

# When is Debt Odious?

## A Theory of Repression and Growth Traps<sup>†</sup>

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### Abstract

How is a developing country affected by its government's ability to borrow in international markets? We examine the dynamics of a country's growth, consumption, and sovereign debt, assuming that the government's objective is to maximize short-term, typically socially unproductive, expenditures. Sovereign debt can extend the government's effective horizon; the government's ability to borrow hinges on its convincing investors they will be repaid, which gives it a stake in the future. The lengthening of the government's effective horizon can incentivize it to adopt policies that result in higher steady-state household consumption than if it could not borrow. However, access to borrowing does not always improve government behavior. In a developing country that saves little, the government may engage in repressive policies to enhance its debt capacity, which only ensures that successor governments repress as well. This leads to a "growth trap" where household steady-state consumption is lower than if the government had no access to debt. We argue that such a model can us understand the well-known negative correlation between a developing economy's reliance on external financing and its economic growth. We also analyze the effects of debt relief, debt ceilings, and fiscal transfers in helping a developing economy emerge out of a growth trap.

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# 1 Introduction

Is the ability to borrow in international markets good for a country, especially for a developing one? Many theories of international borrowing emphasize the better risk-sharing a country can achieve – in case of economic or natural calamity, it can borrow to smooth consumption – as well as its ability to draw on international savings to finance domestic growth (see, for example, Kletzer and Wright [2000]). Yet it is hard empirically to see a positive correlation between a developing country’s use of foreign financing and good outcomes such as stronger economic growth (see Aizenman, Pinto and Radziwill [2004], Prasad et al. [2006], and Gourinchas and Jeanne [2013]). What might explain the divergence between theory and evidence?

One weakness with many existing models is that they tend to assume that the government of the country in question maximizes the utility of its citizenry over the long run. Yet an important reality in many developing countries is that their governments are often myopic, wasteful, and even rapacious, one reason these countries are poor. A second weakness is related to the first. Once the government is assumed to maximize the welfare of its citizenry, often the best thing it can do is to default on its foreign debt (Bulow and Rogoff [1989a], Bulow and Rogoff [1989b], and Tomz [2012]). To explain the existence of sovereign debt, researchers then have to appeal to a variety of mechanisms that enforce sovereign repayment such as a government’s concern for its reputation or punishment strategies by other countries. Unfortunately, there is little empirical evidence for these mechanisms (Eichengreen [1987], Özler [1993], Flandreau and Zumer [2004], Sandleris et al. [2004] and Arellano [2008]).

In this paper, we start with an extreme view of a government, that it is myopic and all of its spending does little for the welfare of its citizenry. It turns out that in this setting it is relatively easy to explain the enforceability of sizeable amounts of sovereign borrowing even with small costs of default. We then ask whether access to sovereign borrowing, taking into account the need for international investors to be confident that the borrowing will be repaid, is welfare-improving for the country’s citizens.

An emerging literature on “odious” debt takes the view that it is not (see Buchheit, Gulati and Thompson [2006] and Jayachandran and Kremer [2006]). The ability to borrow essentially gives the government more resources to waste or steal, with the repayment eventually extracted by international lenders from the citizens. Therefore, some commentators advocate declaring debt issued by such governments odious, and recommend limiting the enforcement of such debt in international courts. The value of such proposals turn on whether a country’s citizens will be better off when it no longer has the ability to borrow (see, for example, Rajan [2004]). The answer, it turns out, is not straightforward.

Under some circumstances, citizens can be better off when their government can borrow.

While the government certainly will waste the resources it can collect, its ability to borrow gives it a stake in the future – for future resources are necessary to assure investors of future repayment. The horizon-lengthening aspects of debt can actually ameliorate the government’s behavior, leading it to set lower taxes when it can borrow, so that citizens have higher endowments and consumption in the long run.

Interestingly, though, under different parameters, the government’s desire to expand its debt capacity can also make its behavior towards citizens even worse for them than if it did not have the capacity to borrow – leading the country into a growth trap. This double-edged nature of sovereign borrowing, and the parameters it depends upon, allows us to frame testable implications on the kinds of countries that might benefit from access to sovereign borrowing, and the kinds of countries for whom it might be odious, keeping the nature of the government fixed (as uniformly myopic and self-interested). Of course, our implications would be altered if governments worked more in the long-term public interest, but so would the enforceability of debt, as we will argue. At any rate, our analysis offers a baseline from which we can examine a number of issues such as debt limits, debt restructuring, and debt forgiveness.

Let us explain in more detail. Consider an overlapping generations model of a country with a representative young citizen each period – the citizen is a composite of the households and the productive private sector, and we will use these terms interchangeably. The other agents in the model are the government and international investors.

The representative citizen has an initial endowment (smaller if a poorer country)<sup>1</sup> that she can either consume, save in domestic government bonds, or invest in private enterprise. She maximizes the sum of her consumption this period and the discounted endowment left behind for the next generation, a proxy for the future stream of her descendants’ consumption.

The myopic government rules only one period, and thus has a short horizon. Initially, it is assumed to spend in ways that do not enhance citizen welfare. This could be thought of as wasteful populist spending (such as election propaganda), white elephant projects (such as gigantic power plants that are not economic to run), or plain theft (luxury flats in Miami or London or Cayman Island bank accounts). The precise nature of the spending does not matter, only that it does not add to citizen utility. The government maximizes the resources it can raise for spending, which consist of the sum of the taxes it levies on private sector output and the amount it can raise through debt issuance (net of repayment of past debt).

Government debt is issued to both domestic investors and foreign investors in the form of bearer bonds, and we assume the government cannot tell who holds its debt.<sup>2</sup> Successor

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<sup>1</sup>Throughout the paper, we refer to “poorer” country as one that has a smaller initial endowment, *ceteris paribus*, that is, holding constant all other parameters of the economy.

<sup>2</sup>See, for example, [Broner, Martin and Ventura \[2010\]](#). This assumption ensures the government cannot default selectively on foreigners.

governments inherit the obligation to repay sovereign debt, though they can default. If the government defaults on past debt, it pays the default cost and cannot issue new debt for the rest of the period. In that case, which we term “debt autarky”, it will set the tax rate on private output at the level corresponding to the maximal point of the Laffer Curve – the level that trades off the disincetivizing effect of higher taxes on private investment against the revenue impact.

International investors do not care about the quality of spending, but will lend only if they expect to get their money back with interest. Therefore, given the model has no uncertainty, there will be no over-lending, and no default. This allows us to highlight the central tradeoffs.

Thus far, this is a fairly standard set up. What is somewhat different is the source of the incentive for the government to repay. The government cannot default selectively on foreign investors alone since it cannot tell domestic holders of government bonds apart from foreign holders. So if it defaults, domestic investors experience significant losses:<sup>3</sup> If these are banks, the government will have to bail them out to have any reasonable economic output; if these are traders, there may be a dearth of safe assets in the economy with which to do collateralized transactions; if these are individuals, the government will have to assuage them so they do not take to the streets to throw it out immediately. Regardless of the precise reason for the deadweight cost of default, we assume it rises in the size of sovereign bonds held by domestic investors. So the government does not default on the debt for two reasons. First, it will incur the deadweight cost immediately. Second, it has a short horizon, so it does not trade off the present value of the outstanding debt against the deadweight cost but only the net debt repayments it has to make in its period in power against the deadweight cost. This implies that a sizeable amount of debt issuance can be supported with modest deadweight costs.

The government’s ability to borrow alters the tax it will impose on the real sector – as we will see later, this tax could also be seen as a measure of the financial repression in the economy, which channels savings to the government. The higher the tax it imposes, the lower the amount that the private sector allocates to real investment, leaving more of its endowment to consumption and financial savings (in government bonds). So when a government has access to sovereign debt issuance, its taxation is driven by two sets of opposing concerns. A higher tax rate will curb real investment, resulting in lower future revenues to be taxed. It will also reduce the surplus available to future governments to repay debt, thus lowering how much debt can be raised today. A higher tax rate therefore reduces the government’s *ability to pay*. But it will also raise the domestic private sector’s financial savings in government debt, increasing the *willingness* of a future government to repay the debt, and thus increasing how much debt can be issued today.

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<sup>3</sup>See, for example, Bolton and Jeanne [2011], Acharya, Drechsler and Schnabl [2014], Gennaioli, Martin and Rossi [2014], Andrade and Chhaochharia [2018], and Farhi and Tirole [2018].

So whether the country's ability to issue debt raises or lowers the tax that the myopic rapacious government imposes on the private sector depends on which of these incentives predominates. For instance, if the citizen has a high propensity to save, the government may have little need to channel more into financial savings, and it will lower tax rates relative to autarky. The government's ability to issue debt here tends to be beneficial for the citizen over the long run because the need to convince debt holders of repayment limits the government's rapacity and enhances steady-state consumption. Conversely, if the citizen has a low propensity to save, the government may set higher-than-autarky tax rates. This could push the country into a lower consumption "growth trap", precisely because the rapacious government tries to enhance its debt issuance. For the citizens of such countries, sovereign debt is truly odious.

Another key factor driving whether a country gets trapped is the size of the citizen's initial endowment. The higher the endowment (that is, the richer the country), the greater the amounts the citizen will invest in government bonds after meeting consumption and real investment needs, and the easier external debt will be to enforce without repressive government taxation. All this then suggests a potential explanation to the puzzle of the weak or negative correlation between developing country growth and the use of foreign borrowing; governments of countries that already have a high domestic propensity to save will likely be most growth-friendly in their policies, as will richer countries. But these are precisely the countries that will rely least on foreign borrowing. As a result, the absence of a positive correlation between foreign borrowing and economic growth for developing countries may stem from the endogenous selection of which countries rely more on foreign borrowing.

Typically, debt relief in this model will do little for a country's citizens, even when it is in a growth trap. The government will simply use the expanded space to borrow, and spend it quickly. It will soon be back to pre-relief levels of debt – this was a common concern with the debt relief measures undertaken in developing countries in the late 1990s and early 2000s. Indeed, in a number of cases, debt relief has had little long-term effect as governments continue to mispend.<sup>4</sup> These countries have returned to situations of debt stress.

In contrast to the ineffectiveness of debt relief in the model, debt ceilings that also bind on future governments can be desirable. In particular, countries could decide to limit their own ability to borrow through a constitutional debt ceiling. Alternatively, well-intentioned international lenders could set informal debt limits for countries – though this requires a collective agreement since the country is capable of servicing more debt at the ceiling. We examine the consequences of such debt limits, and discuss why they might be more effective in developing countries than in rich countries, even if the quality of their governments is similar.

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<sup>4</sup>See <https://www.worldbank.org/en/topic/debt-sustainability#2> for a list of countries and the risk of debt distress prepared by the World Bank.

We also show that governments of developing countries may find it desirable to transfer some of the funds raised by debt to the citizen to bolster her endowment; when the citizen's propensity to save is high enough, this can boost the government's debt capacity, and at the same time, set the economy on a higher growth path. Importantly, however, for debt ceilings and fiscal transfers to be effective, we need some additional source of government commitment than present in the basic model.

Of course, our assumption that the government is both myopic and unconcerned about citizen welfare is a caricature, perhaps on par with the more traditional assumption that the government cares only about its citizenry and is benevolent. We trace the consequences of moving away from this assumption. If we allow government spending to be useful to the economy, then the outcome depends on the specific nature of how it is useful: if government spending leads to output that accrues only to future governments for spending (e.g., toll roads), then myopic governments may not invest in such spending when they lack the willingness to repay (at low endowments) and are therefore unable to borrow against such enhanced output; in contrast, if government spending leads to future output that augments household endowment (for example, allocations to social security endowments), then the myopic government may invest in such spending when it increases its debt capacity (at low endowments) but not otherwise (at high endowments). Finally, if the government is sufficiently far-sighted in nature (as characterized by its discount rate on future spending), then its capacity to borrow can collapse leading to autarky. The collapse in access to borrowing naturally improves economic outcomes when access to debt leads to a growth trap and worsens them when such access leads instead to a growth boost. This suggests what is crucial to our model and the broad thrust of its conclusions is not the assumption of wasted spending but of myopic government objectives.

Despite the fact that government defaults are costly by design in our model, we observe also that countries in a growth trap can benefit from default – potentially caused by unanticipated changes in parameters such as the interest rate. Because growth is suppressed by the government's repressive policies, a significant one-period boost to growth can arise from the economy entering debt autarky post-default (see [Levy-Yeyati and Panizza \[2011\]](#) for empirical evidence on the positive effects of sovereign default on growth). In some cases, the boost can be even larger than the cost of default such that in the medium run the economies outgrow their original endowment levels. They may even emerge out of the trap.

The rest of the paper is as follows. In Section 2, we discuss the baseline model and the main Bellman equation capturing the model dynamics. In Section 3, we present an in-depth analysis of the properties of the baseline model solution and explain how a growth trap arises. In Section 4, we analyze what happens when government spending is not entirely wasteful. In Section 5, we analyze various policy instruments that can help the economy escape the growth trap. In

Section 6, we discuss the impact of unanticipated permanent small shocks to the economy in the steady state. In Section 7, we offer concluding remarks and possible future extensions.

## 2 Baseline Model

We consider an overlapping generations model. Time is discrete and the horizon is infinite. The world consists of a single country and the rest of the world. The country is a small open economy with two agents, the private sector and the government. Foreign investors invest in the country's sovereign debt as well as its private sector's debt. We assume all debt issued is single-period maturity at a required world interest rate of  $r > 0$ . The timeline of the model is shown in Fig. 1.

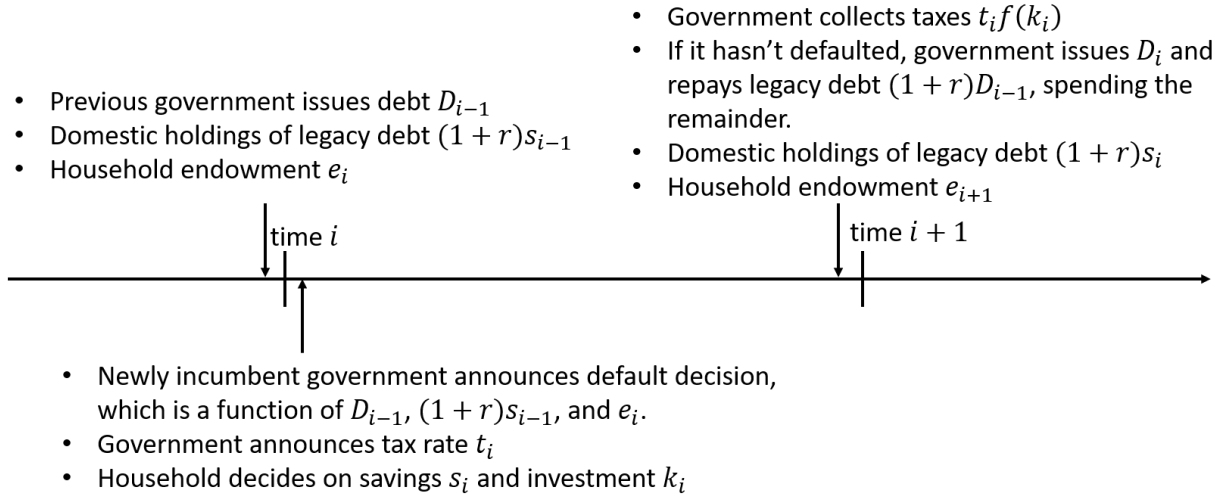


Figure 1: Timeline of the model.

Consider period  $i$ . The private sector is a representative household that maximizes the sum of the log of current period consumption  $c_i$  and the log of next period endowment  $e_{i+1}$  (representing the endowment it leaves for the next generation) times a parameter  $\rho$ , where  $\rho \in (0, \frac{1}{r})$  captures the overall preference for savings (bequest) of the household. At the beginning of the period  $i$ , the household inherits an endowment  $e_i$ , which it allocates to financial savings  $s_i$  and physical investment  $k_i$  so as to maximize utility. The household has a mild home bias so financial savings are invested in domestic government bonds at the rate  $r$  (rather than internationally) whenever the government borrows. Physical investment produces  $f(k_i)$  at the end of the period, where  $f' > 0$  and  $f'' < 0$ . The government can potentially tax the production at a rate  $t_i$ , in which case the total proceeds for the household from production is  $(1 - t_i)f(k_i)$ .

We assume the private household's financial savings into government debt are not taxed (equivalently, it bears a relatively lower tax than household investment in real assets). This is a key assumption. Consider three justifications. First, fixed hard assets are easier to tax than fungible financial savings. Since financial savings are more mobile and also easily converted to concealable assets like gold, the government typically keeps taxes on financial savings relatively low. Second, we have in mind here both actual taxes as well as the implicit taxes the government collects through corruption, which usually falls more heavily on business enterprise. Most important, though, governments that need resources tend to direct flows toward themselves through *financial repression*. For instance, financial institutions are required to allocate a significant part of their assets to government debt, crowding out the private sector's access to finance (effectively a tax).<sup>5</sup> For simplicity, we do not model any of these effects, assuming they are fully captured by the tax falling only on real investment. It should be kept in mind, though, that real repression (high taxes on private sector real investment) and financial repression (guiding financial savings into government instruments) are two sides of the same coin.

The private sector's problem can be summarized by the following constrained optimization problem:

$$\max_{c_i, e_{i+1}, k_i, s_i} \ln c_i + \rho \ln e_{i+1} \quad (2.1)$$

$$s.t. \quad c_i + s_i + k_i \leq e_i, \text{ and} \quad (2.2)$$

$$e_{i+1} \leq (1 + r)s_i + (1 - t_i)f(k_i). \quad (2.3)$$

The government in our model is incumbent for only one period and its sole objective is to maximize its wasteful spending, wasteful in that it does not directly augment the economy's endowment or private consumption. The spending could be on itself (high government salaries or corruption), on grandiose white elephant projects, or on political propaganda. It finances the spending by imposing a tax on the private sector, as well as issuing debt which is sold to both domestic and foreign investors. We assume the government cannot default selectively on foreign debt holders, which would be true if it issued bearer bonds. All we really need, however, is that a default on external sovereign debt spills over to domestic debt. This is hardwired in the model by assuming the two forms of debt are indistinguishable, but there is a variety of other sources of spillover that could be invoked.<sup>6</sup>

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<sup>5</sup>In this vein, [Gennaioli, Martin and Rossi \[2018\]](#) find that there is a negative and statistically significant correlation between a bank's holding of domestic government bonds and its ratio of loans to asset, especially in developing countries.

<sup>6</sup>There is evidence consistent with such spillovers. [Borensztein and Panizza \[2009\]](#) show that public defaults are associated with banking crises; [Brutti \[2011\]](#) finds more financially dependent sectors tend to grow relatively

The government can decide whether to default or to repay the maturing debt that the previous government issued. If it defaults, the economy's infrastructure incurs direct damage – for instance, banks holding government debt are “run” upon, the payment system freezes, and repo markets collateralized by government debt are disrupted. To ensure the private sector produces (and can be taxed) the government has to commit a part of its spending on cleaning up the disruption. We model this cost as  $C + zD^{Dom}$ , where  $C > 0$ ,  $z > 1$  are constant parameters and  $D^{Dom}$  is the face value of government debt held by the domestic residents at the time of default;  $C$  captures a fixed cost of default, whereas  $zD^{Dom}$  captures the idea that the default cost is increasing in the face value amount that the domestic private sector has invested in the government debt.<sup>7</sup> In addition to this cost, the government is excluded post default from debt markets for that period – this could be thought of as the period the debt is being renegotiated (down to zero for simplicity in our model); the government thus experiences “debt autarky” with no access to the sovereign debt market. We assume that the investors – both domestic and foreign – are fully rational and are therefore willing to lend to the government only to the extent that the debt will be fully repaid in the next period.

## 2.1 Household problem

The government decides whether to service legacy debt, sets the tax rate, and issues the maximum new debt consistent with these decisions, while expecting the household to maximize its utility in reaction to government policy. Start with the household's problem in period  $i$ . The representative household receives an endowment  $e_i$  from the past generation, and takes the tax rate  $t_i$  as given.<sup>8</sup> It solves the maximization problem in (2.1). Let us set  $\lambda$  and  $\mu$  as the Lagrangian multipliers for constraints in (2.2) and (2.3), respectively. The corresponding Lagrangian is the following:

$$\mathcal{L} = \ln c_i + \rho \ln e_{i+1} - \lambda(e_i - c_i - s_i - k_i) - \mu[(1+r)s_i + (1-t)f(k_i) - e_{i+1}].$$

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less after sovereign default; [De Paoli, Hoggarth and Saporta \[2009\]](#) show that sovereign default is associated with substantial output costs for the domestic economy; [Arteta and Hale \[2008\]](#) use firm-level data to show that syndicated lending by foreign banks to domestic firms declines after default; [Ağca and Celasun \[2012\]](#) also use firm-level data to show the corporate borrowing costs increase after default.

<sup>7</sup>Because household savings  $s$  can be negative in our model, we need a high enough  $C$  to ensure that the default cost itself never becomes negative.

<sup>8</sup>To keep the timing straight, we assume only financial investment held between periods accrues interest, real investment takes place at the beginning of the period, real production pays taxes to the government at the end of the period, and the after-tax production is returned to households at the beginning of the next period as part of their endowment.

Obtaining the first order conditions (FOC's) for our four choice variables yields:

$$c_i : 0 = \frac{1}{c_i} + \lambda; \quad (2.4)$$

$$s_i : 0 = \lambda - (1 + r)\mu; \quad (2.5)$$

$$k_i : 0 = \lambda - (1 - t_i)f'(k_i)\mu; \text{ and} \quad (2.6)$$

$$e_{i+1} : 0 = \frac{\rho}{e_{i+1}} + \mu. \quad (2.7)$$

We show in Lemma B.1 in the appendix that FOC's (2.4) - (2.7) lead to the following set of decision functions for the households:

$$k_i = f'^{-1}\left(\frac{1+r}{1-t_i}\right), \quad (2.8)$$

$$c_i = \kappa_0[(1+r)(e_i - k_i) + (1-t_i)f(k_i)], \quad (2.9)$$

$$e_{i+1} = \kappa_1[(1+r)(e_i - k_i) + (1-t_i)f(k_i)], \text{ and} \quad (2.10)$$

$$s_i = \kappa_1(e_i - k_i) - \kappa_0(1-t_i)f(k_i); \text{ where} \quad (2.11)$$

$$\kappa_0 := \frac{1}{(1+\rho)(1+r)}; \text{ and } \kappa_1 := \frac{\rho}{1+\rho}.$$

The government chooses the tax rate and debt issuance knowing the household will react according to (2.8), (2.9), (2.10), and (2.11).

*Remark 2.1.* We discuss some properties of the solutions (2.8) - (2.11).

- (1) Note from (2.10) that the next-period endowment depends on the current-period endowment linearly with a coefficient  $\kappa_1(1+r)$ . In order to rule out exploding economies, we impose a condition that  $\kappa_1(1+r) < 1 \Leftrightarrow \rho < 1/r$ .
- (2) Note from (2.8) that the total amount of tax collected by the government is  $tf(k(t))$ , a single variable function of  $t$ . We denote this function as  $\tau(t)$ .
- (3) Note from (2.9) and (2.10) that  $\forall i, c_i = \frac{1}{\rho(1+r)}e_{i+1}$ . This implies that there is a one-to-one relationship between the level of endowment and consumption in the model.
- (4) Note in (2.11) that household financial savings is increasing in the tax rate  $t$ , and in (2.8) that investment is decreasing in  $t$ .<sup>9</sup>

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<sup>9</sup>Under the log-utility assumption for households, both investment declines and savings increase with the tax rate  $t$ ; in other words, economic and financial repression map one-for-one in this case. With a more general utility function for households, the impact of the tax rate on savings would depend on the elasticity of inter-temporal substitution (EIS); what is crucial to note is that the government's incentive in the willingness-to-pay region is to channel domestic savings to its bonds, for which it may in general have to employ financial repression explicitly, beyond economic repression.

## 2.2 Benchmark case without debt: Debt autarky

The benchmark case is one where the government cannot issue any debt. Since this government can only spend what it raises from tax, it will simply choose a tax rate that maximizes tax revenues  $\tau(t)$ . Let  $**$  denote this benchmark “debt autarky” case:

$$\begin{aligned} t^{**} &:= \text{benchmark tax rate} = \underset{t}{\operatorname{argmax}} \tau(t), \\ k^{**} &:= \text{benchmark investment} = k(t^{**}), \text{ and} \\ \tau^{**} &:= \text{benchmark tax revenue} = \tau(t^{**}). \end{aligned}$$

For instance, in the case of a power production function  $f(k) = Ak^\gamma$ ,  $t^{**} = 1 - \gamma$ .

## 2.3 Optimization problem of myopic government with debt

Consider now the government’s problem. It has legacy debt  $(1+r)D_{i-1}$  due, of which  $(1+r)D_{i-1}^{Dom}$  is held domestically. Suppose for now that the government finds default suboptimal and decides to pay back the legacy debt. It finances its spending by issuing debt  $D_i$  and collecting taxes from the private sector at rate  $t_i$ . It expects the household to react as in (2.8), (2.10), and (2.11). Suppose that the maximum resource that next period’s government can raise – through taxation and borrowing – is  $S_{i+1}$ . Debt issuance  $D_i$  today is then constrained by the next-period government’s *ability to pay*:

$$D_i(1+r) \leq S_{i+1}. \quad (2.12)$$

Consider now the next-period government’s *willingness to pay*. In the event that the next-period government defaults, its tax revenues are at the autarky level  $\tau^{**}$ . It follows that in order for the next-period government to be willing to pay, the amount it can spend if it doesn’t default should be more than  $\tau^{**}$  minus the spending to clean up the post-default financial disruption:

$$\begin{aligned} \underbrace{S_{i+1} - D_i(1+r)}_{\text{net spending on no default}} &\geq \underbrace{\tau^{**}}_{\text{revenues in autarky}} - \underbrace{(C + zD_i^{Dom}(1+r))}_{\text{spending to clean up default}} \quad (2.13) \\ \Rightarrow D_i(1+r) &\leq S_{i+1} + zD_i^{Dom}(1+r) + C - \tau^{**} \\ \Rightarrow D_i(1+r) &\leq S_{i+1} + zs_i(1+r) + C - \tau^{**}. \quad (\text{in equilibrium}) \end{aligned}$$

Since both the ability-to-pay constraint as well as the willingness-to-pay constraint must be

met, the effective constraint on current-period debt is

$$\begin{aligned} D_i(1+r) &\leq \min\{S_{i+1}, S_{i+1} + z s_i(1+r) + C - \tau^{**}\} \\ \Rightarrow D_i(1+r) &\leq S_{i+1} - \max\{0, \tau^{**} - C - z s_i(1+r)\}. \end{aligned} \quad (2.14)$$

It can be seen that  $\tau^{**} - C - z s_i(1+r) = 0$  traces the threshold between willingness-to-pay and ability-to-pay constraint; when  $\tau^{**} - C - z s_i(1+r)$  is positive, the willingness-to-pay constraint is binding, whereas when it is negative, the ability-to-pay constraint is binding. Notice from (2.11) that  $s_i$  is increasing linearly in  $e_i$ . This implies that for sufficiently high endowments,  $\tau^{**} - C - z s_i(1+r) < 0$ , implying that the ability-to-pay constraint is binding. Conversely, for sufficiently low levels of endowment, the willingness-to-pay constraint is binding

Constraint (2.14) highlights the double-edged nature of sovereign debt that is at the heart of our model. On the one hand, the willingness-to-pay constraint implies  $D_i$  increases in  $s_i$ , which incentivizes the myopic government to repress investment with higher taxation in order to boost financial savings in government debt. On the other hand, when focusing on the next-period government's resources available to pay debt (its ability to pay), it turns out that  $D_i$  increases in  $S_{i+1}$ , which increases in  $e_{i+1}$ . In this case, the current-period government has an incentive to increase  $e_{i+1}$  by lowering taxation and boosting growth. As we show in the following sections, the government can under-tax or over-tax – relative to our benchmark case, which is the debt autarky optimum ( $\arg\max_t t f(k(t))$ ) – depending on which term is more sensitive. If  $S_{i+1}$  is more sensitive to current-period taxation than the penalty term  $\max\{0, \tau^{**} - C - z s_i(1+r)\}$ , then the myopic government will choose a lower-than-benchmark tax rate, otherwise it will choose a higher-than-benchmark tax rate. Since for the current-period government,

$$\text{spending} = S_i - \text{legacy debt} = \max_t [D_i(t) + \tau(t)] - D_{i-1}(1+r), \quad (2.15)$$

and the debt capacity  $D_i(t)$  depends on  $S_{i+1}$ , the problem is inherently infinite-horizon, even though the myopic government only optimizes a one-period problem. This is why debt is potentially a horizon-lengthening device.

Let us formulate this problem recursively. Note that a myopic government  $i$  takes  $e_i$ ,  $D_{i-1}^{Dom}$ , and  $D_{i-1}$  as given, and maximizes (2.15). This implies that the natural set of state variables is  $(e_i, D_{i-1}^{Dom}, D_{i-1})$ ; however, since  $D_{i-1}$  enters (2.15) only additively, the maximization problem is independent of  $D_{i-1}$ . Moreover,  $D_{i-1}^{Dom}$  only governs the government's decision to default or not. Therefore, conditional on the government finding default suboptimal, the only state variable is  $e_i$ . Furthermore, since a myopic government will always choose  $D_i$  at the maximum, we can replace  $D_i$  with the expression in (2.14). Note that since the maximum is derived from no-

default condition for the next government, there will be no government defaults in our model on the equilibrium path. Therefore, we have:

**Lemma 2.1.** *(Main Bellman equation)*

$$S(e) = \max_t \left[ \frac{1}{1+r} [S(e') - \max\{0, \tau^{**} - C - zs(1+r)\}] + \tau(t) \right] \quad (2.16)$$

$$s.t. \quad e' = \kappa_1 [(1+r)(e - k(t)) + (1-t)f(k(t))], \quad (2.17)$$

$$s = \kappa_1(e - k(t)) - \kappa_0(1-t)f(k(t)), \text{ and} \quad (2.18)$$

$$k(t) = f'^{-1}\left(\frac{1+r}{1-t}\right). \quad (2.19)$$

The value function  $S(e)$  as well as the policy function  $t(e)$ , i.e., the decision rule conditional on the myopic government finding default suboptimal, constitute the complete solution for (2.16).

The decision rule encompassing default can be obtained by revisiting the two constraints, (2.12) and (2.13); for endowment  $e$ , legacy domestic debt holdings  $D_{-1}^{Dom}$  (the face value of which is  $(1+r)D_{-1}^{Dom}$ ), and legacy debt  $D_{-1}$  (the face value of which is  $(1+r)D_{-1}$ ),

- (i) If  $S(e) - (1+r)D_{-1} < 0$ , the government cannot pay back the legacy debt and defaults. Upon default, it enters autarky and charges the autarkic tax rate  $t^{**}$ .
- (ii) If  $S(e) - (1+r)D_{-1} < \tau^{**} - C - z(1+r)D_{-1}^{Dom}$ , the government potentially can pay back the legacy debt, but finds defaulting more advantageous. In other words it defaults strategically, enters autarky, and charges the autarkic tax rate  $t^{**}$ .
- (iii) If neither of the above two conditions apply, then the government pays back the legacy debt, charges tax  $t(e)$  and issues  $D(e) := S(e) - \tau(t(e))$  amount of debt. Government spending is  $S(e) - (1+r)D_{-1}$ .

Finally, note that the debt issuance  $D(e)$  can be further decomposed into domestic and foreign debt:

$$D^{Dom} := \text{Domestic debt} = s(e, t(e)), \text{ and} \quad (2.20)$$

$$D^{For} := \text{Foreign debt} = \text{Total debt} - \text{Domestic debt} = D(e) - s(e, t(e)). \quad (2.21)$$

## 2.4 Model solution

Fig. 2 shows a solution from the model specialized to  $f = 3k^{.65}$ ,  $r = 10\%$ ,  $z = 4$ ,  $\rho = 2.3$ , and  $C = 1$ . We have

$$e_+(e, t) := \kappa_1 [(1+r)(e - k(t)) + (1-t)f(k(t))]; \quad (2.22)$$

$$s(e, t) := \kappa_1(e - k(t)) - \kappa_0(1 - t)f(k(t)); \quad (2.23)$$

$$\pi(e, t) := (1 - t)f(k(t)) - (1 + r)k(t). \quad (2.24)$$

as the next-period endowment, financial savings, and private profit from investment, respectively, in reaction to current-period endowment  $e$  and tax rate  $t$ . We prove formally in Proposition 2.2 that the solution possesses the following properties which are illustrated in Fig. 2:

- There exists a low- $e$  region (see Fig. 2, regions annotated “WTP”) where only the willingness-to-pay constraint is binding. Depending on the parameter set, there can be a steady state below a threshold endowment  $\bar{e}^1$  such that for  $\forall e < \bar{e}^1$ ,  $t(e) = t^W > t^{**}$ . This is driven by the fact that the willingness-to-pay constraint potentially incentivizes the government to repress investment so much that the economy never escapes that region, and the ability-to-pay constraint is rendered irrelevant.
- There exists a middle- $e$  region (see Fig. 2, regions annotated “WTP & ATP”) where the optimal solution for the government is to “slide” between the two constraints, *i.e.*, setting  $\tau^{**} - z(1 + r)s = 0$ . In this region, the policy tax rate  $t(e)$  is always strictly decreasing in  $e$  (see Fig. 2(b)). Financial savings are flat over this region (see Fig. 2(c)) while the falling tax rate incentivizes allocation to real investment (see Fig. 2(d)).
- There exists a high- $e$  region (see Fig. 2, regions annotated “ATP”) where only the ability-to-pay constraint is binding. Depending on the parameter set, there can be a steady state after a threshold endowment  $\bar{e}^1$  such that for  $\forall e > \bar{e}^2$ ,  $t(e) = t^{**} := \arg\max_t \tau(t)$ . This is driven by the fact that large-endowment economies have so much domestic savings that default is ruled out. However, when the willingness-to-pay constraint is not binding, the size of the government’s surplus and its ability to borrow does not vary with the private sector endowment – the sensitivity of debt to endowment tapers off at high levels of endowment in Fig. 2(e). Since the government wants to lower taxes only so as to enhance future private sector endowments (in order to affect future government surplus and hence current borrowing capacity), this channel is no longer operative. Therefore, the government’s taxation incentives are not affected by access to borrowing here. In other words, a myopic government with a wealthy private sector taxes as if it has no access to debt, *i.e.*, our benchmark autarkic case.

[Fig. 2 about here]

We formalize the findings above in Proposition 2.2. Note that the proposition requires a set of regularity conditions set out in Definition 2.1, imposed mainly to ensure convexity and single-crossing properties of the derived functions. Any power production function of the form

$f(k) = Ak^\gamma$  automatically meets regularity conditions A and B, and therefore will be used in all our numerical exercises throughout (as in Fig. 2).

**Definition 2.1.** We assume that the following regularity conditions are met:

- A. (Convexity of investment in  $t$ )  $k(t)$  is decreasing and convex in  $t$ , from which it follows that private profit  $\pi(t)$  is also decreasing and convex in  $t$ .
- B. (Single-crossing properties)  $\frac{k'(t)}{\pi'(t)}$  is decreasing in  $t$ , and  $\frac{\tau'(t)}{\pi'(t)}$  is strictly increasing in  $t$ .
- C. (Minimal government feasibility in autarky)  $\tau^{**} > C$ .

**Proposition 2.2.** *There is a unique bounded and weakly monotonic value function  $S(e)$ , and a corresponding policy function  $t(e)$ , that solve (2.16). Suppose that model's specifications satisfy the regularity conditions in Definition 2.1. Then, the solution has the following properties:*

- (i)  $S(e)$  is weakly concave, and  $S'(e) \rightarrow 0$  as  $e \rightarrow \infty$ .
- (ii)  $\exists \hat{e}^1 \leq \hat{e}^2$  such that for  $e < \hat{e}^1$ , only the willingness-to-pay constraint binds; for  $e > \hat{e}^2$ , only the ability-to-pay constraint binds; and, for  $e \in [\hat{e}^1, \hat{e}^2]$ , both constraints bind.
- (iii)  $t(e)$  is continuous, (weakly) increasing in the region  $e \in [0, \hat{e}^1]$ , (weakly) decreasing in the region  $[\hat{e}^1, \hat{e}^2]$ , and (weakly) increasing in the region  $[\hat{e}^2, \infty)$ . Also,  $t(e) \rightarrow t^{**}$  as  $e \rightarrow \infty$ .

### 3 Solution Steady States

As the economy grows in our model (higher  $e$ ), the willingness-to-pay constraint eventually may not bind. At the same time, because of consumption, there is a natural saturation point for the endowment, above which the private agents simply consume the excess. Therefore, if the parameter set is such that this saturation point is lower than the point at which the willingness-to-pay constraint is relaxed, then the economy may never escape from the consequences of government myopia, and may be stuck in a growth trap. To see this formally, we first need some definitions regarding the growth path:

**Definition 3.1.** Given the solution program  $t(e)$  from the Bellman equation (2.16) and the private sector reaction function (2.17)–(2.19), we define

- An endowment path  $\{e_i\}_{i=0}^\infty$  as  $e_{i+1} := e_+(e_i, t(e_i))$  starting at  $e_0$ . In addition, we define  $e_\infty(e_0)$  as the limit (if it exists) of this endowment path:  $e_\infty(e_0) := \lim_{i \rightarrow \infty} e_i$ .

- Steady state  $(e^{ss}, t^{ss})$  as a pair satisfying

$$t^{ss} = t(e^{ss}), \text{ and} \quad (3.1)$$

$$e^{ss} = e \text{ such that } e = e_+(e, t^{ss}). \quad (3.2)$$

In addition, a no-saddle-point condition is imposed as follows:  $\exists \epsilon > 0$  such that for all  $e \in (e^{ss} - \epsilon, e^{ss} + \epsilon)$ ,  $e_\infty(e) = e^{ss}$ . This excludes the measure-zero set of fixed-point endowments on which a small shock can push the endowment path away from the fixed point in the long run.

- From Remark 2.1(3), consumption at the steady state  $c^{ss} = \frac{1}{\rho(1+r)}e^{ss}$ .

From Proposition 2.2, it must be the case that  $e^{ss}$  is (i) interior in the willingness-to-pay constraint region; or, (ii) interior in the ability-to-pay constraint region; or, (iii) interior in the “sliding” region; or, (iv) at the boundary of these regions. We show in Lemma B.5 in the appendix that there are no possible steady states in (iii) and (iv) because any candidate steady state in this region will always be a saddle point.

Suppose next that  $e^{ss}$  exists and it is in case (i) where  $e^{ss}$  is interior in the willingness-to-pay constraint region. We note first that using the envelope condition as well as the definition  $e^{ss} = e_+(e^{ss}, t^{ss})$ , we can get the exact  $\frac{dS}{de}$  at this point:

$$\begin{aligned} \frac{dS}{de} &= \kappa_1 \frac{dS}{de} + z\kappa_1 \\ \Rightarrow \frac{dS}{de} &= z \frac{\kappa_1}{1 - \kappa_1} = \rho z. \end{aligned} \quad (3.3)$$

Also, the optimal  $t$  should also satisfy the FOC:

$$\frac{1}{1+r} \left[ \frac{de_+}{dt} \frac{dS}{de} + z(1+r) \frac{ds}{dt} \right] + \tau' = 0. \quad (3.4)$$

Plugging (3.3) into (3.4), we get the following characteristic equation:

$$\frac{de_+}{dt} \underbrace{\frac{dS}{de}}_{=\rho z} + z(1+r) \frac{ds}{dt} + (1+r)\tau' = 0. \quad (3.5)$$

It is straightforward to see that the equation above is independent of  $e$ . Therefore, it follows that if such a steady state were to exist, the tax rate  $t^{ss}$  can be completely characterized from the model primitives, which we define as  $t^W$ . Then, the corresponding  $e^{ss}$  can be derived simply by solving  $e^W = e_+(e^W, t^W)$ .

On the other hand, suppose instead that  $e^{ss}$  is in case (ii) where it is interior in the ability-to-pay constraint region. The corresponding envelope condition and the FOC yield respectively

$$\frac{dS}{de} = \kappa_1 \frac{dS}{de} \Rightarrow \frac{dS}{de} = 0, \text{ and} \quad (3.6)$$

$$\frac{de_+}{dt} \underbrace{\frac{dS}{de}}_{=0} + (1+r)\tau' = 0. \quad (3.7)$$

Following the same logic as for case (i), it follows that, if such a steady state were to exist, the tax rate  $t^{ss}$  must be equal to  $t^A = \operatorname{argmax}_t \tau = t^{**}$ . Again,  $e^{ss}$  in this region can be derived by solving  $e^A = e_+(e^A, t^{**})$ . Note that the steady-state taxation will be set at the debt autarky level, even though the government will be borrowing.

The remaining piece of logic is whether the  $e^{ss}$  derived above indeed fall under the correct regions. That is,

$$\text{Steady state exists in the ability-to-pay region only if } \tau^{**} - C - zs(e^A, t^{**}) \leq 0, \text{ and} \quad (3.8)$$

$$\text{Steady state exists in the willingness-to-pay region only if } \tau^{**} - C - zs(e^W, t^W) > 0. \quad (3.9)$$

We show in Lemma B.6, via an application of the contraction-mapping theorem, that conditions in (3.8) and (3.9) are not only necessary, but also sufficient for the existence of each of the steady states, respectively.

Next, we show in Lemma 3.1 that (i) all endowment paths in our model have a limit; and, (ii) the limit must be one of the steady states  $e^{ss}$  characterized above. To this end, and in the following analyses, we make use of the intermediate functions  $e^{sat}$ :

**Definition 3.2.** Define the following function:

$$\begin{aligned} e^{sat}(t) &:= e \quad \text{s.t.} \quad e_+(e, t) = e \\ \Rightarrow e^{sat}(t) &= \frac{(1-t)f(k(t)) - (1+r)k(t)}{1/\kappa_1 - (1+r)}. \end{aligned} \quad (3.10)$$

In intuitive terms,  $e^{sat}(t)$  is the point towards which the economy “saturates” under the given  $t$ :  $\left[ \lim e_n = e_+(e_+(\dots(e_+(e, t), \dots), t), t) = e^{sat}(t) \right]$ . It also follows that for a given  $t$ , at  $e > e^{sat}(t)$  the economy is “contracting” ( $e_+(e, t) < e$ ), and at  $e < e^{sat}(t)$ , the economy is “growing” ( $e_+(e, t) > e$ ). This discussion is formalized below:

**Lemma 3.1.** Any endowment path  $\{e_i\}_{i=0}^\infty$  is a monotone sequence (increasing or decreasing) and has a limit. It follows that  $e_\infty(e_0)$  is always well-defined. Furthermore,  $e_\infty(e_0)$  is always one of three possible steady states:

- (Ability-to-pay region steady state, denoted hereafter as **steady state A**) Steady state is interior in the ability-to-pay constraint region  $(\hat{e}^2, \infty)$ , and  $e^{ss} = e^A := e^{sat}(t^{**})$ .
- (Willingness-to-pay region steady state, denoted hereafter as **steady state W**) Steady state is interior in the willingness-to-pay constraint region  $[0, \hat{e}^1)$ , and  $e^{ss} = e^W := e^{sat}(t^W)$  where

$$t^W = t \quad \text{such that} \quad \rho z \frac{de_+}{dt} + z(1+r) \frac{ds}{dt} + (1+r)\tau' = 0.$$

- (Sliding-region steady state, denoted hereafter as **steady state S**) Steady state is interior in the interim region  $[\hat{e}^1, \hat{e}^2]$ , and the pair  $(e^S, t^S)$  simultaneously solve

$$\begin{aligned} e &= e_+(e, t), \text{ and} \\ 0 &= \tau^{**} - C - z(1+r)s(e, t). \end{aligned}$$

In general, in the case where there are multiple steady states in the model,  $e_\infty(e_0)$  is not independent of  $e_0$ . In particular,  $e_\infty(e_1) \leq e_\infty(e_2)$  if  $e_1 < e_2$ .

We can then prove Proposition 3.2, the central result in the paper. For comparison with the benchmark case, we have used the notation  $\{e_n^{**}\}_{n=0}^\infty$  where  $e_{n+1}^{**} = e_+(e_n^{**}, t^{**})$  and the corresponding steady state as  $e_\infty^{**}$ . We exclude measure zero events in our analysis as even a small perturbation would remove the possibility of their existence.

**Proposition 3.2.** *Access to sovereign borrowing can lead the government to set steady-state taxation at levels that are below or above the benchmark. Steady-state endowments and consumption vary correspondingly. Specifically :*

- Suppose that  $t^{**} < t^W$ . Then,  $e_\infty(e_0)$  is in general not independent of  $e_0$ , and  $e_\infty(e_0) \leq e_\infty^{**}$  always. In particular, for a set of parameters of strictly positive measure,  $\exists \bar{e}$  such that
  - $\forall e_0 < \bar{e}$ ,  $e_\infty(e_0) < e_\infty^{**}$  (**Growth Trap**), and
  - $\forall e_0 \geq \bar{e}$ ,  $e_\infty(e_0) = e_\infty^{**}$  (**Benchmark**).
- Suppose instead that  $t^{**} \geq t^W$ . Then,  $e_\infty(e_0)$  is independent of  $e_0$  and  $e_\infty(e_0) \geq e_\infty^{**}$  always. Depending on the parameter set,
  - $e_\infty$  is either equal to  $e_\infty^{**}$  (**Benchmark**), or
  - $e_\infty$  is strictly greater than  $e_\infty^{**}$  (**Growth Boost**).

The proofs are in the appendix. In order to graphically illustrate the growth dynamics under the myopic government in the presence of sovereign debt, we show in Fig. 3 the simulated

endowment paths. In Fig. 3(a), economies starting at sufficiently low endowments may never escape the lower endowment region where willingness-to-pay constraint is always binding, and it makes the government highly repressive; the repression leads to a growth trap (in fact, the growth in endowment can be negative as seen in Fig. 3(a) for some starting endowments) and the economy never converges to endowment in the benchmark steady state. However, if it were to start at a higher endowment, then the willingness-to-pay constraint is never binding, and the economy converges to the “better” steady state. In Fig. 3(b), there is no growth trap, and all economies eventually converge to the benchmark steady state. Obviously, poorer economies take longer to reach there. Finally, in Fig. 3(c), the willingness-to-pay constraint lengthens the horizon of myopic government, and makes it tax less. Access to borrowing acts as a growth boost, and all economies converge to a better-than-benchmark equilibrium, no matter what endowment they start with.

[Fig. 3 about here]

Whether the myopic government potentially achieves a better final endowment or not – as compared to the benchmark no-debt case – depends solely on the comparison between two constants which are model primitives ( $t^W \geq t^{**}$ ). This condition can be further simplified to a bound on  $\rho$ , where the default cost parameter  $z$  is irrelevant. The intuition behind this can be observed by analyzing the  $t^W$  condition:

$$t^W := \operatorname{argmax}_t \left[ \frac{1}{1+r} [S(e') - \max\{0, \tau^{**} - C - zs(1+r)\}] + \tau(t) \right].$$

Note that  $e' = \kappa_1[\pi(t) + (1+r)e]$ . Differentiating, we get

$$\underbrace{\frac{dS}{de}}_{=\rho z} \frac{\rho}{1+r} \pi'(t) - z \left[ \rho k'(t) + \frac{1}{1+r} \frac{d}{dt} (1-t)f(k(t)) \right].$$

Whether  $t^W$  is lower or higher than  $t^{**} = \operatorname{argmax}_t t f(k(t))$  depends on whether this expression, evaluated at  $t = t^{**}$ , is positive or not. The two conflicting incentives for the myopic government follow:

$$\begin{aligned} & \underbrace{\frac{dS}{de} \frac{\rho}{1+r} \pi'(t)}_{=\rho z} \\ & - \underbrace{z \left[ \rho k'(t) + \frac{1}{1+r} \frac{d}{dt} (1-t)f(k(t)) \right]}_{\text{Incentive to repress investment with higher taxes to increase next-period government's willingness-to-pay}}. \end{aligned}$$

Incentive to lower taxes to boost growth to increase next-period government's spendable

In the equation above, we observe that (i)  $z$  enters linearly in both terms, so that when determining the sign of the expression,  $z$  is irrelevant; (ii)  $\rho$  enters as a quadratic term in the first term (+ incentive to grow), and as a linear term in the second term (− incentive to grow). This is because the savings parameter  $\rho$  influences both the marginal sensitivity of the future endowment to current tax rate ( $\frac{de_+}{dt}$ ) and the marginal sensitivity of next period government's repayment capacity to endowment ( $\frac{dS}{de}$ ). For high enough  $\rho$ , the first term dominates and the myopic government chooses an even lower tax rate than benchmark. For low enough  $\rho$ , the second term dominates and the opposite occurs. We formalize this argument as:

**Proposition 3.3.** *A necessary and sufficient condition for  $t^{**} < t^W$ , which can lead to the growth trap, is an upper bound on the savings parameter  $\rho$ :*

$$t^{**} < t^W \Leftrightarrow \rho < \frac{1}{t^{**}}. \quad (3.11)$$

We can also show the following:

**Proposition 3.4.** *A sufficient condition for the economy to converge to the benchmark steady state is a lower bound on the propensity to save parameter  $\rho$ :*

$$\rho \in \left( \bar{\rho}, \frac{1}{r} \right), \text{ where } \bar{\rho} < \frac{1}{r}. \quad (3.12)$$

The intuition is that with a high savings parameter, household endowments grow quickly, enabling the economy to escape from the willingness-to-pay region to the ability-to-pay region swiftly, and in turn, leading to convergence to the benchmark case. Combining the two results above (Propositions 3.3 and 3.4), we conclude that when private agents have a high propensity to save, sovereign debt can be (weakly) beneficial to growth even in the presence of a myopic and wasteful government.

Whether growth is strictly boosted by access to borrowing depends on whether the default cost parameter  $z$  (which, as discussed earlier, can also be interpreted as the centrality of sovereign debt to the domestic financial sector's functioning) is sufficiently small. To see this, recall that (i) growth boost in our model occurs only when the economy's steady state remains in the willingness-to-pay region, and (ii) the willingness-to-pay constraint is binding when  $\tau^{**} - C - zs(1+r) > 0$ , with the ability-to-pay constraint binding otherwise. Therefore, when  $z$  is low,  $\tau^{**} - C - zs(1+r)$  stays positive and the willingness-to-pay constraint can remain binding for a longer duration; conversely, when  $z$  is high, the willingness-to-pay region is small and the steady state moves quickly to the benchmark steady state which is in the ability-to-pay region. These results on how the savings parameter  $\rho$  and the default cost parameter  $z$

affect the nature of the steady state (growth trap, benchmark or growth boost) are illustrated in Fig. 4.

[Fig. 4 about here]

From this point on, we focus our main attention on cases where both steady states W and A exist:

**Lemma 3.5.** *Suppose that the model parameters admit two steady states, depending on the starting endowment  $e_0$ . Consider a steady state where all subsequent governments choose the same policies  $(t, D)$  with none defaulting. Then, equilibrium quantities chosen at the two steady states can be derived as the following, where NPV stands for the “net present value of”:*

**Steady state A.** *In the ability-to-pay region steady state, the tax rate is  $t^{**}$  and the corresponding endowment is  $e^A = e^{\text{sat}}(t^{**})$ . The debt  $D^A$ , its domestic and foreign components, and government spending are:*

- $D^A = \frac{\tau^{**}}{r} = \text{NPV}(\text{future period tax revenue}),$
- $D^{\text{Dom}} = s(e^A, t^{**}),$
- $D^{\text{For}} = \frac{\tau^{**}}{r} - s(e^A, t^{**}),$  and
- $\text{Government spending} = 0.$

**Steady state W.** *In the willingness-to-pay region steady state W, the tax rate is chosen at  $t^W > t^{**}$  and the corresponding endowment is  $e^W = e^{\text{sat}}(t^W) < e^{**}$ . The debt  $D^W$ , its domestic and foreign components, and government spending are:*

- $D^W = \frac{\tau(t^W) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{r} = \text{NPV}(\text{future period tax revenue} - \text{spending}),$
- $D^{\text{Dom}} = s(e^W, t^W),$
- $D^{\text{For}} = \frac{\tau(t^W) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{r} - s(e^W, t^W),$  and
- $\text{Government spending} = \tau^{**} - C - z(1+r)s(e^W, t^W).$

Interestingly, in the ability-to-pay region, the borrowing by the previous government leaves the current government with no room to spend. In contrast, the government in the willingness-to pay-region can spend  $\tau^{**} - C - z(1+r)s(e^W, t^W)$ . In steady state, all future governments will act in the exact same way, collecting taxes  $\tau(t^W)$  and spending  $\tau^{**} - C - z(1+r)s(e^W, t^W)$ . It follows that the debt capacity of the government in this steady state equals to the net present value of tax revenues, net of spending.

### 3.1 Discussion and Related Literature

#### 3.1.1 Why the Weak or Negative Correlation between Foreign Finance and Growth

The distortions we consider — government myopia and wasteful nature of its expenditures — affect the steady state of the economy. In particular, we have shown that access to debt in the presence of such distortions can lead developing countries to a growth or poverty trap (see, for example, [Kharas and Kohli \[2011\]](#)). The dynamic setting is key to this result: developing country governments repress the economy to channel savings to sovereign bonds in a bid to improve their willingness to repay foreign creditors, but in the process leave the next period government also with a low-endowment economy that is heavily indebted, and this way the repression gets entrenched *ad infinitum*.

Whether access to foreign borrowing is good or bad for a country depends on the country's characteristics, and not just on the nature of its government. Specifically, the household's propensity to save plays a crucial role in determining which of the dynamics described above is the more powerful force. Indeed, in Fig. 5(a), we see that as the household's propensity to save increases, the steady-state tax rate set by the repressive government initially falls in the willingness-to-pay region, as does the steady-state foreign debt as a fraction of endowment,  $\frac{D^{For}}{e}$  (see Fig. 5(b)), while the steady-state endowment rises (see Fig. 5(c)). Over this region, the cross-sectional correlation between foreign debt as a fraction of endowment and steady-state endowment (a proxy for long-run growth<sup>10</sup>) across countries with different propensities to save is negative. At somewhat higher propensities to save, while still in the willingness-to-pay region, steady-state tax rates rise, steady-state foreign debt normalized by endowment increase while the steady-state endowment falls. Once again in this region, the correlation between normalized foreign debt and long-run economic growth is negative. Finally, at yet higher propensities to save, countries set the tax rate at the benchmark steady-state tax rate  $t^{**}$  as they enter the ability-to-pay region, the steady-state normalized foreign debt falls with higher propensities to save, while steady-state endowment and long-run growth increases. Thus in each of the regions, there is a negative correlation between foreign debt to endowment and long-run growth as propensities to save increase.

[Fig. 5 about here]

Our model might explain a puzzling empirical finding in the literature. [Prasad et al. \[2006\]](#) find that over the period 1970-2004, there is no positive correlation for nonindustrial countries between current account balances and growth, or equivalently, that developing countries that have relied more on foreign finance have not grown faster in the long run, and have typically

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<sup>10</sup>In our model, steady-state endowment maps one-for-one into steady-state consumption. See Definition 3.1.

grown more slowly. They conclude this runs counter to the predictions of standard theoretical models. Similarly, [Aizenman, Pinto and Radziwill \[2004\]](#) construct a “self-financing” ratio for countries in the 1990s and find that countries with higher ratios grew faster than countries with lower ratios. Our model suggests that it is not that foreign financing is bad for developing country growth, but the countries that tend to need more foreign financing (because of low endowments or low propensities to save) also have governments that are likely to repress more, whence the puzzling correlation.

There are, of course, other explanations. [Gourinchas and Jeanne \[2013\]](#) conclude that poorer countries are poor because they have lower productivity or more distortions than richer countries, not because capital is scarce in them—the implication being that access to foreign capital by itself would not generate much additional growth in these countries. While our model also draws on policy distortions to explain the differential effects of foreign capital, it also explains why distortions are lower in countries that have substantial domestic savings. Another insightful explanation is in [Aguiar and Amador \[2011\]](#): High outstanding debt, either due to improved country access or aid, can lead to an underinvestment problem for myopic governments that also have the ability to expropriate capital in future, giving rise in their model to a reduction in the accumulation of capital stock and in the speed of convergence to the steady state; the steady state, however, remains unaffected by these government distortions. In contrast, our model’s implication is that when government myopia is combined with wasteful expenditures, there can in fact be a *permanent* impact on endowments for developing economies: The steady-state endowment can be trapped below the debt autarky levels, as the government taxes heavily and discourages private investment.

### 3.1.2 Odious Debt

It might be tempting to declare the debt issued by myopic rapacious government “odious” and non-enforceable going forward. Yet, as we have just seen, it is possible that an “odious” government’s incentives could be improved by access to borrowing. The key to the change in its behavior on gaining access to debt may not be the nature of the government (they are uniformly odious in our model thus far) but the nature of the country’s environment – for instance, the propensity to save of households, their endowment, or the centrality of government debt to the private sector’s functioning, as captured in the default cost parameter  $z$ . Governments may choose growth-enhancing policies (relative to the debt “autarky” benchmark) in order to boost their successor government’s willingness to repay, and in turn, borrow more today; this dynamic enables the economy to experience a growth boost in the form of a steady-state endowment that is above the autarkic one. Odious government does not always imply that access

to borrowing has odious consequences.

Moreover, even if the rapacious government represses in order to borrow more, an obvious situation where its access to borrowing hurts the citizenry, a declaration that the debt issued by the government is odious and unenforceable will immediately trigger default (since the government cannot borrow to repay legacy debt), which may be costly to the country's citizens. Indeed, even if the current government is not rapacious, the increased possibility that one of its successors could be deemed "odious" could reduce prospects for rolling over debt then, and thus constrict the market for new debt issuance today. This too could precipitate costly default.

The broader point is that proposals to declare newly issued debt odious should take into account both their effects on repayment of past debt, as well as the uncertainty they may create for regimes that are perfectly reasonable today, but could be followed at a future date by odious regimes. Since few countries can guarantee the quality of successor governments, the unintended consequences of proposals to declare debt "odious" on curtailing country access to borrowing and precipitating default could be quite substantial. We will return to these issues shortly when we examine debt ceilings and debt relief.

### 3.1.3 Implications for Repression

We have examined repression from the perspective of developing countries attempting to grow, and have testable implications on the extent of repression (the steady state tax rate) and country characteristics like its propensity to save, the size of its endowment, and the extent of sovereign default costs. Our model allows for both real and financial repression, but has little to say on the relative magnitude of each. Empirical work to highlight patterns in the data on economic and financial repression policies could be useful to inform further theoretical modeling.

Relatedly, [Reinhart, Kirkegaard and Sbrancia \[2011\]](#), [Reinhart \[2012\]](#), [Reinhart and Sbrancia \[2015\]](#), and [Chari, DAVIS and Kehoe \[2020\]](#) look at financial repression as a way to ease the debt repayment burden for a rich country that has suddenly experienced a large accumulation of debt (due to crisis or war). [Roubini and Sala-i Martin \[1992\]](#) model financial repression as a way for governments to raise "easy" resources for the public budget when tax evasion by the private sector is high, with consequent effects on efficiency of the financial sector and long-run growth. An interesting avenue for research is to compare the nature of repression in industrial countries with repression in developing countries, and to compare their relative deadweight costs in terms of effects on long-run growth.

### 3.1.4 Relationship to the Literature

There is a vast literature on sovereign debt that we have benefited from but cannot do justice to.<sup>11</sup> Our paper is most related to an emerging literature that embeds a cost of sovereign default that is tied to the extent to which the economy's private sector is entangled with sovereign debt. Specifically, we build on [Acharya and Rajan \[2013\]](#), who present a two-period (three-date) model of sovereign debt with a myopic wasteful government. Given their model, they cannot examine long-run or steady-state equilibria, nor do they address the choice between consumption, investment, and savings by the household sector. Our model enables us to examine dynamics, wherein lie the key results of our paper.

[Basu \[2009\]](#), [Bolton and Jeanne \[2011\]](#) and [Gennaioli, Martin and Rossi \[2014\]](#) relate the costs of sovereign default to the amount of debt held by domestic banks. They examine the trade-offs between more credible sovereign borrowing (when domestic banks hold more sovereign bonds) against the greater costs when the sovereign defaults. A version of this trade-off is also in our model, but our focus is on how access to sovereign borrowing can alter long-run growth. Moreover, our fundamental assumption – of myopic wasteful governments – is different from these papers.

## 4 Extension with a productive government

Thus far, we have assumed the government wastes all the resources it collects. Now consider an alternative setting where the government has access to a productive technology which yields, in return for today's investment  $I$ ,  $g(I)$  in the next period. We assume that the investment is made at the end of current period, when the government undertakes other spending, and the return of the investment is at the end of the next period. We assume that the government technology  $g$  satisfies Inada conditions, *i.e.*,  $g'(0) \rightarrow \infty$ ,  $g' > 0$ ,  $g'' < 0$ .

### 4.1 Cash flow to government

First, we assume that an investment of  $I$  creates a cash flow of  $g(I)$  for the government in the next period – so this is best thought of as investment in a state-owned steel plant or a toll road. Since  $g(I)$  is created only in the next period, the myopic current government does not enjoy the future cash flow *per se*. However, non-zero investment may still be in the government's

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<sup>11</sup>Including, but not limited to, [Eaton and Gersovitz \[1981\]](#), [Bulow and Rogoff \[1989a\]](#), [Bulow and Rogoff \[1989b\]](#), [Fernandez and Rosenthal \[1990\]](#), [Eaton and Fernandez \[1995\]](#), [Cole and Kehoe \[1998\]](#), [Guembel and Sussman \[2009\]](#), [Reinhart and Rogoff \[2010\]](#), [Amador \[2012\]](#), and [Tomz \[2012\]](#).

incentive if it increases its debt capacity. Importantly, the government will invest if it is in the ability-to-pay region, but not if it is in the willingness-to-pay region.

To see this, suppose for simplicity that the next period government's total surplus is fixed at  $S$  and the option to invest in technology  $g$  is only available to the current government. Note that the next period government's ability-to-pay constraint, with respect to the current government's debt issuance  $D$  and investment  $I$  is now :

$$D(1+r) \leq S + g(I) \Rightarrow D \leq \frac{1}{1+r}(S + g(I)) . \quad (4.1)$$

In contrast, the next government's willingness-to-pay constraint is:

$$S + g(I) - D(1+r) \geq \tau^{**} - \text{default cost} + g(I) \quad (4.2)$$

$$\Rightarrow D \leq \frac{1}{1+r} [S - \tau^{**} + \text{default cost}] . \quad (4.3)$$

Clearly, if the next period government is constrained by the ability to pay, an investment in government technology  $I$  increases the debt capacity of the current government by  $\frac{1}{1+r}g(I)$ .

In contrast, if the next period government is constrained by the willingness to pay, investment does not help the current government's debt capacity at all. Although the incremental cash flow  $g(I)$  increases the net spending by the future government in case it honors the legacy debt, it also increases its net spending in the default state by exactly the same amount. The two effects offset each other so that the debt capacity is left unchanged in the willingness-to-pay region.

We formalize the argument above in the following proposition and illustrate it in the left panel of Fig. 6<sup>12</sup>:

**Proposition 4.1.** *The government's problem, with access to a technology that for investment  $I$  generates cash flow  $g(I)$  accruing to the next-period government, is characterized by the following Bellman equation:*

$$S(e) = \max_{t,I} \left[ \frac{1}{1+r} [S(e') + \min\{g(I), C + zs(1+r) - \tau^{**}\}] + \tau(t) \right] - I.$$

*The optimal investment function  $I(e)$  has the following property:  $\exists \bar{e}_{gcf}^1 < \bar{e}_{gcf}^2$  such that  $\forall e < \bar{e}_{gcf}^1$ ,  $I(e) = 0$ , and  $\forall e > \bar{e}_{gcf}^2$ ,  $I(e) = \text{argmax}_i [g(i) - i]$ . In other words, governments in economies with low endowments may not see any value in spending productively, even if the technology exists.*

<sup>12</sup>Fig. 6 shows the numerical solutions from models with same parameters as the one in Fig. 2, with  $g(\cdot) = \alpha \times f(\cdot)$ , where  $\alpha$  takes three different values.

[Fig. 6 about here]

The government of the developing country cannot take advantage of public investment opportunities, not because it is less capable or more corrupt than a rich-country government, but because the willingness-to-pay constraint binds more strongly. Effectively, public investment does nothing to alleviate this constraint, so it sees no value in such investments.

## 4.2 Cash flow to household

Now assume that the investment  $I$  creates a cash flow of  $g(I)$  which accrues, or is returned, to the households at the end of the next period. This could be thought of as setting aside investments to fund future social security spending. By similar reasoning as above, even though this does not increase the government's spending capacity directly, it may do so indirectly by increasing the household endowment which increases the government debt capacity. Debt as a horizon-lengthening device is still at work in this set up.

Interestingly, however, the effects are different than when  $g(I)$  accrues to the next government; Recall from Fig. 2, the government's debt capacity is more sensitive to the level of household endowment in low-endowment regions than it is in high-endowment regions, with the sensitivity approaching zero at high endowments. It follows that governments of poorer countries have stronger incentives to invest in technologies that raise household endowments. In fact, we show in the next proposition and illustrate in the right panel of Fig. 6 that when the economy's endowment crosses a certain threshold, the government chooses not to make an investment of this kind:

**Proposition 4.2.** *The government's problem, with access to a technology that for investment  $I$  generates cash flow  $g(I)$  accruing to the households next period, is characterized by the following Bellman equation:*

$$S(e) = \max_{t,I} \left[ \frac{1}{1+r} [S(e' + g(I)) - \max\{0, \tau^{**} - C - zs(1+r)\}] + \tau(t) \right] - I.$$

*The optimal investment function  $I(e)$  has the following property:  $\exists \bar{e}_{hcf}$  such that  $\forall e > \bar{e}_{hcf}$ ,  $I(e) = 0$ .*

It may seem from this proposition that developing country governments have a comparative advantage in setting up social security schemes, which seems to contradict reality. Yet there is an important caveat here to the theory. Unlike government spending on public sector assets, whose returns naturally accrue to successor governments, there is no guarantee that the successor government will turn over the returns from assets set aside in the social security fund to

households. A strong institutional mechanism is required to ensure that such a commitment is respected. In the absence of such commitment, this kind of household-directed spending will simply not occur.

## 5 Policy Instruments for Escaping the Trap

We now discuss possible policy instruments to help economies escape from, or remove, the poverty or growth trap we have identified earlier.

### 5.1 Debt ceiling

The primary reason economies are trapped is because their governments adopt repressive policies in order to enhance borrowing. Therefore, a natural policy instrument would be to cap the government's ability to borrow with a constitutional debt ceiling (as, for example, in Germany) or through a common understanding imposed by external lenders (as, for instance, in the call for multilateral agencies like the IMF to monitor and limit debt build up in poor countries). An extreme version would be to declare all new debt "odious" and set the debt ceiling at zero.

Suppose that debt ceiling takes the general form  $\{\bar{D}_i\}_{i=0}^{\infty}$  where each government  $i$  faces the debt ceiling  $\bar{D}_i$ . Conditional on not defaulting, government's actions are independent of past government debt ceilings and legacy debt, but not of future debt ceilings. Let us denote the current government's spendable surplus as  $S(e; \bar{D}_0, \bar{D}_1, \dots)$ . We can show  $S(e; \bar{D}_0, \bar{D}_1, \dots)$  exhibits the following intuitive property:

**Proposition 5.1.**  *$S(e; \bar{D}_0, \bar{D}_1, \dots)$  is weakly decreasing in all debt ceilings,  $\bar{D}_i$ , current ( $i = 1$ ) and future ( $i > 1$ ). It follows that lowering the debt ceiling – whether for the government itself or future governments – weakly decreases the current government's ability to spend.*

We now consider a special form of debt ceiling where  $D_i = \bar{D} \forall i$  (flat debt ceiling). Let us define  $e_{\infty}(e_0; \bar{D})$  as the limit of the endowment sequence under debt ceiling  $\bar{D}$ . We first prove that

**Proposition 5.2.** *(Optimal debt ceiling). Suppose that  $t^{**} < t^W$  (corresponding to the trap case). Then, in general  $e_{\infty}(e_0) \leq e_{\infty}(e_0; \bar{D})$ . In particular, there exists a threshold debt ceiling  $\bar{\bar{D}} = D^W$  such that for all  $\bar{D} < \bar{\bar{D}}$ ,  $e_{\infty}(e_0; \bar{D}) = e_{\infty}^{**}$  for all  $e_0$ , completely removing the trap. Recall, from Proposition 3.2, that  $e_{\infty}(e_0) \leq e_{\infty}^{**}$  without the debt ceiling.*

Suppose instead that  $t^{**} > t^W$ . Then, in general  $e_{\infty}(e_0) \geq e_{\infty}(e_0; \bar{D})$ . Similarly,  $\exists \bar{\bar{D}}$  such that for all  $\bar{D} < \bar{\bar{D}}$ ,  $e_{\infty}(e_0; \bar{D}) = e_{\infty}^{**}$  for all  $e_0$ . Recall, from Proposition 3.2, that  $e_{\infty}(e_0) \geq e_{\infty}^{**}$  in the original problem without debt ceiling.

*In summary, the best that the debt ceiling can achieve when there is a growth trap is the benchmark steady-state endowment  $e_{\infty}^{**}$ . In this case, it can help enhance long-run growth; conversely, when debt in the presence of government myopia boosts growth, a debt ceiling can hurt long-run growth.*

One way to see this intuitively is to analyze the marginal incentives for a myopic government in the short run. Recall the original Bellman equation and suppose for simplicity that  $e$  is in the willingness-to-pay region:

$$t(e) = \operatorname{argmax}_t \frac{1}{1+r} \left[ S(e') - \tau^{**} + C + z(1+r)s \right] + \tau(t).$$

Recall that the myopic governments' optimal taxation was chosen by trading off the incentive to boost ( $\frac{de'}{dt} \frac{dS}{de} < 0$ ) and to repress ( $\frac{ds}{dt} > 0$ ). We consider two cases:

- The debt ceiling is imposed only on the current government. In this case, the problem is changed to

$$t(e) = \operatorname{argmax}_t \frac{1}{1+r} \left[ \min\{S(e') - \tau^{**} + C + z(1+r)s, \bar{D}\} \right] + \tau(t).$$

If  $\bar{D}$  is low enough so that  $S(e') - \tau^{**} + C + z(1+r)s$  is greater than or equal to  $\bar{D}$ , then the government's marginal incentives to both boost or repress disappear. Therefore, the government would simply choose  $t = t^{**}$  that maximizes  $\tau(t)$ .

- The debt ceiling is imposed on all future governments but not on the current government. In this case, the problem is changed to

$$t(e) = \operatorname{argmax}_t \frac{1}{1+r} \left[ S(e'; \bar{D}) - \tau^{**} + C + z(1+r)s \right] + \tau(t).$$

The incentive to repress remains unchanged; however, because  $S(e')$  is constrained by  $\bar{D}$  in some states of the world, the incentive to boost is lower. Therefore, the government engages in even higher repression than without debt ceiling.

Given that a flat ceiling is a combination of the debt ceiling now and a debt ceiling starting tomorrow for ever, it follows that a debt ceiling either moves the tax rate to the benchmark tax rate  $t^{**}$ , or induces the government to repress even more. It follows that if  $t^{**} < t^W$ , then the debt ceiling could improve the steady state by achieving the benchmark steady state instead. On the other hand if  $t^{**} > t^W$ , then the debt ceiling always hurts when it is binding. Fig. 7 offers an illustration.

[Fig. 7 about here]

Finally, we should note that a debt ceiling is a less abrupt way of nudging an irresponsible borrowing government into responsibility than simply declaring its debt odious. It is likely to embed a lower expected cost of default. Indeed, when combined with debt relief which we explore next, the default costs can be avoided entirely.

## 5.2 Debt relief

Consider now debt relief, that is, forgiveness of a certain amount of the face value of debt. Debt relief alone is inconsequential in our model. It simply allows the current-period government to increase spending by the amount of the relief.

**Lemma 5.3.** *In an equilibrium path, any debt relief in a period is transferred one-to-one to government spending in that period. The ensuing tax rates and endowment paths remain unchanged.*

This is not very far from reality. Of the 36 countries that received significant official debt relief under the Highly Indebted Poor Country (HIPC) Initiative and Multilateral Debt Relief Initiative (MDRI) in the early 2000s, 15 were either back in debt distress or had a high risk of debt distress by 2019. Another 13 had a moderate risk of debt distress.<sup>13</sup> Even the remaining did not all have a low risk of debt distress – some simply did not produce the data to compute debt sustainability.

However, when coupled with a debt ceiling, debt relief can be beneficial in moving a country to a better equilibrium. Suppose, that the debt ceiling was not initially in place and governments are trapped in steady-state  $W$  equilibrium (*i.e.*, the scenario analyzed in Lemma 3.5). Only a debt ceiling below the steady-state level of debt will have effect, but imposing it will cause the country to default, thus causing it to incur the deadweight costs.<sup>14</sup> Therefore, if default is a dominated option, any attempt to impose a debt ceiling should first be preceded by debt relief so as to avoid immediate default.

Formally, let the debt amount be reduced by fraction  $\lambda$ . Our debt restructuring scheme then can be summarized by a pair  $(\lambda, \bar{D})$ . We analyze how various restructuring schemes  $(\lambda, \bar{D})$  can affect the utilities of different interested parties.

We first take the perspective of external creditors. Clearly, creditors want no debt relief since their claims are being serviced, and their utility is decreasing in the amount of debt relief. Therefore, assuming a debt reduction has to be undertaken, they would want to minimize  $\lambda$  given  $\bar{D}$ , such that relief is enough to prevent default. Intuitively,  $\lambda$  required to prevent default

<sup>13</sup>See <https://www.worldbank.org/en/topic/debt-sustainability#2> for a list of countries and the risk of debt distress prepared by the World Bank.

<sup>14</sup>A call to declare government debt as odious will act like a low debt ceiling in that it will inhibit fresh borrowing.

is a decreasing function of the debt ceiling  $\bar{D}$ , as a lower ceiling constrains the government's resources more. By Proposition 5.2, lowering  $\bar{D}$  eventually gets the economy out of the trap. It follows, then, that finding an efficient scheme can be reduced to finding the threshold debt ceiling  $\bar{D}$  at or below which the economy escapes the trap. It is intuitive to conjecture that the threshold  $\bar{D}$  is smaller than the debt issued in steady state  $W$ , as anything higher is not going to change the current and subsequent government's behavior.

We formalize this argument in Proposition 5.4.

**Proposition 5.4.** *For any debt ceiling  $\bar{D}$ , debt relief  $\lambda$  prevents government default if and only if*

$$\lambda \geq \lambda^{\min}(\bar{D}) := 1 - \frac{S(e^W; \bar{D}) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{(1+r)D_{-1}^W}.$$

Since  $S(e^W; \bar{D})$  is increasing and continuous in  $\bar{D}$ ,  $\lambda^{\min}(\bar{D})$  is decreasing and continuous in  $\bar{D}$ .

A debt restructuring scheme that minimizes  $\lambda$  while ensuring no default as well as no growth trap ( $e_\infty = e^{**}$ ) can be characterized as choosing the debt ceiling  $\bar{D}$  that is arbitrarily smaller than the current level of debt

$$\bar{D} := D^W = \frac{\tau^W - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{r},$$

and choosing a  $\lambda$  arbitrarily close to 0. At this debt ceiling, the tax rate is initially arbitrarily close to  $t^W$  as well.

The top panel in Fig. 8 illustrates the patterns exhibited by  $\lambda^{\min}(\bar{D})$  and  $e_\infty(\bar{D})$ . Note first a sharp discontinuity of  $e_\infty(\bar{D})$ ; for  $\bar{D}$  higher than the steady-state level  $D^W$ , the trap is unchanged. For  $\bar{D}$  slightly lower than  $D^W$ , the trap is suddenly removed. However,  $\lambda^{\min}(\bar{D})$  is continuous in  $\bar{D}$ , and need only be vanishingly small. In sum, while some debt relief is required, when coupled with a debt ceiling just below  $D^W$ , the debt relief can be an arbitrarily small amount to get the economy out of the growth trap without default.

Next, we take the perspective of the long-run interest of the private sector. It cares about the discounted sum of consumption by the households. This depends on how fast the economy converges to the ability-to-pay steady state  $A$  after the debt ceiling has been placed. Interestingly, while the levels of debt ceilings do not affect the level of long-run endowment once the debt ceiling is below the threshold  $\bar{D}$  – as stated in Proposition 5.2 – lower debt ceilings induce faster convergence to the long-run endowment. The bottom-left panel in Fig. 8 illustrates this point. At a debt ceiling just below the threshold (99.95% of level of debt in steady state  $W$  ( $D^W$ ) in this parameter set), it takes about 100 periods for the economy to reach the benchmark steady state, whereas a lower debt ceiling (80% in the figure) achieves it in 40 periods.

Intuitively, governments do not start charging the autarkic tax rate right away; if the debt ceiling is just below  $D^W$ , they will set the tax rate just below  $\tau^W$  and only slowly will it decline to the autarkic tax rate. Convergence is faster when the debt ceiling is set lower and debt relief is set accordingly higher, as can be seen in the bottom-right panel in Fig 8.

[Fig. 8 about here]

Formalizing the preceding argument:

**Proposition 5.5.** *Suppose that model parameters admit a trap equilibrium, and that the economy initially is trapped at endowment  $e^W$ . Suppose now that a permanent debt ceiling  $\bar{D}$  is placed at  $t = 0$ , along with adequate levels of debt relief such that the debt ceiling does not trigger default. Let  $\{t_i^{\bar{D}}\} := \{t_0^{\bar{D}}, t_1^{\bar{D}}, \dots\}$  denote the collection of tax rates that the governments in periods  $i = 0, 1, 2, \dots$  charge, and similarly, let  $\{e_i^{\bar{D}}\} := \{e_0^{\bar{D}}, e_1^{\bar{D}}, \dots\}$  be the corresponding endowments. Then, for two debt ceilings  $\bar{D}^1 < \bar{D}^2$ ,  $t_i^{\bar{D}^1} \leq t_i^{\bar{D}^2}$  holds for all  $i \in \mathbb{Z}_+$ . This immediately implies that  $e_i^{\bar{D}^1} \geq e_i^{\bar{D}^2}$  for all  $i$  as well.*

Propositions 5.4 and 5.5 show that there is an understandable conflict of interest between creditors and the domestic private sector on the extent of government debt haircuts. While creditors would prefer the minimum debt relief that allows the country to escape the growth trap, the domestic private sector would prefer higher levels of debt relief for faster convergence to the steady state. In reality, debt renegotiation will be a bargaining process, taking these and other factors into account.

It should also be noted that debt ceilings are inherently time-inconsistent. While a suitable debt restructuring scheme is in the present government's incentive, it is not in the future governments' incentive; future governments benefit, if possible, from removing or relaxing the debt ceilings and increasing their spending by borrowing more. And future creditors have an incentive to lend. Therefore, the bargaining between creditors and the present government may potentially break down should the creditors anticipate that there is a lack of commitment on future governments' or creditors' behavior in complying with the debt ceilings.

Finally, the knife-edged nature of debt ceilings and debt relief (no effect above a threshold ceiling, large effects below so minor debt relief is enough) are largely driven by the fact that in the model there is no uncertainty and all parameters are exactly known. In the presence of various forms of uncertainties, the minimum debt relief would likely be higher.

### 5.3 Fiscal transfer

If private endowments matter, can the government transfer some of its funds to households to get the economy out of a growth trap? Assume at the end of the period, the government

simultaneously engages in three actions we have already considered so far, as well as a new one: (a) raises debt by selling bonds; (b) raises taxes; (c) pays back its legacy debt; and, in addition, (d) shares some of the surplus with the households, spending the rest. We assume the sharing is not foreseen in prior periods and one-off, meant to dislodge the economy from the repressive steady state. We also assume that the present government is perfectly committed to the announced transfer at the end of the period, and this is understood by households at the beginning of the period when they choose investment.

The myopic government may have a private incentive to engage in the fiscal transfer, because the anticipated increase in the household endowment increases the government's debt capacity, which ultimately increases its spending today.

Recall that a government with endowment  $e$  has the objective function to maximize:

$$\begin{aligned} \text{spending} &= S(e) - D^{\text{legacy}}(1+r) \\ &= \max_t \left[ \frac{1}{1+r} [S(e') - \max\{0, \tau^{**} - C - zs(1+r)\}] + \tau(t) \right] - D^{\text{legacy}}(1+r). \end{aligned}$$

Suppose that the government can take out  $\Delta e \geq 0$  from its spending and transfer it to households at the end of the period. Under the assumption that  $e = e^W$ , we know that (i) the next period endowment is also  $e^W$ , and (ii) from (3.3), the marginal sensitivity of optimal  $t$  to endowment is zero, so that  $\frac{dS}{de} = \rho z$ . Based on this information, collecting only the terms dependent on  $\Delta e$ , we have that

$$\text{spending} = \frac{1}{1+r} S(e^W + \Delta e) - \Delta e.$$

This immediately implies that there is a positive  $\Delta e$  that increases the objective function if and only if  $\frac{\rho z}{1+r} > 1$ . In Fig. 9, we plot the spending as a function of  $\Delta e$ . Clearly, for some parameters, there is a non-zero fiscal transfer that increases the government's spending. Therefore:

**Proposition 5.6.** *Suppose the model parameters admit a trap equilibrium. There is a non-zero fiscal transfer to the households that increases the government's spending if and only if  $\rho > \frac{1+r}{z}$ . Depending on other parameters, the fiscal transfer that maximizes the government's spending can be large enough that the economy escapes the growth trap.*

[Fig. 9 about here]

Notice again the importance of household savings. We have established in Proposition 3.3 that the trap occurs only if  $\rho < \frac{1}{t^{**}}$ . Proposition 5.6 shows that as the savings parameter falls even further, such that  $\rho < \frac{1+r}{z} < \frac{1}{t^{**}}$ , the myopic government will not engage in growth-friendly fiscal transfer, even given a chance; it will do so only for  $\rho \in \left(\frac{1+r}{z}, \frac{1}{t^{**}}\right)$ .

Note also that a substantial degree of commitment is required for the government to find these fiscal transfers worthwhile. For after announcing the transfer and affecting household investment, the government has an incentive to renege on the transfer. As such, this exercise suggests the very high degree of commitment required to get away from the growth trap. Implicitly, it also suggests some robustness to the baseline model and results in Section 2.

## 6 Shocks to Steady State

Finally, we analyze the effects of unexpected permanent shocks to model parameters. We focus on our benchmark case where the model exhibits both steady states A and W, as defined in Lemma 3.1. We again assume that the model economy has stayed at either of the steady states for a long enough time, such that the endowment, taxes, and debt issuances all follow quantities defined in Lemma 3.5.

Specifically, we consider a shock to the current endowment  $e$ ; a permanent shock to the propensity to save  $\rho$ ; a permanent shock to private sector productivity  $\phi$  which level-shifts the production function  $f(k) \rightarrow \phi \times f(k)$ ; and a permanent shock to the interest rate  $r$ . We analyze the effects of these shocks on (i) the current government's decision to default, and (ii) the steady states.

**Proposition 6.1.** *Consider a parametrized spendables function  $S(e; \rho, \phi, r)$  where  $\rho$ ,  $\phi$ , and  $r$  are savings parameter, productivity parameter, and interest rate, respectively. For sufficiently low  $r$ , partial derivatives of the spendables function with respect to  $e$ ,  $\rho$ , and  $\phi$ , at steady state A and steady state W, are as follows:*

$$\begin{aligned} \frac{\partial S}{\partial e} \Big|_{e^W} &> 0, & \frac{\partial S}{\partial \rho} \Big|_{e^W} &> 0, & \frac{\partial S}{\partial \phi} \Big|_{e^W} &< 0, & \frac{\partial S}{\partial r} \Big|_{e^W} &< 0; \text{ and} \\ \frac{\partial S}{\partial e} \Big|_{e^A} &= 0, & \frac{\partial S}{\partial \rho} \Big|_{e^A} &= 0, & \frac{\partial S}{\partial \phi} \Big|_{e^A} &> 0, & \frac{\partial S}{\partial r} \Big|_{e^A} &< 0. \end{aligned}$$

*At steady states, a shock triggers default if and only if it decreases current spendables  $S$ . It follows then that*

1. *In steady state W, a negative shock to endowment  $e$ , a negative shock to savings  $\rho$ , and a positive shock to productivity  $\phi$ , all trigger default.*
2. *In steady state A, a negative shock to productivity  $\phi$  triggers default.*
3. *A positive shock to interest rate  $r$  triggers default in both steady states.*

4. *Endowments in both steady states are positively related to savings  $\rho$  and productivity  $\phi$ :*

$$\frac{\partial e^W}{\partial \rho}, \quad \frac{\partial e^W}{\partial \phi} > 0; \text{ and}$$

$$\frac{\partial e^A}{\partial \rho}, \quad \frac{\partial e^A}{\partial \phi} > 0.$$

Perhaps the most intriguing part in Proposition 6.1 is the fact that government spendable in steady state W is negatively related to the productivity parameter. This is driven by two forces: (i) An increase in productivity induces a decrease in financial savings by the private sector; in steady state W, this drives down the government debt capacity. (ii) An increase in productivity also increases tax revenue in case of default, which weakens the government's commitment to not default, thereby further reducing the debt capacity. Lower debt capacity will trigger default if the government had previously maximized borrowing.

## 7 Conclusion

We analyzed the effects of access to debt under the assumption that the government is myopic and spends wastefully. The key takeaway that emerged is that sovereign debt is a double-edged sword. When the economy is poor or has a low propensity to save, access to debt can lead to a growth trap where the economy's steady state is worse than under debt autarky (without access to debt) as the government adopts repressive policies to channel domestic savings to government bonds; in other cases, however, access to debt can extend a myopic government's horizon, resulting in steady states that are the same as or even better than autarky. When debt induces a growth trap, policy instruments such as debt ceilings and fiscal transfers can be effective, provided there is adequate commitment to enforce them. These implications of our model are worthy of further empirical investigation, and could account for the puzzle that there is little positive correlation between a developing country's growth and its use of foreign finance.

Our model considered sovereign debt only in the form of short-term or one-period contract. It turns out that long-term debt does not lead to any different outcomes under the assumptions that (i) any default by the government on any portion of the debt that is due in a period triggers cross-default clauses on all other debt; and (ii) the resulting default costs are therefore linked to the domestic portion of *all* outstanding debt. Since governments are myopic and care only about the current-period spending, it is immaterial to outcomes whether their ability to spend is reduced by their having to repay all legacy short-term debt, or whether their ability to issue debt is lowered by the stock of legacy long-term debt. In either case, the government can tap

all debt capacity into the indefinite future regardless of the maturity of debt issued.

An interesting extension could be to allow uncertainty in the model. The key difference in this extension would be the optimal choice of the myopic government between issuing large quantities of risky debt or small quantities of riskless debt. We conjecture that a similar double-edged sword feature would arise with risky debt. On the one hand, when the government issues risky debt, the level of endowment in the future high-endowment states matters for the government, and therefore the government will have an extra incentive to boost growth by lowering tax rates. However, risky debt exposes the economy to the costs of government default in low-endowment states. Risky debt can also lead to other adverse spillovers by reducing the ability of real and financial sectors to use government bonds as safe collateral in borrowing contracts. On the other hand, when the government issues riskless debt, the economy will not default. However, future endowments in high states are now of no concern for the myopic government, and therefore the incentive to boost growth will be absent.

Finally, while we have focused on the sustainability of external sovereign debt in developing countries, our paper may have some bearing on sovereign debt more generally, including in industrial countries. It may be argued that the lack of commitment to repay is a problem irrelevant to developed countries; however, with a large negative shock in household endowment as witnessed during the Global Financial Crisis and the recent COVID-19 outbreak, and the ensuing rise of public debt, this argument has perhaps weakened. While our model may not quite fit rich industrial countries at the time of writing, aspects of it may be very pertinent in the not-distant future. There is clearly scope for more research.

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## A Figures

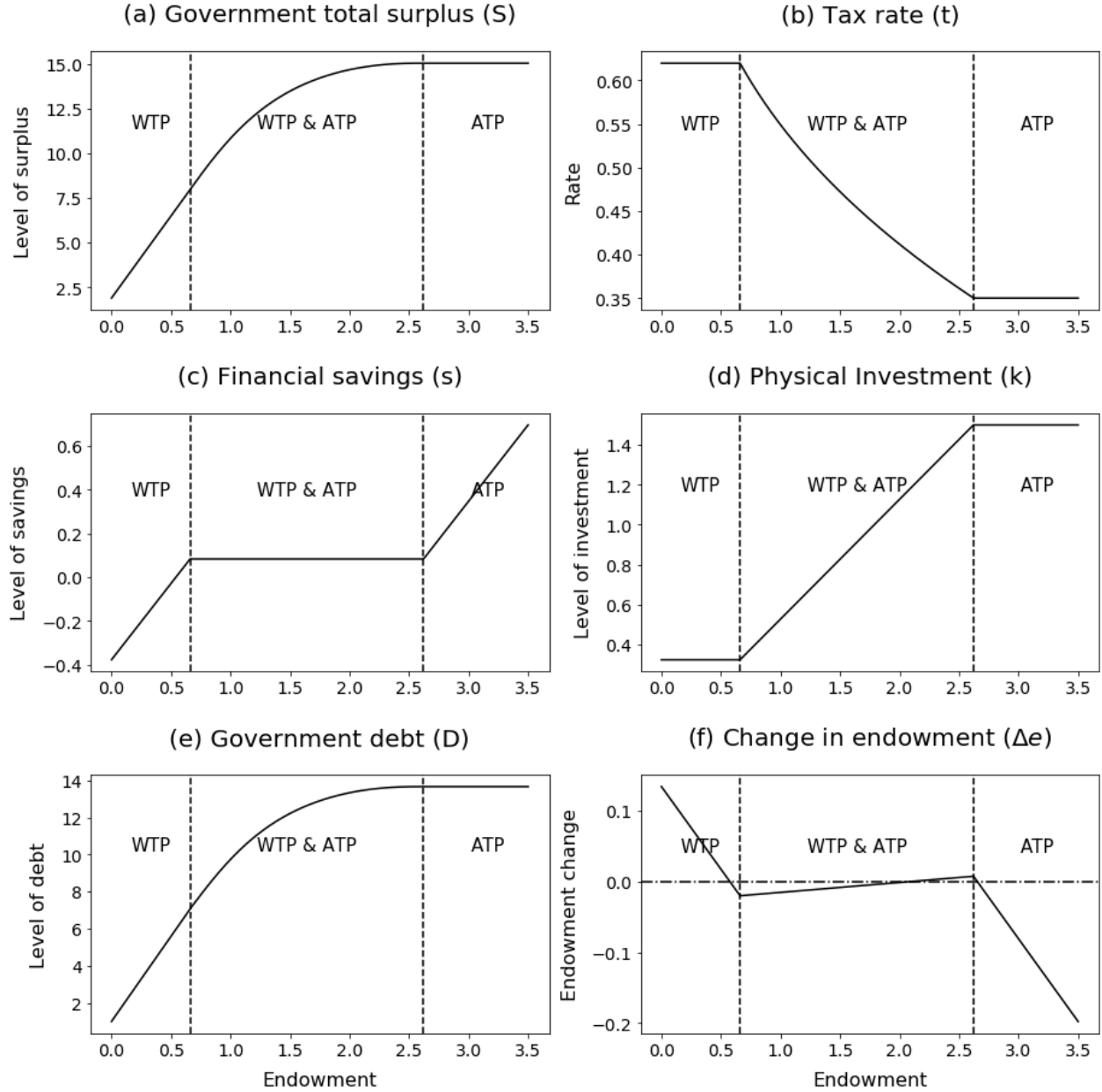


Figure 2: Solution from the baseline model, with parameters  $f = 3k^{.65}$ ,  $r = 10\%$ ,  $z = 4$ ,  $\rho = 2.3$  and  $C = 1.0$ . “WTP” stands for willingness-to-pay region; “ATP” for the ability-to-pay region; and “WTP & ATP” for the sliding region where both willingness-to-pay and ability-to-pay constraints bind.

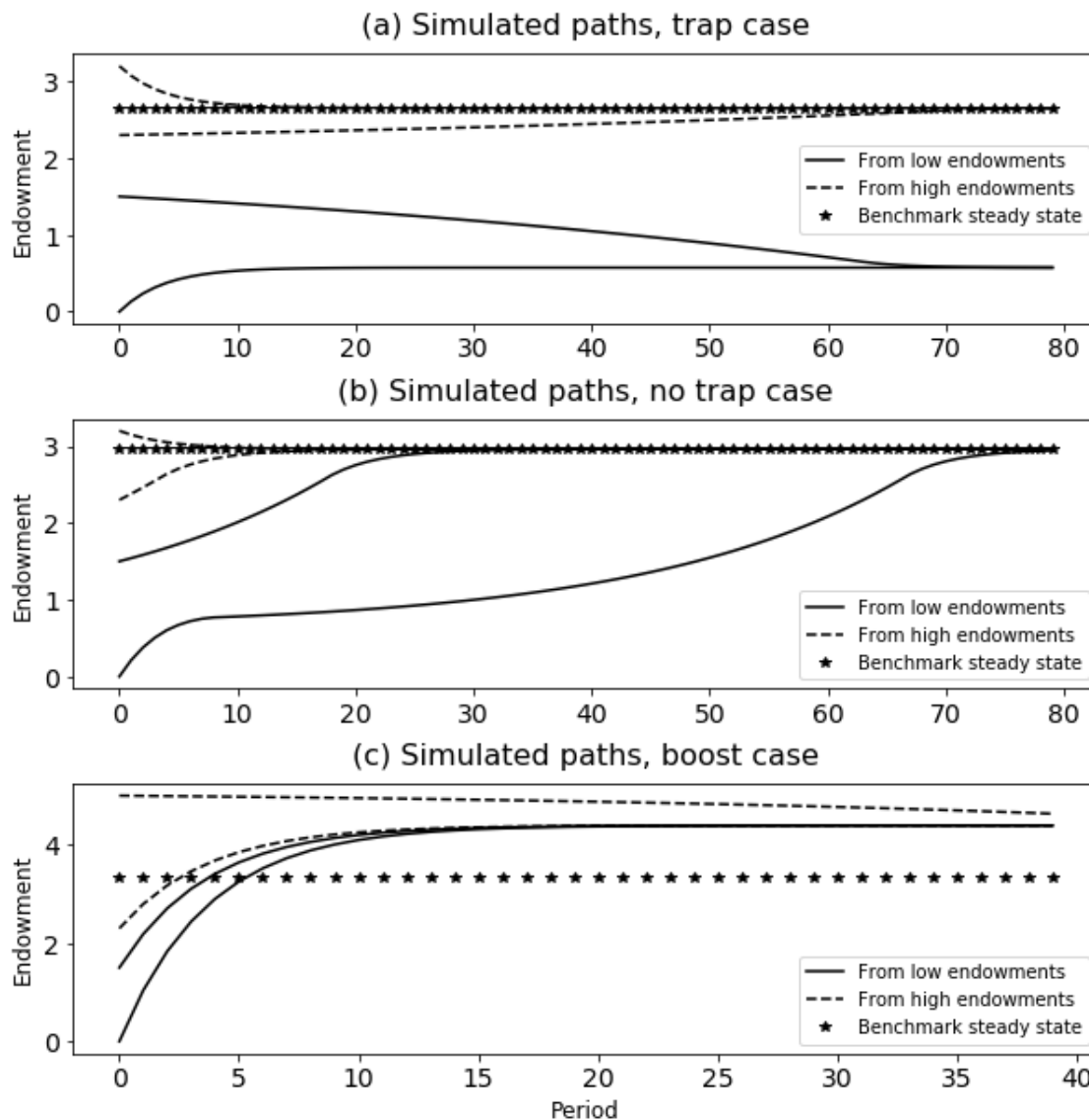


Figure 3: Simulated endowment paths for three different parameter sets. The model in panel (a) exhibits two steady states, W and A. Endowment paths starting from low endowments (solid lines) converge to steady state W (lower), whereas those starting from high endowments (dashed lines) converge to steady state A (higher). The model in panel (b) exhibits only one steady state (steady state A). All endowment paths converge to the same endowment regardless of the starting endowment. The model in panel (c) exhibits only steady state W. Contrary to other parameter configurations, steady state W in this case is at a higher endowment level than the benchmark autarky case. All endowment paths converge to the same endowment regardless of the starting endowment. Parameters used:  $f = 3k^{.65}$ ,  $C = 1$ , (a)  $r = 10\%$ ,  $\rho = 2.3$ , and  $z = 4$ . (b)  $r = 10\%$ ,  $\rho = 2.5$ , and  $z = 4$ . (c)  $r = 1\%$ ,  $\rho = 3.1$ , and  $z = 1.1$ .

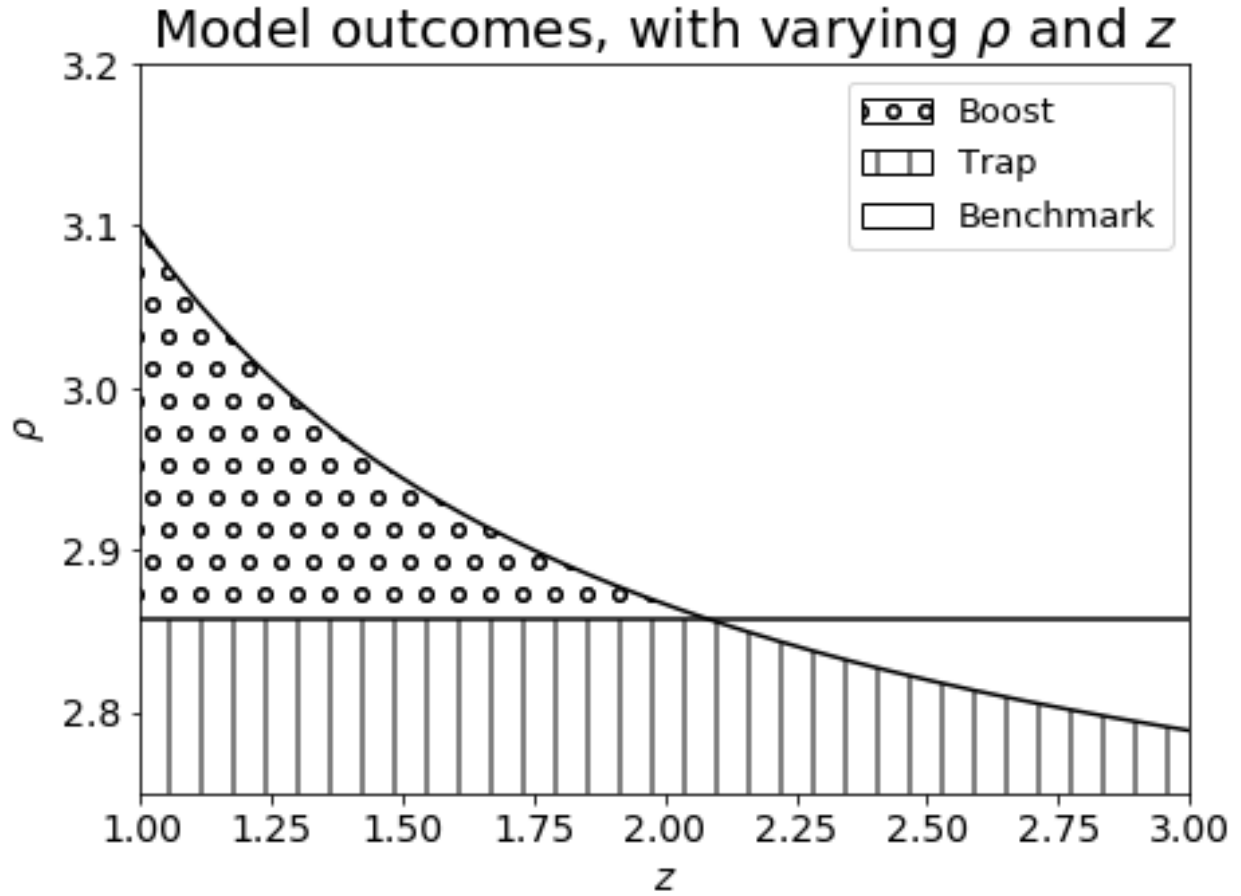


Figure 4: Model outcomes in terms of steady states.  $\rho$  and  $z$  are varied, while the following parameters have been used:  $f = 3k^{.65}$ ,  $r = 3\%$ , and  $C = 1.0$ . The straight horizontal line is at  $\rho = \frac{1}{t^{**}}$ , markedly separating the boost and trap cases.

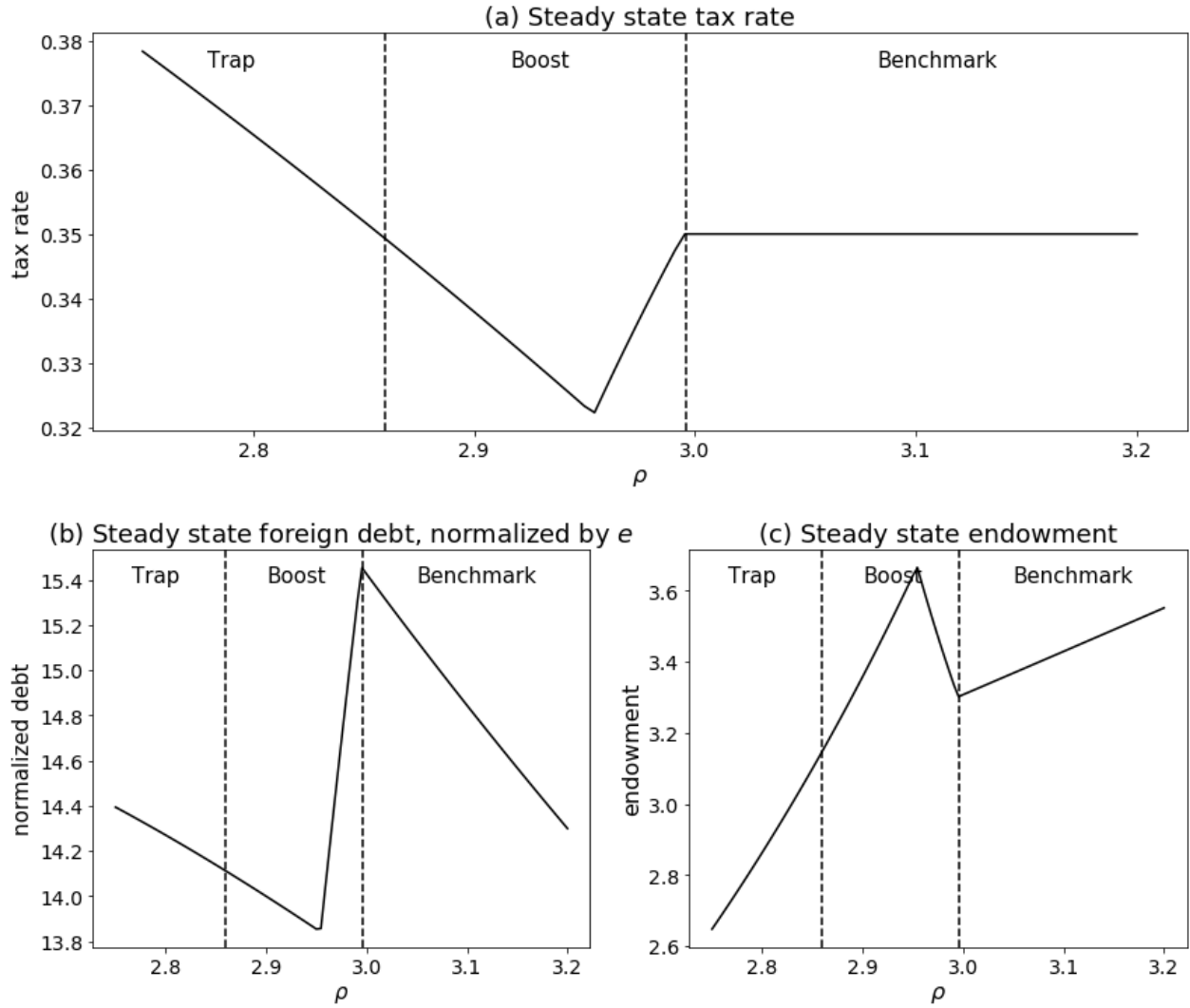


Figure 5: Comparative statics on  $\rho$  – households' propensity to save – to various steady state variables. The following parameters are used:  $f = 3k^{.65}$ ,  $r = 10\%$ ,  $z = 1.25$ , and  $C = 1.0$ .

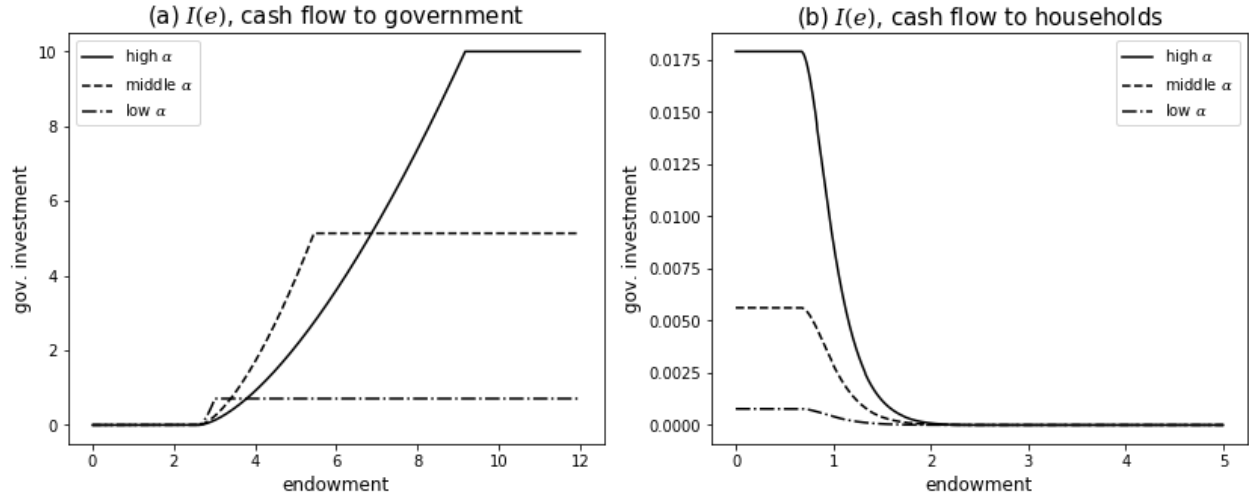


Figure 6: Numerical solution for the two extensions with government technology. Panel (a) correspond to the case where the government production accrues to the next period government, whereas panel (b) correspond to the case where it is returned to the households.  $\alpha$  is the varied parameter, where  $g(\cdot) = \alpha \times f(\cdot)$ . All other parameters are the same as in Fig. 2;  $f = 3k^{.65}$ ,  $r = 10\%$ ,  $z = 4$ ,  $\rho = 2.3$  and  $C = 1.0$ .

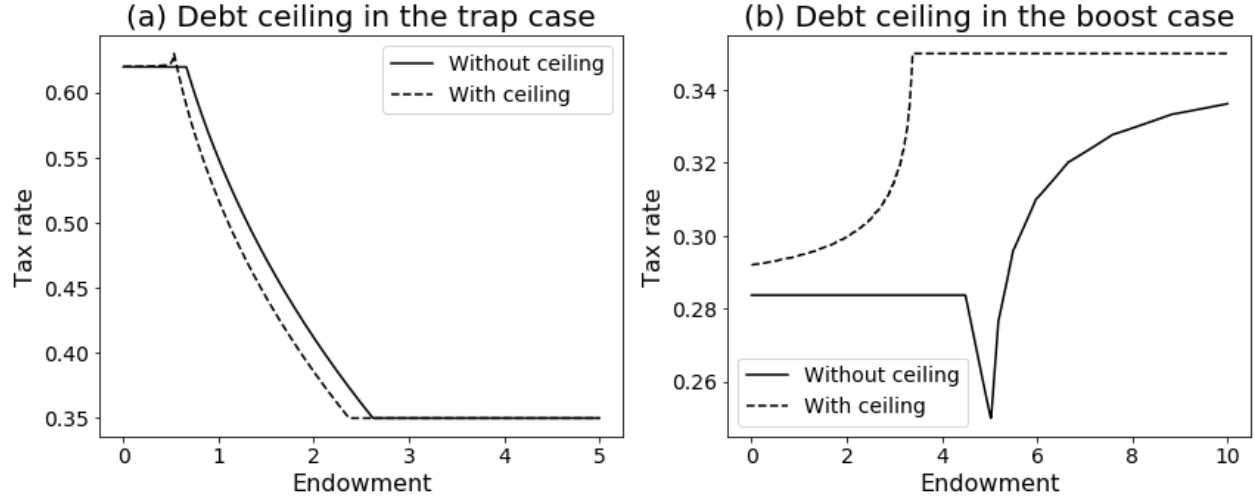


Figure 7: Tax policy of a myopic government facing a debt ceiling equal to 95% of the debt amount taken at steady state  $W$ ,  $D^W$ . In panel (a), the debt ceiling is placed on a model which originally exhibited a growth trap. It can be seen that the debt ceiling lowers the tax rate for the most part. In panel (b), the debt ceiling is placed on a model which originally exhibited a growth boost. In this case, the debt ceiling raises the tax rate uniformly. Parameters used: (a)  $f = 3k^{.65}$ ,  $r = 10\%$ ,  $z = 4$ ,  $\rho = 2.3$  and  $C = 1.0$ . (b)  $f = 3k^{.65}$ ,  $r = 1\%$ ,  $z = 1.1$ ,  $\rho = 3.1$  and  $C = 1.0$

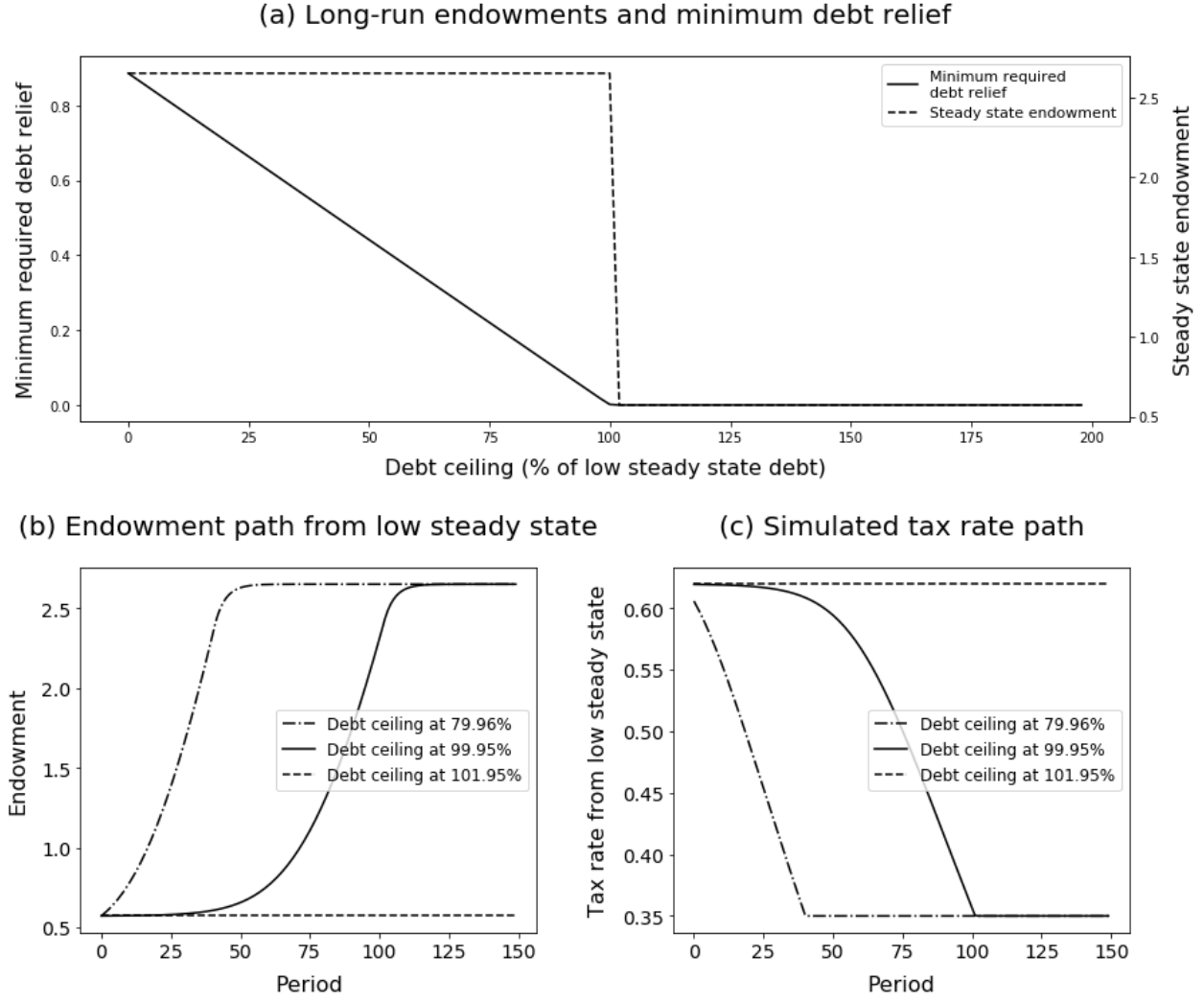


Figure 8: (a) Minimum required relief (left scale) and steady-state endowment (right scale), as functions of debt ceiling. Simulated endowment (b) and tax rate (c) paths after different levels of debt ceilings are placed on a trapped economy. In all figures, The debt ceilings are expressed as % of the level of debt in steady state  $W$ ,  $D^W$ . Parameters used:  $f = 3k^{.65}$ ,  $r = 10\%$ ,  $z = 4$ ,  $\rho = 2.3$  and  $C = 1.0$ .

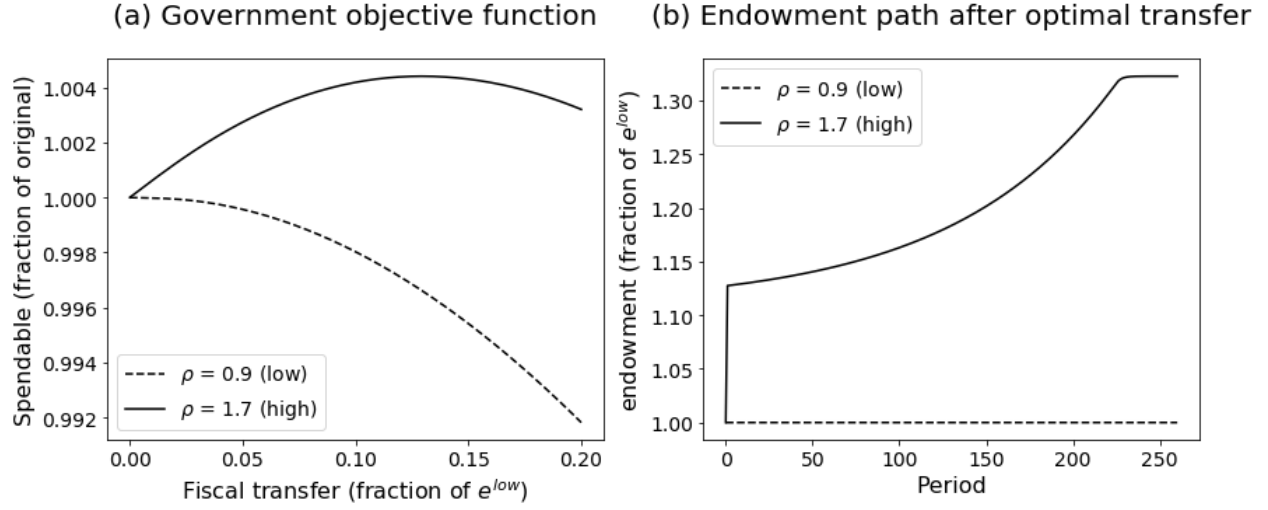


Figure 9: Panel (a) plots the government objective functions against fiscal transfers, for low and high savings parameters  $\rho^{low} = 0.9$ ,  $\rho^{high} = 1.7$ . They are low and high in a relevant sense to  $z$ , *i.e.*, in the “low” parameter configuration,  $\rho^{low} < \frac{1}{z^{low}}$ , and in the “high” parameter configuration,  $\rho^{high} > \frac{1}{z^{high}}$ . Both parameter configurations admit a trap equilibrium. It can be seen that a non-zero fiscal transfer can increase the objective function for the model with high savings parameter, whereas it does not for the one with low savings parameter. Panel (b) plots endowments paths after optimal transfers for the two models. Notice that the fiscal transfer at  $t = 1$  by the government with high savings parameter leads an eventual escape of the trap, whereas it does not happen for the government with low savings parameter. Other parameters used:  $f = 3k^5$ ,  $r = 10\%$ ,  $z = 1.1$ , and  $C = 1.0$ .

## B Mathematical Appendix

**Lemma B.1.** *Household's optimization problem in (2.1) - (2.3) and the associated FOC's (2.4) - (2.7) is solved by the following set of decision functions:*

$$\begin{aligned} k_i &= f'^{-1}\left(\frac{1+r}{1-t_i}\right), \\ c_i &= \kappa_0[(1+r)(e_i - k_i) + (1-t_i)f(k_i)], \\ e_{i+1} &= \kappa_1[(1+r)(e_i - k_i) + (1-t_i)f(k_i)], \text{ and} \\ s_i &= \kappa_1(e_i - k_i) - \kappa_0(1-t_i)f(k_i); \text{ where} \\ \kappa_0 &:= \frac{1}{(1+\rho)(1+r)}; \text{ and } \kappa_1 := \frac{\rho}{1+\rho}. \end{aligned}$$

**Proof:** Combining (2.5) and (2.6), we get the investment decision as a function of tax rate  $t_i$  only:

$$k_i = f'^{-1}\left(\frac{1+r}{1-t_i}\right). \quad (\text{B.1})$$

Combining (2.4), (2.5), and (2.7), we obtain the following marginal condition between the next-period endowment  $e_{i+1}$  and the current-period consumption  $c_i$ :

$$\frac{1}{c_i} - (1+r)\frac{\rho}{e_{i+1}} \Rightarrow e_{i+1} = \rho(1+r)c_i. \quad (\text{B.2})$$

Given our four equations (two each from resource constraints and FOC's), we solve for the four unknowns.  $c_i$  can be solved by adding (2.3) to  $(1+r) \times$  (2.2) and plugging in (B.2):

$$\begin{aligned} (1+r)c_i + \cancel{(1+r)s_i} + (1+r)k_i + \underbrace{e_{i+1}}_{=\rho(1+r)c_i} &= (1+r)e_i + \cancel{(1+r)s_i} + (1-t_i)f(k_i) \\ \Rightarrow (1+r)(1+\rho)c_i &= (1+r)(e_i - k_i) + (1-t_i)f(k_i) \\ \Rightarrow c_i &= \underbrace{\frac{1}{(1+\rho)(1+r)}}_{=\kappa_0} [(1+r)(e_i - k_i) + (1-t_i)f(k_i)]. \end{aligned}$$

and  $k_i$  is determined in (B.1). Similarly, we can derive conditions for  $e_{i+1}$  and  $s_i$ :

$$\begin{aligned} e_{i+1} &= \kappa_1[(1+r)(e_i - k_i) + (1-t_i)f(k_i)], \text{ and} \\ s_i &= \kappa_1(e_i - k_i) - \kappa_0(1-t_i)f(k_i). \end{aligned} \quad \blacksquare$$

**Proof of Proposition 2.2:** It suffices to show that the mapping  $T$  implied by the Bellman

equation preserves monotonicity and concavity. In what follows, we denote  $F : \mathbb{R}_+ \rightarrow \mathbb{R}$  as a generic weakly increasing and concave function. In addition, we let  $e_1$  and  $e_2$  denote generic real values of endowments where  $e_1 < e_2$ , and  $t_1, t_2$  the respective optimal tax rates.

**Monotonicity.** Observe first that both  $e_+(e, t)$  and  $s(e, t)$ , defined respectively in (2.22) and (2.23), are increasing in  $e$ . Next, note that

$$\begin{aligned} TF(e_2) &= \max_t \frac{1}{1+r} [F(e_+(e_2, t)) - \max\{0, \tau^{**} - C - z(1+r)s(e_2, t)\}] + \tau(t) \\ &\geq \frac{1}{1+r} [F(e_+(e_2, t_1)) - \max\{0, \tau^{**} - C - z(1+r)s(e_2, t_1)\}] + \tau(t_1) \\ &\geq \frac{1}{1+r} [F(e_+(e_1, t_1)) - \max\{0, \tau^{**} - C - z(1+r)s(e_1, t_1)\}] + \tau(t_1) \\ &= TF(e_1). \end{aligned}$$

This proves the preservation of monotonicity under the mapping  $T$ . ■

**(i) Concavity.** Take some  $(e_1, t_1)$ ,  $(e_2, t_2)$  and  $\alpha \in (0, 1)$ . Let

$$\begin{aligned} e_\alpha &:= (1-\alpha)e_1 + \alpha e_2; \\ t_\alpha &: e_+(e_\alpha, t_\alpha) = (1-\alpha)e_+(e_1, t_1) + \alpha e_+(e_2, t_2). \end{aligned}$$

It is immediate that such a  $t_\alpha$  always exists. We prove the following lemma first:

**Lemma B.2.** For  $(e_1, t_1)$ ,  $(e_2, t_2)$ , and  $(e_\alpha, t_\alpha)$  defined as above,

$$\begin{aligned} \tau(t_\alpha) &\geq (1-\alpha)\tau(t_1) + \alpha\tau(t_2); \\ s(e_\alpha, t_\alpha) &\geq (1-\alpha)s(e_1, t_1) + \alpha s(e_2, t_2). \end{aligned}$$

**Proof:** From the definition of  $t_\alpha$ , denoting  $k_\alpha := k(t_\alpha)$ ,  $f_\alpha := f(k(t_\alpha))$ ,  $s_\alpha := s(e_\alpha, t_\alpha)$ , and  $\pi_\alpha := \pi(t_\alpha)$ , and recognizing that by definition  $e_\alpha = (1-\alpha)e_1 + \alpha e_2$ , it follows that

$$\begin{aligned} e_+(e_\alpha, t_\alpha) &= (1-\alpha)e_+(e_1, t_1) + \alpha e_+(e_2, t_2) \\ \Rightarrow (1-t_\alpha)f_\alpha - (1+r)k_\alpha &= (1-\alpha)[(1-t_1)f_1 - (1+r)k_1] + \alpha[(1-t_2)f_2 - (1+r)k_2] \\ \Rightarrow \pi(t_\alpha) &= (1-\alpha)\pi(t_1) + \alpha\pi(t_2), \end{aligned}$$

where  $\pi$  is defined in (2.24). From Lemma 1.1 in the Online Appendix, assumptions stated in Definition 2.1 imply that

$$k(t_\alpha) \leq (1-\alpha)k(t_1) + \alpha k(t_2); \tag{B.3}$$

$$\tau(t_\alpha) \geq (1-\alpha)\tau(t_1) + \alpha\tau(t_2). \quad (\text{B.4})$$

In addition, from the definition of  $\pi$  in (2.24), we also have that

$$\begin{aligned} \pi_\alpha &= (1-\alpha)\pi_1 + \alpha\pi_2 \\ \Rightarrow (1-t_\alpha)f_\alpha - (1+r)k_\alpha &= (1-\alpha)(1-t_1)f_1 + \alpha(1-t_2)f_2 - (1+r)((1-\alpha)k_1 + \alpha k_2) \\ \Rightarrow (1-t_\alpha)f_\alpha &= (1-\alpha)(1-t_1)f_1 + \alpha(1-t_2)f_2 - \underbrace{(1+r)((1-\alpha)k_1 + \alpha k_2 - k_\alpha)}_{\geq 0} \\ \Rightarrow (1-t_\alpha)f_\alpha &\leq (1-\alpha)(1-t_1)f_1 + \alpha(1-t_2)f_2, \end{aligned}$$

which leads to

$$\begin{aligned} s_\alpha &= \kappa_1(e_\alpha - k_\alpha) - \kappa_0(1-t_\alpha)f_\alpha \\ &\geq (1-\alpha)s_1 + \alpha s_2. \end{aligned} \quad \blacksquare$$

To show that concavity is preserved under  $T$ , we need to show that

$$TF(e_\alpha) \geq (1-\alpha)TF(e_1) + \alpha TF(e_2).$$

First, by the definition of  $t_\alpha$  and the concavity of  $F$ ,

$$\begin{aligned} e_+(e_\alpha, t_\alpha) &= (1-\alpha)e_+(e_1, t_1) + \alpha e_+(e_2, t_2) \quad (\because \text{Construction of } t_\alpha) \\ \Rightarrow F(e_+(e_\alpha, t_\alpha)) &\geq (1-\alpha)F(e_+(e_1, t_1)) + \alpha F(e_+(e_2, t_2)). \end{aligned} \quad (\text{B.5})$$

Second, since  $\max(x, y) + \max(a, b) \geq \max(x+a, x+b)$ , we have

$$\begin{aligned} &(1-\alpha)\max\{0, \tau^{**} - Cz(1+r)s_1\} + \alpha\max\{0, \tau^{**} - Cz(1+r)s_2\} \\ &\geq \max\{0, \tau^{**} - C - z(1+r)[(1-\alpha)s_1 + \alpha s_2]\} \\ &\geq \max\{0, \tau^{**} - C - z(1+r)s_\alpha\}. \end{aligned} \quad (\text{B.6})$$

Then,

$$\begin{aligned} TF(e_\alpha) &= \max_t \frac{1}{1+r} [F(e_+(e_\alpha, t)) - \max\{0, \tau^{**} - C - z(1+r)s(e_\alpha, t)\}] + \tau(t) \\ &\geq \frac{1}{1+r} [F(e_+(e_\alpha, t_\alpha)) - \max\{0, \tau^{**} - C - z(1+r)s(e_\alpha, t_\alpha)\}] + \tau(t_\alpha) \\ &\geq (1-\alpha)TF(e_1) + \alpha TF(e_2), \end{aligned}$$

where the last step comes from the combination of (B.5), (B.6), and (B.4). ■

**(ii) (Binding constraints).** We prove the following logically equivalent statement: let  $e_1 < e_2$ . If at  $e_1$  the ability-to-pay constraint is binding, that so it must at  $e_2$  also. If instead at  $e_2$  the willingness-to-pay binds, then so it must at  $e_1$  also.

**Proof:** First let us set forth the associated first-order conditions (FOC's). If at  $e$  the ability-to-pay constraint is binding, then the following FOC is satisfied:

$$\begin{aligned} & \underbrace{\frac{de_+}{dt} \frac{dS}{de}}_{=\pi'(t)} + (1+r)\tau'(t) = 0 \\ \Rightarrow & \underbrace{\frac{dS}{de} + (1+r)\frac{\tau'(t)}{\pi'(t)}}_{:=FOC^{ability}(e,t)} = 0. \end{aligned}$$

If instead at  $e$  the willingness constraint is binding, then the following FOC is satisfied:

$$\begin{aligned} & \underbrace{\frac{de_+}{dt} \frac{dS}{de}}_{=\pi'(t)} + z(1+r)\frac{ds}{dt} + (1+r)\tau'(t) = 0 \\ \Rightarrow & \underbrace{\frac{dS}{de} + z(1+r)\frac{s'(t)}{\pi'(t)} + (1+r)\frac{\tau'(t)}{\pi'(t)}}_{:=FOC^{willingness}(e,t)} = 0. \end{aligned}$$

Since  $s' > 0$  and  $\pi' < 0$ , it follows that  $FOC^{willingness}(e, t) < FOC^{ability}(e, t)$  always.

If both are binding, then it must be that  $\tau^{**} - C - z(1+r)s = 0$  and

$$\begin{aligned} & FOC^{ability}(e, t) > 0, \text{ and} \\ & FOC^{willingness}(e, t) < 0. \end{aligned}$$

as increasing  $t$  by  $dt$  would enter the region where only the ability-to-pay constraint is binding ( $\tau^{**} - C - z(1+r)s < 0$ ) and increase the objective function by  $\pi' FOC^{ability}(e, t)dt$ . Since  $\pi' < 0$  and  $dt > 0$ ,  $FOC^{ability}$  must be greater than 0 for this not to be a perturbation that increases the objective function. Similar argument applies in the opposite direction ( $dt < 0$ ) for  $FOC^{willingness}$ .

We then prove the following lemma:

**Lemma B.3.** Both  $FOC^{ability}(e, t)$  and  $FOC^{willingness}(e, t)$  are (weakly) decreasing in  $e$  and (strictly) increasing in  $t$ .

**Proof:** For  $FOC^{ability}(e, t)$ , observe that  $e_+(e, t)$  is increasing in  $e$  and decreasing in  $t$ . Combined with the fact that  $S$  is concave, it follows that  $dS/de$  is decreasing in  $e$  and increasing in  $t$ . From the assumptions stated in Definition 2.1,  $\frac{\tau'}{\pi'}$  is increasing in  $t$ . This proves the properties for  $FOC^{ability}(e, t)$ .

For  $FOC^{willingness}(e, t)$ , it only remains to be proved that  $\frac{s'}{\pi'}$  is increasing in  $t$  as the function is independent of  $e$ . Notice that since  $\pi = (1-t)f - (1+r)k$  and  $s = \kappa_1(e-k) - \kappa_0(1-t)f$ ,

$$\begin{aligned}\frac{s'}{\pi'} &= \frac{-\kappa_1 k' - \kappa_0(\pi' + (1+r)k')}{\pi'} \\ &= -[\kappa_1 + \kappa_0(1+r)] \frac{k'}{\pi'} - \kappa_0.\end{aligned}$$

Since  $\frac{k'}{\pi'}$  is assumed to be decreasing in  $t$  in Definition 2.1, this proves the properties for  $FOC^{willingness}(e, t)$ . ■

Now, consider the first case where at  $e_1$  the ability-to-pay constraint is binding and suppose per contra that at  $e_2$  the ability-to-pay constraint is non-binding. This implies that

$$\begin{aligned}\tau^{**} - C - z(1+r)s(e_1, t_1) &\leq 0, \text{ and} \\ \tau^{**} - C - z(1+r)s(e_2, t_2) &> 0.\end{aligned}$$

Observe that since  $s$  is increasing in both  $e$  and  $t$ ,

$$\begin{aligned}\tau^{**} - C - z(1+r)s(e_2, t_2) &> 0 \geq \tau^{**} - C - z(1+r)s(e_1, t_1) \\ \Rightarrow z(1+r)s(e_2, t_2) &< z(1+r)s(e_1, t_1) \\ \Rightarrow z(1+r)s(e_1, t_2) &< z(1+r)s(e_1, t_1) \\ \Rightarrow t_1 &> t_2.\end{aligned}$$

At  $e_1$ , the FOC should be met, which implies that  $FOC^{ability}(e_1, t_1) = 0$  and accordingly  $FOC^{willingness}(e_1, t_1) < 0$ . At  $e_2$ ,  $FOC^{willingness}(e_2, t_2) = 0$  and accordingly  $FOC^{ability}(e_2, t_2) > 0$ . Comparing  $FOC^{ability}$  evaluated at different parameters,

$$FOC^{ability}(e_2, t_2) > 0 = FOC^{ability}(e_1, t_1) > FOC^{ability}(e_2, t_1) \Rightarrow t_2 > t_1,$$

leading to a contradiction. The proof of the second case is a mirror image. ■

**(iii) (Continuity).** By the theorem of the maximum, we only have to prove that for each  $e$ , there is a unique  $t$  that maximizes the objective function. First observe that, since  $s(e, t)$  is concave in  $t$ , the penalty function  $-\max\{0, \cdot\}$  is concave in  $t$ . Next, Let  $e$  be an arbitrary number and

consider  $t_1 < t_2$  and suppose per contra that  $t_1$  and  $t_2$  both achieve the maximum. Consider an arbitrary  $\alpha \in (0, 1)$  and pick  $t_\alpha$  as in Lemma B.2. By the stated lemma and the fact that  $S$  is concave, we know respectively that

$$\begin{aligned}\tau(t_\alpha) &\geq (1-\alpha)\tau(t_1) + \alpha\tau(t_2), \text{ and} \\ S(e'(t_\alpha, e)) &\geq (1-\alpha)S(e'(t_1, e)) + \alpha S(e'(t_2, e)).\end{aligned}$$

Since this holds true for any arbitrary  $\alpha$ , by picking  $t_\alpha$  we should achieve a larger objective function. The claim is then proved by contradiction. ■

$t(e)$  **increasing in**  $[0, \bar{e}^1]$ : Suppose not, and suppose that  $e_1 < e_2$  and  $t_1 > t_2$ . This creates the following contradiction:

$$0 = FOC^{willingness}(t_1, e_1) \geq FOC^{willingness}(t_1, e_2) > FOC^{willingness}(t_2, e_2) = 0.$$

$t(e)$  **decreasing in**  $[\bar{e}^1, \bar{e}^2]$ : In this region, the optimal  $t$  is such that  $\tau^{**} - C - z(1+r)s = 0$ . The proof follows from the fact that  $s$  is increasing in both  $e$  and  $t$ .

$t(e)$  **increasing in**  $[\bar{e}^2, \infty]$ : Suppose not, and suppose that  $e_1 < e_2$  and  $t_1 > t_2$ . This creates the following contradiction:

$$0 = FOC^{ability}(t_1, e_1) \geq FOC^{ability}(t_1, e_2) > FOC^{ability}(t_2, e_2) = 0.$$

**(iv) (Asymptotics).** We first prove that  $S(e)$  is bounded. First observe that, since  $\max\{0, \tau^{**} - C - zs(1+r)\} \geq 0$ ,  $S(e)$  is bounded from above by an alternative value function  $\tilde{S}(e)$

$$\tilde{S}(e) := \max_t \frac{1}{1+r} \tilde{S}(e') + \tau(t)$$

for which the solution is simply  $\tilde{S} = \frac{\tau^{**}}{r}$ . Therefore, we conclude that  $S(e) \leq \frac{\tau^{**}}{r} \forall e$ . Combined with the fact that  $S(e)$  is weakly increasing and concave in  $e$ , we have that  $S'(e) \rightarrow 0$  as  $e \rightarrow \infty$ . Note, then, at sufficiently high  $e$ , the optimal  $t = \operatorname{argmax}_t \frac{1}{1+r} S(e') + \tau(t) = t^{**}$ . ■

**Proof of Lemma 3.1:** In order to prove this lemma, we prove Lemmas B.4 - B.6 first.

**Lemma B.4.** Any endowment path  $\{e_i\}_{i=0}^\infty$  is a monotone sequence (increasing or decreasing). This immediately implies that any growth path has a limit, and it must be a fixed point of the policy function  $h(e) := e_+(e, t(e))$ .

**Proof:** It suffices to prove that  $h(e)$  is a monotonic increasing function, because  $e_i < e_{i+1} = h(e_i)$  would imply that  $e_{i+2} = h(e_{i+1}) > h(e_i) = e_{i+1}$ , which leads by induction that  $e_{j+1} > e_j$

for  $\forall j \geq i$ . We have proved in Proposition 2.2 that there are three regions to consider:  $[0, \hat{e}^1]$ ,  $[\hat{e}^1, \hat{e}^2]$ , and  $[\hat{e}^2, \infty]$ . We prove piecewise monotonicity in each of these regions, which suffices for overall monotonicity given the continuity of  $t(e)$  proved in Proposition 2.2. Recall from (2.22) that  $e_+(e, t)$  is increasing in  $e$  and decreasing in  $t$ .

- (Region 1) Take  $e_1 < e_2$ ,  $e_1, e_2 \in [0, \hat{e}^1]$  and suppose per contra  $h(e_1) > h(e_2)$ . This must imply that  $t_1 < t_2$ . Note that  $FOC^{willingness}$  must be met at both points and recall that both  $\frac{s'}{\pi'}$  and  $\frac{\tau'}{\pi'}$  are strictly increasing in  $t$  (Lemma B.3). This leads to

$$\begin{aligned}
0 &= \frac{dS}{de} \Big|_{h(e_1)} + z(1+r) \frac{s'(t_1)}{\pi'(t_1)} + (1+r) \frac{\tau'(t_1)}{\pi'(t_1)} \\
&< \frac{dS}{de} \Big|_{h(e_1)} + z(1+r) \frac{s'(t_2)}{\pi'(t_2)} + (1+r) \frac{\tau'(t_2)}{\pi'(t_2)} & (\because t_1 < t_2) \\
&\leq \frac{dS}{de} \Big|_{h(e_2)} + z(1+r) \frac{s'(t_2)}{\pi'(t_2)} + (1+r) \frac{\tau'(t_2)}{\pi'(t_2)} & (\because h(e_1) > h(e_2) \text{ and concavity of } S) \\
&= 0.
\end{aligned}$$

which is a contradiction.

- (Region 2) Take  $e_1 < e_2$ ,  $e_1, e_2 \in [\hat{e}^1, \hat{e}^2]$ . We have proved in Proposition 2.2 that  $t_1 > t_2$  in this region. Therefore  $h(e_1) < h(e_2)$  immediately follows.
- (Region 3) This part is similar to region 1. ■

Lemma B.4 allows us limit the analysis of only the fixed points of the policy function  $h(e)$ . Essentially, these are steady states defined in Definition 3.1 plus the saddle fixed points. Saddle fixed points are limiting endowments of a measure zero starting endowment - only if it starts at that exact point - and therefore we exclude them from our analysis.

Next we characterize all possible steady states. Recall that  $e^{sat}(t)$  is defined in (3.10). In addition, we define an additional auxiliary function  $e^{abil}(t)$  in (B.7):

**Definition B.1.** Define the following function:

$$\begin{aligned}
e^{abil}(t) &:= e \quad \text{s.t.} \quad \tau^{**} - C - z(1+r)s(e, t) = 0 \\
\Rightarrow e^{abil}(t) &= k(t) + \frac{(1-t)f(k(t))}{\rho(1+r)} + \frac{\tau^{**} - C}{z\kappa_1(1+r)}; \text{ and,} & (B.7)
\end{aligned}$$

In intuitive terms, for any given  $t$ ,  $e^{abil}(t)$  is the boundary endowment at which both constraints are binding ( $\tau^{**} - C - z(1+r)s(e, t) = 0$ ).

**Lemma B.5.**  $e^{ss}$  must satisfy one of the following:

- (Steady state W)  $e^{ss} \in [0, \hat{e}^1)$  and is characterized by

$$\begin{aligned} t^W := t \quad \text{such that} \quad \rho \frac{de_+}{dt} z + z(1+r) \frac{ds}{dt} + (1+r)\tau' &= 0; \\ e^{ss} &= e^{sat}(t^W). \end{aligned} \quad (\text{B.8})$$

- (Steady state A)  $e^{ss} \in (\hat{e}^2, \infty)$  and is characterized by  $e^{ss} = e^{sat}(t^{**})$ .
- (Steady state S)  $e^{ss} = \hat{e}^1 = \hat{e}^2$ , and is characterized by  $t^{ss}$  such that  $e^{ss} = e^{abil}(t^{ss}) = e^{sat}(t^{ss})$ .

**Proof:** It is straightforward to see that  $e^{ss}$  must belong in one of the three regions  $[0, \hat{e}^1)$ ,  $[\hat{e}^1, \hat{e}^2]$ ,  $(\hat{e}^2, \infty)$ . We first prove that in the interior in the region  $([0, \hat{e}^1))$  and region  $((\hat{e}^2, \infty))$ , the fixed points must take the aforementioned form. Suppose that  $e^{ss} \in [0, \hat{e}^1)$ . Then, in the neighborhood of  $e^{ss}$ , the Bellman equation is

$$S = \max_t \frac{1}{1+r} \left[ S(e') - \tau^{**} + C + zs(1+r) \right] + \tau(t).$$

From the envelope condition, we get that  $\frac{dS}{de} = \rho z$ . Then, the optimal  $t$  can be derived by solving the following isolated equation:

$$\rho \frac{de_+}{dt} z + z(1+r) \frac{ds}{dt} + (1+r)\tau' = 0. \quad (\text{B.9})$$

Finally, since  $e^{ss}$  must be a fixed point, it follows that  $e^{ss} = e^{sat}(t^W)$  where  $t^W$  is the solution to (B.9). The steady-state endowment in the region  $((\hat{e}^2, \infty))$  can be obtained similarly.

Next, we prove that if  $\hat{e}^1 < \hat{e}^2$ , then  $e^{ss}$  cannot belong to the middle region  $([\hat{e}^1, \hat{e}^2])$ . We prove that in order for a fixed point  $t^{ss} : e^{abil}(t^{ss}) - e^{sat}(t^{ss}) = 0$  to be a stable point,  $\frac{d}{dt}e^{abil}(t) - \frac{d}{dt}e^{sat}(t)$  must be non-positive at  $t^{ss}$ . Suppose per contra that  $\frac{d}{dt}e^{abil}(t) - \frac{d}{dt}e^{sat}(t) > 0$ . Note that in a small neighborhood of  $e^{ss}$ , the two functions can be approximated as

$$\begin{aligned} e^{abil}(t) &= e^{ss} + \frac{d}{dt}e^{abil}(t)(t - t^{ss}) \Rightarrow e_{abil}^{-1}(e) = t^{ss} + \left( \frac{d}{dt}e^{abil}(t) \right)^{-1} (e - e^{ss}); \\ e^{sat}(t) &= e^{ss} + \frac{d}{dt}e^{sat}(t)(t - t^{ss}) \Rightarrow e_{sat}^{-1}(e) = t^{ss} + \left( \frac{d}{dt}e^{sat}(t) \right)^{-1} (e - e^{ss}). \end{aligned}$$

Note that in this neighborhood  $e < e^{ss} \Rightarrow e_{abil}^{-1}(e) > e_{sat}^{-1}(e)$ .

Suppose now WLOG<sup>15</sup> that in the left neighborhood of  $e^{ss}$ , the optimal policy is sliding

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<sup>15</sup>without loss of generality

between the two constraints, i.e.,  $t(e) = e_{abil}^{-1}(e)$ . Consider  $e$  in this neighborhood  $e \in (e^{ss} - \epsilon, e^{ss})$  and consider  $e_+(e, t(e))$ . By definition of  $e^{sat}$ ,  $e_+(e, t) < e$  if and only if  $t > e_{sat}^{-1}(e)$ . Therefore, it follows that  $e_+(e, t(e)) = e_+(e, e_{abil}^{-1}(e)) < e$ . Since this applies to all elements of the left neighborhood of  $e^{ss}$ , combined with the fact from Lemma B.4  $h(e)$  is a monotonic increasing function, that endowment paths are it follows that  $e$  can never converge to  $e^{ss}$ . Therefore,  $e^{ss} \notin [\hat{e}^1, \hat{e}^2]$  if  $\hat{e}^1 < \hat{e}^2$ .

We next prove that the derivative condition  $\frac{d}{dt}e^{abil}(t) - \frac{d}{dt}e^{sat}(t) \leq 0$  is impossible. Recall that  $e^{abil}(t) - e^{sat}(t) = \psi_1\pi(t) + \psi_2k(t) + \psi_3$  where  $\psi_2$  and  $\psi_3$  are positive. By the definition of  $t^{ss}$ ,

$$\begin{aligned} \psi_1\pi(t^{ss}) + \psi_2k(t^{ss}) + D = 0 &\Rightarrow \psi_1\pi(t^{ss}) + \psi_2k(t^{ss}) < 0 \Rightarrow \psi_1 < -\psi_2 \frac{k(t^{ss})}{\pi(t^{ss})}, \text{ so that} \\ \frac{d}{dt}e^{abil}(t^{ss}) - \frac{d}{dt}e^{sat}(t^{ss}) &= \psi_1\pi'(t^{ss}) + \psi_2k'(t^{ss}) \\ &> -\psi_2 \frac{k(t^{ss})}{\pi(t^{ss})} \pi'(t^{ss}) + \psi_2k'(t^{ss}). \quad (\because \pi' < 0) \end{aligned}$$

Note that

$$\begin{aligned} -\psi_2 \frac{k(t^{ss})}{\pi(t^{ss})} \pi'(t^{ss}) + \psi_2k'(t^{ss}) \geq 0 &\Leftrightarrow -\frac{\pi'(t^{ss})}{\pi(t^{ss})} + \frac{k'(t^{ss})}{k(t^{ss})} \geq 0 \quad (\because \psi_2, k > 0) \\ &\Leftrightarrow -\frac{d}{dt} \log(\pi(t^{ss})) + \frac{d}{dt} \log(k(t^{ss})) \geq 0 \\ &\Leftrightarrow \frac{d}{dt} \log\left(\frac{k(t^{ss})}{\pi(t^{ss})}\right) \geq 0 \\ &\Leftrightarrow \frac{d}{dt} \frac{k(t^{ss})}{\pi(t^{ss})} \geq 0 \\ &\Leftrightarrow \frac{k(t)}{\pi(t)} \text{ is weakly increasing.} \end{aligned}$$

Therefore, the assumption that  $\frac{k(t)}{\pi(t)}$  is weakly increasing (it is constant for power production function) is a sufficient condition for any fixed point in  $[\hat{e}^1, \hat{e}^2]$  not to be a stable fixed point. ■

**Lemma B.6.** *The following facts are true:*

- A. Steady state  $W$  ( $e^{ss} \in [0, \hat{e}^1)$ ) exists if and only if  $e^{abil}(t^W) \geq e^{sat}(t^W)$ .
- B. Steady state  $A$  ( $e^{ss} \in (\hat{e}^2, \infty)$ ) exists if and only if  $e^{abil}(t^{**}) \leq e^{sat}(t^{**})$ .

- C. If either of conditions A and B are met, then  $\hat{e}^1 < \hat{e}^2$  almost always, implying that steady state S cannot exist.
- D. If neither of conditions A and B are met, then  $\hat{e}^1 = \hat{e}^2$  and the only steady state is steady state S:  $e^{ss} = \hat{e}^1 = \hat{e}^2$ .

**Proof:** The proof follows four steps A-D below.

- A. The “only if” part is proved in Lemma B.5. To show the “if” part, recall the Bellman equation

$$\begin{aligned} S(e) = \max_t \quad & \left[ \frac{1}{1+r} [S(e') - \max\{0, \tau^{**} - C - zs(1+r)\}] + \tau(t) \right] \\ \text{s.t.} \quad & e' = \kappa_1[(1+r)(e - k(t)) + (1-t)f(k(t))], \\ & s = \kappa_1(e - k(t)) - \kappa_0(1-t)f(k(t)), \text{ and} \\ & k(t) = f'^{-1}\left(\frac{1+r}{1-t}\right). \end{aligned} \quad (\text{B.10})$$

Now conjecture that  $S(e) = \alpha + \beta e$  and  $t(e) = t^W \forall e \leq e^{abil}(t^W)$ . It can be verified that the conjecture is correct if

$$\begin{aligned} \alpha &= \frac{1+r}{r} - r(\tau^{**} - C), \text{ and} \\ \beta &= \rho z. \end{aligned}$$

owing to the fact that  $e'(e, t^W) < e^{abil}(t^W)$  if  $e < e^{abil}(t^W)$  and thus the ability-to-pay constraint is never binding in this region.

- B. Similar to A., we can verify a conjectured partial solution  $S(e) = \frac{1+r}{r}\tau^{**}$  and  $t(e) = t^{**} \forall e \geq e^{abil}(t^{**})$ , owing to the fact that  $e'(e, t^{**}) > e^{abil}(t^{**})$  if  $e < e^{abil}(t^{**})$  and thus the willingness-to-pay constraint is never binding in this region.
- C. Suppose per contra that steady state A exists, and that  $\hat{e}^1 = \hat{e}^2$ . Note that steady state W cannot exist as it would directly violate the continuity of  $t(e)$  proved in Proposition 2.2. Now suppose that it does not, and consider an endowment  $e$  arbitrarily lower than  $\hat{e}^1$ . Because steady state W does not exist, the next-period endowment must be over  $\hat{e}^2$ , at which point the spendables function  $S$  is a constant value. Note that this would imply the optimal tax rate  $t$  to be the solution of:

$$t = \operatorname{argmax}_t \left[ \frac{1}{1+r} [S(e') - \tau^{**} + C + zs(1+r)] + \tau(t) \right] \quad (\because e < \hat{e}^1)$$

$$= \operatorname{argmax}_t [zs(e, t) + \tau(t)] \quad (\because S(e') \text{ is constant})$$

which is almost surely different from  $t^{**} := \operatorname{argmax} \tau(t)$ . This violates the continuity of  $t(e)$ . The proof of the case where steady state  $W$  exists is a mirror image. ■

D. This immediately follows from Lemma B.5. ■

**Proof of Proposition 3.2:** The following corollary of Lemma B.6 is a sufficient condition for the proposition:

**Lemma B.7.** *We analyze six different parameter cases, which span all possible cases due to the fact that  $e^{abil}(1) > e^{sat}(1)$  always, and the single-crossing properties implied by the assumptions in Definition 2.1. [Refer to Figs. 1–4 of the Online Appendix for the solution characteristics for each of the six cases.]*

- **Case A.**  $t^{**} < t^W$ , and
  - **A1. (Benchmark)**  $e^{sat}(t) \geq e^{abil}(t)$  for both  $t^{**}$  and  $t^W$ : Regardless of the starting endowment  $e_0$ , the economy converges to  $e_{\infty}^{**}$  ( $\forall e_0, e_{\infty}(e_0) = e_{\infty}^{**}$ ).
  - **A2. (Trap)**  $e^{sat}(t) \leq e^{abil}(t)$  for both  $t^{**}$  and  $t^W$ : Regardless of  $e_0$ , the economy converges to the same point **lower** than the benchmark limit ( $\forall e_0, e_{\infty}(e_0) = e^{sat}(t^W) < e_{\infty}^{**}$ ).
  - **A3. (Trap or Benchmark)**  $e^{sat}(t^{**}) > e^{abil}(t^{**})$  and  $e^{sat}(t^W) < e^{abil}(t^W)$ : There is a unique crossing point for the two functions  $e^{sat}$  and  $e^{abil}$ , say  $\bar{e}_A$ . Then,

$$e_{\infty}(e_0) = \begin{cases} e^{sat}(t^W) & \text{if } e_0 < \bar{e}_A; \text{ and} \\ e_{\infty}^{**} & \text{if } e_0 \geq \bar{e}_A. \end{cases}$$

- **Case B.**  $t^{**} \geq t^W$ , and
  - **B1. (Benchmark)**  $e^{sat}(t) \geq e^{abil}(t)$  for both  $t^{**}$  and  $t^W$ : Regardless of  $e_0$ , the economy converges to  $e_{\infty}^{**}$  ( $\forall e_0, e_{\infty}(e_0) = e_{\infty}^{**}$ ).
  - **B2. (Boost)**  $e^{sat}(t) \leq e^{abil}(t)$  for both  $t^{**}$  and  $t^W$ : Regardless of  $e_0$ , the economy converges to the same point **higher** than the benchmark limit ( $\forall e_0, e_{\infty}(e_0) = e^{sat}(t^W) > e_{\infty}^{**}$ ).
  - **B3. (Boost)**  $e^{sat}(t^{**}) < e^{abil}(t^{**})$  and  $e^{sat}(t^W) > e^{abil}(t^W)$ : There is a unique crossing point for the two functions  $e^{sat}$  and  $e^{abil}$ , say  $\bar{e}_B$ . Then, regardless of  $e_0$ , the economy converges to  $\bar{e}_B$  which is **higher** than the benchmark limit ( $\forall e_0, e_{\infty}(e_0) = \bar{e}_B > e_{\infty}^{**}$ ). Also, it is only at this singleton point that both constraints are binding.

**Proof of Proposition 3.3:** Note that  $t^W$  maximizes

$$t^W = \operatorname{argmax}_t \rho z \frac{\kappa_1}{1+r} \pi(t) - z[\kappa_1 k(t) + \kappa_0(1-t)f(k(t))] + \tau(t). \quad (\text{B.11})$$

Note that

$$\rho z \frac{\kappa_1}{1+r} \pi(t) - z[\kappa_1 k(t) + \kappa_0(1-t)f(k(t))] + \tau(t) = \tau(t) - z\left(\frac{1-\rho}{1+r} \pi(t) + k(t)\right)$$

Since by assumption  $\pi$  and  $k$  are convex, and  $\tau$  is concave, expression in (B.11) is concave.

This implies that  $t^W > t^{**}$  if and only if the FOC at  $t^{**}$  is positive. This translates to

$$\frac{\rho z \kappa_1}{1+r} \pi'(t^{**}) - z \kappa_1 k'(t^{**}) + z \kappa_0 f(k(t^{**})) - z \kappa_0(1+r)k'(t^{**}) + \tau'(t^{**}) > 0, \quad (\text{B.12})$$

It is sufficient to derive conditions for (B.12) to hold. Using  $\pi'(t) = -f(k)$  as well as

$$\begin{aligned} (tf(k(t)))'|_{t^{**}} &= 0 \\ \Rightarrow f(k(t^{**})) + t f'(k)k'(t^{**}) &= 0 \\ \Rightarrow f(k(t^{**})) &= -t \frac{1+r}{1-t} k'(t^{**}), \end{aligned}$$

we can simplify the expression in (B.12) as the following:

$$\begin{aligned} &\frac{\rho z \kappa_1}{1+r} \pi'(t^{**}) - z \kappa_1 k'(t^{**}) + z \kappa_0 f(k(t^{**})) - z \kappa_0(1+r)k'(t^{**}) + \tau'(t^{**}) > 0 \\ \Rightarrow &z \kappa_0 \left[ \rho^2 t^{**} \frac{1+r}{1-t^{**}} - \rho(1+r) - t^{**} \frac{1+r}{1-t^{**}} - (1+r) \right] k'(t^{**}) > 0 \\ \Rightarrow &z \kappa_0 \frac{1+r}{1-t^{**}} [t^{**} \rho^2 - (1-t^{**})\rho - 1] < 0. \end{aligned}$$

The characteristic quadratic equation has two roots:

$$\frac{(1-t^{**}) \pm \sqrt{((1-t^{**})^2 + 4t^{**})}}{2t^{**}} = \left\{ \frac{1}{t^{**}}, -1 \right\}.$$

Since  $\rho > 0$ , the second root is economically irrelevant and therefore we get that

$$t^W > t^{**} \Leftrightarrow \rho < \frac{1}{t^{**}}. \quad \blacksquare$$

**Proof of Proposition 3.4:** First, we prove that  $t^{**} < 1$ . Recall that  $t^{**} = \operatorname{argmax}_t \tau(t)$  and  $\tau(t) \geq 0$ . Since  $\tau(1) = 0$  always, it cannot be the case that  $1 = \operatorname{argmax}_t \tau(t)$ . Therefore,  $t^{**} < 1$ . Further,  $t^{**}$  does not vary with  $\rho$ .

Next, we prove that for any  $t < 1$ ,  $\exists \hat{\rho}$  such that  $e^{abil}(t) < e^{sat}(t)$ . Recall that

$$e^{abil}(t) = k(t) + \frac{(1-t)f(k(t))}{\rho(1+r)} + \frac{\tau^{**} - C}{z\left(\frac{\rho}{1+\rho}\right)(1+r)}, \text{ and}$$

$$e^{sat}(t) = \frac{(1-t)f(k(t)) - (1+r)k(t)}{\frac{1}{\rho} - r}.$$

Note that for  $t < 1$ ,  $(1-t)f(k(t)) - (1+r)k(t) > 0$ , and that keeping all else equal,  $e^{sat}(t)$  is monotonically increasing in  $\rho$ , reaching infinity as  $\rho \rightarrow \frac{1}{r}$ , whereas  $e^{abil}$  is monotonically decreasing in  $\rho$ . It follows that for any given  $t < 1$ , there must exist a threshold  $\hat{\rho}(t) < \frac{1}{r}$  such that  $e^{sat}(t) > e^{abil}(t)$ .

Finally, it suffices to consider the case where  $\rho > \frac{1}{t^{**}}$ , under which case  $t^W < t^{**}$ . Notice that due to the single-crossing properties of  $e^{abil}$  and  $e^{sat}$ ,  $e^{sat}(t^{**}) > e^{abil}(t^{**}) \Rightarrow e^{sat}(t^W) > e^{abil}(t^W)$  in this case. Given that  $t^{**}$  does not vary with  $\rho$ , it follows that for  $\rho > \bar{\rho} = \hat{\rho}(t^{**})$ ,  $e^{sat}(t) > e^{abil}(t)$  for both  $t^{**}$  and  $t^W$ . From Lemma B.7, this implies that model outcomes are either A1 or B1, where endowments always converge to the benchmark steady state. ■

**Lemma B.8.** *Conditional on not defaulting, government's actions are independent of past government debt ceilings and legacy debt. Suppose that the debt ceiling that the government in period  $i$  faces is  $\bar{D}_i$ ,  $\forall i \in \mathbb{Z}_+$ . Then, the current government's problem can be summarized as solving the following Bellman equation:*

$$S(e; \bar{D}_0, \bar{D}_1, \dots) = \max_t \left[ \min \left[ \frac{1}{1+r} (S(e'; \bar{D}_1, \bar{D}_2, \dots) - \max\{0, \tau^{**} - C - zs(1+r)\}), \bar{D}_0 \right] + \tau(t) \right] \quad (\text{B.13})$$

$$\begin{aligned} \text{s.t. } e' &= \kappa_1[(1+r)(e - k(t)) + (1-t)f(k(t))], \\ s &= \kappa_1(e - k(t)) - \kappa_0(1-t)f(k(t)), \text{ and} \\ k(t) &= f'^{-1}\left(\frac{1+r}{1-t}\right). \end{aligned}$$

Then, similarly to Lemma 2.1, the decision rule encompassing default for government  $i$  which has inherited an economy with endowment  $e_i$ , legacy debt  $D_{i-1}$ , and legacy domestic debt  $D_{i-1}^{Dom}$  can be characterized as the following. For the sake of brevity, we use the notation  $S_i(\cdot) := S(\cdot; \bar{D}_i, \bar{D}_{i+1}, \dots)$  and  $t_i(\cdot) := t(\cdot; \bar{D}_i, \bar{D}_{i+1}, \dots)$ .

- (i) If  $S_i(e_i) - (1+r)D_{i-1} < 0$ , the government cannot pay back the legacy debt and defaults. Upon default, it enters autarky and charges autarkic tax rate  $t^{**}$ .
- (ii) If  $S_i(e_i) - (1+r)D_{i-1} < \tau^{**} - C - z(1+r)D_{i-1}^{Dom}$ , the government potentially can pay back the legacy debt, but finds defaulting more advantageous. In other words it strategically

defaults, enters autarky, and charges the autarkic tax rate  $t^{**}$ .

- (iii) If neither of the above two conditions apply, then the government pays back the legacy debt, charges tax  $t_i(e_i)$  and issues  $S_i(e_i) - \tau(t_i(e_i))$  amount of debt. Total spending of the government is  $S_i(e_i) - (1+r)D_{i-1}$ .

**Proof of Proposition 5.1:** Let us first prove that the mapping  $T(\bar{D})$ :

$$F \rightarrow T(\bar{D})F = \max_t \frac{1}{1+r} \min \left[ F(e') - \max\{0, \tau^{**} - C - zs(1+r)\}, \bar{D} \right] + \tau(t),$$

is monotonic:

$$F \leq G \quad \forall e \Rightarrow TF \leq TG \quad \forall e; \text{ and} \tag{B.14}$$

$$\bar{D}^1 \leq \bar{D}^2 \Rightarrow T(\bar{D}^1)F \leq T(\bar{D}^2)F \quad \forall e. \tag{B.15}$$

In the interest of brevity, let us define:

$$T^t(\bar{D})F := \frac{1}{1+r} \min \left[ F(e') - \max\{0, \tau^{**} - C - zs(1+r)\}, \bar{D} \right] + \tau(t),$$

so that  $T(\bar{D}) = \max_t T^t(\bar{D})$ . Note that fixing  $t$ ,  $T^t$  is a monotonic transformation:  $F \geq G \Rightarrow T^t F \geq T^t G$ ,  $\bar{D}^1 \leq \bar{D}^2 \Rightarrow T(\bar{D}^1)F \leq T(\bar{D}^2)F$ . Next, we prove (B.14) and (B.15).

**Proof of (B.14).** Suppose per contra that for some  $e$ ,  $TF > TG$ . Let the associated tax rates be  $t_F$  and  $t_G$ . This leads to the following contradiction:

$$\begin{aligned} T^{t_F}F(e) &> T^{t_G}G(e) && \text{(by assumption)} \\ &\geq T^{t_F}G(e) && (\because \text{optimality of } t_G) \\ &\geq T^{t_F}F(e). && \text{(monotonicity of } T^t) \end{aligned}$$

**Proof of (B.15).** Similarly, suppose per contra that  $T(\bar{D}^1)F > T(\bar{D}^2)F$  for some  $e$ . Let the associated tax rates be  $t_1$  and  $t_1$ . This leads to the following contradiction:

$$\begin{aligned} T^{t_1}(\bar{D}^1)F(e) &> T^{t_2}(\bar{D}^2)F(e) && \text{(by assumption)} \\ &\geq T^{t_1}(\bar{D}^2)F(e) && (\because \text{optimality of } t_G) \\ &\geq T^{t_1}(\bar{D}^1)F(e). && \text{(monotonicity of } T^t) \end{aligned}$$

Now consider two generic value functions  $S^1 := S(\cdot; \bar{D}_1, \dots, \bar{D}_n^1, \dots)$  and  $S^2 := S(\cdot; \bar{D}_1, \dots, \bar{D}_n^2, \dots)$  where the debt ceiling is different for only one period  $i = n$ , and suppose WLOG that  $\bar{D}_n^1 < \bar{D}_n^2$ .

Note that

$$S^1 = \left( \prod_{i=1}^{n-1} T(\bar{D}_i) \right) T(\bar{D}_n^1) S^{n+1}, \text{ and}$$

$$S^2 = \left( \prod_{i=1}^{n-1} T(\bar{D}_i) \right) T(\bar{D}_n^2) S^{n+1};$$

where  $S^{n+1} := S(\cdot; \bar{D}_{n+1}, \bar{D}_{n+2}, \dots)$ . Note that from (B.15),

$$S_n^1 := T(\bar{D}_n^1) S^{n+1} \leq T(\bar{D}_n^2) S^{n+1} =: S_n^2.$$

Then, by successive application of (B.14) for  $i = 1, \dots, n-1$ , we derive that  $S^1 \leq S^2$ . ■

**Proof of Proposition 5.2:** First note that in this special case the Bellman equation takes the following form:

$$S(e; \bar{D}) = \max_t \left[ \frac{1}{1+r} \min \left[ S(e'; \bar{D}) - \max\{0, \tau^{**} - C - zs(1+r)\}, \bar{D} \right] + \tau(t) \right] \quad (\text{B.16})$$

$$\text{s.t. } e' = \kappa_1[(1+r)(e - k(t)) + (1-t)f(k(t))],$$

$$s = \kappa_1(e - k(t)) - \kappa_0(1-t)f(k(t)), \text{ and}$$

$$k(t) = f'^{-1}\left(\frac{1+r}{1-t}\right).$$

It follows similarly to Lemma B.6 that there are only two possible steady states, A and W, which must satisfy conditions specified in Lemma B.5. What remains to be proved is that the necessary and sufficient condition for the willingness-to-pay region steady state W to exist is that  $\bar{D} \geq \bar{\bar{D}}$  for some  $\bar{\bar{D}}$ .

Let us conjecture that  $\bar{\bar{D}} = D^W$  defined in Lemma 3.5, and suppose first that  $\bar{D} > D^W$ . Note that in steady state W, the current and all future governments on the equilibrium path take on the debt of amount  $D^W$  which is below the debt ceiling. Using this logic, we can verify that a conjectured partial solution  $S(e; \bar{D}) = S(e) \forall e \leq \hat{e}^1$  solves the Bellman equation in (B.16), similarly to Lemma B.6. By the uniqueness of the solution, this proves that  $\bar{D} > D^W$  does not alter the behavior of the model economy for  $e < \hat{e}^1$ .

Now suppose instead that  $\bar{D} < D^W$ . We know that if the steady state were to exist, the tax rate must satisfy (B.8), and that  $e^{ss} = e^{sat}(t^W)$ . We then verify the impossibility of the existence by observing the fact that at  $(e^{ss}, t^W)$ , the optimality condition is violated because of the debt ceiling binding.

It can be seen that once the debt ceiling starts binding, the marginal sensitivity of the first

term  $(\min\{\cdot, \bar{D}\})$  to the tax rate is zero. Therefore, the government's choice of tax rate in this case would be  $t^{**}$ . Therefore, if steady state W is removed, the only steady state that can survive is  $e^A = e^{sat}(t^{**})$ . ■

**Proof of Proposition 5.4:** In a steady state, the government defaults if and only if the new government spendings under the debt restructuring scheme,  $(S(e^W; \bar{D}) - (1+r)(1-\lambda)D_{-1}^W)$ , is lower than the original spending  $(\tau^{**} - C - z(1+r)s(e^W, t^W))$ , the expression for which is derived in Lemma 3.5. Observe that

$$\begin{aligned} S(e^W; \bar{D}) - (1+r)(1-\lambda)D_{-1}^W &\geq \tau^{**} - C - z(1+r)s(e^W, t^W) \\ \Rightarrow (1-\lambda) &\leq \frac{S(e^W; \bar{D}) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{(1+r)D_{-1}^W} \\ \Rightarrow \lambda &\geq 1 - \frac{S(e^W; \bar{D}) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{(1+r)D_{-1}^W}. \end{aligned}$$

**Proof of Proposition 5.5:** First observe that for all endowment paths starting from the trap endowment, the debt ceiling is binding. Therefore, there are only three possible choices of tax rate: choose tax rate such that either (i)  $S(e'; \bar{D}) - \tau^{**} + C + zs(1+r) = \bar{D}$ , (ii)  $S(e'; \bar{D}) = \bar{D}$  or (iii)  $S(e'; \bar{D}) - \tau^{**} + C + zs(1+r) > \bar{D}$  and  $\tau'(t) = 0$ .

We show that in all possible cases,  $t(e; \bar{D})$  is weakly decreasing in  $\bar{D}$ , having  $e$  fixed. Observe that using the envelope theorem – given that the debt ceiling is binding – yields  $\frac{\partial S(e; \bar{D})}{\partial \bar{D}} < 1$ . Using this, and supposing  $\bar{D}^1 < \bar{D}^2$ , we assess the property in each case:

- (i)  $S(e'; \bar{D}) - \tau^{**} + C + zs(1+r) - \bar{D} = 0$ . Note that the LHS is decreasing in  $\bar{D}$ , and therefore  $t$  has to increase the LHS to counteract. The LHS is decreasing in  $t$  implying that  $t$  should be decreasing as  $\bar{D}$  is decreasing.
- (ii)  $S(e'; \bar{D}) - \bar{D} = 0$ . This case is similar to case (i) above.
- (iii)  $S(e'; \bar{D}) - \tau^{**} + C + zs(1+r) > \bar{D}$  and  $\tau'(t) = 0$ . In this case  $t = t^{**}$  and therefore the stated condition that  $t(e; \bar{D})$  is weakly decreasing condition in  $\bar{D}$  is preserved. ■

**Proof of Proposition 5.6:** The optimality condition for the fiscal transfer can be expressed as

$$\max_{\Delta e} \quad \frac{1}{1+r} S(e^W + \Delta e) - \Delta e \quad \text{s.t.} \quad \Delta e \geq 0$$

Notice that due to the concavity of  $S$ , the optimal  $\Delta e > 0$  if and only if  $\frac{dS}{de}(e^W) > 1+r$ .

Therefore, we conclude that

$$\Delta e > 0 \Leftrightarrow 1 + r < \frac{dS}{de}(e^W) = \rho z \Leftrightarrow \rho > \frac{1+r}{z} . \quad \blacksquare$$

**Proof of Proposition 6.1:** The partial derivatives of  $S$  and  $e$  in the two steady states were proved in Lemma B.6. For the savings parameter  $\rho$ , notice first that an application of envelope theorem on the Bellman equation in (2.16) yields, in steady state W:

$$\begin{aligned} \frac{\partial S}{\partial \rho} &= \frac{1}{1+r} \left[ \frac{\partial S}{\partial \rho} - \frac{\partial \tau^{**}}{\partial \rho} + z \frac{\partial s}{\partial \rho} (1+r) \right] + \frac{\partial \tau(t)}{\partial \rho} \\ \Rightarrow \frac{\partial S}{\partial \rho} &= \frac{1+r}{r} z \frac{\partial s}{\partial \rho} (1+r) > 0 \quad (\because \frac{\partial \tau}{\partial \rho} = 0) \end{aligned}$$

It follows similarly that at steady state A,  $\frac{\partial S}{\partial \rho} = 0$ . For the productivity parameter  $\phi$ , an application of envelope theorem yields, in steady state W:

$$\begin{aligned} \frac{\partial S}{\partial \phi} &= \frac{1}{1+r} \left[ \frac{\partial S}{\partial \phi} - \frac{\partial \tau^{**}}{\partial \phi} + z \frac{\partial s}{\partial \phi} (1+r) \right] + \frac{\partial \tau(t)}{\partial \phi} \\ \Rightarrow \frac{r}{1+r} \frac{\partial S}{\partial \phi} &= z \frac{\partial s}{\partial \phi} (1+r) - \frac{1}{1+r} \frac{\partial \tau^{**}}{\partial \phi} + \frac{\partial \tau(t)}{\partial \phi} \\ \Rightarrow \frac{r}{1+r} \frac{\partial S}{\partial \phi} &= \underbrace{z \frac{\partial s}{\partial \phi} (1+r)}_{<0} - \left[ \frac{1}{1+r} \tau^{**} - \tau(t) \right] \quad (\because \frac{\partial \tau(t)}{\partial \phi} = \tau(t)) \end{aligned}$$

Now notice that since  $\tau^{**} = \max_s \tau(s) \geq \tau(t)$ , the second term  $\left[ \frac{1}{1+r} \tau^{**} - \tau(t) \right] > 0$  for sufficiently low  $r$ . The partial derivative in steady state A  $\frac{\partial S}{\partial \phi} > 0$  follows similarly.  $\blacksquare$