Online Appendix to
Limits to Arbitrage and Hedging:
Evidence from Commodity Markets

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January 8, 2013

Abstract

In this Online Appendix, we solve a general equilibrium version of the model in the main text and show that the predictions of the model are robust to this extension. We also solve a general equilibrium model where the managerial costs of default are the motivation for firm hedging. We show that the implications of this calibrated model are qualitatively the same as in the model with risk averse managers. We also solve a model where default leads to supply disruptions, which alters the spot price dynamics and thus has implications for the futures risk premium. In this model higher default risk will tend *decrease* the futures risk premium as a supply disruption will benefit the long side of the futures contract. This is counter to our empirical results, which thus are consistent with the hedging story. Finally, we give some additional empirical results that were not included in the main paper and describe in more detail the micro data set used in the paper.

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1 General equilibrium version of main model

In general equilibrium, the consumers' consumption of other goods will typically be affected by the frictions in the commodity market. Thus, both the commodity 'demand' shocks, C_t, and the marginal intertemporal rate of substitution will be affected when varying the frictions in the commodity market. Further, a general equilibrium model allow us to calibrate the model to gauge the likely magnitudes of the effect of the model's frictions.

We follow the same setup as in the partial equilibrium model given in the main paper, but now also solve the consumers' problem. Let consumers' preferences be given by:

$$V = u(C_0, Q_0) + \beta E_0 [u(C_1, Q_1)], \tag{1}$$

where the felicity function is of the constant elasticity of substitution (CES) form:

$$u(x, y) = \frac{1}{1-\gamma} \left\{ \left(x^{(\varepsilon-1)/\varepsilon} + \omega y^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \right\}^{1-\gamma}, \tag{2}$$

where ε is the intratemporal elasticity of substitution and γ is the level of relative risk-aversion. The standard intratemporal first order condition implies that the equilibrium commodity spot price S_t is given by:

$$S_t = \omega \left(\frac{C_t}{Q_t} \right)^{1/\varepsilon}, \tag{3}$$

as assumed earlier in the partial equilibrium version of the model. However, in the general equilibrium case we assume that the consumers own a Lucas tree producing the numeraire good A_t, as well as the commodity producing firms which produce the aggregate supply of the commodity Q_t. However, the consumers must hire managers to manage the firms (their inventory and hedging decisions). The manager's objective function is as in the partial equilibrium case (see Equation (3) in the main paper).[1] Consumers can also invest in the commodity futures markets, but only through specialized funds who provide an aggregate number of contracts h_s as the solution to the problem given in Equation (8) in the main paper. Denote the cost charged per contract by these funds as c. We assume the costs are incurred at time 0. The consumers' equilibrium consumption of other goods in the first period will equal $C_0 = A_0 - c \times h^*$, where h^* is the equilibrium open interest in the futures market, while in the last period $C_1 = A_1$.

In equilibrium, consumers' net present value of a marginal investment in a commodity futures must be zero and so we have that $c = E[\Lambda(S_1 - F)]$. Therefore, the aggregate loss due to intermediation is $h^* E[\Lambda(S_1 - F)]$. Given the optimal position in futures contracts from Equation (9) in the main paper, we have that the equilibrium aggregate cost is $\frac{E[\Lambda(S_1-F)]^2}{\gamma_s \sigma_s^2}$. The reason the consumers are willing to incur this cost is the utility gain from moving to more optimal Q_0 and Q_1 as the futures price affects the commodity producers' inventory decisions. In equilibrium, we have that $E[\Lambda(S_1 - F)] = \frac{\gamma_p \gamma_s}{\gamma_s + \gamma_p} \sigma_s^2 Q_1$ and so, substituting out σ_s^2, we have:

$$\text{Aggregate cost } = c \times h^* = \frac{1}{\gamma_s} \left(\frac{\gamma_p \gamma_s}{\gamma_s + \gamma_p} \right)^2 \omega^2 Q_1^{2(1-1/\varepsilon)} k, \tag{4}$$

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[1] We do not model the managers' consumption, but instead argue that this is a reasonable abstraction as there are very few managers relative to the total population and so their consumption is a minuscule component of aggregate consumption.

where k is a positive constant defined earlier. In sum, *both the supply of the commodity and consumption of other goods are affected by the combination of hedging demand and limits to arbitrage.* It is clear then that the intertemporal marginal rate of substitution for consumers is explicitly a function of *both* equilibrium inventory and the frictions that give rise to hedging demand and costly intermediation in the futures market:

$$\Lambda\left(I,\gamma_p,\gamma_s\right) = \beta\left(\frac{C_1}{C_0}\right)^{-\gamma}\left(\frac{1+\omega\left(\frac{Q_1}{C_1}\right)^{(\varepsilon-1)/\varepsilon}}{1+\omega\left(\frac{Q_0}{C_0}\right)^{(\varepsilon-1)/\varepsilon}}\right)^{(1/\gamma-\varepsilon)/((\varepsilon-1)/\gamma)}, \tag{5}$$

where equilibrium consumption of other goods is $C_0 = A_0 - \frac{1}{\gamma_s}\left(\frac{\gamma_p\gamma_s}{\gamma_s+\gamma_p}\right)^2\omega^2 Q_1^{2(1-1/\varepsilon)}k$ and $C_1 = A_1$.

This analysis shows that the frictions assumed in this paper (limits to arbitrage and producer hedging demand) in general equilibrium also affect the consumption of other goods and the equity market pricing kernel. Thus, the covariance term in Equation (12) in the main paper is affected, not only through the volatility of the commodity price as in the model presented in the main text, but also through the dynamics of the pricing kernel (Λ). However, the identifying component of the frictions lies in the risk compensation of the second term in Equation (12) in the main paper, which is related to the magnitude of desired hedging and the total volatility (including any idiosyncratic components) of the futures price.

To illustrate the model implications, we calibrate the model using key moments of the data: in particular, the volatility of the commodity futures returns, commodity expenditure relative to aggregate endowment (GDP) and aggregate endowment growth. All moments are quarterly, corresponding to the empirical exercise in the next section. We calibrate the demand shock A_1 to roughly correspond with aggregate GDP growth and set $\mu = 0.004$, $\sigma = 0.02$ and the initial demand shock $A_0 = 1$. We set the depreciation rate, δ, to 0.01 and the coefficient of relative risk-aversion, γ, to 2.5 which is within the range typically used in macro economic models. Next, we calibrate the intratemporal elasticity of substitution ε and the constant ω jointly such that the standard deviation of futures returns $(S_1 - F)/F$ is about 20% per quarter, as in the data, and the average commodity expenditure is about 10% of expenditures on other goods. Given the volatility of demand shocks, this is achieved when $\varepsilon = 0.1$ and $\omega = 0.01$. That is, consumers are relatively inelastic in terms of substituting the commodity good for other goods, which is reasonable given our focus on oil and gas in the empirical section.[2] We let period 0 and period 1 production, g_0 and g_1, be 0.8 and 0.75, respectively, such that inventory holdings are positive as in the data.

[Figure 1 about here]

The severity of the model's frictions are increasing in the variance aversion of producers and speculators, γ_p and γ_s. As we posit mean-variance preferences, the values of these coefficients do not directly correspond to easily interpretable magnitudes (such as might be the case with relative risk-aversion). We therefore use the following two economic measures to calibrate a reasonable range

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[2]This also justifies our implicit model assumption that price risk outweighs quantity risk for the producers. As pointed out by, e.g., Hirshleifer (1988), if the opposite is the case, producers would hedge by going *long* the futures contract.

for each of these parameters. First, we let the loss to the firm from hedging, $h^* E\left[\Lambda\left(S_1 - F\right)\right]$, be between 0.1% and 1% of firm value and, second, we let the abnormal quarterly Sharpe ratio earned by speculators be between 0.05 and 0.25. The variation in these quantities is shown in the two top graphs of Figure 1, where $\gamma_p \in \{2, 4, ..., 20\}$ on the horizontal axis and $\gamma_s \in \{8, 40\}$ is shown as a dashed line for high speculator risk-aversion and a solid line for low speculator risk-aversion.

The remaining plots in Figure 1 show that the futures risk premium is indeed increasing in producer and speculator risk-aversion, while the spot price and inventory are decreasing. The calibration implies economically significant variation in both spot and futures risk premiums. In particular, for high speculator risk-aversion (corresponding to their earning a quarterly Sharpe ratio of about 0.25) and high hedging demand (corresponding to a loss of about 0.8% of firm value due to hedging), the abnormal quarterly futures risk premium is about 6%, whereas for low producer hedging demand and speculator risk-aversion (Sharpe ratio of about 0.05 and loss from hedging of about 0.1% of firm value), the abnormal futures risk premium is less than 1%. The standard risk component of the futures risk premium, captured by the covariance term in Equation (12) in the main text, shows a much more modest increase in response to changes in γ_p and γ_s, indicating an important role for market specific variables that capture the constraints in the model that will not be captured by standard controls for time-varying risk premiums. The decrease in inventory is about a 1% to 9% change in the level. The effect on percentage spot price changes is about the same as that for the futures risk premium, since the cost of carry relation holds when there is no stock-out. In sum, reasonable levels of the costs of hedging and required risk compensation leads to economically significant abnormal returns in the futures market, and concomitant changes in inventory and spot prices that are consistent with the intuition from the partial equilibrium model.

Figure 1 also illustrates an intuitive interaction between speculator risk tolerance and producer hedging demand. In particular, the response of the abnormal futures risk premium to changes in producer hedging demand is smaller when speculator risk tolerance is high. These are times when speculators are willing to meet the hedging demand of producers with small price concessions. If, conversely, speculators are more risk-averse, the price concession required to meet an additional unit of hedging demand is high.

[Figure 2 about here]

Figure 2 shows a slighty different calibration of the model that sets $\varepsilon = 0.08$ and $\omega = 0.012$. While the futures risk premium and spot price implications qualitatively are the same, the figure shows that equilibrium inventory in some cases in fact *increases* when producers' fundamental hedging demand increases, contrary to the intuition in the partial equilibrium model. This happens even though the spot price is still decreasing in producers' fundamental hedging demand as before. The reason for this is that it is the ratio of C_t to Q_t that matters for spot prices and that C_t in the general equilibrium case is endogenous and also a function of the magnitude of the frictions in the futures market. We have checked a large variety of reasonable parameter configurations and, with the exception of the inventory prediction, the predictions of the model with respect to the futures risk premium and the link to the spot price are robust and as described above.

3

2 Model with costs of default as hedging motive

In the model in the main paper, firm hedging is modeled as being due to managerial risk aversion. In the following, we present an alternative model where costs of default explicitly makes default risk the determinant of hedging behavior. As will be clear, such a model is somewhat less transparent than the mean-variance framework applied in the model in the main text. In particular, the default risk model does not yield an analytical expression of the risk premium in terms of fundamental variables. Also, debt default and limited liability of equity gives an incentive to risk shift and increase risk in bad states. Thus, this model introduces a trade-off between managerial costs of default and risk shifting in terms of the propensity to hedge. In the following, we show that the implications of the model with explicit default costs and the ensuing hedging demand qualitatively yield the same implications as the model with managerial risk aversion, given a model calibration corresponding to that in the main paper and for levels of firm leverage as that found for our sample of firms. The latter is shown in Table 2 in the main paper.

The speculators' objective function and the exogenous shocks are assumed to be the same as in the model in the main text and so these components are not repeated here. The producers' problem, however, is the key difference.

2.1 The producers' problem

The manager of the producing firm now maximizes firm value and has no preference-induced hedging demand. However, upon default, the manager bears a deadweight cost C (for instance, due to a loss in human capital). The firm has debt due in period 1 with face value m. This debt is taken to be 'old' debt and exogenous to the problem.

In the following, we let lower case i and h denote the individual manager's inventory and hedging decisions, while I and H denote the corresponding aggregate quantities. Otherwise, the notation is the same as that in the main paper. The managers' problem is then:

$$\max_{i,h} S_0 \left(g_0 - i\right) + E\left[\Lambda \left\{S_1 \left(g_1 + \left(1 - \delta\right) i\right) + h \left(F - S_1\right) - m\right\} | NoDefault\right] \times \left(1 - \Pr\left\{Default\right\}\right)$$

$$\tag{6}$$

$$-E\left[\Lambda C | Default\right] \times \Pr\left\{Default\right\}$$

$$s.t. \quad i \geq 0.$$

Default happens if:

$$S_1 \left(g_1 + \left(1 - \delta\right) i\right) + h \left(F - S_1\right) \quad < \quad m$$

$$\Updownarrow$$

$$S_1 \quad < \quad \frac{m - hF}{g_1 + \left(1 - \delta\right) i - h} \tag{7}$$

assuming that $g_1 + \left(1 - \delta\right) i - h > 0$ in equilibrium. Varying m is our comparative static and we will link m to our measures of default risk later.

The speculators' FOC is the same as in the model in the main text:

$$h_s = \frac{E\left[\Lambda \left(S_1 - F\right)\right]}{\gamma_S \sigma_S^2}, \tag{8}$$

4

where in equilibrium $h = h_s$. We can the write the futures risk premium as:

$$E\left[\Lambda\left(\frac{S_1 - F}{F}\right)\right] = E\left[\Lambda\right]E\left[\frac{S_1 - F}{F}\right] + cov\left(\Lambda, \frac{S_1}{F}\right)$$

$$\Updownarrow$$

$$E\left[\frac{S_1 - F}{F}\right] = -R_f cov\left(\Lambda, \frac{S_1}{F}\right) + R_f E\left[\Lambda\left(\frac{S_1 - F}{F}\right)\right]$$

$$= -R_f corr\left(S_1, \Lambda\right)\sigma_\Lambda \sigma_F + R_f \gamma_S F H^* \sigma_F^2, \qquad (9)$$

where $\sigma_F \equiv \sqrt{E\left[\left((S - F)/F\right)^2\right]} = \frac{\sigma_S}{F}$ is the standard deviation of the futures' returns. Unlike the model with managerial risk aversion, the model considered here does not yield an analytical expression for aggregate open interest H^*, and therefore we unfortunately do not obtain an analytical expression of the futures risk premium in terms of fundamental parameters. We solve the model numerically and show the model's implications for relevant parameterizations.

2.2 Equilibrium and model solution

We will solve a partial equilibrium version of this model where for ease of exposition we let the net risk-free rate equal zero, and where we assume that the exogenous pricing kernel, relevant for the value of the producer firms, is constant. Thus, $\Lambda = 1$. In this case:

$$E\left[\Lambda C | Default\right] \times \Pr\left\{Default\right\} = C \times \Pr\left\{Default\right\}$$

$$= C \int_{-\infty}^{\frac{m - hF}{g_1 + (1 - \delta)i - h}} f(S_1) dS_1. \qquad (10)$$

Given the demand function, $S_t = \omega\left(\frac{A_t}{Q_t}\right)^{1/\varepsilon}$ and the log-normality of A_1 ($\ln A_1 \sim N\left(\mu, \sigma^2\right)$), and since inventory and new supply is known at time 0, S_1 is lognormally distributed. Thus:

$$C \int_{-\infty}^{\frac{m - hF}{g_1 + (1 - \delta)i - h}} f(S_1) dS_1 = C \times \Phi\left(\frac{\varepsilon \ln\left(\max\left[0, \frac{m - hF}{g_1 + (1 - \delta)i - h}\right]\right) - \varepsilon \ln \omega + \ln Q_1 - \mu}{\sigma}\right), \qquad (11)$$

where $\Phi\left(\cdot\right)$ is the cumulative density function of the standard Normal distribution. In the following, we consider the case where $m > hF$ and $g_1 + (1 - \delta)i - h > 0$ (ie, the case where the firm is not fully hedged and default is possible), and so we will drop the max operator in the above expression. Define $x_2 \equiv \frac{\varepsilon \ln\left(\frac{m - hF}{g_1 + (1 - \delta)i - h}\right) - \varepsilon \ln \omega + \ln Q_1 - \mu}{\sigma}$.

Note that:

$$\ln\frac{S_1}{S_0} = \frac{1}{\varepsilon}\ln a_1 + \frac{1}{\varepsilon}\ln\frac{1}{A_0 Q_1 / Q_0} \sim N\left(\left(\mu - \ln\left(A_0 Q_1 / Q_0\right)\right)/\varepsilon, \sigma^2/\varepsilon^2\right). \qquad (12)$$

Thus, we can apply the Black-Scholes formula to show that:[3]

$$\left(g_1 + (1 - \delta)i - h\right)E\left[\max\left[S_1 - \frac{m - hF}{g_1 + (1 - \delta)i - h}, 0\right]\right] = \qquad (13)$$

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[3]In Black-Scholes the mean growth of the log price change is $r - \frac{1}{2}\sigma^2$. This drift term is then the counterpart of $\left(\mu - \ln A_0 Q_1 / Q_0\right)/\varepsilon$ in the model at hand.

$$(g_1 + (1 - \delta) i - h) \times S_0 \times \Phi(x_1) - (m - hF) \times \Phi(-x_2),$$

where we have used the fact that the risk-free rate is set to zero, and where:

$$
\begin{aligned}
x_1 &= \frac{\varepsilon \ln \frac{S_0(g_1 + (1-\delta)i - h)}{m - hF} + \mu - \ln(A_0 Q_1/Q_0) + \sigma^2/\varepsilon}{\sigma} \\
&= \frac{\varepsilon \ln \frac{g_1 + (1-\delta)i - h}{m - hF} + \mu - \ln Q_1 + \varepsilon \ln \omega + \sigma^2/\varepsilon}{\sigma},
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
x_2 &= -\frac{\varepsilon \ln \frac{S_0(g_1 + (1-\delta)i - h)}{m - hF} + \mu - \ln(A_0 Q_1/Q_0)}{\sigma} \\
&= \frac{\varepsilon \ln \frac{m - hF}{g_1 + (1-\delta)i - h} - \mu + \ln Q_1 - \varepsilon \ln \omega}{\sigma}.
\end{aligned}
\tag{15}
$$

Note that the last equality coincides with the original definition of x_2. Since $\Phi(-x) = 1 - \Phi(x)$, the producers' objective function can now be written:

$$\max_{i,h} S_0 (g_0 - i) - m + hF + (g_1 + (1 - \delta) i - h) \times S_0 \times \Phi(x_1)$$

$$- (C + hF - m) \times \Phi(x_2) \tag{16}$$

$$s.t. \quad i \geq 0.$$

The first order condition with respect to firm level inventory then implies that:

$$\frac{1}{1 - \delta} = \Phi(x_1) + \phi(x_1) \frac{\varepsilon}{\sigma} + \phi(x_2) \frac{\varepsilon}{\sigma} \frac{C + hF - m}{S_0 (g_1 + (1 - \delta) i - h)} - \lambda, \tag{17}$$

where $\phi(\cdot)$ is the probability density function of the standard Normal distribution and λ is the LaGrange multiplier on the inventory constraint.

The producers' first order condition with respect to short hedging h is then:

$$\frac{F}{S_0} - \Phi(x_1) + \phi(x_1) \frac{\varepsilon}{\sigma} \left(\frac{F(g_1 + (1 - \delta) i - h)}{m - hF} - 1 \right)$$

$$= \frac{F}{S_0} \times \Phi(x_2) + \phi(x_2) \frac{\varepsilon}{\sigma} \frac{C + hF - m}{S_0 (g_1 + (1 - \delta) i - h)} \frac{m - F(g_1 + (1 - \delta) i)}{m - hF}. \tag{18}$$

Combing these first order conditions yield (setting $\lambda = 0$, as we will focus on the case of no stock-out here), we get the somewhat simpler equation:

$$\frac{F}{S_0} - \frac{1}{1 - \delta} + \phi(x_1(F, i, h, Q_1)) \frac{\varepsilon}{\sigma} \frac{F(g_1 + (1 - \delta) i - h)}{m - hF}$$

$$= \frac{F}{S_0} \times \Phi(x_2(F, i, h, Q_1)) - \frac{F}{S_0} \phi(x_2(F, i, h, Q_1)) \frac{\varepsilon}{\sigma} \frac{C + hF - m}{m - hF} \tag{19}$$

Solving for equilibrium, we first note that Equation (19) gives the futures price F as a function of aggregate inventory, after we substitute out the hedging demand using the speculators' first order condition, imposing market clearing:

$$H = \frac{E[S_1] - F}{\gamma_s \sigma_S^2}, \tag{20}$$

6

noting also that $E[S_1] = \omega Q_1^{-1/\varepsilon} e^{\mu/\varepsilon + \frac{1}{2}\sigma^2/\varepsilon^2}$. Next, we solve for equilibrium inventory using Equation (17) substituting out H and F so the only unknown is inventory. With equilibrium inventory in hand, finding the equilibrium hedging and futures price is immediate using Equations (18) and (20).

2.3 Comparative statics

The two parameters of interest is the firm's debt level m (really, firm leverage), which is the driver of producer hedging demand, and speculator capital constraints, γ_s. We calibrate the parameters of the model as described in the main text of the paper. In particular, we let $\varepsilon = 0.1$, $\omega = 0.01$, $\gamma_s \in \{8, 40\}$, $\delta = 0.01$, $\mu = 0.004$, $\sigma = 0.03$, $A_0 = 1$, $g_0 = 0.8$, $g_1 = 0.75$, $C = 0.05$. Our measures of default risk, the Zmijewski score and the EDF, are both positively related firm leverage, which we use as the aggregate default risk measure from the model. In the data and as given in Table 2 in the main text, leverage is computed using the market value of equity and the book value of debt: $Leverage = \frac{m}{E+m}$, where E denotes the value of equity given the optimal inventory and hedging decision and m is the book value of debt.

Figure 3 reproduces this alternative model's counterparts to Figures 1 with the leverage measure of default risk on the horizontal axis. The changing default risk is achieved by varying the net debt level m. We vary the debt level such that the costs of hedging remains typically less than 1% of firm value and focus on the cases where leverage and default probabilities are such that the hedging demand exceeds the risk shifting motive and producers therefore are short in the futures market, as in the data.

[Figure 3 about here]

As shown by the figure, an increase in default risk, increases the futures risk premium, and decreases the spot price and inventory. This effect is stronger when speculator risk aversion is high, consistent with the model with managerial risk aversion presented in the main paper. It should be noted that for leverage levels lower than 20%, the futures risk premium goes negative, and producers choose to go *long* the futures contract in order to *increase* risk due to the risk shifting motive given limited liability. As we show in the empirical micro study, increasing default risk is associated in the data with larger *short* hedging positions. This is the case we consider in the above calibration, which also has leverage levels on par with those in the data, as given in Table 2.

3 Model where producer default leads to supply disruption

In this section we consider a default risk model similar to that in the previous section, but where upon default a fraction α of the period 1 supply of the commodity is lost. This supply disruption will impact the spot price and therefore its conditional mean and variance, which in turn will affect the cost of hedging.

7

3.1 The producers' problem

The manager of the producing firm now maximizes firm value and has no preference-induced hedging demand. However, upon default, the manager bears a deadweight cost C (for instance, due to a loss in human capital). The firm has debt due in period 1 with face value m. This debt is taken to be 'old' debt and exogenous to the problem.

In the following, we let lower case i and h denote the individual manager's inventory and hedging decisions, while I and H denote the corresponding aggregate quantities. Otherwise, the notation is the same as that in the main paper. The managers' problem is then:

$$\max_{i,h} S_0 \left(g_0 - i\right) + E\left[\Lambda \left\{S_1 \left(g_1 + \left(1 - \delta\right) i\right) + h\left(F - S_1\right) - m\right\} | NoDefault\right] \times \left(1 - \Pr\left\{Default\right\}\right) \tag{21}$$

$$-E\left[\Lambda C | Default\right] \times \Pr\left\{Default\right\}$$

$$s.t. \quad i \geq 0.$$

In the event of a default, a fraction α of the period 1 supply is lost. That is, upon default $\alpha\left(g_1 + \left(1 - \delta\right) I\right)$ of the commodity is in the aggregate not brought to the market period 1 equilibrium supply is therefore $Q_{1|default} = \left(1 - \alpha\right)\left(g_1 + \left(1 - \delta\right) I\right)$. Default happens if:

$$S_1^- \left(g_1 + \left(1 - \delta\right) i\right) + h\left(F - S_1^-\right) \quad < \quad m$$

$$\Updownarrow$$

$$S_1^- \quad < \quad \frac{m - hF}{g_1 + \left(1 - \delta\right) i - h} \tag{22}$$

assuming that $g_1 + \left(1 - \delta\right) i - h > 0$ in equilibrium, and where $S_1^- \equiv \omega \left(\frac{A_1}{Q_{1|NoDefault}}\right)^{1/\varepsilon}$, where $Q_{1|NoDefault} \equiv g_1 + \left(1 - \delta\right) I$. That is, the default event is determined based on the price that would prevail if there is no aggregate default event. This assumption is necessary in order to obtain equilibrium in the model. Otherwise, there is a region of outcomes for A_1 where the competitive equilibrium does not exist. For instance, consider the case where S_1 given no default would be so low as to trigger default, but where the supply disruption in the case of a default would increase the spot price S_1 to a level where the firms are solvent. Again, varying m is again our comparative static and we assume in the following that $\Lambda = 1$.

Notice that the producer's problem is exactly the same as before, as the loss upon default is borne by debt-holders. However, speculator's are still exposed to the supply disruption event in the case of default. Thus, the conditional volatility and expectation of S_1 is different in this model. In particular:

$$E\left[S_1\right] \quad = \quad E\left[S_1 | NoDefault\right] \Pr\left\{NoDefault\right\} + E\left[S_1 | Default\right] \Pr\left\{Default\right\}$$

$$= \quad S_0 \times \Phi\left(x_1\right) + \omega \left(g_1 + \left(1 - \delta\right)\left(1 - \alpha\right) I\right)^{-1/\varepsilon} e^{\mu/\varepsilon + \frac{1}{2}\sigma^2/\varepsilon^2} - S_0 \times \Phi\left(\tilde{x}_1\right), \tag{23}$$

where[4]

$$x_1 \quad = \quad \frac{\varepsilon \ln \frac{g_1 + \left(1 - \delta\right) I - H}{m - hF} + \mu - \ln\left(g_1 + \left(1 - \delta\right) I\right) + \varepsilon \ln \omega + \sigma^2/\varepsilon}{\sigma}, \tag{24}$$

$$\tilde{x}_1 \quad = \quad \frac{\varepsilon \ln \frac{g_1 + \left(1 - \delta\right) I\left(1 - \alpha\right) - H}{m - hF} + \mu - \ln\left(g_1 + \left(1 - \delta\right)\left(1 - \alpha\right) I\right) + \varepsilon \ln \omega + \sigma^2/\varepsilon}{\sigma}, \tag{25}$$

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[4]To see the this, first note that $E\left[S_1 | NoDefault\right] \Pr\left\{NoDefault\right\}$ is the same as in the model with no supply

and as before $\Phi\left(\cdot\right)$ is the standard normal cumulative density function. Note the presence of $1-\alpha$ in the above equations, which takes into account the supply disruption.

The variance of the period 1 spot price is then:

$$\sigma^2\left(S_1\right) = E\left[S_1^2\right] - E\left[S_1\right]^2$$

note that $\ln S_1$ is normally distributed and that we need to find $E\left[e^{2s_1}\right]$ where $s_1 = \ln\omega + a_1/\varepsilon - \frac{1}{\varepsilon}\ln Q_1$. We have that $2s_1 - 2s_0 = 2\left(\mu - \ln\left(Q_1 A_0/Q_0\right)\right)/\varepsilon + 2\frac{\sigma}{\varepsilon}\eta_1$, where $\eta_1 \sim N\left(0,1\right)$. Again, using results from the Black-Scholes formula, we have that:

$$E\left[S_1^2|NoDefault\right] \times \Pr\left\{NoDefault\right\} = S_0^2 \times \Phi\left(x_3\right), \tag{26}$$

where

$$x_3 = \frac{\varepsilon\ln\frac{S_0^2(g_1+(1-\delta)I-H)}{m-hF} + 2\left(\mu - \ln\left(\left(g_1 + (1-\delta)I\right)A_0/Q_0\right)\right) + 4\sigma^2/\varepsilon}{2\sigma}. \tag{27}$$

Similarly, we have that:

$$E\left[S_1^2|Default\right] \times \Pr\left\{Default\right\} = \omega^2\left(g_1 + (1-\delta)(1-\alpha)I\right)^{-2/\varepsilon}e^{2\mu/\varepsilon + 2\sigma^2/\varepsilon^2} - S_0^2 \times \Phi\left(\tilde{x}_3\right), \tag{28}$$

where

$$\tilde{x}_3 = \frac{\varepsilon\ln\frac{S_0^2(g_1+(1-\delta)I(1-\alpha)-H)}{m-hF} + 2\left(\mu - \ln\left(\left(g_1 + (1-\delta)(1-\alpha)I\right)A_0/Q_0\right)\right) + 4\sigma^2/\varepsilon}{2\sigma}. \tag{29}$$

Using these expressions, we have that:

$$\begin{aligned}\sigma^2\left(S_1\right) &= E\left[S_1^2\right] - E\left[S_1\right]^2 \\ &= S_0^2 \times \Phi\left(x_3\right) + \omega^2\left(g_1 + (1-\delta)(1-\alpha)I\right)^{-2/\varepsilon}e^{2\mu/\varepsilon + 2\sigma^2/\varepsilon^2} - \dots \tag{30} \\ &\quad S_0^2 \times \Phi\left(\tilde{x}_3\right) - E\left[S_1\right]^2. \tag{31}\end{aligned}$$

With the mean and variance of S_1 in hand, we use the speculator's first order condition and the market clearing condition in the futures market to substitute out H of the equilibrium equations as before:

$$H = \frac{E\left[S_1\right] - F}{\gamma_s\sigma^2\left(S_1\right)}. \tag{32}$$

In sum, the supply disruption does not directly affect the producer's problem. However, the speculators face a different mean and variance of the spot price, which affects their demand. Thus, the supply disruption will affect the cost of hedging.

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disruption. Second, note that

$$\begin{aligned}E\left[S_1|Default\right]\Pr\left\{Default\right\} &= E\left[S_1|S_1^- < \frac{m-HF}{g_1+(1-\delta)I-H}\right]\Pr\left\{S_1^- < \frac{m-HF}{g_1+(1-\delta)I-H}\right\} \\ &= E\left[S_1|Q_1 = g_1 + (1-\delta)I\right] - \dots \\ &\quad E\left[S_1|Q_1 = g_1 + (1-\delta)I \text{ AND } S_1^- \geq \frac{m-HF}{g_1+(1-\delta)I-H}\right]\Pr\left\{S_1^- \geq \frac{m-HF}{g_1+(1-\delta)I-H}\right\} \\ &= \omega\left(g_1 + (1-\delta)(1-\alpha)I\right)^{-1/\varepsilon}e^{\mu/\varepsilon+\frac{1}{2}\sigma^2/\varepsilon^2} - S_0 \times \Phi\left(\tilde{x}_1\right).\end{aligned}$$

3.2 Comparative statics

The relevant comparative static for this model is with respect to α – the amount of supply lost in the event of default. The case $\alpha = 0$ corresponds to the case of no supply disruption as analyzed in the first section of this online Appendix. Otherwise, we keep the parameters the same as in the model with no supply disruption.

Figure 4 shows the futures risk premium versus α for the case where speculator risk aversion is high ($\gamma_S = 40$) and the firm leverage is around 25%, as is the mean value in the data. This corresponds to $m = 0.0517$. The top plot shows that the futures risk premium is *decreasing* in the supply disruption. This is intuitive: a negative supply shock means the spot price increases. An increase in the price benefits the long side of the futures contract and thus decreases the futures risk premium.

The lowe plot in Figure 4 shows that the variance of the next period's spot price is decreasing in the supply disruption α. This occurs as the supply disruption is an implicit hedge for the demand shock: a supply disruption, which only happens in default when the demand shock is low, leads to a higher spot price. As in the previous model, for low levels of leverage it is possible to get the opposite effect on the risk premium as producers will go long when the risk shifting effect dominates their futures demand, as discussed for the model in the previous section.

[Figure 4 about here]

In sum, a supply disruption related to default risk in the model calibrated to the mean leverage level in the data leads to a *lower* futures risk premium. This is the opposite of what we find in the data, we conclude that this effect, if present, is not the dominating factor for determining the empirical relation between the futures risk premium and aggregate producer default risk.

4 Additional empirical results

In this section, we present additional empirical results and robustness tests relative to those presented in the main paper.

4.1 Time-varying equity factor exposures

[Figure 5 about here]

Table 1 shows the effect of default risk after controlling for the realized covariance of the futures return and the Fama-French (1993) three factor model. We choose these factors as this model is the most common factor model used in studies of the cross-section of equity returns. This is thus a robustness test in terms of controlling for standard risk factors, allowing for time-varying factor exposures.

The realized covariances are constructed quarterly using daily data on the commodity futures return, as well as the factor returns. The realized covariance is simply the sum of the product

10

of the commodity return and the factor over the quarter. These tests effectively allow for time-varying correlation between the commodity futures return and the risk factors, as well as time-varying volatility of both commodity and factor returns. Figure 5 shows that there indeed are large variation in these correlations. In fact, they often flip signs. Notably, however, the covariances are very small, hovering around zero except for in recession periods where commodity prices have been strongly affected. For instance, during the recession in 1991 with the onset of the first Iraq war, market covariances become large and negative as the market dropped while oil prices increased. In the recent financial crisis, the covariances with the market sky-rocketed, but had the opposite sign (see Tang and Xiong (2009)).

Table 1 shows that including these controls do not change the main results - the hedging demand channel is still a statistically and economically significant predictor of commodity futures returns in our sample. The commodity futures tend to move negatively with the market factor, negatively with the size factor, but positively with the book-to-market factor. We conclude that time-varying factor exposure cannot account for the variation in the commodity futures risk premiums related to time-variation in commodity sector aggregate default risk.

[Table 1 about here]

4.2   Additional results: inventory

We next turn to the implications for variation in the default risk measures on aggregate commodity inventory levels. The impact of producers' inventory decision of increased fundamental hedging demand is typically negative, but per the discussion of the general equilibrium model in Section 2 in the main paper, this implication does not hold for all reasonable parameterizations of this model. Thus, the sign here is an empirical question.

An instrumental variables approach is needed to answer this question. In particular, transitory demand shocks (the A_t's in the model) will affect both measures of default risk and inventory with opposite signs, even though there is no causal relation between the two. For instance, a transitory negative shock to demand will increase inventory holdings even in a frictionless model such as Deaton and Laroque (1992). At the same time, the resulting lower spot price will affect the measures of default risk negatively. Thus, we need an instrument for default risk at time $t + 1$ that's unrelated to the demand shock at time $t + 1$, but related to the inventory decision at time $t + 1$. Since the default risk measures are quite persistent, with use the fitted measure of default risk from an AR(1) as the instrumented default risk variable: $DefRisk_{i,t+1}^{IV} = \hat{\alpha}_i + \hat{\beta}_i DefRisk_{i,t}$ where $\hat{\alpha}_i$ and $\hat{\beta}_i$ are the AR(1) regression coefficients obtained from the full sample regression and adjusting the $\hat{\beta}_i$ for the well-known small-sample bias in the AR(1) coefficient. We run the individual regressions:

$$\Delta i_{i,t+1} = \beta_i \times DefRisk_{i,t+1}^{IV} + \sum_{j=1}^{3} \gamma_{i,j} \Delta i_{i,t+1-j} + ControlVariables_t + u_{i,t+1}, \qquad (33)$$

and the pooled regression:

$$\Delta i_{i,t+1} = \beta \times DefRisk_{i,t+1}^{IV} + \sum_{j=1}^{3} \gamma_{i,j} \Delta i_{i,t+1-j} + ControlVariables_t + u_{i,t+1}, \qquad (34)$$

where i is the log inventory level and Δ denotes the first difference operator. Table 2 shows that high default risk indeed is associated with lower levels of inventory in every regression, for all commodities and both default risk measures. The effect is significant at the 5% level fro Crude Oil and in the pooled regressions for both default risk measures, as well as for Gasoline for the Zmijewski-score. The effect is significant at the 10% level for heating oil for the EDF measure of default risk. For the remainder the coefficients on default risk are not significant, although all have a negative sign. A one standard deviation increase in aggregate default risk decreases the inventory by 7% to 10% of its standard deviation in the pooled regressions, which in turn tends to depress current spot prices and increase future spot prices, consistent with the forecasting results shown in Table 3. For Crude Oil, which is the largest market, this inventory response is 16% to 17% of its standard deviation for a one standard deviation increase in the default risk measures.

[Table 2 about here]

4.3   Additional results: the futures basis

The commodity basis spread, defined in our setting as the difference between the spot price and the futures price relative to the spot price:

$$basis_t^\tau \equiv \frac{S_t - F_t^{(\tau)}}{S_t} \tag{35}$$

, is often used as an indicator of future expected spot prices and also of expected returns in the futures market. In particular, Hong and Yogo (2010) show that the aggregate basis across all commodities has some forecasting power for future commodity returns. Gorton, Hayashi, and Rouwenhorst (2010) show that the basis is a powerful predictor of the cross-section of commodity returns. The model in Section 2 in the main paper, however, predicts that the individual commodity basis is not affected by the commodity sector default risk unless there is an inventory stock-out. This predictions is a bit stark and comes from the two period set up. Routledge, Seppi, and Spatt (2000) show that the probability of a stock-out does affect the basis in a multiperiod setting.

To investigate whether the default risk measures can help explain the commodity basis spread, we apply the same instrumental variables approach as we did for the inventory regressions. In particular, a transitory negative demand shock will tend to increase the basis as well as default spread, even if there is no causal relationship between the two. We run the individual commodity regressions:

$$basis_{i,t+1} = \beta_i \times DefRisk_{i,t+1}^{IV} + \gamma_i basis_{i,t} + ControlVariables_t + u_{i,t+1}. \tag{36}$$

and the pooled regression:

$$basis_{i,t+1} = \beta \times DefRisk_{i,t+1}^{IV} + \gamma_i basis_{i,t} + ControlVariables_t + u_{i,t+1}. \tag{37}$$

Table 3 shows that in the pooled regressions for the EDF measure, the basis indeed increases by 14% of its standard deviation for a one standard deviation in aggregate default risk, significant

at the 5% level. For the Zmijewski-score, the estimated increase is 15%, but this number is not significant at conventional levels. For Crude Oil, which is the largest market, the effect is stronger (21% and 22%), where the EDF is significant at the 5% level and the Zmijewski-score is significant at the 10% level. Otherwise, Heating Oil has the EDF positive and significant at the 5% level as well. The remaining coefficients are not significant at conventional levels, but, with the exception of Gasoline for the EDF, they are all positive. Thus, overall there is a positive relation between the basis as we define it in Equation (35) and the default risk measures, but the regressions reveal that a large fraction of the variation in the basis is not related to variation in producer default risk, consistent with the model.

[Table 3 about here]

4.4 Additional results: CFTC hedger positions

In the main paper, we show that the default risk measures in pooled regressions across all commodities are positively, significantly related to the CFTC measures of net hedger positions. The net hedger position is calculated as net commercial traders' short futures positions divided by the sum of net, lagged commercial traders' and non-commercial traders' positions. Table 4 gives the same regressions for the individual commodities in two ways. First, we simply run contemporaneous regressions of hedger positions for each commodity on the corresponding default risk measures. All right hand side and left hand side variables have been normalized to have zero mean and unit variance. As the table shows, a one standard deviation increase in hedger positions is in the pooled regression significantly associated with on average a 10% increase in the CFTC hedger positions. At the individual commodity level, however, the significance is only there for heating oil, while the magnitude of the coefficients are overall comparable across commodities.

The regressions in Panel A of Table 4 may, however, not pick up the pure effect of increased default risk on the CFTC hedger positions. In particular, higher default risk is associated with negative shocks to the spot price. The latter means that speculators have incurred losses which may make them more risk averse which, all else equal, means the futures risk premium increases. The ensuing higher cost of hedging may be sufficiently large so as to make producer scale back on their hedge, even though their default risk increases. To isolate the effect of default risk alone, we apply an instrumental variable approach. In particular, we regress the default risk measure on its lag and use the predicted default risk from this regression as the instrument for current default risk. We also include the lagged CFTC hedger position in the regression. Panel B of Table 4 shows the results of running this regression. In this case the coefficients are somewhat higher – in the pooled regression the coefficients imply a CFTC hedger measure response of 17% to 23% of its standard deviation to a one standard deviation increase in the default risk measures. Also, all coefficients in the individual regressions are now positive and higher than in the contemporaneous regression.

[Table 4 about here]

13

5   Description of Micro Data

The EDGAR database contains quarterly or annual statements for 94 firms with SIC code 1311. These firms' reports available following the issuance of the Financial Accounting Standards Board's (FAS) 133 regulation "Accounting for Derivative Instruments and Hedging Activities" in June 1998. The reports are available for fiscal years ending after 15th of June 2000 – we therefore begin our sample for quarterly reports in Q1 of 2000 and end in Q4 of 2010.

Since the introduction of FAS 133, firms are required to measure all financial assets and liabilities on company balance sheets at fair value. In particular, hedging and derivative activities are usually disclosed in two places. Risk exposures and the accounting policy relating to the use of derivatives are included in "Market Risk Information." Any unusual impact on earnings resulting from accounting for derivatives should be explained in the "Results of Operations." Additionally, a further discussion of risk management activity is provided in a footnote disclosure titled "Risk Management Activities & Derivative Financial Instruments."

While firms are required to recognize derivative positions as assets or liabilities on their balance sheet given this accounting standard, the current market value of a derivative in most cases does not allow one to infer the notional exposure or even the direction of trade (for example, consider an exchange traded futures contract). Thus, it is necessary to read the management's notes on hedging behavior which gives qualitative information of the amount and nature of hedging. Our 94 firm sample required the reading of around 2,500 quarterly and annual reports over the sample period.

To make the onerous task of manually reading and deciphering all of these reports manageable and also to quantify the qualitative information given in the reports, we created a set of fields to be filled out for each firm quarter with a "0" for "No", a "1" for "Yes" and missing if there is no clear information in the report. The fields we employ in the paper are:

For each quarter and firm:

use derivatives?
futures or forwards?
swaps?
options?
significant short crude?
significant long crude?

The first question is simply "Does the firm allow for using derivatives to manage risk?" There are 2,400 firm-quarter reports where we were able to determine from the report whether they did, and these firm-quarters constitute the main sample. In 88% of the firm-quarters the answer was affirmative. Note that this does not necessarily mean that a firm is using derivatives that quarter – it may just mean that the management wrote in the report a generic statement that the firm *may* use derivatives to manage risk. Furthermore, in 47.7% of the firm-quarters forwards or futures were used, in 80.6% of the firm-quarters swaps were used, and in 81.8% of the firm-quarters options (typically a short collar position) were used.[5]

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[5]We focus on short-term commodity futures - the most liquid derivative instruments in the commodity markets -

The dominant commodity exposure hedged by firms is Crude Oil. We therefore concentrate our efforts on determining whether the firm had a significant long or short position in Crude Oil derivatives. In some cases, the actual notional positions in the derivatives were given, and in such cases, we specify that a significant position is one in which the management hedged at least 25% of production. In many cases, however, we infer the existence of a significant position from management statements that 'most' or 'a large part of' the production is hedged using a particular type of derivatives position.

We also filled in the following fields not used in the paper:

For each quarter and firm:

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speculating using derivatives?
hedging using derivatives?
short crude derivatives exposure?
increased short derivatives crude oil exposure?
decreased short derivatives crude oil exposure?
minor short crude derivatives exposure?
no crude derivatives exposure?
minor long crude derivatives exposure?

These fields are used as checks relative to the main fields used in the analysis, so as to minimize reporting errors – either from the report itself or in the interpretation of the reports. If an inconsistency is apparent, the report is asked to be re-read. For instance, if a firm is tagged as having increased the short crude derivatives exposure, but at the same time being tagged for having no crude derivatives exposure, an inconsistency is noted. If a firm is tagged for having short crude derivatives exposure but is not tagged for hedging or is tagged for speculation, an inconsistency is noted. If a firm goes from minor crude to significant crude short exposure, the firm must be tagged also for the 'increased short derivatives exposure' field. If not, an inconsistency is noted.

Minor short or long derivatives exposure corresponds to less than 25% of current set production plans. There is not always sufficient information to determine exactly the percentage of production and inventory that is hedged, and qualitative judgements are then made. For instance, if the management writes that 'a large (or significant or substantial or considerable) fraction of Crude Oil production and inventory is hedged using derivatives, this is deemed as significant short crude derivatives exposure. It is important to note that in most cases, a qualitative judgment must be made. Finally, we also did a number of random checks where we read the underlying reports to ensure that the manual data collection and codification was executed with high accuracy and consistency.

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in our empirical analysis. However, a considerable fraction of the hedging is done with swaps, which are provided by banks over-the-counter, and often are longer term. On the one hand, this indicates that a significant proportion of producer's hedging is done outside the futures markets that we consider. On the other hand, banks in turn hedge their aggregate net exposure in the underlying futures market and in the most liquid contracts. For instance, it is common to hedge long-term exposure by rolling over short-term contracts (e.g., Metallgesellschaft). A similar argument can be made for the net commodity option imbalance held by banks in the aggregate. Therefore, producers' aggregate net hedging pressure is likely to be reflected in trades in the underlying short-term futures market.

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[5] Routledge, Bryan, Duane Seppi, and Chester Spatt (2000), "Equilibrium Forward Curves for Commodities," *Journal of Finance* 55: 1297-1338.

[6] Tang, Ke and Wei Xiong (2009), "Index Investing and the Financialization of Commodities," Working paper, Princeton University.

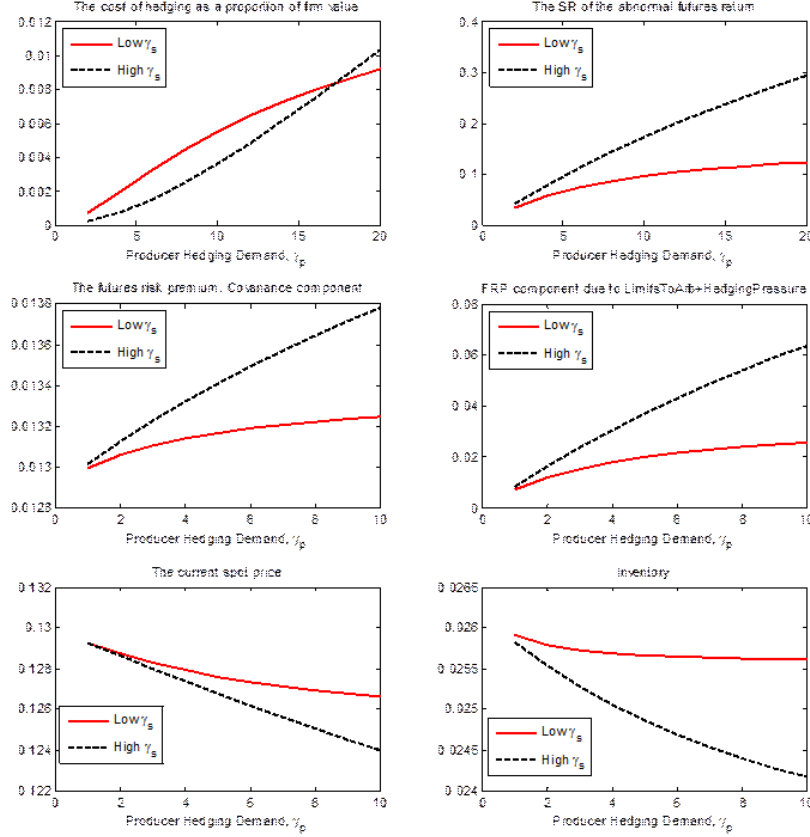**Figure 1 - General Equilibrium Model Predictions**

Figure 1: All plots have producer fundamental hedging demand ($\gamma_p = 2, 4, ..., 20$) on the horizontal axis. The dashed line corresponds to high speculator capital constraints ($\gamma_s = 40$), while the solid line is the case of low speculator capital constraints ($\gamma_s = 8$). The two top plots show the cost of hedging as a proportion of firm value (left) and the quarterly Sharpe ratio of the abnormal returns earned by speculators. The two middle plots show (on the left) the component of the futures risk premium due to the covariance with the equity market pricing kernel and (on the right) the component due to the combination of hedging pressure and limits to arbitrage. The two lowest graphs show the current spot price and the optimal inventory. Here the elasticity of intratemporal substitution (ϵ) equals 0.1 and the weight of the commodity in the utility function (ω) equals 0.01. The other numbers used in the calibration of the model are given in the text.
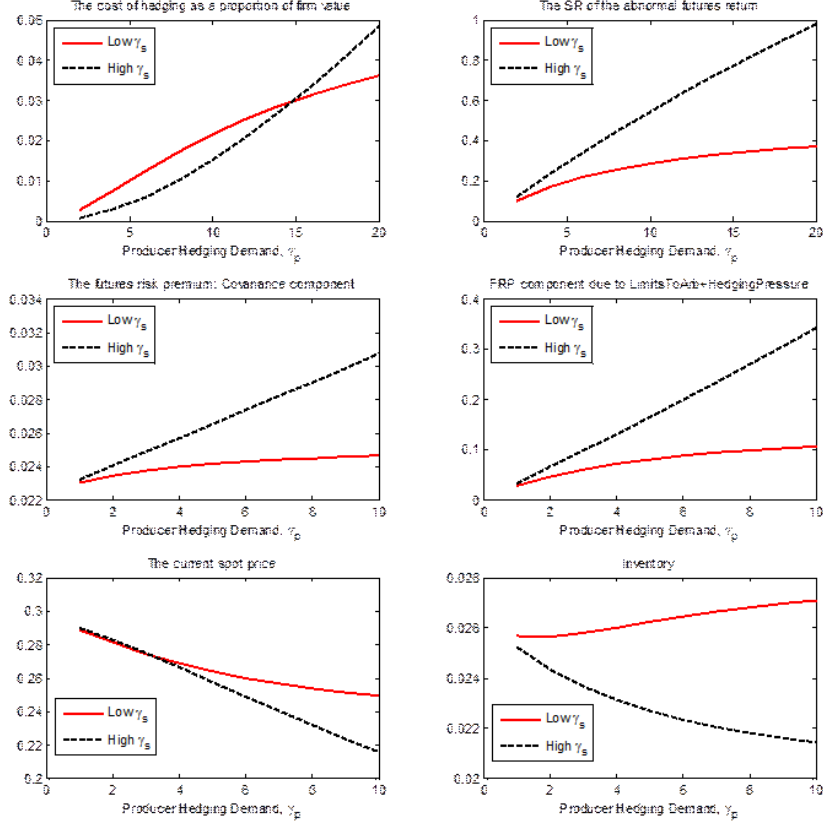
Figure 2 - Comparative statics from default risk model

Figure 2: All plots have producer fundamental hedging demand ($\gamma_p = 2, 4, ..., 20$) on the horizontal axis. The dashed line corresponds to high speculator capital constraints ($\gamma_s = 40$), while the solid line is the case of low speculator capital constraints ($\gamma_s = 8$). The two top plots show the cost of hedging as a proportion of firm value (left) and the quarterly Sharpe ratio of the abnormal returns earned by speculators. The two middle plots show (on the left) the component of the futures risk premium due to the covariance with the equity market pricing kernel and (on the right) the component due to the combination of hedging pressure and limits to arbitrage. The two lowest graphs show the current spot price and the optimal inventory. In this case, the elasticity of intratemporal substitution (ϵ) equals 0.08 and the weight of the commodity in the utility function (ω) equals 0.012 - slightly different from the case shown in Figure 1. The other numbers used in the calibration of the model are given in the text.

Figure 3 - Comparative statics from default risk model
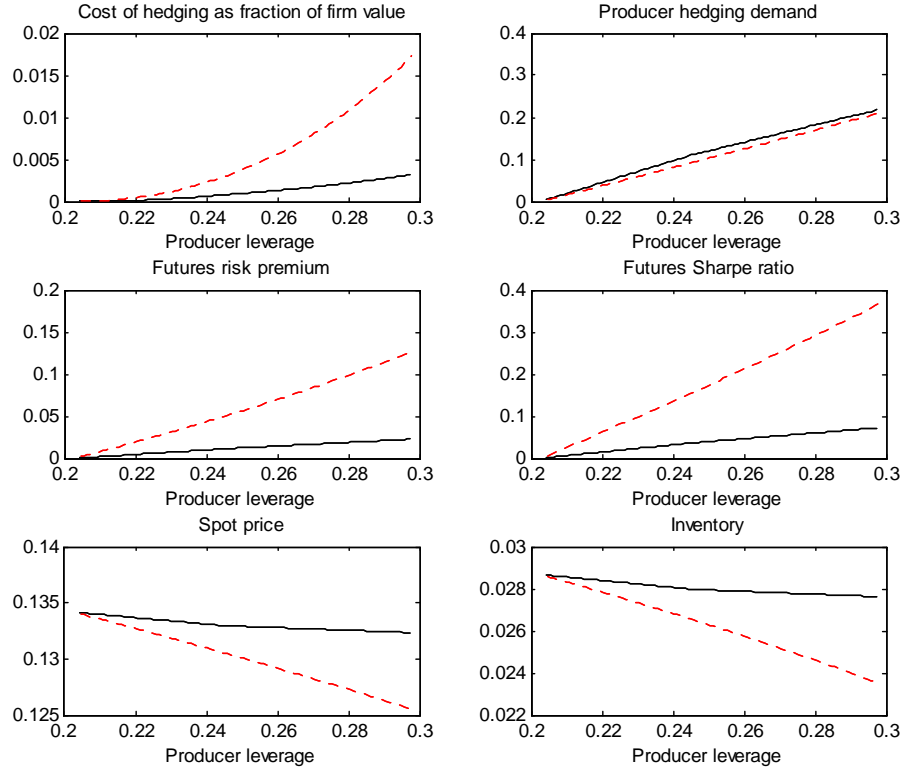
Figure 3: The figure shows moments from the default risk model, corresponding to the moments shown for the model with managerial risk averison in the main text. The x-axis is always producer leverage, measured as the book value of debt over the sum of the book value of debt and the market value of equity. The parameters of the model is given in the text. The solid line corresponds to the case with low speculator risk aversion, while the dashed line corresponds to the case with high speculator risk aversion.

Figure 4 - Supply disruption versus futures risk premium and return variance
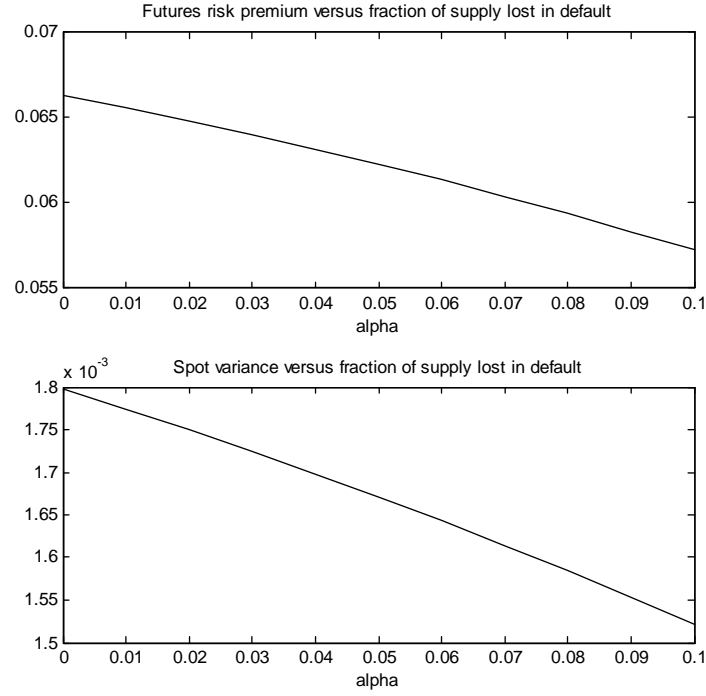
Figure 4: The top plot shows the futures risk premium, defined as $E[\frac{S_1-F}{F}]$ versus the fraction of period 1 supply lost in the event of a default. The case shown here corresponds to a prducer firm leverage of 24% as is the average in the sample we use in the empirical analysis. The speculator risk aversion is high ($\gamma_s = 40$), but the results are qualitatively the same for low speculator risk aversion (not shown). The bottom plot shows the variance of period 1's spot price versus the fraction of supply lost in the case of a default, for the same economy as in the top plot. The parameters of the model is given in the text.

Figure 5 - Realized covariances between Fama-French factors and commodity futures returns

Figure 5: The figure shows the realized covariance between the commodity futures returns and the three Fama-French factors; the market return, the return to small firms minus the return to large firms (SMB), and the return to high book-to-market firms minus the return to low book-to-market firms (HML). The realized covariance is calculated using daily returns to both the factors and the futures positions (next to nearest to maturity contract). The sample periods vary as different futures contracts were introduced at different times. The vertical axis gives the realized covariance, while the horizontal axis gives the year.

Table 1 - Futures return forecasting regressions:

Covariance controls

Table 1: The table shows the results from regressions of Crude Oil, Heating Oil, Gasoline, and Natural Gas futures returns on lagged default risk of oil and gas producers, as measured by the Expected Default Frequency (EDF) and Zmijewski-score (Zm) of firms with SIC code 1311. In addition to the usual controls (aggregate default risk, forecasted GDP growth, and the risk-free rate), the regression also has on the r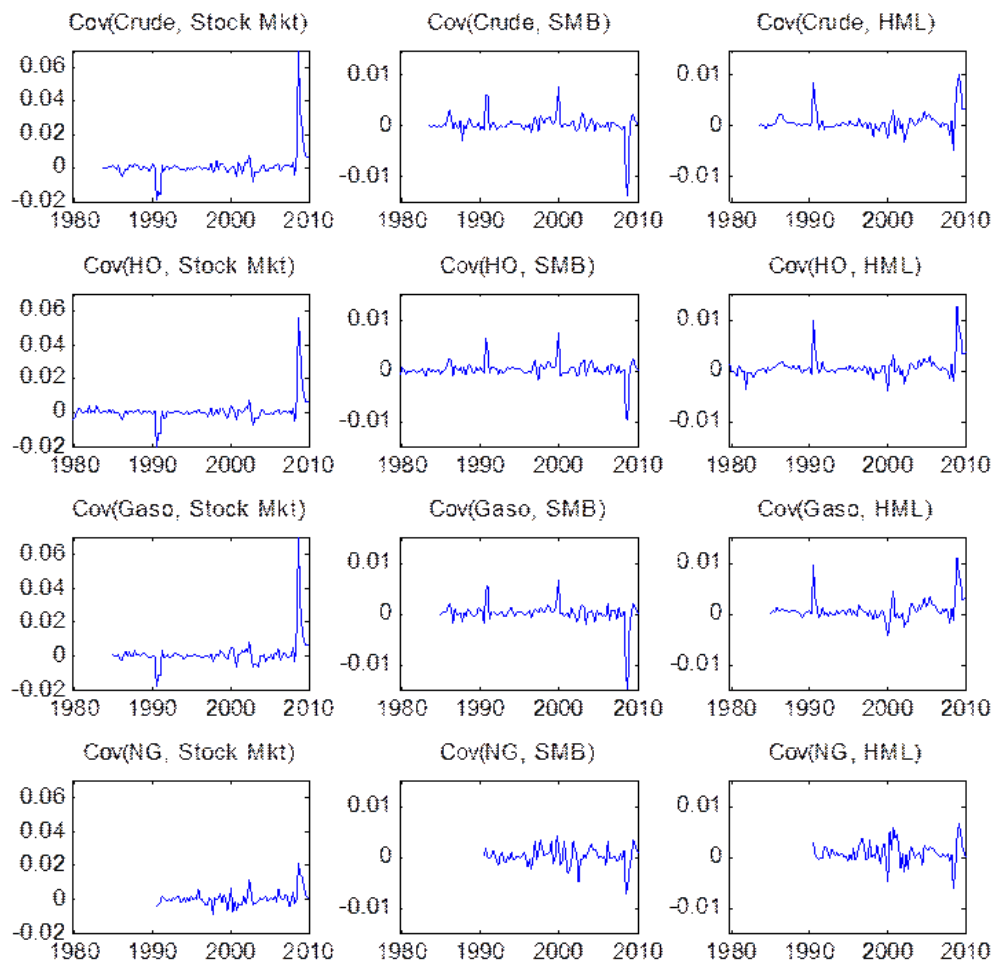ight hand side the realized covariance between the futures return and the three Fama-French factors: The market return, the return to the size factor (SMB), and the return to the book-to-market factor (HML). The realized covariance in a quarter is calculated from daily returns to the futures and the factors. Heteroskedasticity and autocorrelation adjusted (Newey-West; 3 lags) standard errors are used. In the joint regression in the right-most column, Rogers standard errors are used to control for heteroskedasticity, cross-correlation, and autocorrelation (3 lags). * denotes significance at the 10% level, ** denotes significance at the 5% level, and *** denotes significance at the 1% level.

	Crude Oil		Heating Oil		Gasoline		Natural Gas		All	
	EDF	Zm	EDF	Zm	EDF	Zm	EDF	Zm	EDF	Zm
$Cov(r_M, r_i)$	-15.1^{***}	-13.5^{***}	-11.1^{**}	-10.7^{**}	-11.7^{**}	-11.4^{**}	2.27	4.41	-10.3^{**}	-9.58^{**}
	(5.21)	(4.60)	(5.30)	(5.00)	(4.89)	(4.47)	(8.22)	(6.48)	(4.32)	(3.87)
$Cov(r_{SMB}, r_i)$	-35.3	-28.1	-8.02	-5.20	-8.23	-6.01	7.48	4.83	-9.68	-6.95^{**}
	(27.2)	(23.8)	(28.2)	(26.0)	(21.3)	(18.5)	(22.6)	(20.1)	(22.5)	(20.0)
$Cov(r_{HML}, r_i)$	30.2	32.2	15.8	15.4	24.6	24.1	17.2	20.4^{*}	19.8	21.2
	(20.2)	(20.3)	(19.5)	(19.8)	(18.9)	(19.7)	(13.6)	(12.7)	(13.8)	(14.1)
$DefRisk$	0.077^{**}	0.051^{**}	0.048^{**}	0.034^{**}	0.046	0.031	0.044	0.053^{*}	0.047^{*}	0.038^{**}
	(0.031)	(0.021)	(0.021)	(0.017)	(0.031)	(0.022)	(0.043)	(0.035)	(0.026)	(0.017)
controls?	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
R^2	25.9%	20.0%	15.9%	15.3%	22.4%	21.6%	7.8%	10.2%	13.8%	13.4%
N	107	110	122	125	101	104	79	82	404	421

Dependent variable: next quarter futures return ($r^i_{t,t+1}$)

Table 2 - Inventory vs. default risk

Table 2: The table shows the results from regressions of log changes in Crude Oil, Heating Oil, Gasoline, and Natural Gas inventories on three lags of the respective inventory changes, and lagged default risk of oil and gas producers, as measured by the Expected Default Frequency (EDF) and Zmijewski-score (Zm) of producers for Crude and Natural Gas and refiners for Heating Oil and Gasoline. The other controls in the regressions are lagged futures basis, aggregate default risk, forecasted GDP growth, the risk-free rate, as well as quarterly dummy variables (seasonality), Heteroskedasticity and autocorrelation adjusted (Newey-West; 3 lags) standard errors are used. In the joint regression in the right-most column, Rogers standard errors are used to control for heteroskedasticity, cross-correlation, and autocorrelation (3 lags). * denotes significance at the 10% level, ** denotes significance at the 5% level, and *** denotes significance at the 1% level.

	Dependent variable: next quarter log inventory change $\left(\Delta I^i_{t,t+1}\right)$									
	Crude Oil		**Heating Oil**		**Gasoline**		**Natural Gas**		**All**	
	EDF	Zm	EDF	Zm	EDF	Zm	EDF	Zm	EDF	Zm
$DefRisk_t$	-0.170^{**}	-0.155^{**}	-0.106^{*}	-0.034	-0.120	-0.062^{**}	-0.006	-0.015	-0.096^{**}	-0.067^{**}
	(0.081)	(0.065)	(0.058)	(0.035)	(0.096)	(0.068)	(0.023)	(0.016)	(0.047)	(0.038)
controls?	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
R^2	38.0%	39.5%	80.8%	80.7%	29.9%	32.0%	92.4%	92.7%	43.0%	44.8%
N	107	110	122	125	100	103	79	82	408	420

Table 3 - The Futures Basis

Table 3: The table shows the results from regressions of the futures basis for Crude Oil, Heating Oil, Gasoline, and Natural Gas on lagged default risk of oil and gas producers, as measured by the Expected Default Frequency (EDF) and Zmijewski-score (Zm). The futures basis is defined as $\frac{S_t - F_t}{F_t}$. The controls in the regressions are lagged futures basis, aggregate default risk, forecasted GDP growth, the risk-free rate, as well as quarterly dummy variables (seasonality). Heteroskedasticity and autocorrelation adjusted (Newey-West; 3 lags) standard errors are used. In the joint regression in the right-most column, Rogers standard errors are used to control for heteroskedasticity, cross-correlation, and autocorrelation (3 lags). * denotes significance at the 10% level, ** denotes significance at the 5% level, and *** denotes significance at the 1% level.

| | Dependent variable: The Futures Basis (IV) | | | | | | | | | |
| | **Crude Oil** | | **Heating Oil** | | **Gasoline** | | **Natural Gas** | | **All** | |
	EDF	Zm	EDF	Zm	EDF	Zm	EDF	Zm	EDF	Zm
DefRisk	0.214**	0.222*	0.190**	0.164	−0.016	0.035	0.073	0.109	0.138**	0.151
	(0.100)	(0.117)	(0.093)	(0.104)	(0.127)	(0.122)	(0.110)	(0.100)	(0.069)	(0.093)
controls?	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
R^2	25.3%	27.7%	9.3%	9.4%	9.5%	9.8%	5.2%	5.4%	9.1%	10.1%
N	107	110	122	125	100	103	79	82	408	420

Table 4 - CFTC net hedger positions

Table 4: Panel A shows the results from regressions of net short hedger positions relative to total hedger positions for Crude Oil, Heating Oil, Gasoline, and Natural Gas, as recorded by the CFTC (commercial positions), on contemporaneous default risk of oil and gas producers, as measured by the Expected Default Frequency (EDF) and Zmijewski-score (Zm). The controls in the regressions are the futures basis, aggregate default risk, forecasted GDP growth, the risk-free rate, the lagged CFTC net short hedger position, as well as quarterly dummy variables (seasonality). Panel B shows the same regression but where the net short hedger positions are regressed on controls and the lagged default risk measures. Heteroskedasticity and autocorrelation adjusted (Newey-West; 3 lags) standard errors are used. In the joint regression in the right-most column, Rogers standard errors are used to control for heteroskedasticity, cross-correlation, and autocorrelation (3 lags). * denotes significance at the 10% level, ** denotes significance at the 5% level, and *** denotes significance at the 1% level.

| | Dependent variable: CFTC short hedger positions (contemporaneous) | | | | | | | | | |
| **Panel A:** | **Crude Oil** | | **Heating Oil** | | **Gasoline** | | **Natural Gas** | | **All** | |
	EDF	Zm	EDF	Zm	EDF	Zm	EDF	Zm	EDF	Zm
$Def Risk$	0.090	0.140	0.259**	0.253***	−0.010	0.083	0.035	0.076	0.097**	0.152***
	(0.138)	(0.109)	(0.121)	(0.107)	(0.062)	(0.102)	(0.115)	(0.066)	(0.048)	(0.044)
controls?	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
R^2	10.4%	14.4%	9.0%	10.7%	39.7%	39.8%	51.7%	64.4%	16.3%	20.1%
N	91	95	103	107	96	100	64	68	354	370

| | Dependent variable: CFTC short hedger positions (IV) | | | | | | | | | |
| **Panel B:** | **Crude Oil** | | **Heating Oil** | | **Gasoline** | | **Natural Gas** | | **All** | |
	EDF	Zm	EDF	Zm	EDF	Zm	EDF	Zm	EDF	Zm
$Def Risk$	0.179	0.259**	0.307**	0.394***	0.121	0.083	0.038	0.085	0.172*	0.225***
	(0.146)	(0.128)	(0.139)	(0.122)	(0.125)	(0.085)	(0.108)	(0.053)	(0.096)	(0.067)
controls?	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
R^2	11.2%	16.6%	10.8%	18.4%	42.4%	41.6%	51.1%	71.8%	17.6%	21.8%
N	92	95	104	107	97	100	65	71	358	370