# A Theory of Income Smoothing When Insiders Know More Than Outsiders*

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**Abstract**

We develop a theory of income and payout smoothing by firms when insiders know more about income than outside shareholders, but property rights ensure that outsiders can enforce a fair payout. Insiders set payout to meet outsiders' expectations and underproduce to manage downward future expectations. The observed income and payout process are smooth and adjust partially and over time in response to economic shocks. Underproduction is more severe the smaller is the inside ownership and results in an "outside equity Laffer curve".

J.E.L.: G32, G35, M41, M42, O43, D82, D92

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# Introduction

The practice of income smoothing has a long tradition in corporate finance. For example, Harold Geneen ran ITT for 18 years (1959-77), during which the company reported earnings increases for 58 consecutive quarters. It was widely assumed that this streak depended on a certain amount of gray-area fiddling with the numbers, but it was also accepted that investors were not being misled about the big picture. ITT was in fact growing steadily during his years and the figures were, on average, a fair reflection of the company's performance. More recently, Microsoft, General Electric and American Express have all been labelled as "smoothers".

Why do firms smooth income?[1] We argue in this paper that a primary reason for income smoothing is the pressure imposed on managers to meet the market's (i.e. analysts') earnings expectations.While shuffling cash flows backwards and forwards ("financial smoothing") to level out income fluctuations may be harmless, it is widely acknowledged that income smoothing has a darker side as it can lead firms to under-produce.[2]

Graham et al. (2005) published the results of a survey among more than 400 executives on the factors that drive reported earnings and disclosure decisions. 80% of survey participants report that they would decrease discretionary spending on R&D, advertising, and maintenance to meet an earnings target.[3] More than half (55.3%) state that they would delay starting a

[1]According to Investopedia "Companies indulge in this practice because investors are generally willing to pay a premium for stocks with steady and predictable earnings streams, compared with stocks whose earnings are subject to wild fluctuations". Related reasons often cited for income smoothing are: risk-averse insiders with limited access to external markets trying to insure themselves (Lambert (1984), Dye (1988)), managers aiming to maximize their tenure (Fudenberg and Tirole (1995)) or to minimize taxes (Graham (2003)). Income smoothing can signal good prospects (Ronen and Sadan (1981)) or low volatility to reduce the cost of debt (Trueman and Titman (1988)). Income smoothing can also encourage liquidity trading by uninformed investors (Goel and Thakor (2003)). We refer to section 5 for a detailed literature review.

[2]Jensen (2005) (page 8) notes:"Indeed, earnings management has been considered an integral part of every top managers job for at least the last two decades. But when managers smooth earnings to meet market projections, they are not creating value for the firm; they are both lying and making poor decisions that destroy value... when numbers are manipulated to tell the markets what they want to hear (or what managers want them to hear) rather than the true status of the firm it is lying, and when real operating decisions that would maximize value are compromised to meet market expectations real long-term value is being destroyed."

[3]Related theories that explain income manipulation (but not smoothing) are linked to insiders' myopia

new project to meet an earnings target, even if such a delay entailed a small sacrifice in value. (Graham et al., 2005, pp. 30-31). Their survey results are supported by a series of empirical studies that show that managers are prepared to destroy value in order to meet the market's expectations.[4]

While this interaction of market expectations and managerial behavior are widely acknowledged, it begs the question as to how is it possible that in equilibrium firms can keep on managing earnings and expectations, and get away with it in many cases indefinitely? Why do investors not intervene, or why does the smoothing equilibrium not unravel? If income and expectation management lead to value destruction, why then do insiders and outsiders engage in this game in the first place? Our theory answers these questions by providing a rational expectations equilibrium featuring income smoothing and expectations management that are driven by the pressure imposed on managers to meet income expectations.

The intuition behind our model can be illustrated by the following stylized example. Consider a firm that each period realizes profits of either 0 or 200 according to a flip of a coin. All income must be distributed each period among the shareholders, and outsiders enjoy legal protection to enforce payout. Only insiders who own, say, 10% of the firm, can observe the profit realization. Outsiders merely have beliefs about the income generating process. We argue that, in equilibrium, insiders pay out each period according to what outsiders expect to get, i.e. each period outsiders get paid 90 (90% of 100). While each period outsiders get the wrong amount (they should either get 0 or 180), on average they get a fair deal. This type of

(Stein (1989), Bebchuk and Stole (1993)) or career concerns (Gibbons and Murphy (1992), Holmström (1999)).

[4] In an early study, Baber et al. (1991) examine whether firms cut R&D expenses to avoid missing earnings benchmarks. They show that managers forfeit positive net present value R&D investments to avoid reporting a loss or earnings declines. Perry and Grinaker (1994) extend the Baber et al. (1991) study. Their results confirm that managers deliberately cut R&D expenditures to meet earnings expectations. Subsequent studies on R&D investments and earnings targets (e.g. Bange and DeBondt (1998), Bushee (1998), Cheng (2004) and Gunny (2010)) generally confirm the results of Baber et al. (1991) and Perry and Grinaker (1994).

financial smoothing is harmless. Insiders may, however, also have an incentive to "manage" outsiders' expectations. Once the equity is issued (and assuming insiders compensation is not linked to the stock price), insiders benefit if they could make outsiders believe that the probability of success is not 0.5 but only, say, 0.4 because in that case insiders only have to pay out 90% of 80 (i.e. 72) each period.

But how can insiders influence outsiders' beliefs and expectations about current and future income? As only actions (and not words) are credible, insiders distort key determinants of the income generating process that are observable to outsiders, incurring a real cost for the firm as it implies that the firm deviates from first-best decision making. In our model, insiders underproduce to downplay the firm's fundamentals. Of course, outsiders rationally anticipate what insiders are up to, but nevertheless this type of value destroying manipulation persists in this signal-jamming equilibrium because both parties are "trapped" in some kind of a prisoners' dilemma. Conditional on outsiders believing that insiders will "behave" it is optimal for insiders to manipulate (i.e. underproduce). As a result, under-investment and expectations management always prevail in equilibrium.

Formally, we consider a neo-classical firm that sets output on the basis of marginal revenues and marginal costs. Marginal revenues are constant and exogenously given (the firm is a price-taker), but marginal costs follow an AR(1) process. Shocks to marginal costs are therefore persistent. Only insiders observe marginal costs. Outsiders can, however, observe a noisy measure of the output level, which we call sales. The "noise" is value-irrelevant, for instance, due to measurement error, and is transitory, normally distributed, and i.i.d. over time. When observing an increase in sales (i.e., the noisy proxy of output) outsiders cannot distinguish whether the increase is due to a reduction in marginal costs (and therefore represents a real increase in income), or whether the increase is due to value-irrelevant measurement error. Since measurement errors are transitory and shocks to costs persistent, the underlying source of change becomes clear only as time passes by. Therefore, outsiders calculate their best estimate of income on the basis of not only current sales but also past sales, by solving a Kalman filtering problem.

Then, in a rational expectations equilibrium outsiders form their expectation of actual income on the basis of the complete history of sales and of what they believe insiders' optimal output policy to be. Conversely, insiders determine each period their optimal output policy given outsiders' beliefs. We obtain a perfect Bayesian equilibrium in which insiders' actions are consistent with outsiders' beliefs and outsiders' expectations are unbiased conditional on the information available. Each period outsiders receive a payout that equals their share of what they expect income to be. Insiders also get a payout but they have to soak up any under (over) payment to outsiders as some kind of discretionary remuneration (charge): if actual income is higher (lower) than outsiders' estimate then insiders cash in (make up for) the difference in outsiders' payout.

Consequently, income and payout are smooth compared to actual income *not* because insiders want to smooth income, but because insiders have to meet outsiders' expectations to avoid intervention. Two types of income smoothing take place simultaneously: "financial" smoothing and "real" smoothing. The former is value-neutral and merely alters the time pattern of reported income without changing the firm's underlying cash-flows as determined by insiders' production decision. Insiders also engage in real smoothing by manipulating production in an attempt to manage outsiders' expectations. In particular, insiders under-invest and make output less sensitive to changes in the latent variable affecting marginal costs. This type of smoothing is value destroying.[5]

Importantly, smoothing has an *inter-temporal* dimension. The first-best output level is determined in our model by considerations regarding the *contemporaneous* level only of the

[5] We do not model *how* real and financial smoothing are implemented in practice. In Ronen and Sadan (1981), various smoothing mechanisms are discussed and illustrated in great detail. For empirical evidence regarding real smoothing, we refer to section 4.2.

latent marginal cost variable. But, the current output decision not only affects current sales levels but also outsiders' expectations of current and all future income. This exacerbates the previously discussed underinvestment problem for insiders because bumping up sales now means the outsiders will expect higher income and payout not only now but also in future. Even though the spillover effect of a one-off increase in sales on outsiders' future expectations wears off over time, it still causes insiders to underproduce even more.

Besides describing the type of behavior informally described in Jensen (2005) (see footnote 2), our model has implications for a number of areas in corporate finance. First, our model explains key dynamics of corporate payout. We show that in equilibrium payout follows a distributed lag model and has features as in Lintner (1956). For example, the effect on payout of a positive shock in sales is distributed over time because outsiders do not immediately know whether the increase in sales is due to transitory noise or whether it is caused by a persistent improvement in the firm's fundamental. Importantly, the higher the degree of incomplete information, the more payout is smoothed. Our model provides closed-form expressions for the Lintner constant and speed of adjustment (SOA), allowing these to be linked to economic determinants such as the volatility and growth of income, the persistence of income shocks, the firm's ownership structure and the variance of income measurement error.

Second, our model has implications for the firm's ownership structure and the role of independent audited disclosure. We show that smoothing and underproduction in particular increase with outside shareholders' ownership stake because it increases insiders' incentives to manage outsiders' expectations. Conversely, a higher level of inside ownership leads to less real smoothing. Indeed, the under-investment problem disappears as insiders move towards 100% ownership. These effects lead to an "outside equity Laffer curve": the value of the total outside equity is an inverted U-shaped function of outsiders' ownership stake.[6]

[6]The analogy with the taxation literature is straightforward: outsiders' ownership stake acts ex post like a proportional tax on distributable income and undermines insiders' incentives to produce. Note that our

Morck, Shleifer, and Vishny (1988) document a non-monotonic relation between Tobin's Q and managerial stock ownership, and McConnell and Servaes (1990) report an "inverted-U" or "hump-shaped" relation between Q and managerial ownership. Our model provides a new theoretical explanation for this empirical phenomenon.

Finally, our model provides new insights as to why firms go public or are taken private. It is well known that firms go public to raise more outside equity capital. However, consistent with empirical evidence by Asker, Farre-Mensa, and Ljungqvist (2012) our model predicts that public firms invest less and are less responsive to changes in investment opportunities compared to private firms. Furthermore, we predict that public firms that have accumulated ample internal sources of funds may be taken private in order to eliminate the investment distortions and costly disclosure requirements public firms are subject to. Our model also implies that public firms smooth payout more than private firms. This implication is consistent with Michaely and Roberts (2012) who show that private firms smooth dividends less than their public counterparts.

Section 1 presents the benchmark case with symmetric information between outsiders and insiders. Section 2 analyzes the asymmetric information model and its implications for income and payout smoothing. Section 3 discusses the robustness and extensions, in particular, the effect of stock-based and sales-based insider compensation. Section 4 presents novel empirical implications for (1) the time-series and cross-sectional properties of corporate income, (2) real smoothing by firms, (3) corporate ownership structure, and (4) public versus private firms. Section 5 relates our paper to existing literature. Section 6 concludes. Proofs are in the appendix. A complementary online appendix provides elements of the proofs that have been omitted and a brief discussion of insiders' participation constraint.

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under-investment result does *not* require the presence of costly effort by insiders.

# 1 Symmetric information case

Consider a firm with an open-ended (infinite) horizon that has access to a productive technology. The output from the technology is sold at a fixed unit price, but its scale can be varied. Marginal costs of production follow an AR(1) process and are revealed each period before the output scale is chosen. A part of the firm is owned by risk-neutral shareholders (outsiders) and the rest by risk-neutral insiders who also act as the technology operators. To start with, we focus on the first-best scenario in which there is congruence of objectives between outsiders and insiders, and information about marginal costs is known symmetrically to both outsiders and insiders.

Formally, we consider a firm with the following income function:

$$\pi_t \; = \; q_t \; - \; \frac{q_t^2}{2x_t} \tag{1}$$

$$where \;\; x_t \; = \; A\,x_{t-1} \; + \; B \; + \; w_{t-1} \quad with \; w_{t-1} \sim N(0,Q) \;\; , \tag{2}$$

where $q_t$ denotes the chosen output level. The (inverse) marginal production cost variable $x_t$ follows an AR(1) process with auto-regressive coefficient $A \in [0,1)$, a drift $B$, and an i.i.d. noise term $w_{t-1}$ with zero mean and variance $Q$.[7] The output level $q_t$ is implemented after the realization of $w_{t-1}$ is observed.

All shareholders are risk-neutral, can borrow and save at the risk-free rate, and have a discount factor $\beta \in (0,1)$. Therefore -unlike Stein (1989)- changing the time pattern of cash flows (without changing their present value) through more borrowing or saving is costless. The value of the firm is given by the present value of discounted income:

$$V_t \; = \; \max_{q_{t+j}, j=0\ldots\infty} E_t[\sum_{j=0}^{\infty} \beta^j \pi_{t+j}] \; = \; \max_{q_{t+j}, j=0\ldots\infty} E_t\left[\sum_{j=0}^{\infty} \beta^j \left(q_{t+j} \; - \; \frac{q_{t+j}^2}{2x_{t+j}}\right)\right] \tag{3}$$

[7] Mean reversion (i.e. $A < 1$) is a realistic assumption for production costs. For example, commodity prices (which constitute a large component of production costs in some industries) are often mean reverting due to the negative relation between interest rates and prices.

Then, the first-best production policy that maximizes firm value is as follows.

**Proposition 1** *The first-best production policy is* $q_t^o = x_t$ *. The firm's realized income and total payout under the first-best policy are given by:* $\pi_t^o = \frac{x_t}{2}$ *.*

The first-best output level $q_t^o$ equals $x_t$. Recall that a higher value for $x_t$ implies lower marginal costs. Therefore, the output level rises with $x_t$. As $x_t$ goes to zero, marginal costs spiral out of control and the first-best output quantity goes to zero.[8]

## 1.1 Inside and outside shareholders

We now introduce inside and outside shareholders who, respectively, own a fraction $(1 - \varphi)$ and $\varphi$ of the shares, $\varphi \in [0, 1]$. For example, insiders (managers and even board members involved in the firm's operating decisions) typically own the majority of shares of private firms ($\varphi < 0.5$), whereas for public firms it is more common that outsiders own the majority of shares ($\varphi > 0.5$). Insiders set the production ($q_t$) and payout ($d_t$) policies. Analogous to Myers (2000), Jin and Myers (2006), Lambrecht and Myers (2007, 2008, 2012), and Acharya,

[8] Since the shocks that drive $x_t$ are normally distributed, marginal costs could theoretically become negative. The solution in proposition 1 no longer makes sense for negative $x_t$ because marginal costs can, of course, not be negative. Given that the stationary distribution for $x_t$ is normal with mean $B/(1-A)$ and variance $Q/(1-A^2)$ it follows that the probability of $x$ being negative equals $N(-(B/(1-A))/\sqrt{Q/(1-A^2)}) \approx N(-19.5) \approx 5.5*10^{-85}$ for the parameter values used in figures 1 and 2. Given the tiny probability of $x$ being negative, and given exponential discounting, the effect of any negative $x_t$ on today's value is negligibly small, and therefore our approximation is (almost) perfect if the mean to standard deviation ratio, $(B/(1-A))/\sqrt{Q/(1-A^2)}$, is sufficiently large. If, for example, we allow the probability of $x_t$ being negative to be at most 0.001 in order to maintain a high degree of accuracy then the mean to standard deviation ratio must exceed 3 (since, $N(-3) \approx 0.001$). To rule out negative values for $x_t$ altogether one could assume that $x_t$ is log-normally distributed. This would, however, make the Bayesian updating process deployed in next section completely intractable. The normality assumption is standard in the information economics literature (for example, Grossman (1976) and papers that originated from this seminal paper).

Myers and Rajan (2011), we assume that insiders operate subject to a threat of collective action. Outsiders' payoff from collective action is given by $\varphi\alpha V_t$ where $\alpha$ ($\in (0,1)$) reflects the degree of investor protection (or specificity of the firm's technology).[9] Therefore, the value of the outside equity, $S_t$, must at all times satisfy the following constraint:

$$S_t \;\geq\; \alpha\phi V_t \;\equiv\; \theta V_t \tag{4}$$

Equation (4) is a *governance constraint* that ensures outside equityholders get a share of the income generated by the firm. How big the share is depends on insiders' effective ownership stake as summarized by the parameter $\theta$ with $0 < \theta < 1$.[10]. Outsiders can force the firm to pay out by taking collective action. Condition (4) states that insiders will at all times set the payout $d_t$ high enough so that outsiders are willing to postpone intervention for one more period.[11]

The governance constraint captures parsimoniously a repeated game between insiders and outsiders. At each time $t$ insiders propose to outsiders (e.g. at the annual general meeting) a payout and rent level ($d_t$, $r_t$). If outsiders reject this offer then they get the payoff from

[9] When we have a public corporation with a large outside ownership stake, then collective action is as described in the Myers (2000) "corporation model". Outsiders take over the firm and displace insiders. The cost of collective action reflects the loss in managerial human capital, deadweight costs of getting organized, etc. If we have a private company with a small outside ownership stake then outsiders are minority stakeholders and the inside majority rules. Minority shareholders are, however, not entirely impotent as company law or commercial code grants minority shareholders either a judicial venue to challenge the decisions of management or the right to step out of the company by requiring the company to purchase their shares. The payoff from collective action to outside minority shareholders under this "oppressed minorities mechanism" (see La Porta et al. (1998)) is therefore the fair value of their stake, net of any costs of intervention (such as a possible minority discount or legal costs).

[10] For $\theta = 0$ shareholders have no stake in the firm and the capital market constraint disappears. For $\theta = 1$ managers can no longer capture rents and their objective function is no longer defined. Therefore $\theta \in (0,1)$.

[11] Graham et al. (2005) provide convincing evidence of how capital market pressures induce managers to meet earnings targets at all costs. As one surveyed manager put it: "I miss the target, I'm out of a job." Mergenthaler et al. (2012) find that CEOs are penalized via bonus cuts, fewer equity grants, and forced turnover when they just miss the latest consensus analyst forecast.

intervention, $\theta V_t$, insiders get $(1-\theta-c)V_t$, and the game ends. $cV_t$ reflects the cost of intervention to insiders. If outsiders accept then insiders and outsiders respectively get $r_t$ and $d_t$, and insiders stay in charge for one more period, at which point the game is repeated at $t+1$. In equilibrium insiders always remain in charge as they propose a pair $(d_t\ ,\ r_t)$ for which outsiders are indifferent between intervening and leaving insiders in charge, i.e.:

$$\theta\left[\pi_t+\sum_{j=1}^{\infty}\beta^j E_t\left[\pi_{t+j}\right]\right] = d_t+\theta\beta\left[E_t\left[\pi_{t+1}\right]+\sum_{j=1}^{\infty}\beta^j E_t\left[\pi_{t+1+j}\right]\right] \tag{5}$$

$$\Longleftrightarrow \quad d_t = \theta\pi_t \ \ (\text{and therefore } r_t = \pi_t - d_t = (1-\theta)\pi_t) \tag{6}$$

The left (right) handside of (5) equals outsiders' payoff from intervention (continuation). Equation (6) can be interpreted as a capital market constraint that requires insiders to provide an adequate return to outside investors.

It follows that the payouts to outsiders ($d_t$) and insiders ($r_t$) are respectively given by $\theta\pi_t$ and $(1-\theta)\pi_t$. Income ($\pi_t$) is shared between insiders and outsiders according to their real ownership stake. Outsiders' and insiders' claim values are, respectively, $\theta V_t$ and $(1-\theta)V_t$. Under symmetric information insiders' payoff from intervention, $(1-\theta-c)V_t$, is always less than their payoff from continuation, $(1-\theta)V_t$. Therefore, insiders avoid triggering collective action.[12] The following corollary results at once.

**Corollary 1** *With symmetric information, insiders adopt the first-best production policy, and payout to outsiders (insiders) equals a fraction $\theta$ $(1-\theta)$ of realized income $\pi_t$.*

# 2 Asymmetric information

We now add two new ingredients to the model. First, we assume that the actual realizations of the stochastic marginal cost variable $x_t$ are observed by insiders only. All model parameters

[12] It is possible for insiders' participation constraint to be violated under asymmetric information (see online appendix C for further details).

remain common knowledge, however. Outsiders also have an unbiased estimate $\hat{x}_0$ of the initial value $x_0$.[13] Second, outsiders cannot observe the output level $q_t$, but have to rely on some noisy proxy. This introduces *measurement error.* Instead of observing $q_t$, outsiders observe $s_t \equiv q_t + \epsilon_t$ where $\epsilon_t$ is an i.i.d. normally distributed noise term with zero mean and variance $R$ (i.e., $\epsilon_t \sim N(0, R)$). The measurement error is uncorrelated with the marginal cost variable $x_t$ (i.e., $E(w_k \epsilon_l) = 0$ for all $k$ and $l$). In what follows we refer to $s_t$ as the firm's "sales" as perceived by outsiders, i.e. outsiders perceive the firm's revenues to be $s_t$, whereas in reality they are $q_t$. $s_t$ can, for example, be interpreted as analysts' estimate of output. We assume this estimate to be noisy, but unbiased (i.e. $E(\epsilon_t) = 0$).

Outsiders are aware that sales are an imperfect proxy for economic output and they know the distribution from which $\epsilon_t$ is drawn. Importantly, insiders implement output ($q_t$) *after* the realization of $x_t$ but *before* the realization of $\epsilon_t$ is known. Since $\epsilon_t$ is value-irrelevant noise, the firm's actual income is still given by $\pi(q_t) = q_t - \frac{q_t^2}{2x_t}$. However, as $q_t$ and $x_t$ are unobservable outsiders have to estimate income on the basis of noisy sales figures. Therefore measurement errors can lead to a discrepancy between the firm's stock price and its fundamental value (unlike Stein (1989) where the stock price always equals its fundamental value).

We know from previous section that there is a mapping from the latent variable $x_t$ to both $q_t$ and $\pi_t$. The presence of the noise term $\epsilon_t$ obscures, however, this link and makes it impossible for outsiders exactly to infer $x_t$ and $\pi_t$ from sales. Assuming that insiders cannot trade in the firm's stock and that the information asymmetry cannot be mitigated through monitoring or some other mechanism, the best outsiders can do is to calculate a probability distribution of income, $\pi_t$, on the basis of all information available to them. This information set $I_t$ is given by the full history of current and past sales, i.e., $I_t \equiv \{s_t\, ,\, s_{t-1}\, ,\, s_{t-2}\, ...\}$. We show that on the basis of the initial estimate $\hat{x}_0$ and the sales history, $I_t$, outsiders can infer a probability distribution for the latent marginal cost variable $x_t$, which in turn maps into

[13]E.g. $\hat{x}_0$ is revealed to outside investors when the firm is set up at time zero.

a probability distribution for income $\pi_t$. Outsiders then use this distribution to calculate their estimate $\hat{\pi}_t$ of the firm's income, i.e. $\hat{\pi}_t = E[\pi_t|I_t] \equiv E_{S,t}(\pi_t)$, where the subscript $S$ in $E_{S,t}[\pi_t]$ emphasizes (outside) shareholders' expectation at time $t$ of $\pi_t$ based on the information set $I_t$.

The capital market constraint requires that $d_t$ satisfies the following constraint:

$$\theta \left[ E_{S,t}(\pi_t) + \sum_{j=1}^{\infty} \beta^j E_{S,t}\left[\pi_{t+j}\right] \right] = d_t + \theta\beta \left[ E_{S,t}\left[\pi_{t+1}\right] + \sum_{j=1}^{\infty} \beta^j E_{S,t}\left[\pi_{t+1+j}\right] \right] \quad (7)$$

$$\Longleftrightarrow \quad d_t = \theta E_{S,t}(\pi_t) \;\; (\text{and therefore } r_t = \pi_t - \theta E_{S,t}(\pi_t)) \quad (8)$$

Therefore, to avoid collective action insiders set the payout equal to $d_t = \theta E_{S,t}(\pi_t)$. In other words, outsiders want their share of the income they believe has been realized according to all information available to them. While insiders cannot manage outsiders' expectations through words (which are not credible) they can do so through their actions. Managers can influence observable sales ($s_t$) and therefore $\hat{\pi}_t$ by their chosen output level ($q_t$).

Insiders' optimization problem can now be formulated as follows:

$$M_t = \max_{q_{t+j};j=0..\infty} E_t \left[ \sum_{j=0}^{\infty} \beta^j \left(\pi(q_{t+j}) - \theta E_{S,t+j}\left[\pi\left(q_{t+j}\right)\right]\right) \right] \quad (9)$$

with insiders' optimal output policy $q_t$ being an equilibrium once outsiders' beliefs are fixed. The complete derivation of the solution is given in the appendix. We briefly present a heuristic derivation of the rational expectations equilibrium here. Outsiders conjecture that insiders' production policy is given by $q_t = Hx_t$, where $H$ is some constant. Therefore, $E_{S,t+j}\left[\pi(q_{t+j})\right] = \left(H - \frac{H^2}{2}\right) E_{S,t+j}\left[x_{t+j}\right] \equiv hE_{S,t+j}\left[x_{t+j}\right]$. Define $\hat{x}_t \equiv E_{S,t}[x_t]$ as outsiders' estimate of the latent variable $x_t$ conditional on the information available at time $t$. Since $s_t = q_t + \epsilon_t$ and $q_t = Hx_t$, sales are an imperfect (noisy) measure of the latent variable $x_t$, as is clear from the following "measurement equation":

$$s_t = H\,x_t + \epsilon_t \quad with \;\; \epsilon_t \sim N(0, R) \quad (10)$$

Outsiders also know the variance $R$ of the noise, $\epsilon_t$, and the parameters $A$, $B$ and $Q$ of the "state equation":

$$x_t \;=\; A\,x_{t-1} \;+\; B \;+\; w_{t-1} \quad with \;\; w_t \sim N(0,Q) \; for \; all \; t \tag{11}$$

Outsiders now solve what is known as a "filtering" problem. Using the Kalman filter (see appendix), the measurement equation can be combined with the state equation to make inferences about $x_t$ on the basis of current and past observations of $s_t$. This allows outsiders to form an estimate of actual income $\pi_t$. While the measurement equation is usually exogenously given, our Kalman filter has the novel feature that the constant slope coefficient $H$ in the measurement equation is set *endogenously* by insiders.

The solution is formulated in terms of the steady state or "limiting" Kalman filter which is the estimator $\hat{x}_t$ for $x_t$ that is obtained after a sufficient number of measurements $s_t$ have taken place over time for the estimator to reach a steady state.[14] The steady-state estimator $\hat{x}_t$ allows us to analyze the long-run behavior of income and payout and is given by (Chui and Chen (1991), p78):

$$\hat{x}_t \;=\; (A\hat{x}_{t-1} + B)\,\lambda \;+\; K s_t \tag{12}$$

where $\lambda$ and $K$ are as defined in proposition 2. K is called the "Kalman gain" and it plays a crucial role in the updating process. Substituting $\hat{x}_{t-1}$ in (12) by its estimate, one obtains after repeated substitution:

$$\begin{aligned}
\hat{x}_t \;&=\; B\lambda\left[1 + \lambda A + \lambda^2 A^2 + \lambda^3 A^3 + ...\right] \;+\; K\left[s_t + \lambda A s_{t-1} + \lambda^2 A^2 s_{t-2} + \lambda^3 A^3 s_{t-3} + ...\right] \\
&=\; \frac{B\lambda}{1-\lambda A} \;+\; K\sum_{j=0}^{\infty} \lambda^j A^j s_{t-j}\;. 
\end{aligned} \tag{13}$$

Thus, outsiders' income estimate is not only determined by their observation of current sales but also by *the whole history of past sales.* Hence, insiders' optimization problem is no longer

[14] Under mild conditions (see footnote 25 in the appendix) the Kalman filter converges to its steady state. Convergence is of geometric order and therefore fast.

static but inter-temporal and dynamic. Indeed, the current production decision not only affects insiders' expectations about current but also future income.

Substituting outsiders' beliefs $E_{S,t+j}[\pi(q_{t+j})] = E_{S,t+j}[Hx_t - H^2x_t/2] = h\hat{x}_{t+j}$ into insiders' objective function (9), insiders optimize:

$$M_t = \max_{q_{t+j};j=0..\infty} E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \pi(q_{t+j}) - \theta h \hat{x}_{t+j} \right) \right] \tag{14}$$

Using (13) and the fact that $s_t = q_t + \epsilon_t$ gives the following first-order condition:

$$\frac{\partial M_t}{\partial q_t} = 1 - \frac{q_t}{x_t} - \theta hK - \theta hK\beta\lambda A - \theta hK\left(\beta\lambda A\right)^2 - \theta hK\left(\beta\lambda A\right)^3 - ... = 0 \tag{15}$$

Or equivalently, since $0 \leq \beta\lambda A < 1$:

$$q_t = \left[ 1 - \frac{\theta hK}{1 - \beta\lambda A} \right] x_t \tag{16}$$

Outsiders' conjectured output policy $q_t = Hx_t$ is a perfect Bayesian equilibrium if and only if:

$$H = 1 - \frac{\theta hK}{1 - \beta\lambda A} \tag{17}$$

$$\iff H = 1 - \frac{\theta H^2 P\left(1 - \frac{H}{2}\right)}{H^2P + R\left(1 - \beta A\right)} \tag{18}$$

At the fixed point, we have a perfect Bayesian equilibrium in which outsiders' expectations are rational given insiders' output policy, and insiders' output policy is optimal given outsiders' expectations. Equation (18) has a unique positive root for $H$, which pins down the equilibrium value for $H$. This root is less than 1 (i.e. $H < 1$) and therefore insiders underproduce compared to what is first best (see proof proposition 2 and online appendix A for further details). The results are summarized in the following proposition.

**Proposition 2** *The insiders' optimal production plan is given by:*

$$q_t = H\,x_t = Hq_t^o \quad for\ all\ t \tag{19}$$

*Payout to outside shareholders equals a fraction $\theta$ of expected income: $d_t = \theta\hat{\pi}_t$ where*

$$\hat{\pi}_t = \left(H - \frac{H^2}{2}\right)\hat{x}_t \equiv h\hat{x}_t \ , \tag{20}$$

$$\textit{and where } \hat{x}_t = (A\hat{x}_{t-1} + B)\,\lambda + K\,s_t \tag{21}$$

$$= \frac{\lambda B}{1-\lambda A} + K\sum_{j=0}^{\infty}(\lambda A)^j s_{t-j} \ . \tag{22}$$

*with $K \equiv \frac{HP}{H^2P+R}$, $\lambda \equiv (1-KH)$ and $P$ is the positive root of the equation:*

$$P = A^2P - \frac{A^2H^2P^2}{H^2P+R} + Q \ . \tag{23}$$

*$H$ is the (unique) positive root to (18) and lies in the interval $]0,1[$. The error of outsiders' income estimate $(\pi_t - \hat{\pi}_t)$ is normally distributed with mean zero (i.e., $E_{S,t}[\pi_t - \hat{\pi}_t] = 0$) and variance $\hat{\sigma}^2 \equiv E_{S,t}[(\pi_t - \hat{\pi}_t)^2] = h^2\,P$.*

## 2.1 Production Policy

We know from Proposition 2 that insiders' optimal production policy is given by $q_t = H\,x_t$ where $H$ is a solution to equation (18). There exists a unique positive root for $H$ which lies in the interval $]0,1[$. We therefore obtain the following corollary.

**Corollary 2** *If outsiders indirectly infer income from sales ($s_t$) then insiders underproduce (i.e., $q_t = Hx_t = Hq_t^o \leq q_t^0$).*

Insiders underproduce because outsiders do not observe $x_t$ directly but estimate its value indirectly from sales. This gives insiders an incentive to manipulate sales (engage in "signal-jamming") by underproducing. In equilibrium, outsiders correctly anticipate this manipulation and incorporate it into their expectations. Nevertheless, insiders are "trapped" into behaving myopically. The situation is analogous to what happens in a prisoner's dilemma. The preferred cooperative equilibrium would be efficient production by insiders and no conjecture of manipulation by outsiders. This can, however, not be sustained as a Nash equilibrium because insiders have an incentive to underproduce whenever outsiders believe the efficient production policy is being adopted.

The unconditional long-run mean for $q_t$ under the first-best and actual production policies are, respectively, $E[q_t^o] = E[x_t] = B/(1-A)$ and $E[q_t] = HE[x_t] = BH/(1-A)$. Lost output, in turn, translates into a loss of income. The unconditional mean income under the first-best and actual production policies are, respectively, $E[\pi_t^o] = \frac{1}{2}E[x_t]$ and $E[\pi_t] = hE[x_t]$.

## 2.2 The dynamics of income and payout

Proposition 2 leads to the following corollary:

**Corollary 3** *Underproduction reduces the variance of income and leads to "real smoothing":*

$$var(\pi_t) = \left(H - \frac{H^2}{2}\right)^2 var(x_t) < \frac{1}{4}var(x_t) = var(\pi_t^o) \tag{24}$$

Underproduction implies that income becomes less sensitive to changes in the state variable than would be the case under the first-best production policy ($H = 1$). This effect can be economically significant if outside ownership or income volatility are high, or if economic shocks are highly persistent (see figure 1 in section 2.5.1).

Proposition 2 allows us to derive the dynamics of the firm's expected income and its payout:

**Proposition 3** *The firm's expected income, $\hat{\pi}_t (= E_{S,t}[\pi_t] = h\hat{x}_t)$, is described by the following partial adjustment model:*

$$\hat{\pi}_t = \lambda A \hat{\pi}_{t-1} + Khs_t + h\lambda B \, . \tag{25}$$

*The firm's payout to outside shareholders is described by the following target adjustment model:*

$$d_t \;=\; d_{t-1} \;+\; (1-\lambda A)\,(d_t^* \;-\; d_{t-1}) \tag{26}$$

$$\;=\; \lambda A d_{t-1} \;+\; \theta K h s_t \;+\; \theta h \lambda B \;\equiv\; \gamma_2\, d_{t-1} \;+\; \gamma_1\, s_t \;+\; \gamma_0 \;. \tag{27}$$

*The payout "target" $d_t^*$ is given by:*

$$d_t^* \;=\; \frac{\theta h \lambda B}{1-\lambda A} \;+\; \left(\frac{\theta K h}{1-\lambda A}\right) s_t \;\equiv\; \gamma_0^* \;+\; \gamma_1^*\, s_t \;. \tag{28}$$

*The speed of adjustment coefficient is given by $SOA \equiv (1-\lambda A)$ with $0 < SOA \leq 1$.*

Payout ($d_t$) follows a target ($d^*$) that is determined by the contemporaneous level of sales. However, as equation (26) shows, payout only gradually adjusts to changes in sales because the SOA coefficient $(1-\lambda A)$ is less than unity. This leads to payout smoothing in the sense that the effect on payout of a shock to sales is distributed over time. In particular, a dollar increase in sales leads to an immediate increase in payout of only $\theta h K$. The lagged incremental effects in subsequent periods are given by $\theta h K \lambda A$, $\theta h K (\lambda A)^2$, $\theta h K (\lambda A)^3$,... The long-run effect of a dollar increase in sales on payout equals $\theta h K \sum_{j=0}^{\infty} (\lambda A)^j \;=\; \frac{\theta h K}{1-\lambda A}$, which is the slope coefficient $\gamma_1^*$ of the payout target $d_t^*$ (see equation (28)). In contrast, with symmetric information, the impact of a shock to sales is fully impounded into payout immediately.

Intuitively, payout only partially adjusts to a contemporaneous shock in sales because in the short run outsiders cannot distinguish between a transitory measurement error and a persistent shock to the latent cost variable. However, as subsequent sales are observed the transitory or persistent nature of the shock is gradually revealed. Payout can therefore also be expressed as a distributed lag model in which it is a function of current and past sales, by repeated backward substitution of equation (25):

$$d_t \;=\; \frac{\theta h \lambda B}{1-\lambda A} \;+\; \theta K h \sum_{j=0}^{\infty} (\lambda A)^j\, s_{t-j} \;. \tag{29}$$

Expected income $\hat{\pi}_t$ displays similar dynamics as payout. In fact, one merely needs to set $\theta = 1$ in the above expression for payout to obtain the corresponding expressions for expected income. Finally, given that (i) expected income is smooth relative to actual income and (ii) payout is based on expected income, it follows that insiders soak up the variation.

## 2.3 Payout smoothing and the Lintner model

The payout model (27) is very similar to the well known Lintner (1956) dividend model.[15] Hundreds of papers have tested the Lintner model and estimated the Lintner constant and SOA. However, in the absence of a formal theoretical underpinning for the Lintner model, little is known a priori regarding the magnitude and behavior of the Lintner constant and SOA.

Lintner (1956) notes that "The constant will be zero for some companies but will generally be positive to reflect the greater reluctance to reduce than to raise dividends ... as well as the influence of the specific desire for a gradual growth in dividend payments found in about a third of the companies visited."

Lintner (1956) finds a $SOA$ of about 0.3 using aggregate data on corporate earnings and dividends. Fama and Babiak (1968) test Lintner's model for individual firms over a 20-year period and report a mean $SOA$ of 0.32. Skinner (2008) finds a SOA for total payout of 0.4 and 0.55 for the periods 1980 to 1994 and 1995 to 2005, respectively. Recall that SOA=1 implies instantaneous adjustment and therefore no smoothing, whereas SOA=0 means that payout no longer changes from one period to the next. The Lintner $SOA$ implies a half-life for adjustment of payout to changes in income. Half-life is the time needed to close the gap

[15] The only difference is that in Lintner (1956) the payout target is determined by the firm's net income, whereas in our model the target is a function of sales because net income is not directly observed by outsiders. Payout in our model is not smoothed relative to income but relative to a proxy variable observable by outsiders, i.e., sales. Payout smoothing in the strict Lintner sense obtains, e.g., if insiders are risk-averse and subject to habit formation. Lambrecht and Myers (2012) show that insiders of this type smooth payout relative to income by borrowing and lending.

between the actual and target payout by 50%, after a one-unit shock to the error term in the Lintner model equation. When payout follows an AR(1) process, half-life is log(0.5)/ log(1 - $SOA$). If the $SOA$ equals, say, 0.3, then the half-life is about two years and it would take the firm about 6.5 years to close the gap between the actual payout and the target by 90%. Thus, payout is history dependent, and for reasonable parameters the history extends back several years.

Our model provides closed-form structural expressions for the SOA (given by $1 - \lambda A$) and the Lintner constant (given by $\theta h \lambda B$). This allows us to give an economic interpretation to the Lintner model and to explore how the coefficients in the Lintner model depend on key economic variables.

For example, our model confirms that the constant ($\theta h \lambda B$) is positive provided that income has a positive drift, $B$. This is consistent with empirical evidence and Lintner's observation that dividends tend to grow over time. More importantly, our model explains that this growth in dividends is linked to growth in the firm's income. The Lintner constant is a non-linear function of the main model parameters. Numerical model simulations (available upon request) reveal, for example, that the constant $\theta h \lambda B$ is an inverted U-shaped function of outsiders' ownership stake $\theta$. We refer the reader to section 2.5.3 for more details regarding ownership structure and its effect on firm value. Figure 2 in the comparative statics section 2.5.2 illustrates and summarizes the effect of the main model parameters ($\theta, A, R$ and $Q$) on the speed of adjustment (SOA) of payout to the payout target.

## 2.4 Dynamic versus static model

Our model is fully dynamic and technically more demanding than a static model. What do we gain from the added complexity, and how do the results in a dynamic setting differ from the ones in a static setting? Consider the following static version of our model: $x \sim$

$N(B,Q)$ and $\epsilon \sim N(0,R)$. This static model is equivalent to the special case of our dynamic model for which $A = 0$. Indeed, if the latent variable has zero autocorrelation then insiders' optimal decision becomes a "myopic" one. If $A = 0$ the present has no bearing for the future production decision, and therefore the problem becomes essentially a static one (even for the intertemporal, infinite-horizon case).

What are the key differences between the solution of the static and the dynamic model? First, in a static setting the dividends are given by: $d_t = \theta h \lambda B + \theta h K s_t$. Consequently, while in a dynamic model dividends are smooth relative to sales and sales have a lagged effect on dividends, this is no longer the case in a static model. In fact, payout smoothing no longer occurs in a static model. Similarly, sales only have a contemporaneous effect on outsiders' income expectations in a static model. As such, a static model is unable to capture and explain inter-temporal income and payout smoothing.

Second, outsiders' estimate of the latent variable $x$ (and therefore also their estimate of $\pi_t$) has no memory, i.e. $\hat{x}_t = B\left(1 - \frac{H^2Q}{H^2Q+R}\right) + \left(\frac{HQ}{H^2Q+R}\right) s_t$. In the dynamic model $\hat{x}_t$ (and $\hat{\pi}_t$) depends on the previous estimate of $x_t$, and therefore on the whole history of $s_t$. As a result, a static model is unable to capture inter-temporal expectations management by insiders.

Third, if $A = 0$ then $P = Q$. While in the dynamic model $P$, the variance of outsiders' estimate of the latent variable $x$, depends on insiders' production policy ($H$) and all other model parameters, this is no longer the case in the static version. In fact, in the static version the variance of outsiders' estimate of $x$ equals $Q$, the actual variance of $x$. Therefore, the static model does not capture the effect of insiders' actions on the accuracy of outsiders' income estimate.

Finally, insiders underproduce also in the static model. However, as figure 1 in the comparative statics section 2.5.1 illustrates, the underproduction problem is more severe in a dynamic context. A myopic insider or an insider who engages in a one-shot production decision does not consider the effect of his decision for future production decisions. In a dynamic setting insiders know that their current production decision affects not only outsiders' current but also future payout expectations. As a result the static model underestimates insiders' incentive to manage expectations and to underproduce.

## 2.5 Comparative statics and further results

### 2.5.1 Asymmetric information and the production decision

The following corollary explains the effect of asymmetric information on production.

**Corollary 4** *The noisier the link between the latent variable ($x_t$) and its observable proxy ($s_t$), the weaker insiders' incentive to manipulate the proxy by underproducing. In particular, insiders' production decision converges to the first-best one as the variance of measurement errors becomes infinitely large ($R \to \infty$) or as uncertainty with respect to the latent variable $x_t$ decreases ($Q \to 0$), i.e., $\lim_{Q\to 0} H = \lim_{R\to\infty} H = 1$. Conversely, the more precise the link between $s_t$ and $x_t$, the higher the incentive to underproduce. The lower bound for $H$ is achieved for the limiting cases $Q \to \infty$ and $R \to 0$, i.e., $\lim_{Q\to\infty} H = \lim_{R\to 0} H = 1 - \frac{\theta}{2-\theta}$.*

When $x_t$ becomes deterministic ($Q = 0$) then the estimation error with respect to $x_t$, goes to zero (i.e., $P \to 0$). This means that the Kalman gain coefficient $K$ becomes zero too (there is no learning). But if there is no learning ($K = 0$ and $\lambda = 1$) then insiders' output decision $q_t$ no longer affects outsiders' estimate of the cost variable, as illustrated by equation (12). As a result the production policy becomes efficient (i.e., $H = 1$ and $q_t = x_t$).

Similarly, if there are measurement errors then the link between sales and the latent cost variable becomes noisy. This mitigates the under-investment problem, because the noise obscures insiders' actions and therefore their incentive to cut production.

In the absence of measurement errors ($R = 0$) the link between sales $s_t$ and the contemporaneous level of the latent variable $x_t$ becomes deterministic.[16] Outsiders know for sure that an increase in sales results from a fall in marginal costs. Therefore, when observing higher sales, outsiders want higher payout. In an attempt to manage outsiders' expectations downward, insiders underproduce. If $R = 0$ then we get the efficient outcome ($H = 1$) only if insiders get all the income ($\theta = 0$); otherwise we get under-investment ($H < 1$). As the insiders' stake of income goes to zero ($\theta \to 1$) also production goes to zero (i.e., $H \to 0$). Both outsiders and insiders get nothing, even though the firm could be highly profitable! This result is in sharp contrast with the symmetric information case where the efficient outcome is obtained no matter how small the insiders' share of the income. Thus, for firms where insiders have a very small ownership stake (e.g. public firms with a highly dispersed ownership structure), asymmetric information and the resulting indirect inference-making process by outsiders could undermine the firm's very existence, an issue we return to in section 3.

Figure 1 illustrates the effect of the key model parameters ($R, Q, A$ and $\theta$) on production efficiency. Efficiency is measured with respect to two different variables: the unconditional mean output ($E[q_t]$), and unconditional mean income ($E[\pi_t]$). The degree of efficiency is determined by comparing the actual outcome with the first-best outcome, i.e., $E[q_t]/E[q_t^o] = H$ (dashed line), and $E[\pi_t]/E[\pi_t^o] = 2h$ (solid line).

The figure shows that the efficiency loss is larger with respect to output than income because the loss in revenues due to underproduction is to some extent offset by lower costs of production. Panel A and B confirm that full efficiency is achieved as $R$ moves towards $\infty$ and for $Q = 0$. Panel C shows that a higher autocorrelation in marginal costs substantially reduces efficiency because it allows outsiders to infer more information about the latent cost variable from sales and therefore gives insiders stronger incentives to distort production.

[16] For $R = 0$ we get $P = Q$, $K = 1/H$ and $\lambda = 0$. Therefore, from Proposition 2 it follows that $\hat{x}_t = s_t/H$ and $s_t = Hx_t$. Consequently, $\hat{x}_t = x_t$.

Finally, panel D shows that production is fully efficient if outsiders have no real stake in the firm's income (i.e., $\theta = 0$). Efficiency severely declines as outsiders' stake increases. For $\theta = 1$, insiders achieve only 28% of the first-best output level. However, one can show that as $Q/R \to 0$ incentives are fully restored, and the first-best outcome can be achieved even for $\theta = 1$. This confirms that the root cause of underproduction is the process of indirect inference and not the outside ownership stake per se. The firm's ownership structure serves, however, as a transmission mechanism through which inefficiencies can be amplified.

### 2.5.2 Asymmetric information and payout smoothing

The following corollary summarizes how asymmetric information affects payout:

**Corollary 5** *Measurement errors create asymmetric information, which in turn leads to payout smoothing. A lower degree of information asymmetry (i.e., $R$ falls relative to $Q$) leads to less smoothing. In the limiting case where outsiders can accurately infer income (i.e., $R = 0$ or $Q \to \infty$) payout is always on target and coincides at all times with outsiders' share of actual income (i.e., $d_t = d_t^* = \theta\pi_t$ for all $t$).*

No financial smoothing whatsoever occurs when $R = 0$ because in that case all information asymmetry is eliminated. In the absence of measurement errors, it is possible to infer the marginal cost variable $x_t$ with 100% accuracy from the observed sales figure $s_t$. The same result obtains when $Q \to \infty$ because in that case measurement errors are negligibly small compared to the variance of the latent cost variable. This important result confirms again that *asymmetric information and not uncertainty per se is the root cause of payout smoothing.* The corollary also confirms that as the degree of information asymmetry goes to zero, our rational expectations equilibrium converges to the simple sharing rule that prevails under symmetric information. Indeed: $\lim_{R\to 0} d_t = \theta \lim_{R\to 0} \hat{\pi}_t = \theta\pi_t$.

Next, Figure 2 illustrates and summarizes the effect of the main model parameters ($\theta, A, R$ and $Q$) on the speed of adjustment (SOA) of payout to the payout target. Recall that no smoothing (i.e., $SOA = 1$) occurs under symmetric information. Our symmetric information benchmark case corresponds therefore with $SOA = 1$ (represented by a solid horizontal line at $SOA = 1$ in the figure). The dotted line plots the SOA that results from the actual production policy (as determined by $H$) derived under asymmetric information. While this gives us an idea of the total amount of intertemporal payout smoothing, it does not tell us how much of this is due to the suboptimal production policy that results from indirect inference and how much is due to mere financial smoothing that results from asymmetric information. We refer to the former as *"real"* smoothing and to the latter as *"financial"* smoothing.

The financial smoothing component is measured by evaluating the SOA at the first-best production policy $H = 1$, i.e., $SOA = 1 - A\lambda[H = 1]$ (as represented by the dashed line). Therefore $A\lambda[H = 1]$ reflects the amount of smoothing that would take place under asymmetric information but assuming that insiders were to adopt the efficient production policy. Financial smoothing is therefore measured in figure 2 by the distance between the horizontal solid line at SOA=1 and the dashed line. Since the dotted line represents the total amount of smoothing (i.e., financial plus real smoothing), the difference between the dashed line and the dotted line (given by $A\lambda - A\lambda[H = 1]$) captures the amount of "real smoothing".

The distinction between the two types of smoothing is clearly illustrated in panel A which plots the SOAs as a function of the (real) outside ownership stake $\theta$. Changing $\theta$ does not alter the degree of asymmetric information between insiders and outsiders and, as a result, the amount of financial smoothing remains constant. The corresponding SOA of 0.86 (dashed line) implies a half-life of about 0.35 years for adjustment of reported income to changes in sales. Increasing $\theta$ introduces, however, additional real smoothing and this reduces the SOA from 0.86 (for $\theta = 0$) to 0.49 (for $\theta = 1$) corresponding, respectively, to a half-life of 0.35 years and 1.03 years. The plot confirms our earlier results that reducing inside ownership leads to severe underproduction, which in turn leads to a smoother payout flow because payout becomes less sensitive to sales.

Panel B shows that smoothing also increases with the degree of autocorrelation in the latent cost variable. No *intertemporal* smoothing takes place when $A = 0$ because in that case current and past realizations of $x_t$ are irrelevant for the future. As a result, insiders' private information about $x_t$ is also irrelevant for the future. Note that higher autocorrelation raises both real and financial smoothing substantially.

Finally, panels C and D confirm that the total amount of smoothing increases with the degree of information asymmetry (as reflected by a higher $R$ or lower $Q$). Paradoxically, more intertemporal smoothing coincides with higher production efficiency (see figure 1): when outsiders can infer less from sales, there is also less of an incentive to manipulate production. Note that a higher degree of information asymmetry unambiguously increases the amount of financial smoothing.

To summarize, our model captures the history dependence of payout and allows us to link the SOA to key economic determinants. Figure 2 illustrates that the SOA decreases with outsiders' ownership stake ($\theta$), the degree of income persistence ($A$), and the variance of measurement errors ($R$). The SOA increases with the variance of income ($Q$). These are predictions that could be empirically tested. The figure also shows that for reasonable parameters our model tends to generate SOAs in the 0.5 to 0.8 range, well above the empirically observed estimates. Our model is, however, not capturing features such as risk aversion and habit formation which induce further smoothing and reduce the SOA (see Lambrecht and Myers (2012)).

### 2.5.3 Ownership structure and firm value

The following proposition states the outside equity value, $\theta V(\hat{x}_t)$.

**Proposition 4** *The outside equity value of the firm is given by:*

$$\theta V(\hat{x}_t;\theta) \;=\; \frac{\theta h}{(1-\beta A)}\left(\hat{x}_t + \frac{B\beta}{1-\beta}\right) \tag{30}$$

We know that, for a given value of $\hat{x}_t$, the firm value $V(\hat{x}_t;\theta)$ monotonically declines in the ownership stake $\theta$ and that the first-best firm value is achieved when the outside ownership stake is zero (i.e., $\theta = 0$). Numerical simulations (available upon request) show that as much as half of the firm value can be lost as $\theta$ varies from 0 to 1. Numerical simulations also show that the outside equity value $\theta V_(\hat{x}_t;\theta)$ is an inverted U-shaped function of $\theta$ that reaches a unique maximum, hereby resembling an "outside equity Laffer curves".[17] This result has important empirical implications for the relation between ownership structure and firm value (see section 4.3) and the behavior of public versus private firms (see section 4.4).

# 3 Robustness, extensions and discussion

## 3.1 Stock-based compensation

Stein (1989) argues that stock-based compensation induces insiders to inflate income. How does stock-based compensation affect insiders' production incentives in our setting where market pressures apply not only with respect to the current stock price but also with respect to future payout? To explore this question we now consider the scenario where insiders get

[17]The traditional Laffer curve is a graphical representation of the relation between government revenue raised by taxation and all possible rates of taxation. The curve resembles an inverted U-shaped function that reaches a maximum at an interior rate of taxation.

each period a fraction $\delta$ of the existing *outside* equity. Insiders get the shares *cum* dividend and must sell them in the market upon receipt (in contrast to their existing stockholding $1-\varphi$ which they are not allowed to sell).[18] Outsiders know that their equityholding will be diluted each period by a fraction $1-\delta$, and take this into account when pricing the outside equity, $S_t$. Managers' optimization problem is now given by:

$$M_t \;=\; \max_{q_{t+j};j=0..\infty} E_t\left[\sum_{j=0}^{\infty}\beta^j\left(\pi(q_{t+j}) \;-\; \theta E_{S,t+j}\left[\pi\left(q_{t+j}\right)\right] \;+\; \delta S_{t+j}\right)\right] \tag{31}$$

Solving this problem gives the following proposition:

**Proposition 5** *If insiders get each period the cash equivalent of a fraction $\delta$ of the outside equity then their optimal production decision is given by $q_t = Hx_t$ where $H$ is the solution to:*

$$H \;=\; 1 \;-\; \frac{hK\theta\left(1 \;-\; \frac{\delta}{1-\beta(1-\delta)A}\right)}{1-\beta\lambda A} \tag{32}$$

*The value of the outside equity (cum dividend) at time $t$ is:*

$$S_t \;=\; \theta E_{S,t}\left[\sum_{i=0}^{\infty}\beta^i\left(1-\delta\right)^i\pi(q_{t+j})\right] \;=\; \frac{\theta h}{1-\beta A(1-\delta)}\left(\hat{x}_t \;+\; \frac{B\beta(1-\delta)}{1-\beta(1-\delta)}\right) \tag{33}$$

*Stock based compensation mitigates, but does not eliminate the underinvestment problem except if outsiders in effect own 100% of the firm (i.e. $\delta = 1$).*

Equation (32) shows that increasing stock-based compensation is similar to reducing $\theta$, outsiders' stake in the firm. Therefore stock-based compensation unambiguously improves efficiency and mitigates the under-investment problem.[19] From (32) it is clear that $H = 1$ if

[18] It is not crucial for the analysis that shares are sold immediately. The key restriction is that insiders do not have discretion regarding the timing of the sale, as this would introduce an adverse selection and an optimal stopping problem.

[19] Stock-based compensation may, however, introduce other (dis)incentives not considered in this paper. E.g. Benmelech, Kandel, and Veronesi (2010) show that stock-based compensation not only induces managers to exert costly effort, but also induces them to conceal bad news about future growth options and to choose suboptimal investment policies to support the pretense.

$\delta = 1$, i.e., the efficient outcome is achieved if outside equityholders get 100% diluted each period.

Unlike Stein (1989) insiders do not have an incentive to inflate income in the presence of stock compensation because market pressures do not only apply to the current stock price but also to future payout. By inflating income insiders not only inflate the current stock price, but also outsiders' expectations regarding future dividend payout. Therefore insiders' immediate gain with respect to their stock-based compensation is more than offset by the loss from paying higher future dividends (unless insiders own 100% of the firm).

How then can incentives to inflate income arise? High powered compensation mechanisms (such as stock options, or other contracts that are convex in income) that lever up the effect of income changes may be a possible explanation. Giving insiders a tenure of limited duration may also encourage them to inflate income because they escape the market discipline with respect to future dividend payout once they are retired and they leave it to their successors to meet the raised expectations. Similarly, incentives to inflate income may arise in the run-up to an anticipated cash offer that allows insiders to cash in their shares and flee.

### 3.2 Sales-based compensation

Given that stock-based compensation does not fully eliminate the under-production problem, a natural question to ask is whether there exists a contractual compensation scheme that leads to the efficient outcome. We show below that introducing a bonus for insiders that is proportional to the observable sales can induce full efficiency.

In particular, suppose that insiders get each period a cash bonus equal to $b\, s_{t+j}$, then managers' optimization problem is now given by:

$$M_t \;=\; \max_{q_{t+j};j=0..\infty} E_t\left[\sum_{j=0}^{\infty}\beta^j\left(\pi(q_{t+j})\;-\;\theta E_{S,t+j}\left[\pi\left(q_{t+j}\right)\right]\;+\;bs_{t+j}\right)\right] \tag{34}$$

Solving this problem gives the following proposition:

**Proposition 6** *Insiders implement the first-best production decision $q_t = x_t$ if they get each period a cash bonus equal to $b\,s_{t+j}$ where the constant $b$ is given by:*

$$b \;=\; \frac{(1-\beta)\theta P}{2\left[P + R(1-\beta A)\right]} \tag{35}$$

The value for the optimal level of $b$ turns out to be surprisingly small. E.g. if the discount factor $\beta$ equals, say, 0.95 then $b$ must be below 0.025 (since $\theta < 1$). A relatively small cash bonus linked to sales can therefore eliminate the underinvestment, even if insiders have little or no stake in the firm. For example, for the parameter values used in the figures the optimal value for $b$ equals 0.0197 or almost 2% of sales. Note that the optimal value for $b$ increases with outsiders' ownership stake ($\theta$) and the variance of outsiders' income estimate (as determined by $P$).

Although sales-based compensation results in the first-best outcome, there are a number of potential issues that make this type of contract problematic in practice. The relevant decision variable in the model is output. Output is, however, not observable, and instead the contract is written on sales, an observable but noisy measure of output. The noise component in sales (e.g. due measurement error) makes it, however, problematic to verify this variable in court, which in turn makes it difficult to enforce the contract. One can show that a myopic outsider will always want to renege on the sales compensation. Even an outsider with a long term perspective may find it optimal to renege if the measurement error is sufficiently positive. Since $s_t \;=\; q_t \;+\; \epsilon_t$, each dollar of measurement error in sales costs outsiders $b$ dollars in sales compensation. Therefore, if the measurement error is too large, outsiders will refuse to pay and try to prove in court that the sales figure is unreliable. These issues might explain why sales-based compensation is less prevalent than stock compensation.[20]

# 4 Empirical implications

Our paper provides empirical implications for a variety of literatures in financial economics.

## 4.1 Time-series and cross-sectional implications

The time-series properties of income and payout were discussed in great detail in sections 2.2 and 2.3. In terms of cross-sectional analysis, our model predicts that the speed of adjustment towards the payout target should decrease with the degree of information asymmetry between inside and outside investors and with the degree of persistence (autocorrelation) in income. Our predictions are novel and can be easily tested using panel data on income and payout.

## 4.2 Real smoothing

Our model predicts that if insiders face capital market pressure then asymmetric information and the resulting inference process lead to underproduction by firms.

There is convincing empirical evidence that firms engage in real smoothing, and are pre-

[20]Even though pure stock compensation appears somewhat less effective in mitigating the underinvestment problem, it is easier to implement and enforce. First, the stock price is not only observable but also verifiable in court. Therefore, stock based compensation should be enforceable for public firms (provided the penalty from reneging on the stock compensation is sufficiently large). Second, the stock price (and therefore the stock compensation component) is less sensitive to the noise term than sales-based compensation. The reason is that the stock market partially filters out the transitory effect of measurement error.

pared to sacrifice value in order to meet earnings targets. Baber, Fairfield, and Haggard (1991) find that firms cut R&D spending to avoid reporting losses. Several studies subsequently confirm that managers deliberately cut expenditures to meet earnings expectations (see footnote 4 for further references). Bartov (1993) provides empirical evidence that managers strategically time the disposal of long-lived assets and investments to smooth earnings. Survey-based evidence by Graham, Harvey, and Rajgopal (2005) indicates that: (i) insiders (managers) always try to meet outsiders' earnings per share (EPS) expectations at all costs to avoid serious repercussions; and, (ii) many managers under-invest by postponing or forgoing positive NPV projects to smooth earnings and therefore engage in real smoothing. Roychowdhury (2006) finds that firms discount product prices to boost sales and thereby meet analyst earnings forecasts. Bhojraj, Hribar, Picconi, and McInnis (2009) find evidence suggesting that firms which cut discretionary expenditures and/or manage accruals to achieve the latest analyst forecast benchmark achieve a short-run stock price benefit, but destroy long-run firm value. Finally, Daniel, Denis, and Naveen (2012) analyze situations in which the firm's cash flow from operations is insufficient to meet its expected levels of dividends and investment. They find that among dividend-paying firms with a cash flow shortfall, over two-thirds reduce investment (relative to median industry levels). These investment cutbacks are economically significant - they constitute 65% of the shortfall.

## 4.3 Corporate ownership structure

(i) Our model predicts that the degree of income smoothing should increase in the cross-section of firms as outside ownership increases. Kamin and Ronen (1978) and Amihud, Kamin, and Ronen (1983) show that owner-controlled firms do not smooth as much as manager-controlled firms. Prencipe, Bar-Yosef, Mazzola, and Pozza (2011) also provide direct evidence for this. They find that income smoothing is less likely among family-controlled companies than non-family-controlled companies in a set of Italian firms.

(ii) In our model underproduction is more severe the smaller is the inside ownership and this results in an "outside equity Laffer curve". Morck, Shleifer, and Vishny (1988) document a non-monotonic relation between Tobin's Q and managerial stock ownership, and McConnell and Servaes (1990) report an "inverted-U" or "hump-shaped" relation between Q and managerial ownership. Numerous successors investigate the ownership-performance relation using different data, various measures of performance and ownership structure, and alternative empirical methods. The standard interpretation of the hump-shaped performance-ownership relation is that incentive alignment effects dominate for low inside ownership but, as managerial ownership increases, these incentive benefits eventually are overtaken on the margin by the cost of an increased managerial ability to pursue non-value-maximizing activities without being disciplined by shareholders. Our paper provides a new explanation for the non-monotonic relation between Tobin's Q and managerial stock ownership.[21]

## 4.4 Public versus private firms

Public (private) firms tend to have a high outside (inside) ownership. Our model therefore has a number implications for the behavior of public versus private firms.

(i) The model's main prediction is that public firms underproduce and that their output is less sensitive to economic shocks. Asker, Farre-Mensa, and Ljungqvist (2012) evaluate differences in investment behavior between stock market listed and privately held firms in the U.S. Listed firms invest less and are less responsive to changes in investment opportunities compared to matched private firms, especially in industries in which stock prices are particularly sensitive to current earnings. They show that the observed patterns are consistent with theoretical models emphasizing the role of managerial myopia. Their result is consistent with

[21] Note that the firm's replacement value is a constant in our model. Therefore, Tobin's Q is the outside equity value scaled down by a constant.

what is predicted by our model, in that firms with a higher outside ownership produce less and production is less sensitive to changes in the marginal cost variable. This result follows from the fact that insiders become increasingly concerned about "ratcheting up" outsiders' expectations as outsiders' stake in the firm increases.

(ii) Since smoother income leads to smoother payout, one would expect, all else equal, that public firms also smooth payout more than private firms. This implication is consistent with Michaely and Roberts (2012) who show that private firms smooth dividends less than their public counterparts.

(iii) Our model shows that as real ownership of outside shareholders approaches 100% the existence of the firm is in doubt. How can public firms with a low ownership stake then exist? One solution may be the introduction of audited disclosure. Income figures that are independently provided by auditors improve production efficiency because it reduces insiders' incentives to manipulate income through their production policy. Thus, all else equal higher quality accounting information should increase firm productivity, stock market capitalization, and, more generally, economic growth (as confirmed, for instance, by Rajan and Zingales, 1998). It seems implausible, however, that the existence of auditing alone resolves all inefficiencies. A standard approach in finance research suggests that managers are paid based on a stock based incentive compensation in which the share price captures all relevant information for future cash flows. Such schemes resolve the problems occurring under fixed compensation and missing incentives.

Our model (see section 3.4 on stock-based compensation) shows that stock-based compensation can act as a substitute for inside ownership, and that it is not merely a desirable but necessary component for the proper functioning of public corporations when asymmetric information is prevalent. Furthermore, while output and sales based compensation schemes may be difficult to enforce in court (output based measures may be hard to verify in court), stock-based compensation is easily enforceable for public firms. Our theory is consistent with the widespread use of stock based compensation in public corporations. Gao, Lemmon, and Li (2000) compare CEO compensation in public and private firms and find that public-firm pay - but not private-firm pay - is sensitive to measureable performance variables such as stock prices and profitability, and that when a firm goes public, pay becomes more performance-sensitive.

# 5 Related literature

We now briefly review the related literature. In a seminal paper concerning the firm and capital market interaction, Stein (1989) considers an environment where insiders can pump up current earnings by secretly borrowing at the expense of next period's earnings. When the implicit borrowing rate is unfavorable, such earnings manipulation is value destroying. Stein (1989) shows that insiders do not engage in manipulation if they only care about current and future earnings. Incentives to manipulate arise, however, if insiders also care about the firm's stock price. The market anticipates, however, that insiders engage in this form of "signal jamming" and is not fooled. Despite the fact that stock prices instantaneously reveal all information, insiders are "trapped" into behaving myopically. Thus, stock market pressures can have a dark side, even if markets are fully efficient.

There are several important differences between our model and Stein's. In Stein (1989), myopic managerial behavior takes the form of an attempt to inflate earnings so as to boost stock prices. In contrast, in our model, insiders are not directly concerned about stock prices, but fear intervention by outsiders when their expectations are not met; as a result, myopic behavior by insiders takes the form of managing earnings *downward* and underproducing so as not to set outsiders' expectations about future income too high. Further, in Stein (1989) the time-series properties of observed earnings and unmanipulated earnings are essentially the same (the difference between the two happens to be constant at all times, allowing original earnings to be reconstructed from observed earnings). In contrast, in our model outsiders' income is smooth compared to actual income and follows a simple partial adjustment model that can be linked to the underlying economic fundamentals in a very transparent and empirically testable fashion.[22] Finally, our model has important implications for payout smoothing and corporate ownership structure, whereas Stein (1989) remains silent on these matters.

In our model market pressures imply that insiders must meet payout expectations and disgorge cash to outside investors. To this end, we call upon the investor protection framework described in Fluck (1998, 1999), Myers (2000), Jin and Myers (2006), Lambrecht and Myers (2007, 2008, 2012), Acharya, Myers and Rajan (2011), among others. With the exception of Jin and Myers (2006) these papers assume symmetric information between insiders and outsiders. While under symmetric information outsiders know exactly what they are due, under asymmetric information outsiders refrain from intervention for as long as the reported income (and corresponding payout) meet their expectations. Therefore, in Jin and Myers (2006) insiders pay out according to outsiders' expectations of cashflows and absorb the residual variation, as is also the case in our model.[23] But Jin and Myers (2006) also differs from our model in a number of fundamental ways. While in their model the actual income process is completely exogenous, in our model income is endogenously determined through insiders' output decision. This allows us to identify the effect of asymmetric information on insiders' production decisions (real smoothing). Also, in Jin and Myers (2006) outsiders base their income estimates at each moment in time on their initial prior information and they do not

[22] Another difference is that in Stein (1989) stock prices are strong-form efficient at all times because outsiders can reconstruct the original earnings stream from the observed earnings. In contrast, stock prices are unbiased but only semi-strong efficient in our model because outsiders constantly learn and update their expectations on the basis of observable signals that act as a noisy proxy for the unobserved output variables seen only by the insiders.

[23] Other important but less closely related papers on smoothing include Ronen and Sadan (1981), Lambert (1984), Trueman and Titman (1988), Dye (1988), Fudenberg and Tirole (1995), Kanodia and Mukherji (1996) and Tucker and Zarowin (2006), among others.

learn about the evolution of the latent income component. As a result, there is no *intertemporal* smoothing in their model. In our model outsiders observe sales, a noisy proxy for output, which allows them to update their expectations regarding the latent marginal cost variable.

Note that the basic mechanism in our model can be considered similar to that in a strand of signal-jamming equilibrium models in which the indirect inference process distorts corporate choices. This informational effect is similar to the ones discussed (albeit in different economic settings) in Milgrom and Roberts (1982), Riordan (1985), Gal-Or (1987), Stein (1989), Holmström (1999), and more recently Bagnoli and Watts (2010).[24] The learning process (which we model as a filtering problem) and the resulting intertemporal smoothing are, however, quite different from existing papers.

Finally, our paper is also linked to a small but growing literature on payout smoothing. Kumar (1988) derives a coarse signaling equilibrium in which a firm's dividends are more stable than its performance and prospects. Guttman, Kadan, and Kandel (2010) derive an equilibrium in a Miller and Rock (1985) setup in which dividends are constant over a range of earnings. In DeMarzo and Sannikov (2011) the agent and the firm start out with zero cash, but accumulate cash in order to build a buffer stock to absorb cash and avoid inefficient liquidation. Once sufficient cash is accumulated, dividends are paid, and the optimal dividends are smoother than earnings. Lambrecht and Myers (2012) derive a Lintner model of payout based on managerial risk aversion and habit formation. Unlike these papers, our model delivers income and payout smoothing jointly, and these are associated with under-investment and therefore a real cost for the firm.

[24]While in our model insiders have an incentive not to raise outsiders' expectations regarding income, opposite incentives arise in Bagnoli and Watts (2010) who examine the interaction between product market competition and financial reporting. They show that Cournot competitors bias their financial reports so as to create the impression that their production costs are lower than they actually are.

# 6 Conclusion

The theory of income smoothing developed in this paper assumes that (i) insiders have information about income that outside shareholders do not, but (ii) outsiders are endowed with property rights that enables them to take collective action against insiders if they do not receive a fair payout that meets their expectations. We showed that insiders try to manage outsiders' expectations. Furthermore, insiders report income consistent with outsiders' expectations based on available information rather than the true income. This gives rise to a theory of inter-temporal smoothing – both real and financial – in which observed income and payout adjust partially and over time in response to economic shocks, and insiders under-invest in production. The primary friction driving the smoothing is information asymmetry as insiders are averse to choosing actions that would unduly raise outsiders' expectations about future income.

Interestingly, this problem is more severe the smaller is the inside ownership and thus should be a greater hindrance to the functioning of publicly (or dispersedly) owned firms. We show that the firm's outside equity value is an inverted U-shaped function of outsiders' ownership stake. This "outside equity Laffer curve" shows that the under-investment problem severely limits the firm's capacity to raise outside equity. However, a disclosure environment with adequate quality of independent auditing can help mitigate the problem, leading to the conclusion that accounting quality can enhance investments, size of public stock markets and economic growth. While this theory of inter-temporal smoothing of income and payout conforms to several existing findings (such as the Lintner (1956) model of payout policy), it also leads to a range of testable empirical implications in the cross-section of firms as information asymmetry and ownership structure are varied. These implications are worthy of empirical investigation.

# 7 Appendix

**Proof of Proposition 1:** The firm value is given by:

$$V_t = E_t \left[ \sum_{j=0}^{\infty} \beta^j \left[ q_{t+j} - \frac{q_{t+j}^2}{2x_{t+j}} \right] \right] \tag{36}$$

The first-order and second-order conditions with respect to $q_t$ are, respectively,

$$\frac{\partial V_t}{\partial q_t} = 1 - \frac{q_t}{x_t} = 0 \quad and \quad \frac{\partial^2 V_t}{\partial q_t^2} = -\frac{1}{x_t} < 0 \tag{37}$$

Solving the first-order condition gives the expressions for $q_t$ as in the proposition. The second-order condition is always satisfied under the reasonable (see footnote 8) assumption that production costs are positive (i.e. $x_t > 0$).

**Proof of Proposition 2:** Insiders' optimization problem can be formulated as:

$$M_t = \max_{\{q_{t+j};j=0..\infty\}} E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \pi(q_{t+j}) - \theta E_{S,t+j}(\pi(q_{t+j})) \right) \right] \tag{38}$$

We guess the form of the solution and use the method of undetermined coefficients (and subsequently verify our conjecture). The conjectured solution for outsiders' rational expectations based on the information $I_t$ is as follows:

$$E_{S,t}\left[\pi(q_t)\right] = b + \sum_{j=0}^{\infty} a_j s_{t-j} \tag{39}$$

where the coefficients $b$ and $a_j (j = 0, 1, ...)$ remain to be determined.

The first-order condition is

$$\frac{\partial M_t}{\partial q_t} = 1 - \frac{q_t}{x_t} - \theta \left( a_0 + \beta a_1 + \beta^2 a_2 + \beta^3 a_3 + ... \right) = 0. \tag{40}$$

$$\iff q_t = \left[ 1 - \theta \sum_{j=0}^{\infty} a_j \beta^j \right] x_t \equiv H x_t. \tag{41}$$

Outsiders rationally anticipate this policy and can therefore make inferences about the latent variable $x_t$ on the basis of their observation of current and past sales $s_{t-j}$ $(j = 0, 1, ...)$. We know that $s_t = q_t + \epsilon_t$. This measurement equation can be combined with the state equation (11) to make inferences about $x_t$ on the basis of current and past observations of $s_t$. This, in turn, allows outsiders to form an estimate of realized income $\pi_t$. It can be shown that the Kalman filter is the optimal filter (in terms of minimizing the mean squared error) for the type of problem we are considering (see Chui and Chen (1991)).

One can show (see Chui and Chen (1991), p78) that the error of the steady state estimator, $x_t - \hat{x}_t$, is normally distributed with zero mean and variance $P$, i.e., $E_{S,t}[x_t - \hat{x}_t] = 0$ and $E_{S,t}[(x_t - \hat{x}_t)^2] = P$, or $p(x_t|I_t) \sim N(\hat{x}_t, P)$, where $\hat{x}_t$ is given by:

$$\hat{x}_t = A\hat{x}_{t-1} + B + K\left[s_t - H\left(A\hat{x}_{t-1} + B\right)\right] = \left(A\hat{x}_{t-1} + B\right)\lambda + Ks_t \tag{42}$$

$$= \frac{B\lambda}{1-\lambda A} + K\sum_{j=0}^{\infty} \lambda^j A^j s_{t-j}\ , \text{ where} \tag{43}$$

$$\lambda \equiv (1 - KH) \qquad \text{and} \qquad K \equiv \frac{HP}{H^2P + R} \tag{44}$$

and where $P$ is the positive root of the equation (23) (the online appendix B proves that (23) has one positive and one negative root).

K is called the “Kalman gain” and it plays a crucial role in the updating process.[25] Using the conjectured solution for $q_t$ it follows that outsiders’ estimate of income at time $t$ is given by:

$$E_{S,t}[\pi_t] = E_{St}\left[Hx_t - \frac{H^2 x_t}{2}\right] = \left(H - \frac{H^2}{2}\right)\hat{x}_t \tag{45}$$

$$= \left(H - \frac{H^2}{2}\right)\left[\frac{\lambda B}{1-\lambda A} + K\sum_{j=0}^{\infty} (\lambda A)^j s_{t-j}\right] \tag{46}$$

$$= b + \sum_{j=0}^{\infty} a_j s_{t-j} \tag{47}$$

[25]If there is little prior history regarding sales $s_t$ then $K_t$ itself will vary over time because $P_t$, the variance of the estimation error, initially fluctuates over time. Once a sufficient number of observations have occurred $P_t$, and therefore $K_t$, converge to their stationary level $P$ and $K$. A sufficient condition for the filter to converge is that $\lambda\ A < 1$. The order of convergence is geometric (see Chui and Chen, 1991, Theorem 6.1 on Page 88).

where the last step follows from our original conjecture given by equation (39). This allows us to identify the coefficients $b$ and $a_j$:

$$b = \left(H - \frac{H^2}{2}\right)\left[\frac{\lambda B}{1-\lambda A}\right] \tag{48}$$

$$a_j = \left(H - \frac{H^2}{2}\right) K\,(\lambda A)^j \tag{49}$$

For this to be a rational expectations equilibrium it has to be that (see equation (41)):

$$H = 1 - \theta \sum_{j=0}^{\infty} a_j \beta^j = 1 - \frac{\theta\left(H - \frac{H^2}{2}\right)K}{1-\beta\lambda A} \tag{50}$$

Simplifying gives the condition for $H$ in the proposition. Fixing outsiders' beliefs (i.e. $E_{S,t}[\pi(q_{t+j})] = \left(H - \frac{H^2}{2}\right)\hat{x}_{t+j} \equiv h\hat{x}_{t+j}$) and solving for insiders' optimal production it follows from (14)–(16) that insiders' output strategy is a perfect Bayesian equilibrium. One can also immediately verify that the second order condition for a maximum is satisfied (assuming $x_t$ is positive).

Next, we prove that there exists a unique positive value for $H$ that satisfies (50). Substituting for $\lambda$ and $K$, equation (50) becomes:

$$f(H) \equiv 1 - H - \frac{\theta H^2 P\left(1-\frac{H}{2}\right)}{H^2 P + R(1-\beta A)} \equiv 1 - H - g(H) = 0 \tag{51}$$

Noting that $f(0) = 1 > 0$ and $f(1) = -\frac{\theta P}{2(P+R(1-\beta A))} < 0$, it follows that there exists a $H \in ]0,1[$ for which $f(H) = 0$. In the online appendix A we prove that $f(H)$ is a decreasing function, and therefore the root is unique.

Finally, we calculate the expected value and variance of the estimate's error: $\pi_t - \hat{\pi}_t$. We make use of the result that the error with respect to the steady state estimator for $x_t$ is normally distributed with zero mean and variance $P$. Hence,

$$E_{S,t}[\pi_t - \hat{\pi}_t] = E_{S,t}\left[h(x_t - \hat{x}_t)\right] = 0 \tag{52}$$

$$E_{S,t}[(\pi_t - \hat{\pi}_t)^2] = E_{S,t}\left[h^2(x_t - \hat{x}_t)^2\right] = h^2 P \tag{53}$$

**Proof of Proposition 3:** Actual income under insiders' production policy is given by:

$$\pi_t \;=\; q_t - \frac{q_t^2}{2x_t} \;=\; hx_t \tag{54}$$

We know from the proof of proposition 2 that $\hat{\pi}_t = E_{S,t}[\pi_t] = b + \sum_{j=0}^{\infty} a_j s_{t-j}$ (where the values for $b$ and $a_j$ are defined there). Lagging this expression by one period, it follows that $\hat{\pi}_t - \lambda A \hat{\pi}_{t-1} = hKs_t + h\lambda B$.

Since $d_t = \theta\hat{\pi}_t$ it follows immediately that $d_t = \lambda A d_{t-1} + \theta Khs_t + \theta h\lambda B$. Substituting this expression into the target adjustment model (26) gives:

$$\lambda A\, d_{t-1} + \theta Khs_t + \theta h\lambda B \;=\; d_{t-1} + (1-\lambda A)d_t^* - d_{t-1} + \lambda A d_{t-1} \tag{55}$$

Simplifying and solving for $d_t^*$ gives equation (28).

**Proof of Proposition 4:** We know that $E_{S,t}[x_{t+1}] = A\hat{x}_0 + B$; $E_{S,t}[x_{t+2}] = A^2\hat{x}_t + AB + B$; $E_{S,t}[x_{t+3}] = ....$

Therefore, the firm's outside equity value is:

$$\begin{aligned}
\theta V(\hat{x}_t) \;&=\; \theta E_t[\sum_{j=0}^{\infty}\beta^j \pi_{t+j}] \;=\; \theta\left[h\,\hat{x}_t + \beta\,(hA\hat{x}_t + hB) + \beta^2\left(hA^2\hat{x}_t + hAB + hB\right) + ...\right] \\
&=\; \theta\left[h\hat{x}_t\left(1 + \beta A + \beta^2 A^2 + \beta^3 A^3 + ...\right) + hB\beta\left(1 + \beta A + \beta^2 A^2 + \beta^3 A^3 + ...\right)\right] \\
&\quad +\theta\left[hB\beta^2\left(1 + \beta A + \beta^2 A^2 + ...\right) + \frac{hB\beta^3}{1-\beta A} + \frac{hB\beta^4}{1-\beta A} + ...\right] \\
&=\; \frac{\theta h}{(1-\beta A)}\left(\hat{x}_t + \frac{B\beta}{1-\beta}\right)\;.\;\diamondsuit
\end{aligned} \tag{56}$$

**Proof of Proposition 5:** The valuation equation for $S_t$ follows immediately from proposition 4 (by substituting $\beta$ by $\beta(1-\delta)$). The derivation of the equilibrium value for $H$ is as given in the proof to proposition 2 but with $\theta$ replaced by $\theta\left(1 - \frac{\delta}{1-\beta(1-\delta)A}\right)$.

**Proof of Proposition 6:** The derivation of the equilibrium value for $H$ is analogous as in the proof to proposition 2, and leads to the following equilibrium condition for $H$ (for a given value of $b$):

$$H \;=\; 1 \;-\; \frac{\theta H^2 P\left(1-\frac{H}{2}\right)}{H^2 P \,+\, R\,(1-\beta A)} + \frac{b}{1-\beta} \tag{57}$$

Setting $H = 1$ in the above equation, and solving for $b$ gives the expression in proposition 6

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# Figure 1: Production efficiency

The figure examines how production efficiency is affected by the variance of measurement errors ($R$), the variance of the latent cost variable $x_t$ ($Q$), the autocorrelation at lag one of the latent cost variable ($A$) and outsiders' real ownership stake ($\theta$). Production efficiency is measured by comparing unconditional mean output ($E(q_t)$) and unconditional mean income ($E(\pi_t)$) relative to their first-best level. The baseline parameter values used to generate the figures in this paper are: $A = 0.9$, $B = 10$, $Q = 5$, $R = 1$, $\beta = 0.95$ and $\theta = 0.8$.

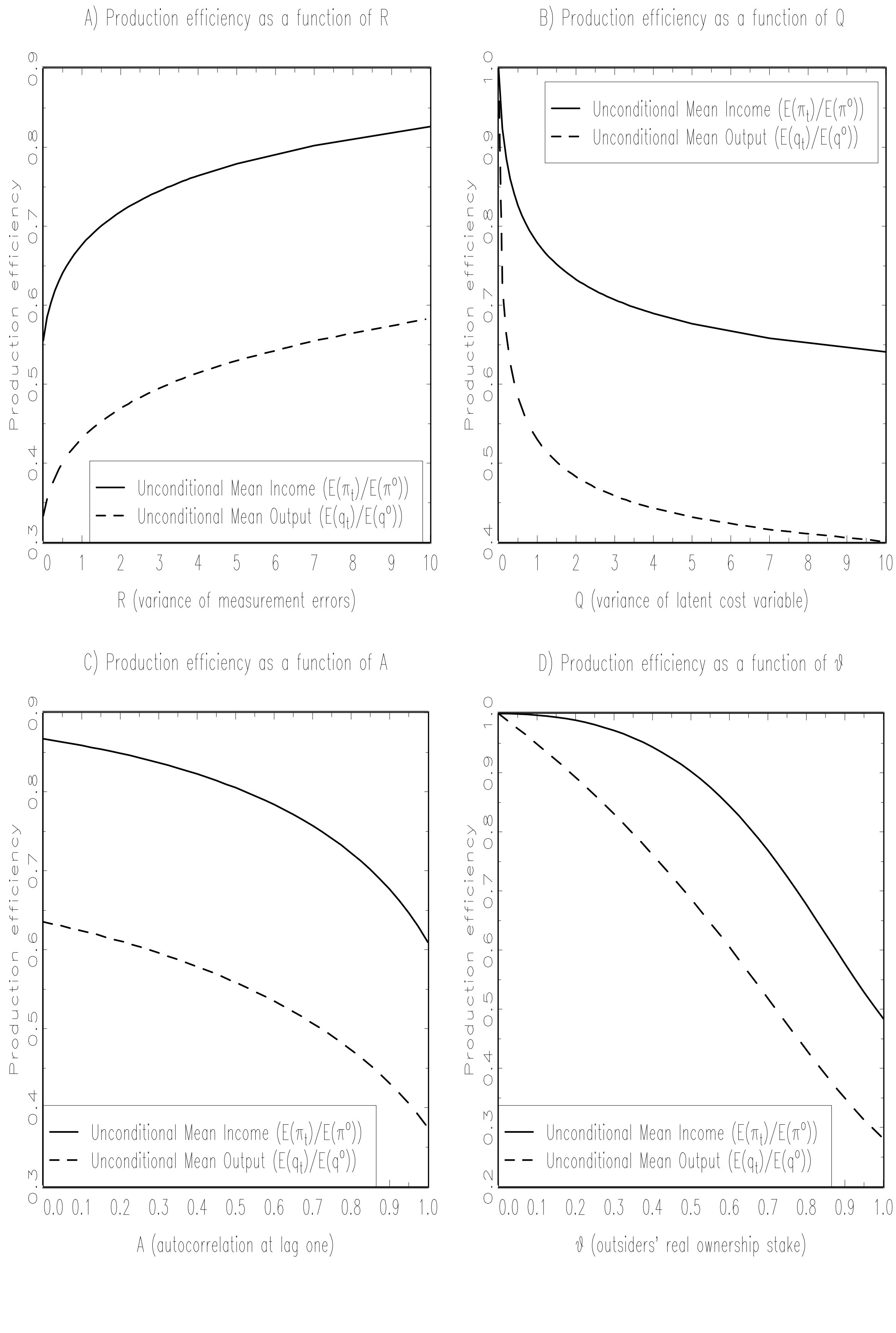

Figure 2: Speed of Adjustment

The figure examines how outsiders' real ownership stake ($\theta$), the autocorrelation at lag one of the latent cost variable ($A$), the variance of measurement errors ($R$) and the variance of the latent cost variable $x_t$ ($Q$) affect the speed of adjustment (SOA) of reported income to the income target. The speed of adjustment is given by $SOA = 1 - \lambda A$. The total amount of smoothing (measured by $A\lambda$) is split up in its two components: financial smoothing (measured by $A\lambda[H = 1]$) and real smoothing (measured by $A\lambda - A\lambda[H = 1]$). The baseline parameter values used to generate the figure are the same as before, i.e., $A = 0.9$, $B = 10$, $Q = 5$, $R = 1$, $\beta = 0.95$ and $\theta = 0.8$.