Disasters with Unobservable Duration and Frequency: Intensified Responses and Diminished Preparedness*

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Abstract

We study an economy subject to recurrent disasters when the frequency and duration of the disasters are unobservable parameters. Imprecise information about transition intensities increases the probability of the current state effectively lasting forever. In a disaster, uncertainty about duration makes disasters subjectively much worse and can make the welfare value of information extremely high. However, in advance of a disaster, uncertainty about the arrival rate can be welfare-increasing. Agents optimally invest less in mitigation than under full information and pay less for insurance against the next disaster.

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1 Introduction

This paper studies the real effects of parameter uncertainty in a model of repeated disasters. Among the many deeply alarming aspects of the COVID-19 pandemic was the realization of *how little we knew* about what would happen. Structural uncertainty about the forces at work encompassed many dimensions. We focus on two of these that seemed especially salient: uncertainty about the persistence (or duration) of the crisis, and uncertainty about its recurrence (or frequency) in future. Such uncertainty appears pervasive and spans economic disasters beyond pandemics. Uncertainty about the duration and frequency of recessions and financial crises is also realistic and potentially important. The same considerations are also likely to be relevant in the context of climate-related disasters.

Our model depicts disasters as regimes in which the stock of wealth (potentially including human wealth) is subject to exogenous destruction. The economy transitions stochastically between these episodes and "normal times." Agents update their beliefs by observing the frequencies of transitions, and optimally solve their investment/consumption problem given that information. We derive closed-form expressions for belief dynamics, and we obtain the value function and optimal policies under generalized preferences up to a tractable system of difference/differential equations. We contrast agents' welfare, policies, and incentives in the partial-information or parameter-uncertainty setting to the full-information setting. Our main finding is that uncertainty about the persistence of states makes agents appear to overweight the likelihood of the current state lasting effectively forever. This can lead to seemingly pessimistic behavior in bad times and optimistic behavior in good times.

The mechanism driving this is that Bayesian updating implies *negative duration dependence*, i.e., that the longer the current state lasts, the longer it is expected to last. This dynamic is very general: the absence of a transition in a given observation interval is information that must rationally shift the posterior density for the transition intensity towards zero regardless of the form of prior beliefs. Negative duration dependence is equivalent to transition times subjectively exhibiting a decreasing hazard function. Unconditionally, this means that beliefs about future regime durations are described by heavy tailed distributions. Most importantly, when agents form expectations about future consumption and wealth, they give extra weight to the possibility of the *current* regime being very long-lived. These are "best case scenarios" prior to a disaster, and "worst case scenarios" during one.

Moreover, these effects are strongly increasing as parameter uncertainty increases. This property is also not due to a particular specification of the prior. As precision declines, negative duration dependence increases, and the unconditional expected waiting time for the next transition, T, can become unbounded even holding fixed the mean belief about the transition probability. This is depicted graphically in Figure 1 when observers have belief about the switching probability per unit time, λ, that is described by a gamma distribution with mean $E[\lambda] = 1$ and variance $1/a$. The figure illustrates how, for low values of the precision parameter the expected waiting time, $E[T]$, becomes increasingly determined by the possibility that the true value of the intensity is close to zero. These effects are very general: The pattern in Figure 1 is similar if the belief distribution is log-normal instead of gamma, for example.

The paper's analysis is based on comparisons of the representative agent's lifetime value function across states or economies. We express these comparisons in terms of welfare costs, meaning the fraction of wealth the agent would be willing to surrender to exchange one state for another. We first compute the value of ending a disaster, and show that the benefit is much higher with partial information compared to the full information benchmark. Similarly, the welfare gain from reducing the severity of an on-going disaster increases strongly with parameter uncertainty.

These welfare differences map directly to investment incentives. If the economy is augmented to include a mitigation technology, it is straightforward to show that investment in this activity at the onset of a disaster would optimally be higher under parameter uncertainty.[1]

The observation that disasters are subjectively much worse under incomplete information raises the topic of the value of information. How much would the economy benefit from increasing precision about the transition parameters, even without altering the current disaster state? The answer is surprisingly large. The welfare gain from removing parameter uncertainty is as large as (and, in some cases, larger than) the benefit of ending the disaster. Compared to the full information economy, imprecision acts as an amplification mechanism for perceived risk, leading agents to respond to a disaster with extreme

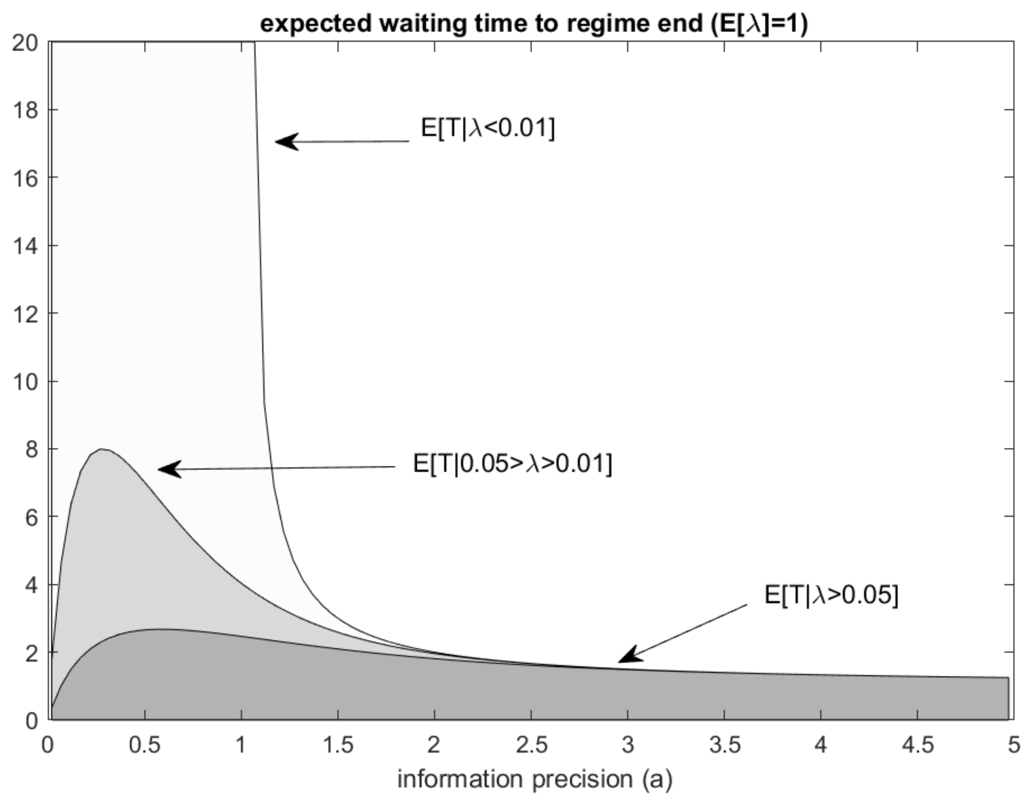
[1]In a similar vein, Barnett et al. (2023) show that uncertainty about infectious parameters within a pandemic leads a central planner with ambiguity averse preferences to impose stricter quarantine measures compared to the full-information benchmark.

conservatism in their investment/consumption behavior.

As with mitigation incentives, our findings have strong implications for investment in information production. We again consider endowing the economy with a technology to increase information precision. This technology is itself extremely valuable due to the very high *marginal* value of information when precision is low.

The paper thus contributes to the literature that assesses the welfare costs of disaster risk (see Barro (2009), Martin (2008), Pindyck and Wang (2013), Martin and Pindyck (2015), Jordà et al. (2020), Martin and Pindyck (2021), and Hong et al. (2022)). Reducing uncertainty about the evolution of disasters (e.g., through epidemiological modeling in the case of pandemics) may be an extremely valuable mechanism for reducing their perceived harm.

Figure 1: Information Precision and Expected Transition Time



Note: The top line plots the expected waiting time in years for the end of a regime when observers have belief about the intensity per unit time of a switch, λ, that are described by a gamma distribution with mean $E[\lambda] = 1$ and variance $1/a$, where a is the variable on the horizontal axis. The lower lines depict the contribution to this expectation of different components of the belief distribution.

The information dynamics of the model, as outlined above, imply, however, that the welfare and incentive effects reverse in advance of a disaster. Beliefs about the arrival rate also exhibit negative duration dependence, which increases when information is imprecise. Repeating the welfare computations in normal times, we show that, in some settings, information about the arrival rate can be welfare-destroying: agents may be subjectively better off with imprecise beliefs. We show that this effect can coexist with the strong positive value of information during a disaster. Turning again to incentives, when agents have the real option to invest in a mitigation technology prior the onset of a disaster, we show that agent with imprecise information about the disaster frequency optimally choose less mitigation than those with full information. As a parallel result, we show that agents with less precise information value disaster insurance less.

Taken together, the model describes a belief dynamic across regimes that may feature simultaneous optimism and pessimism. There is, in fact, empirical support for this implication. A well established branch of behavioral economics takes as given the observation that economic decision makers tend to ignore the risk of rare adverse events in good times and exaggerate them in bad times. The theory of *diagnostic expectations* has been formulated precisely to account for the evidence of this pattern (see Bordalo et al. (2022) for a recent overview). Moreover, in common with the implication in our model, that theory stresses that agents overreact *more* to recent news when it is more *salient*, which could be viewed as equivalent to settings in which there is less precision of prior information. While agents are not overreacting in our model, they would appear to be doing so to an observer with full information. Their consumption/investment behavior would appear increasingly optimistic prior to a crisis, and then increasingly pessimistic during one. In business cycle terms, their forecasts (e.g., for future output) would be highest at peaks and lowest at troughs.

2 Related Literature

A number of papers study learning problems in the context of models with disasters. It may be helpful to highlight distinguishing features of our setting and the focus of our contribution. A feature common to many models is an exogenous shock process (hitting consumption, or output, or the capital stock) whose intensity is unobservable and possibly time-varying. (Such models include Benzoni et al. (2011), Wachter and Zhu (2019), and Hong et al. (2022).) We also have such a shock process, and its intensity varies over time: it is zero in normal times and positive in a disaster regime. However, we assume that agents do know this shock intensity.

Collin-Dufresne et al. (2016) also study a 2-regime rare disaster economy with learning about the switching parameters. They show that, when risk aversion exceeds the inverse of the elasticity of intertemporal substitution, even small amounts of persistence uncertainty can produce large effects on the equity premium and Sharpe ratio. The mechanism they highlight is the increase due to learning in the subjective volatility of consumption growth and marginal utility. In contrast, while our setting is similar, the real effects we document are driven by the drift of the parameter estimates (the duration dependence), not their revisions.[2]

In emphasizing uncertainty about persistence, our paper also shares similarities with Gillman et al. (2014) and Ghaderi et al. (2022) in which regimes of differing growth differ in their expected duration. These models assume the regime itself is unobservable.[3] Hence agents' beliefs about the persistence of current conditions is formed from a mixture over exponential distributions. In our model, agents do know whether or not they are in a disaster regime; but, in contrast, they do not know the switching intensities conditional on the regime. Another related work is Andrei et al. (2019) in which agents do not observe the mean-reversion speed of current consumption shocks and thus face persistence risk. In their model, as in ours, the persistence risk is asymmetric: increasing news about persistence is positive in good times and negative in bad.

Most of the above papers focus on the implications of their specifications for asset pricing. An exception is Hong et al. (2022) who study implications of time-varying disaster beliefs for willingness to pay for mitigation efforts in the presence of externalities. Our focus too is on welfare effects. We highlight, in particular, the interaction between unobservable persistence and the current state of the economy in determining the value of information and investment incentives.

3 Model

In this section, we introduce a regime-switching model of disasters under partial information. Our fundamental view of a disaster is as a process that destroys household wealth, as in Gourio (2012), with consumption responding endogenously. For this reason, we work with a production-based framework rather than an endowment economy. Our goal

[2] In addition, many of our findings are *larger* in magnitude when the elasticity of intertemporal substitution is less than the inverse of the coefficient of risk aversion.

[3] David and Veronesi (2013) model learning about unobservalble inflation regimes. In Bianchi et al. (2022), agents are uncertain about the duration of monetary policy regimes.

is to study how the representative agent's value and consumption functions vary with information precision.

3.1 Disaster Dynamics

Following Nakamura et al. (2013), we consider the state of the economy to be either in a "non-disaster" regime or in a "disaster" regime, and denote the state as $s \in \{0, 1, \}$. Let η denote the probability per unit time (or, intensity) of a disaster arrival, and let λ denote the probability per unit time of a disaster ending.

The model's depiction of the disaster consists of a state-specific stochastic process for the accumulation of wealth. Specifically, let q denote the quantity of productive capital of an individual household (which could be viewed as both physical and human capital, the latter reflecting health as well as intangible capital). We assume that the stock of q is freely convertible into a flow of consumption goods at rate C per unit time. Then our specification is that q evolves according to the process

$$dq = \mu(s)q dt - C dt + \sigma(s)q dB_t - \chi(s)q dJ_t \tag{1}$$

where B_t is a standard Brownian Motion and J_t is a Poisson process with intensity $\zeta(s)$. We set $\chi(0) = 0$ and $\chi(1) > 0$ for the disaster state. The Poisson shock captures the risk of an economic loss to the household. While we refer to the occurrence of the state $s = 1$ as the "disaster" (i.e., independent of whether or how many wealth shocks actually occur), this is a matter of semantics. Somewhat more common in the literature would be to label these dJ shocks themselves as the "disasters", in which case our model maps to a particular specification of time-varying disaster risk, being either "on" or "off" depending on the regime.[4]

An assumption worth highlighting concerns the long-run effects of the disaster. Our specification is pessimistic in the sense that loss of wealth due to the J shocks is permanent. Productive capital q does not get restored when the disaster ends. On the other hand, the model is optimistic in the sense that the productive *process*, dq, does fully revert to pre-disaster dynamics. After the disaster, the world looks stochastically the same as it did before. In particular, there are no long-run scarring effects, e.g., on the economy's growth rate, μ. Both assumptions are important for tractability. In Section 4 we will consider augmenting the economy to include real options to mitigate the disaster or acquire

[4]Besides Gourio (2012), important contributions to the literature on time-varying disaster risk include Gabaix (2012), and Tsai and Wachter (2015).

information.

3.2 Information Structure

As discussed in the introduction, within a disaster there is likely to be deep uncertainty about *all* the governing parameters. Our focus on the timing parameters is motivated by the experience of COVID-19 in which the likely duration of the pandemic and the frequency of future pandemics were especially urgent questions to resolve. In our context, the switching intensities η and λ, are assumed unobservable.

The paper's main findings stem from very general properties of Bayesian learning in this setting. First, within a regime, the non-occurrence of a transition over an interval Δt is information that must rationally shift probability mass for beliefs about the transition intensity towards zero. Hence, the longer a regime lasts, the longer it is expected to last. This *negative duration dependence* happens regardless of the form of the prior distribution, but, notably, is stronger when prior information is less precise: Bayes' rule gives more weight to recent information ("the data") and less to old ("the prior") when the latter is relatively uninformative. A key consequence of negative duration dependence is that the expected transition time is always greater than the inverse of the (expected) intensity – which is what it would be under full information – and increases with imprecision.

We will assume that at time zero the agent has beliefs about the two intensity parameters that are described by gamma distributions, which are independent of each other. Each gamma distribution has a pair of non-negative hyperparameters, a^η, b^η and a^λ, b^λ, that are related to the first and second moments via

$$\mathbb{E}[\eta] = \frac{a^\eta}{b^\eta}, \qquad \mathrm{Std}[\eta] = \frac{\sqrt{a^\eta}}{b^\eta}, \tag{2}$$

and likewise for λ. The *relative precision* about η, defined as its mean divided by its standard deviation, is $\sqrt{a^\eta}$.

By Bayes' rule, under this specification, as the agent observes the switches from one regime to the next, her beliefs remain in the gamma class with the hyperparameters updating as follows

$$\begin{aligned} a_t^\eta &= a_0^\eta + N_t^\eta \\ b_t^\eta &= b_0^\eta + t^\eta \end{aligned}$$

where t^η represents the cumulative time spent in state 0 and N_t^η represents the total number of observed switches from 0 to 1. Analogous expressions apply for a^λ and b^λ. Thus, while in $s = 0$, the only information that arrives (on a given day, say) is whether or not we have switched to $s = 1$ on that day. If that has occurred, the counter N^η increments by one and the clock t^η turns off and t^λ turns on. The system is assumed to start in the state $s = 0$ with $N^\eta = N^\lambda = 0$.

The model thus pastes together two linked learning regimes. In each regime, we have a finite dimensional filter in the sense that the two updated parameters fully characterize beliefs about that regime. Further, $\hat{\eta}_t \equiv \mathbb{E}_t[\eta] = a^\eta / b^\eta$, and it remains the case that the agent views this number as the probability per unit time of an instantaneous switch from $s = 0$ to $s = 1$ (again with equivalent expressions for the other regime.)

This type of gamma-exponential conjugate system is well studied in stochastic process theory (e.g., see Harris and Singpurwalla (1968) and Rubin (1972)). Under these beliefs, the explicit measure for the switching time is described by a Lomax distribution (Lomax (1954)), whose expectation (in the $s = 0$ regime) is $1/\hat{\eta}$ times $a^\eta/(a^\eta - 1)$. This can be infinite when the relative precision of knowledge of η is low (as illustrated in Figure 1). Similarly the variance of the waiting time explodes for low precision. As we will see, these features have important consequences for the agents' welfare and optimal behavior in our economy.

3.3 Preferences

We assume the economy has a unit mass of identical agents (households). Each agent has stochastic differential utility or Epstein-Zin preferences (Duffie and Epstein, 1992; Duffie and Skiadas, 1994) based on consumption flow rate C, given as

$$\mathbb{J}_t = \mathbb{E}_t \left[\int_t^\infty f(C_{t'}, \mathbb{J}_{t'}) dt' \right] \tag{3}$$

and aggregator

$$f(C, \mathbb{J}) = \frac{\rho}{1 - \psi^{-1}} \left[\frac{C^{1-\psi^{-1}} - [(1-\gamma)\mathbb{J}]^{\frac{1}{\theta}}}{[(1-\gamma)\mathbb{J}]^{\frac{1}{\theta}-1}} \right] \tag{4}$$

where $0 < \rho < 1$ is the discount factor, $\gamma \geq 0$ is the coefficient of relative risk aversion (RRA), $\psi \geq 0$ is the elasticity of intertemporal substitution (EIS), and

$$\theta \equiv \frac{1-\gamma}{1-\psi^{-1}} \tag{5}$$

The use of recursive preferences is standard in macro-finance models because of their ability to match financial moments. We recognize the limitations of using a utility specification driven by consumption goods, particularly within a crisis when other considerations (e.g., health, social interaction, the safety of others) so strongly affect well-being. However, using a familiar formulation ensures that our findings are not driven by non-standard assumptions about utility.

The representative agent's problem is, in each state s, to choose optimal consumption $C(s)$ that maximizes the objective function $\mathbb{J}(s)$.

3.4 Solution

Under the model's information setting, the economy is characterized by a six-dimensional state vector consisting of the stock of wealth, q, $a^\eta, b^\eta, a^\lambda, b^\lambda$ and the regime indicator S. However this six-dimensional space can be reduced to three when solving the agent's optimization problem.

Since the switches between states alternate, we can define an integer index M_t to be the total number of switches $N_t^\eta + N_t^\lambda$ and then $N_t^\eta = M_t/2$ when M is even, and $N_t^\lambda = (M_t+1)/2$ when M is odd. Knowing M (along with the priors a_0^η and a_0^λ) is equivalent to knowing a_t^η and a_t^λ. Given these values, specifying the current mean estimates $\hat{\eta}_t$ and $\hat{\lambda}_t$ is equivalent to specifying the remaining hyperparameters b_t^η and b_t^λ.

Within each regime the only changes to the state (apart from q) come through variation in the estimates $\hat{\eta}_t$ and $\hat{\lambda}_t$ which change deterministically with the respective clocks t^η and t^λ. Holding M fixed, the dynamics of $\hat{\eta}_t$ are given by

$$\begin{aligned} d\hat{\eta}_t &= d\frac{a_t^\eta}{b_t^\eta} = a_t^\eta d\frac{1}{b_t^\eta} \\ &= -\frac{a_t^\eta}{(b_t^\eta)^2}\, dt \\ &= -\frac{(\hat{\eta}_t)^2}{a_t^\eta}\, dt. \end{aligned} \tag{6}$$

The latter expression says that, until new information arrives, $\hat{\eta}$ decays quadratically and deterministically to zero at a rate that is faster when a^η is small. This dynamic defines the negative duration dependence of the system and illustrates its dependence on the degree of information precision.[5]

The agent's Hamilton-Jacobi-Bellman (HJB) equation links the value functions for states with successively more history. For large M, the estimation errors for both η and λ go to zero:

$$\frac{\mathrm{Std}[\eta]}{\mathbb{E}[\eta]} = \frac{1}{\sqrt{a^{\eta}}} = \frac{1}{\sqrt{a_0^{\eta} + M_t}}.$$

Thus the system converges to the full-information solution, which is characterized by two coupled algebraic equations. The appendix establishes the following:

Proposition 1. *Let $H(\hat{\eta}, \hat{\lambda}, M)$ denote the solutions to system of coupled first-order differential equations in the appendix. Assuming these are positive, optimal consumption is*

$$C = \rho^{\psi} \ (H)^{-\frac{\psi}{\theta}} q, \tag{7}$$

and the value function of the representative agent is

$$\mathbb{J} \equiv \frac{H(s) q^{1-\gamma}}{1-\gamma}. \tag{8}$$

Note: All proofs appear in the appendix.

The appendix also describes a straightforward and fast solution algorithm for the system, and discusses necessary conditions for existence of a unique positive solution. (We verify that these our satisfied in our numerical work below.)

4 Results

We now turn to numerical analysis to illustrate the real effect of parameter uncertainty on the economy. Our baseline calibration fixes the growth rate $\mu(s)$ and Gaussian volatility $\sigma(s)$ across regimes to be 0.04 and 0.05. (The values are chosen to approximately capture the growth rate and volatility of aggregate dividends in non-disaster times.) The disaster shock size is set to $\chi = 0.04$. We fix the disaster shock intensity to be 1.0 in order to

[5] The ODE in (6) has the exact solution

$$\hat{\eta}_t = \frac{1}{\frac{1}{\hat{\eta}_0} + \frac{t}{a_0^{\eta}}}$$

where t is the time since the regime began.

interpret χ as the expected loss of wealth per year. We use baseline preference parameters ($\gamma = 4$, $\psi = 1.5$, $\rho = 0.04$) that are broadly consistent with the macro-finance literature under stochastic differential utility. We explore the role of these choices below.

4.1 Information Precision in a Disaster

To start, we examine the welfare consequences of parameter uncertainty within a disaster. Since Lucas (1987), a large literature has analyzed the welfare costs of aggregate risks in business cycle models in order to quantify incentives to reduce such risks. Here, we extend this line of research to encompass the *perceived* risk that stems from parameter unobservability. We address two main questions. First, comparing partial information to full information, how much worse is the disaster compared to the non-disaster state? Second, how much would agents pay to gain information about the unknown parameters?

For any pair of economies or states, $\{i, j\}$, we report the fraction of wealth that the representative agent would be willing to pay for a one-time transition from the worse (j) to the better state (i). The welfare gain is computed as the certainty equivalent change in the representative agent's lifetime value function :

$$1 - \left(\frac{H(j)}{H(i)}\right)^{\frac{1}{1-\gamma}}$$

This definition is standard in the literature.

4.1.1 Welfare Gain from Curtailing a Disaster

To quantify the severity of disasters under our base parameterization, Table 1 reports the welfare gain for ending a disaster, that is, to transitioning from $s = 1$ to $s = 0$ holding everything else fixed. In the context of a pandemic, this could be viewed as the value of a perfectly effective cure or vaccine. Each cell of the table shows this gain for three values of $\hat{\lambda}$ and two values of $\hat{\eta}$. The top panel shows the result when there is no uncertainty about the parameters. Here the upper left cell shows that, in this benchmark case, agents would be willing to pay between roughly 5% and 20% of wealth to return to the normal economic state. The values are intuitively reasonable in the sense that, for $\eta = 0.01$ say, they are not too far from just the *expected duration* of the disaster ($1/\lambda$) times the expected loss of wealth per year, $\chi = 0.04$. Reading across the top panel, the preference parameters do not have large effects on the the full-information values. The bottom panel shows the same computation when agents' current uncertainty about the timing parameters (their posterior standard deviation) is equal to their mean belief about each of them, or their relative precisions are 1.0 for both. This is our baseline case of partial information.[6] Compared to the top panel, the partial information situation is subjectively much worse.

Adding parameter uncertainty greatly increases the resources that the economy would be willing to expend to find a cure or otherwise limit the damage. Indeed, an analogous computation (omitted for brevity) shows that the welfare benefit from lowering the disaster severity (χ) by any fixed amount is also much larger under partial information. To see how these welfare differentials map into investment incentives, suppose now that the economy is endowed with a real option to undertake a lump-sum expenditure, I, to reduce the severity according to $\chi = g(I/q)$ for an arbitrary function $g > 0$ with $g' < 0$. By an argument that we formalize in the online appendix, the sensitivity of the welfare function H to χ effectively pins down the marginal benefit of I. Hence, for any parameterization of the mitigation technology, we can assert that the optimal investment will be strictly greater under partial information than under full information.

4.1.2 Welfare Gain from Resolving Parameter Uncertainty

The results above immediately raise the question of how much agents would be willing to pay to resolve parameter uncertainty, even without curtailing the current disaster. Panel (A) of Table 2 answers this question. For each of the preference configurations considered and for nearly all values of $\hat{\eta}$ and $\hat{\lambda}$, the value of resolving the parameter uncertainty is as large or larger than the value of resolving the ongoing disaster.[7]

It is perhaps not surprising that risk averse agents would be willing to pay to resolve parameter uncertainty. However, as we will see below, this need not always be the case. Moreover, here, it is the *magnitude* of the value that is surprising. The numbers are much larger than typically found in analogous calculations in the literature for other types of risk. In a similar setting, Collin-Dufresne et al. (2016) show that, using a myopic utility benchmark, uncertainty about the persistence of the bad state is an order of magnitude more important than uncertainty about other parameters, e.g., growth rates and volatilities in the two regimes.

Comparing the results in Panel (A) across preference specifications, the value of resolving parameter uncertainty increases with higher risk-aversion (γ), and is lower with a lower time discount factor (ρ). The γ effect is intuitive: parameter risk increases the

[6] In this case the gamma prior is an exponential distribution. Results are similar for differing initial precisions.

[7] Note that the welfare gain is an understatement in that it excludes any "instrumental value" of information, for example, upon agents' ability to avert future disasters. The model contains no mechanism by which *knowing more about* λ and η allows agents to affect them.

subjective volatility of wealth, which agents dislike. Less apparent is the effect of ρ. Why should agents with a longer time horizon (lower subjective discount rate) care so strongly about information? Recall that when agents do not know the true value of λ, their expected time until the end of the disaster is governed by a Lomax distribution whose expectation explodes as the precision of their information declines. Put differently, with parameter uncertainty, *worst case scenarios* come into play. When there is insufficient evidence to rule them out, the current disaster may look effectively permanent.[8] Agents cannot rule out that $\lambda \sim 0$, i.e., that the disaster will effectively last for their entire lifetime. Hence, its impact on welfare can be enormous when agents have a long time horizon (low discount rate).

The largest effects in Panel (A) come from lowering the elasticity of intertemporal substitution. This is noteworthy because there is a common understanding of Epstein-Zin preferences under which agents with $\psi \leq 1/\gamma$ can be viewed as having a preference for "later resolution of uncertainty," which might suggest that they value information *less* than high EIS agents, whereas here the result is precisely the opposite.[9]

To understand this, note that, with recursive preferences, agents with low EIS cut consumption when the economy enters the disaster state. This is because a low EIS implies strong consumption smoothing motives, and the prospect of lower future wealth motivates a sharp increase in savings. By contrast, a higher EIS implies relatively more concern with investment risk than consumption smoothing. Agents with a high EIS therefore decrease investment in a disaster, since investment is exposed to greater risk. However, the differing consumption responses do not make disasters worse *per se* for agents with a low EIS: the top panel of Table 1 shows little effect of the EIS under full information. Instead, it is the extreme *decrease* in consumption as information precision declines that leads to the large welfare losses for these agents. This is again due to the time horizon effect. With low precision of information about λ, there is a chance that the withdrawal of consumption will be effectively permanent.[10]

As with mitigation, there is a direct mapping from the welfare costs of information

[8]Note that this dynamic is *not* driven by our distributional assumptions. For any positively valued density, decreasing precision while holding the mean fixed necessarily implies placing more mass near zero.

[9]See Epstein et al. (2014) for an examination of the welfare consequences of varying the timing of the resolution of uncertainty.

[10]In Van Nieuwerburgh and Veldkamp (2006) and Kozlowski et al. (2020) learning effects within downturns endogenously cause the downturns to last longer. In our case, the uncertainty-induced investment and consumption distortions do not affect the length of the disaster. However, negative duration dependence implies that the *perceived* duration lengthens the longer the episode goes on.

to investment incentives. The findings here imply that the ability to produce information about the underlying determinants of disaster duration would be an extremely valuable real option. Indeed, we can again consider augmenting the economy to have a one-time investment opportunity to increase information precision. In the online appendix, we show that, under a simple linear information production technology, agents endogenously attain almost the full value of information with only a fraction of the expenditure. This value-added of the technology stems from the rapid increase in agents' value function from even moderate increases in information quality. From a policy perspective, the implication of the extreme *marginal* value of information in a disaster is that fundamental research can crucially complement (or perhaps even substitute for) efforts to directly affect the course of the disaster.

4.2 Parameter Uncertainty Prior to a Disaster

The analysis above immediately suggest an unexpected corollary: all of the conclusions may be *reversed* prior to a disaster. Low precision of information about the disaster intensity in normal times could lead to agents acting as if they overweight *best* case scenarios, namely, that a disaster will never materialize. We now show that, indeed, this can be the case. Moreover, we will see that *both* types of effects – pessimistic in a disaster and optimistic beforehand – may co-exist.

4.2.1 Value of Information

We start by examining the welfare effect of uncertainty about η when $s = 0$. This effect can be isolated by setting the prior precision for λ to be very high, so that, effectively agents know its value. Panel (B) of Table 2 shows the value of information under these conditions. In the baseline case, the value of information about η indeed can be negative, although the magnitude is not large economically. Working against the effect of longer subjective waiting time until a disaster is the effect of risk aversion: the value function is concave in $\hat{\eta}$, so higher posterior variance lowers welfare through this channel. With $\gamma = 2$ the effect can be economically significant: when the point estimate $\hat{\eta}$ is large the representative agent would be willing to give up to 2.1% of wealth to *not* learn the true disaster frequency.

When information about both λ and η is imprecise, the former typically matters more in the sense that full information is overall welfare improving in both states. Intuitively, the worst-case scenarios still loom large prior to a disaster. However, we can vary the degree to which duration dependence operates in each regime by observing that the percentage drift in the means (which drives the effect) scales with the ratio of the mean to the precision. Thus, when $\hat{\eta}/a_0^{\eta}$ and $\hat{\lambda}/a_0^{\lambda}$ are similar, we obtain similar belief dynamics in the two states. The top panel of Figure 2 illustrates this co-existence of pessimism and optimism in terms of growth rate expectations. Using the parameters in that figure together with $\gamma = 1$, the welfare cost of parameter uncertainty is 3.2% of wealth in the disaster and -3.5% before it. Hence, the incentives to acquire information alternate sign in the two states.

4.2.2 Disaster Mitigation Incentives

We saw above that, when information about disaster duration was imprecise, agents had stronger incentive to end or curtail the disaster. But that logic would now also be expected to flip. When agents place more weight on best-case scenarios, their incentives to invest in mitigation are weaker. To make this explicit, again consider endowing the economy with a one-time real option to expend resources to lower the disaster severity. But now the investment decision is made prior to the onset of a disaster. We argued above that, for any given mitigation technology, the optimal amount invested will scale with the sensitivity of the (log) value function to χ.

The lower panel Figure 2 plots $\log H$ as a function of the disaster severity under full and partial information.[11] When $s = 1$ (right panel) we verify our assertion above that the slope is steeper under partial information. However, with these parameters, when $s = 0$ (left panel) the relation is reversed. We can conclude that, for any smooth specification of the mitigation technology, lower precision of information will result in underinvestment or underpreparedness relative to full-information in advance of a disaster.

4.2.3 Pricing of Disaster Insurance

Another way of capturing preparedness incentives is via willingness to pay for insurance against a disaster. So consider the price of a financial contract which pays 1 upon the arrival of the next disaster. This contract is in net zero supply and does not affect real outcomes. However, its price provides a measure of agents' assessment of the likelihood and timing of a disaster, as well as its consequences in marginal utility terms. Then,

Proposition 2. *The price, P, in the non-disaster state of the claim which pays 1 upon the arrival*

[11] Recall the full value function is negative, so higher values of H are worse.

of the next disaster, satisfies the equation

$$-\frac{(\hat{\eta})^2}{a^\eta}\frac{\partial P}{\partial \hat{\eta}} + \hat{\eta}\,\frac{H(\hat{\eta},\hat{\lambda},M+1)}{H(\hat{\eta},\hat{\lambda},M)}\,(1-P) - r_0\,P = 0 \tag{9}$$

where r_0 is the riskless rate.[12]

Given the value function solutions, this is a first-order differential equation in $\hat{\eta}$, with boundary condition $P(0) = 0$. Figure 3 plots the solutions for the parameter set we have been considering. In line with the intuition that partial information leads to longer expected waiting times, we see that the contract is substantially underpriced relative to its full-information value.

This section has shown that information about disaster frequency can be welfare reducing because, with less information, agents rationally believe a disaster may never materialize (the expected waiting time becomes unbounded) even when the mean intensity of disasters is held fixed. The other phenomena that we have illustrated (optimistic forecasts, underinvestment in mitigation, undervaluing insurance) are all manifestations of the same belief dynamic. This negative value of information may shed light on failure to prepare adequate for disasters and on "don't look up" behavior of seemingly willful ignorance towards their threat.[13]

5 Conclusion

This paper considers a regime-switching disaster model when agents do not know the true transition probabilities. We find a surprising dichotomy: the welfare benefit of information about the unobservable parameters can be extreme within a disaster and yet small or even negative prior to one. The finding stems from two general properties of Bayesian updating in this situation. First, beliefs about transition intensities exhibit negative duration dependence, resulting in a decreasing hazard function for regime changes. This implies that the unconditional distribution of the exit time from the current regime is fatter tailed, with a higher expectation than under full information. Second, imprecision heightens the possibility that the probability of change is small. Agents in the economy then act, in effect, as if the current state may never end, even holding fixed their estimate

[12] The rate and the pricing kernel are derived in terms of the model primitives in the Appendix.

[13] Models with costly information processing have also been used to explain failure to prepare for disasters. See Maćkowiak and Wiederholt (2018). Aversion to information is explicitly modelled in the preference specification of Andries and Haddad (2020).

of the instantaneous probability of it ending. This can result in seemingly exaggerated pessimistic behavior within a disaster (e.g., extreme reductions in consumption) at the same time as seemingly exaggerated optimistic behavior (e.g. reduced expenditure on mitigation or insurance) in non-disaster times.

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Table 1: Welfare Gain to Ending Disaster

(A) Full Information

Benchmark

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.188	0.090	0.046
	0.05	0.147	0.081	0.0447

$\psi = 0.20$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.205	0.093	0.046
	0.05	0.162	0.085	0.045

$\gamma = 2$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.160	0.079	0.042
	0.05	0.135	0.072	0.04

$\rho = 0.02$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.225	0.100	0.048
	0.05	0.166	0.091	0.047

(B) Partial Information

Benchmark

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.340	0.288	0.235
	0.05	0.246	0.225	0.201

$\psi = 0.20$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.777	0.808	0.827
	0.05	0.646	0.700	0.739

$\gamma = 2$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.292	0.214	0.156
	0.05	0.232	0.186	0.147

$\rho = 0.02$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.519	0.531	0.537
	0.05	0.350	0.378	0.405

The table shows the fraction of wealth the agent would be willing to surrender for a one-time transition out of the disaster state. In Panel (A), agents in the economy know the parameters λ and η. In Panel (B), they have posterior standard deviation equal to their point estimates of these quantities. The benchmark parameters are given in Section 4.

Table 2: The Value of Information

(A) During a Disaster

Benchmark

		$\hat{\lambda}$		
		0.2	0.5	1.0
$\hat{\eta}$	0.01	0.235	0.234	0.220
	0.05	0.182	0.239	0.232

$\psi = 0.20$

		$\hat{\lambda}$		
		0.2	0.5	1.0
$\hat{\eta}$	0.01	0.934	0.931	0.921
	0.05	0.922	0.928	0.922

$\gamma = 2$

		$\hat{\lambda}$		
		0.2	0.5	1.0
$\hat{\eta}$	0.01	0.178	0.164	0.131
	0.05	0.148	0.169	0.152

$\rho = 0.02$

		$\hat{\lambda}$		
		0.2	0.5	1.0
$\hat{\eta}$	0.01	0.641	0.688	0.666
	0.05	0.485	0.619	0.645

(B) Prior to Disaster

Benchmark

		$\hat{\lambda}$		
		0.2	0.5	1.0
$\hat{\eta}$	0.01	0.001	0.001	0.000
	0.05	-0.008	-0.000	0.001

$\psi = 0.20$

		$\hat{\lambda}$		
		0.2	0.5	1.0
$\hat{\eta}$	0.01	0.012	0.002	0.001
	0.05	0.079	0.026	0.007

$\gamma = 2$

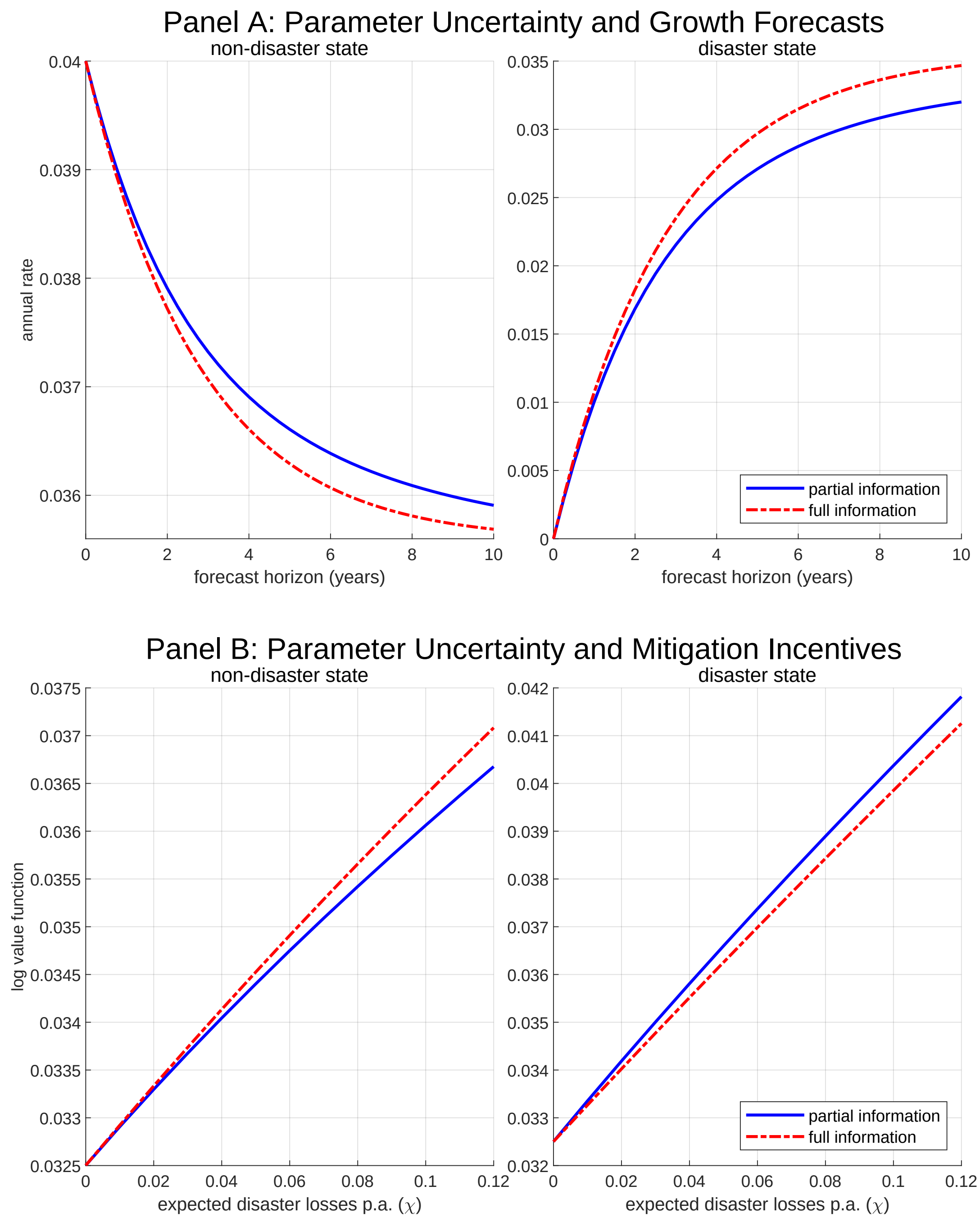
		$\hat{\lambda}$		
		0.2	0.5	1.0
$\hat{\eta}$	0.01	-0.001	-0.000	-0.000
	0.05	-0.021	-0.007	-0.002

$\rho = 0.02$

		$\hat{\lambda}$		
		0.2	0.5	1.0
$\hat{\eta}$	0.01	0.050	0.015	0.004
	0.05	0.044	0.047	0.023

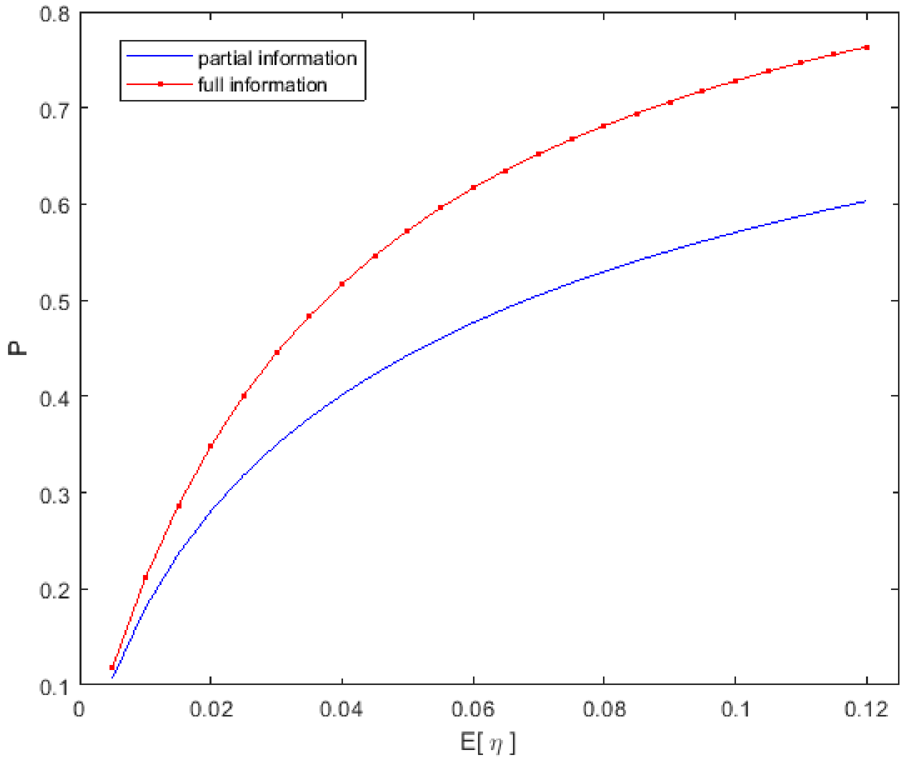
Panel (A) shows the fraction of wealth that the representative agent would be willing to surrender for a transition from partial information to full information (as defined in Table 1) while remaining in the disaster state. Panel (B) shows the fraction of wealth the agent would surrender for a transition from partial information to full information about the disaster intensity η while remaining in the non-disaster state. The agent is assumed to have full information about λ. Benchmark parameters are given in Section 4.

Figure 2: Optimism and Pessimism



Panel (A) plots subjective expectations for the growth of wealth to different horizons, T. The left panel shows agents' forecasts when in normal times. The right panel shows forecasts during a disaster. The plots take the agent's posterior expected switching intensities for the two states to be $(0.05, 0.20)$ with respective posterior standard deviations of $(0.05, 0.10)$. Panel (B) plots the log value function multiplier, H, as a function of the disaster severity χ also within the disaster (right) and nondisaster (left) states. For each plot, the full-information economy's values are plotted as dotted (red) lines and the partial information ones as solid (blue) lines. Panel (B) uses the benchmark parameters given in the text with $\gamma = 1.01$.

Figure 3: Disaster Insurance Pricing



The figure plots the price of a contract paying 1 upon the arrival of the next disaster as a function of the mean arrival intensity, $\hat{\eta}$. Other parameter values are the same as in Figure 2.

Online Appendix

A Proofs and Derivations

A.1 Full Information

To prove Proposition 1, we first treat the case of full-information in which the only state variables are $s \in \{0,1\}$ and q. For ease of notation, define the following combination of preference parameters:

$$e_0 \equiv \frac{\theta}{\psi}\, \rho^{\psi} \quad \text{and} \quad e_1 \equiv -\frac{\psi}{\theta}. \tag{A.1}$$

Also define $\lambda(0) = \eta, \lambda(1) = \lambda$.

Lemma *Denote*

$$g(s) \equiv \theta\, \rho - (1-\gamma)\left(\mu(s) - \frac{1}{2}\gamma\sigma(s)^2\right) - \zeta(s)\, \left(\left[1-\chi(s)\right]^{1-\gamma} - 1\right) \tag{A.2}$$

for $s \in \{0,1\}$. Let $H(s)$'s denote the solution to the following system of recursive equations:

$$g_0 \equiv g(0) = e_0\,(H(0))^{e_1} + \eta\left[\frac{H(1)}{H(0)} - 1\right] \tag{A.3}$$

$$g_1 \equiv g(1) = e_0\,(H(1))^{e_1} + \lambda\left[\frac{H(0)}{H(1)} - 1\right] \tag{A.4}$$

Assuming the solutions are positive, optimal consumption in state s is

$$C(s) = \rho^{\psi}\, (H(s))^{e_1}\, q, \tag{A.5}$$

and the value function of the representative agent is

$$\mathbb{J}(s) \equiv \frac{H(s) q^{1-\gamma}}{1-\gamma}. \tag{A.6}$$

Proof. Using the evolution of capital stock for the representative agent (1) the Hamilton-

Jacobi-Bellman (HJB) equation for each state s can be written:

$$
\begin{aligned}
0 = \max_C \Big[& f(C,\mathbb{J}(s)) - \rho\mathbb{J}(s) + \mathbb{J}_q(s)(q\mu(s) - C) \\
& + \frac{1}{2}\mathbb{J}_{qq}(s)q^2\sigma(s)^2 + \zeta(s)\left[\mathbb{J}(s)\left(q(1-\chi(s))\right) - \mathbb{J}(s)(q)\right] \\
& + \lambda(s)\left[\mathbb{J}(s')(q) - \mathbb{J}(s)(q)\right]\Big]
\end{aligned} \tag{A.7}
$$

for $s = \{0,1\}$ and $s' = \{1,0\}$.

Taking the first-order condition with respect to $C(s)$ in (A.7), we obtain

$$
f_c(C,\mathbb{J}(s)) - \mathbb{J}_q(s) = 0. \tag{A.8}
$$

Using $f(C,\mathbb{J})$ from (4) and taking the derivative with respect to C, we obtain

$$
f_c = \frac{\rho C^{-\psi^{-1}}}{[(1-\gamma)\mathbb{J}(s)]^{\frac{1}{\theta}-1}}. \tag{A.9}
$$

Substituting the conjecture $\mathbb{J}(s)$ in equation (8) yields

$$
f_c = \frac{\rho C^{-\psi^{-1}}}{H(s)^{\frac{\gamma-\psi^{-1}}{1-\gamma}} q^{\gamma-\psi^{-1}}}. \tag{A.10}
$$

Then, for state s, we obtain by substituting $\mathbb{J}_q(s) = H(s)q^{-\gamma}$ in (A.8), and simplifying:

$$
C(s) = \frac{H(s)^{-\theta\psi^{-1}} q}{\rho^{-\psi}} \tag{A.11}
$$

which agrees with (7) using the definitions of the constants in (A.1).

To verify the conjectured form of the value function, we plug it in to the HJB equation (A.7) and reduce it to the recursive system in the proposition via the following steps:

1. substitute the optimal policy $C(s)$ into the HJB equation (A.7);
2. cancel the terms in q which have the same exponent; and
3. group constant terms not involving Hs and define them to be $g(0)$ for state 0 and $g(1)$ for state 1.

The third step yields the system of recursive equations A.3, A.4. □

Regularity Conditions

The functions $H(s)$ are necessarily bounded by the limiting solutions in which the economy is never in a disaster, H_0^{min}, or is always in a disaster, H_1^{max}. It is straightforward to show that these constants are given by

$$H_0^{min} = \left(\frac{g_0}{e_0}\right)^{1/e_1} \quad \text{and} \quad H_1^{max} = \left(\frac{g_1}{e_0}\right)^{1/e_1}.$$

These quantities are real and positive if g_0, g_1, and e_0 all have the same sign. Given this, it can be shown that a necessary and sufficient condition for existence of a unique solution is that $g_1 < g_0$.

A.2 Proposition 1: HJB System with Parameter Uncertainty

Proof. As noted in the text, the model can be parameterized in terms of the state variables $M, \hat{\eta}, \hat{\lambda}$, and q, where $M = M_t$ is an integer counter that increases on a state switch such that $M_0 = 0$ and even numbered states are the non-disaster epochs and odd numbered states are the disasters. Also, in the non-disaster states, $\hat{\lambda}$ is constant, while $\hat{\eta}$ is constant in disasters. As a consequence, compared with the derivation above for the full-information case, there is now only one additional source of variability in each regime. The dynamics of $\hat{\eta}$ are given in (6) with an analogous expression for and $\hat{\lambda}$. And note that, under the agents' information set, the dynamics of the wealth variable q are identical to the full information dynamics.

As a result, the HBJ equations under partial information are the same as (A.7) above (with state 0 and state 1 being replaced by M and $M+1$) with the addition of a single term on the right side:

$$-\frac{(\hat{\eta})^2}{a^{\eta}}\frac{\partial \mathbb{J}(0)}{\partial \hat{\eta}} \tag{A.12}$$

for $s = 0$, and

$$-\frac{(\hat{\lambda})^2}{a^{\lambda}}\frac{\partial \mathbb{J}(1)}{\partial \hat{\lambda}} \tag{A.13}$$

for $s = 1$. Since, under the agent's information set, the state switches are a point-process with instantaneous intensities $\hat{\eta}$ and $\hat{\lambda}$, these quantities also replace their full information counterparts, η and λ, in multiplying the jump terms in the respective equations.

The next steps in the derivation involving the first order condition for optimal consumption are unchanged from the full-information case. This follows because consumption does not enter into any of the new terms involving the information variables. Replace $\mathbb{J}$ by the conjecture $\frac{q^{1-\gamma}}{1-\gamma}\, H(\hat{\eta}, \hat{\lambda}, M)$, then a common power of q term is cancelled, and the whole equation is divided by H. These manipulations lead to the above two terms showing up on the right hand side, in a system that is otherwise identical to the full-information system (A.3) and (A.4).

$$g_0 \;=\; e_0 H_M^{e_1} + \hat{\eta}\left(\frac{H_{M+1}}{H_M} - 1\right) - \frac{(\hat{\eta})^2}{a^\eta H_M}\,\frac{\partial H_M}{\partial \hat{\eta}} \tag{A.14}$$

$$g_1 \;=\; e_0 H_{M+1}^{e_1} + \hat{\lambda}\left(\frac{H_{M+2}}{H_{M+1}} - 1\right) - \frac{(\hat{\lambda})^2}{a^\lambda H_{M+1}}\,\frac{\partial H_{M+1}}{\partial \hat{\lambda}} \tag{A.15}$$

where the constants g_0 and g_1 are as defined in Lemma 1 above.

□

A.2.1 Solution Algorithm

In the full information case, solution of the algebraic system over a grid in the $(\hat{\eta}, \hat{\lambda})$ plane is straightforward. The unknown constants $H(s)$ are bounded by the limiting solutions in which the economy is never in a disaster, H_0^{min}, or is always in a disaster, H_1^{max}. The former corresponds to $\eta = 0$ and the latter to $\lambda = 0$.

For the general case, we pick a large even integer M^{max} and assume that the economy has converged to the full information solution with $s = 0$ at M^{max} and $s = 1$ at $M^{max} - 1$. Given these solutions, the HBJ system for $M = M^{max} - 2$ is just a first order ODE, since the jump terms in (A.14)-(A.15) can be explicitly evaluated. For even values of M, the boundary condition at $\hat{\eta} = 0$ is again the full-information solution because the posterior standard deviation $\sqrt{a^\eta}\hat{\eta}$ is also zero. (Note that the value of $\hat{\lambda}$ is immaterial if disasters cannot arise.) Likewise, for odd values of M, the boundary condition at $\hat{\lambda} = 0$ is given by the full-information solution. Hence, the first-order ODEs can be explicitly solved in alternating directions. The procedure is then repeated for all lower values of M.

A.3 Pricing Kernel, Riskless Rate and Proposition 2

This section first derives the pricing kernel and riskless rate under partial information. The results are then used to prove Proposition 2 Section 4.2 which describes the pricing equation of insurance against a disaster.

Under stochastic differential utility, the kernel can be represented as

$$\Lambda_t = e^{\int_0^t f_{\mathbb{J}} du} f_C \tag{A.16}$$

where the aggregator function is given in (4). With the form of the value function and the optimal consumption rule from Proposition 1, evaluating the partial derivatives yields (after some rearrangement)

$$\Lambda_t = q^{-\gamma} \; H(\hat{\eta}, \hat{\lambda}, M) \; e^{\int_0^t [c_u \; (\theta-1) - \rho\theta] du} \tag{A.17}$$

where $c = c(\hat{\eta}, \hat{\lambda}, M) \equiv C/q$ is the marginal propensity to consume.

The riskless rate is minus the expected rate of change of $d\Lambda_t/\Lambda_t$ under the agents' information set. Applying Itô's lemma, for even values of M, the expected change is

$$c\;(\theta - 1) - \rho\theta \; - \gamma(\mu - c) + \frac{1}{2}\gamma(\gamma+1)\sigma^2$$

$$-\frac{(\hat{\eta})^2}{a^{\eta}} \; \frac{1}{H}\frac{\partial H}{\partial \hat{\eta}} \; + \hat{\eta}\left(\frac{H(M+1)}{H(M)} - 1\right).$$

A key simplification is to observe that, by the HJB equation derived above (see (A.14)), the latter two terms in this expression can be replaced by $g_0 - \frac{\theta}{\psi}c$. This causes all of the terms involving c to exactly cancel. Using the definition of g_0 in (A.2), the remaining terms are just $-\mu + \gamma\sigma^2$. Hence we have shown

$$r_0 = \mu - \gamma\sigma^2.$$

Repeating the above steps for odd values of M and applying the same trick yields

$$r_1 = \mu - \gamma\sigma^2 - \zeta\chi(1-\chi)^{-\gamma}.$$

Turning to the insurance claim, the asset is assumed to make a terminal payout of 1.0 upon the occurrence of the next disaster. Proposition 3 characterizes its price in normal-times prior to that disaster.

Proof. We conjecture that the price, P, of the insurance is not a function of wealth, q. Moreover, when $s = 0$, the state variables a^{η}, a^{λ}, and $\hat{\lambda}$ are all fixed, and $\hat{\eta}$ evolves deterministically according to (6).

By the definition of the pricing kernel, for any claim in the economy, its instantaneous payout per unit time (in this case, zero) times Λ must equal minus the expected change of the product process $P\Lambda$, or

$$\mathcal{L}(\Lambda(q_t, s_t, \hat{\eta}_t)\ P(s_t, \hat{\eta}_t))/\Lambda_t = 0, \tag{A.18}$$

where $\mathcal{L}(X)$ is the drift operator $\mathrm{E}[dX]/dt$ under the agents' information set.

Using Itô's lemma for jumping processes to expand the expected change,

$$-\frac{(\hat{\eta})^2}{a^\eta}\frac{\partial P}{\partial \hat{\eta}} + \mu_\Lambda P + \hat{\eta}\left(\frac{H(M+1)}{H(M)} - P\right) = 0$$

where we have written μ_Λ for the deterministic terms in $d\Lambda_t/\Lambda_t$ and used the fact that $P(M+1) = 1$.

Next, add and subtract $\hat{\eta}(\frac{H(M+1)}{H(M)} - 1)P$ and use the fact that the expected growth rate of the pricing kernel is minus the riskless rate:

$$r_0 = -\mu_\Lambda - \hat{\eta}\left(\frac{H(M+1)}{H(M)} - 1\right)$$

to get (9):

$$-\frac{(\hat{\eta})^2}{a^\eta}\frac{\partial P}{\partial \hat{\eta}} - r_0 P + \hat{\eta}\frac{H(M+1)}{H(M)}(1-P) = 0.$$

□

A.4 Real Options

A.4.1 Mitigation

The text in Section 4 describes endowing the model economy with a one-time real option to invest in a mitigation technology to alter a structural parameter, χ, via $\chi = g(i)$ where I is a lump-sum investment and $i = I/q$. Since the option is a one-shot decision, the post-investment economy is identical to the original model (without the technology) and hence its value function is as derived in the main propositions.

Then, the assertion is that, for two otherwise equal economies E1 and E2, if the sensitivity of the value function, H, to χ is weaker in E1 than in E2, then, if a solution to the real-options problem exists in E2, a solution also exists in E1 with smaller optimal investment.

To see this, view H as a function of χ, and the problem is to choose i to maximize the $H(g(i))(1-i)^{1-\gamma}/(1-\gamma)$ with first order condition $-g'(i)\,\partial \log H(g(i))/\partial\chi = (\gamma-1)/(1-i)$. Assume $\gamma > 1$. Then the right side (the marginal cost) is an unbounded increasing function of i on $[0,1)$ which is the same for both economies. Call it $RHS(i)$. On the left side (the marginal benefit), the first term is the same for both economies. The hypothesis is that $\partial \log H(\chi)/\partial\chi$ is smaller in E1 than in E2 for all χ implying that the second term is smaller. Hence $LHS1(i) < LHS2(i)$ for all i. Assume $LHS2$ is continuous and declining. Then, if an interior solution, i_2^*, exists, it follows that on $[i_2^*,1)$ we have $LHS1 < LHS2 < RHS$, meaning that there cannot be a solution for E1 in this region. Hence, either there is a solution $i_1^* < i_2^*$ or no interior optimum exists and $i_1^* = 0$ in E1.

A.4.2 Information Production

The top panel of the table below presents the optimal information investment as a fraction of wealth when the economy contains the technology that alters information precision according to $a(i) = a_0 + 100\,i$, where $i = I/q$ is the lump-sum investment. The option to make this investment is a one-time occurence at the on-set of a disaster.

The lower panel reports the welfare gain, in units of wealth, of the investment. The difference between the respective panels can be interpreted as the value-added of the technology.

Table A.1: Information Production

(A) Optimal Investment

	Benchmark				$\psi = 0.20$		
		$\hat{\lambda}$				$\hat{\lambda}$	
	0.2	0.5	1.0		0.2	0.5	1.0
$\hat{\eta}$ 0.01	0.06	0.07	0.04	$\hat{\eta}$ 0.01	0.12	0.14	0.07
$\hat{\eta}$ 0.05	0.06	0.07	0.04	$\hat{\eta}$ 0.05	0.12	0.15	0.08
	$\gamma = 2$				$\rho = 0.02$		
		$\hat{\lambda}$				$\hat{\lambda}$	
	0.2	0.5	1.0		0.2	0.5	1.0
$\hat{\eta}$ 0.01	0.03	0.03	0.02	$\hat{\eta}$ 0.01	0.12	0.15	0.07
$\hat{\eta}$ 0.05	0.03	0.03	0.03	$\hat{\eta}$ 0.05	0.12	0.15	0.15

(B) Welfare Gain

	Benchmark				$\psi = 0.20$		
		$\hat{\lambda}$				$\hat{\lambda}$	
	0.2	0.5	1.0		0.2	0.5	1.0
$\hat{\eta}$ 0.01	0.182	0.227	0.189	$\hat{\eta}$ 0.01	0.921	0.928	0.917
$\hat{\eta}$ 0.05	0.137	0.201	0.182	$\hat{\eta}$ 0.05	0.899	0.922	0.914
	$\gamma = 2$				$\rho = 0.02$		
		$\hat{\lambda}$				$\hat{\lambda}$	
	0.2	0.5	1.0		0.2	0.5	1.0
$\hat{\eta}$ 0.01	0.120	0.124	0.094	$\hat{\eta}$ 0.01	0.544	0.667	0.636
$\hat{\eta}$ 0.05	0.095	0.116	0.108	$\hat{\eta}$ 0.05	0.398	0.573	0.614