansion, Activity and Financial Fragility

Thus far we have taken the central bank's monetary motives in injecting reserves as given, and focused on the consequences of commercial bank decisions. Since there are many rationales for why central bank reserve expansion could directly affect date-0 activity, let us focus on an obvious one – it alleviates an explicit or implicit reserve requirement on the commercial bank's deposit issuance. Explicit reserve requirements used to be the norm, but more recently, implicit requirements such as a liquidity coverage ratio (LCR), wherein liquid assets must exceed some "run-offs" on deposits (and other demandable claims on the bank), have taken their place. We will show that with a binding reserve requirement, deposits issued can be a multiple greater than 1 of reserves, and we no longer need $\tau > 0$ for

$\frac{dr_1}{dS_0} > 0$. Interestingly, real activity need not be enhanced by additional reserves, even when the reserve

constraint is binding.

Suppose we add to the model in section I a requirement that a bank's deposits cannot exceed a multiplier $\varsigma > 1$ of its reserves holding, i.e., $D_0 \leq \varsigma S_0$. The requirement likely binds when date-0 equity issue costs, α_0, are so high that no additional equity is issued. We assume this for simplicity, though by continuity, the results will hold when equity issuance costs are merely high. When the requirement binds, deposits are no longer the residual from the date-0 funding constraint (equation 1.4). Formally, denoting the Lagrangian on the reserve constraint on deposits as $\Lambda \geq 0$, the cost of deposits when the constraint binds is $(1 + qr_1 + \Lambda)$. Then $(1 + \lambda L_0) = (1 + qr_1 + \Lambda)^{-1}(1 + q\gamma + qr_1)$. As before, we have

$$\frac{r_1}{\alpha_1} = \frac{\theta}{[\varphi(1-\theta)+\theta]}\left[I_0 + I_1 - L_0 - W_0^F + D_0\right] - S_0(1-\tau) \equiv f(r_1, S_0). \text{ Totally differentiating,}$$

$\left(\frac{1}{\alpha_1} - \frac{\partial f}{\partial r_1}\right)\frac{dr_1}{dS_0} = \frac{\partial f}{\partial S_0}$. We know $\frac{\partial I_0}{\partial r_1} < 0$ and $\frac{\partial I_1}{\partial r_1} < 0$. Also, when the deposit constraint binds, i.e.,

$D_0 = \varsigma S_0$, and with no date-0 equity issuance, the date-0 resource constraint for the bank takes the form

$L_0 + \frac{1}{2}\lambda(L_0)^2 = (\varsigma - 1)S_0$, whereby $\frac{\partial D_0}{\partial r_1} = 0$ and $\frac{\partial L_0}{\partial r_1} = 0$, so we have $\frac{\partial f}{\partial r_1} < 0$ as before. Therefore,

$Sign\left(\frac{dr_1}{dS_0}\right) = Sign\left(\frac{\theta}{[\varphi(1-\theta)+\theta]}\left(\frac{\partial D_0}{\partial S_0} - \frac{\partial L_0}{\partial S_0}\right) - (1-\tau)\right)$. Furthermore, $\frac{\partial D_0}{\partial S_0} = \zeta$ and

$$\frac{\partial L_0}{\partial S_0} = \frac{(\zeta - 1)}{1 + \lambda L_0} < (\zeta - 1). \text{ So } \frac{dr_1}{dS_0} > 0 \text{ if and only if } \theta > \frac{\varphi(1 - \tau)}{\left(\zeta - \frac{(\zeta - 1)}{1 + \lambda L_0}\right) - (1 - \varphi)(1 - \tau)}, \text{ which can also}$$

be expressed as the condition $\theta > \dfrac{\varphi(1 - \tau)}{\tau + \varphi(1 - \tau) + (\zeta - 1)\dfrac{\lambda L_0}{1 + \lambda L_0}}$, the same as earlier except for the extra

positive term in the denominator $(\zeta - 1)\dfrac{\lambda L_0}{1 + \lambda L_0}$.

Note that even if $\tau = 0$, the condition can now hold because the right hand side of the condition is always smaller than 1. When this condition does not hold, the date-1 rate (weakly) falls with reserves and date-0 investment (weakly) rises. This is the typical monetary effect of reserve expansion. When the condition does hold, however, the date-1 demand on liquidity increases with reserves more than one for one. So somewhat paradoxically, date-0 corporate investment falls (because r_1 is higher), date-0 firm borrowing L_0 increases, and the excess is stored as a deposit with the bank (equivalently, this is a committed line of credit). Then, central bank balance sheet expansion raises term lending and corporate leverage but this translates into greater corporate deposits or savings rather than real investment.

It has been hard to discern the net positive macroeconomic effects of quantitative easing (see Fabo, Jancokova, Kempf, and Pastor (2021), Greenlaw et al. (2018) and Moreno (2019)). One reason is perhaps because so much else is going on over the period of the interventions. Our analysis suggests another possible reason – the possible offsetting effects of central bank balance sheet expansion via an even greater expansion of banking sector's demandable deposits; the latter can lead to financial fragility and raise the interbank premium r_1, which dampens investment. This may partly account for why the effects of unconventional monetary policy are hard to discern.

5.2. Nature of Liquidity Shock

We have assumed the liquidity shock at date 1 is a shock to firm fundamentals – and thereby affects both sides of the funding bank's balance sheet (in terms of loan demands and deposit withdrawals). Yet to the extent that liquidity shocks precipitate solvency concerns and hoarding, any large-scale flows out of the financial system such as firms drawing down credit lines (following the collapse of Lehman Brothers, see Ivashina and Scharfstein (2010), and at onset of the pandemic, see Acharya, Engle and Steffen (2021)) would trigger similar effects.

An important simplification is that the share of stressed banks, θ, is invariant to the build-up in reserves. It might seem that the risks to commercial bank balance sheets should fall as the share of reserves composing those balance sheets increases. If so, our model would hold for only the range of reserve expansion where commercial bank credit risk is not swamped by reserve expansion. Yet this neglects three possible sources of risk.

First, Acharya et al (2022) find that incremental reserves are more than funded by demandable deposits (with a decline in time deposits accounting for the difference, presumably because liquidity ratios and capital requirements do not bind with accumulation of reserves). So absent commercial bank capital-raising, even small amounts of credit risk relative to the size of the commercial bank's assets can have large consequences. Second, the un-modeled effects of central bank balance sheet expansion on activity, if sizeable, should expand corporate borrowing and increase the risk thereof (that is, it should increase I_0). Third, we have assumed the encumbrance on reserves from speculative activity to be risk-free. In practice, some of it will be risky (speculators will go bust). Our model in Section IV (also see Appendix IV) of speculation determining the encumbrance share τ suggests that a prolonged period of easy money could cause the encumbrance rate to rise. The same factors causing greater speculation could also cause θ to rise. Thus, a steady infusion of reserves ex ante could switch the system from alleviating future liquidity stress to exacerbating it. A deeper analysis of the underpinnings of θ and τ, and their related dynamics, is therefore an important avenue for future research.

Two recent examples of greater need for liquidity in the non-bank sector due to hedging or speculation are (i) the demand for UK pension funds for loans as their Liability-Driven Investment (LDI) strategies led to large margin calls in October 2022, prompting intervention by the Bank of England, and (ii) the spectacular implosion of crypto-exchange FTX in November 2022 as its speculative bets fizzled and it could find no one that would lend.

5.3. Other Extensions

Our simple model allows for many other possible extensions and explorations. Two are worth sketching. First, what if banks were not forced to hold S_0? It turns out, not surprisingly, that banks will privately not have the same incentives as the planner/central bank and will want to optimally hold different levels of reserves (often lower) in a variety of circumstances (see Appendix V). Note that the same analysis can be applied to voluntary holdings of Treasuries.

Finally, what if the central bank issues reserves directly to the non-bank financial sector? Here again (see Appendix V), a desire to match the duration of liabilities with assets to reduce risk will result in

non-banks financing with short-term liabilities. Many of the consequences we have documented will follow. In April 2021, the Federal Reserve reinstated the supplementary leverage ratio (SLR) for commercial banks. This is a regulatory capital requirement that was suspended in April 2020 in the wake of the pandemic (see Covas, 2021). Given the increased cost to banks of funding reserves with long-term capital, they released reserves. Interestingly, money market funds, themselves funded with short-term liabilities, took on the reserves, redepositing them at Fed through reverse-repo facilities. This suggests the natural way for intermediaries, even from the non-bank financial sector, to fund reserves is short-term and will likely lead to concerns of financial fragility akin to the ones we have analyzed for the banking sector.

VI. Conclusion

The significant expansion of central bank balance sheets in recent years should have reduced liquidity stress, and even perhaps increased real activity. We propose that that the ex-ante supply of reserves affects the ex-post demand for them and that this is a key reason why central bank balance sheet expansion may be less helpful in stress situations. Consequently, its usefulness in enhancing real activity may be more dampened, than one might think a priori. We have likely only scratched the surface in sketching out implications of this phenomenon.

While we have used the liquidity stress in Treasury repo markets in September 2019 as a motivating example, our paper should not be seen as an alternative to explanations based on micro-disruptions (for example, the buildup of reserves in the Treasury account as in Copeland, Duffie, and Yang (2021)) or the effects of new implicit regulations on reserve maintenance, as in D'Avernas and Vanderweyer (2021)). Instead, it suggests a more general consequence of central bank balance sheet expansion, the issuance of claims on liquidity by commercial banks, which might leave the system prone to disruptions. In that sense, it should be seen as a complementary argument that has broad implications.

Our paper and the subsequent empirical work suggest numerous avenues worth exploring. For instance, Acharya et al. (2022) find that when quantitative easing ceased, and even reversed, bank-issued claims on liquidity did not shrink and even rose for many banks, further exacerbating liquidity tightness. Some of this may be because reserve expansion is the instigator, and the expanded claims on liquidity are the lagged response. It may also be that it is hard for banks to shrink the businesses that issue these claims, or once issued, banks feel they can collectively rely on the central bank to make up liquidity shortfalls. Finally, it may be that some banks see high quality financial assets as sufficient substitutes for reserves, not internalizing the fact that the ultimate form of settlement is reserves, and when everyone rushes to get reserves, some will be left high and dry. The causes of liquidity dependence need to be explored carefully. There is clearly more work to be done.

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Exhibit 1. Time-Series of Credit Lines, Deposits and Reserves

This figure plots the time-series of credit lines, deposits and reserves of the 2008 to 2021 period using data from the Federal Reserves' Flow of Funds. Panel A plots credit lines (left y-axis), deposits (right y-axis) and reserves (left y-axis) as percentage of gross domestic product (GDP) for all commercial banks. Panel B shows demand and other liquid deposits (right y-axis), time deposits (left y-axis) and reserves (left y-axis) all as percentage of GDP. Time deposits are the sum of small and large time deposits (H6 and H8 release). Demand and other liquid deposits are from the H6 release. All data are obtained from the Federal Reserve Economic Data (FRED) online database. Panel C shows the break-up of demand and time deposits into insured and uninsured time series using FDIC's Call Reports Data. Estimation of Insured and Uninsured Domestic Deposits are based on the items in the call report schedule RC-O. Insured deposits are defined as deposits lying below the FDIC deposit insurance thresholds of $100,000 before 2008Q4 and $250,000 after 2008Q4. Uninsured deposits are domestic deposits above the aforementioned deposit insurance thresholds and all foreign deposits. Insured deposits are adjusted for the FDIC Transaction Account Guarantee (TAG) program. Split of Time Deposits into Insured vs. Uninsured Deposits are based by splits of Time Deposits by the aforementioned deposit insurance thresholds in schedule RC-E. Non-time Insured and Uninsured deposits are estimated by taking the difference of Total Insured/Uninsured Deposits and Insured/Uninsured Time Deposits respectively. Non-time Deposits are labelled as Demand+Savings Deposits. The vertical lines correspond to the beginning of the different Federal Reserve QE / QT phases: (1) Nov 2008 (QE I), (2) Nov 2010 (QE II), (3) Nov 2012 (QE III), (4) Oct 2014 (Post-QE III), (5) QT period, (6) Sept 2019 (Pandemic QE). These exhibits are respectively reproduced from Figures 1A, 1Cand 1E of Acharya, Chauhan, Rajan and Steffen (2022).
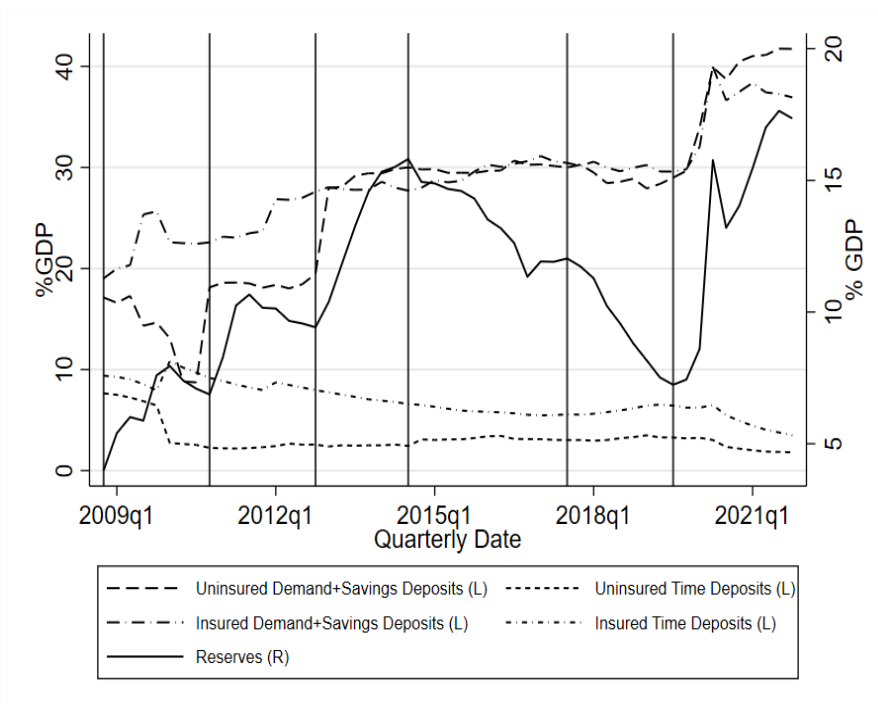
A. Reserves, Deposits, and outstanding Credit Lines as a percentage of GDP
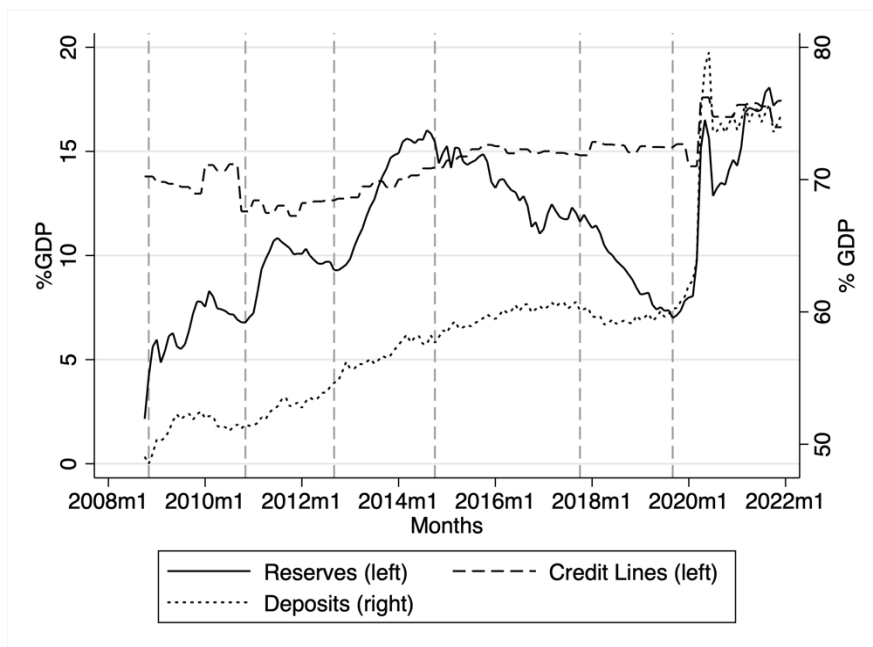
B. Demand (and other Liquid) Deposits and Time Deposits vs. Reserves
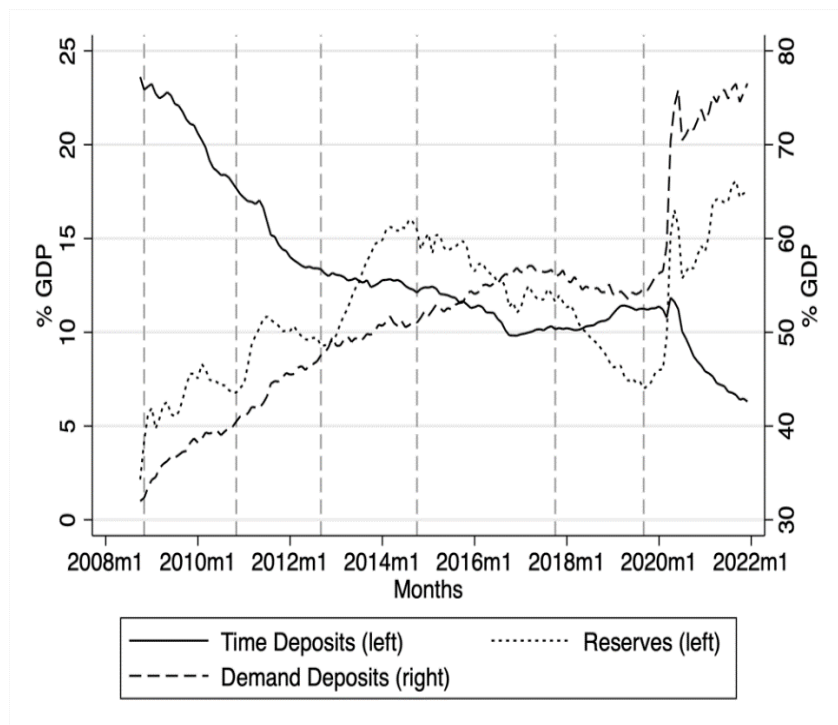
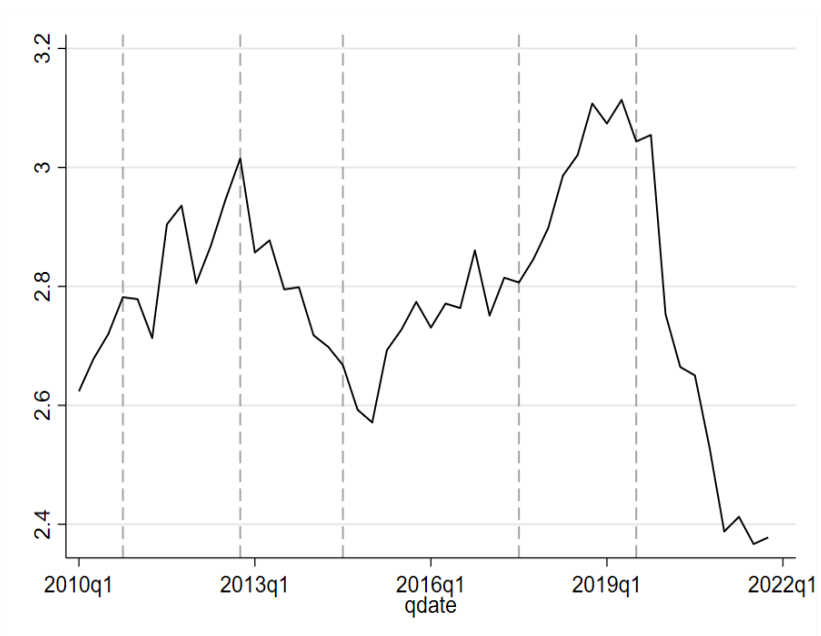**C. Uninsured and Insured Demand and Time Deposits vs. Reserves**

D. Ratio of Credit Lines plus Demand Deposits to Reserves plus Eligible Assets

Exhibit 2: Bank and Firm Balance Sheets

Firm Balance Sheet at Date 0	
Assets	**Liabilities**
I_0	$L_0^F \quad (= L_0^B)$
D_0^F	W_0^F
	Net worth

Bank Balance Sheet at Date 0	
Assets	**Liabilities**
$L_0^B + \tfrac{1}{2}\lambda (L_0^B)^2$	D_0
S_0	e_0
	Net worth

Firm Balance Sheet at Date 1 if stressed	
Assets	**Liabilities**
I_1	l_1^F
	L_0^F
	Net worth

Bank Balance Sheet at Date 1 if bank stressed	
Assets	**Liabilities**
$L_0^B + \tfrac{1}{2}\lambda (L_0^B)^2$	Possible interbank borrowing $= b_1$
τS_0	e_1
$l_1^B \quad (= l_1^F)$	e_0
	Net worth

Firm Balance Sheet at Date 1 if healthy	
Assets	**Liabilities**
I_0	L_0^F
D_0^F	W_0^F
	Net worth

Bank Balance Sheet at Date 1 if economy stressed, bank healthy but "tainted" (makes interbank loans)	
Assets	**Liabilities**
$L_0^B + \tfrac{1}{2}\lambda (L_0^B)^2$	D_0
Interbank loans of up to $e_1 + (1-\tau)S_0$	e_1
Reserves of $(S_0 + e_1$-interbank loans$)$	e_0
	Net worth

Figure 3: Numerical example for the model of section 2.2 with endogenous interbank entry:
The effect of varying the size of the shock θ

Figure 3A: Market Rate and Reserves

$$\delta = 0.2, \; \tau = 0.2 \;, \theta = 0.8$$

Figure 3B: Market Rate and Reserves

$$\delta = 0.2, \; \tau = 0.2 \;, \theta = 0.6$$

Figure 4: Numerical example for the model of section 2.2 with endogenous interbank entry:
The effect of varying the convenience yield on reserves δ
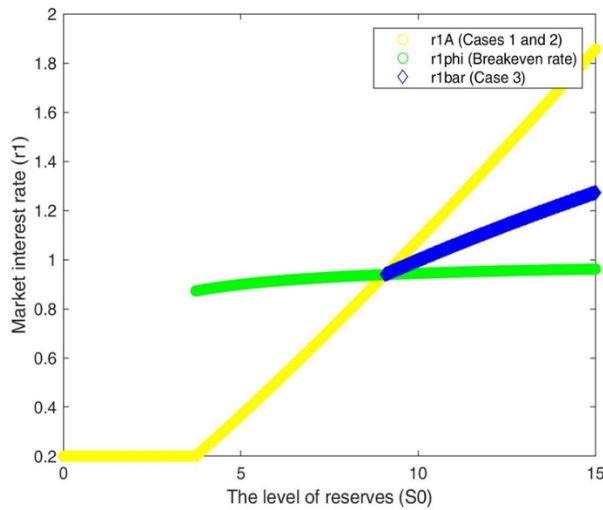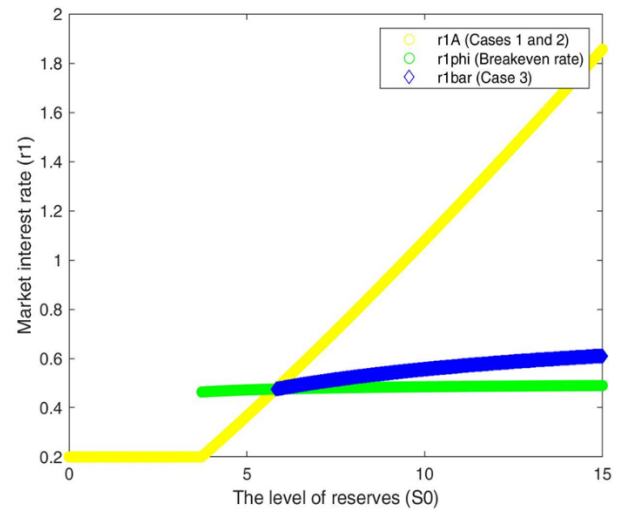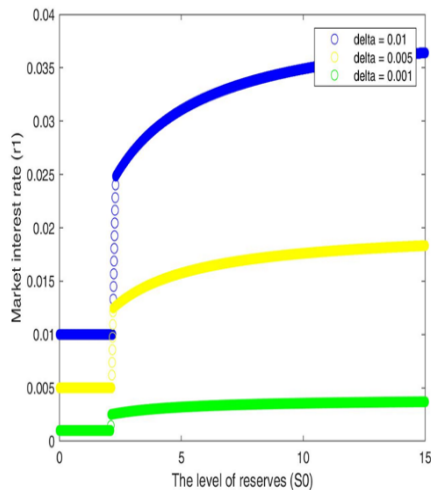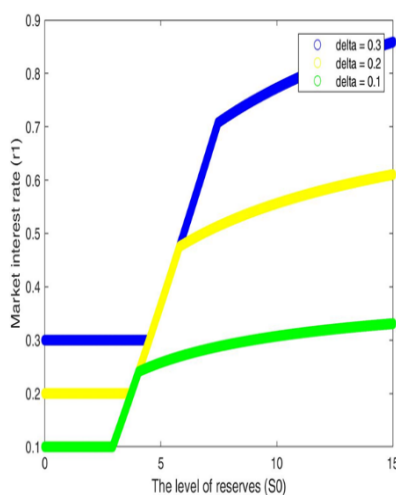

Figure 4A: Market Rate and
Reserves for low δ

Figure 4B: Market Rate and
Reserves for high δ
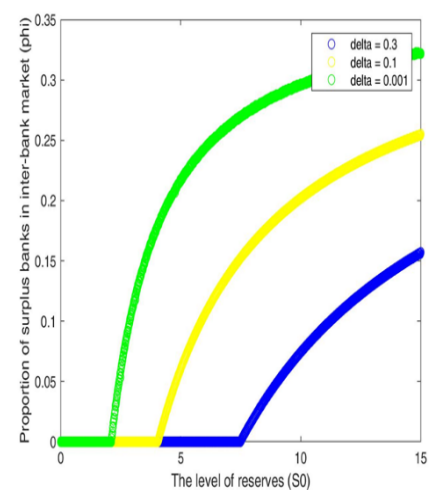
Figure 4C: φ and reserves for
various δ

Appendix I – Proofs for Section 2.1

Proof of Lemma 1: The proof essentially demonstrates that the net need for capital issuance, $f(r_1, S_0)$, falls in r_1. To see this, note that the right hand side of (1.10) is decreasing in r_1 (we will see this shortly). The left hand side is obviously increasing in r_1. If so, if the right hand side of (1.10) is positive when $r_1 = 0$ then there is excess demand for funds in the inter-bank market when the premium is zero, and hence there is an unique positive crossing point, the equilibrium r_1. If the right hand side is non-positive when $r_1 = 0$, there is (weakly) excess supply, and r_1 is zero. So it remains to show the right hand side of (1.10) is decreasing in r_1. From (1.3), $I_1 = g_1'^{-1}(1 + \gamma + r_1)$, which is decreasing in r_1. Turn next to the second term in the square brackets, $(D_0 - D_0^F)$. This equals

$$\left[I_0 - e_0 + (S_0 - W_0^F) + \tfrac{1}{2}\lambda(L_0)^2 \right]. \text{ We know } I_0 = g_0'^{-1}\left(\frac{1 + q\gamma + qr_1}{(1-q)} \right) \text{ which is decreasing in } r_1. \text{ Also,}$$

$-e_0 = -\dfrac{qr_1}{\alpha_0}$, which is decreasing in r_1. The next term, $(S_0 - W_0^F)$, is a constant. That leaves the last

term, in the expression for $(D_0 - D_0^F)$, $\tfrac{1}{2}\lambda(L_0)^2$. From (1.9), $L_0^B = \dfrac{1}{\lambda}\left(\dfrac{R_0^{DF} - R_0^{DB}}{R_0^{DB}} \right) = \dfrac{q}{\lambda}\left(\dfrac{\gamma}{1 + qr_1} \right)$,

which decreases in r_1 whence given L_0 is positive, $\tfrac{1}{2}\lambda(L_0)^2$ also decreases in r_1. So we have $(D_0 - D_0^F)$, the deposits the bank raises from the public, decreasing in r_1. Finally, the last term on the right hand side of (1.10), $-\alpha_1 S_0(1 - \tau)$, is a constant. So the right hand side of (1.10) is decreasing in r_1 and the equilibrium r_1 is unique.

Next, we examine how does the possible positive equilibrium rate, $\overline{r_1}$, implicitly determined by (1.10), vary with central bank reserves. Totally differentiating (1.10), we have $\dfrac{1}{\alpha_1}\dfrac{d\overline{r_1}}{dS_0} = \dfrac{\partial f}{\partial r_1} \cdot \dfrac{d\overline{r_1}}{dS_0} + \dfrac{\partial f}{\partial S_0}$.

Therefore, $\left(\dfrac{1}{\alpha_1}-\dfrac{\partial f}{\partial r_1}\right)\dfrac{d\overline{r_1}}{dS_0}=\dfrac{\partial f}{\partial S_0}=\dfrac{\theta}{\left[\varphi(1-\theta)+\theta\right]}-(1-\tau)$. [20] Since $\dfrac{\partial f}{\partial r_1}$ is negative as shown in the

proof of Lemma 1, $\left(\dfrac{1}{\alpha_1}-\dfrac{\partial f}{\partial r_1}\right)$ is positive, and $Sign\left(\dfrac{d\overline{r_1}}{dS_0}\right)=Sign\left(\dfrac{\theta}{\left[\varphi(1-\theta)+\theta\right]}-(1-\tau)\right)$. Q.E.D.

Proof of Theorem 1: Because $f(r_1,S_0)$ is decreasing in r_1, it must be that $\overline{r_1}$ is positive iff

$f(0,S_0)>0$. We have

$$f(r_1,S_0)=\dfrac{\theta}{\left[\varphi(1-\theta)+\theta\right]}\left(g_1'^{-1}\left(1+\gamma+r_1\right)+g_0'^{-1}\left(\dfrac{1+q\gamma+qr_1}{(1-q)}\right)-\dfrac{qr_1}{\alpha_0}-W_0^F+\dfrac{1}{2}\dfrac{q^2}{\lambda}\left(\dfrac{\gamma}{1+qr_1}\right)^2\right)$$
$$+S_0\left(\dfrac{\theta}{\left[\varphi(1-\theta)+\theta\right]}-(1-\tau)\right)$$

So, for $f(0,S_0)>0$, it must be that

$$S_0\left(\dfrac{\theta}{\left[\varphi(1-\theta)+\theta\right]}-(1-\tau)\right)>\dfrac{\theta}{\left[\varphi(1-\theta)+\theta\right]}\left(-g_1'^{-1}\left(1+\gamma\right)-g_0'^{-1}\left(\dfrac{1+q\gamma}{(1-q)}\right)+W_0^F-\dfrac{1}{2}\dfrac{q^2}{\lambda}(\gamma)^2\right)\equiv NLS.$$

The theorem then follows. Q.E.D.

Threshold level of reserves when *NLS* > 0: Recall that NLS is the net liquidity supplied by the corporate sector anticipating a date-1 interbank premium of zero (and adjusting for any cost to lending).

When $NLS>0$ and the risk of liquidity stress in the economy is high, *that is,* $\theta>\dfrac{\varphi(1-\tau)}{\tau+\varphi(1-\tau)}$ (this is

the region to the right of the vertical axis in Figure 2B), $\hat{S}_0$ is positive, and falls in θ. Intuitively,

because higher ex-ante reserves tighten liquidity in the stressed state, and a higher θ consumes more

liquidity per dollar of reserves, the reserve threshold at which the net liquidity supplied by the corporate

sector is fully consumed is positive and falls in θ. Furthermore, $\overline{r_1}$ increases in S_0, and the unhatched

region above the $\hat{S}_0$ curve is where $\overline{r_1}$ is positive. When $NLS>0$ and the risk of liquidity stress in the

---

[20] This requires substituting in (1.10)

$$\left(D_0-D_0^F\right)=\left(S_0+L_0+\tfrac{1}{2}\lambda(L_0)^2-e_0\right)-\left(W_0^F+L_0-I_0\right)=I_0-e_0+(S_0-W_0^F)+\tfrac{1}{2}\lambda(L_0)^2.$$

economy is low, *that is,* $\theta < \dfrac{\varphi(1-\tau)}{\tau+\varphi(1-\tau)}$ (that is, in the region to the left of the vertical axis in Figure

2B), $\hat{S}_0$ is negative, and falls in θ. Because higher ex-ante reserves loosen liquidity conditions, $\overline{r}_1$ falls

in S_0, and the unhatched region below the $\hat{S}_0$ curve is where $\overline{r}_1$ is positive. The hatched area is where $\overline{r}_1$

is zero.

$NLS > 0$ $\hat{S}_0$ $\overline{r}_1 > 0$

$\overline{r}_1 = 0$

$\theta = 0$ $\theta = 1$

$\hat{S}_0$ $\overline{r}_1 = 0$

$\overline{r}_1 > 0$
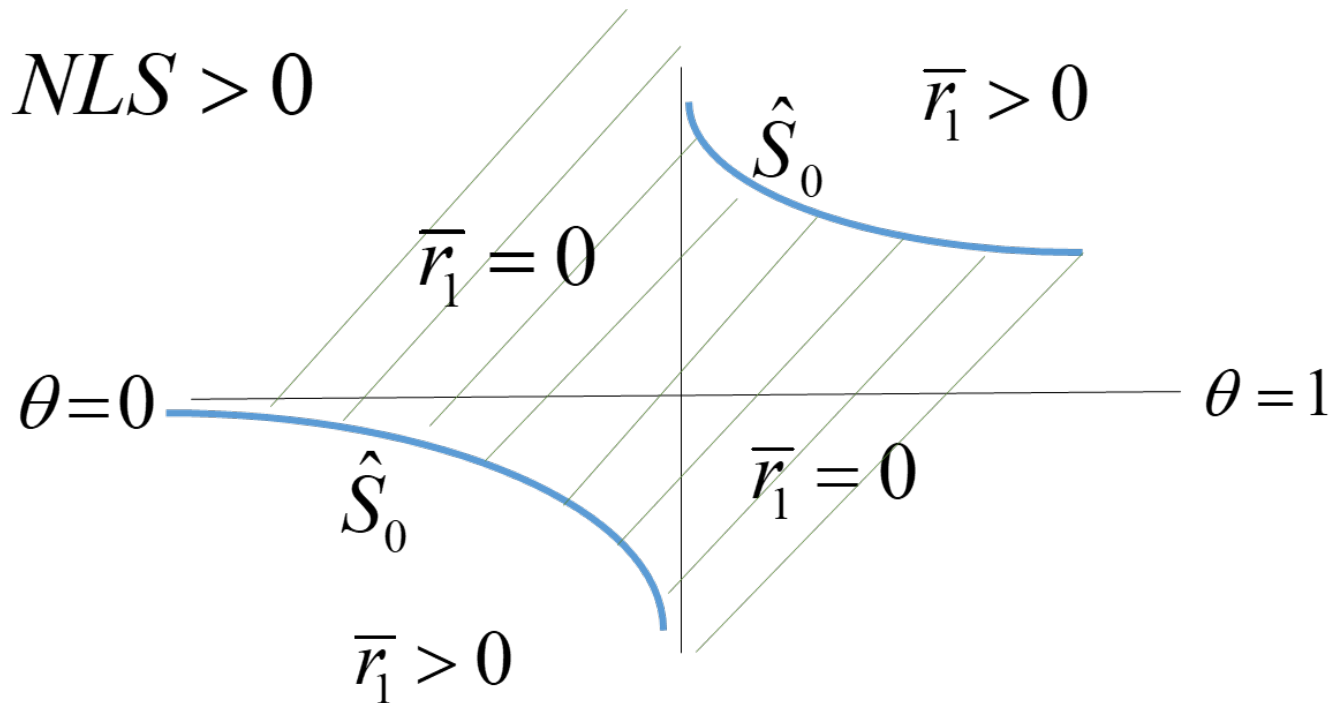
Figure 2B: Reserves are on the y axis, θ on x axis, with the two axes intersecting at $\theta = \dfrac{\varphi(1-\tau)}{\tau+\varphi(1-\tau)}$

Appendix II – Proofs and Analysis for Section 2.2

Bank Choices at Date 1 (in the presence of a convenience yield δ)

Consider the three cases for the aggregate liquidity condition at date 1. We will denote the incremental

value of a bank at date 1 as $V_1(y,z)$ where recall that $y=1$ if the economy is liquidity stressed and zero

otherwise, while $z=1$ if the bank is stressed and zero otherwise.

Case 1: *Stressed banks have enough liquidity to meet the needs of deposit outflows and to fund rescue investment without accessing the inter-bank market.*

Since reserves have a convenience yield δ, stressed banks will issue some capital e_1 to add to reserves even if they do not need to use it for loans or deposit outflows. Furthermore, no bank will loan out reserves without earning at least the convenience yield. Finally, since liquidity is in surplus, any competition to make bank loans would push the bank lending rate down to the convenience yield. The

bank solves $\max_{e_1} V_1(y=1, z=1) \equiv \left[r_1\left(I_1(r_1) - D_0^F\right) - \delta\left(D_0 - D_0^F + I_1(r_1) - e_1\right) - \frac{\alpha_1}{2}e_1^2 \right]$ where the

first term of the maximization is the return on loans, the second term the cost of the reserve outflow reduced by the inflow of capital, while the last term is the incremental cost of raising capital over and above the gross cost of 1. Since $r_1 = \delta$, the stressed bank makes no profit from the rescue loan. Solving,

$e_1 = \frac{\delta}{\alpha_1} > 0$ even if the stressed bank has no need to use the funds to meet depositor outflows or loan

demand (details are in proofs of Theorems 2-3 below but follows the structure in Section 2.1).

Safe banks will not issue capital since they know capital issuance will not alter their reserves on net – any investor in capital will first acquire reserves from the safe banks to buy the capital.[21] Since there is no need for interbank loans, no healthy bank will become tainted. This means that the reserve outflows from the stressed banks are spread across all the healthy banks, and their incremental date-1 value is

$V_1(y=1, z=0) = \dfrac{\delta\theta(D_0 - D_0^F + I_1 - e_1)}{(1-\theta)}$ where the numerator is the value of flight-to-safety deposit

outflows (plus new deposits created by purchases less capital issued) to the healthy banks, and the

denominator is the measure of healthy banks. It also follows then at date 0, $e_0 = \dfrac{q\delta}{\alpha_o}$.

Finally, this case arises when the stressed bank's reserves are enough to meet the demands on it, that is, $S_0(1-\tau) \geq \left(D_0 - D_0^F + I_1 - e_1\right)$. Substituting for the endogenous $(D_0 - D_0^F)$, we see this case

arises when $\tau S_0 \leq \left[\left(\dfrac{q\delta}{\alpha_0} + \dfrac{\delta}{\alpha_1}\right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2 \right]$ where I_1 is the optimized value

evaluated at $r_1 = \delta$, I_0 at $R_0^L = (1 + q\gamma + q\delta)$ and $L_0 = \dfrac{q\gamma}{\lambda(1+q\delta)}$ (see proofs of Theorems 2-3 for all

steps). Note that since r_1 is a constant, as S_0 increases deposits increase dollar for dollar since no

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[21] Of course, safe banks may issue capital assuming it will come from reserve flows from other safe banks. If everyone does this, no one will have any additional reserves, but everyone will have issued capital commensurate with the size of the convenience yield and incurred the associated costs. Allowing for this adds little to the analysis.

additional capital is issued. Since a fraction τ of the reserves will be encumbered, the distressed bank's net need for date-1 funds grows as S_0 grows. Eventually, it will exhaust available own funds at date 1, and have to issue more capital (compared to the amount that would be optimal considering only the convenience yield). This is when the economy moves into Case 2.

<u>Case 2:</u> *The liquidity needs of each stressed bank can be entirely met by its raising date-1 capital (beyond that warranted by the convenience yield).*

Now, the rate at which the stressed bank lends to the firm, r_1, rises above δ to incentivize further date-1 capital-raising. However, the rate stays too low for any of the healthy banks to lend in the interbank market. Essentially, the stressed bank is in *autarky* and has to issue costly capital even though there is plentiful lending capacity in the system. Let the equilibrium bank lending rate in autarky be r_1^A.

The stressed bank maximizes

$$V_1(y=1, z=1) = \max_{e_1} \left[r_1^A \left(I_1(r_1^A) - D_0^F \right) - \delta S_0 (1-\tau) - \frac{\alpha_1}{2} e_1^2 \right] \quad \text{such that}$$

$e_1 = \left(D_0 + I_1(r_1^A) - D_0^F - S_0(1-\tau) \right)$. It follows that $e_1 = \dfrac{r_1^A}{\alpha_1}$ (and $e_0 = \dfrac{q r_1^A}{\alpha_0}$). As before, a rise in the date-1 interest rate equilibrates the demand and supply of liquidity by decreasing the size of the rescue investment and increasing the capital raised. Expanding the constraint for the maximization, we get

$\dfrac{r_1^A}{\alpha_1} = \left(I_0 + I_1 + \frac{1}{2} \lambda L_0^2 - W_0^F - \dfrac{q r_1^A}{\alpha_0} + \tau S_0 \right)$. Furthermore, because the stressed banks are on their own, once again an increase in ex-ante reserves S_0 always raises r_1^A, regardless of the size of τ (so long as $\tau > 0$).[22]   Since the stressed banks just meet liquidity demand using all their unencumbered reserves, the healthy banks get all of it. So $V_1(y=1, z=0) = \dfrac{\delta \theta S_0 (1-\tau)}{(1-\theta)}$.

<u>Case 3:</u> *The liquidity needs of the stressed banks are high enough that at the equilibrium autarkic interest rate, some of the healthy banks are willing to lend in the interbank market and become tainted. The equilibrium rate then is lower than the (now counterfactual) autarkic rate.*

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[22] Since all the endogenous variables on the right hand side are decreasing in r_1^A while the left hand side is increasing in r_1^A, there is a unique equilibrium r_1^A, and S_0 shifts it up whenever $\tau > 0$.

Given the analysis in Section 2.2 for this case, for Case 3 to occur, it must be that $\varphi > 0$, that is,

$$\frac{\delta S_0(1-\tau)}{(1-\theta)\left(r_1 S_0(1-\tau) + {r_1^2}\big/{2\alpha_1}\right)} < 1 .$$ Rearranging, this requires $\left[{r_1^2}\big/{2\alpha_1} + r_1 S_0(1-\tau) - \frac{\delta S_0(1-\tau)}{(1-\theta)}\right] > 0$.

Since the expression on the left hand side of the inequality is increasing in r_1, it must be that the threshold value or the "breakeven interbank rate" r_1^{φ} that induces banks to lend in the interbank market is the positive root of the quadratic equation obtained by setting the expression to zero. So

$$r_1^{\varphi} = \alpha_1 S_0(1-\tau)\left[\sqrt{1 + \frac{2\delta}{\alpha_1(1-\theta)S_0(1-\tau)}} - 1\right].$$ Since this increases in S_0, we know that in Case 2, an

increase in S_0 expands both the autarky rate r_1^A as well as the rate r_1^{φ} necessary for the system to move

into Case 3. However, under reasonable assumptions, we show in proofs of Theorems 2-3 that r_1^{φ}

increases at a decreasing rate while r_1^A does not, so at a high enough S_0, $r_1^A > r_1^{\varphi}$ and the interbank

market will open.

Proofs of Theorems 2-3: We now provide remaining steps of the proofs by first detailing the date-0 maximization problems in the presence of a convenience yield on reserves at date 1. The firm's maximization problem remains unchanged. The bank's maximization problem is

$$\underset{L_0^B, e_0}{Max} R_0^L L_0^B + S_0 - e_0 - \frac{\alpha_0}{2} e_0^{\,2} - D_0 + E_0\left[V(y,z)\,\middle|\,L_0, e_0\right]$$

s.t. $D_0 + e_0 = L_0 + \tfrac{1}{2}\lambda(L_0)^2 + S_0$

Case 1: The convenience yield associated with reserves in the stressed state, δ, is an opportunity cost for stressed banks, and they pass it on while lending to their firm at date 1. So they lend at rate $(1+\gamma+\delta)$ where γ is their monitoring cost. Therefore,

$$V(y=1, z=1) = \underset{e_1}{Max}\left[-\delta(D_0 - e_1) - \frac{\alpha_1}{2} e_1^{\,2}\right]$$

In turn, $e_1 = {\delta}\big/{\alpha_1}, e_0 = {q\delta}\big/{\alpha_0}$. The $(1-\theta)$ healthy banks divide the deposit outflows from the stressed banks so

$$V(y=1,z=0) = \frac{\theta\delta(D_0 - D_0^F + I_1 - \delta/\alpha_1)}{(1-\theta)}$$. Note that in making decisions at date 0, the inflows that

come into the bank if it were healthy at date 1 are unrelated to any decision it takes at date 0 – it stems from decisions (on the size of loans, capital raise, and deposit funding) taken by other banks.[23] So, maximizing at date 0 w.r.t. L_0, we get $R_0^L = (1+q\delta)(1+\lambda L_0)$. From the firm's maximization, we know

$$R_0^D = (1+q\gamma+q\delta) = R_0^L, \text{ so } L_0 = \frac{q\gamma}{\lambda(1+q\delta)}.$$ We now derive when $\left(D_0 - D_0^F + I_1 - e_1\right) < S_0(1-\tau)$.

Since $\left(D_0 - D_0^F\right) = \left(S_0 + L_0 + \frac{1}{2}\lambda(L_0)^2 - e_0\right) - \left(W_0^F + L_0 - I_0\right) = I_0 - e_0 + (S_0 - W_0^F) + \frac{1}{2}\lambda(L_0)^2$, where

the second equality uses $L_0^B = L_0^F$, the condition simplifies to

$$\tau S_0 \leq \left[\left(\frac{q\delta}{\alpha_0} + \frac{\delta}{\alpha_1}\right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2\right].$$

Case 2: Here the opportunity cost of lending at date 1 is r_1 (since this is the marginal cost of raising capital, the source of incremental funding at date 1), and it replaces δ in the bank's maximization in Case 1. The stressed bank sees

$$V(y=1,z=1) = \underset{e_1}{Max}\left[-r_1(D_0 - e_1) - \frac{\alpha_1}{2}e_1^2\right].$$ The healthy banks receive

$$V(y=1,z=0) = \frac{\theta\delta S_0(1-\tau)}{(1-\theta)}.$$ Furthermore, $\tau S_0 = \left[\left(\frac{qr_1}{\alpha_0} + \frac{r_1}{\alpha_1}\right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2\right]$ for

liquidity demand to equal liquidity supply. Since the right hand side increases in r_1, a higher S_0 always induces a higher r_1, whatever the level of τ so long as it is positive.

Case 3: For the bank, the date-0 maximization is similar to the one in Case 2. In this case, if healthy, the bank may use its reserves to lend at date 1. However, this will not enter its maximization since it takes the reserves as given. The bank's maximization problem at date 0, and the stressed bank's problem at date 1 then is as in case 2, where it takes r_1 as given.

Now, recall that S_0^* is the level of reserves at which the stressed bank can just meet liquidity needs with the (shadow) rate δ and reserves having a convenience yield δ. That is,

$$S_0^* = \frac{1}{\tau}\left[\left(\frac{q\delta}{\alpha_0} + \frac{\delta}{\alpha_1}\right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2\right] \text{ where } g_0'(I_0) = \frac{1+q(\gamma+\delta)}{1-q},$$

$g_1'(I_1) = (1+\gamma+\delta)$, and $L_0 = \frac{q\gamma}{\lambda(1+q\delta)}$. Note the right hand side is increasing in δ so S_0^* is

increasing in δ. Furthermore, the net rate the stressed banks charge firms is $(\gamma+\delta)$ for $S_0 < S_0^*$.

When S_0 rises from S_0^*, the (shadow autarky) rate r_1^A rises from δ. It solves

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[23] Put differently, all the variables in this expression should have a superscript O to signify they are decisions made by other banks. In the symmetric equilibrium, however, they will be equal to the values chosen by the bank whose maximization decisions we are studying.

$$\tau S_0 = \left[\left(\frac{qr_1}{\alpha_0} + \frac{r_1}{\alpha_1} \right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2 \right] \text{ where } g_0{}'(I_0) = \frac{1+q(\gamma+r_1)}{1-q},$$

$g_1{}'(I_1) = (1+\gamma+r_1)$, and $L_0 = \dfrac{q\gamma}{\lambda(1+qr_1)}$. Once again, since the right hand side increases in r_1, r_1^A is

increasing in S_0. If we further assume $g_0{}'''$, $g_1{}'''$ are both positive, then it is convex.

Also $r_1^\varphi = \alpha_1 S_0(1-\tau)\left[\sqrt{1+\dfrac{2\delta}{\alpha_1(1-\theta)S_0(1-\tau)}} - 1 \right]$. So $r_1^\varphi > 0$ for $S_0 > 0$. Furthermore, it is

straightforward to show that r_1^φ is increasing in S_0 and it is concave. Assume for now, and we will revisit

later, that at S_0^*, $r_1^\varphi > r_1^A$. Since both rates are increasing in S_0, and r_1^A is convex in S_0 while r_1^φ is

concave, they can intersect only once at S_0^{**}. So the (shadow) rate is r_1^A as S_0 increases from S_0^* to S_0^{**}

after which it becomes the rate dictated by the interbank market. Finally, r_1^φ increases in δ (as does r_1^A,

see above). So S_0^{**} increases in δ. Finally, since the equilibrium φ falls in δ, the required

equilibrating interbank rate also increases in δ. Now, it can be shown using the 2nd order Taylor-series

expansion of $\sqrt{(1+x)}$ in r_1^φ, that at S_0^*, $r_1^\varphi - r_1^A > \dfrac{\delta\theta}{(1-\theta)} - \dfrac{\delta^2}{2\alpha_1(1-\theta)^2 S_0^*(1-\tau)} > 0$, if

$\delta < 2\alpha_1\theta(1-\theta)(1-\tau)S_0^*$. Substituting for S_0^*, and doing some algebra, it can be shown that a sufficient

condition for this is that $\theta(1-\theta) > \dfrac{\tau}{2(1-\tau)}$ and $\delta > \delta^*$ where δ^* satisfies

$$\frac{\tau}{2\alpha_1\theta(1-\theta)(1-\tau)}\delta^* = \left[\left(\frac{q\delta^*}{\alpha_0} + \frac{\delta^*}{\alpha_1} \right) + W_0^F - I_0(\delta^*) - I_1(\delta^*) - \frac{1}{2}\lambda\left[L_0(\delta^*) \right]^2 \right]. \text{ This}$$

guarantees that there exists a unique $S_0^{**} > S_0^*$ such that $r_1^\varphi < r_1^A$ (interbank market is open) if and only

if $S_0 > S_0^{**}$. Q.E.D.

Condition for r_1 to be increasing in S_0 in Case 3: Recognize that in this region, r_1 is determined by
equating the demand by stressed banks for loans in the inter-bank market to the supply by tainted banks of
those loans. So $\theta\left[D_0 - D_0^F + I_1 - S_0(1-\tau) - e_1 \right] = \varphi(1-\theta)\left[S_0(1-\tau) + e_1 \right]$. Substituting

$\left(D_0 - D_0^F \right) = \left[I_0 - e_0 + (S_0 - W_0^F) + \frac{1}{2}\lambda(L_0)^2 \right]$ and $e_1 = \dfrac{r_1}{\alpha_1}$ and rearranging, we get

$$\frac{r_1}{\alpha_1} = \frac{\theta}{\left[\varphi(1-\theta)+\theta\right]}\left[I_0 + I_1 - e_0 + \tfrac{1}{2}\lambda(L_0^B)^2 - W_0^F\right] + \frac{\left[\theta\tau - \varphi(1-\theta)(1-\tau)\right]}{\left[\varphi(1-\theta)+\theta\right]}S_0.$$ Denoting the right hand

side of this equality as f as before and totally differentiating, we get $\left[\dfrac{1}{\alpha_1} - \dfrac{\partial f}{\partial r_1}\right]\dfrac{dr_1}{dS_0} = \dfrac{\partial f}{\partial \varphi}\dfrac{\partial \varphi}{\partial S_0} + \dfrac{\partial f}{\partial S_0}.$

Since $\dfrac{\partial f}{\partial r_1} < 0$, $\dfrac{dr_1}{dS_0} > 0$ if $\dfrac{\partial f}{\partial \varphi}\dfrac{\partial \varphi}{\partial S_0} + \dfrac{\partial f}{\partial S_0} > 0$. But $\dfrac{\partial f}{\partial \varphi} < 0$ by inspection, and we argued in the text that

$\dfrac{\partial \varphi}{\partial S_0} < 0$. So $\dfrac{\partial f}{\partial \varphi}\dfrac{\partial \varphi}{\partial S_0} > 0$ and a sufficient condition for $\dfrac{dr_1}{dS_0} > 0$ is that $\dfrac{\partial f}{\partial S_0} > 0$. This then requires

$\left[\theta\tau - \varphi(1-\theta)(1-\tau)\right] > 0$, which on simplifying requires $\theta > \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$. Note that this is only

sufficient, since even if it does not hold, it may still be that $\dfrac{dr_1}{dS_0} > 0$. Intuitively, there is now a new

channel through which a higher S_0 leads to a higher r_1: a higher S_0 leads to a lower φ ceteris paribus, since healthy banks have more reason to stay on the sideline given the larger flight to safety flows, which in turn leads to a greater net need for liquidity from capital-raising, and hence a higher r_1.

Section 2.2.4: Endogenous δ.

If $\delta(r_1) = \delta^A + \delta^B r_1^2$ where $\delta^A \geq 0$, $\delta^B \geq 0$, then it follows from (1.12) that

$$(1-\varphi) = \frac{(\delta^A + \delta^B r_1^2)S_0(1-\tau)}{(1-\theta)\left(r_1 S_0(1-\tau) + \frac{r_1^2}{2\alpha_1}\right)}.$$ Let $A = \dfrac{\delta^B}{(1-\theta)}$, $B = \dfrac{1}{2\alpha_1 S_0(1-\tau)}$, $C = \dfrac{\delta^A}{(1-\theta)}$. Then the

previous expression is $(1-\varphi) = \dfrac{C + Ar_1^2}{r_1 + Br_1^2}$. Note that A, B, C are all positive.

We want to see when the RHS ≥ 1 so that there is no r_1 for which $\varphi > 0$. This requires

$(A-B)r_1^2 - r_1 + C \geq 0$. The roots of the quadratic are $r_1 = \dfrac{1 \pm \sqrt{1 - 4C(A-B)}}{2(A-B)}$.

Case 1: If $4C(A-B) > 1$, there are no real roots to the quadratic. So $C + Ar_1^2$ always lies above $r_1 + Br_1^2$ (as at $r_1 = 0$), which means φ is always zero. The interbank market never opens in this case.

Case 2: If $4C(A-B) \leq 1$ and $A > B$, there are two positive real roots, $r_1^+ > r_1^-$, and $\varphi > 0$ if and only if $r_1 \in (r_1^-, r_1^+)$. Essentially, the two curves $C + Ar_1^2$ and $r_1 + Br_1^2$ intersect at two points, and $\varphi > 0$ in between. The interbank market opens only in a range of rates and is closed both above and below.

Case 3: If $A < B$, then there is only one positive root, which is $r_1^+ = \dfrac{-1 + \sqrt{1 + 4C(B-A)}}{2(B-A)}$, and $\varphi > 0$

iff $r_1 > r_1^+$. Essentially, the slope of $r_1 + Br_1^2$ is higher than $C + Ar_1^2$, so it intersects once from below, after which $\varphi > 0$. The interbank market opens only above a specific rate.

Appendix III – Proofs for Section III

Proof that $\dfrac{\partial U}{\partial r_1} < 0$ **:** Substituting $D_0^F = \left(L_0 + W_0^F - I_0 \right)$ in (1.13), the planner maximizes objective $U=$

$$\left((1-q)g_0(I_0) - I_0(1+q\gamma)\right) + q\left(g_1(I_1) - I_1(1+\gamma)\right) - \tfrac{1}{2}\alpha_0 e_0^2 - \frac{q}{\theta}\left[\varphi(1-\theta)+\theta\right]\left(\tfrac{1}{2}\alpha_1 e_1^2\right) - \tfrac{1}{2}\lambda\left(L_0\right)^2 + q(L_0 + W_0^F)\gamma$$

Differentiating U w.r.t. r_1, we get $\dfrac{\partial U}{\partial r_1} =$

$$\left((1-q)g_0' - (1+q\gamma)\right)\frac{dI_0}{dr_1} + q\left(g_1' - (1+\gamma)\right)\frac{dI_1}{dr_1} - \alpha_0 e_0 \frac{de_0}{dr_1} - \frac{q}{\theta}\left[\varphi(1-\theta)+\theta\right]\alpha_1 e_1 \frac{de_1}{dr_1} + (q\gamma - \lambda L_0)\frac{dL_0}{dr_1}.$$

Substituting from the firm's FOC, that is, $(1-q)g_0' = (1 + q(\gamma + r_1))$, $g_1' = \left(1 + \gamma + r_1\right)$, and

$\lambda L_0 = \dfrac{q\gamma}{1 + qr_1}$, inspection reveals that the first 4 elements are all negative, so $\dfrac{\partial U}{\partial r_1} < 0$ if

$(q\gamma - \lambda L_0)\dfrac{dL_0}{dr_1} \leq 0$. But $L_0 = \dfrac{q}{\lambda}\left(\dfrac{\gamma}{1 + qr_1}\right)$. So $(q\gamma - \lambda L_0) \geq 0$. Since $\dfrac{dL_0}{dr_1} < 0$, $\dfrac{\partial U}{\partial r_1} < 0$. Q.E.D.

Proof that with exogenous φ, $\dfrac{dU}{de_0} = 0$ **at the privately optimal** $e_0 = \dfrac{qr_1}{\alpha_0}$. Recall from (1.13) that

$$U = \left((1-q)g_0(I_0) - I_0(1+q\gamma)\right) + q\left(g_1(I_1) - I_1(1+\gamma)\right) - \tfrac{1}{2}\alpha_0 e_0^2$$
$$- \frac{q}{\theta}\left[\varphi(1-\theta)+\theta\right]\left(\tfrac{1}{2}\alpha_1 e_1^2\right) - \tfrac{1}{2}\lambda\left(L_0\right)^2 + q(L_0 + W_0^F)\gamma$$

Assume e_0 is set exogenously (so it does not respond to the interest rate). Now we know that

$$\frac{dU}{de_0} = \frac{\partial U}{\partial e_0} + \frac{\partial U}{\partial r_1}\frac{dr_1}{de_0} = -\alpha_0 e_0 + \frac{\partial U}{\partial r_1}\frac{dr_1}{de_0}.$$

$$\frac{\partial U}{\partial r_1} = [(1-q)g_0' - (1+q\gamma)]\frac{\partial I_0}{\partial r_1} + q[g_1' - (1+\gamma)]\frac{\partial I_1}{\partial r_1} - \frac{q}{\theta}\left[\varphi(1-\theta)+\theta\right]\alpha_1 e_1 \frac{\partial e_1}{\partial r_1} + \left[q\gamma - \lambda L_0\right]\frac{\partial L_0}{\partial r_1}.$$

Substituting from the firm's FOC, that is, $(1-q)g_0' = (1+q(\gamma+r_1))$, $g_1' = (1+\gamma+r_1)$, and

$\lambda L_0 = \dfrac{q\gamma}{1+qr_1}$, as well as recognizing that $e_1 = \dfrac{r_1}{\alpha_1}$, we get

$$\frac{\partial U}{\partial r_1} = qr_1\left[\frac{\partial I_0}{\partial r_1} + \frac{\partial I_1}{\partial r_1} + \frac{q\gamma}{1+qr_1}\frac{\partial L_0}{\partial r_1} + \frac{\varphi(1-\theta)+\theta}{\alpha_1\theta}\right].$$

From (1.10), $r_1 = \dfrac{\alpha_1\theta}{[\varphi(1-\theta)+\theta]}\left[I_1 + (D_0 - D_0^F)\right] - S_0(1-\tau)$.

Therefore, $\dfrac{dr_1}{de_0} = \dfrac{\alpha_1\theta}{[\varphi(1-\theta)+\theta]}\left[\lambda L_0 \cdot\dfrac{dL_0}{dr_1}\cdot\dfrac{dr_1}{de_0} - 1 + \dfrac{dI_0}{dr_1}\cdot\dfrac{dr_1}{de_0} + \dfrac{dI_1}{dr_1}\cdot\dfrac{dr_1}{de_0}\right].$

Rearranging, and substituting $\lambda L_0 = \dfrac{q\gamma}{1+qr_1}$, $\dfrac{dr_1}{de_0} = \dfrac{-\dfrac{\alpha_1\theta}{[\varphi(1-\theta)+\theta]}}{1 - \dfrac{\alpha_1\theta}{[\varphi(1-\theta)+\theta]}\left[\dfrac{q\gamma}{1+qr_1}\dfrac{dL_0}{dr_1} + \dfrac{dI_0}{dr_1} + \dfrac{dI_1}{dr_1}\right]}.$

Returning to the expression, we obtain that $\dfrac{dU}{de_0} = -\alpha_0 e_0 + \dfrac{\partial U}{\partial r_1}\dfrac{dr_1}{de_0}$. We know $\alpha_0 e_0 = qr_1$ at the private

optimal. Also, multiplying the earlier expressions, we get $\dfrac{\partial U}{\partial r_1}\dfrac{dr_1}{de_0} = qr_1$, so $\dfrac{dU}{de_0}\Big|_{e_0=\frac{qr_1}{\alpha_0}} = 0$. Q.E.D.

Proof of Theorem 4 (endogenous φ). Recall from (1.13) that

$$U = \left((1-q)g_0(I_0) - I_0(1+q\gamma)\right) + q\left(g_1(I_1) - I_1(1+\gamma)\right) - \tfrac{1}{2}\alpha_0 e_0^2$$

$$- \frac{q}{\theta}[\varphi(1-\theta)+\theta]\left(\tfrac{1}{2}\alpha_1 e_1^2\right) - \tfrac{1}{2}\lambda\left(L_0\right)^2 + q(L_0 + W_0^F)\gamma$$

Since φ is not influenced directly by e_0 but only indirectly via r_1, we have

$$\frac{dU}{de_0} = \frac{\partial U}{\partial e_0} + \left(\frac{\partial U}{\partial r_1}\Big|_{\varphi=const}\right)\frac{dr_1}{de_0} + \frac{\partial U}{\partial\varphi}\frac{\partial\varphi}{\partial r_1}\frac{dr_1}{de_0} = -\alpha_0 e_0 + \left(\left(\frac{\partial U}{\partial r_1}\Big|_{\varphi=const}\right) + \frac{\partial U}{\partial\varphi}\frac{\partial\varphi}{\partial r_1}\right)\frac{dr_1}{de_0}.$$

Also $r_1 = \alpha_1 f(r_1,\varphi,e_0)$. Therefore, $\dfrac{dr_1}{de_0} = \alpha_1\left[\dfrac{\partial f}{\partial e_0} + \dfrac{\partial f}{\partial r_1}\cdot\dfrac{dr_1}{de_0} + \dfrac{\partial f}{\partial\varphi}\cdot\dfrac{\partial\varphi}{\partial r_1}\dfrac{dr_1}{de_0}\right].$

Rearranging, $\dfrac{dr_1}{de_0} = \dfrac{\alpha_1\dfrac{\partial f}{\partial e_0}}{\left[1-\alpha_1\left(\dfrac{\partial f}{\partial r_1} + \dfrac{\partial f}{\partial\varphi}\cdot\dfrac{\partial\varphi}{\partial r_1}\right)\right]}$. Substituting in the earlier expression, we get

$$\frac{dU}{de_0} = -\alpha_0 e_0 + \left(\left(\frac{\partial U}{\partial r_1}\bigg|_{\varphi=const} \right) + \frac{\partial U}{\partial \varphi}\frac{\partial \varphi}{\partial r_1} \right) \frac{\alpha_1 \dfrac{\partial f}{\partial e_0}}{\left[1 - \alpha_1 \left(\dfrac{\partial f}{\partial r_1} + \dfrac{\partial f}{\partial \varphi} \cdot \dfrac{\partial \varphi}{\partial r_1} \right) \right]} \qquad (A)$$

Now we can see that

$$\frac{\partial U}{\partial \varphi} = -\frac{q}{\theta}(1-\theta)\frac{1}{2}\alpha_1 e_1^2 < 0 \qquad \text{(the direct effect of a higher } \varphi \text{ is more dissipative date-1 equity}$$

issuance by tainted banks). Also from (1.12), $(1-\varphi) = \dfrac{\delta S_0 (1-\tau)}{(1-\theta)\left(r_1 S_0 (1-\tau) + {r_1^2}\big/{2\alpha_1} \right)}$.

So $\dfrac{\partial \varphi}{\partial r_1} = \dfrac{\delta S_0 (1-\tau)}{(1-\theta)} \dfrac{\left(S_0 (1-\tau) + {r_1}\big/{\alpha_1} \right)}{\left(r_1 S_0 (1-\tau) + {r_1^2}\big/{2\alpha_1} \right)^2} > 0$, and $\dfrac{\partial f}{\partial \varphi} = \dfrac{-\theta(1-\theta)}{(\varphi(1-\theta)+\theta)^2}\left[I_1 + \left(D_0 - D_0^F \right) \right] < 0$.

We can then understand the effect of internalizing φ on how the social planner would set e_0 from scrutinizing the terms containing φ on the right hand side of (A) above. In the numerator,

$\dfrac{\partial U}{\partial \varphi}\dfrac{\partial \varphi}{\partial r_1}\alpha_1\dfrac{\partial f}{\partial e_0} > 0$. An increase in e_0 reduces the liquidity shortfall f , reducing the rate r_1 , reducing the mass of tainted banks φ , and reducing dissipative date-1 equity issues by tainted banks, thus increasing welfare. However, there is an offsetting dampening effect from the term in the denominator,

$-\alpha_1 \dfrac{\partial f}{\partial \varphi} \cdot \dfrac{\partial \varphi}{\partial r_1} > 0$, because a fall in the ex-post inter-bank rate increases the degree of liquidity hoarding,

and thus increases the degree of liquidity shortfall that has to be met by date-1 dissipative capital issues.

To check whether the social planner's capital choice is higher than the private optimal (which ignores the dependence of φ on the bank's capital issuance), we need to see whether

$$\frac{dU}{de_0} = -\alpha_0 e_0 + \left(\left(\frac{\partial U}{\partial r_1}\bigg|_{\varphi=const} \right) + \frac{\partial U}{\partial \varphi}\frac{\partial \varphi}{\partial r_1} \right) \frac{\alpha_1 \dfrac{\partial f}{\partial e_0}}{\left[1 - \alpha_1 \left(\dfrac{\partial f}{\partial r_1} + \dfrac{\partial f}{\partial \varphi} \cdot \dfrac{\partial \varphi}{\partial r_1} \right) \right]} \text{ is greater or less than zero at the}$$

private optimal $e_0 = \dfrac{q r_1}{\alpha_0}$. In other words, we need to check whether

$$\left(\left(\left.\frac{\partial U}{\partial r_1}\right|_{\varphi=const}\right)+\frac{\partial U}{\partial \varphi}\frac{\partial \varphi}{\partial r_1}\right)\frac{\alpha_1\frac{\partial f}{\partial e_0}}{\left[1-\alpha_1\left(\frac{\partial f}{\partial r_1}+\frac{\partial f}{\partial \varphi}\cdot\frac{\partial \varphi}{\partial r_1}\right)\right]}>qr_1.$$ Multiplying both sides by the denominator on

the left hand side, and recognizing from our earlier analysis with exogenous φ that

$$\left(\left.\frac{\partial U}{\partial r_1}\right|_{\varphi=const}\right)\alpha_1\frac{\partial f}{\partial e_0}=qr_1\left(1-\alpha_1\frac{\partial f}{\partial r_1}\right),$$ we require that $\dfrac{\partial U}{\partial \varphi}\dfrac{\partial f}{\partial e_0}>-qr_1\dfrac{\partial f}{\partial \varphi}$.

Now $\dfrac{\partial U}{\partial \varphi}\dfrac{\partial f}{\partial e_0}=\left(-\dfrac{q}{\theta}(1-\theta)\dfrac{1}{2}\alpha_1 e_1^{2}\right)\cdot\left(\dfrac{-\theta}{\varphi(1-\theta)+\theta}\right)$. We substitute $e_1=\dfrac{r_1}{\alpha_1}$.

Note also that $-qr_1\dfrac{\partial f}{\partial \varphi}=-qr_1\cdot\left(\dfrac{-\theta(1-\theta)}{(\varphi(1-\theta)+\theta)^{2}}\left(I_1+\left(D_0-D_0^{F}\right)\right)\right)$. Substituting on both sides of the

inequality and recognizing that $r_1=\dfrac{\alpha_1\theta}{[\varphi(1-\theta)+\theta]}\left[I_1+(D_0-D_0^{F})\right]-\alpha_1 S_0(1-\tau)$, we get

$\dfrac{\partial U}{\partial \varphi}\dfrac{\partial f}{\partial e_0}>-qr_1\dfrac{\partial f}{\partial \varphi}$ iff $\dfrac{r_1}{2}>r_1+\alpha_1 S_0(1-\tau)$ which is impossible. Indeed, the inequality goes the other

way, so $\left.\dfrac{dU}{de_0}\right|_{e_0=\frac{qr_1}{\alpha_0}}<0$. The social planner prefers lower capital than the privately optimal level. Q.E.D.

Appendix IV: Additional extensions and Proofs for Section IV

Endogenizing Encumbrance Share due to Speculation (section 4.2)

Consider, for example, the prime brokerage services that banks offer. Let each bank serve one speculator. Let the speculator put on trades at date 0 of size x. In normal economic times, the bets pay off and return ηx to the speculator and fees of ρx to the bank. Conditional on the economy getting liquidity stressed (with probability $\dfrac{q}{\theta}$), the bank has to meet margin calls on the speculator, putting up reserves of κx. These calls have priority over all other claims on the bank (else it will have to default on exchanges, and see its brokerage business shut down).[24]

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[24] Alternatively, if the trades are centrally cleared to reduce any risk of contagion from such speculative positions, the clearinghouse would require the clearing members (dealer banks) to over-collateralize their positions and contribute variation margins. The resulting funds with clearinghouses face significant limits on rehypothecation, and a large fraction of it is in the form of reserves deposited with the central bank, thereby being unavailable for further private use.

Finally, assume each speculator's search costs of putting on a profitable trade is increasing in the size of a trade (that is, there are fewer remaining low hanging fruit as they trade more) and decreasing in the unencumbered liquidity of the system, so it is $\dfrac{v}{2}\dfrac{x^2}{(S_0 - \kappa \overline{x})}$, where v is a parameter and $\overline{x}$ is the equilibrium level of trade per bank. This captures the notion that liquidity facilitates speculation, but speculators are aware that liquidity gets tied up as there is more speculative trade. Assume that $\eta > \rho$ which ensures that speculation is profitable net of fees. The speculator's maximization problem is then:

$$\underset{x}{Max} \quad (1-\frac{q}{\theta})[\eta - \rho]x - \frac{v}{2}\frac{x^2}{(S_0 - \kappa \overline{x})}$$

The first order condition is $(1-\frac{q}{\theta})[\eta - \rho] = \dfrac{vx}{(S_0 - \kappa \overline{x})}$. Recognizing that $x = \overline{x}$ in equilibrium, we have

$$\kappa \overline{x} = \frac{S_0 \kappa \left(1-\dfrac{q}{\theta}\right)(\eta - \rho)}{v + \kappa \left(1-\dfrac{q}{\theta}\right)(\eta - \rho)} = \tau S_0 \text{ where } \tau = \frac{\kappa \left(1-\dfrac{q}{\theta}\right)(\eta - \rho)}{v + \kappa \left(1-\dfrac{q}{\theta}\right)(\eta - \rho)}.$$

Assuming that the market for provision of prime-brokerage services to speculators is competitive among banks at date 0, the fee ρ per unit of speculative activity is set such that in expectation banks are compensated for the cost of providing the per-unit margin call κ. This zero-profit condition implies then that $\left(1-\dfrac{q}{\theta}\right)\rho = \dfrac{q}{\theta}[\varphi(1-\theta) + \theta]\overline{r_1}\kappa$.[25] Substituting above for the implied $\rho(\overline{r_1})$, we obtain that the encumbrance per unit of reserves is a function of the date-1 interbank rate premium; it is $\tau(\overline{r_1})$, such that $\tau'(\overline{r_1}) \leq 0$. Theorem 5 in the text (derived analogously to Theorem 1) implies that $\tau(\overline{r_1})$ never falls to zero, and our assumption that prime-brokerage fee is lower than the speculative return, $\eta > \rho$, always holds in equilibrium. Importantly, the lower is the average or expected margin requirement κ on speculative activity, the greater the ex-ante speculative activity (all else equal), and in turn, the range of parameters for which more reserves can tighten interbank markets.

Embedding Liquidity Regulations (section 4.3)

In the context of our framework, suppose that after reserves are set and speculation is under way, regulators can affect overall $\tau \ (= \tau^{Spec} + \tau^{Reg})$ by setting τ^{Reg}. Let the fraction of banks that suffer withdrawals at date 1 be $K(\tau^{Reg})\theta$ instead of θ, with $K' < 0$, $K'' > 0$ and $K(0) = 1$. This means the share of banks that are stressed falls in mandatory regulatory reserve holdings (in part because that also

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[25] Note that there is no opportunity cost of reserves encumbrance to safe banks that hoard liquidity.

curbs the effects of speculation). However, this also hampers the liquidity available from healthy banks in times of liquidity stress. Hence, if regulators are narrowly focused on maximizing overall liquidity available per dollar of reserves ex post, given the central bank has set reserves, they would maximize

$(1-\tau) - K(\tau^{\text{Reg}})\theta.$ So they would optimally choose $\tau^{\text{Reg}*} = K'^{-1}(-\frac{1}{\theta})$. On inspection, and bearing in

mind that risk reduction has diminishing returns so that $K'' > 0$, the higher is θ the greater will be the regulatory encumbrance $\tau^{\text{Reg}*}$. Depending on functional forms, that is, how effectively a higher τ^{Reg} reduces the share of banks that are stressed, it can be shown that all the cases we have discussed earlier could still be possible with optimal regulation. Our model easily allows for an analysis of alternative formulations of the regulatory requirement. For instance, if banks are required to maintain τD_0 of deposits as reserves at all times (that is, a traditional reserve requirement), we can show easily that once again $\tau^{\text{Reg}*} = K'^{-1}(-\frac{1}{\theta})$ since deposit issuance moves one for one with reserves.

Fixed Encumbrance on Reserves ($\tau S_0 \equiv \overline{\text{E}}$) (section 4.4)

Consider the full model of Section 2.2 with the endogenized share of surplus banks in the interbank market. Case 1 in which each stressed bank is self-sufficient in liquidity at the convenience yield δ arises whenever $\overline{\text{E}} \leq \overline{\text{E}}^* \equiv \left[\left(\frac{q\delta}{\alpha_0} + \frac{\delta}{\alpha_1} \right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2 \right]$, a condition that is

independent of the level of reserves; note that I_1 is the optimized value evaluated at $r_1 = \delta$, I_0 at

$R_0^L = (1+q\gamma+q\delta)$ and $L_0 = \frac{q\gamma}{\lambda(1+q\delta)}$. For $\overline{\text{E}} > \overline{\text{E}}^*$, the interbank market may be shut (autarky) or

open; when shut, the autarkic rate $r_1^A(\overline{\text{E}})$ satisfies $\overline{\text{E}} = \left[\left(\frac{qr_1^A}{\alpha_0} + \frac{r_1^A}{\alpha_1} \right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2 \right]$,

and is now a function of the fixed level of encumbrance and not of the level of reserves, with I_1, I_0 and

L_0 set accordingly. Naturally, $r_1^A(\overline{\text{E}})$ is increasing in the encumbrance $\overline{\text{E}}$, and $r_1^A(\overline{\text{E}}) > \delta$ for $\overline{\text{E}} > \overline{\text{E}}^*$.

A key question then is when is the autarkic rate above or below the breakeven rate r_1^φ that induces banks to lend in the interbank market. With fixed encumbrance, this rate is given by

$r_1^\varphi = \alpha_1 \left(S_0 - \overline{\text{E}} \right) \left[\sqrt{1 + \frac{2\delta}{\alpha_1(1-\theta)\left(S_0 - \overline{\text{E}}\right)}} - 1 \right]$, which as before is increasing and concave in S_0. Note

also that the endogenous share φ of surplus banks that lend in the interbank market for a given rate r_1

satisfies $(1-\varphi) = \dfrac{\delta\left(S_0 - \overline{E}\right)}{(1-\theta)\left[r_1\left(S_0 - \overline{E}\right) + \dfrac{r_1^2}{2\alpha_1}\right]}$ and the equilibrium interbank rate $\overline{r_1}$ is given by the

usual market-clearing condition adjusted for encumbrance being now at a fixed level:

$$\frac{r_1}{\alpha_1} = \frac{\theta}{\left[\varphi(1-\theta)+\theta\right]}\left[I_0 + I_1 - e_0 + \tfrac{1}{2}\lambda(L_0^B)^2 - W_0^F\right] - \frac{\varphi(1-\theta)}{\left[\varphi(1-\theta)+\theta\right]}S_0 + \overline{E} .$$ It can then be shown that

Theorem 6: For $S_0 > \overline{E} > \overline{E}^*$, there exists a critical threshold $\overline{S}_0 > \overline{E}$ such that

(i)   For $S_0 \in \left(\overline{E}, \overline{S}_0\right)$, the autarkic rate $r_1^A\left(\overline{E}\right)$ exceeds the breakeven rate $r_1^\varphi\left(S_0\right)$, the

interbank market is open $(\varphi > 0)$, and the equilibrium interbank rate $\overline{r_1} \in \left(r_1^\varphi(S_0), r_1^A\left(\overline{E}\right)\right)$.

(ii)   For $S_0 \geq \overline{S}_0$, the autarkic rate $r_1^A\left(\overline{E}\right)$ is at or below the breakeven rate $r_1^\varphi\left(S_0\right)$, the

interbank market is shut $(\varphi = 0)$, and the equilibrium interbank rate $\overline{r_1}$ equals the autarkic

rate $r_1^A\left(\overline{E}\right) > \delta$.

(iii)   When fixed encumbrance $\overline{E}$ is sufficiently small such that $r_1^A\left(\overline{E}\right) < r_1^\varphi(S_0 \to \infty) = \dfrac{\delta}{(1-\theta)}$,

then both cases (i) and (ii) arise and $\overline{r_1}$ is strictly increasing in S_0 for at least some range of

S_0 in $\left(\overline{E}, \overline{S}_0\right]$; otherwise, when $r_1^A\left(\overline{E}\right) \geq \dfrac{\delta}{(1-\theta)}$, only case (i) arises and $\overline{S}_0 \to \infty$.

Proof of Theorem 6: Following earlier derivations, but with a fixed encumbrance, we know

$$r_1^\varphi = \alpha_1\left(S_0 - \overline{E}\right)\left[\sqrt{1 + \frac{2\delta}{\alpha_1(1-\theta)\left(S_0 - \overline{E}\right)}} - 1\right].$$ Clearly $r_1^\varphi \to 0$ as $S_0 \to \overline{E}$. Also, $r_1^\varphi \to \dfrac{\delta}{(1-\theta)}$ as

$S_0 \to \infty$. Finally, r_1^φ is increasing in S_0. We also know that r_1^A is the value of r_1 that solves

$$\overline{E} = \left[\left(\frac{qr_1}{\alpha_0} + \frac{r_1}{\alpha_1}\right) + W_0^F - I_0 - I_1 - \tfrac{1}{2}\lambda L_0^2\right],$$ where I_0, I_1, L_0 depend on r_1 in the usual manner.

Since none of the elements on the right hand side change with S_0, r_1^A does not change with S_0.

Therefore, if $r_1^A < \dfrac{\delta}{(1-\theta)}$ because $\overline{E}$ is small, there is an $\overline{S}_0 > \overline{E}$ such that $r_1^\varphi = r_1^A$ at $S_0 = \overline{S}_0$, and

$r_1^\varphi > r_1^A$ for $S_0 > \overline{S}_0$. So the equilibrium interbank rate $\overline{r}_1 \in \left(r_1^\varphi(S_0), r_1^A\left(\overline{E}\right) \right)$ for $\overline{S}_0 > S_0 > \overline{E} > \overline{E}^*$ and

$\overline{r}_1 = r_1^A\left(\overline{E}\right)$ for $S_0 \geq \overline{S}_0$. If, however, $r_1^A \geq \dfrac{\delta}{(1-\theta)}$, then $r_1^\varphi < r_1^A$ for all finite S_0, and the equilibrium

interbank rate $\overline{r}_1 \in \left(r_1^\varphi(S_0), r_1^A\left(\overline{E}\right) \right)$ for all finite S_0. Q.E.D.
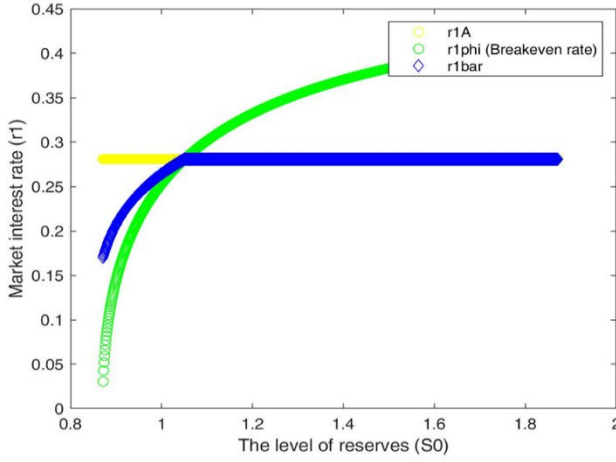
Figure 5A: Market Rate and Reserves

Figure 5B: Market Rate and Reserves

$\overline{E} = 0.87, \theta = 0.6$

$\overline{E} = 1.25, \theta = 0.6$

Figure 5: Numerical example for the effect of varying the fixed encumbrance on reserves $\overline{E}$ in the model of section 2.2 [26]

We illustrate the result with examples in Figures 5A and 5B, with the same parameters as in Figures 3-4 ($\theta = 0.6$). When $\overline{E}$ is low, the interbank market is open for low levels of S_0 but shuts down at high levels (Panel A); when $\overline{E}$ is high, the interbank market is always open regardless of the level of S_0. In both parameterizations, $\overline{r}_1$ is strictly increasing in S_0. It is therefore a robust feature of the equilibrium that the interbank market may remain shut and the interbank rate can increase in the level of reserves when the interbank market is open.

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[26] Note that in Figures 5A-B, the equilibrium interbank rate is rising in the level of reserves; in general, this holds when the systemic extent of liquidity shock θ is sufficiently small and the convenience yield δ is sufficiently high; when this is not the case, it can be shown that an increase in the supply of reserves can cause the rate to decrease as it starts out high, close to the autarkic rate, when reserves are close to the fixed encumbrance, and then decreases towards the breakeven rate.

Appendix V – Extensions in Section V not in the main body of the paper

Private Incentives to Hold Reserves (section 5.3)

What if banks were not forced to hold S_0, that is, what if banks can choose S_0 in addition to term lending and financing at date 0 (see representative bank's objective function in section 2.2)? To make the problem relevant, we assume there is a cost to carrying liquidity due to agency problems or a capital cost of holding excess reserves of $C(S_0)$ such that $C'(S_0) > 0$, $C''(S_0) > 0$. Focusing only on the bank's optimal choice of reserve holdings, and assuming that $\delta \to 0$ so that $\varphi \to 1$ (all surplus banks lend in the interbank market in the stressed state of the economy), this choice – based on bank's objective function in Section 2.1 – boils down to

$$\underset{S_0}{Max} \quad S_0 - D_0 - C(S_0) - \frac{q}{\theta}\theta\big(r_1\big(D_0 - S_0(1-\tau)\big)\big) + \frac{q}{\theta}(1-\theta)r_1 S_0(1-\tau)$$

Recognizing that deposits D_0 increase one for one in reserves S_0, the FOC w.r.t. S_0 is

$-C'(S_0) - qr_1\tau + \frac{q}{\theta}(1-\theta)r_1(1-\tau) = 0$. Simplifying

$$S_0^{pvt} = C'^{-1}\left[qr_1\left(\frac{(1-\tau)}{\theta} - 1\right)\right]$$

There is thus no guarantee that the level of (per bank) reserves that the central bank wants to issue (given its other concerns for conducting unconventional monetary policy) are at, or below, the level that commercial banks want to optimally hold. Suppose for instance that the central bank wants to initially place higher level of reserves than S_0^{pvt}. If $\theta > (1-\tau)$, banks privately do not wish to hold any reserves; furthermore, an increase in anticipated interest rates reduces the S_0^{pvt} that the commercial bank would optimally like to hold. Since under this same condition, we know that anticipated interbank rates are rising in S_0, the divergence, between what commercial banks are willing to hold and what the central bank has to place, grows with S_0. Put differently, the prospect of greater liquidity shortages need not increase the commercial banks' private incentives to hold more reserves; indeed, recognizing that the source of the shortage is the financing of reserves, they may want to hold less or none at all.

Maturity Matching or Short-term Financing of Reserves by Shadow Banks (section 5.3)

We have assumed that the reserves end up on bank balance sheets. What if the central bank departs from normal practice and allows non-bank financial firms to hold reserves directly? Unless the central bank buys money-like assets from the non-bank private sector, we may not get significantly different outcomes; if the central bank buys long-term financial assets and pays with reserves, for standard risk management reasons the non-bank private sector may want to match the maturity of their liability structure to their shorter-maturity asset holdings.

To see this, let us focus on the healthy state (that is, assume $q = 0$), and assume that economy-wide date-1 short-term (gross) interest rates in the healthy state are $(1 + r)$ with probability p and $(1 - r)$ with probability $(1 - p)$. The net rates (+r and –r) represent the state-contingent cost of rolling over each bank's liquidity shortfall given by $(D_0 - S_0)$. Further, assume the financial firm holding reserves wants to finance it so as to minimize costs, but it also dislikes the variability of its date-2 profits given by the variance of profits, $p (1-p) 4r^2 (D_0 - S_0)^2$, with aversion parameter $\psi / 2$. Finally, the cost of capital issuance at date 0 is $R_0^E = \left[p(1+r) + (1-p)(1-r) + \Delta_0^E \right]$, where Δ_0^E is a capital risk premium. So ignoring the other activities of the financial firm, its objective function for choosing the maturity structure of its liabilities, given the need to finance reserve holdings, is as follows (where variables have their earlier connotation):

$$\underset{D_0}{Max} \quad \left[-R_0^E e_0 - p(1+r)(D_0 - S_0) - (1-p)(1-r)(D_0 - S_0) - \frac{\psi}{2} p(1-p)4r^2 (D_0 - S_0)^2 \right]$$

$$s.t. \quad e_0 = S_0 - D_0$$

It is straightforward from the maximization that $D_0 = \left[S_0 + \dfrac{\Delta_0^E}{\psi \, p(1-p)4r^2} \right]$. So deposits increase one for one with reserves and also increase with the capital premium – the point is that longer term financing for reserves can increase the variability of profits by locking in financing costs while leaving returns on reserves variable. Financial firms will match maturity to avoid this variability. Put differently, so long as central-bank-issued reserves have to be financed somewhere in the economy rather than resting in household balance sheets, there will be some offsetting short-term liabilities.