Financing Infrastructure in the Shadow of Expropriation*

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Abstract

We examine the optimal financing of infrastructure when governments have limited financial commitment and can expropriate rents from private sector firms that manage infrastructure. While private firms need incentives to implement projects well, governments need incentives to limit expropriation. This double moral hazard limits the willingness of outside investors to fund infrastructure projects. Optimal financing involves government guarantees to investors against project failure to incentivize the government to commit not to expropriate which improves private sector incentives and project quality. The model captures several other features prevalent in infrastructure financing such as government co-investment, tax subsidies, development rights, and cross-guarantees.

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1 Introduction

Investment in infrastructure is known to contribute valuably to economic productivity and growth.[1] Yet, there are significant gaps in the financing of infrastructure all over the world. For instance, the 2017 Global Infrastructure Outlook provides estimates for infrastructure gaps between 2016 and 2040 that can be as high as $1.9 and $1.2 trillion in China and Brazil, respectively; in the U.S. too, the infrastructure investment gaps between 2016-2025 are estimated to be $2.1 trillion [see Katseff, Peloquin, Rooney and Winter (2020) and Tankersley (2021)]. One view is that there is not enough private capital to fund these infrastructure needs. However, given the size of global capital markets and the impact of infrastructure on economic growth, this seems difficult to fully reconcile. Hence, we entertain the alternate view that it may be difficult for private capital to ensure its returns from infrastructure projects are adequate even if the underlying returns on infrastructure are sufficiently high.

Why might this be the case? A proximate cause, we argue, is the risk of expropriation of project returns by the government. Governments tend to opportunistically limit user fees that can be charged by private operators who build and manage infrastructure; for instance, governments can limit tariffs or give "toll holidays" to appease the voting public. Indeed, with some exceptions, user fees on infrastructure are invariably subsidized at levels well below marginal costs.[2] Additionally, governments may reduce the cash flows received by the private sector through changes in ownership (such as privatizations or early termination of concessions) or through regulatory changes (such as unexpected phasing out of technology); such changes may be motivated by politics or regulatory capture.[3] Regardless of its form and motivation, we will refer to the possibility of the government's rent extraction from the private sector broadly as *expropriation risk*. In this paper, we theoretically analyze how infrastructure should be financed in the shadow of such expropriation risk. While expropriation by governments with defaultable debt is known in the sovereign debt literature to affect sovereign debt capacity, the rate of economic growth, and the steady-state output [see Myers (1977), Aguiar, Amador and Gopinath (2009a,b), Aguiar and Amador (2011), Acharya and Rajan

[1] Aschauer (1989) finds that core infrastructure investments have significant explanatory power for productivity of the economy. Barro (1990) provides international evidence that *"the typical country comes close to the quantity of public investment that maximizes the growth rate."* Rioja (2013) shows that investment in existing public infrastructure can have a positive effect on GDP (using the Latin American countries as the context). Roller and Waverman (2001) show that an increase in the penetration rate of telecommunications generated significant aggregate economic output in twenty OECD countries over the period 1970 to 1990. Finally, Gibson and Rioja (2020) note that *"public infrastructure is one of the foundations for the economic growth of a country"* and show that, irrespective of how infrastructure investments are financed, they lead to significant welfare effects.

[2] Alm (2010) notes that the problems that lead to this outcome include inadequate billing and collection procedures, insufficient attention to operations and maintenance, and political constraints. Expropriation by the government is also documented in Butler, Fauver and Mortal (2009). Lewis and Bajari (2014) show that government moral hazard is a central issue in highway procurement projects. Liu and Mikesell (2014) provide quantitative estimates of expropriation (siphoning off resources and using of sub-standard inputs) by public officials in state-funded projects in the United States. Similarly, Gardner and Henry (2021) identifies government moral hazard – in particular, a lack of willingness on part of the government to honor the terms of underlying project agreement – as one of the important constraining factors in infrastructure investment.

[3] The 2015 World Economic Forum report on strategic infrastructure (pp. 14-18) documents such instances in the context of infrastructure for developing and developed countries.

(2013), Acharya, Rajan and Shim (2020), among others], we instead ask the question of how a sovereign's limited debt capacity can be used in an incentive-efficient financing design for infrastructure and how that impacts the debt capacity of the infrastructure project. We find that such design relates to several salient features that are prevalent in the observed practice of infrastructure financing.

Our important starting observation is that in contrast to other types of private and public investment, typical infrastructure financing is structured most commonly as public-private partnerships. Distinctively, infrastructure projects involve (i) the government, (ii) private sector operators that build and manage, and (iii) private outside financiers who supply financing. The model features these three players with investments in an infrastructure project funded by private financiers and potentially also co-investment by the government, provided that their opportunity cost of capital is met by the expected repayments. In addition to the cash flows directly generated by the infrastructure project, there can be additional cash flows in the form of externalites (for example, increased tax revenues due to increased productivity and growth after the infrastructure investment) and development rights (for example, housing development around a highway, subway or bridge). The payoffs from the project, including externalities and development rights, are distributed among the three participating parties according to contracts between the private parties and the government. The operator's compensation is determined by an *operational contract* while the financiers are compensated according to a *financial contract*, contracts whose incentive-efficient design is the focus of our analysis.

Now, whether the private investors' individual rationality constraint is met or not depends on the efforts expended by the other two players. The private sector operator develops and maintains the project and it can shape the project's quality with its inputs. Given its private benefits from shirking, a standard corporate-finance problem [Jensen and Meckling (1976)], the operator will only provide inputs at the efficient level if it has incentives to do so. The government has incentives to expropriate the project cash flows once realized (divert, retroactively tax, restrict price-setting, etc.) as it has direct or indirect benefits from doing so; such expropriation of economic rents would weaken the private operator's incentives. However, the government can commit in its operational contracts with the private firms not to engage in such expropriation (agreeing contractually to reasonable user fees, e.g.) provided it has the incentives to do so given its financial contract with infrastructure financiers. Finally, the government has some but only limited commitment or willingness to pay in its financial contracts. This commitment arises from (unmodeled) costs associated with renegotiating or defaulting on financial contracts that can have reputational consequences with the suppliers of capital and that can also inflict collateral damage on the domestic financial sector.[4]

[4] The sovereign debt literature justifies such costs theoretically [Eaton and Gersovitz (1981), Sandleris (2008), Broner, Martin and Ventura (2010), Bolton and Jeanne (2011), Acharya, Drechsler and Schnabl (2014), Farhi and Tirole (2018), among others] and provides some empirical support [see Basu (2009), Acharya et al. (2014), Gennaioli, Martin and Rossi (2014)]. Nevertheless, government commitment to repay is argued to be potentially ineffective from a theoretical standpoint [notably in Bulow and Rogoff (1989a,b)] and also found to be limited empirically [e.g., Eichengreen (1987) and Arellano (2008)].

We solve for the optimal infrastructure financing contract in the presence of this *double moral hazard* problem – the confluence of private sector and government moral hazards. The second-best contract maximizes the expected profit from the infrastructure investment subject to the individual rationality constraints of financiers and the government, the incentive compatibility constraints of the private operator and the government, and given the government's limited commitment in making financial promises. The principal insight that emerges from the solution to this constrained optimization problem is that for a given level of government commitment, if the moral hazard of the private sector is more severe, then so is the government's moral hazard in that its incentives to commit not to expropriate are weak. An implication is that the stronger the private sector moral hazard, the stronger is the incentive that the government needs to commit not to expropriate; in other words, the strength of the double moral hazard is intimately linked to the strength of the private sector moral hazard. This is because then less resources are available to repay the suppliers of funds, which limits the scale (*intensive margin*) and potentially also the feasibility (*extensive margin*) of the infrastructure project.[5] We show that to ameliorate the resulting inefficiencies in feasibility and scale, the optimal design of infrastructure financing features the following salient characteristics:

1. **Government guarantees** to financiers against project failure. Such guarantees expose the government to the risk of project failure and induce it to commit in the operational contracts to not expropriate, which in turn improves the private sector's incentives to provide effort. This way, government guarantees to financiers expand the project's cash flows available to pay the private sector operator, which in turn, expands the overall size of project's cash flows, including to repay financiers. The extent of commitment over fiscal resources that the government can set aside for the provision of such guarantees naturally affects the feasibility and the scale up to which infrastructure projects will be funded by private financiers.

2. **Co-investment** between the government and the private sector. Such co-investment arises, however, only when the return of the infrastructure project is high relative to the severity of the double moral hazard. On the one hand, investment by the government reduces the resources available to provide guarantees to financiers; this makes it harder for the government to internalize the failure of the infrastructure project and commit not to expropriate. On the other hand, government investment increases the scale and therefore the payoff from the infrastructure project available to repay private financiers. This trade off implies that when the double moral hazard is severe, government guarantees are more valuable than government co-investment in financing the project; however, when the severity of moral hazard is low, co-investment by the government can dominate the provision of guarantees.

[5]The lack of willingness on the part of financiers to fund infrastructure arises in our model even absent any risk-premium considerations; it arises purely due to the impact of private sector and government moral hazard problems on the expected cash flows from infrastructure projects.

3. **Tax subsidies** to the private sector operator and financiers funded out of infrastructure externalities. The government can meet the tax subsidies, for instance, by collecting taxes from beneficiaries of the externalities. If such tax subsidies can be credibly provided by the government (over and above its limited commitment to provide guarantees), then they dampen the adverse impact of the government's limited commitment in pledging fiscal resources and help ameliorate the private sector operator's moral hazard and improve the ability to repay financiers.

4. **Bundling of development rights** for the private sector operator and private financiers. Infrastructure typically also leads to growth opportunities beyond the immediate project. If the rights to such opportunities can be bundled with the infrastructure project, then it is easier to incentivize the private sector operator to exert effort and the financiers to participate, in that they require a lower fraction of the project's direct cash flows (assuming that such development rights cannot be fully expropriated by the government). Such bundling therefore reduces the inefficiencies generated by the government moral hazard; the limited commitment to provide guarantees via fiscal resources is now partly counteracted by the provision of additional cash flows to the private sector operator and the financiers.

5. **Cross-guarantees** between projects. Cross-guarantees improve the incentive of financiers to participate in individual projects by expanding the cash flows available to them. However, whether cross-guarantees strengthen or weaken the government's moral hazard in individual projects depends on the probability of success of the projects: on the one hand, by increasing the government's transfers to the financiers when a project fails, cross-guarantees decrease the government's incentives to expropriate; on the other hand, cross-guarantees decrease the government's payoff from an individual project when it succeeds in the event of other projects' failure, increasing the incentives to expropriate. The first effect dominates and cross-guarantees are valuable only when projects are high quality. The cross-guarantees can be interpreted as "general obligation" (GO) financing in municipal bond issuance in the United States supported by the overall pool of taxes; the absence of cross-guarantees can be interpreted as "revenue only" (RO) financing as is common for utilities and competitive enterprises where municipal bonds are paid off only from project-specific revenue collections.

We document in Section 7 that these features of optimal financing of infrastructure in the shadow of expropriation risk by governments are indeed prevalent in infrastructure projects and their financing in practice. The mapping from the second-best contract to practice suggests to us that expropriation risk by governments likely affects the ability of infrastructure to attract private capital, in scale and in feasibility, and may thus be an important candidate for explaining infrastructure gaps observed the world over. While some of the contracting features above can be rationalized in other frameworks for infrastructure financing (such as to reduce the risk-premium charged by private investors), our double moral hazard setting provides a simple unified explanation for the presence of these seemingly distinct features. The framework also suggests unique testable implications for future research. For instance, government guarantees are the incentive-efficient use of limited fiscal resources in case of financing of low-quality (high moral hazard) infrastructure projects, whereas government co-investment plays this role for high-quality projects; and, while cross-guarantees in the form of GO financing are incentive-efficient for high-quality infrastructure projects, low-quality projects are more efficiently financed with RO bonds or without cross-guarantees.[6]

The rest of the paper is organized as follows. In Section 2 we discuss the related literature. In Section 3 we introduce our model with double moral hazard and government limited commitment, considering direct government investment and externalities. In Section 4, we present our main results. In Sections 5 and 6 we extend our baseline model to consider development rights and tax subsidies, and general obligation and revenue only financing, respectively. Section 7 discusses features of infrastructure financing in practice and Section 8 concludes. All omitted proofs are in the Appendix.

2 Related Literature

Our paper relates to three important strands of literature: first, on the implications of expropriation by governments on growth; second, on infrastructure financing in the context of public-private partnerships; and, third on the specific framework we employ to study infrastructure financing, viz., double moral hazard.

A small but growing literature on sovereign debt explores how the risk of expropriation and lack of commitment by governments affects economic outcomes. Aguiar, Amador and Gopinath (2009b) and Aguiar and Amador (2011) extend the Myers (1977) idea of debt overhang in a neoclassical model of investment and growth, showing that in the presence of sovereign debt, there is a natural ex-post risk of expropriation by governments; this reduces ex-ante sovereign debt capacity and investment, and myopic governments can adversely affect the level of, or the rate of convergence to, the steady state endowment of the economy. Gourinchas and Jeanne (2013) attribute weak growth rate associated with higher (foreign) sovereign debt to correlated underlying conditions of these economiesthat can also result in a poor investment environment. In Acharya and Rajan (2013) and Acharya et al. (2020), technology is owned by the private sector and governments are not only myopic but endowed with a preference for wasteful diversion and expenditures; sovereign debt can lengthen government horizons and even produce growth boosts in some cases, but otherwise lead to economic and/or financial repression of private investments. In a recent paper, DeMarzo, He and Tourre (2021) examine myopic governments with limited commitment and ability to trade dynami-

[6]We show in an extension of the model in Section C in the Online Appendix that when infrastructure projects also involve an early-stage requirement of government clearances (land acquisition, for example, is often the bottleneck in emerging markets), then the term-structure of guarantees in the optimal financing of projects reflects the relative severity of the early-stage government moral hazard relative to the late-stage one relating to cash flow expropriation. Specifically, if the early-stage government moral hazard is more severe, then the optimal financing of the infrastructure project features a higher government guarantee against the risk of project failure initially, with the guarantee tapering off to a lower level once the project is off the ground.

cally in debt markets, wherein patient citizenry is left worse off as a result of sovereign debt ratcheting; the underlying cash flow (income) structure is, however, exogenous to debt dynamics.

Our paper is related to this important strand but differs in significant ways. Unlike this literature, we focus on infrastructure and hence consider public-private partnerships rather than investments that are entirely public or private. In our setting with public-private partnership, expropriation by governments aggravates private sector moral hazard, amplifying the impact on cash flows and in turn the willingness of financiers to fund projects. Furthermore, we take limited sovereign debt capacity as given but ask the question of how it can be used in an incentive-efficient manner to finance public-private partnerships in infrastructure. Indeed, we find an optimal role for the state-contingent use of government debt capacity in the form of provision of guarantees to infrastructure financiers; we also derive other features of optimal financing (government co-investment, tax subsidies, bundling of development rights, and cross-guarantees) that are observed in practice. This way, our framework helps understand why the risk of expropriation can limit scale and feasibility of infrastructure financing, and simultaneously shows that the second-best contract maps into the institutional details prevalent in infrastructure financing.

The existing theoretical literature on infrastructure financing mostly focuses on partnerships between the private and public sectors either as co-owners/co-managers of the projects or as co-investors. For example, Perotti (1995) and Martimort and Sand-Zantman (2006) focus on whether projects should be organized/owned by the government or by the private sector. In Perotti (1995) partial privatization allows the government to credibly signal that it will not behave opportunistically upon privatization (such as decreasing or even eliminating tolls, once the toll-highways are privatized). In his model, the government's type determines whether it will behave opportunistically and thus it cannot be incentivized not to misbehave– there is no moral hazard. In this context, privatization serves as a way of revealing the government's private information, namely its type. Relatedly, Martimort and Sand-Zantman (2006) consider the classic infrastructure problem in which the government can deliver a public good or service under public ownership or outsource the activity to the private sector. They examine the optimal delegated management contract when the government has private information about the project's quality and the private sector's effort is not verifiable.[7]

In both of these papers, the ownership structure of the project partially resolves a problem of asymmetric information between the government and the private sector. In contrast, we abstract from asymmetric information and instead consider how infrastructure financing should be optimally designed given that the government has incentives to expropriate upon project completion (which weakens incentives of operators who build and maintain infrastructure), but the government can use its limited fiscal resources in a state-contingent manner to commit not to do so. To emphasize the implications of the double moral hazard on the

[7]More recently, Fay, Martimort and Straub (2018) provide conditions under which both public and private finance of infrastructure projects coexist in a costly state verification contracting model.

financing of infrastructure projects, we take the need for delegated management as given.[8]

Finally, our double moral hazard model of infrastructure financing is an application of the literature of contracting under agency problems [see Tirole (2006) and Bolton and Dewatripont (2004) for textbook analyses]. More specifically, contracts under double moral hazard are considered, for example, in Repullo and Suarez (1998), Hellmann and Puri (2000), Casamatta (2003), and Inderst and Mueller (2009) in the context of venture capital; in Cooper and Ross (1985), Eswaran and Kotwal (1985), and Demski and Sappington (1991) applied to product warranties, agricultural risk-sharing arrangements and labor buyouts, respectively; and in Tirole (2003) looking at foreign lending, among others. These papers focus mostly on the risk-sharing features of the contracts. Surprisingly, in spite of the importance of infrastructure to the long-run growth of economies there is only a sparse literature focused on providing the agency-theoretic foundations of infrastructure financing when there is a risk of expropriation by governments which endangers private sector participation and can explain the observed infrastructure gaps globally.

3 Model

In this section we develop our baseline model of public-private partnership in the shadow of to understand how the interaction between the government's moral hazard and the private sector's moral hazard affects private incentives to finance infrastructure projects. The model takes into account possible spillovers from infrastructure projects such as externalities.

3.1 Setup

We consider an infrastructure project to be run by a private sector project operator, to whom we will refer simply as the "private sector operator" or the "operator". The project can be financed by private investors (or financiers) and the government. The project is constant returns to scale. We denote the scale of the project by $I = I_f + I_g$, where I_f and I_g denote the amounts invested by the financiers and the government, respectively. The project is risky and has a per unit payoff $R > 1$ if it is successful and zero otherwise.

The project is implemented in several stages as shown in Figure 1. First, in the investment stage, the *financial contract* between the financiers and the government is determined; and the financiers and the government then invest in the project. The second stage is a "gestation" period that includes the appointment of a private sector operator. The *operational contract* between the government and operator is determined in

[8] Another part of the literature has focused on the risk-sharing implications of jointly owned investment options in infrastructure. Banerjee, Gucbilmez and Paulina (2014) provide a real-options framework to investigate the optimal investment timing in the presence of such joint ownership, bargaining and side payments. On the private equity and venture capital front, recent empirical research by Andonov, Kraussl and Rauh (2021) suggests that infrastructure assets display cash flow properties akin to private equity investments. They report that public institutional investors implicitly subsidize infrastructure investments in a significant way.

Investment Stage	Gestation Stage	Operating Stage (Moral Hazard)	Payoff Stage
Government and financiers enter into a financial contract Government and private financiers invest	Private sector operator is appointed Government and operator enter into an operational contract (determines expropriation)	Private sector operator undertakes project development (Effort choice)	Project's payoffs are realized & distributed Govt. guarantees are honored from fiscal capacity (Figure 2)

Figure 1: Timeline of the model

this stage. Next, in the "operating stage", the government agrees to a fraction of the project return to share with the operator – its expropriation policy as the residual project return is diverted by the government – and the private sector operator chooses the effort level while developing the project. Finally, in the "cash flow" stage the payoffs are realized and distributed among the involved parties according to the government's sharing or expropriation policy and the financing contract. The relative timing of the first two stages is not crucial in our model. Since there is no information revealed and no action is taken between these stages, we can reverse the order of the investment and operating stages or make them simultaneous without altering our analysis.

The operator's effort choice and the government's expropriation policy in the operating stage lead to a double moral hazard problem, which is the center of our analysis. More specifically, the private sector operator can affect the probability of the project's success. If the operator exerts high effort and provides a high input, the project's probability of success is $p_h \in (0,1)$, else it is p_l, where $0 < p_l < p_h$. We denote by Δp the difference in these probabilities: $\Delta p \equiv (p_h - p_l)$ and we assume that $p_l R < 1 < p_h R$ so that the project is worth pursuing only if the operator exerts high effort and provides a high input. Moreover, if the operator does not exert high effort, it derives a non-pecuniary private benefit of BI, where $B > 0$. We assume that the effort exerted by the operator is not observable and therefore the operator is subject to a moral hazard problem in the operating stage.

Additionally, in the operating stage, the government can expropriate the cash flows coming from the project from the operator and keep the project's return to itself. For example, the government can limit tariffs or give "toll holidays" to appease the voting public. However, in the operational contract with the private operator the government can choose to commit not to expropriate a given amount of the project's return from the private operator by setting "user fees" $R_b I$ as part of the operational contract. An important limitation is that the government cannot be forced into any specific contract and therefore will only agree to an expropriation policy if it has the incentives to do so. Therefore, the possibility of the government's expropriation is akin to the government facing a moral hazard problem in the operating stage, which we will show interacts with the private sector moral hazard and government's commitment in financial contracts, and in general, limits the scale and the feasibility of the project.

Prior to the operating stage of the project is its investment stage in which the government and the financiers enter a state-contingent financial contract. In each state of the world, financiers are paid out from the project's return and possibly also by the government. The payment to financiers in the event of the project's success $R_f I$ can be considered a "coupon". The payment to financiers in the event of the project's failure $K_g I$ can be thought of as government guarantees. These guarantees compensate the financiers for their anticipated loss of return in case of an eventual project failure, which is more likely if the government engages in expropriation and induces low effort by the private sector operator. Therefore, government guarantees can ameliorate the government's moral hazard problem in the operating stage. As it will be clear in the analysis below, by providing these guarantees, the government internalizes the downside of the project and has incentives to limit its expropriation so as to induce the private sector operator to supply inputs efficiently.

The government has fiscal resources $\overline{K}_0$ available in the investment stage, of which it can use I_g to invest directly in the infrastructure project and save the rest to make payments to financiers. Additionally, the government has fiscal resources $\overline{K}_1$ available when the payoffs of the project are realized and these can only be used pay financiers. As in the literature on sovereign debt, the government faces limited commitment on its promises to financiers in the financial contract. More specifically, the government's willingness to pay is determined by the costs that it incurs from defaulting on financial contracts, which we denote by Φ. This default penalty can arise, for example, from exclusion from debt markets [Eaton and Gersovitz (1981)], trade sanctions [Bulow and Rogoff (1989a,b)], and collateral damage costs to the domestic financial sector [Broner et al. (2010), Acharya et al. (2014), Gennaioli et al. (2014), and Farhi and Tirole (2018)] interbank markets [Bolton and Jeanne (2011)], and the domestic non-financial sector [Almeida et al (2017) and and Du and Schreger (2016)]. Given the government's lack of commitment, the amount the government can credibly provide is limited by its willingness to pay Φ since the government will choose to default on any payment above Φ.

To allow for spillovers generated by the infrastructure project, we consider the possibility that if the project succeeds it generates additional payoffs in the form of externalities X per unit scale that only accrue to the government. The payoffs from the infrastructure contracts distributed among the financiers, the government, and the private operator are denoted as follows: If the project succeeds, the private financiers receive a payoff $R_f I$, the operator receives a payoff $R_b I$, and the government receives the residual payoff from the project and the externalities, $(R_g + X)I$, where $R_g \equiv (R - R_f - R_b)$. The fiscal implications for the government are accounted for separately below. If the project fails, the private financiers receive $K_g I$, the government receives $-K_g I$, and the operator receives no payoff. This payoff structure assumes that there is

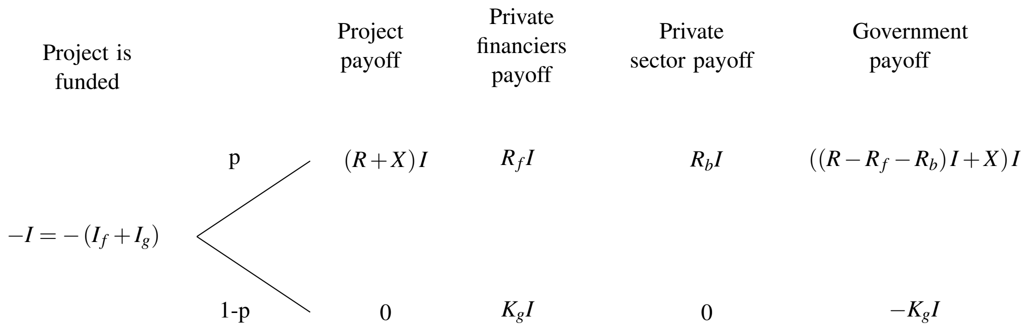


Figure 2: State space of possible outcomes and corresponding payoffs for various economic agents under the baseline model

no government default, which will be the case in equilibrium. The resulting state space of outcomes for the projects, project payoffs and spillovers, and the payoffs to the various parties (the private financiers, the private sector project operator, and the government) are summarized in Figure 2. Finally, we assume that both the private financiers and the government require a net rate of return $r > 1$ on their respective investments in the project.

3.2 Discussion of Assumptions

There are three assumptions of our model that are worth discussing further: the government's moral hazard, the government's limited commitment, and the absence of government guarantees for the private sector operator in the event of the project's failure.

Government's moral hazard. The government's choice of "user fees" in the operational contract is what leads to the government's moral hazard problem. If the government could be forced to commit to any user fees ex ante, the second-best contract would leave just enough of the return of the infrastructure project for the private operator to exert high effort and the remaining return to the financiers to incentivize them to finance the project. In this case, the scale of the project would not be restricted whenever feasible. Only the private moral hazard would affect the region of parameters in which the project is undertaken.

Government's limited commitment. The government is assumed to have limited commitment in its financial contracts but can commit to the terms of its operational contract with the operator. These assumptions reflect differences in the costs the government faces for defaulting on financial and operational contracts. As mentioned above and in the Introduction [footnote 4], defaulting or renegotiating on a financial contract can have reputational costs for the government and also trigger other forms of collateral damage; however, the literature has found these default costs to be limited as seen in the ability of defaulting governments, even serially defaulting ones such as Argentina, to re-enter the credit markets at reasonable borrowing costs [Arellano (2008)]. In contrast, defaulting on operational contracts can lead to a perception of low ease of doing business in the country, affecting its rankings for investment attractiveness by multilateral institutions such as the World Bank; in turn, this can have long-lasting repercussions for business and financial investments in the economy. While our assumption that the government's operational contracts are fully binding is stark, this can be relaxed. All that we require is that the government's limited commitment in financial contracts affects its incentives to limit expropriation in the operational contract.

Guarantees to the private sector operator. In our model setup, we set the payoff to the private sector operator in the event of project failure to zero. Suppose instead we allow the government to give a guarantee to the private sector operator in the event of project failure. Since all guarantees are funded by the government's fiscal resources $\overline{K}$, offering a guarantee to the operator implies that there are fewer resources left to offer a guarantee to the financiers, leading in general to a smaller project scale. Moreover, a guarantee to the private sector operator exacerbates its moral hazard problem and requires a higher payoff to the operator when the project succeeds to induce high effort. Therefore, the optimal contract sets guarantees to the operator in case of project failure to zero and our assumption is without loss of generality.[9]

4 Analysis

We are interested in the socially optimal contract that is feasible given the double moral hazard and limited commitment problems introduced in the previous section. In the analysis that follows, we consider in turn the two incentive compatibility constraints (one each for the private sector operator and the government), the two individual rationality constraints (one each for the private financiers and the government), and the government's no-default conditions on the financing contract given its fiscal capacity and default costs.

Incentive compatibility for the operator. The private sector operator will exert effort as long as the expected payoff from exerting the high effort is not dominated by the expected payoff (inclusive of the private benefits) from exerting the low effort. If the project is successful, the operator receives a payoff R_bI. If the operator exerts effort, the probability of success of the project is p_h. Otherwise, the probability of success is p_l and the operator experiences additional private benefits BI. Therefore, the incentive compatibility constraint of the private sector operator is given by

$$p_h R_b I \geq p_l R_b I + BI \ . \tag{ICP}$$

The operator's incentive compatibility constraint imposes a limit on the amount the government will be

[9] Medda (2007), argues that in the case of large-scale public-private partnerships, if the guarantees provided exceed the potential financial losses of the private sector, it can lead to strategic behavior and lead to problems of moral hazard emanating from the guarantees. In our model, government provides guarantees to financiers and *not* to operators.

willing to commit to not expropriating from the operator. To see this, note that the incentive compatibility constraint ICP can be written as the following familiar Holmstrom and Tirole (1998) condition:

$$R_b \geq \frac{B}{\Delta p}\ .$$

Incentive compatibility for the government. Given the moral hazard problem of the operator, the minimum R_b that the government has to give the operator to ensure that the operator implements the high success probability p_h is $\frac{B}{\Delta p}$. This amount gives just enough cash flow to the operator for her to be incentivized to exert effort. Therefore, the government will never commit to giving the operator more that $\frac{B}{\Delta p}$ and the government's payoff from the project's cash flows for the operator to be induced to exert effort is $R_g = \left(R - \frac{B}{\Delta p} - R_f\right)$. Moreover, if the government sets R_b less than $\frac{B}{\Delta p}$, the operator will not exert effort and the probability of success of the project will be p_l. Therefore, the government might as well give nothing to the operator if it expects her to exert no effort.

Then, to implement p_h by the private sector operator, the contract needs to satisfy the following incentive compatibility constraint for the government to commit to not expropriating too much from the operator:

$$p_h\left(R - \frac{B}{\Delta p} - R_f + X\right) - (1-p_h)K_g \geq p_l\left(R - R_f + X\right) - (1-p_l)K_g \tag{ICG}$$

or

$$(K_g - R_f) \geq \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) - X.$$

Note that when deciding whether to expropriate from the operator, the financial contract, viz., K_g and R_f, is already determined in the investment stage. A higher government guarantee K_g makes it easier to satisfy the incentive compatibility constraint of the government. Intuitively, the higher the guarantee, the more the government cares about the project not failing, and hence, the higher its incentive to not expropriate the entire payoff and thereby ensure the private sector operator exerts high effort. Analogously, the higher the amount R_f of the project's payoff the government commits to sharing with the private financiers, the less the government cares about the project succeeding and the lower its incentives to induce the private sector operator to exert high effort.

Participation constraint for financiers. The private financiers must also be left with an adequate share of the project payoff plus expected guarantees to be willing to finance the infrastructure project. The financiers' expected share of the project's payoff, $p_h R_f I$, and the expected value of the government guarantee, $(1-p_h)K_g$, together need to compensate the financiers for an adequate rate of return on their investment in the project. This yields the private financiers' individual rationality constraint:

$$rI_f \leq p_h R_f I + (1-p_h)K_g I \tag{IRP}$$

or

$$\frac{r}{(1-p_h)}\frac{I_f}{(I_f+I_g)} \leq K_g + \frac{p_h}{(1-p_h)}R_f \,.$$

Participation constraint for the government. At the time of investment, the government has to have incentives to participate in the financial contract with the financiers and undertake the infrastructure project. The individual rationality constraint for the government states that the expected payoff from the project, inclusive of externalities, has to be greater than the government's outside option, which without loss of generality we normalize to 0. Formally,

$$[p_h(R_g+X)-(1-p_h)K_g]I \geq rI_g \,, \tag{IRG}$$

or using that $R_g = \left[R - \frac{B}{\Delta p} - R_f\right]$,

$$\left(K_g + \frac{p_h}{1-p_h}R_f\right) \leq \frac{p_h}{(1-p_h)}\left[\left(R-\frac{B}{\Delta p}\right)+X\right] - \frac{r}{(1-p_h)}\frac{I_g}{(I_f+I_g)}.$$

No default and feasibility constraints for the government. The government's limited commitment and fiscal resources impose limits to the terms of the financial contract to which the government can agree. More specifically, the promised payments to the financiers, either in the form of guarantees or shared returns, must meet both the government's ability-to-pay and willingness-to-pay constraints. Formally, the payment $R_f I$ to the financiers and the government guarantee $K_g I$ and need to satisfy

$$R_f I \leq \min\left\{\Phi, \left[\left(R-\frac{B}{\Delta p}\right)\right]I + \overline{K}_0 + \overline{K}_1 - I_g\right\} \quad \text{and} \tag{NDR}$$

$$K_g I \leq \min\left\{\Phi, \overline{K}_1 + \overline{K}_0 - I_g\right\} \,. \tag{NDK}$$

Recall that Φ is the cost the government incurs from defaulting on financial contracts. NDR and NDK stand for no default on R and K payments to financiers, respectively.[10]

Finally, the government's investment I_g cannot exceed its available fiscal resources, i.e.,

$$I_g \leq \overline{K}_0 \,.$$

The fiscal constraints will limit the scale of the investment directly by limiting the amount the government can invest directly and also indirectly by restricting the size of the (per-unit) guarantee provided by the government to the private financiers.

[10] Note that we allow the government to use its idle fiscal capacity to repay the financiers regardless of whether the project is successful. Therefore, in principle one could have $R_f > \left(R - \frac{B}{\Delta p}\right)$.

4.1 Benchmarks

Before characterizing the second-best contract in general, we briefly discuss how the moral hazard of the private sector operator and the moral hazard of the government, each in isolation, affect the financing contract to highlight the importance of their interaction–the double moral hazard–in leading to the results that follow.

Private operator's moral hazard. Suppose that the government has adequate commitment in its financial contracts and therefore can be forced to commit to the incentive-efficient sharing (expropriation) policy with the operator. Then, the only friction in the model is given by the moral hazard of the operator who will be willing to exert high effort as long as she receives $\frac{B}{\Delta p}$. In this case, the scale of the project is optimal as long as the project is financed. However, the moral hazard of the operator implies that some low return infrastructure projects will not be undertaken even if their NPV is positive, i.e., $p_h(R+X) > r$ but $R < \frac{B}{\Delta p}$. For these projects, the return would be enough to compensate the financiers for their outside option but there wouldn't be enough left to incentivize the operators to exert high effort.

Government's moral hazard. Suppose that the government's moral hazard is the only friction in the economy. This is equivalent to setting $B = 0$. In this case, the government's expropriation does not affect the probability of success of the project; the feasibility and scale of the project are now affected only by the extent of government's commitment in financial contracts, and in particular, the first best is attained if $\Phi \to \infty$.

We will see next, however, that when *both* the private operator's moral hazard and the government's moral hazard are present, then they interact and the resulting moral hazard problem affects the optimal financing contract in intricate ways, helping us relate to several features that are observed in infrastructure financing practices.

4.2 Optimal financing contract

While our model is streamlined and conceptually easy to understand, there are multiple linear constraints whereby there are many cases that need to be considered when characterizing the optimal financing contract. For the sake of clarity, we focus on the economic properties of the solution and relegate all technical details to the Appendix.

The objective of the planner is to choose a financing contract, inclusive of government investment, to maximize the net present value of the infrastructure project, $[p_h(R+X)-r]I+(r-1)I_g$, that is, its expected payoff inclusive of externalities and net of the cost of investment (as all other payoffs are simply transfers between the government and the private sector), subject to the constraints above:

$$\max_{R_f \geq 0, I_g \in \left[0, \overline{K}_0\right], K_g \geq 0} \quad \left[p_h \left(R + X\right) - r\right] I$$

subject to

$$K_g + \frac{p_h}{(1-p_h)} R_f \geq \frac{r}{(1-p_h)} \frac{I_f}{(I_f + I_g)} \ , \tag{IRP}$$

$$K_g + \frac{p_h}{(1-p_h)} R_f \leq \frac{p_h}{(1-p_h)} \left[\left(R - \frac{B}{\Delta p}\right) + X\right] - \frac{r}{(1-p_h)} \frac{I_g}{(I_f + I_g)}, \tag{IRG}$$

$$(K_g - R_f) \geq \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) - X, \tag{ICG}$$

$$R_f I \leq \min\left\{\Phi, \left[\left(R - \frac{B}{\Delta p}\right))\right] I + \overline{K}_0 + \overline{K}_1 - I_g\right\}, \quad \text{and} \tag{NDR}$$

$$K_g I \leq \min\left\{\Phi, \overline{K}_1 + \overline{K}_0 - I_g\right\} \ , \tag{NDK}$$

where we used the observation that the payoff of the private sector operator is enough to incentivize it to exert effort, i.e., $R_b = \frac{B}{\Delta_p}$.

The total scale of the project is limited by the willingness-to-pay and the ability-to-pay constraints of the government in the financial contract, stated in the no-default conditions NDR and NDK. This implies that at least one of the government's no-default conditions on the financial contract will be binding, i.e., I is characterized implicitly by the solution to

$$I = \min\left\{\frac{\min\left\{\Phi, \left(R - \frac{B}{\Delta p}\right) I + \overline{K}_1 + \overline{K}_0 - I_g\right\}}{R_f}, \frac{\min\left\{\Phi, \overline{K}_1 + \overline{K}_0 - I_g\right\}}{K_g}\right\}.$$

The expression above shows that the total scale of the project is determined by the promised payments to the financiers, either in the form of shared return or government guarantees, which in turn need to satisfy the individual rationality constraint of the financiers, IRP, and the incentive compatibility constraint of the government, ICG.

A key result is that the interaction between the moral hazard of the private sector operator and the moral hazard of the government shapes the constrained-efficient financing contract and imposes limits on the scale of the infrastructure project. The proposition below characterizes the resulting inefficiencies:

Proposition 1. ***(Inefficiency of double moral hazard)*** *The double moral hazard affects both the feasibility and the scale of the infrastructure project:*

a. ***(Feasibility)*** *There exist thresholds $\underline{\Gamma}$, $\overline{\Gamma}$, and $\Gamma^{\star}$, where $\overline{\Gamma}$ and $\Gamma^{\star}$ are increasing in the severity of the*

$\underline{\Gamma}$	$\frac{r}{p_h} - X$
$\overline{\Gamma}$	$(1 - p_h)\,\frac{p_l B}{(\Delta p)^2} - X$
Γ_I	$\frac{p_l B}{(\Delta p)^2} - X - \frac{r}{1-p_h}$
Γ_R	$\frac{p_l B}{(\Delta p)^2} - X - \frac{r}{\left(1-p_h+r\frac{\left(\overline{K}_1+\overline{K}_0\right)-\min\left\{\overline{K}_1,\Phi\right\}}{\min\left\{\overline{K}_1,\Phi\right\}}\right)}$
$\Gamma^{\star}$	$\frac{p_l B}{(\Delta p)^2} - X + \frac{\left(\Phi-\min\left\{\overline{K}_1,\Phi\right\}\right)r}{\left(\left(\Phi-\overline{K}_1\right)p_h+r\overline{K}_0+\overline{K}_1\right)}$

Table 1: Equilibrium thresholds

moral hazard measured by $\frac{B}{\Delta p}$*, such that:*

i. If $\left(R - \frac{B}{\Delta p}\right) < \underline{\Gamma}$*, the project is not funded even in the absence of any government moral hazard.*

ii. If $\underline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right) < \overline{\Gamma}$*, the project is not funded in the presence of government moral hazard.*

iii. Otherwise, the project is funded.

b. ***(Scale)*** *If the project is funded, the optimal scale is weakly decreasing in the severity of the moral hazard measured by* $\frac{B}{\Delta p}$ *and increasing in the return of the project R. If* $\left(R - \frac{B}{\Delta p}\right) > \Gamma^{\star}$*, the scale of the project is maximal and determined by the willingness-to-pay constraint of the government.*

The proposition shows that the double moral hazard problem imposes inefficiencies in the financing of infrastructure projects, either by rendering the projects infeasible (extensive margin) or by limiting their scale (intensive margin). The type and magnitude of these inefficiencies depends on the size of the project's return R relative to the severity of the double moral hazard, which is measured by $\frac{B}{\Delta p}$. The ratio $\frac{B}{\Delta p}$ measures the opportunity cost for the private sector operator of exerting effort relative to the increase in the project's probability of success if effort is high. The thresholds that determine the regions described in Proposition 1 (and in Proposition 2 below) are explicitly characterized in Table 1.

When $\frac{B}{\Delta p}$ is high, it is tempting for the operator to reap the private benefits of providing low effort. In this case, the operator requires a high payoff to be incentivized to exert high effort. In turn, this implies that the payoff to the government from not expropriating is low, and hence, a high level of guarantees is needed to incentivize the government not to expropriate, making the scale of the project small. Therefore, a high ratio $\frac{B}{\Delta p}$ implies that *both* moral hazard problems in the infrastructure financing are severe – the private sector operator moral hazard (effort aversion) as well as the government moral hazard (expropriation).

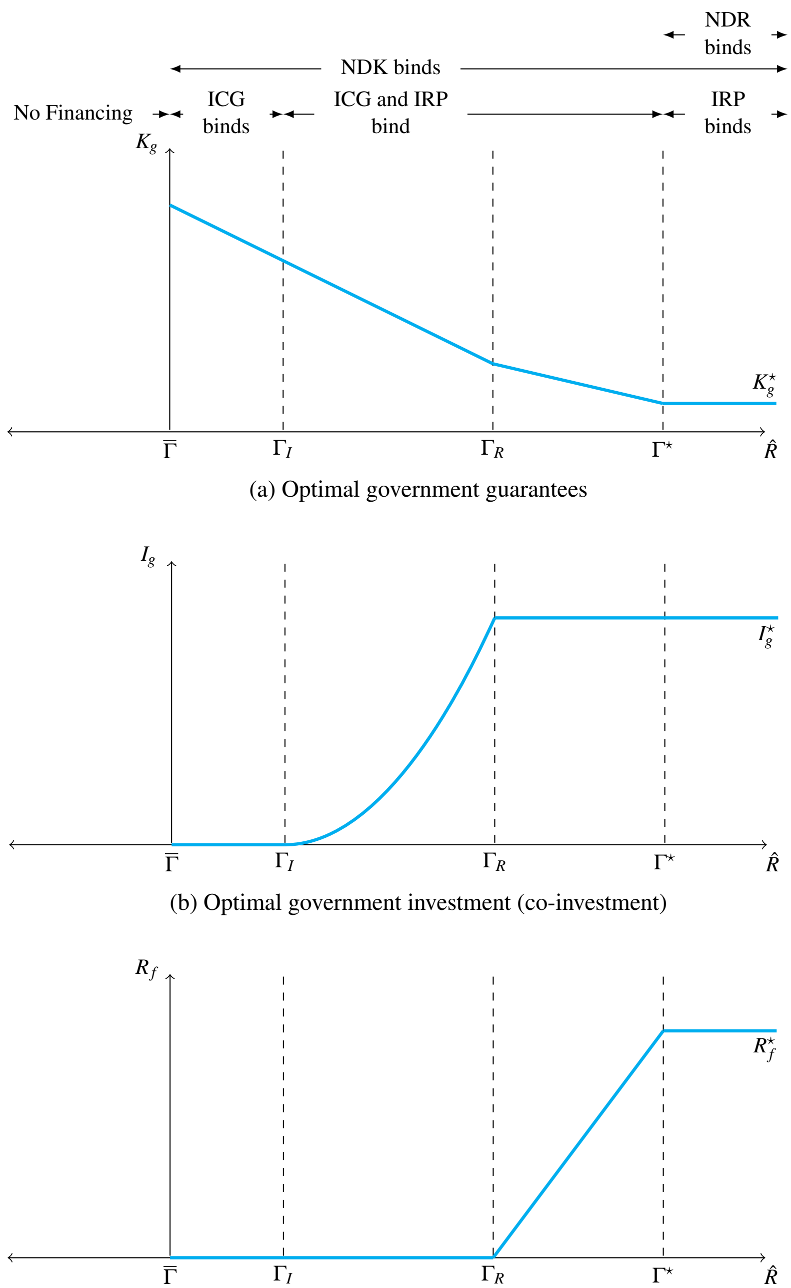
Next, note that when the return of the project is low relative to the severity of the moral hazard, i.e., when $\underline{\Gamma} \equiv \frac{r}{p_h} > \left(R - \frac{B}{\Delta p}\right)$, it is not possible to incentivize the financiers to fund the project even if the government could commit in the operational contract to giving its entire payoff from the project. In this case, the project is not funded, even in the absence of government moral hazard.

Conversely, when the return of the project is high enough to be funded in the absence of government moral hazard, i.e., $\underline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right)$, the government's moral hazard imposes further limits on the project's feasibility. In particular, for returns of the project such that $\underline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right) < \overline{\Gamma}$, the project is not funded when the government can expropriate the project's return from the private operator but it would be funded otherwise.

Finally, at even higher levels of the project's return, i.e., $\overline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right)$, the project is undertaken even in the presence of government moral hazard. However, government guarantees are needed for the project to be funded, i.e., $K_g > 0$, and the scale of the project is limited. In this case, as the severity of the moral hazard increases, higher guarantees are needed to incentivize the government not to expropriate the entire return of the project from the operator. Moreover, as the project's payoff increases, the government's payoff from not expropriating increases and it can provide lower guarantees while still satisfying its incentive compatibility constraint. Eventually, when $\left(R - \frac{B}{\Delta p}\right) \geq \Gamma^{\star}$, the government's willingness-to-pay constraints bind and the scale of the project is maximal.

Recall that there are two ways in which the financiers can be compensated for their investment in the infrastructure project: government guarantees and a share of the return of the project if it is successful. These two instruments have opposite effects on the government's moral hazard. On the one hand, higher government guarantees ameliorate the moral hazard of the government by increasing the costs to the government if the project fails and incentivizing it to commit to higher user fees (R_b) in the operational contract. On the other hand, a larger share of the project's return assigned to the financiers increases the government's incentives to expropriate from the operator. Therefore, financiers receive part of the project's return only if the financiers' individual rationality constraint is binding.

Note also that the direct government investment (or co-investment) also affects the participation constraint of the financiers. As can be seen from constraint IRP above, the government's investment in the project relaxes the individual rationality constraint of the private financiers: the larger the government investment, the greater is the project scale and therefore more resources there are to pay private financiers. However, government investment also increases the cost to the government from participating, as seen from constraint IRG. Moreover, for each unit the government invests directly there is one less unit of resources available to provide guarantees to the private financiers in the event of project failure, which makes it harder for the government to internalize the project's downside and decreases its incentives to induce high effort from the operator. In this case, government investment makes its moral hazard more severe. The optimal government investment reflects this trade-off. The following proposition characterizes the resulting pecking order of the tools used in the optimal financing contract:



(a) Optimal government guarantees

(b) Optimal government investment (co-investment)

(c) Optimal return to financiers

Figure 3: Optimal guarantee, co-investment, and return to financiers as a function of the adjusted return of the project $\hat{R} \equiv \left(R - \frac{B}{\Delta p}\right)$

Proposition 2. ***(Pecking order)***

1. The optimal financing contract always requires government guarantees $(K_g > 0)$. *The optimal government guarantee* K_g *is decreasing in* $\left(R - \frac{B}{\Delta p}\right)$.

2. There exist thresholds Γ_I *and* Γ_R, $\Gamma_I < \Gamma_R$, *such that:*

i. Government investment I_g *is positive if and only if* $\left(R - \frac{B}{\Delta p}\right) > \Gamma_I$ *and* $I_g = \overline{K}_0$ *if and only if* $\left(R - \frac{B}{\Delta p}\right) > \Gamma_R$. *The optimal government investment is increasing in* $\left(R - \frac{B}{\Delta p}\right)$.

ii. Sharing of project return with financiers (R_f) *is positive if and only if* $\left(R - \frac{B}{\Delta p}\right) > \Gamma_R$.

Figure 3 shows the optimal government guarantees, optimal government investment, and optimal return to financiers as a function of the adjusted return of the project, $\hat{R} \equiv \left(R - \frac{B}{\Delta p}\right)$

Government guarantees are always needed ($K_g > 0$) in the optimal financing contract when the government has limited commitment, and they are decreasing in the project's adjusted return $\hat{R}$. If government guarantees were zero, the scale of the project would be determined by the willingness-to-pay constraint of the government on R_f (NDR). Therefore, any decrease in the return promised to financiers R_f would increase the scale of the project. In particular, it is always possible to keep the compensation of financiers constant by decreasing R_f and increasing the government guarantees K_g, which would increase the scale of the infrastructure project while relaxing the incentive compatibility constraint of the government.

Whether the government invests in the project directly depends on the value of the optimal guarantee and the relative return of the project relative to the severity of the moral hazard. When the return of the project is low, the incentive compatibility constraint of the government binds and the government's direct investment tightens the constraint by restricting the size of the government guarantees. In this case, it is optimal to set the government's investment to zero ($I_g = 0$).

As the project's return increases, it is easier to satisfy the government's incentive compatibility constraint with a lower guarantee, which leads to an increase in the scale of the infrastructure project. However, as the government guarantee becomes lower, the payoff shared with the private financiers when the project succeeds needs to increase to satisfy the individual rationality constraint of the financiers. Therefore, it becomes optimal for the government to invest in the infrastructure project ($I_g > 0$). Once the government exhausts all its available resources to invest in the project directly, the financiers receive a share of the project's return when it succeeds ($R_f > 0$).

4.3 Limited commitment

We now turn to understanding the role of limited commitment that the government faces in meeting its financial promises in affecting infrastructure financing. We show that as limits on the commitment are relaxed, government guarantees would eventually be zero and the project would become self-financing.

Limited commitment does not allow that to happen:

Proposition 3. ***(Limited commitment)*** *The government's limited commitment in financial contracts limits the scale of the project. i. Greater commitment, measured by the size of the default penalty,* Φ*, increases the return the government can commit to share with the financiers, i.e.,* $\frac{\partial R_f}{\partial \Phi} > 0$*, and*
ii. If default penalties are large enough, the infrastructure project becomes self-financing, in that no government guarantees are needed for the project to be undertaken, i.e., $\lim_{\Phi \to \infty} K_g = 0$*.*

4.4 Externalities

We now turn our attention to the role of externalities. Recall that, while the government can expropriate the direct return of the project from the private parties, externalities accrue directly only to the government and increase its payoff if the project succeeds. This strengthens the government's incentives to implement high effort from the private operator by committing to expropriate less from the direct return of the project, reducing the severity of the government's moral hazard. In other words, externalities are effectively an increase in the return of the project R. Proposition 4 summarizes the resulting effect of externalities X on the optimal financing contract:

Proposition 4. ***(Externalities)*** *The externalities generated by the infrastructure project,* X*,*
i. decrease the optimal government guarantee, i.e., $\frac{\partial K_g}{\partial X} \leq 0$*;*
ii. increase the optimal government investment i.e., $\frac{\partial I_g}{\partial X} \geq 0$*; and*
iii. increase the share of the project's return received by the financiers, i.e., $\frac{\partial R_f}{\partial X} \geq 0$*.*
Externalities increase the size of the region in which the infrastructure project is financed and its scale when financed.

Essentially, externalities ameliorate the double moral hazard problem, which implies lower government guarantees are necessary to incentivize the government not to expropriate from the operator. In turn, this translates into more resources being available for the government to invest directly in the infrastructure project and allows a larger fraction of the project's return to be credibly promised to the financiers. All these effects increase both the feasibility and the scale of the infrastructure project:

5 Extensions: Development Rights and Tax Subsidies

Beyond externalities that only accrue to the government, there can be spillovers from the infrastructure that accrue to the private parties involved in the financing and operation of the project or to both. For example, infrastructure projects in transportation, particularly those that build new connection to otherwise isolated areas, increase the value of land and private property as well as create new business opportunities, which are often awarded by governments as “development rights.” We extend the model to consider such spillovers that accrue to the private sector upon project’s success and study their optimal distribution between financiers and the operator. First, we take the size of the development rights as given. Then, we endogeneize the size of the spillovers that accrue to the private sector by thinking of them as tax subsidies that the government can commit to transferring to either the financiers or the operator.

5.1 Development Rights

We extend the model to include development rights of economic value D (per unit scale) that can be distributed between the financiers and the operator if the project succeeds, and which cannot be expropriated by the government. We denote by D_f the amount of the development rights assigned to the financiers in the financial contract and the residual $(D-D_f)$ goes to the operator in the operational contract.

Development rights have a direct effect on the incentive compatibility constraints of the operator and the government, which respectively become

$$(R-R_g-R_f) \geq \frac{B}{\Delta p}-(D-D_f) \quad \text{and} \tag{ICP-D}$$

and

$$(K_g-R_f) \geq \frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}(D-D_f)-\left(R-\frac{B}{\Delta p}\right)-X;. \tag{ICG-D}$$

Every dollar from the development rights assigned to the operator ameliorates the moral hazard of the operator by increasing the payoff it receives if the project succeeds. In turn, this decreases the payoff from the project required by the operator to exert high effort and decreases the guarantees to be provided by the government so as to have incentives not to to expropriate from the operator.

Moreover, any fraction of the development rights assigned to the financiers is analogous to decreasing their outside option by $p_h D_f$, and hence, relaxes their individual rationality constraint, requiring lower government guarantees (K_g) or the promised payment if the project succeeds (R_f) for the infrastructure project to be financed. The individual rationality constraint of the financiers in the presence of development rights becomes

$$(1-p_h)K_g+p_h(R_f+D_f) \geq r\frac{I_f}{I_f+I_g}. \tag{IRP-D}$$

These effect of development rights on these constraints helps us characterize the optimal distribution of development rights between the financiers and the operator:

Proposition 5. ***(Distribution of development rights)*** *There exists a threshold $\Gamma_D > \overline{\Gamma}$ such that the development rights are assigned entirely to the private sector operator if and only if $\left(R-\frac{B}{\Delta p}\right) \leq \Gamma_D$.*
Development rights increase the size of the region in which the infrastructure project is financed (formally,

the threshold $\overline{\Gamma}$ is decreasing in D) and increase the scale of the infrastructure project when financed.

The distribution of the development rights between the financiers and the private sector operator depends on which marginal agent is restricting the size of the project. Intuitively, when the payoff of the project is low, the government has more incentives to expropriate since inducing high effort from the operator does not increase its payoff much. In this case, compensating the operator is relatively harder than providing guarantees to the financiers. By allocating all the development rights to the operator, the government can induce high effort from the operator while increasing its payoff from not expropriating.

On the other hand, when the payoff of the project is high, the government has low incentives to expropriate. In this case, it is relatively easy to induce high effort from the operator and harder to provide guarantees to the financiers. By allocating all the development rights to the financiers, the planner reduces the payoff the government needs to commit not to expropriate in order to incentivize the financiers to participate in the project.

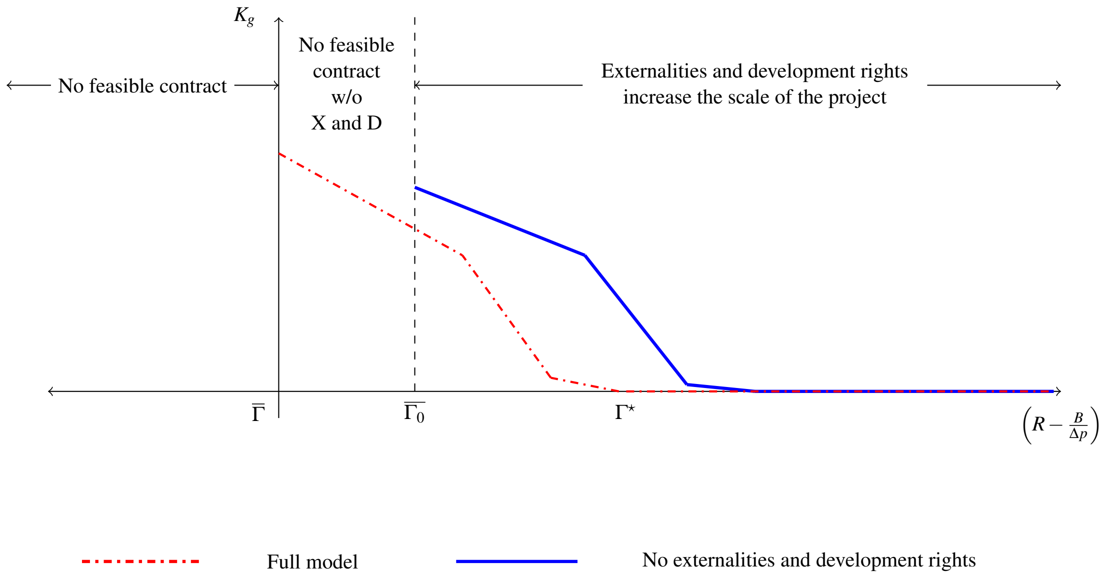
This way, development rights, like externalities, ameliorate the inefficiencies posed by the double moral hazard by making it easier for the government to commit not to expropriate or by strengthening the incentives of financiers to participate in the infrastructure project. Figure 4 illustrates the effect of development rights and externalities on the optimal government guarantees and the optimal distribution of development rights.

5.2 Tax subsidies

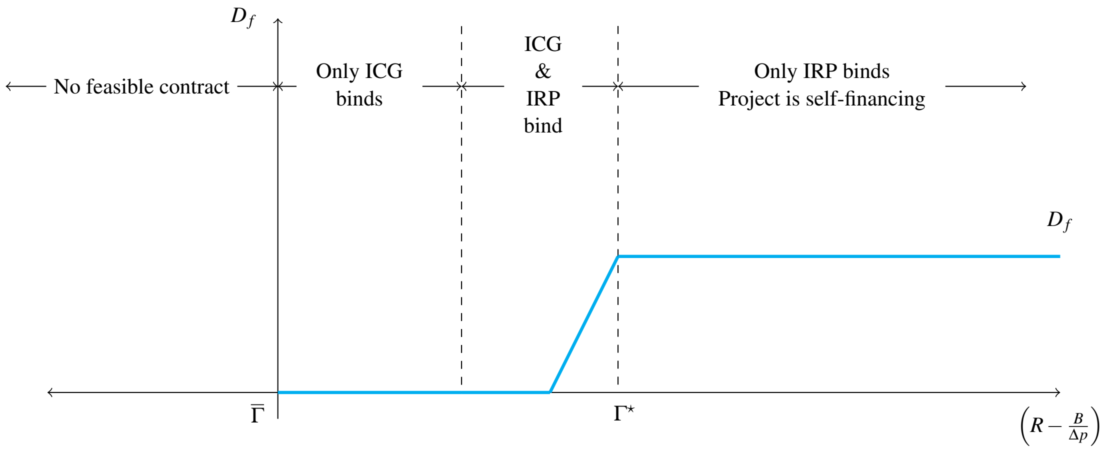
So far, we have taken as given the externalities for the government from infrastructure's success and also the size of the spillovers from the infrastructure project that are shared as development rights for the private sector. Suppose now that the entire additional revenue generated by the infrastructure project is given by an increase in the government's tax revenue and that the government can commit to share a fraction of it by providing tax subsidies to the private financiers and the private sector operator. Once the government agrees to giving tax subsidies to the private parties, we assume that it cannot expropriate these payoffs from them ex post.

Formally, we assume that there are additional tax revenues T (per unit scale) that the government receives from the project if the project succeeds. The government can choose to commit to transferring a fraction τ of the return T to the private sector either in the form of a tax subsidy to the private financiers or a tax abatement for the private sector operator. For a given value of τ, this setup nests the model in the Section 5.1 with development rights $D = \tau T$ and externalities $X = (1-\tau)T$. Proposition 6 below characterizes the optimal tax subsidy:

Proposition 6. ***(Tax subsidies)*** *The optimal financial and operational contracts always require positive tax subsidies when they are available. Whenever government guarantees are necessary to undertake the project,*



(a) Optimal government guarantees under the models with and without development rights (D) and externalities (X).



(b) Distribution of development rights under the optimal contract (D_f = allocation of development rights to financiers

Figure 4: Optimal government guarantees and the distribution of development rights

the optimal tax subsidy is maximal and $\tau = 1$.

A larger tax subsidy to the financiers reduces the government guarantee required by them to participate in the project while increasing the government's incentives to expropriate from the operator. However, analogous to the distribution of development rights, the tax subsidy will only be fully assigned to the financiers when the government's incentive to expropriate is weak and its incentive compatibility constraint is not binding. In this case, the optimal contract requires the subsidy to be as high as possible.

When it is optimal to assign the tax subsidy fully to the private sector operator, a higher tax subsidy decreases the incentives of the government to expropriate and therefore allows for a lower government guarantee. Since the tax subsidy will be fully assigned to the private sector operator only when the financiers' participation constraint is slack, the optimal tax subsidy is again as high as possible.

Under our maintained assumptions, the government can commit not to expropriate τT while it cannot commit to sharing $(1-\tau)T$. As the proposition above shows, it is always (weakly) optimal to make as much as possible of the additional return pledgeable to the private sector ($\tau = 1$) to reduce the severity of the government's moral hazard. The higher the pledgeable return, the lower the government guarantees needed for the project to be undertaken and the larger the scale of the project.

6 Financing Multiple Infrastructure Projects

Finally, we analyze the case in which there are multiple projects that need to be financed and allow for the government to pool the resources from both projects to raise funds and invest in them.

Specifically, governments may have access to multiple sources of cash flows to pay the financiers. So far, we have implicitly considered "revenue only" (RO) financing, i.e., only the cash flows associated with the infrastructure project (including attendant spillovers) and the government's fiscal capacity for guarantees can be used to pay financiers. However, in many instances, cash flows from other projects are also used to pay financiers, for example, in "general obligation" (GO) financing which is supported by overall tax collections at municipality or city level. To encompass this feature we expand the set of projects and financing contracts.

We consider two infrastructure projects and model general obligation financing as a cross-guarantee between the projects. Formally, we consider two ex-ante identical, independent infrastructure projects, $i = a, b$. Each project is subject to moral hazard from the respective private sector operator. The government can choose to expropriate the returns of the projects after they are realized and decides whether to do so in each project independently of what it does in the other project (say, the projects are under different operating arms of the government bureaucracy). To finance the projects, the government offers to financiers of project i a guarantee $K_g^i I^i$, $i = a, b$ if the project fails, and an additional transfer or cross-guarantee $K^i I^i$ from the cash flows from project $j \neq i$ if project i fails and project j succeeds. We denote by $\alpha \equiv \frac{I^a}{I^a+I^b}$ the fraction of total investment in project a. To simplify the analysis and focus on the interaction between the double moral hazard and the choice of infrastructure financing, we ignore the possibility of direct government investment by setting $\overline{K}_0 = 0$ and assume the government can commit to any feasible finite terms of the financing contract by letting $\Phi \to \infty$ (i.e., there is always the willingness to pay).

Note that since cross-guarantees can always be chosen to be zero, general obligation financing can only increase the scale of the project relative to revenue only financing (benchmark model). Hence, the analysis of interest is when general obligation financing features positive cross-guarantees, or in other words, strictly dominates revenue only financing.

The incentive compatibility constraint of the private sector operator in each project i is the same as the one considered in the benchmark model, i.e.,

$$\left[R - \left(R_I^i + R_g^i\right)\right] \geq \frac{B}{\Delta p}, \quad i = a, b,$$

where R_I^i and R_g^i are, respectively, the return to the financiers and the government from project i if it succeeds. As in the benchmark model, the government will expropriate all it can from the private sector operator while providing the private sector incentives to exert effort. Hence, $R_g^i = \overline{R}_g^i \equiv \left[\left(R - \frac{B}{\Delta p}\right) - R_I^i\right]$ for $i = a, b$. Moreover, the government will only commit to sharing part of the project's payoff with the financiers if their participation constraint is binding.

The incentive compatibility constraint of the government in each project now takes into account the expected transfers made and received from the other project. Formally,

$$p_h \overline{R}_g^i I^i - (1 - p_h) K_g^i I^i - p_h (1 - p_h) K^i I^i - p_h (1 - p_h) K^j I^j \geq$$
$$p_l \overline{R}_g^i I^i + \frac{p_l B}{\Delta p} I^i - (1 - p_l) K_g^i I^i - p_h (1 - p_l) K^i I^i - p_l (1 - p_h) K^j I^j ,$$

or

$$\left(p_h K^i - (1 - p_h) K^j \frac{1 - \alpha}{\alpha} + K_g^i - R_I^i\right) \geq \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right), \qquad \text{(ICG–GO)}$$

for $i \neq j$ and $i, j = a, b$. Cross-guarantees, which appear in the terms with $p(1 - p)$, have two opposing effects on the government's incentives to expropriate. On the one hand, providing a guarantee K^i when project i fails and project j succeeds makes it more costly for the government to expropriate and have the private sector operator not exert effort in project i. This mechanism decreases the government's incentives to expropriate. On the other hand, providing a guarantee K^j to financiers from project i to project j (when project j fails and i succeeds) lowers the government's payoff from not expropriating and tightens its incentive compatibility constraint. It turns out that which of these two effects dominates, depends on the probability of success of the projects.

The participation constraints of the financiers and the government when cross-guarantees are allowed are analogous to the ones in the baseline model with the innovation that they now take into account the cross-transfers among projects. We formally state these participation constraints and the feasibility constraints on guarantees in the Appendix.

Since rojects are identical and the optimal financing contract maximizes the sum of the expect payoff of the projects, it is optimal to maximize the joint scale of the projects. Since the scale of the projects is determined by the government guarantee required by the financiers, it is optimal to undertake the project with the lowest required guarantee. Therefore, if both projects are undertaken it has to be the case that the government guarantees required for financiers in both projects are the same. We then obtain the following result on when general obligation financing, i.e., cross-guarantees, are desirable:

Proposition 7. ***(RO vs. GO)*** *Whether general obligation financing is preferred to revenue only financing depends on* $\left(R-\frac{B}{\Delta p}\right)$*, each project's return net of the moral hazard:*

a. If $\left(R-\frac{B}{\Delta p}\right)<\underline{\Gamma}$*, projects are not funded even in the absence of any government moral hazard.*

b. If $\underline{\Gamma}\leq\left(R-\frac{B}{\Delta p}\right)<\overline{\Gamma}$*, projects are not funded in the presence of government moral hazard.*

c. Otherwise, projects are funded as follows:

i. If $p_h\geq\frac{1}{2}$*, general obligation financing is strictly preferred to revenue only financing; in other words, the optimal cross-guarantees are positive (*$K^a=K^b>0$*).*

ii. If $p_h<\frac{1}{2}$*, general obligation financing is strictly preferred to revenue only financing only if the return of the project is high enough; in particular, the optimal cross-guarantees are positive (*$K^a=K^b>0$*) if*

$$\left(R-\frac{B}{\Delta p}\right)>\left[\frac{\overline{\Gamma}}{(1-p_h)}-\frac{r}{(1-p_h)}\right].$$

Furthermore, if the return on projects is high enough, they do not require any additional government guarantees, i.e., $K_g^a=K_g^b=0$*, when cross-guarantees are chosen optimally.*

Proposition (7) shows that general obligation financing (weakly) increases the scale of the project. However, it does not increase the likelihood of the project being financed, at least in the symmetric case.

Intuitively, cross-guarantees mainly affect the government's incentives to expropriate from the operator. By expropriating, the government cannot induce high effort from the private sector operator. Low effort by the operator in project a implies a higher probability of paying the cross-guarantee from project b to project a. The increase in this probability is $[p_h(1-p_l)-p_h(1-p_h)]$, where p_h and p_l are the project's success probabilities when the private sector operator exerts high and low effort, respectively. At the same time, it decreases the probability with which the cross-guarantees will be paid to project b from project a. The decrease in this probability is $[p_l(1-p_h)-p_h(1-p_h)]$. When both projects are symmetric, the cross-guarantees are the same from a to b and from b to a. Therefore, the increase in the incentives to expropriate based on the cross-guarantees is given by

$$-p_h\Delta p + \Delta p\,(1-p_h) = \Delta p\,(1-2p_h)\ ,$$

where $\Delta p \equiv (p_h - p_l)$. When $p_h < \frac{1}{2}$, cross-guarantee exacerbates the moral hazard of the government and revenue only financing (setting the cross-guarantees to zero) is optimal whenever the payoff of the project is low and the incentive compatibility of the government binds. Alternatively, when $p_h > \frac{1}{2}$, cross-guarantees mitigate the moral hazard of the government and general obligation financing (positive cross-guarantees) are optimal as long as the return of the project is high enough to satisfy the government's participation constraint.

Finally, when the cash flow from the project is high enough, the project is self-financing regardless of whether $p_h \gtrless \frac{1}{2}$. Any dollar pledged in the cross-guarantees cannot be expropriated by the government. As a result, if the expected return of the projects is high enough, then the cross-guarantees are enough to satisfy the individual rationality constraint of the financiers and no additional government guarantee is needed for the project to be undertaken.

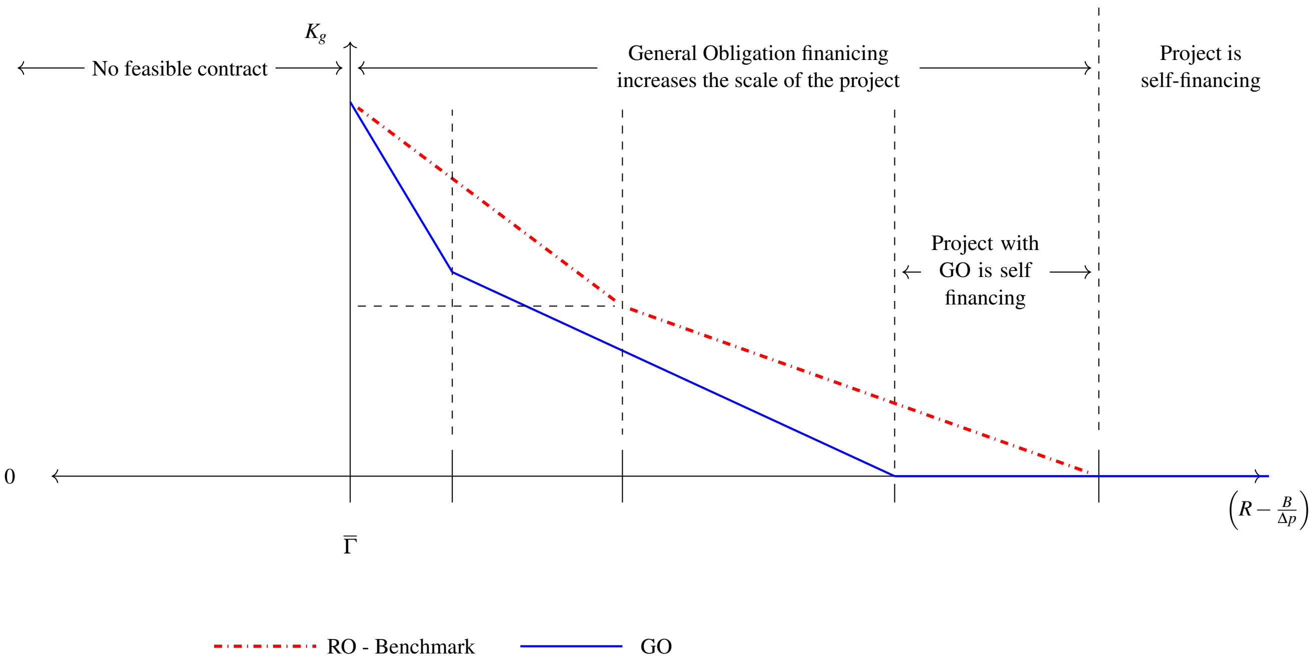
In summary, revenue only financing can only be optimal when the quality and the return of the projects are low, and in this case, having general obligation financing available does not affect the scale of the project. However, when the return of the project is high, cross-guarantees create value and are positive whenever the project is implemented; in this case, general obligation financing increases the scale of the project.

Figures 5 show the optimal government guarantees with cross-guarantees compared with the optimal guarantees in the benchmark model without cross-guarantees. Figure 5a shows the case when $p_h > \frac{1}{2}$ and Figure 5b shows the case when $p_h < \frac{1}{2}$.

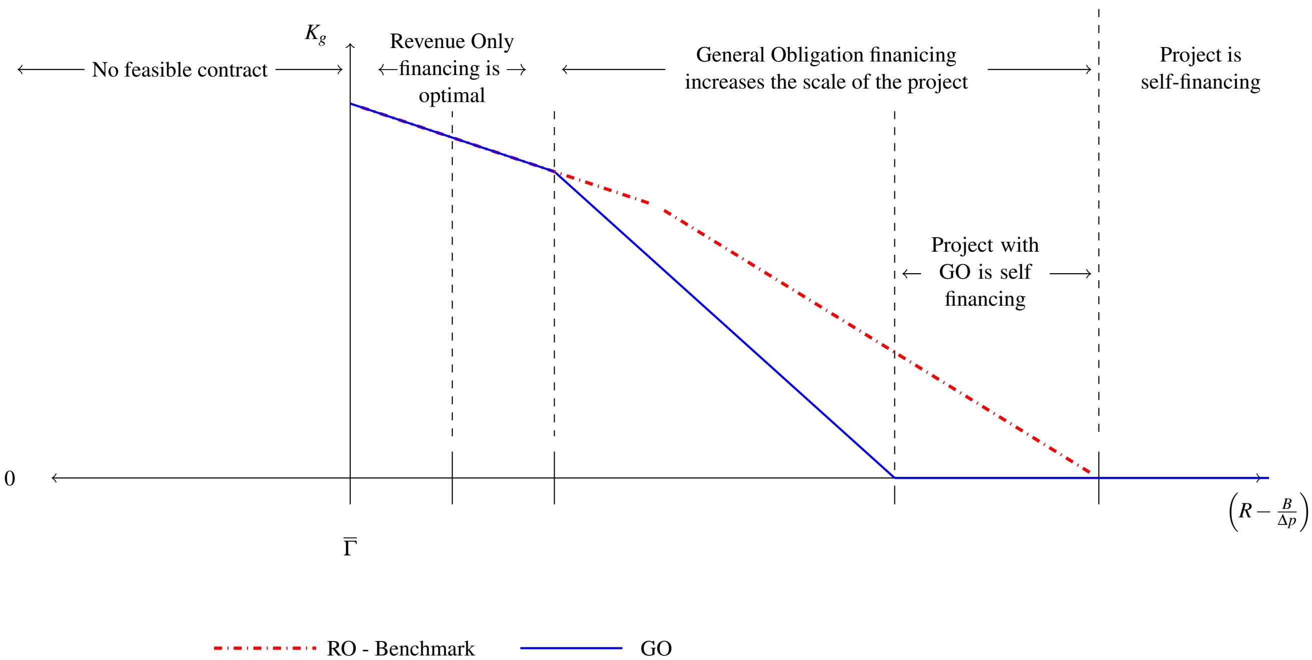
We discuss in Section 7 that revenue only bonds account for nearly two thirds of the municipal bond market. Our model suggests that this may be the case because of a high severity of moral hazard in these lower quality projects; in these cases, providing cross-guarantees across projects in the form of general obligation bonds would lead to an increase in the inefficiency by weakening the government's incentives to participate or conditional on participation not to expropriate.

7 Infrastructure Financing in Practice

Over a period of time, governments and private sector firms in many infrastructure investments have come together in different countries with varying contractual arrangements to design and execute projects. We document below that the salient features identified in our (second-best) solution to the problem of optimal financing of infrastructure in the shadow of expropriation finds immediate counterparts in these contractual



(a) Optimal government guarantees under cross-guarantees (GO) and under the benchmark (RO) model when $p_h > 0.5$.



(b) Optimal government guarantees under cross-guarantees (GO) and under the benchmark (RO) model when $p_h < 0.5$.

Figure 5: Optimal government guarantees when there are cross-guarantees available

arrangements observed in practice: First, the government is directly involved either as a guarantor (and occasionally also as a co-lender or as a supplier) of capital. This role is played explicitly by the government or its agencies or supranational institutions with implicit or explicit backing. Second, some infrastructure projects (such as low income housing, slum clearance, sanitation, health services, etc.) can deliver "public goods" that may be valued by the government but not necessarily by the private parties, which in turn can help finance tax subsidies for infrastructure financiers. Third, some infrastructure projects can deliver development rights for private parties expanding their pecuniary gains from building and operating infrastructure and/or financing it. Finally, cross-guarantees feature in several municipal bond financing instruments for infrastructure projects while others are financed individually. Let us elaborate.

Government guarantees. A key result of our analysis is the need of government guarantees to offset the threat of government expropriation. In practice, government guarantees are used in the financing of infrastructure across the world. For example, in the United States, "loan guarantees" to institutional investors who fund transportation projects of national or regional significance are available through the Transportation Infrastructure Finance and Innovation Act (TIFIA) of 1998.[11] In France, a two-pronged approach is used to offer guarantees to investors in infrastructure. First, the French government provides guarantees to bank loans that are directed specifically towards infrastructure projects. Second, the government has established another guarantees program to promote debt financing, allowing infrastructure projects to be funded at relatively low costs.[12] Similar contractual arrangements are used in Australia through a government-designed guarantee program to address the funding gap in infrastructure financing.[13]

Government co-investment. There are many examples of governments co-investing with the private sector to help the infrastructure projects achieve the closure of their initial financing. For instance, under TIFIA the United States government offers secured direct loans to the private sponsors of infrastructure projects in the transportation industry. In the United Kingdom too, the Treasury has established since 2009 a unit that co-lends along with private sector lenders to fund privately financed infrastructure initiatives, the stated goal being to exit the investment by selling the loans in the private capital markets once the projects become self-sustaining. The Australian government also has co-lending facilities, whereby it lends on commercial terms along with private sector banks to fill the funding gap in infrastructure projects.

Tax subsidies. A unique innovation in infrastructure financing in the United States has been the tax treatment of municipal bonds: the interest income from municipal bonds is tax-exempt from the perspective of private investors, and the bonds were (up until the global financial crisis) typically insured by monoline

[11] TIFIA was passed by Congress in 1998 with the goal of leveraging federal dollars and attract private and non-federal capital into transportation infrastructure. See "Transportation Infrastructure Finance and Innovation Act" (2011) for details.

[12] See "Public and private financing of infrastructure Policy challenges in mobilizing finance", EIB Papers Volume 15 No 2, 2010.

[13] See "Infrastructure Partnerships Australia: Financing Infrastructure in the Global Financial Crisis," March 2009.

insurance companies.[14] Together, these two features have allowed the development of a fairly big municipal bond market, which, as we discuss below, offers a major source of funding of infrastructure projects in the United States.[15]

Bundling of development rights. Some infrastructure projects can result in significant development rights of lands and buildings adjoining the project. Future cash flows from such development rights can increase the overall attractiveness to different contracting parties. Gupta, Van Nieuwerburgh and Kontokosta (2022) show that the new transit infrastructure project in New York city resulted in significant spillover benefits to local real estate prices through reduction in transit times for commuters in those localities. Some infrastructure projects can also deliver significant development rights that can affect ex ante the way in which the project may be financed and reduce the operating costs of using the infrastructure. Hong Kong Mass Transit Railway Corporation (MRTC) which covers slightly over 200 kilometers with 84 stations and 68 light train stops resulted in significant increase in land and property values close to the stations. Such rights were deemed valuable to all parties in the contracting arrangements. According to one study, during the period 1998-2013, property-related development operations generated nearly twice the amount of money spent on railway line construction.[16] Our model implies that the bundling of such development rights can play an important role in expanding the feasibility and the scale of infrastructure projects.

General obligation versus revenue-only financing. Many projects at state and city levels, such as public hospitals, highways, bridges, etc., are funded in the United States through the issuance of municipal bonds. The municipal bond market is large and diverse with $4.0 trillion of municipal debt outstanding and over 50,000 issuers. The municipal bonds typically fall into two categories: the so-called general obligation (GO) bonds, which depend on the overall tax revenues of the State or City for cash flow integrity, effectively providing cross-guarantees across different municipal projects; and, the so-called revenue only (RO) bonds, which depend only on user-fees such as tolls. The revenue bonds account for nearly two-thirds of the municipal bond market as measured by their share of the Bloomberg Barclays Municipal Index, as of June 30, 2019.[17] It is an open question, to the best of our knowledge, as to what determines the choice across infrastructure projects between financing with GO bonds (cross-guarantees) and RO bonds (no cross-guarantees),

[14] Since the global financial crisis and in the aftermath of monoline insurer default/distress, while monoline insurance guarantees are no longer the norm for municipal bonds, the bonds are increasingly held by the following groups in their order of ownership: Households and Nonprofit Organizations, Money Market Mutual Funds, Mutual Funds, Closed-End Funds, U.S. Chartered Depository Institutions and Banks Brokers and Dealers and Exchange-Traded Funds. See MSRB, "Trends in Municipal Bond Ownership," (2019).

[15] Given households are the largest holders of municipal debt, the tax treatment of municipal bonds has attracted some attention from researchers such as Green (1993), Ang, Bhansali and Xing (2010), and Longstaff (2011).

[16] See "Land Value Capture Mechanism: The Case of the Hong Kong Mass Transit Railway" by Mathieu Verougstraete and Han Zeng (July 2014).

[17] See "Why Municipal Revenue Over General Obligation Bonds," August (2019), by Marques and Barton, BNY Mellon, Investment Managment.

and our framework provides preliminary guidance on this choice.

Beyond this mapping between the second-best contract in our model and the financing arrangements that one observes in practice for infrastructure projects, there are two testable implications from our model that can be drawn about the cross-section of infrastructure financing arrangements. First, our model implies that government guarantees are an incentive-efficient use of limited fiscal resources in case of financing of low-quality (high moral hazard) infrastructure projects, whereas government co-investment plays this role in financing of high-quality projects. Second, GO financing should be more common in high-quality (low moral hazard) infrastructure projects, whereas RO financing should be more common for low-quality infrastructure projects. We leave the empirical testing of these implications for further work.

8 Conclusion

We analyze the optimal design of infrastructure financing in the presence of private moral hazard and the threat of government expropriation. The private sector operators need incentives to exert effort to implement projects well and governments that can expropriate cash flows from such projects need incentives to commit to sharing the projects' returns with the private sector (for instance, by not restricting the user fees). This double moral hazard problem limits the willingness of outside investors to fund infrastructure projects; given limited financial commitment of governments, the shadow of expropriation limits in turn feasibility and scale of infrastructure, explaining potentially the large infrastructure gaps observed globally. The optimal (second-best) design of infrastructure finance can ameliorate these two moral hazards using (I) government guarantees to investors; (II) direct government investment for projects with high returns; (III) tax subsidies to the private parties; (IV) bundling of development rights for the private parties; and, (V) "general obligation" financing for high-quality projects and "revenue only" financing for low-quality projects. All of these features are prevalent in the practice of infrastructure financing, highlighting the relevance of the double moral hazard we considered as a unifying framework to understand infrastructure finance.

Indeed, our framework appears relevant also to the provision of public goods such as public health infrastructure (sufficient capacities of hospital beds, medical equipment, and human capital in healthcare professionals). These were found critical – and wanting – in the wake of the pandemic, and yet it is difficult to elicit private investments in public health infrastructure. A clear recent example in healthcare of success in public-private partnerships has been investment in the development and production of vaccines. Typically, patents or monopoly power are thought to be ex-ante desirable to incentivize efficient technological innovation. However, public health concerns are likely to lead the government to lower the user fees of vaccines ex post, discouraging private investment. To restore the willingness of the private sector to invest, the government can then either commit to paying the difference between the market user fee and the public cap on the fee or co-invest. Our theoretical framework suggests that one alternate way to subsidize investment in vaccine development is for the government to provide guarantees in the event of a failure of the investment in such projects. There seems ample scope for such applications of our primary insights in other settings.

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APPENDIX

A Baseline Model

In this section, we characterize the optimal contract for the model presented in Section 5.1, which nests the model in Section 3 by setting $D = 0$.

Characterization of optimal contract in the baseline model

The optimal financing contract solves

$$\max_{I_g \in \{0, K_0\}, I_f \geq 0, K_g \geq 0, R_f \geq 0, D_f \in [0, D]} \left(p_h \left(R + X + D\right) - r\right)\left(I_g + I_f\right)$$

subject to

$$\begin{aligned}
(1 - p_h) K_g + p_h \left(R_f + D_f\right) &\geq r \frac{I_f}{I_f + I_g}, && \text{(IRP)} \\
(1 - p_h) K_g + p_h \left(R_f + D_f\right) + r \frac{I_g}{I_f + I_g} &\leq p_h \left(R - \frac{B}{\Delta p}\right) + p_h (X + D), && \text{(IRG)} \\
K_g - R_f &\geq \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}\left(D - D_f\right) - \left(R - \frac{B}{\Delta p}\right) - X, && \text{(ICG)} \\
K_g \left(I_f + I_g\right) &\leq \min\left\{\Phi, \overline{K}_1 + \overline{K}_0 - I_g\right\}, \quad \text{and} && \text{(NDK)} \\
R_f \left(I_f + I_g\right) &\leq \min\left\{\Phi, \left(R - \frac{B}{\Delta p}\right)\left(I_f + I_g\right) + \overline{K}_1 + \overline{K}_0 - I_g\right\}, && \text{(NDR)}
\end{aligned} \tag{1}$$

where IRP is the individual rationality constraint of the financiers, IRG is the individual rationality constraint of the government, ICG is the incentive compatibility constraint of the government, and NDK and NDR combine the no-default conditions for the government and the feasibility constraints on government guarantees and promised returns. Note that to have IRG and IRP satisfied at the same time it has to be the case that

$$(1 - r) \frac{I_g}{I} \leq p_h \left(R - \frac{B}{\Delta p}\right) + p_h (X + D) - r.$$

Moreover, the no-default conditions impose upper bounds on the total scale of the infrastructure project. Therefore, since the infrastructure project is positive NPV, either the constraint NDK or the constraint NDR or both have to bind, which implies

$$I = \min\left\{\frac{\min\left\{\Phi, \left(\left(R - \frac{B}{\Delta p}\right) + \left(D - D_f\right)\right) I + \overline{K}_1 + \overline{K}_0 - I_g\right\}}{R_f}, \frac{\min\left\{\Phi, \overline{K}_1 + \overline{K}_0 - I_g\right\}}{K_g}\right\}.$$

Therefore, the scale of the project will be determined either by the promised return to financiers R_f or by the government guarantees K_g or by both. Which is the case depends on whether the incentive compatibility constraint of the government and the participation constraint of the financier bind. As it is the case in this type of problems with multiple linear constraints, there are several possible cases to consider depending on the parameter values. To keep the Appendix brief, we provide a characterization of each of these cases and the parameter regions in which they are relevant in the Online Appendix.

Optimal financial contract

The optimal scale of the project is given by

$$
I^{\star}=\begin{cases}
0 & \left(R-\frac{B}{\Delta p}\right)<\overline{\Gamma}\\
\frac{\min\left\{\Phi,\overline{K}_1+\overline{K}_0\right\}}{\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X} & \text{if}\quad \overline{\Gamma}\le\left(R-\frac{B}{\Delta p}\right)<\Gamma_I\\
\frac{\min\left\{\Phi,\overline{K}_1+\overline{K}_0-I_g^{\star}\right\}}{\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X} & \text{if}\quad \Gamma_I\le\left(R-\frac{B}{\Delta p}\right)<\Gamma_R\\
\frac{\min\left\{\Phi,\overline{K}_1\right\}+r\overline{K}_0}{r+p_h\left(\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X\right)} & \text{if}\quad \Gamma_R\le\left(R-\frac{B}{\Delta p}\right)<\Gamma_D\\
\frac{\overline{K}_1+r\overline{K}_0}{r-p_h D_f^{\star}+p_h\left(\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}\left(D-D_f^{\star}\right)-\left(R-\frac{B}{\Delta p}\right)-X\right)} & \text{if}\quad \Gamma_D\le\left(R-\frac{B}{\Delta p}\right)<\Gamma^{\star}\\
\frac{r\overline{K}_0+(1-p_h)\min\left\{\Phi,\overline{K}_1\right\}+p_h\Phi}{(r-p_h D)} & \text{if}\quad \Gamma^{\star}\le\left(R-\frac{B}{\Delta p}\right),
\end{cases}
\tag{2}
$$

the optimal government guarantee is given by

$$
K_g^{\star}=\begin{cases}
\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X & \text{if}\quad \overline{\Gamma}\le\left(R-\frac{B}{\Delta p}\right)<\Gamma_R\\
r\frac{\min\left\{\Phi,\overline{K}_1\right\}}{\min\left\{\Phi,\overline{K}_1\right\}+r\overline{K}_0}+\frac{p_h\min\left\{\Phi,\overline{K}_1\right\}}{\min\left\{\Phi,\overline{K}_1\right\}+r\overline{K}_0}\left(\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X\right) & \text{if}\quad \Gamma_R\le\left(R-\frac{B}{\Delta p}\right)<\Gamma_D\\
r-p_h D_f^{\star}-\frac{r\overline{K}_0}{I^{\star}}+p_h\left(\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}\left(D-D_f^{\star}\right)-\left(R-\frac{B}{\Delta p}\right)-X\right) & \text{if}\quad \Gamma_D\le\left(R-\frac{B}{\Delta p}\right)<\Gamma^{\star}\\
\frac{\min\left\{\Phi,\overline{K}_1\right\}}{r\overline{K}_0+(1-p_h)\min\left\{\Phi,\overline{K}_1\right\}+p_h\Phi}\left(r-p_h D\right) & \text{if}\quad \Gamma^{\star}\le\left(R-\frac{B}{\Delta p}\right),
\end{cases}
\tag{3}
$$

the optimal government investment is given by

$$
I_g^{\star}=\begin{cases}
0 & \text{if}\quad \overline{\Gamma}\le\left(R-\frac{B}{\Delta p}\right)<\Gamma_I\\
\frac{\left(r-(1-p_h)\left(\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X\right)\right)}{\left(\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X\right)}\min\left\{\frac{\Phi}{r},\frac{\left(\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X\right)\left(\overline{K}_1+\overline{K}_0\right)}{\left(\left(\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X\right)(r-(1-p_h))+r\right)}\right\} & \text{if}\quad \Gamma_I\le\left(R-\frac{B}{\Delta p}\right)<\Gamma_R\\
\overline{K}_0 & \text{if}\quad \Gamma_R\le\left(R-\frac{B}{\Delta p}\right),
\end{cases}
\tag{4}
$$

the optimal return promised to financiers is

$$
R_f^{\star}=\begin{cases}
0 & \text{if}\quad \overline{\Gamma}\le\left(R-\frac{B}{\Delta p}\right)<\Gamma_R\\
r\frac{\min\left\{\Phi,\overline{K}_1\right\}}{\min\left\{\Phi,\overline{K}_1\right\}+r\overline{K}_0}-\left(\frac{r\overline{K}_0+(1-p_h)\min\left\{\Phi,\overline{K}_1\right\}}{\min\left\{\Phi,\overline{K}_1\right\}+r\overline{K}_0}\right)\left(\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X\right) & \text{if}\quad \Gamma_R\le\left(R-\frac{B}{\Delta p}\right)<\Gamma_D\\
r-p_h D_f^{\star}-\frac{r\overline{K}_0}{I^{\star}}-(1-p_h)\left(\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}\left(D-D_f^{\star}\right)-\left(R-\frac{B}{\Delta p}\right)-X\right) & \text{if}\quad \Gamma_D\le\left(R-\frac{B}{\Delta p}\right)<\Gamma^{\star}\\
\frac{\Phi}{r\overline{K}_0+(1-p_h)\min\left\{\Phi,\overline{K}_1\right\}+p_h\Phi}\left(r-p_h D\right) & \text{if}\quad \Gamma^{\star}\le\left(R-\frac{B}{\Delta p}\right),
\end{cases}
\tag{5}
$$

and the optimal development rights assigned to the financiers are

$$D_f^\star = \begin{cases} 0 & \text{if} \quad \overline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right) < \hat{\Gamma}_D \\ \dfrac{\left(R - \frac{B}{\Delta p}\right) - \left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p} D - X\right)}{\frac{p_h}{\Delta p}} & \text{if} \quad \Gamma_D \leq \left(R - \frac{B}{\Delta p}\right) < \Gamma^\star \quad \text{and} \quad \Phi < \overline{K}_1 \\ \dfrac{\left(p_h \Phi + r\overline{K}_0 + \overline{K}_1(1-p_h)\right)\left(\left(R - \frac{B}{\Delta p}\right) - \left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p} D - X\right)\right) - \left(\Phi - \overline{K}_1\right) r}{\left(p_l \Phi + r\overline{K}_0 + \overline{K}_1(1-p_l)\right)\frac{p_h}{\Delta p}} & \text{if} \quad \Gamma_D \leq \left(R - \frac{B}{\Delta p}\right) < \Gamma^\star \quad \text{and} \quad \Phi > \overline{K}_1 \\ D & \text{if} \quad \Gamma^\star \leq \left(R - \frac{B}{\Delta p}\right), \end{cases} \tag{6}$$

where the thresholds $\underline{\Gamma}$, $\overline{\Gamma}$, Γ_I, Γ_R, Γ_D, and $\Gamma^\star$ are given by

$$\begin{aligned}
&\underline{\Gamma} \equiv \frac{r}{p_h} - X - D, \\
&\overline{\Gamma} \equiv (1-p_h)\frac{p_l B}{(\Delta p)^2} - p_h \frac{(1-p_l)}{\Delta p} D - X, \\
&\Gamma_I \equiv \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p} D - X - \frac{r}{1-p_h}, \\
&\Gamma_R \equiv \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p} D - X - \frac{r \min\left\{\overline{K}_1, \Phi\right\}}{\left(r\left(\overline{K}_1 + \overline{K}_0\right) - (r - (1-p_h)) \min\left\{\overline{K}_1, \Phi\right\}\right)}, \\
&\Gamma_D \equiv \max\left\{ \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p} D - X + \frac{\Phi - \min\left\{\Phi, \overline{K}_1\right\}}{\left(r\overline{K}_0 + (1-p_h)\min\left\{\Phi, \overline{K}_1\right\} + p_h \Phi\right)} r, \right. \\
&\qquad \left. \frac{\Phi - \overline{K}_1}{\overline{K}_1 + r\overline{K}_0 + p_h\left(\Phi - \overline{K}_1\right)}\left(r + p_h\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p} D - X\right)\right) - \frac{\left(\overline{K}_1 + r\overline{K}_0\right)}{\left(\overline{K}_1 + r\overline{K}_0 + p_h\left(\Phi - \overline{K}_1\right)\right)} D \right\}, \\
&\Gamma^\star \equiv \max\left\{ \frac{\left(\Phi - \overline{K}_1\right)}{\left(\left(\Phi - \overline{K}_1\right) p_h + r\overline{K}_0 + \overline{K}_1\right)}(r - p_h D) + \frac{p_l B}{(\Delta p)^2} - X, \frac{p_l B}{(\Delta p)^2} - X \right\}.
\end{aligned}$$

Proof of Proposition 1 (Inefficiency of double moral hazard)

a) **(Feasibility)** Define $\underline{\Gamma}$, $\overline{\Gamma}$ and $\Gamma^\star$ as in the section above. Then, for the participation constraints of the investors and the government to be satisfied at the same time it must be the case that $\left(R - \frac{B}{\Delta p}\right) \geq \underline{\Gamma}$. Otherwise, the contract is not funded even in the absence of moral hazard,which proves the statement in part i. of the proposition.

Using the definitions of $\underline{\Gamma}$ and $\overline{\Gamma}$ and Eq. (2) in the analysis above, we have that, in the presence of moral hazard, the project will be financed as long as $\max\left\{\underline{\Gamma}, \overline{\Gamma}\right\} \leq \left(R - \frac{B}{\Delta p}\right)$. Hence, using part a) i., it follows that if $\underline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right) < \overline{\Gamma}$ the project is not funded in the presence of moral hazard. These two statements prove parts ii. and iii. of the proposition.

b) **(Scale)** The results follow directly from the characterization of the optimal scale and optimal guarantee in Equations (2) and (3).

Proof of Proposition 2 (Pecking order)

The proof follows directly from the characterization of the optimal contract in Equations (3) (4), and (5).

Proof of Proposition 3 (Limited commitment)

From Equation 2, we have that $\frac{\partial I}{\partial \Phi}$ is increasing in Φ. i. follows from the characterization of R_f in Equation 5 and ii. from the characterization of K_g in Equation 3 taking limits as $K_g \to \infty$.

Proof of Proposition 4 (Externalities)

From the characterization of the optimal guarantee in Equation (3) it follows that $\frac{\partial K_g^\star}{\partial X} \leq 0$, which proves part i.
ii. follows from the characterization of the optimal government investment in Equation (4) noticing that $\frac{\partial I_g^\star}{\partial X} \geq 0$ iii. follows from the characterization of the optimal return to the financiers in Equation (5) we have $\frac{\partial R_f^\star}{\partial X} \geq 0$. Finally, statements on feasibility and scale follow from noting that $\frac{\partial \overline{\Gamma}}{\partial X} < 0$ and $\frac{\partial I}{\partial X} > 0$.

Proof of Proposition 5 (Distribution of development rights)

The proof follows from the characterization of the optimal development rights assigned to the financier characterized in Equation (6).
The results on scale and feasibility follow from the characterization of the threshold $\overline{\Gamma}$ in Equation (9) and the optimal scale of the infrastructure project in Equation (2), by noting that $\frac{\partial \overline{\Gamma}}{\partial D} < 0$ and $\frac{\partial I}{\partial D} \geq 0$.

Proof of Proposition 6 (Tax subsidies)

The proof follows setting $X = (1-\tau)T$ and $D = \tau T$ and noticing that I is increasing in τ, hence the optimal $\tau = 1$.

B Revenue Only vs. General Obligation Financing

In this section we characterize the optimal contract for the model in Section 6 when there are cross-guarantees and provide the proofs of the results in this section.

Characterization of optimal contract

In this case, the government solves

$$\max_{\substack{K^i I^i \in \left[0, \left(R - \frac{B}{\Delta p}\right) I^j\right], \\ \alpha \in [0,1],\, K_g^j \geq 0}} \quad (p_h R - r) I$$

s.t.

$$\frac{r}{1-p_h} \le p_h K^a + K_g^a + \frac{p_h}{1-p_h} R_I^a\,, \tag{IRPA–GO}$$

$$\frac{r}{1-p_h} \le p_h K^b + K_g^b + \frac{p_h}{1-p_h} R_I^b\,, \tag{IRPB–GO}$$

$$(1-p_h)\left[p_h\left(K^a\alpha + K^b(1-\alpha)\right) + \left(K_g^a\alpha + K_g^b(1-\alpha)\right)\right] \le p_h\left(R - \frac{B}{\Delta p}\right) - \alpha R_I^a - (1-\alpha)R_I^b, \tag{IRG–GO}$$

$$p_h K^a - (1-p_h)K^b\frac{1-\alpha}{\alpha} + K_g^a - R_I^a + \left(R - \frac{B}{\Delta p}\right) \ge \frac{p_l B}{(\Delta p)^2}\,, \tag{ICGA–GO}$$

$$p_h K^b - (1-p_h)K^a\frac{\alpha}{1-\alpha} + K_g^b - R_I^b + \left(R - \frac{B}{\Delta p}\right) \ge \frac{p_l B}{(\Delta p)^2}\,, \quad \text{and} \tag{ICGB–GO}$$

$$\alpha I K_g^a + (1-\alpha) I K_g^b \le \overline{K}\,. \tag{Fiscal-Constraint–GO}$$

Fiscal-Constraint–GO holds with equality in equilibrium. Therefore, one can rewrite the objective function as follows

$$(p_h R - 1)\frac{\overline{K}}{\alpha K_g^a + (1-\alpha)K_g^b}\,.$$

Lemma. *If both projects are undertaken, then the government guarantee is the same for both projects, i.e.,* $K_g^a = K_g^b = K_g$ *and the scale of each project is* $\frac{1}{2}\frac{\overline{K}}{K_g}$.

Proof. Suppose to the contrary that $K_g^b > K_g^a$ and $\alpha \in (0,1)$. Then, one could increase α and increase K^b while still satisfying all the constraints and increasing the objective function. Note that an increase in α would relax ICGA–GO and the upper bound on K^b while tightening ICGB–GO. But one could increase K^b to guarantee that IRPB–GO and ICGB–GO hold while still satisfying the rest of the constraints. Analogously if $K_g^b < K_g^a$. Hence, if both projects are undertaken, then we must have $K_g^a = K_g^b$. □

Using the Lemma above, the problem becomes

$$\max_{\substack{K^i I^i \in \left[0, \left(R - \frac{B}{\Delta p}\right) I^j\right], \\ \alpha \in [0,1],\, K_g \ge 0}} (p_h R - r)\frac{\overline{K}}{K_g}$$

$$\frac{r}{1-p_h} \le p_h K^a + K_g + \frac{p_h}{1-p_h} R_I^a\,, \tag{IRA–GO}$$

$$\frac{r}{1-p_h} \le p_h K^b + K_g + \frac{p_h}{1-p_h} R_I^b\,, \tag{IRB–GO}$$

$$p_h(1-p_h)\left(\alpha K^a + (1-\alpha)K^b\right) + (1-p_h)K_g \le p_h\left(R - \frac{B}{\Delta p}\right) - p_h\alpha R_I^a - p_h(1-\alpha)R_I^b, \tag{IRG–GO}$$

$$p_h K^a - (1-p_h)K^b\frac{1-\alpha}{\alpha} + K_g - R_I^a \ge \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right), \text{ and} \tag{ICGA–GO}$$

$$p_h K^b - (1-p_h)K^a\frac{\alpha}{1-\alpha} + K_g - R_I^b \ge \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right). \tag{ICGB–GO}$$

To be able to satisfy the three individual rationality constraints it must be the case that

$$\frac{r}{p_h} \le \left(R - \frac{B}{\Delta p}\right). \tag{7}$$

Moreover, the individual rationality constraints of the investor and the incentive compatibility constraints of the government impose lower bounds on the government guarantee K_g as follows:

$$K_g \geq \max\left\{\frac{r-p_h R_I^a}{1-p_h} - p_h K^a, \frac{r-p_h R_I^b}{1-p_h} - p_h K^b, \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) + R_I^a - \left(p_h K^a - (1-p_h) K^b \frac{1-\alpha}{\alpha}\right),\right.$$
$$\left.\frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) + R_I^b - \left(p_h K^b - (1-p_h) K^a \frac{\alpha}{1-\alpha}\right), 0\right\}.$$

Note that this constraint is minimized at $\alpha = \frac{1}{2}$, which implies

$$K_g \geq \max\left\{\frac{r-p_h R_I^a}{1-p_h} - p_h K^a, \frac{r-p_h R_I^b}{1-p_h} - p_h K^b, \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) + R_I^a - \left(p_h K^a - (1-p_h) K^b\right),\right.$$
$$\left.\frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) + R_I^b - \left(p_h K^b - (1-p_h) K^a\right), 0\right\}.$$

Moreover, the constraint is also minimized when $K^a = K^b$ and $R_I^a = R_I^b$. Hence,

$$K_g \geq \max\left\{\frac{r-p_h \hat{R}_b}{1-p_h} - p_h \hat{K}, \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) + \hat{R}_b - (2p_h - 1)\hat{K}, 0\right\},$$

where $K^a = K^b = \hat{K}$, $R_I^a = R_I^b = \hat{R}_b$, and

$$0 \leq \hat{K} \leq \left(R - \frac{B}{\Delta p}\right) - \hat{R}_b.$$

1) If only the individual rationality constraint of the investors binds, then

$$K_g = \frac{r}{1-p_h} - \frac{p_h}{1-p_h}\hat{R}_b - p_h \hat{K}.$$

Then, it is optimal to set $\hat{R}_b = \frac{r}{p_h} - (1-p_h)\hat{K}$ and the project is self-financing, i.e., $K_g = 0$.

The incentive compatibility constraint of the government will be satisfied as long as

$$\left(R - \frac{B}{\Delta p}\right) \geq \frac{p_l B}{(\Delta p)^2} + \frac{r}{p_h} - p_h \hat{K}$$

and the feasibility constraint on $\hat{R}_b$ implies $\hat{K} \leq \frac{1}{p_h(1-p_h)} r$.

If $\left(R - \frac{B}{\Delta p}\right) \geq \frac{1}{1-p_h}\frac{r}{p_h}$ all constraints are slack and $\hat{R}_b = 0$ and $K_g = 0$.

2) If the incentive compatibility constraint of the government is binding and the individual rationality constraint of the investors is slack, then

$$K_g = \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) + \hat{R}_b - (2p_h - 1)\hat{K}.$$

The incentive compatibility constraint of the government with respect to the private sector implies $\hat{R}_b = 0$. Moreover, whether $\hat{K}$ increases or decreases the scale of the project depends on the value of $2p_h - 1$.

a) If$2p_h < 1$, then K_g is increasing in $\hat{K}$ and it is optimal to set $\hat{K} = 0$. This will be the case when

$$(1-p_h)\frac{p_l B}{(\Delta p)^2} \leq \left(R - \frac{B}{\Delta p}\right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h},$$

which implies that the individual rationality constraints of investors and the government, and the feasibility constraints are satisfied. In this case,

$$K_g = \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) .$$

b) If $2p_h > 1$, then K_g is decreasing in $\hat{K}$ and it is optimal to set $\hat{K}$ as large as possible. To satisfy the incentive compatibility constraint of the government and the feasibility constraints it must be the case that

$$\hat{K} = \max\left\{0, \min\left\{\left(R - \frac{B}{\Delta p}\right), \frac{1}{1-p_h}\left(\frac{1}{1-p_h}\left(R - \frac{B}{\Delta p}\right) - \frac{p_l B}{(\Delta p)^2}\right), \frac{1}{2p_h - 1}\left(\frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right)\right)\right\}\right\}.$$

If

$$(1-p_h)\frac{p_l B}{(\Delta p)^2} \leq \left(R - \frac{B}{\Delta p}\right) \leq \frac{(1-p_h)}{p_h(2-p_h)}\frac{p_l B}{(\Delta p)^2}$$

then

$$\hat{K} = \frac{1}{1-p_h}\left(\frac{1}{1-p_h}\left(R - \frac{B}{\Delta p}\right) - \frac{p_l B}{(\Delta p)^2}\right) \quad \text{and} \quad K_g \quad = \frac{p_h}{1-p_h}\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{1-p_h}\left(R - \frac{B}{\Delta p}\right)\right) .$$

If

$$\frac{(1-p_h)}{p_h(2-p_h)}\frac{p_l B}{(\Delta p)^2} \leq \left(R - \frac{B}{\Delta p}\right) \leq \frac{1}{2p_h}\frac{p_l B}{(\Delta p)^2}$$

then

$$\hat{K} = \left(R - \frac{B}{\Delta p}\right) \quad \text{and} \quad K_g = \frac{p_l B}{(\Delta p)^2} - 2p_h\left(R - \frac{B}{\Delta p}\right) .$$

If

$$\frac{1}{2p_h}\frac{p_l B}{(\Delta p)^2} \leq \left(R - \frac{B}{\Delta p}\right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{(2p_h - 1)}{(1-p_h)}\frac{r}{p_h} ,$$

then

$$\hat{K} = \frac{1}{2p_h - 1}\left(\frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right)\right) \quad \text{and} \quad K_g = 0 .$$

3) If the individual rationality constraint of the investors and the incentive compatibility constraint of the government bind, then

$$\hat{R}_b = (1-p_h)\left(\left(R - \frac{B}{\Delta p}\right) + \frac{r}{1-p_h} - \frac{p_l B}{(\Delta p)^2} - (1-p_h)\hat{K}\right) ,$$

which implies

$$K_g = r - p_h\left(\left(R - \frac{B}{\Delta p}\right) - \frac{p_l B}{(\Delta p)^2} + p_h \hat{K}\right) .$$

To satisfy the feasibility constraint on K_g and $\hat{R}_b$, using that $\left(R - \frac{B}{\Delta p}\right) < \frac{p_l B}{(\Delta p)^2} - \frac{(2p_h - 1)}{p_h(1-p_h)} r$, it has to be the case that

$$\frac{1}{1-p_h}\left(\left(R - \frac{B}{\Delta p}\right) + \frac{(2p_h - 1)}{p_h(1-p_h)} r - \frac{p_l B}{(\Delta p)^2}\right) \leq \hat{K} \leq \frac{1}{1-p_h}\left(\left(R - \frac{B}{\Delta p}\right) + \frac{r}{1-p_h} - \frac{p_l B}{(\Delta p)^2}\right) .$$

Note that if $2p_h > 1$, $\left(R - \frac{B}{\Delta p}\right) \leq \frac{1}{1-p_h}\left(\left(R - \frac{B}{\Delta p}\right) + \frac{r}{1-p_h} - \frac{p_l B}{(\Delta p)^2}\right)$ and it can't be the case that bot constraints bind at the same time unless the inequality above holds with equality .

If $2p_h < 1$, since K_g is decreasing in $\hat{K}$ and it must be the case that $0 \leq \hat{K} \leq \frac{r}{p_h(1-p_h)}$ and $K_g \geq 0$, we have

$$\hat{K} = \frac{1}{1-p_h}\left(\left(R - \frac{B}{\Delta p}\right) + \frac{r}{1-p_h} - \frac{p_l B}{(\Delta p)^2}\right), \hat{R}_b = 0\,,$$

and

$$K_g = r - \frac{p_h}{1-p_h}\left(\left(R - \frac{B}{\Delta p}\right) + \frac{r}{1-p_h} - \frac{p_l B}{(\Delta p)^2}\right)$$

when

$$\frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} \leq \left(R - \frac{B}{\Delta p}\right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{(2p_h - 1)}{p_h(1-p_h)} r\,.$$

Note that the individual rationality constraint of the government will be satisfied as long as $\left(R - \frac{B}{\Delta p}\right) \geq \frac{r}{p_h}$.

Proof of Proposition 7 (RO vs. GO)

a. Using that $\underline{\Gamma} = \frac{r}{p_h}$ and Eq. (7) it follows that the project is not undertaken even in the absence of moral hazard when $\left(R - \frac{B}{\Delta p}\right) < \underline{\Gamma}$.

b. From the analysis above, it follows that if $\overline{\Gamma} \equiv (1-p_h)\frac{p_l B}{(\Delta p)^2} > \left(R - \frac{B}{\Delta p}\right)$ there is no feasible contract that satisfies the incentive compatibility of the government and the individual rationality constraint of the investors and the projects are not financed in the presence of moral hazard.

c. If $\overline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right)$, the projects are financed.

i) If $2p_h > 1$, from the analysis above it follows that the optimal cross-guarantee satisfies

$$\hat{K} = \min\left\{\left(R - \frac{B}{\Delta p}\right), \frac{1}{1-p_h}\left(\frac{1}{1-p_h}\left(R - \frac{B}{\Delta p}\right) - \frac{p_l B}{(\Delta p)^2}\right), \frac{1}{2p_h - 1}\left(\frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right)\right)\right\},$$

which is greater than 0 for $\overline{\Gamma} < \left(R - \frac{B}{\Delta p}\right)$. Then, cross-guarantees are always positive and general obligation financing is always preferred in this case.

ii) If $2p_h < 1$, from the analysis above is follows that the optimal cross guarantees will be positive if

$$\frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} < \left(R - \frac{B}{\Delta p}\right)$$

and revenue only financing is preferred if

$$(1-p_h)\frac{p_l B}{(\Delta p)^2} \leq \left(R - \frac{B}{\Delta p}\right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h}\,.$$

d. If $2p_h > 1$ and

$$\overline{\Gamma}^* \equiv \frac{1}{2p_h}\frac{p_l B}{(\Delta p)^2} \leq \left(R - \frac{B}{\Delta p}\right),$$

we have $K_g = 0$ and the projects are self-financing when cross-guarantees are chosen optimally.

If $2p_h \leq 1$, $K_g = 0$ and the projects are self-financing when cross-guarantees are chosen optimally if

$$\underline{\Gamma}^* \equiv \frac{p_l B}{(\Delta p)^2} - \frac{(2p_h - 1)}{p_h(1-p_h)} r \leq \left(R - \frac{B}{\Delta p}\right).$$

ONLINE APPENDIX

A Characterization of optimal financial contract

In this section we provide a detailed characterization of the optimal financial contract for the model presented in Section 5.1, which nests the model in Section 3 by setting $D = 0$.

The optimal financing contract solves

$$\max_{I_g \in \{0, K_0\}, I_f \geq 0, K_g \geq 0, R_f \geq 0, D_f \in [0, D]} \left(p_h (R + X + D) - r\right)\left(I_g + I_f\right)$$

subject to

$$(1 - p_h) K_g + p_h \left(R_f + D_f\right) \geq r \frac{I_f}{I_f + I_g}, \tag{IRP}$$

$$(1 - p_h) K_g + p_h \left(R_f + D_f\right) + r \frac{I_g}{I_f + I_g} \leq p_h \left(R - \frac{B}{\Delta p}\right) + p_h (X + D), \tag{IRG}$$

$$K_g - R_f \geq \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}\left(D - D_f\right) - \left(R - \frac{B}{\Delta p}\right) - X, \tag{ICG}$$

$$K_g \left(I_f + I_g\right) \leq \min\left\{\Phi, \overline{K}_1 + \overline{K}_0 - I_g\right\}, \quad \text{and} \tag{NDK}$$

$$R_f \left(I_f + I_g\right) \leq \min\left\{\Phi, \left(R - \frac{B}{\Delta p}\right)\left(I_f + I_g\right) + \overline{K}_1 + \overline{K}_0 - I_g\right\}, \tag{NDR}$$

$$\tag{8}$$

where IRP is the individual rationality constraint of the financiers, IRG is the individual rationality constraint of the government, ICG is the incentive compatibility constraint of the government, and NDK and NDR combine the no-default conditions for the government and the feasibility constraints on government guarantees and promised returns. Note that to have IRG and IRP satisfied at the same time it has to be the case that

$$0 \leq p_h \left(R - \frac{B}{\Delta p}\right) + p_h (X + D) - r.$$

Moreover, the no-default conditions impose upper bounds on the total scale of the infrastructure project. Therefore, since the infrastructure project is positive NPV, either the constraint NDK or the constraint NDR or both have to bind, which implies

$$I = \min\left\{\frac{\min\left\{\Phi, \left(\left(R - \frac{B}{\Delta p}\right) + \left(D - D_f\right)\right) I + \overline{K}_1 + \overline{K}_0 - I_g\right\}}{R_f}, \frac{\min\left\{\Phi, \overline{K}_1 + \overline{K}_0 - I_g\right\}}{K_g}\right\}.$$

To characterize the optimal financing contract one needs to consider three possible cases, that depend on which constraints are binding. Below, we consider each case separately and characterize the parameter regions in which they are relevant.

Case 1: ICG binds and IRP and IRG are slack

If ICG binds, the government guarantees are given by

$$K_g = \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}\left(D - D_f\right) - \left(R - \frac{B}{\Delta p}\right) - X + R_f$$

and total investment is given by

$$I = \min\left\{ \frac{\min\left\{\Phi, \left(\left(R-\frac{B}{\Delta p}\right)+\left(D-D_f\right)\right)I+\overline{K}_1+\overline{K}_0-I_g\right\}}{R_f}, \frac{\min\left\{\Phi, \overline{K}_1+\overline{K}_0-I_g\right\}}{\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}\left(D-D_f\right)-\left(R-\frac{B}{\Delta p}\right)-X+R_f} \right\},$$

which is decreasing in R_f, D_f, and I_g. Then, it is optimal to set R_f, D_f, and I_g as small as possible to maximize the size of the investment. More specifically, it is optimal to set $R_f^\star = 0$, $D_f^\star = 0$, and $I_g^\star = 0$, which implies

$$K_g^\star = \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p} D - \left(R - \frac{B}{\Delta p}\right) - X \quad \text{and} \quad I^\star = \frac{\min\left\{\Phi, \overline{K}_1+\overline{K}_0\right\}}{\frac{p_l B}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X}.$$

Finally, to satisfy the IRP and IRG it has to be the case that $\overline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right) \leq \Gamma_I$, where

$$\overline{\Gamma} \equiv \max\left\{\frac{r}{p_h} - X - D, (1-p_h)\frac{p_l B}{(\Delta p)^2} - p_h \frac{(1-p_l)}{\Delta p} D - X\right\} \quad \text{and} \tag{9}$$

$$\Gamma_I \equiv \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p} D - X - \frac{r}{1-p_h}. \tag{10}$$

Note that in this region, we have $K_g^\star > 0$.

Case 2: ICG and IRP bind

First, note that if IRP binds, IRG is given by

$$R_f^\star = r\frac{I^\star - I_g^\star}{I^\star} - p_h D_f^\star - (1-p_h)\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}\left(D - D_f^\star\right) - \left(R - \frac{B}{\Delta p}\right) - X\right) \tag{11}$$

and

$$K_g^\star = r\frac{I^\star - I_g^\star}{I^\star} - p_h D_f^\star + p_h\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}\left(D - D_f^\star\right) - \left(R - \frac{B}{\Delta p}\right) - X\right). \tag{12}$$

In this case, the total scale of the project is given by

$$I^\star = \min\left\{\frac{\min\left\{\Phi, \left(\left(R-\frac{B}{\Delta p}\right)+\left(D-D_f^\star\right)\right)I^\star+\overline{K}_1+\overline{K}_0-I_g^\star\right\}}{R_f^\star}, \frac{\min\left\{\Phi, \overline{K}_1+\overline{K}_0-I_g^\star\right\}}{K_g^\star}\right\}, \tag{13}$$

where $I_g^\star$ and $D_f^\star$ are constrained by the non-negativity constraints on K_g and R_f. Since $I^\star$ is increasing in $I_g^\star$, it is optimal to set $I_g^\star$ as high as possible conditional on satisfying these constraints. Since $I^\star$ is decreasing in $R_f^\star$, it is optimal to set $I_g^\star$ such that $R_f^\star = 0$ as long as $I_g^\star < \overline{K}_0$. Setting $R_f^\star = 0$ in Equation (11) implies

$$r - p_h D_f^\star + (1-p_h)\left(\left(R - \frac{B}{\Delta p}\right) + X - \frac{p_l B}{(\Delta p)^2} + \frac{p_h}{\Delta p}\left(D - D_f^\star\right)\right) = r\frac{I_g^\star}{I^\star}. \tag{14}$$

If the scale of the project is given by $K_g^\star$, i.e., $I = \frac{\min\left\{\Phi, \overline{K}_1 + \overline{K}_0 - I_g\right\}}{K_g}$ we have

$$I^\star = \frac{\min\left\{\Phi, \overline{K}_1 + \overline{K}_0 - I_g^\star\right\} + rI_g^\star}{\left(r - p_h D_f^\star + p_h\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}\left(D - D_f^\star\right) - \left(R - \frac{B}{\Delta p}\right) - X\right)\right)},$$

which together with Equation (14) gives

$$I^\star = \frac{\min\left\{\Phi, \overline{K}_1 + \overline{K}_0 - I_g^\star\right\}}{\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}\left(D - D_f^\star\right) - \left(R - \frac{B}{\Delta p}\right) - X\right)}, \tag{15}$$

where $I_g^\star$ is given by the solution to

$$\frac{rI_g^\star}{\min\left\{\Phi, \overline{K}_1 + \overline{K}_0 - I_g^\star\right\}} = \frac{\left(r - p_h D_f^\star - (1 - p_h)\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}\left(D - D_f^\star\right) - \left(R - \frac{B}{\Delta p}\right) - X\right)\right)}{\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}\left(D - D_f^\star\right) - \left(R - \frac{B}{\Delta p}\right) - X\right)}. \tag{16}$$

Note that $I^\star$ in the Equation (15) is decreasing in $D_f^\star$. Therefore, it is optimal to set $D_f^\star = 0$ as long as $I_g^\star \leq \overline{K}_0$.

Solving for $I_g^\star$ in Equation (16) setting $D_f^\star = 0$ implies

$$I_g^\star = \begin{cases} \frac{\left(r-(1-p_h)\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}D - \left(R - \frac{B}{\Delta p}\right) - X\right)\right)}{\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}D - \left(R - \frac{B}{\Delta p}\right) - X\right)} \frac{\Phi}{r} & \text{if } \left(R - \frac{B}{\Delta p}\right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}D - X - \frac{r\Phi}{\left(r\left(\overline{K}_1 + \overline{K}_0\right) - (r-(1-p_h))\Phi\right)} \\ \frac{\left(r-(1-p_h)\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}D - \left(R - \frac{B}{\Delta p}\right) - X\right)\right)}{\left(\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}D - \left(R - \frac{B}{\Delta p}\right) - X\right)(r-(1-p_h)) + r\right)} \left(\overline{K}_1 + \overline{K}_0\right) & \text{if } \left(R - \frac{B}{\Delta p}\right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}D - X - \frac{r\overline{K}_1}{\left(r\overline{K}_0 + (1-p_h)\overline{K}_1\right)} \\ & \text{and } \left(R - \frac{B}{\Delta p}\right) > \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}D - X - \frac{r\Phi}{\left(r\left(\overline{K}_1 + \overline{K}_0\right) - (r-(1-p_h))\Phi\right)} \end{cases} \tag{17}$$

where the second case is only feasible when $\Phi > \overline{K}_1$. Since $I_g^\star \leq \overline{K}_0$, we have that $D_f^\star = 0$ if and only if $\left(R - \frac{B}{\Delta p}\right) \leq \Gamma_R$, where

$$\Gamma_R \equiv \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}D - X - \frac{r \min\left\{\overline{K}_1, \Phi\right\}}{\left(r\left(\overline{K}_1 + \overline{K}_0\right) - (r - (1 - p_h))\min\left\{\overline{K}_1, \Phi\right\}\right)}. \tag{18}$$

In this region, $R_f^\star = 0$ and investment is positive as long as $\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}D - X \geq \left(R - \frac{B}{\Delta p}\right)$. If $\left(R - \frac{B}{\Delta p}\right) \geq \Gamma_R$, then $I_g^\star = \overline{K}_0$, $D_f^\star = 0$ and $R_f^\star$ is given by

$$R_f^\star = r - r\frac{\overline{K}_0}{I^\star} - (1 - p_h)\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}D - \left(R - \frac{B}{\Delta p}\right) - X\right),$$

where

$$I^\star = \frac{\min\left\{\Phi, \overline{K}_1\right\} + r\overline{K}_0}{\left(r + p_h\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p}D - \left(R - \frac{B}{\Delta p}\right) - X\right)\right)}.$$

Putting this two equations together gives

$$R_f^\star = r\frac{\min\left\{\Phi,\overline{K}_1\right\}}{\min\left\{\Phi,\overline{K}_1\right\}+r\overline{K}_0} - \left(\frac{r\overline{K}_0+(1-p_h)\min\left\{\Phi,\overline{K}_1\right\}}{\min\left\{\Phi,\overline{K}_1\right\}+r\overline{K}_0}\right)\left(\frac{p_lB}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-\left(R-\frac{B}{\Delta p}\right)-X\right),$$

which will satisfy ND-2, the willingness-to-pay and ability-to-pay constraints for R_f as long as $\left(R-\frac{B}{\Delta p}\right)\leq\Gamma_D$, where

$$\begin{aligned}\Gamma_D \equiv \max\{ & \frac{p_lB}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-X+\frac{\Phi-\min\left\{\Phi,\overline{K}_1\right\}}{\left(r\overline{K}_0+(1-p_h)\min\left\{\Phi,\overline{K}_1\right\}+p_h\Phi\right)}r, \\ & \frac{\Phi-\overline{K}_1}{\overline{K}_1+r\overline{K}_0+p_h\left(\Phi-\overline{K}_1\right)}\left(r+p_h\left(\frac{p_lB}{(\Delta p)^2}-\frac{p_h}{\Delta p}D-X\right)\right)-\frac{\left(\overline{K}_1+r\overline{K}_0\right)}{\left(\overline{K}_1+r\overline{K}_0+p_h\left(\Phi-\overline{K}_1\right)\right)}D\},\end{aligned} \tag{19}$$

which implies $I^\star\geq 0$ and $K_g^\star>0$.

If $\left(R-\frac{B}{\Delta p}\right)>\Gamma_D$, we have $I_g^\star=\overline{K}_0$ and the no-default conditions for R_f and K_g bind. In this case, $D_f^\star$ is given by the solution to

$$\frac{\min\left\{\Phi,\left(\left(R-\frac{B}{\Delta p}\right)+\left(D-D_f^\star\right)\right)I^\star+\overline{K}_1+\overline{K}_0-I_g^\star\right\}}{R_f^\star}=\frac{\min\left\{\Phi,\overline{K}_1+\overline{K}_0-I_g^\star\right\}}{K_g^\star}, \tag{20}$$

where $R_f^\star$ is given by Equation (11) and $K_g^\star$ is given by Equation (12). This holds as long as $\left(R-\frac{B}{\Delta p}\right)\leq\Gamma^\star$, where

$$\Gamma^\star\equiv\max\left\{\frac{\left(\Phi-\overline{K}_1\right)}{\left(\left(\Phi-\overline{K}_1\right)p_h+r\overline{K}_0+\overline{K}_1\right)}\left(r-p_hD\right)+\frac{p_lB}{(\Delta p)^2}-X,\frac{p_lB}{(\Delta p)^2}-X\right\}. \tag{21}$$

At $\left(R-\frac{B}{\Delta p}\right)=\Gamma^\star$, there are four constraints binding: IRP, ICG and the no-default constraints on $R_f^\star$ and $K_g^\star$, and we have $D_f^\star=D$.

Case 3: Only IRP binds

If $\left(R-\frac{B}{\Delta p}\right)>\Gamma^\star$, then IRP binds and ICG is slack. In this case

$$I^\star=\min\left\{\frac{\min\left\{\Phi,\left(\left(R-\frac{B}{\Delta p}\right)+\left(D-D_f^\star\right)\right)I^\star+\overline{K}_1+\overline{K}_0-I_g\right\}}{R_f^\star},\frac{(1-p_h)\left(\min\left\{\Phi,\overline{K}_1\right\}+\overline{K}_0-I_g\right)+rI_g}{r-p_h\left(R_f^\star+D_f\right)}\right\}$$

so it is optimal to choose $D_f^\star=D$, $I_g^\star=\overline{K}_0$ and $R_f^\star$ such that the no-default conditions for $R_f^\star$ and $K_g^\star$ are binding, i.e.,

$$\min\left\{\Phi,\left(R-\frac{B}{\Delta p}\right)I^\star+\overline{K}_1\right\}\left(r-p_h\left(R_f^\star+D\right)\right)=\left((1-p_h)\min\left\{\Phi,\overline{K}_1\right\}+r\overline{K}_0\right)R_f^\star.$$

Note that $\Phi<\left(R-\frac{B}{\Delta p}\right)I^\star+\overline{K}_1$. Otherwise, we would have $\Phi>\overline{K}_1$ and

$$R_f^\star=\frac{\overline{K}_1\left(r-p_hD\right)+\left((1-p_h)\overline{K}_1+r\overline{K}_0\right)\left(R-\frac{B}{\Delta p}\right)}{\left(\overline{K}_1+r\overline{K}_0\right)},$$

which would imply $\Phi < \left(R - \frac{B}{\Delta p}\right) I^\star + \overline{K}_1$, contradicting our initial assumption.

Therefore, when $\left(R - \frac{B}{\Delta p}\right) \geq \Gamma^\star$ it has to be the case that $\Phi < \left(R - \frac{B}{\Delta p}\right) I^\star + \overline{K}_1$ and we have

$$\begin{aligned}
R_f^\star &= \frac{\Phi}{r\overline{K}_0 + (1-p_h)\min\left\{\Phi, \overline{K}_1\right\} + p_h\Phi}(r - p_h D), \\
K_g^\star &= \frac{\min\left\{\Phi, \overline{K}_1\right\}}{r\overline{K}_0 + (1-p_h)\min\left\{\Phi, \overline{K}_1\right\} + p_h\Phi}(r - p_h D), \quad \text{and} \\
I^\star &= \frac{r\overline{K}_0 + (1-p_h)\min\left\{\Phi, \overline{K}_1\right\} + p_h\Phi}{(r - p_h D)}.
\end{aligned}$$

To be in this case we need $\Phi < \left(R - \frac{B}{\Delta p}\right) I^\star + \overline{K}_1$ which is the same as

$$\frac{\left(\Phi - \overline{K}_1\right)}{r\overline{K}_0 + (1-p_h)\min\left\{\Phi, \overline{K}_1\right\} + p_h\Phi}(r - p_h D) < \left(R - \frac{B}{\Delta p}\right).$$

This will be the case as long as ICG is satisfied, which is the same as

$$\left(R - \frac{B}{\Delta p}\right) \geq \frac{p_l B}{(\Delta p)^2} - X + \left(\frac{\Phi(1-p_h) - \min\left\{\Phi, \overline{K}_1\right\}}{r\overline{K}_0 + (1-p_h)\min\left\{\Phi, \overline{K}_1\right\} + p_h\Phi}\right)\frac{1}{1-p_h}(r - p_h D).$$

B Development Rights and General Obligation Financing

We further explore the choice between general obligation and revenue only financing in the presence of development rights. Formally, we consider the same model as in Section 6 with the addition that each project i generates an additional payoff DI^i with $D > 0$ that is accrued if the project is successful, which we refer to as development rights. As in Section 5.1, this payoff can only be distributed to the financiers and the private sector operator and cannot be extorted by the government. We denote by $D_I^i I^i$ the portion of the payoff from the development rights that is assigned to the financiers in project i. The residual, $\left(D - D_I^i\right) I^i$, is distributed to the private sector operator of project i. With respect to the model with general obligation financing, development rights affect the incentive compatibility constraint of the financiers, which in turn changes the maximum amount that the government can extort from the private sector operator while still inducing high effort; development rights also affect the individual rationality constraint of the financiers.

The incentive compatibility constraint of the private sector operator in project i now takes the form:

$$p_h\left[R - \left(R_I^i + R_g^i\right) + \hat{R} - D_I^i\right] I \;\geq\; p_l\left[R - \left(R_I^i + R_g^i\right) + \hat{R} - D_I^i\right] I + BI\,, \tag{ICP–GO-DR}$$

or,

$$\left(R - \frac{B}{\Delta p}\right) - R_I^i + \left(D - D_I^i\right) \;\geq\; \overline{R}_g^i\,.$$

Since the government cannot extort any proceeds from the development rights, the maximum payoff for the government from project i is

$$\overline{R}_g^i \;=\; \left(R - \frac{B}{\Delta p}\right) - R_I^i + \left(D - D_I^i\right)\,.$$

The individual rationality constraint of the financiers in project i now becomes

$$p_h\left(R_I^i + D_I^i\right) + (1-p_h)K_g^i + (1-p_h)\,p_h K^i \geq r \tag{IRP–GO-DR}$$

or,

$$K_g^i \geq \frac{r - p_h\left(R_I^i + D_I^i\right)}{1 - p_h} - p_h K^i \,.$$

We then obtain the following result on how development rights affect infrastructure financing in the presence of cross-guarantees.

Proposition 8. ***(RO vs. GO with Development Rights)*** *Development rights ($D > 0$) reduce the inefficiencies imposed by double moral hazard, i.e., they increase the scale of the projects and the parameter space in which the projects are financed, even in the presence of cross-guarantees. Moreover, when the quality of the projects is low, i.e., $2p_h < 1$, the parameter region over which general obligation financing is strictly preferred is increasing in the value of the development rights, D.*

Proposition 8 shows that, as in the benchmark model in Section 3, development rights decrease the inefficiencies imposed by the double moral hazard by increasing the scale of the projects and expanding the set of parameters under which the projects are undertaken. Moreover, if the quality of the projects is low, i.e., $2p_h < 1$, their presence can affect whether infrastructure financing involves positive cross-guarantees. In particular, development rights increase the parameter region over which projects are financed with general obligation financing (positive cross-guarantees). Intuitively, development rights ameliorate the government's moral hazard problem on each project, and in turn, increase the size of the cross-guarantees that can be provided.

C Early-stage Government Moral Hazard

We extend the benchmark model to consider infrastructure projects with multiple stages. In the first "early" stage, the project requires government "input", which can represent project approval, land acquisition, clearance of existing properties on the land, provision of public utilities, etc. In the second stage, once the project has gone past the government input stage, the private sector can shape the quality of the project based on its own inputs. As in the benchmark model, the government can extort the cash flows from the operator.

In this case, there are two instances in which the project can fail, after the government input stage or after the private sector operator's input stage. The government may offer guarantees to the financiers in the event the project fails after each of these instances. These guarantees expose the government to the risk of project failure and potentially ameliorate the government moral hazard problems in the two stages. As in the model considering GO and RO financing, we abstract from direct government investment by setting $\overline{K}_0$ to focus on how the double moral hazard affects the financing of the infrastructure project.

Formally, in the first stage, the government can affect the probability e of the project's success through its input. If the government input is high, the project succeeds with probability $e_h \in (0,1)$, else it succeeds with probability $e_l, \quad 0 < e_l < e_h$. We denote as Δe the difference in these probabilities, that is, $\Delta e \equiv (e_h - e_l)$. If the government does not provide the high input, the associated officials are assumed to derive a non-pecuniary private benefit of bI with $b > 0$. In case the project fails in the first stage, it has no further chance of success and its payoff is zero. If the first stage of the project does not fail, the model is exactly the same as the benchmark model. The private sector can affect the probability of success of the project by exerting effort and the government can extort the project's cash flows once they are realized at the end of the second stage.

We denote by $K_g^e I$ and $K_g^p I$ the government guarantees if the project fails after the first and the second stages, respectively. As before, the size of these guarantees is constrained by the fiscal capacity of the government, which we take to be fixed at $\overline{K}$. Since either the first-stage guarantee or the second-stage guarantee is paid but not both, the fiscal constraint is

$$\max\left\{K_g^e, K_g^p\right\} I \leq \overline{K}. \qquad \text{(Fiscal-constraint–GI)}$$

The state space of outcomes for the projects, and project payoffs as well as payoffs to various parties (the private sector operator, the private financiers, and the government) are summarized in Figure A.1.

The presence of the first-stage government moral hazard reduces the parameter space in which the project is financed. Moreover, the scale of the project depends on the highest guarantee offered by the government, which in

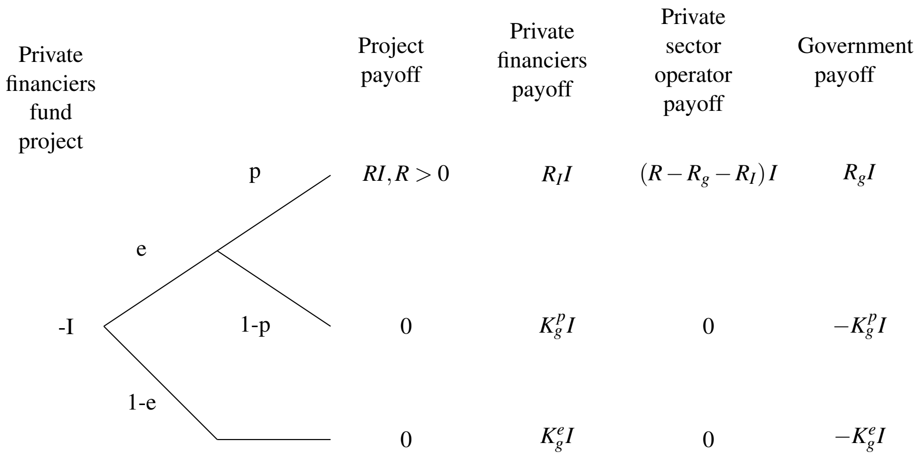


Figure A.1: State space of possible outcomes and corresponding payoffs for the economic agents in the model with government moral hazard in the first stage.

turn depends on the relative severity of the government's moral hazard in the two stages. When the government's moral hazard in the first stage, measured by $\frac{b}{p_h \Delta e}$, is more (less) severe than that in the second stage, measured by $\frac{p_l B}{(\Delta p)^2}$, it is harder (easier) to incentivize the government to provide high input than to incentivize it not to extort. In this case, the punishment for failing earlier needs to be higher (lower) than the one for failing in the second stage which is achieved by having the first-stage guarantee be larger (smaller) than the second-stage one. When the return of the project is high enough, the government gets a high enough return to incentivize it to provide high input and not to extort, and the government guarantees are determined by the participation constraint of the financiers. In this case, the guarantees in the first and second stage are equal to maximize the scale of the project while satisfying the financiers' participation constraint. The Online Appendix formally characterizes the optimal contract and the optimal guarantee structure.

Figure A.2 shows the optimal government guarantees. Panels a and b respectively show the optimal guarantees for the cases in which the moral hazard of the government in the first stage is more severe than its moral hazard in the second stage and vice versa.

D Proofs

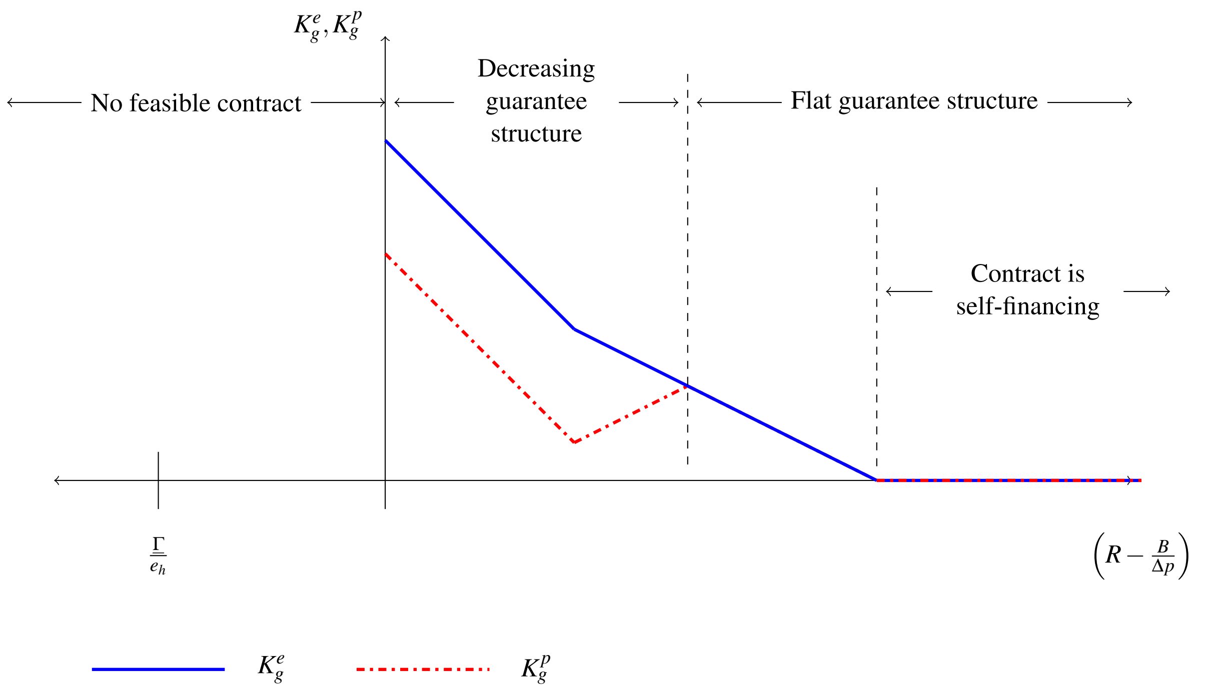
Proof of Proposition 8

Development rights decrease the outside option of investors by $p_h D_I$ and decrease the moral hazard of the private sector operators by $p_h (D - D_I)$, where $D_I \in [0, D]$. Analogous to the analysis of development rights in Section 5.1, it is optimal to set $D_I = 0$ if only the incentive compatibility constraint of the government binds and $D_I = D$ if only the individual rationality constraint of the investors binds. Then, using the results in the previous section we have that

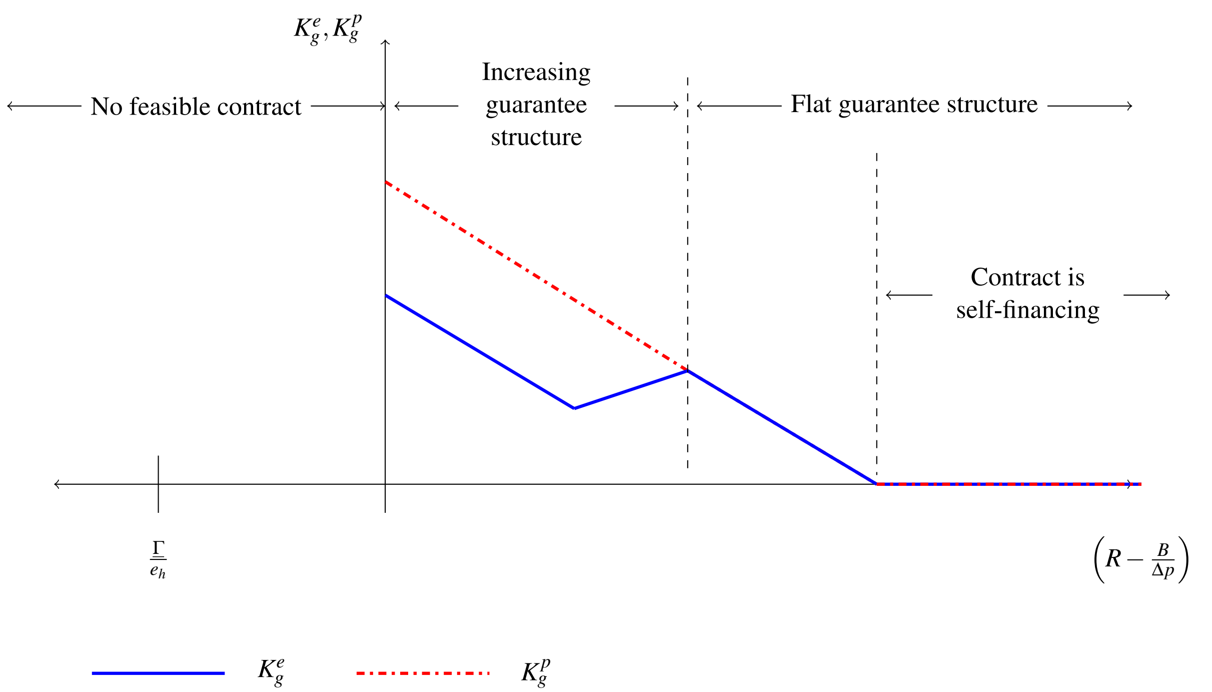
a. If $\left(R - \frac{B}{\Delta p}\right) < \underline{\Gamma} - D$ then there are no contracts that satisfy the individual rationality constraint of the investors and the government at the same time and the projects are not funded even in the absence of moral hazard.

b. From the analysis above, it follows that if $\left(R - \frac{B}{\Delta p}\right) < \overline{\Gamma} - (1 - p_h) \frac{p_h D}{\Delta p}$ there is no feasible contract that satisfies the incentive compatibility of the government and the individual rationality constraint of the investors and the projects are not financed in the presence of moral hazard.

c. If $\overline{\Gamma} - (1 - p_h) \frac{p_h D}{\Delta p} \leq \left(R - \frac{B}{\Delta p}\right)$, the projects are financed.



(a) Case 1: Government guarantee structure when first-stage moral hazard is more severe than the second-stage moral hazard, i.e., $\frac{b}{p_h \Delta e} \geq \frac{p_l B}{(\Delta p)^2}$.



(b) Government guarantee structure when second-stage moral hazard is more severe than the first-stage moral hazard, i.e., $\frac{b}{p_h \Delta e} \leq \frac{p_l B}{(\Delta p)^2}$.

Figure A.2: Optimal guarantee structure as a function of $\left(R - \frac{B}{\Delta p}\right)$.

i) If $2p_h > 1$, $\hat{K} > 0$ for

$$\overline{\Gamma} - (1-p_h)\frac{p_h D}{\Delta p} < \left(R - \frac{B}{\Delta p}\right).$$

Then, cross-guarantees are always positive and general obligation financing is always preferred in this case.

ii) If $2p_h < 1$, the optimal cross guarantees will be positive if

$$\frac{p_l B}{(\Delta p)^2} - \frac{p_h D}{\Delta p} - \frac{r}{1-p_h} < \left(R - \frac{B}{\Delta p}\right)$$

and revenue only financing is preferred if

$$(1-p_h)\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h D}{\Delta p}\right) \leq \left(R - \frac{B}{\Delta p}\right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{p_h D}{\Delta p} - \frac{r}{1-p_h}.$$

d. i) If $2p_h > 1$, $K_g = 0$ if

$$\overline{\Gamma}^* \equiv \frac{1}{2p_h}\left(\frac{p_l B}{(\Delta p)^2} - \frac{p_h D}{\Delta p}\right) \leq \left(R - \frac{B}{\Delta p}\right),$$

and the projects are self-financing when cross-guarantees are chosen optimally.

ii) If $2p_h < 1$, $K_g = 0$ and the projects are self-financing when cross-guarantees are chosen optimally if

$$\underline{\Gamma}^* \equiv \frac{p_l B}{(\Delta p)^2} - \frac{p_h D}{\Delta p} - \frac{(2p_h - 1)}{p_h(1-p_h)} r \leq \left(R - \frac{B}{\Delta p}\right).$$

Therefore, development rights increase the scale of the project and the parameters space in which the project is financed. Moreover, development rights increase the parameter region over which general obligation financing is strictly preferred.

□

D.1 Proofs: Early-stage Government Moral Hazard

In this section we formalize the analysis of the model presented in Section C. We first characterize the constraints faced by the government and private sector when the government faces moral hazard in an early stage and then characterize the optimal financing contract and provide comparative statics for the government guarantees.

Constraints with first-stage government moral hazard

The benchmark model in Section 3 is only played if the first stage succeeds. Therefore, the incentive compatibility constraints of the private sector and the government in the extortion stage remain unchanged. However, the input decision by the government in the first stage imposes an additional incentive compatibility constraint given by

$$e_h p_h \overline{R}_g - (1-e_h) K_g^e - e_h(1-p_h) K_g^p \geq b + e_l p_h \overline{R}_g - (1-e_l) K_g^e - e_l(1-p_h) K_g^p \qquad \text{(ICG–GI-1)}$$

or, using that $\overline{R}_g = \left(R - \frac{B}{\Delta p}\right) - R_I$,

$$K_g^e \geq p_h\left[\frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p}\right) - R_I\right] + (1-p_h) K_g^p.$$

If the government provides a high input, it gets a return $\overline{R}_g$ if neither stage of the project fails, which occurs with probability $e_h p_h$, and pays guarantees if the project fails in either stage. The project fails in the first stage with probability $(1-e_h)$ and then the government pays K_g^e in guarantees per unit of investment; it fails with probability $e_h(1-p_h)$ in the second stage and, in this case, the government pays K_g^p in guarantees per unit of investment. If the government decides not to provide the high input, it gets a private benefit b and the probability of success in the first stage is e_l.

Note that the first and second-period guarantees have opposite effects on the government's incentives to provide input. On one hand, a higher first-stage guarantee increases the penalty for the government if the project fails in the first stage and thus increases the government's incentives to provide high input. On the other hand, a higher second-stage guarantee increases the penalty for the government if the project fails in the second stage, which decreases the government's expected payoff of the project upon succeeding in the first stage. Therefore, a higher second-stage guarantee exacerbates the government's moral hazard in the first stage.

The individual rationality constraints of the investors and the government also change to account for the two guarantees. In particular, the individual rationality constraint of the investors becomes

$$rI \leq (1-e_h)K_g^e I + e_h(1-p_h)K_g^p I + e_h p_h R_I I\,, \tag{IRP–GI}$$

or

$$r \leq (1-e_h)K_g^e + e_h(1-p_h)K_g^p + e_h p_h R_I I\,.$$

Finally, the individual rationality constraint of the government is

$$0 \leq e_h p_h \overline{R}_g I - (1-e_h)K_g^e I - e_h(1-p_h)K_g^p I\,, \tag{IRG–GI}$$

or using that $\overline{R}_g = \left(R - \frac{B}{\Delta p}\right) - R_I$,

$$(1-e_h)K_g^e + e_h(1-p_h)K_g^p + e_h p_h R_I \leq e_h p_h \left(R - \frac{B}{\Delta p}\right).$$

Characterization of optimal contract

When the government also faces moral hazard in the first stage, the contract solves the following problem

$$\max_{K_g^e, K_g^p \geq 0} (e_h p_h R - r)\frac{\overline{K}}{\max\left\{K_g^e, K_g^p\right\}}$$

subject to

$$(1-e_h)K_g^e + e_h(1-p_h)K_g^p + e_h p_h R_I \geq r\,, \tag{IRP–GI}$$

$$(1-e_h)K_g^e + e_h(1-p_h)K_g^p + e_h p_h R_I \leq e_h p_h \left(R - \frac{B}{\Delta p}\right), \tag{IRG–GI}$$

$$K_g^e \geq p_h\left(\frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p}\right)\right) + (1-p_h)K_g^p + p_h R_I\,, \text{ and} \tag{ICG–GI1}$$

$$K_g^p \geq \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) + R_I\,. \tag{ICG–GI2}$$

The iso-curves of the objective function have a minimum at $K_g^e = K_g^p$ and grow towards the origin. Then, the solution to the problem above will be a corner solution within the set of $\left\{K_g^e, K_g^p\right\}$ that satisfies the four constraints above. There are four relevant cases to be considered, depending on which constraints bind.

First, note that to have a contract that satisfies IRP–GI and IRG–GI it has to be the case that

$$\frac{r}{e_h p_h} \leq \left(r - \frac{B}{\Delta p} \right) .$$

Case 1 Suppose that the individual rationality constraint for the private sector IRP–GI binds and all other constraints are slack. In this case, $K_g^e = K_g^p = \overline{K}_g$, where $\overline{K}_g$ is given by

$$\overline{K}_g = \frac{r - e_h p_h R_I}{1 - e_h p_h}.$$

Then, it is optimal to set $R_b = \frac{r}{e_h p_h}$ and the project is be self-financing with $\overline{K}_g = 0$.

For the incentive compatibility constraints of the government to be satisfied we need the following two conditions to be satisfied:

$$0 \geq \frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p} \right) + \frac{r}{e_h p_h} \quad \text{and} \quad 0 \geq \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p} \right) + \frac{r}{e_h p_h}.$$

Then, $K_g^e = K_g^p = 0$ if

$$R - \frac{B}{\Delta p} \geq \max \left\{ \frac{b}{p_h \Delta e} + \frac{r}{e_h p_h}, \frac{p_l B}{(\Delta p)^2} + \frac{r}{e_h p_h}, \frac{r}{e_h p_h} \right\}.$$

Case 2 Consider the case in which the the individual rationality constraint of the private sector binds and only one of the incentive compatibility constraints of the government binds and all the other constraints are slack.

i) If the moral hazard of the government in the first stage is more severe than the one in the second one, that is, if

$$\frac{p_l B}{(\Delta p)^2} < \frac{b}{p_h \Delta e}$$

and the incentive compatibility constraint for the government in the first stage ICG–GI1 binds, it has to be the case that $K_g^e \geq K_g^p$.

If $K_g^e = K_g^p = \overline{K}_g$, then

$$\begin{aligned} (1 - e_h p_h) \overline{K}_g + e_h p_h R_I &= r \\ p_h \overline{K}_g + p_h \left(R - \frac{B}{\Delta p} \right) - p_h R_I &= \frac{b}{\Delta e} . \end{aligned}$$

and

$$\begin{aligned} \overline{K}_g &= r + e_h p_h \left[\frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p} \right) \right] \quad \text{and} \\ R_I &= r - (1 - e_h p_h) \left[\frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p} \right) \right] . \end{aligned}$$

The non-negativity constraints on K_g and R_I imply this will be the case if

$$\frac{b}{p_h \Delta e} + \frac{r}{1 - e_h p_h} \leq \left(R - \frac{B}{\Delta p} \right) \leq \frac{b}{p_h \Delta e} + \frac{r}{e_h p_h}$$

If $K_g^e > K_g^p$, the government guarantees are given by the solution to

$$\begin{aligned}(1-e_h)\,K_g^e + e_h\,(1-p_h)\,K_g^p + p_h R_I &= r \\ K_g^e + p_h\left(R - \frac{B}{\Delta p}\right) - p_h R_I &= \frac{b}{\Delta e} + (1-p_h)\,K_g^p\ .\end{aligned}$$

Then, K_g^p is given by

$$\begin{aligned}K_g^p &= \frac{r - e_h p_h R_I}{(1-p_h)} - \frac{(1-e_h)\,p_h}{(1-p_h)}\left[\frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p}\right) + R_I\right] \\ &= \frac{r}{(1-p_h)} - \frac{(1-e_h)\,p_h}{(1-p_h)}\left[\frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p}\right)\right] - \frac{p_h}{1-p_h} R_I\end{aligned}$$

and K_g^e is

$$K_g^e = r + e_h p_h\left[\frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p}\right) + R_I\right]\ .$$

In this case, it is optimal to set $R_I = 0$ to maximize the scale of the project which is given by $\frac{K}{K_g^e}$. For both guarantees to be positive it must be the case that

$$\frac{r}{e_h p_h} + \frac{b}{p_h \Delta e} \geq \left(R - \frac{B}{\Delta p}\right) \quad \text{and} \quad \left(R - \frac{B}{\Delta p}\right) \geq \frac{b}{p_h \Delta e} - \frac{r}{(1-e_h)\,p_h}\ .$$

For the incentive constraint of the government in the second stage ICG–GI2 to be satisfied it must be the case that

$$K_g^p \geq \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right)$$

which is the same as

$$\left(R - \frac{B}{\Delta p}\right) \geq \frac{1}{1 - e_h p_h}\left[(1-p_h)\,\frac{p_l B}{(\Delta p)^2} - r + (1-e_h)\,\frac{b}{\Delta e}\right]\ .$$

To have $K_g^p < K_g^e$ it has to be the case that

$$\left(R - \frac{B}{\Delta p}\right) < \frac{b}{p_h \Delta e} - \frac{r}{1 - e_h p_h}\ .$$

Hence, we will be in this case if

$$\max\left\{\frac{r}{e_h p_h}, \frac{b}{p_h \Delta e} - \frac{r}{(1-e_h)\,p_h}, \frac{1}{1-e_h p_h}\left[(1-p_h)\,\frac{p_l B}{(\Delta p)^2} - r + (1-e_h)\,\frac{b}{\Delta e}\right]\right\} \leq R - \frac{B}{\Delta p} < \frac{b}{p_h \Delta e} + \frac{r}{1 - e_h p_h}.$$

ii) If the moral hazard problem of the government in the second stage is more severe than the one in the first stage, that is, if

$$\frac{b}{p_h \Delta e} < \frac{p_l B}{(\Delta p)^2}$$

and the incentive compatibility constraint for the government in the second stage ICG–GI2 binds, it has to be the case that $K_g^e \leq K_g^p$.

If $K_g^e = K_g^p = \overline{K}_g$, then

$$\overline{K}_g = \frac{r - e_h p_h R_I}{(1 - e_h p_h)} \quad \text{and}$$
$$\overline{K}_g = \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) + R_I \,.$$

This implies that

$$\overline{K}_g = r + e_h p_h \left[\frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right)\right] \quad \text{and}$$
$$R_I = r - (1 - e_h p_h)\left[\frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right)\right] \,.$$

The non-negativity constraints on K_g and R_I imply this will be the case if

$$\frac{p_l B}{(\Delta p)^2} + \frac{r}{1 - e_h p_h} \leq \left(R - \frac{B}{\Delta p}\right) \leq \frac{p_l B}{(\Delta p)^2} + \frac{r}{e_h p_h}$$

If $K_g^e < K_g^p$, the contract guarantees are given by the solution to

$$(1 - e_h) K_g^e + e_h (1 - p_h) K_g^p = r - e_h p_h R_I$$
$$K_g^p = \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) + R_I \,,$$

which implies $R_I = 0$ and

$$K_g^e = \frac{1}{(1 - e_h)}\left[r - e_h (1 - p_h)\left[\frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right)\right]\right] \,.$$

To have both guarantees be greater than zero, we need

$$\frac{p_l B}{(\Delta p)^2} \geq \left(R - \frac{B}{\Delta p}\right) \geq \frac{p_l B}{(\Delta p)^2} - \frac{r}{e_h (1 - p_h)} \,.$$

For the incentive compatibility constraint of the government in the first stage to be satisfied, it has to be the case that

$$\left(R - \frac{B}{\Delta p}\right) \geq \frac{1}{1 - e_h p_h}\left[(1 - p_h)\frac{p_l B}{(\Delta p)^2} - r + (1 - e_h)\frac{b}{\Delta e}\right].$$

To have $K_g^e < K_g^p$ it has to be the case that

$$\left(R - \frac{B}{\Delta p}\right) < \frac{p_l B}{(\Delta p)^2} - \frac{r}{(1 - e_h p_h)}.$$

Then, we will be in this case if

$$\max\left\{\frac{r}{e_h p_h},\frac{p_l B}{(\Delta p)^2}-\frac{r}{e_h(1-p_h)},\frac{1}{1-e_h p_h}\left[(1-p_h)\frac{p_l B}{(\Delta p)^2}-r+(1-e_h)\frac{b}{\Delta e}\right]\right\}<R-\frac{B}{\Delta p}<\frac{p_l B}{(\Delta p)^2}-\frac{r}{(1-e_h p_h)}.$$

textbfCase 3 Finally, if both incentive compatibility constraints of the government are binding and all other constraints are slack, $R_I = 0$ and the optimal guarantees are given by

$$K_g^p = \frac{p_l B}{(\Delta p)^2}-\left(R-\frac{B}{\Delta p}\right)$$
$$K_g^e = p_h\left(\frac{b}{p_h \Delta e}\right)+(1-p_h)\frac{p_l B}{(\Delta p)^2}-\left(R-\frac{B}{\Delta p}\right).$$

To have both guarantees be positive we need

$$\left(R-\frac{B}{\Delta p}\right)\leq \min\left\{\frac{p_l B}{(\Delta p)^2},p_h\left(\frac{b}{p_h \Delta e}\right)+(1-p_h)\frac{p_l B}{(\Delta p)^2}\right\}$$

For the individual rationality constraint of the private sector to be satisfied it has to be the case that

$$\left(R-\frac{B}{\Delta p}\right)\leq \frac{1}{(1-e_h p_h)}\left[(1-p_h)\frac{p_l B}{(\Delta p)^2}-r+p_h(1-e_h)\frac{b}{p_h \Delta e}\right].$$

For the individual rationality constraint of the government to be satisfied it has to be the case that

$$\left(R-\frac{B}{\Delta p}\right)\geq (1-p_h)\frac{p_l B}{(\Delta p)^2}+p_h(1-e_h)\frac{b}{p_h \Delta e}.$$

Then, we will be in this case if

$$\max\left\{\frac{r}{e_h p_h},(1-p_h)\frac{p_l B}{(\Delta p)^2}+p_h(1-e_h)\frac{b}{p_h \Delta e}\right\}\leq\left(R-\frac{B}{\Delta p}\right)$$

and

$$\left(R-\frac{B}{\Delta p}\right)\leq \min\left\{\frac{p_l B}{(\Delta p)^2},p_h\left(\frac{b}{p_h \Delta e}\right)+(1-p_h)\frac{p_l B}{(\Delta p)^2},\frac{1}{(1-e_h p_h)}\left[(1-p_h)\frac{p_l B}{(\Delta p)^2}-r+(1-e_h)\frac{b}{\Delta e}\right]\right\}.$$

Note that $K_g^{e*} > K_g^{p*}$ if and only if $\frac{p_l B}{(\Delta p)^2} < \frac{b}{p_h \Delta e}$.