

Bankruptcy Exemption of Repo Markets: Too Much Today for Too Little Tomorrow?

Viral V. Acharya, V. Ravi Anshuman and S. Viswanathan*

ABSTRACT

We examine the desirability of granting “safe harbor” provisions to creditors of financial intermediaries in sale-and-repurchase (repo) contracts. Exemption from an automatic stay in bankruptcy can enable financial intermediaries to raise greater liquidity and operate at a higher leverage during normal times. This liquidity creation occurs, however, at the cost of ex-post inefficiency when there are adverse aggregate shocks to the fundamental quality of collateral underlying the contracts. When exempt from bankruptcy, creditors of highly leveraged financial intermediaries respond to such shocks by engaging in collateral liquidations. Financial arbitrage by less leveraged financial intermediaries equilibrates returns from acquiring collateral at fire-sale prices and returns from real-sector lending, inducing a rise in lending rates, a deterioration in endogenous asset quality, and in the extremis, a credit crunch for the real sector. Given this inefficiency, not granting safe harbors, i.e., requiring an automatic stay on repo contracts in bankruptcy, can be not only ex-post optimal, but also ex-ante optimal, especially for illiquid collateral with high exposure to aggregate risk.

Keywords: Automatic stay, safe harbor provisions, sale and repurchase contracts, fire sales, credit crunch, financial crises, systemic risk

JEL Classification: G01, G21, G28, G33, D62, K11, K12

*Viral Acharya is from New York University (Stern School of Business) and a research affiliate of CEPR, ECGI and NBER, V. Ravi Anshuman is from IIM Bangalore, and S. Viswanathan is from Duke University. We acknowledge research assistance of Samidh Pratap Singh, Simrat Singh and especially, Srijith Mohanan. We are grateful to Andrew Metrick (discussant), Zhen Zhou, and participants at the Federal Reserve Bank of New York workshop on Safe Harbors, the AFA 2022 meetings, and the People’s Bank of China School of Finance, Tsinghua University. The usual disclaimer applies. Contact author: V. Ravi Anshuman, C/I 104, IIM Bangalore India 560076, Tel: +91-80-26993062, e-mail: anshuman@iimb.ac.in

1. Introduction

A repurchase agreement – also known as a “sale and repurchase agreement” or more popularly as a “repo” – is a short-term transaction between two parties in which one party effectively borrows cash from the other by pledging a financial security as collateral. One important feature of the repo market in the United States is that a large fraction of transactions falling under the umbrella of repos are exempt from the automatic stay in bankruptcy of the counterparties and, therefore, can be settled with immediacy. For example, if the seller of the asset is unable to repurchase the asset, then the buyer can liquidate the underlying collateral following a bankruptcy filing of the seller. This exemption from bankruptcy, sometimes also called as a “safe harbor” provision, has been extended gradually to different repo markets, starting with Treasuries and Agency (Fannie Mae and Freddie Mac) securities in 1980s, and most recently in 2005, to non-Agency mortgage-backed assets.¹ The failures of financial intermediaries exposed to mortgages or mortgage-backed securities, such as Countrywide, Bear Stearns and Lehman Brothers, all involved in some part a “repo run,” that is, an inability of the borrower to roll over the repo contracts with the financiers. Indeed, since the global financial crisis, there has been stress in the form of fire sales and “repo rate spikes” even in the U.S. Treasuries market, notably during September 2019 and March 2020.²

We develop a model to understand the desirability of granting repo contracts such exemption from bankruptcy. Financial intermediaries (such as, broker dealers or their parent bank-holding companies) borrow funds from financiers (such as, money-market funds) to originate assets. Since the backdrop we have in mind is one of trading-based financial institutions, which are typically highly levered and are primary borrowers in repo markets, we focus on the agency problem of asset substitution or risk-shifting by borrowers as in [Jensen and Meckling \[1976\]](#): financial intermediaries, after raising debt, have incentives to transfer wealth away from financiers by switching to riskier assets unless the expected profits from safer assets are sufficiently high.³

Given the risk-shifting problem and taking a purely partial equilibrium view of the bilateral contract, the ex-ante liquidity of intermediaries would seem to be greater if they grant liquidation rights on underlying assets to the financier (as derived in [Acharya and Viswanathan \[2011\]](#)). The intuition is that if financiers are instead not granted liquidation rights (bankruptcy exemption), financial intermediaries would renegotiate the bilateral contract down to the financier’s reservation

¹See [Acharya and Öncü \[2014\]](#) for a chronology of these exemptions.

²See, in particular, [Copeland et al. \[2021\]](#) and [d’Avernas and Vandeweyer \[2020\]](#).

³Related to the work of [Stiglitz and Weiss \[1981\]](#) and [Diamond \[1989, 1991\]](#), this risk-shifting problem rations potential intermediaries in that it limits the maximum amount of financing they can raise from lenders.

payoff under the inefficient asset choice. Financiers will anticipate this ex ante and provide less liquidity. The implication is that, absent considerations other than the bilateral contracting problem, bankruptcy exemption of collateralized borrowing, as presently accorded to repo contracts should enable financial intermediaries to raise greater liquidity and originate more assets.

A key insight of this paper is that liquidity creation via extension of bankruptcy exemption occurs, however, at potentially significant costs when a general equilibrium view is considered. In particular, financial intermediaries can also originate assets in the future, say in the form of loans to the real asset sector. If adverse economic shocks can lead to forced sale of repo collateral at such times, then the partial equilibrium result on the desirability of bankruptcy exemption for repo contracts can get overturned as asset fire-sales can raise lending rates to the real sector, and even induce a credit crunch. We show that there is an inherent conflict in the choice of bankruptcy exemption between supporting current and future asset originations; complete bankruptcy exemption amplifies this inter-temporal wedge, and can lead to too much origination today for too little asset origination tomorrow.

We consider a three date model in which an aggregate economic shock at the interim date affects the funding liquidity of financial intermediaries. Upon arrival of adverse news about underlying asset quality, highly-leveraged intermediaries face greater funding or rollover stress as their financiers factor in the intermediaries' risk-shifting incentives. Therefore, the ability of these intermediaries to raise new financing to pay off earlier financiers is diminished, prompting them to sell some legacy financial assets. For an adverse enough shock, partial asset sales do not suffice to roll over existing contracts and all assets may have to be liquidated by financiers when given exemption from bankruptcy. Less-leveraged intermediaries, in contrast, have surplus capacity to raise financing and acquire the assets being liquidated. In the industry equilibrium, the market-clearing price of legacy financial assets reflects, in general, fire-sale discounts [Shleifer and Vishny, 1992, Gale and Allen, 1994, Allen and Gale, 1998].

Absent the consideration of new asset origination at the interim date, such a market-based transfer of assets from highly-leveraged intermediaries to less-leveraged ones does not affect ex-post efficiency (in particular, fire-sale discounts may simply reflect welfare-neutral transfers of value). However, if there is a demand from the real sector for intermediation at the interim date, then this result is substantially overturned for the following reasons. Bankruptcy exemption facilitates a greater degree of ex-ante leverage, which, in turn, causes greater consequent liquidations in the event of an adverse economic shock, thereby providing opportunities to less-leveraged intermediaries to

earn excess return from their surplus liquidity. Financial arbitrage implies that the expected return from originating new loans to the real sector must match the expected return from investing in the secondary market for legacy financial assets; therefore, in the new loan market, interest rates rise in tandem with the extent of liquidation. In our model, a moral hazard problem arises as borrowers in the new loan market (say, households) invest less effort when faced with higher interest rates, resulting in (an endogenously determined) lower loan quality (e.g., to maintain the property). The drop in loan quality in turn affects the lender's (i.e., the surplus-liquidity intermediary's) expected profits. Thus, there is an upper bound on the interest rate that intermediaries can charge on new loans; or, in other words, the marginal benefit of increasing the interest rate beyond this level is more than offset by the marginal reduction in loan quality. When bankruptcy exemption causes too much ex-post liquidation, the returns from investing in the financial asset market and the new loan market (both returns being equal) hit this upper bound. Surplus-liquidity intermediaries are no longer interested in deploying additional capital in the new loan market. Instead, they withdraw capital from the real sector and, in the extreme, the market for new loans shuts down.

Next, we show that bankruptcy exemption *can be* sub-optimal in our model, i.e., the negative externality of bankruptcy exemption in the form of fire-sale and credit-crunch effects in future periods *can* overwhelm the positive effect in the form of greater financial intermediation in the current period. The intuition for the result is as follows. While bankruptcy exemption induces more ex-ante asset creation, the *incremental* beneficiaries are intermediaries with larger investment requirements, i.e., a higher leverage, who would not have been financed if there was no safe harbor. These intermediaries are more susceptible to adverse economic shocks, more likely to be liquidated by their financiers, and create financial arbitrage opportunities for less-leveraged intermediaries. This externality diverts the future surplus liquidity of less-leveraged intermediaries toward acquiring assets at fire-sale prices instead of financing real investment activity, thereby inducing adverse welfare consequences that can overwhelm the initial facilitation of financial intermediation by bankruptcy exemption.

Our model derives conditions under which bankruptcy exemption is optimal, and also the conditions under which an automatic stay (the polar opposite policy of bankruptcy exemption) is optimal. An automatic stay on repo contracts in bankruptcy may be especially useful when fire-sale effects in underlying collateral are likely, for instance, in case of less liquid collateral, such as mortgages, that also lose value when aggregate risk materializes. An automatic stay is beneficial also when the real sector funding needs are large and economic downturns are likely to be more severe. On

the other hand, bankruptcy exemption of repo contracts can be ex-ante optimal *only* when there are no fire-sale effects; such a situation arises when the magnitude of the adverse economic shock is mild, the collateral is of unimpeachable quality (potentially benefiting from flight-to-safety or flight-to-quality effects), and the real sector funding needs are small.

Section 2 relates our work to theoretical and empirical literature. Section 3 sets up the basic features of the model. Section 4 analyzes the model and presents the ex-post equilibrium outcomes, taking ex-ante leverage as given. Section 5 augments the model to study the ex-ante leverage of intermediaries. Section 6 derives results on ex-ante welfare analysis, which pins down the optimal level of bankruptcy exemption and its determinants. Section 7 examines the impact of capital requirements on optimal bankruptcy exemption level and Section 8 concludes. Key proofs are in the Appendix, with some additional details relegated to an Internet Appendix.

2. Related Literature

Our paper is motivated by the empirical literature on the role of repo market runs in exacerbating the financial crisis (Copeland et al. [2010, 2014], Gorton et al. [2010], Gorton and Metrick [2010, 2012], Gorton et al. [2020a], Gorton et al. [2020b], and Krishnamurthy et al. [2014]). By and large, this literature points out that the over-dependence of systematically important financial institutions on repo financing in the period before 2008 exposed the financial system to systemic risk, which eventually led to an economic contraction. The institutional arrangements of the repo market model can play a critical role in determining how systemic risk propagates in the economy. Our paper addresses a key design feature of repo markets, namely, bankruptcy exemption of repo creditors, in exacerbating crisis-like situations. The specific model presented in our paper is closely related to three strands of literature: (i) the role of financial frictions in creating inefficient fire sales, (ii) the welfare implications of leverage-induced fire sales when collateral constraints exist, and (iii) the role of bankruptcy exemptions on systemic risk.

The first strand deals with the role of financial frictions in exacerbating the impact of macroeconomic shocks. These frictions limit the ability of a highly-leveraged intermediary from continuing as a going-concern during an adverse economic shock unless it liquidates some of its assets, potentially at fire-sale prices. In addition to the seminal papers referred in the Introduction, this literature is now rather vast. Our model is most closely related to the work of Acharya and Viswanathan [2011] and Lorenzoni [2008]. In Lorenzoni [2008], fire sales are generated by financial frictions that arise due to the limitation of agents to commit credibly to future loan repayments. In Acharya

and Viswanathan [2011], funding liquidity is constrained by financial frictions that arise due to a risk-shifting problem; our model extends their framework and considers the interaction of fire sales generated by rollover risk in the financial sector (as a response to risk-shifting incentives) with a moral hazard problem in the real sector (resulting in lower endogenous asset quality).

The second strand of literature deals with the welfare implications of leverage-induced fire sales. Such liquidations have been argued to cause inefficiencies in the economy (Bordo and Jeanne [2002], Diamond and Rajan [2001], Lorenzoni [2008], Acharya et al. [2010], Shin et al. [2011], and Stein [2012]). The central feature of these studies is that aggregate leverage and fire-sale effects are endogenously related. Bordo and Jeanne [2002] analyze the ex-post consequences of a sharp decline in asset prices (following an asset price boom) on real economic activity and study implications for optimal monetary policy. Diamond and Rajan [2001] show how a fear of fire sales in future can cause a credit freeze today as intermediaries hoard cash to capitalize on fire sales. Lorenzoni [2008] points out there is excess ex-ante borrowing that fails to internalize the ex-post inefficiency due to fire sales and a central planner can improve social welfare by limiting the amount of aggregate leverage in the economy. Stein [2012] examines the financial stability implications of short-term private money creation and how monetary policy and complementary tools such as open-market operations can be deployed to limit the negative externalities arising from fire sales on ex-ante origination.

More recently, Dávila and Korinek [2018] show that financial frictions can lead to both *distributive externalities* (externalities between buyers and sellers of assets) and *collateral externalities* (externalities that depend on the effect of financial constraints on asset prices). Further, Lanteri and Rampini [2023] argue that distributive externalities are much larger than collateral externalities. In our model, there is a large distributive externality in that low price of capital in the second period induces more capital to the financial sector and less capital to the real sector; further the low price of capital (high interest rate) reduces the value of the real sector asset due to moral hazard.

We build on these two strands of literature in the context of bankruptcy exemption of repo contracts, and show how bankruptcy exemption affects the trade-off between ex-ante credit availability and inefficient ex-post fire-sales that limit future credit availability. Two recent studies have also explicitly modeled the bankruptcy exemption provision; both use fundamentally different assumptions from our work. First, Antinolfi et al. [2015] show that fire-sale externalities arise due to bankruptcy exemption. However, as they themselves point out, this externality disappears in their model if the exchange of fire-sale assets arises in a competitive equilibrium. In contrast, fire-sale effects in our model are endogenously determined in a competitive equilibrium and the resulting

welfare implications for the real economy are analyzed. Second, [Ma \[2017\]](#) considers a structural model of the bankruptcy exemption provision to evaluate how it affects the coordination problem of creditors in a repo run and the strategic declaration of bankruptcy by the borrower; the model, however, does not consider the spillovers effects on the real sector, which is the focus of our analysis, whereas we do not focus on coordination issues among repo creditors.⁴

The third strand of related literature discusses the implications of bankruptcy exemption on systemic risk. [Duffie and Skeel \[2012\]](#) recognize the role of bankruptcy exemption in increasing systemic risks and propose limiting the bankruptcy exemption to repos and (centrally cleared) derivative contracts that are backed with highly liquid collateral. [Tuckman \[2010\]](#), too, advocates restricting the safe harbor provision to only those derivatives that are centrally cleared to reduce the risk of fire sales in the event of an adverse shock and to also reduce the incentives of market participants to take up large position in complex, illiquid derivatives whose underlying assets are most susceptible to crashes. [Acharya and Öncü \[2014\]](#) recommend withdrawing the safe harbor exemption from all repo transactions other than those having government-backed claims as collateral. We confirm the intuition of this literature that stronger creditor rights accorded as safe-harbor provisions to repo contracts facilitate ex-ante credit availability, but cause ex-post fire sales and less credit to the real sector in the event of an adverse aggregate shock to the economy.

Our model also sheds lights on the debate among policy makers about the role of bankruptcy exemption – whether it reduces or exacerbates systemic risk (see for example, [Federal Reserve Report \[2011\]](#), written in the aftermath of the global financial crisis). We show that the view that bankruptcy exemption reduces systemic risk is overturned once we take an ex-ante as well as an economy-wide perspective and endogenize the implications of safe harbor on leverage and of fire sales for the real economy. Finally, the legal profession has also discussed the issue of bankruptcy exemption. Several articles in law journals have assessed the costs and benefits of the safe harbor provision. These articles also point out that collateral runs are an important factor in evaluating bankruptcy exemption (e.g., [Edwards and Morrison \[2005\]](#), [Jackson \[2009\]](#), [Skeel and Jackson \[2011\]](#), [Federal Reserve Report \[2011\]](#), [Mooney Jr \[2014\]](#) and [Morrison et al. \[2014\]](#)).

⁴More recently, [Zhong and Zhou \(2021\)](#) endogenize ex-post bankruptcy payoffs to evaluate the ex-ante decision of creditors to stay invested in a firm. Thus, they are able to establish a time-consistent approach to ex-post and ex-ante creditor runs. Such commitment issues of creditor runs are also not a feature of our analysis.

3. Model Setup

We build a model of financial intermediation using repo financing with the objective of determining the optimal extent of bankruptcy exemption for repo contracts. After laying out the model structure in this section, we partition our analysis into two sections: first, in Section 4 we examine the role of bankruptcy exemption on ex-post liquidation effects under an exogenous assumption about the ex-ante leverage in the economy; next, in Section 5 we endogenize the leverage decisions and derive the ex-ante optimal level of bankruptcy exemption in Section 6. Our model follows the setup in [Acharya and Viswanathan \[2011\]](#). Financial intermediaries make investment decisions in a two-period, three-date world – a start date (Date 0), an intermediate date (Date 1), and a terminal date (Date 2). We discuss below the role of financial intermediaries, the available assets in the economy and their Date 2 payoffs, followed by a summary of the sequence of key events in the model. Figure 1 shows the payoffs on the assets (Panel A) and the time line (Panel B).

3.1. Financial Intermediaries

The economy consists of a continuum of financial intermediaries. They start out with differing levels of financial infrastructure and/or human capital, which are required for participating in the intermediation sector. Depending on the accumulation of this capital, intermediaries require differing amounts of investment (shortfall s) to start a business by acquiring a financial asset of unit scale. Similar to the approach followed by [Anderson and Sundaresan \[1996\]](#) in analyzing debt contract design, we assume that the investment shortfall is financed in the short-term debt market; more specifically, in the short-term repo market which provides financing with “sale and repurchase” contract against the financial asset.⁵ Effectively, at Date 0, financial intermediaries operate at the same scale but vary in terms of the degree of leverage in their balance sheets.

3.2. Assets in the Economy

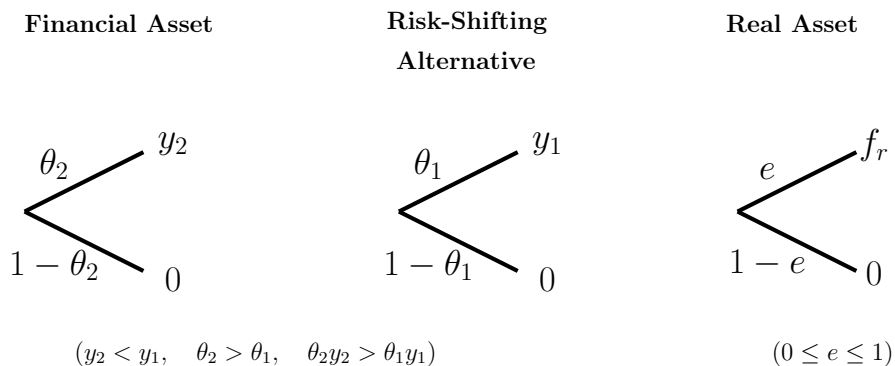
There are two sectors in the economy, the financial sector (consisting of financial assets) and the real sector (consisting of real assets/loans). Financial assets are originated at Date 0, but the real assets are originated at Date 1.

The financial asset could be a legacy loan or a commoditized pool of loans, which produces uncertain cash flows at Date 2, and against which intermediaries can raise leverage at Date 0 in

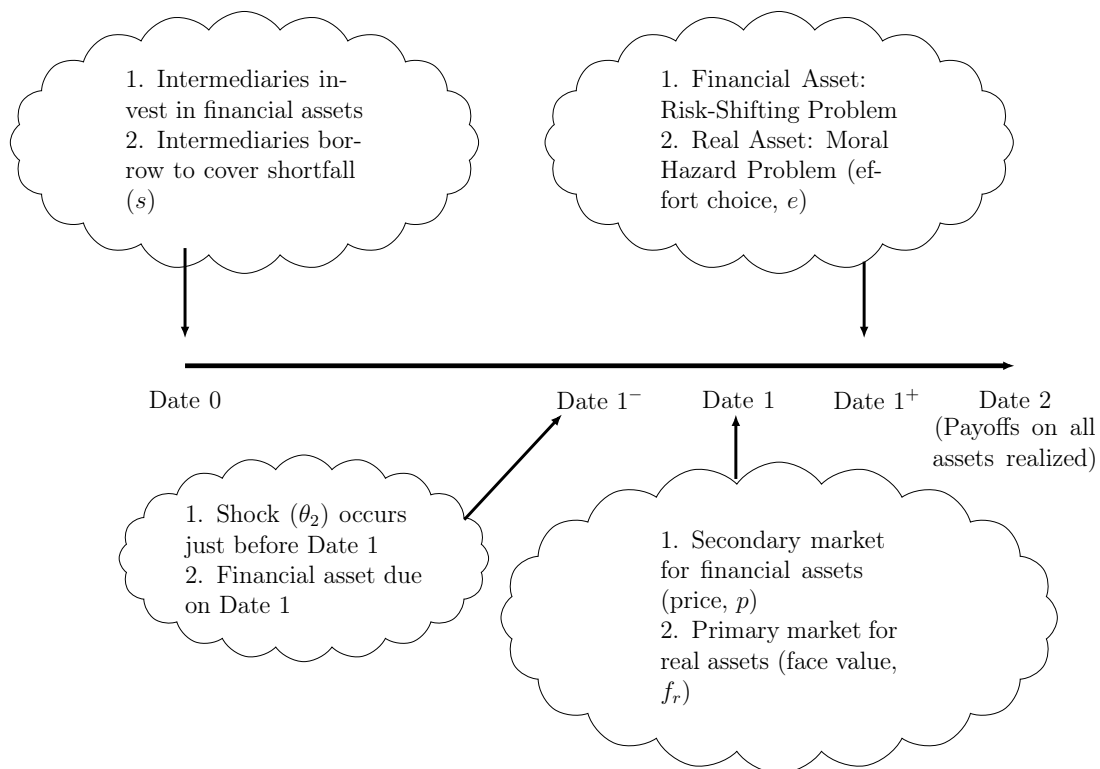
⁵In earlier studies, [Aghion and Bolton \[1992\]](#) and [Hart and Moore \[1994\]](#) have used this approach in the context of security design.

Figure 1: **Description of the Model.** Panel A shows the Date 2 payoffs on the financial asset, the risk-shifting alternative, and the real asset. Panel B show the sequence of events in the model.

PANEL A. Payoffs on the financial asset, the risk-shifting alternative, and the real asset



PANEL B. Sequence of events in the model (Timeline)



the form of repo contracts maturing at Date 1. There is an alternative to increase the risk of the financial asset at Date 1. This risk-shifting alternative will never be taken up in equilibrium but will affect important rollover/liquidation decisions of agents in the economy.

The real sector is characterized by asset-specificity (because of a moral hazard problem that is borrower-specific, as will be elaborated below). The real asset can be thought of as relatively illiquid loans, e.g., a mortgage or small-business loan to households, that are originated at the intermediate date, Date 1, and mature at Date 2. The cash flows from the real asset are not pledgeable by intermediaries (at Date 0 or at Date 1) to raise finances.

3.2.1. *Financial Asset Payoffs*

Payoffs of the financial asset under the risk-shifting alternative and the safer alternative are denoted respectively with subscripts 1 and 2: the safer alternative has a payoff of y_2 with a probability of θ_2 and a payoff of 0 with a probability of $(1 - \theta_2)$; the risk-shifting alternative has a payoff of y_1 with a probability of θ_1 and a payoff of 0 with a probability of $(1 - \theta_1)$. Further, $\theta_1 < \theta_2$, $y_1 > y_2$ and $\theta_1 y_1 \leq \theta_2 y_2$. Thus, while the risk-shifting alternative has a higher payoff in the non-default state, it experiences a higher likelihood of the default state. More importantly, it is riskier in that it has a lower expected payoff as compared to the safer alternative (i.e., $\theta_1 y_1 \leq \theta_2 y_2$) and has a higher variance per unit expected payoff compared to second asset (i.e. $(1 - \theta_1)y_1 > (1 - \theta_2)y_2$). Following [Acharya and Viswanathan \[2011\]](#), we also assume that risk-shifting is costless to implement, and that assets are financial sector specific (such as money-market funds) and cannot be redeployed by financiers in case they choose not to roll over financing at Date 1, i.e., they must be liquidated to other intermediaries.

3.2.2. *Real Asset Payoffs*

The real sector (of size \mathcal{B}) consists of assets such as new mortgage or small business loans taken up by households at Date 1. For each unit of asset, there is an outflow of 1 unit at Date 1 and there is an uncertain binary payoff at Date 2: with a probability e , the payoff is the loan face value (f_r); otherwise, it is 0. The probability e reflects the household effort choice based on a moral hazard problem. Both e and f_r will be endogenously determined. Given that the loan amount is normalized to unity, the face value (f_r) effectively determines the interest rate of household loans.

3.3. Summary of Sequence of Events/Decisions

At Date 0, intermediaries invest in a financial asset after borrowing the required financing (to cover the investment shortfall, s) in the short-term repo market. At Date 1^- , the economy experiences an observable but unverifiable shock (θ_2), which renders intermediaries as either surplus in funding liquidity (less-leveraged intermediaries) that are looking for additional investment opportunities or credit-constrained (highly-leveraged intermediaries) that are unable to roll over their short-term debt claims to the next period, i.e., they are unable to repurchase their financial asset in entirety from the repo-financiers. At Date 1, surplus-liquidity intermediaries face two investment opportunities: first, they could invest in the (financial) asset re-sale market where they can acquire the financial assets of credit-constrained intermediaries at a price p (which will be endogenously determined below); second, they could also consider investing their surplus in the real sector by investing 1 unit in each real asset. Credit-constrained intermediaries have a strategic choice between liquidating an optimally chosen fraction (δ) of their asset to clear their funding deficit or to simply declare bankruptcy.

At Date 1^+ , intermediaries can exercise the risk-shifting alternative and the household makes the effort choice on the real asset. At Date 2, all asset payoffs are realized. While the model relies on the distinction in the sequence of events at Date 1^- , Date 1, and Date 1^+ , for convenience we will often refer to the entire set of events as Date 1 events, e.g., a Date 1 economic shock.

Intermediation decisions are thus made at Date 0 (raising repo financing to enter the financial sector) and Date 1 (repaying repo contracts and extending illiquid loans to the real sector). We refer to intermediary decisions/outcomes at Date 1 as coming from the *ex-post* model and decisions/outcomes at Date 0 as coming from the *ex-ante* model. The ex-ante model must take into account the optimal decision strategies and outcomes of the ex-post Date 1 equilibrium; at the same time, the ex-post equilibrium strategies and outcomes are affected by the strategies of ex-ante optimization, a key feature of the model, as in [Acharya and Viswanathan \[2011\]](#).

3.4. Salient Features of the Model

Our model builds upon but differs from the [Acharya and Viswanathan \[2011\]](#) setup in three significant ways. First, we recognize that not all intermediaries on the verge of bankruptcy are necessarily forced by lenders to liquidate their assets. In practice, we often observe strategic write-downs as a result of renegotiation between the borrower and its lenders. We define a parameter (q) that reflects the probability of a credit-constrained intermediary being unable to renegotiate

successfully with its creditors Date 1 leading to repossession of the asset by the creditors who then liquidate it in the financial asset market. Conversely, $(1 - q)$ is the probability that a credit-constrained intermediary is able to renegotiate with the lender and write-down its obligations. One could view q in the context of how the bankruptcy code treats repo contracts. If $q = 1$, the asset is exempt from an automatic stay and the lender enjoys exclusive rights over the asset in the event of bankruptcy, a feature that allows the lender to always liquidate the asset in the secondary market. We, therefore, refer to q as the bankruptcy exemption or the “safe harbor” parameter; it describes the likelihood of the lender retaining control of the asset in the event of a borrower default.

The second major point of departure from the [Acharya and Viswanathan \[2011\]](#) model is that we allow for the existence of a new loan market at Date 1. After the Date 1 shock has been realized, intermediaries that are not credit-constrained can invest in the primary (origination) market for loans as well as the secondary market for financial assets. This characterization allows us to analyze the important interplay between the financial asset sale market and the real economy, which is at the heart of our welfare analysis of bankruptcy exemption of repo contracts.

The third major point of departure is that we take into account moral hazard in the real economy. Fixed claims, such as debt, exacerbate moral hazard problems in the real sector when loan rates are too high and our model captures this insight. For instance, in the case of mortgage loans, households being residual claimants on levered assets would have lower incentives to maintain the asset if the borrowing rate is too high (as we will show to be the case when an adverse shock occurs in the economy). This effect will also play a crucial role in our model in potentially shutting down the real asset market entirely when the shock is sufficiently adverse.

4. Optimizing Behavior of Agents

In this section, we lay out and solve the ex-post equilibrium at Date 1.

4.1. Lender’s Decision to Roll Over Short-term Debt

At the intermediate date, Date 1, the economy suffers an observable, but unverifiable shock (θ_2). Depending on the shock, financiers demand repayments at Date 1 or agree to roll over debt to Date 2. A financial asset sale market exists where intermediaries can liquidate their claims on the asset in order to service outstanding debt. The counterparties in this asset sale market are intermediaries with surplus liquidity. After the realization of the Date 1 shock, the asset sale market is cleared and (some) debts rolled over, intermediaries that have successfully rolled over can explore the possibility

of making the financial asset riskier by switching to the risk-shifting alternative. Thus, Date 1 financing must account for this risk-shifting possibility. We present the following lemma on the resulting funding liquidity of the financial asset at Date 1:

LEMMA 1: *The funding liquidity at Date 1 per unit of the safer asset is $\rho^* = \theta_2 \frac{(\theta_2 y_2 - \theta_1 y_1)}{(\theta_2 - \theta_1)}$. The reduction in funding liquidity attributable to the risk-shifting problem is given by k_1 , where $k_1 = \theta_2 y_2 - \rho^* = \frac{\theta_2 \theta_1 (y_1 - y_2)}{(\theta_2 - \theta_1)}$. k_1 is decreasing in y_2 and θ_2 .*

The funding liquidity of an asset at Date 1 is the amount of rollover debt that can be raised by pledging the asset. Since the risk-shifting payoff leads to a negative value investment, financiers would want to set the face value (f) in such a way that the borrower has no incentives to risk shift. This requires $\theta_2(y_2 - f) > \theta_1(y_1 - f)$, which implies that $f < f^* \equiv \frac{\theta_2 y_2 - \theta_1 y_1}{\theta_2 - \theta_1}$. The funding liquidity (ρ^*) of the financial asset is given by the loan amount that financiers would be able to finance, is equal to $\theta_2 f^*$, which can also be represented as $\theta_2 y_2 - k_1$. One can think of k_1 as the non-pledgeable portion of expected cash flows ($\theta_2 y_2$) or the funding illiquidity of the asset due to the risk-shifting problem. It can be easily seen that this funding illiquidity (k_1) reduces as the payoff of the asset (y_2) or the economic outlook for the asset (θ_2) improves.

The key implication of the above lemma is that funding liquidity of the financial asset depends on the economic shock to asset quality (θ_2). Because intermediaries differ in the amount of debt assumed at Date 0, the economic shock will have differing implications for them, as we will characterize shortly. Recall that while the financial asset is subject to risk-shifting concerns which affect its funding liquidity, we assume that the real asset cannot be pledged to raise funding. We turn next to the moral hazard problem for the real asset.

4.2. Household's Moral Hazard Problem

Intermediaries that invest in the real asset provide one unit of financing at Date 1 to households in return for a promised payment of f_r at Date 2. Households use this financing to invest in a physical asset that provides a rental income of R at Date 2. Thus, households view their leveraged investment as paying a cash flow of $(R - f_r)$ in the high state (which occurs with a probability of e) and a cash flow of 0 in the low state (which occurs with a probability of $1 - e$). The probability e , which is endogenously determined by the household, reflects its effort choice, and thus the asset quality. The expected benefit from renting is $e(R - f_r)$, and we assume that the pecuniary equivalent of expending effort is quadratic in the level of effort; more specifically, the cost is equal to $\frac{1}{2}\gamma e^2$,

where $\gamma > 0$ captures the intensity of effort aversion. Therefore, the household chooses an effort level e that trades off the benefits of asset quality with effort aversion, and maximizes its net expected payoffs of $e(R - f_r) - \frac{1}{2}\gamma e^2$. Given the bounds on the effort choice ($0 \leq e \leq 1$), the optimal solution is given by,

$$e^* = \min \left[\max \left[0, \frac{1}{\gamma}(R - f_r), 1 \right] \right]. \quad (1)$$

Then, Lemma 2 implies that the moral hazard problem worsens when the interest rate (or the face value of the debt) increases:

LEMMA 2: *The optimal effort level of the representative household (e^*), and, thus, the asset quality, is negatively related to the face value (f_r) of the real asset loan.*

4.3. Liquidation Decisions of Credit-Constrained intermediaries

The continuum of intermediary firms differ from each other in terms of the investment shortfalls (s) required to enter the financial intermediation sector; equivalently, these intermediaries differ in terms of their outstanding liabilities (ρ) due at Date 1. Suppose – and we will verify in Section 5 – the distribution of ρ is given by $\rho \sim G(\rho)$ over $[\rho_{\min}, \rho_{\max}]$, where $\theta_1 y_1 \leq \rho_{\min} < \theta_2 y_2 \leq \rho_{\max}$ and $\rho^* \in [\rho_{\min}, \rho_{\max}]$, where ρ^* is the funding liquidity of the financial asset. At Date 1^- , when the economy-wide shock (θ_2) is realized, intermediaries will either be credit-constrained ($\rho \geq \rho^*$) or will enjoy surplus liquidity ($\rho < \rho^*$). Thus, a market for the financial asset is created in which credit-constrained intermediaries supply the financial asset and surplus-liquidity intermediaries demand it. The market for financial assets clears at a price p , which will be derived keeping in mind that surplus-liquidity intermediaries can also participate in the household loan market (i.e, real asset market), at Date 1.

To raise ρ units to roll over debt, an intermediary can choose a liquidation policy $\delta \geq 0$ such that $[\delta p + (1 - \delta)\rho^*] = \rho$. It follows that $\delta(p, \rho) = \frac{(\rho - \rho^*)}{(p - \rho^*)}$. The creditors get repaid in full (i.e., ρ), while the borrower receives a net payoff of $\delta(p, \rho)\theta_2 y_2 + (1 - \delta(p, \rho))p$. Note that $\delta(p, \rho) > 0$ if and only if $\rho > \rho^*$, i.e., only credit-constrained intermediaries liquidate some of their assets. Further for $\rho > p$, $\delta(p, \rho) > 1$, implying that intermediaries which have $\rho > \rho^*$ are unable to meet their liability even if the entire asset is liquidated and have no choice but to go into bankruptcy.

Now credit-constrained intermediaries that have $\rho^* < \rho \leq p$ face a strategic choice between liquidating δ fraction of the asset to roll over their debt or declaring bankruptcy. In the event they declare bankruptcy, they would lose possession of their asset with a probability q , while with a probability $(1 - q)$ they would get their debt written down to ρ^* resulting in a net payoff of $(1 -$

$q)(\theta_2 y_2 - \rho^*)$ for the defaulting intermediaries. If $\bar{\rho}$ is the level of leverage above which intermediaries choose to default strategically, then Lemma (3) follows:

LEMMA 3: *The leverage level ($\bar{\rho}$) above which intermediaries would seek to default strategically is given by $\bar{\rho} = \rho^* + q(p - \rho^*)$.*

Essentially, $\rho = \bar{\rho}$ is the level of leverage at which the intermediary is indifferent between liquidating δ fraction of the asset to reduce its liability to ρ^* or filing for a strategic default (i.e., $\delta(p, \bar{\rho})\theta_2 y_2 + (1 - \delta(p, \bar{\rho}))p = (1 - q)(\theta_2 y_2 - \rho^*)$). When $\rho > \bar{\rho}$ the intermediary is better off defaulting on its liability, while for $\rho < \bar{\rho}$, it is optimal to liquidate a fraction of the asset to meet the demands of the creditors. Table (1) summarizes the payoffs for intermediaries and repo financiers based on the level of leverage ρ . Note that, when there is full exemption from automatic stay (i.e., $q = 1$), $\bar{\rho} = p$, implying that there is no strategic default. On the other hand, when there is no exemption (i.e., $q = 0$), $\bar{\rho} = \rho^*$, it is optimal for all credit-constrained intermediaries to do a strategic default.⁶

Intermediary Leverage (ρ)	Intermediary Strategy	Financier Payoff	Intermediary Payoff	Asset Fraction Liquidated
$\rho \leq \rho^*$	Use surplus liquidity to acquire new assets.	ρ	$\theta_2 y_2 - \rho$	0
$\rho^* < \rho \leq \bar{\rho}$	Liquidate δ asset to pay back creditor in full.	ρ	$(1 - \delta)\theta_2 y_2 + \delta p - \rho$	δ
$\bar{\rho} < \rho \leq p$	Strategic default.	$qp + (1 - q)\rho^*$	$(1 - q)(\theta_2 y_2 - \rho^*)$	q
$p < \rho$	Involuntary default.	$qp + (1 - q)\rho^*$	$(1 - q)(\theta_2 y_2 - \rho^*)$	q

Table 1: **Intermediary and Financier Payoffs.**

Surplus-liquidity intermediaries ($\rho < \rho^*$) will take long positions in the financial asset. Therefore, the aggregate supply of financial asset is determined as follows. Moderately credit-constrained intermediaries ($\rho^* < \rho \leq \bar{\rho}$), liquidate a fraction δ of their assets. At the same time, for severely credit-constrained intermediaries ($\rho > \bar{\rho}$), only a fraction q go into liquidation. The remaining fraction $(1 - q)$ of severely credit-constrained intermediaries obtain a strategic write-down by entering into negotiations with the financiers. We assume that the liability can be renegotiated downward to the asset's funding liquidity, ρ^* . Thus, given an adverse shock θ_2 at Date 1, a fraction q of the severely credit-constrained intermediaries will be forced to liquidate some or part of their assets.

⁶The bankruptcy exemption parameter (q) can be thought of as an average value that captures the average "style" of heterogeneous judges who interpret the bankruptcy code in their individual style. From a cross-sectional perspective, q can also be thought of as capturing judge fixed effects.

If $g(\rho)$ denotes the p.d.f. of ρ , the aggregate supply of financial assets in the market is given by

$$S(p, \rho^*) = \int_{\rho^*}^{\bar{\rho}} \frac{\rho - \rho^*}{p - \rho^*} g(\rho) d\rho + \int_{\bar{\rho}}^{\rho_{\max}} q g(\rho) d\rho. \quad (2)$$

4.4. Ex-post Equilibrium

Suppose that an intermediary with surplus liquidity acquires α units of the financial asset in the asset sale market and lends β units in the new loan market at Date 1. Such intermediaries would optimally choose α and β , for a given p and f_r and a conjectured household effort choice (e).

Then for a given realization of the economic shock (θ_2) at Date 1, the optimizing behavior of agents with market-clearing results in an ex-post equilibrium which is determined as follows:

- (i) Households maximize their effort given the face value (f_r) of the real asset loan, as given by Equation (1), which is restated below:

$$e^* = \min \left[\max \left[0, \frac{1}{\gamma} (R - f_r), 1 \right] \right]. \quad (3)$$

- (ii) Surplus-liquidity intermediaries maximize the incremental benefits from acquiring α financial assets in the secondary market of legacy financial assets and providing β amount of loans to households in the primary market of real asset loans; they have rational expectations over p and f_r and e^{ast} ; and solve

$$\max_{\alpha \geq 0, \beta \geq 0} (1 + \alpha)(\theta_2 y_2 - \rho^*) + \beta e f_r, \quad (4)$$

subject to the budget constraint

$$\alpha(p - \rho^*) + \beta \leq \rho^* - \rho. \quad (5)$$

- (iii) Denoting the optimal choice for α and β for intermediaries with liquidity ρ be $\alpha^*(\rho)$ and $\beta^*(\rho)$, respectively, the aggregate demand for the financial asset is given by

$$\bar{\alpha} = \int_{\rho_{\min}}^{\rho^*} \alpha^*(\rho) g(\rho) d\rho \leq S(p, \rho^{ast}), \quad (6)$$

and the aggregate demand for the real asset is given by

$$\bar{\beta} = \int_{\rho_{\min}}^{\rho^*} \beta^*(\rho) g(\rho) d\rho \leq \mathcal{B}, \quad (7)$$

where \mathcal{B} denotes the size of the real sector in the economy.

The objective function in (4) captures the incremental benefits associated with acquiring financial and real assets. Acquiring one unit of the financial asset yields an expected payoff of $\theta_2 y_2$, which implies that the incremental benefit over and above the funding liquidity of the financial asset is $(\theta_2 y_2 - \rho^*)$. Since the real asset cash flows cannot be pledged, the incremental benefit of acquiring one unit of the real asset is the same as its expected payoff, i.e., ef_r .

The constraint in (5) is the budget constraint of a surplus-liquidity intermediary. The right hand side reflects the available surplus liquidity. The left hand side represents the allocation of liquidity toward acquiring α financial assets and making β household loans in the real asset market. The other two constraints are that there is a non-negative demand for the financial asset and the real asset. Finally, some technical restrictions on the loan face value (f_r), the effort aversion parameter (γ), and the financial asset price (p) must be satisfied in equilibrium, which are stated in Section A5 of the Appendix.

4.5. Implications of Cross-Market Equilibrium

The optimization exercise of surplus-liquidity intermediaries yields an equilibrium relation between the incremental expected return from investing in the financial asset ($= \frac{k_1}{p - \rho^*}$)⁷ and the real asset ($= ef_r$)⁸, as stated in the lemma below:

LEMMA 4: (i) When both the financial asset market and the real asset market are open:

$$\bar{\beta} > 0 \implies \frac{k_1}{p - \rho^*} = ef_r. \quad (8)$$

(ii) When only the financial asset market is open:

$$\bar{\beta} = 0 \implies \frac{k_1}{p - \rho^*} > ef_r. \quad (9)$$

Equation (8) states that the incremental expected return from investing in two asset markets must be equal. If they are unequal, all surplus liquidity will flow to the market offering higher return, thereby causing a shutdown of the other market. Thus, when both markets are open, it must be the case the returns are equal across the two markets.⁹ Equation (9) states that when only

⁷The numerator and denominator of the expression $\frac{k_1}{p - \rho^*}$ represent the marginal benefit (expected benefits net of funding liquidity) and marginal cost (market price net of funding liquidity) of acquiring the financial asset.

⁸The return per dollar of investment in the real asset market is given by the ratio of the marginal benefit ($ef_r - 0$) and the marginal cost is given by $(1 - 0)$, where 0 indicates the funding liquidity of the real asset and 1 indicates the loan amount of 1 unit.

⁹This feature of the model is a key insight that resonates with the importance of fire sales during a crisis (Diamond and Rajan [2011], Hanson et al. [2011], ?, ?, and Stein [2012]).

the financial asset market is open, the return from investing in the financial asset must necessarily be strictly greater than the return from investing in the real asset. Note that the financial market must necessarily clear (i.e., α is strictly greater than 0) because it is a secondary market of legacy assets. In contrast, the real asset market is a primary market that can be constrained by supply and therefore it may remain closed in equilibrium.

4.6. Solving for the Financial Asset Market Clearing Price (p)

Integrating Equation (5) for intermediaries that are surplus-liquidity, i.e., $\rho < \rho^*$, and using Equation (7) we obtain the following aggregate budget constraint.

$$\bar{\alpha}(p - \rho^*) + \bar{\beta} = \int_{\rho_{\min}}^{\rho^*} (\rho^* - \rho)g(\rho)d\rho, \quad (10)$$

which can be solved using Equation (6) to yield financial asset market-clearing, as given below:

$$\int_{\rho^*}^{\bar{\rho}} \frac{\rho - \rho^*}{p - \rho^*} g(\rho) d\rho + \int_{\bar{\rho}}^{\rho_{\max}} q g(\rho) d\rho + \bar{\beta} \frac{1}{p - \rho^*} = \int_{\rho_{\min}}^{\rho^*} \frac{\rho^* - \rho}{p - \rho^*} g(\rho) d\rho. \quad (11)$$

Equation (11) can be solved to determine the market clearing price of the financial asset (p):

LEMMA 5: *The financial asset market clears at an equilibrium price ($p(\bar{\beta}; \theta_2)$) given by*

$$p = \rho^* + \frac{1}{q} \int_{\rho_{\min}}^{\bar{\rho}} \frac{G(\rho)}{G(\rho_{\max})} d\rho - \frac{\bar{\beta}}{q G(\rho_{\max})}. \quad (12)$$

The first term on the right hand side of Equation (12) represents the funding liquidity of the financial asset, $\rho^* = \theta_2 y_2 - k_1$. The combination of the second and the third terms reflects the spare liquidity in the economy. If the spare liquidity in the economy is sufficiently high and exceeds the funding illiquidity of the asset (k_1), the financial asset will trade at its fair value of $\theta_2 y_2$. This situation would arise when the economic shock (θ_2) is too mild. When the spare liquidity in the economy is lower than k_1 , fire sales arise and the financial asset trades at a discount to its fair value.

PROPOSITION 1: *Conditional on the economic shock (θ_2), the economy lies in either one of two mutually exclusive regions: the Fair Pricing Equilibrium Region, where both the financial asset and the real asset are fairly priced, and the Fire Sale Equilibrium Region, where both the financial asset and the real asset are priced a discount to the fair value. In the Fair Pricing Equilibrium Region, the equilibrium characteristics are given by*

$$p = \theta_2 y_2, \quad (13)$$

$$\bar{f}_r = \frac{R}{2} - \frac{1}{2}\sqrt{R^2 - 4\gamma} < \frac{R}{2}, \quad (14)$$

$$\bar{\beta} = \mathcal{B}. \quad (15)$$

We characterize the Fire Sale Equilibrium below. The critical factor driving the type of equilibrium region is the amount of spare liquidity in the economy. For a given economic shock (θ_2), the spare liquidity depends on the bankruptcy exemption parameter (q).¹⁰ At lower values of q , bankruptcy exemption is rarely applicable and most credit-constrained intermediaries are able to renegotiate their debt to a lower face value and roll over their obligations. There is minimal liquidation in such an economy and the spare liquidity of surplus-liquidity intermediaries is sufficiently high to cause the market-clearing price of the financial asset to hit the fair value of $\theta_2 y_2$ (Fair Pricing Equilibrium Region). For higher values of q , there is greater liquidation of the financial asset subsequent to the economic shock, and the spare liquidity of surplus-liquidity intermediaries is stretched, resulting in a market-clearing price lower than the fair value, i.e., fire sales arise (Fire Sale Equilibrium Region). We can show further that

PROPOSITION 2: *The Fire Sale Equilibrium Region consists of three types of equilibria, depending on the value of the bankruptcy exemption parameter (q), as discussed below.*

(i) *The Real Sector Price Discrimination Equilibrium: Both the financial asset market and the real asset market are open and the real asset loans exhibit price discrimination:*

$$\bar{\beta} = \mathcal{B}, \quad (16)$$

$$f_r = \frac{R}{2} - \frac{1}{2}\sqrt{R^2 - \frac{4\gamma k_1}{p - \rho^*}} > \bar{f}_r, \quad (17)$$

$$p = \rho^* + \frac{1}{q G(\rho_{max})} \left[\int_{\rho_{min}}^{\bar{\rho}} G(\rho) d\rho - \mathcal{B} \right]. \quad (18)$$

(ii) *The Real Sector Liquidity Crunch Equilibrium: Both the financial asset market and the real asset market are open and the real asset market experiences a fire-sale “quantity” constraint:*

$$\bar{\beta} = -q(p - \rho^*) G(\rho_{max}) + \int_{\rho_{min}}^{\bar{\rho}} G(\rho) d\rho < \mathcal{B}, \quad (19)$$

$$f_r = \frac{R}{2}, \quad (20)$$

$$p = \rho^* + \frac{4\gamma k_1}{R^2}. \quad (21)$$

¹⁰In the ex-post equilibrium, we take the economic shock (θ_2) as given on Date 1, but in general, the combination of (θ_2, q) determines the aggregate liquidation of financial assets by credit-constrained intermediaries, as described in Equation (2), which in turn, causes the market price to trade at or below the fair value.

(iii) *The Real Sector Credit Crunch Equilibrium: The real asset market shuts down. Only the financial asset market is open. The equilibrium price (p) is given as below (note that $\bar{\beta} = 0$, although $f_r = \frac{R}{2}$):*

$$p = \rho^* + \frac{1}{q G(\rho_{max})} \int_{\rho_{min}}^{\bar{\rho}} G(\rho) d\rho. \quad (22)$$

For a given level of economic shock (θ_2) as q increases from 0 toward 1, the economy transitions from the Fair Pricing Equilibrium to the Price Discrimination Equilibrium, then to the Liquidity Crunch Equilibrium, and finally to the Credit Crunch Equilibrium. The three fire-sale equilibria are discussed in greater detail below.

4.7. Real Sector Price Discrimination Equilibrium

If q is higher than at the border of the Fair Pricing and Fire Sale Equilibrium Regions, there is enough liquidation of assets to cause the financial asset market clearing price to be lower than the fair value of $\theta_2 y_2$. In this region, there is a fire-sale “price” effect in that as q increases, the price discount from fair value increases. This pricing feature is similar to the “cash-in-the-market” pricing in [Gale and Allen \[1994\]](#) and [Allen and Gale \[1998\]](#).

The fire-sale “price” effect causes the gross return from investing in the financial asset to exceed 1. Cross-market arbitraging activity would then imply that the expected return from investing in the real asset must match that from investing in the financial asset. Consequently, the face value (equivalently, the effective interest rate) on the real asset loans would be increased to offer the same return as on the financial asset. We refer to this equilibrium as the Price Discrimination Equilibrium because surplus-liquidity intermediaries will divert their resources to the real asset market only if they can earn supra-normal rents, i.e., discriminate on price to ensure that they get the same return as on the financial asset.

At a sufficiently high value of q , the economy transitions to the Real Sector Liquidity Crunch Region, as discussed next.

4.8. Real Sector Liquidity Crunch Equilibrium

There is a limit to which surplus-liquidity intermediaries can engage in price discrimination, by increasing the face value on the real asset loan. There is an upper bound on the face value because of the moral hazard problem in the real sector. Borrowers, being residual cash flow claimants, expend less effort as the face value increases, as shown in Equation (1), and the asset quality suffers. The

expected profit from lending in the real sector is, therefore, concave in the face value of the real asset loan. The profit-maximizing face value is $\frac{R}{2}$, and surplus-liquidity intermediaries would never find it incentive compatible to post a higher face value than $\frac{R}{2}$ because the marginal benefit from a higher face value will be lower than the marginal cost in the form of loans with lower asset quality.¹¹ When this upper bound on the loan face value is hit due to an increase in q , the economy transitions from the Real Sector Price Discrimination Equilibrium Region to the Real Sector Liquidity Crunch Equilibrium Region.

In this Liquidity Crunch Equilibrium Region, the financial asset price reflects a fire-sale “price” effect but remains invariant to q because the real asset return has hit an upper bound and cannot increase any further even when q increases. Cross-market arbitraging activity implies that the financial asset return is also arrested, and the price of the financial asset price stays at the same level for all values of q in this region. The financial asset price can no longer adjust to ensure market clearing. Instead, financial market clearing is now ensured by sucking out liquidity from the real sector, i.e., by a reduction in $\bar{\beta}$. This diversion of surplus-liquidity intermediaries’ resources is required to clear the financial asset market, and the real sector contracts with an increase in q in this region. This phenomenon is a fire-sale effect; however, it appears as a quantity discrimination effect in the real asset market, and we refer to it as the fire-sale “quantity” constraint.

The process of shrinking the real sector continues as q increases in this region. At a sufficiently high value of q , the real asset market completely collapses. The economy now transitions to the Real Sector Credit Crunch Equilibrium Region, which is discussed next.

4.9. Real Sector Credit Crunch Equilibrium

In this region, the cross-market equilibrium return condition is irrelevant because the value of q is high enough to cause a breakdown of the real asset market. Only the financial asset market is open and now the financial asset price can adjust freely to ensure financial asset market-clearing. As in the Price Discrimination Equilibrium, there is a fire-sale “price” effect in this region. The return on the financial asset is no longer bounded by the return on the real asset; in fact, the return on the financial asset always exceeds the potential return on the real asset.

To summarize, an interaction between the risk-shifting problem in the financial asset (which

¹¹The expected profit from lending to households (ef_r) is concave in f_r and is maximized at f_r equal to $\frac{R}{2}$. It is worth highlighting that the competitive equilibrium face value (f_r) is the same as the profit-maximizing value for lenders in the real sector. Thus, the equilibrium is stable to off-equilibrium offers because surplus-liquidity intermediaries would make lower profits at any other value of f_r .

limits its funding liquidity) and the moral hazard problem in the real asset market (which affects its asset quality) drives the underlying economics of the model. First, risk-shifting concerns constrain funding liquidity, thereby causing fire sales in the financial sector when an adverse economic shock arises. Cross-market arbitraging activity (which ensures that the expected returns in the two markets are the same) implies that the moral hazard problem in the real sector (effort-aversion) is in sync with risk-shifting problem in the financial sector.

We now move to the ex-ante equilibrium, so that we can evaluate the ex-ante optimal bankruptcy parameter (q) after taking into account the ex-post fire-sale effects.

5. The Ex-Ante Model

In this section, we endogenize the debt obligations assumed by intermediaries who face varying levels of investment shortfall (s) at Date 0. We assume that the investment shortfall (s) is uniformly distributed across intermediaries as $U[s_{min}, s_{max}]$.¹² Financial intermediaries finance this investment shortfall in the short-term repo market, which is subject to rollover risk at Date 1. Let the outstanding liability at Date 1 to finance shortfall (s) be denoted as $\rho(s)$. Financiers can refuse to roll over debt at Date 1 if they calculate that the state of the economy (θ_2) at Date 1 will make it impossible for the intermediary to honor its outstanding liability ($\rho(s)$). In such an event, as discussed in Section 4, intermediaries either liquidate a fraction (δ) of their asset to overcome the funding deficit, or declare bankruptcy leading to either a liquidation of their asset by the financier with a probability q or a negotiated write-down of their liability to ρ^* with a probability of $(1 - q)$.

The key to analyzing the ex-ante model is the observation that the financial asset market-clearing price at Date 1 (i.e., the liquidation price, $p(\theta_2)$), and the liabilities ($\rho(s)$) assumed at Date 0 are endogenously related. The initial liability structure of intermediaries affects the extent of financial asset liquidation at Date 1, and therefore, its price. Financiers anticipate the implied distribution of the liquidation price (p) over θ_2 and accordingly determine the face value of repo financing to be disbursed at Date 0, i.e., the initial liability structure of financial intermediaries.

Formally, while solving the ex-post model, we assumed an exogenous distribution of ρ and derived the ex-post equilibrium outcomes $(\bar{\beta}, f_r, p)$. In the ex-ante model, we begin with a distribution of investment shortfalls (s) at Date 0 which translates into a corresponding distribution of Date 1 liabilities ($\rho(s)$). We denote the resulting distribution of liabilities as $\hat{G}(\rho(s))$. The liquidation price at Date 1 depends on the distribution of ρ across intermediaries. In other words, $\hat{G}(\rho)$ and $p(\theta_2)$

¹² s_{max} is the maximum shortfall at which the asset is still NPV positive.

are determined jointly in equilibrium.

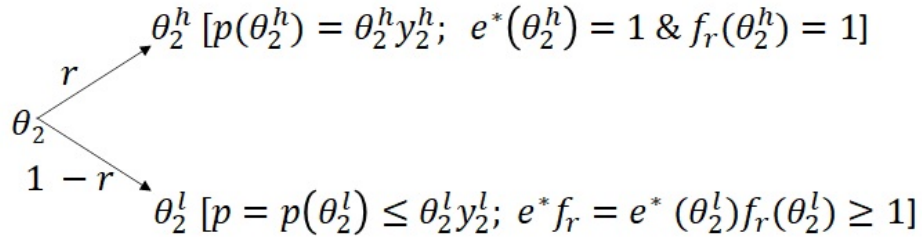
We solve for this equilibrium next and eventually explore the role of the bankruptcy exemption parameter (q) in trading off ex-ante financing against ex-post real outcomes.

5.1. The Set-up

Figure (2) provides the basic set-up for the ex-ante model. As of Date 0, the Date 1 shock, θ_2 , is unknown. For tractability, we consider a discrete two-state distribution for θ_2 : with a probability, r , the state of the economy is described by θ_2^h (which we refer to as the high state), and with a probability, $(1 - r)$, the state of the economy is described by θ_2^l (which we refer to as the low state).

We make the following assumptions regarding the high state (θ_2^h). First, we assume that the asset payoff in the high state is given by y_2^h , while that in the low state is given by y_2^l , where $y_2^h > y_1 > y_2^l$. Consequently, there are no risk-shifting issues in the high state. This assumption is similar to the contention in Gorton and Metrick [2010] regarding the role of adverse selection in repo markets. They rely on arguments in Gorton and Pennacchi [1990] and Dang et al. [2010] that repo securities are “information insensitive” securities during normal times (resulting in high liquidity), but are highly “information sensitive” when the economic shock is severe (resulting in liquidity drying up).

Figure 2: Ex-ante view of the states of the economy (θ_2). The economy is in the high state (θ_2^h) with a probability r and in the low state (θ_2^l) with a probability $1 - r$. In the high state of the economy, both the financial asset and the real asset are fairly priced. However, in the low state of the economy, both assets *could* exhibit fire-sale effects.



Secondly, we also assume that moral-hazard (effort-aversion) in the real asset market is also expected to kick in only in the low state (i.e., $\gamma = 0$ in the high state).¹³ In other words, the funding liquidity of the financial asset in the high state is equal to its fair value ($p = \theta_2^h y_2^h$), and due to arbitraging activity, the real asset would also be fairly priced, i.e., $ef_r = 1$. Furthermore, since

¹³Lack of effort aversion for household borrowers in the high state is assumed to mirror the lack of frictions in the financial asset market. However, the results of the paper follow even in the absence of this assumption.

household borrowers exhibit no effort aversion ($\gamma = 0$), the effort (e) in the high state hits the cap of 1. It follows that the face value of real asset loans (f_r) would be equal to 1 in the high state.

Finally, we assume that the market for real asset loans is fully satiated in the high state, i.e., the surplus-liquidity intermediary supply of real asset loans in the high state meets the maximum potential aggregate loan requirements of household borrowers (\mathcal{B}). In other words, there is no unmet credit demand of household borrowers in the high state.¹⁴

Let us compare the high state and low state properties. In the high state, all intermediaries will be able to roll over their debt because funding liquidity is equal to the fair value of the asset. Consequently, the system is in the Fair Pricing Equilibrium.¹⁵

$$p(\theta_2^h) = \theta_2^h y_2^h ; \quad f_r(\theta_2^h) = 1 ; \quad \bar{\beta}(\theta_2^h) = \mathcal{B} \quad (23)$$

However, in the low state, intermediaries will always be credit-constrained and unable to roll over their debt without liquidating some or all of their assets. Furthermore, the real asset market is not always satiated in the low state. Consequently, any of the four equilibrium types described in Section (4.6) could exist in the low state depending on the severity of the economic shock (θ_2^l).¹⁶ The equilibrium characteristics in the low state are as specified in Propositions (1) & (2). For simplicity of notation, we omit explicit reference of the state when referring to the equilibrium characteristics of the low state in the following sections (i.e., p refers to $p(\theta_2^l)$, f_r refers to $f_r(\theta_2^l)$, $\bar{\beta}$ refers to $\bar{\beta}(\theta_2^l)$, ρ^* refers to $\rho^*(\theta_2^l)$, k_1 refers to $k_1(\theta_2^l)$ and \bar{p} refers to $\bar{p}(\theta_2^l)$). We continue to use explicit references to the high state while discussing its equilibrium characteristics, as in Equation (23).

5.2. Payoff Potential and Investment Shortfall Financing

As shown in Figure (2), the high state occurs with a probability of r and the low state with a probability of $1 - r$. Financiers take into account the payoff potential in both states of the world.

¹⁴In general, one can put an explicit restriction on \mathcal{B} to be strictly less than an endogenously determined $\bar{\beta}$ in the high state, thereby ensuring that there will be no unmet demand. This restriction would essentially result in a constraint on θ_2^h . To avoid clutter, we express this constraint as a simple assumption, which states that there is no unmet demand in the real asset market in the high state.

¹⁵The results for $p(\theta_2^h)$ and $\bar{\beta}(\theta_2^h)$ follow from the equilibrium characteristics of the system in the fair pricing equilibrium as obtained in Proposition (1). However, in the absence of effort aversion in households, households exert maximal effort ($e^* = 1$); implying that a fairly priced real asset loan ($e^* f_r = 1$) would have unit face value ($f_r = 1$).

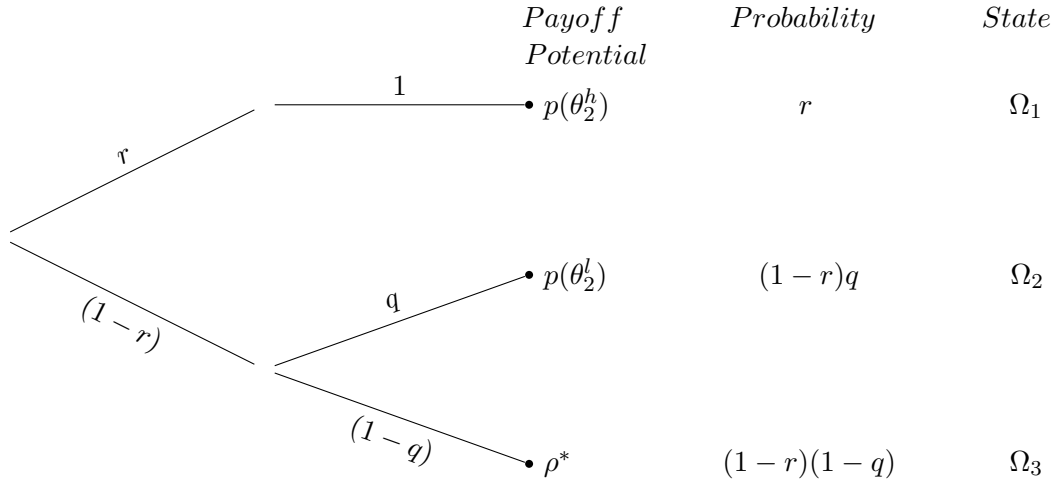
¹⁶Note that fair pricing in the high state is not the same as fair pricing in the low state. First, as $\rho^*(\theta_2^h) = p(\theta_2^h) = \theta_2^h y_2^h$, all intermediaries can roll over their debt in the high state; in the low state, $\rho^*(\theta_2^l) = \theta_2^l y_2^l - k_1$ and intermediaries having $\rho > \rho^*(\theta_2^l)$ will be unable to roll over their debt without partially (or fully) liquidating their financial asset even in the fair pricing equilibrium. Second, due to the absence of effort aversion by households in the high state, $f_r(\theta_2^h) = 1$; whereas in the low state due to non-zero effort aversion, $f_r(\theta_2^l) = \frac{R}{2} - \frac{1}{2}\sqrt{R^2 - 4\gamma}$ in the fair pricing equilibrium, as given by Proposition (1).

In the high state (θ_2^h), the payoff potential is $p(\theta_2^h) = \theta_2^h y_2^h$. In the low state, the payoff potential is determined as follows. For surplus-liquidity ($\rho \leq \rho^*(\theta_2^l)$) or for moderately credit-constrained intermediaries ($\rho^*(\theta_2^l) < \rho \leq \bar{\rho}(\theta_2^l, q)$), financiers are repaid in full. For severely credit-constrained intermediaries ($\rho > \bar{\rho}(\theta_2^l, q)$), with a probability q , financiers take control and liquidate the asset at the market-clearing price of $p(\theta_2^l)$, while with a probability of $(1 - q)$, the liability is renegotiated downward to the asset's funding liquidity, ρ^* ; thus, given an adverse shock θ_2^l at Date 1, financiers can expect a maximum payoff of \bar{p} , given by:

$$\bar{p}(\theta_2^l, q) = qp(\theta_2^l, q) + (1 - q)\rho^*(\theta_2^l). \quad (24)$$

Note that $\bar{p}(\theta_2^l, q) = \bar{\rho}(\theta_2^l, q)$, implying that the maximum payoff the financiers can expect in the low state is exactly equal to the leverage level above which intermediaries would default strategically. Figure (3) summarizes the payoff potential, which helps determine the amount of investment shortfall ($s(\rho)$) that the financier would be willing to finance at Date 0 for a given face value (ρ).

Figure 3: Ex-ante Payoff Potential. The financier's Date 1 payoff potential for a given adverse shock (θ_2^l) in different cases is shown along with the probability of the case.



From the financier's perspective, the maximum shortfall that can be financed based on the asset's payoff potential is given by $\hat{s} = rp(\theta_2^h) + (1 - r)\bar{p}(\theta_2^l)$, which is always less than or equal to s_{max} . Consequently, the range of shortfalls that get financed at Date 0 is given by $[s_{min}, \hat{s}]$, i.e., intermediaries with shortfalls $(\hat{s}, s_{max}]$ are rationed at Date 0. The lemma below discusses the endogenous leverage in the economy at Date 0.

LEMMA 6: *Given a uniform distribution of investment shortfalls in the economy (i.e., $H(\tilde{s})$ is $U[s_{min}, s_{max}]$), the endogenous distribution of leverage ($\rho : \rho \in [\rho_{min}, \rho_{max}]$) at Date 0 that takes*

into account the expected payoff to the financiers at Date 2 is specified by $\hat{G}(\rho)$, as follows:

$$\hat{G}(\rho) = \frac{\tilde{s}(\rho) - s_{min}}{s_{max} - s_{min}},$$

$$\text{where } \tilde{s}(\rho) = \begin{cases} \rho, & \text{if } \rho_{min} \leq \rho \leq \bar{\rho}(\theta_2^l) \\ r\rho + (1-r)\bar{p}(\theta_2^l), & \text{if } \bar{\rho}(\theta_2^l) < \rho \leq p(\theta_2^h) \end{cases} \quad (25)$$

5.3. Ex-ante Dynamic Equilibrium

The ex-ante dynamic equilibrium is (i) a pair of functions $\rho(s)$ and $p(\theta_2^l)$, which respectively give the promised face value ($\rho(s)$) for raising short-term repo financing of s units at Date 0 and the equilibrium price ($p(\theta_2^l)$) at Date 1 given the interim signal of asset quality of θ_2^l ; and (ii) a truncation point \hat{s} , such that $\rho(s)$, $p(\theta_2^l)$ and \hat{s} satisfy the following fixed-point recursion:

1. For a given θ_2^l , the asset's price ($p(\theta_2^l)$) is given by the market-clearing and cross-market arbitrage determined price function in Proposition (1) and Proposition (2).
2. Individual rationality of financiers: Given the price function $p(\theta_2^l)$, for every shortfall $\tilde{s} \in [s_{min}, \hat{s}]$, the promised face value $\rho(s)$ is determined by the requirement that financiers receive in expectation the amount being lent, i.e., $\tilde{s}(\rho(s)) = \tilde{s}$, where $\tilde{s}(\rho(s))$ is given by Equation 25.
3. The derived distribution of leverage, $\hat{G}(\rho)$, depends on $s(\rho) \in [s_{min}, \hat{s}]$ where \hat{s} is the maximal investment shortfall that is financed (Equation (25)).¹⁷

The ex-ante equilibrium is defined for a given θ_2^h and θ_2^l . In the high state, the endogenous distribution of leverage has no impact on the equilibrium characteristics. In the low state, the equilibrium characteristics will mirror the solution provided in Proposition (1) and Proposition (2), except that the exogenously specified distribution of leverage ($G(\rho)$) in Equations (18), (19), and (22) must now be substituted by the endogenously derived distribution ($\hat{G}(\rho)$), as described in Equation (25).¹⁸ The bankruptcy exemption parameter (q) affects the equilibrium characteristics both through its ex-post impact on liquidation and its ex-ante impact on distribution of leverage.¹⁹

¹⁷Because $s(\rho)$ depends on the asset's price ($p(\theta_2^l)$), the derived distribution, $\hat{G}(\rho)$, depends on the asset price.

¹⁸In the ex-ante setup, $\bar{\beta}$ in the Liquidity Crunch Equilibrium as well as p in the Price Discrimination Equilibrium and the Credit Crunch Equilibrium are functions of the distribution of leverage; consequently, the specification of these terms vary, as from that obtained for the ex-post equilibrium (see the Internet Appendix for the closed-form equilibrium solutions of $\bar{\beta}$ and p).

¹⁹In the ex-ante setup, q has the following impact on equilibrium characteristics (Proofs in the Internet Appendix): $\frac{dp}{dq} \Big|_{PD,CC} < 0$, $\frac{dp}{dq} \Big|_{LC} = 0$, $\frac{d\bar{p}}{dq} \Big|_{PD,CC} = 0$, $\frac{d\bar{p}}{dq} \Big|_{LC} > 0$, $\frac{df_r}{dq} \Big|_{PD} > 0$, $\frac{df_r}{dq} \Big|_{LC,CC} = 0$, $\frac{d\bar{\beta}}{dq} \Big|_{PD,CC} = 0$, and $\frac{d\bar{\beta}}{dq} \Big|_{LC} < 0$.

5.4. Equilibrium Regions

Keeping θ_2^h fixed, we vary θ_2^l and analyze the relation between the equilibrium characteristics in the low state and the bankruptcy exemption parameter (q).²⁰

Figure (4) shows the typical demarcation of the feasible (q, θ_2^l) space into the Fair Pricing (FP) region, as shown in white, and the Fire Sale (FS) region, as shown by the gray shade. The Fire Sale region consists of the Price Discrimination (PD), the Liquidity Crunch (LC), and the Credit Crunch (CC) equilibria; we use increasingly darker shades of gray to represent greater fire-sale effects. For different magnitudes of the economic shock (θ_2^l), we see how the type of equilibrium changes with the bankruptcy parameter (q). The solid $\bar{q}(\theta_2^l)$ curve represents the boundary between the FP and PD equilibrium regions. The long dashed $\bar{q}(\theta_2^l)$ curve represents the boundary between the PD and LC equilibrium regions. The dotted $\hat{q}(\theta_2^l)$ curve represents the boundary between the LC and CC equilibrium regions. Consider, for example, the case with $\theta_2^l = 0.48$. The vertical dotted line emanating from this level of θ_2 captures how the system transitions across different types of equilibrium regions, as q increases from 0 to 1 along the dotted vertical line.

5.5. Equilibrium Characteristics for a Given Economic Shock

Figure (5) shows the evolution of equilibrium values of p (Panel A), \bar{p} (Panel B), $\bar{\beta}$ (Panel C), and f_r (Panel D) as we vary q from 0 to 1. The values of q at which the system transitions across each of the equilibrium regions are indicated by dotted vertical lines. Although difficult to detect by observing the figures, the relation between \bar{p} with q is non-monotonic, as shown in Footnote 19. Furthermore, it can be seen in Panel C that $\bar{\beta}$ is (weakly) decreasing q , and in Panel D that f_r is (weakly) increasing in q . Thus, the real sector characteristics are monotonic in q .

Panel E shows the equilibrium return on the financial asset market and the real asset market. The returns in both these markets are the same in the FP, PD, and LC regions, but diverge in the CC region, where the financial asset market returns exceeds that of the real asset market which shuts down. Panel F shows the decreasing relation between effort and bankruptcy exemption; it implies that the real asset quality worsens as bankruptcy exemption parameter (q) increases.

²⁰The interval $[\theta_2^{min}, \theta_2^{max}]$ over which we vary θ_2^l is determined by feasibility constraints. The lower bound θ_2^{min} ensures financial market clearing for all θ_2^l , while the upper bound θ_2^{max} ensures that $\theta_2^l < \theta_2^h$.

Figure 4: Equilibrium Regions. Typical demarcation of the feasible $q - \theta_2^l$ space into the Fair Pricing (FP), as shown by the white region and the Fire Sale (FS) region, as shown by the gray shaded region. The Fire Sale region consists of the Price Discrimination (PD), the Liquidity Crunch (LC) and the Credit Crunch (CC) equilibria. The solid $\bar{q}(\theta_2^l)$ curve is the boundary between the FP and PD equilibrium regions. The long dashed $\bar{q}(\theta_2^l)$ curve is the boundary between the PD and LC equilibrium regions. The dotted $\hat{q}(\theta_2^l)$ curve is the boundary between the LC and CC equilibrium regions. The PD, LC and CC equilibrium regions jointly constitute the Fire Sale Equilibrium Region which is indicated by the differing shades of gray (the darker shades indicate greater fire-sale effects). For a strong economic shock, indicated by $\theta_2^l = 0.48$, as q is increased from 0, the system transitions from FP equilibrium to PD equilibrium at $q = 0.20$, then from PD equilibrium to LC equilibrium at $q = 0.41$ and finally from LC equilibrium to CC equilibrium at $q = 0.79$. For a mild economic shock indicated by $\theta_2^l = 0.75$, the system remains in FP equilibrium for any q . For a severe economic shock, indicated by $\theta_2^l = 0.3$, the system starts in LC equilibrium at $q = 0$ and transitions to CC equilibrium at $q = 0.16$. $\theta_2^l = 0.30$, $\theta_2^l = 0.48$ and $\theta_2^l = 0.75$ are indicated by the three thin vertical dashed lines. Parameter Configuration used: $\theta_2^{min} = 0.15$, $\theta_2^{max} = \theta_2^h = 1$, $\theta_1 = 0.02$, $y_2^l = 15$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$ and $\mathcal{B} = 0.15$.

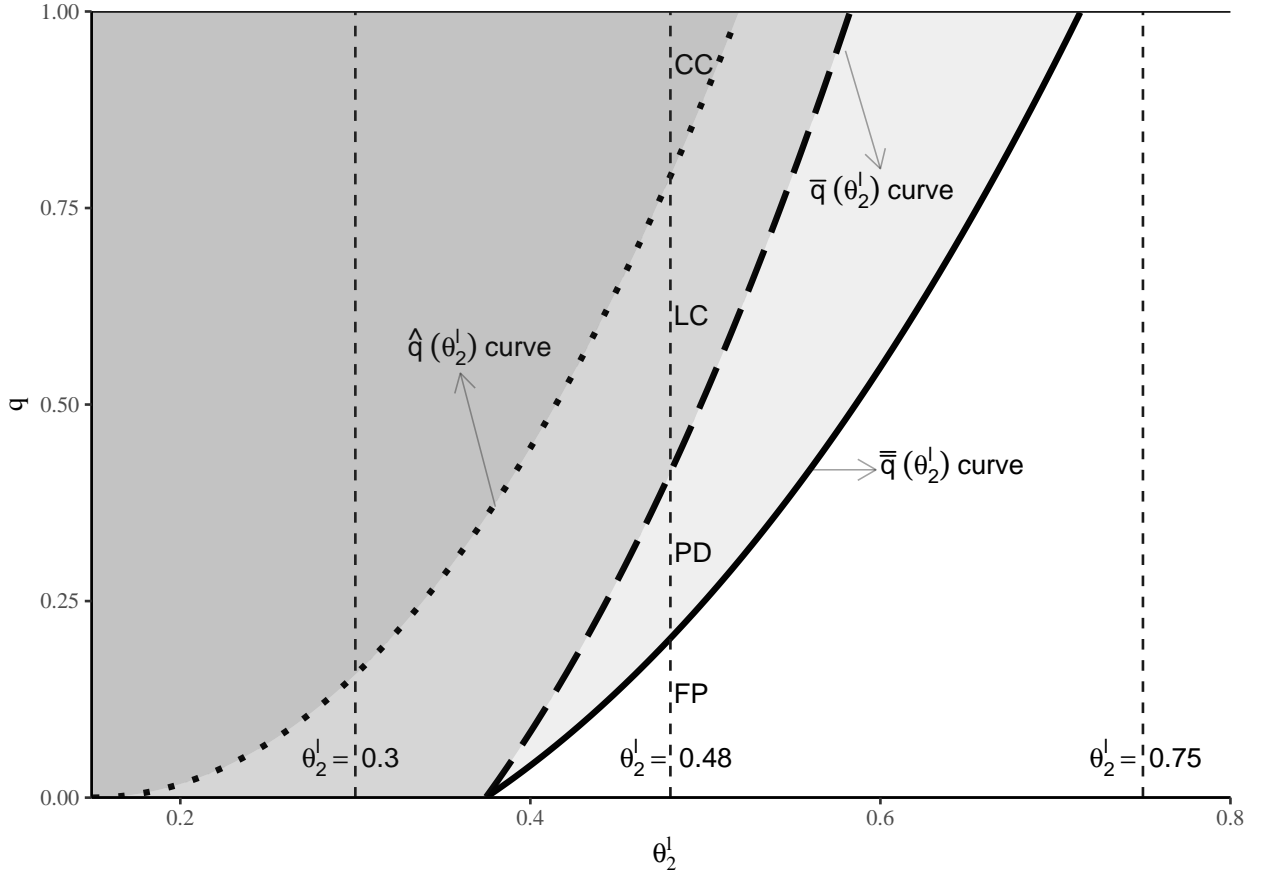
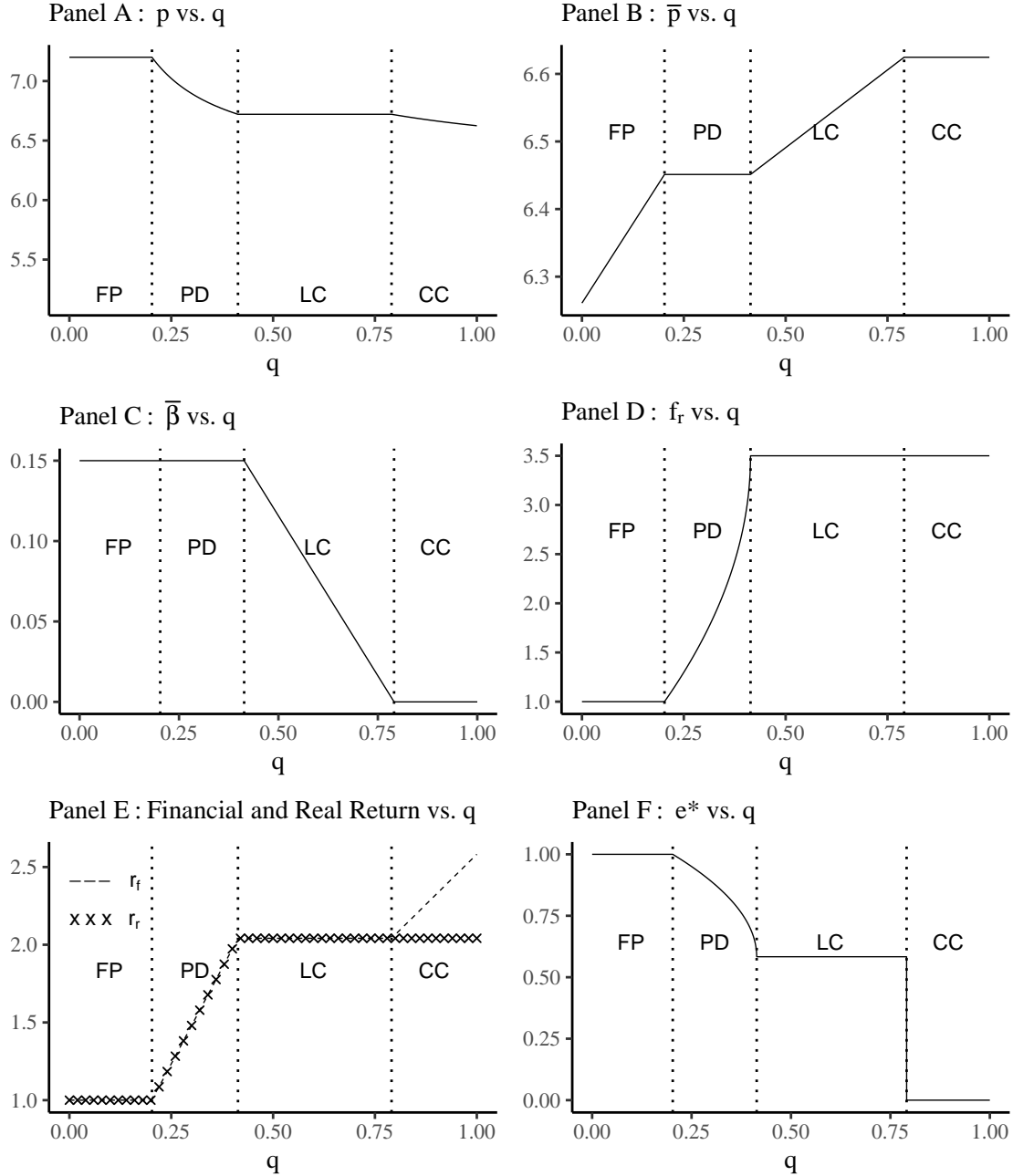


Figure 5: Evolution of equilibrium p , \bar{p} , $\bar{\beta}$, f_r , r_f , r and e^* with q for a given θ_2^l . Panel A depicts the price of the financial asset (p), Panel B depicts the financiers' expected payoff from the financial asset (\bar{p}), Panel C depicts the level of real asset loans made ($\bar{\beta}$), Panel D depicts the face value of real asset loans (f_r), Panel E depicts the returns from the financial (r_f) and real (r_r) asset and Panel F depicts the optimal effort (e^*) exerted by a borrower in the real asset market. The evolution of the equilibrium level of these variables is shown as q is increased from 0 to 1 at $\theta_2^l = 0.48$. The values of q at which the system transitions across each of the equilibrium regions are indicated by dotted vertical lines. Transition points: FP to PD at $\bar{q} = 0.20$, PD to LC at $\bar{q} = 0.41$ and LC to CC at $\hat{q} = 0.79$. Parameter Configuration used: $\theta_2^l = 0.48$, $\theta_2^h = 1$, $\theta_1 = 0.02$, $y_2^l = 15$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$ and $\mathcal{B} = 0.15$.



6. Welfare Analysis

In this section, we examine the welfare implications of bankruptcy exemption for a given $\theta_2^l \in (\theta_2^{min}, \theta_2^{max})$. We evaluate the economic surplus created due to lending at Date 0 and lending at Date 1 as a function of q . We show that surplus due to Date 0 lending surplus is weakly increasing in bankruptcy exemption; while, surplus due to Date 1 lending is weakly decreasing in bankruptcy exemption. Thus, from an overall ex-ante perspective, bankruptcy exemption may create a trade-off between surplus created due to Date 0 lending and Date 1 lending, and bankruptcy exemption can be set at an optimal tradeoff. We begin the analysis with surplus creation due to Date 1 lending.

6.1. Surplus Creation Due to Date 1 Lending

The Date 1 surplus, conditional on θ_2 (θ_2^h or θ_2^l) depends on q through the number of real asset loans supplied ($\bar{\beta}(q; \theta_2)$) and the surplus created per real asset loan ($S_r(q; \theta_2)$), which is given by expected payoff of the real asset created at Date 1, net of pecuniary equivalent of effort (e) expended by households. More specifically, in the high state, $S_r(q; \theta_2^h) = e^*(\theta_2^h)R = R$ as there is no effort aversion. In the low state $S_r(q; \theta_2^l) = e^*(\theta_2^l)R - \frac{1}{2}\gamma[e^*(\theta_2^l)]^2$, where effort, $e^*(\theta_2^l) = \frac{1}{\gamma}[R - f_r(\theta_2^l)]$, is endogenously determined because the equilibrium face value (f_r) depends on q . Using these results for the high state (θ_2^h) and the low state (θ_2^l), the expected Date 1 surplus is

$$S_{D1}(q) = r\mathcal{B}R + (1 - r)\bar{\beta}(q; \theta_2^l)S_r(q; \theta_2^l). \quad (26)$$

In the high state (θ_2^h), the face value is equal to 1 and there is no unmet demand in the real asset loan market, i.e., \mathcal{B} loans are originated. Thus, Date 1 surplus created in the high state is equal to $\mathcal{B}R$, which is independent of q , and the high state occurs with probability r , giving the first term. The second term in Equation (26) reflects the Date 1 surplus, conditional on the low state (θ_2^l), after factoring in the probability of the low state ($1 - r$). This term depends on q through the aggregate loan amount $\bar{\beta}(q; \theta_2^l)$ as well as the surplus created per unit loan $S_r(q; \theta_2^l)$. Furthermore, the dependence on q varies across different types of equilibrium that may arise in the low state.

We rely on the comparative statics (Footnote 19) to show that, for a given θ_2^l and θ_2^h , S_{D1} is invariant to q in the Fair Pricing Equilibrium and Credit Crunch Equilibrium regions but strictly decreasing in q in the Price Discrimination Equilibrium and the Liquidity Crunch Equilibrium regions. The relationship of S_{D1} with q can thus be summarized as weakly decreasing. The first set of rows in Table 2 provides specific insights for understanding this relation across all the different types of equilibrium. In essence, fire-sale “price” effects, which affect f_r , and fire-sale “quantity”

effects, which affect $\bar{\beta}$, cause S_{D1} to be (weakly) decreasing in q . Interestingly, an important implication arising from this result is that the expected Date 1 surplus is *never* increasing in q .

6.2. Surplus Creation Due to Date 0 Lending

The expected surplus created by Date 0 lending (S_{D0}) is calculated as follows. Recall, a financial intermediary creates an asset with an expected payoff of $E_{\theta_2}[\theta_2 y_2]$ by investing an amount s ; thus, the surplus created is the NPV of the financial asset, i.e., $E_{\theta_2}[\theta_2 y_2 - s]$. Then, the expression for S_{D0} is given by aggregating the expected surplus across all intermediaries that have NPV positive projects at Date 0 (i.e., those intermediaries that have investment shortfall, \hat{s} , less than $s_{max} = E_{\theta_2}(\theta_2 y_2)$). Therefore, the expected Date 0 surplus is

$$\begin{aligned} S_{D0}(q) &= \int_{s_{min}}^{\hat{s}} E_{\theta_2}[\theta_2 y_2 - s] dH(s) \\ &= \hat{s} - s_{min} - \frac{1}{2} \frac{(\hat{s} - s_{min})^2}{(s_{max} - s_{min})}. \end{aligned} \quad (27)$$

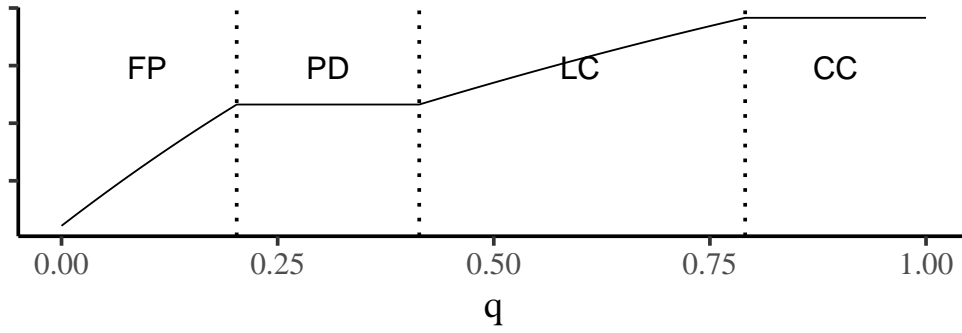
$S_{D0}(q)$ simplifies to Equation (27). It can be shown that $S_{D0}(q)$ is increasing in \hat{s} . Furthermore, since \hat{s} is increasing in \bar{p} , it follows that S_{D0} is increasing in \bar{p} . Thus, the relation between S_{D0} and q depends on the relation between \bar{p} and q .

As discussed earlier, the expected financial asset price (\bar{p}) could be increasing or invariant in q depending on the type of equilibrium. In the Fair Pricing Equilibrium and the Liquidity Crunch Equilibrium regions, \bar{p} is increasing in q , but in the Price Discrimination Equilibrium and the Credit Crunch Equilibrium regions, \bar{p} is invariant in q . The second set of rows in Table 2 provides specific insights for understanding this relation across the different types of equilibrium.

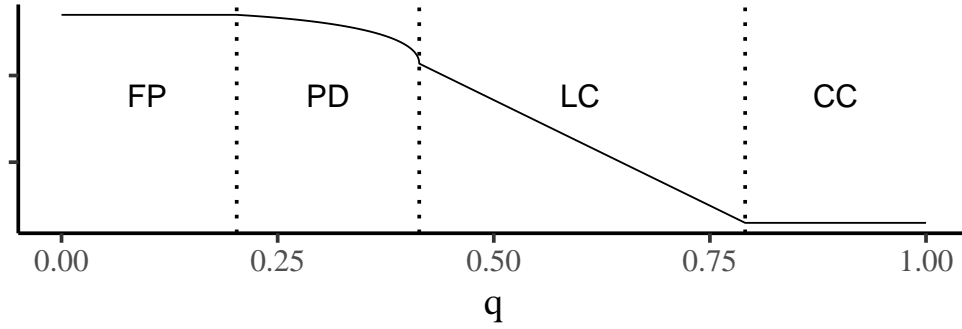
Figure (6) illustrates the evolution of the expected Date 0 surplus (S_{D0}), the expected Date 1 surplus (S_{D1}), and the expected total surplus generated in the economy (S_{Total}), as a function of the bankruptcy exemption parameter (q), conditional on a strong Date 1 shock ($\theta_2^l = 0.48$), as indicated by the marker in Figure (4). We see that the system transitions from the Fair Pricing to the Price Discrimination to the Liquidity Crunch and finally to the Credit Crunch equilibrium regions as q increases. In the Fair Pricing equilibrium, Date 0 surplus (S_{D0}) increases with q while Date 1 surplus (S_{D1}) is invariant in q causing the total surplus (S_{Total}) to increase in q . However, when the system transitions to the Price Discrimination equilibrium at $q = 0.20$, both S_{D0} and S_{D1} decrease with q causing S_{Total} to decrease as well. As q is further increased the system transitions into the Liquidity Crunch equilibrium at $q = 0.41$. While S_{D0} increases with q here, this increase is swamped by the reduction in S_{D1} , leading to an overall reduction in S_{Total} with q in the Liquidity

Figure 6: Ex-ante Equilibrium Total Surplus Evolution. Panel A shows the evolution of the expected Date 0 surplus (S_{D0}), Panel B shows the evolution of the expected Date 1 surplus (S_{D1}) and Panel C shows the evolution of the expected total surplus generated in the economy (S_{Total}), as a function of the bankruptcy exemption parameter (q) for a strong Date 1 shock ($\theta_2^l = 0.48$). As q increases, the system transitions from Fair Pricing (FP) equilibrium to Price Discrimination (PD) equilibrium at $q = 0.20$, then from PD equilibrium to Liquidity Crunch (LC) equilibrium at $q = 0.41$ and finally from LC equilibrium to Credit Crunch (CC) equilibrium at $q = 0.79$. The dotted lines represent the boundaries between the equilibrium regions. The dynamics are obtained for the same parameter configuration for which the demarcation of the feasible $q - \theta_2^l$ space is shown in Figure 4 (i.e., $\theta_2^l = 0.48$, $\theta_1 = 0.02$, $\theta_2^h = 1$, $y_2^l = 15$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$ and $\mathcal{B} = 0.15$.)

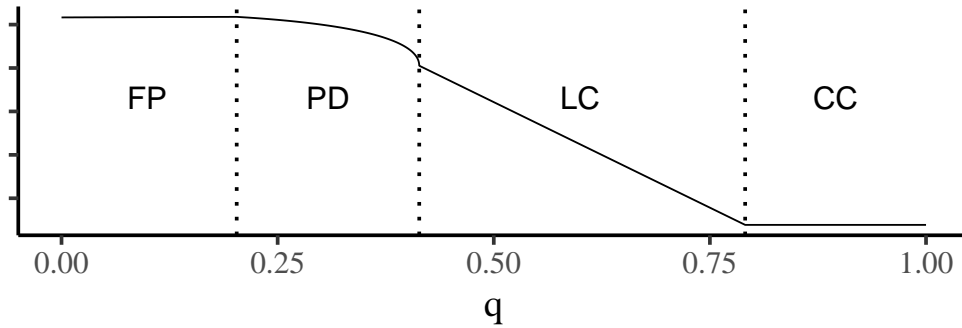
Panel A : S_{D0} vs. q



Panel B : S_{D1} vs. q



Panel C : S_{Total} vs. q



Crunch Equilibrium. Finally, the system transitions to the Credit Crunch Equilibrium at $q = 0.79$, the real asset market shuts down, i.e., S_{D1} is again invariant in q , but S_{D0} decreases with q in this region. Consequently, S_{Total} is decreasing in the Credit Crunch Equilibrium, as well. Therefore, as can be seen Panel C, expected total surplus (S_{Total}) is maximized at the boundary of the Fair Pricing and Price Discrimination equilibrium regions ($q = 0.20$).

6.3. Total Surplus Creation

We can now assess the optimal choice of the bankruptcy exemption parameter (q) by maximizing the sum of the expected surplus created at Date 0 and the expected surplus created at Date 1, i.e., the expected total surplus $S_{Total} = S_{D0} + S_{D1}$. Given that S_{D0} is (weakly) increasing in q and that S_{D1} is (weakly) decreasing in q , it seems reasonable to expect that there is an optimal q that maximizes the S_{Total} .

Fair Pricing Equilibrium	Price Discrimination Equilibrium	Liquidity Crunch Equilibrium	Credit Crunch Equilibrium
$\bar{\beta} \leftrightarrow \text{with } q$ $f_r \leftrightarrow \text{with } q$ $\Rightarrow S_{D1} \leftrightarrow \text{with } q$	$\bar{\beta} \leftrightarrow \text{with } q$ $f_r \uparrow \text{ with } q$ $\Rightarrow S_{D1} \downarrow \text{ with } q$	$\bar{\beta} \downarrow \text{ with } q$ $f_r \leftrightarrow \text{ with } q$ $\Rightarrow S_{D1} \downarrow \text{ with } q$	$\bar{\beta} \leftrightarrow \text{ with } q$ $f_r \leftrightarrow \text{ with } q$ $\Rightarrow S_{D1} \leftrightarrow \text{ with } q$
$\bar{p} \uparrow \text{ with } q$ $\hat{s} \uparrow \text{ with } q$ $\Rightarrow S_{D0} \uparrow \text{ with } q$	$\bar{p} \leftrightarrow \text{ with } q$ $\hat{s} \leftrightarrow \text{ with } q$ $\Rightarrow S_{D0} \leftrightarrow \text{ with } q$	$\bar{p} \uparrow \text{ with } q$ $\hat{s} \uparrow \text{ with } q$ $\Rightarrow S_{D0} \uparrow \text{ with } q$	$\bar{p} \leftrightarrow \text{ with } q$ $\hat{s} \leftrightarrow \text{ with } q$ $\Rightarrow S_{D0} \leftrightarrow \text{ with } q$
$S_{Total} \uparrow \text{ with } q$	$S_{Total} \downarrow \text{ with } q$	$S_{Total} \downarrow \uparrow \text{ with } q$	$S_{Total} \leftrightarrow \text{ with } q$

Table 2: **Equilibrium Characteristics in each Low State Equilibrium Region.** Behavior of the price of the financial asset (p), the expected Date 1 payoff to financiers from the financial asset (\bar{p}), the equilibrium face value of real asset loans (f_r) and the number of real asset loans ($\bar{\beta}$), as a function of the bankruptcy exemption parameter (q) in each of the low state Equilibrium Regions.

Table (2) summarizes the welfare trade-offs under each equilibrium type in the low state, conditional on a given value of (θ_2^l) . At low q , the system is in the Fair Pricing Equilibrium region, as shown in the first column of Table (2). As q increases the system transitions into the fire-sale regions, as shown in the second, third, and fourth columns of Table (2).

As elaborated in Table 2 (bottom row), it is only in the Liquidity Crunch Equilibrium that there exists a trade-off between Date 0 surplus and Date 1 surplus. We show that under a reasonable condition (to be discussed shortly), S_{Total} is decreasing with q in the Liquidity Crunch Equilibrium Region as well. Furthermore, since the expected total surplus (S_{Total}) is invariant to q in the Credit

Crunch Equilibrium region, it follows that the optimal q is always at the boundary of the curve demarcating the Fair Pricing Equilibrium region and the Fire Sale Equilibrium regions (i.e., $q^{opt} = \bar{q}$, see Figure 4).

PROPOSITION 3: *For financial and real assets that satisfy $E_{\theta_2}[\rho^*(\theta_2)] \geq \theta_2^l y_2^l$, the optimal q (q^{opt}) that maximizes total surplus (S_{Total}) is at the border of the Fair Pricing Equilibrium region and the Fire Sale Equilibrium region.*

$$q^{opt} = \frac{-r(\theta_2^h y_2^h - \rho^*) + \sqrt{[r(\theta_2^h y_2^h - \rho^*)]^2 + (1 - 2r)[(\rho^* - s_{min})^2 - 2\mathcal{B}(s_{max} - s_{min})]}}{(1 - 2r)k_1} \quad (28)$$

The intuition behind this finding can be stated as follows. A marginal increase in q results in incremental lending at Date 0; these additional loans are made to those intermediaries who face high investment shortfalls. Two implications follow: (i) the NPV of the assets originated by these intermediaries is necessarily low because of the high investment requirements, and (ii) these intermediaries are also the most leveraged intermediaries because of the large investment requirements that they have to finance with repo financing. As a consequence, Date 0 lending, *at the margin*, results in low NPV asset origination by highly leveraged intermediaries, who will face adverse fire-sale effects at Date 1 when an economic shock occurs. Thus, the loss in Date 1 surplus dominates the low NPV gain from *incremental* assets created at Date 0, provided the condition on asset payoffs in Proposition 3 holds.

The condition on asset payoffs in Proposition 3 simply states that the ex-ante expected funding liquidity should be at least as high as the ex-post payoffs in the adverse state of the economy. Violation of this condition implies that repo-financing would be unattractive for highly leveraged intermediaries. If the ex-ante expectation of funding liquidity is too low, highly leveraged intermediaries realize that they would be unable to roll over their loans at Date 1; this deters all these intermediaries from participating in the economy, and the overall leverage in the economy would be low. As a consequence fire-sale effects are small in terms of economic magnitude, and it might thus be optimal to increase q beyond the border of the Fair Pricing and Fire-sale region to improve social welfare by adding positive NPV projects at Date 0. Appendix (B11) lays out details of the optimal q in this situation where the condition in Proposition 3 is violated.

In numerical analysis of the model, we observe that feasible parameter spaces that violate the condition stated in Proposition (3) rarely occur. This assumption, which also helps in model tractability, is employed for the remainder of the paper.

To summarize, in the fire-sale equilibrium regions, an increase in q increases the expected Date

0 surplus, but it also inhibits the ability of surplus-liquidity intermediaries from servicing the Date 1 real asset market, i.e., an increase in q causes financial instability in the form of an increase in interest rates, or a shrinking (and at worst, a collapsing) real asset market, resulting in a decrease in the expected Date 1 surplus. In other words, our results demonstrate that providing bankruptcy exemption in repo markets (i.e., setting $q = 1$) while creating “too much today” may also provide “too little tomorrow”. There is a trade-off between these two effects that determines the socially optimal bankruptcy exemption parameter (q^{opt}).

We can derive the intuitive relationship of the level of the optimal bankruptcy exemption parameter (q) to three key parameters of the model.

PROPOSITION 4: *The optimal bankruptcy exemption (q^{opt}) is decreasing in the severity of the economic shock (θ_2^l), collateral quality (k_1), and size of the real economy (\mathcal{B}).*

The implication is that bankruptcy exemption is costlier during adverse economic times and when the demand for the real sector is large. Thus, the socially optimal choice could be to provide an automatic stay. The proposition also points out that full bankruptcy exemption is sub-optimal when collateral quality is low, but can be optimal when the quality of collateral is good. Consistent with these arguments, the [Federal Reserve Report \[2011\]](#) presented in the aftermath of the global financial crisis of 2008, and also [Edwards and Morrison \[2005\]](#), [Jackson \[2009\]](#), [Skeel and Jackson \[2011\]](#), and [Duffie and Skeel \[2012\]](#) point out that full repeal of the safe harbor provisions is not desirable. These authors argue that bankruptcy exemption should be continued for Qualified Financial Contracts (QFCs) in which collateral is in the form of cash or cash-equivalent assets but should be removed for QFCs with less liquid assets.

7. Capital Requirements and Optimal Bankruptcy Exemption

Finally we explore the role of capital requirement in our model in the presence of bankruptcy exemption. Intuitively, one would expect that imposing capital requirements may further constrain leverage in the economy and thereby reduce the ex-post adverse effects of excess liquidation by over-leveraged firms. On the other hand, capital requirements would also cause an ex-ante contraction in the financial sector. There could be a tradeoff between these two effects and our model allows us to evaluate this tradeoff.

We model capital requirements as the maximum shortfall (s) that could be financed by a financial

firm.²¹ We refer to this maximum amount as \bar{s} . Recall that intermediaries with shortfall greater than \hat{s} are not financed in equilibrium. Therefore, the only relevant case is if $\bar{s} < \hat{s}$. Our analysis in the appendix establishes that imposing external capital constraints beyond that what is imposed by the equilibrium truncation (\hat{s}) is never optimal. This result is not surprising because \hat{s} internalizes the fire-sale effects of risk-shifting and so long as capital constraints are imposed to eliminate the problem of excessive risk-taking by financial firms, this objective is fully attained through \hat{s} .

PROPOSITION 5: *A social planner aiming to maximize total surplus by imposing external capital constraints can never improve upon the total surplus achieved by setting the bankruptcy exemption parameter at the border of the Fair Pricing region and the Price Discrimination region.*

Proposition (5) implies that optimizing on the bankruptcy exemption parameter in our model never compromises on the total surplus that can be achieved by imposing external capital constraints. This is a useful result in that capital constraints are prone to leakages and the system can be gamed by individual financial firms which can indulge in masking the extent of their leverage. On the other hand, the bankruptcy exemption parameter is a macro-level constraint that is uniformly imposed across all intermediaries and is thus shielded from manipulation.

8. Conclusion

We examine the role of bankruptcy exemption for short-term financing such as “repo” in determining the extent of leverage in the economy, and thereby its consequent impact on financial stability. While bankruptcy exemption is usually seen as facilitating financial sector growth in the hope of priming real sector growth, our model highlights that such a prescription must be viewed with caution. We show that bankruptcy exemption creates upfront leverage-inducing growth, which can cause financial instability in future in the form of fire-sale effects for financial assets and credit crunches in the real sector. We conclude that bankruptcy exemption may require a re-think for repo collateral whose quality is highly sensitive to economic shocks.

The Treasury repo rate spikes and fire sales observed during September 2019 and March 2020 suggest that our conclusions, while derived in the context of risky underlying collateral, may carry over to relatively safe collateral such as Treasuries too. As [Barth et al. \[2021\]](#) note, some of this stress, especially in 2020, can be attributed to a liquidation of speculative positions in the cash-

²¹As the financial asset being considered in the model is same for all the firms, capital requirements that specify a specific percentage of equity to be set aside for acquiring this risky asset would translate into a restriction on the amount of borrowing (s) that can be undertaken to finance the asset.

futures basis trades held by hedge funds and the growing build-up of such positions in the first place. To the extent that bankruptcy exemption in repo markets encourages leverage in these speculative positions, without (at least direct) attendant real benefits, there might be a possible case for revisiting safe harbor provisions in Treasury (and Agency) repo markets as well.

Finally, while our work endogenizes the impact of bankruptcy exemption on leverage in the economy, an interesting research issue would be to consider the role of central bank as a lender of last resort in averting a financial crisis. Expectations about central bank interventions may influence ex-ante leveraging behavior; in particular, while the lender of last resort might be able to diminish the ex-post fire-sale induced spillovers to the real economy, its expectation might raise even greater ex-ante leverage in intermediaries aggravating the fire-sale problem. How such moral hazard would interact with safe harbor provisions in repo financing is a fruitful area for future inquiry.

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Appendix A: Key Results

A1. List of Symbols is provided in Table 3 with respective definitions.

Symbol	Definition	Expansion / Reference
ρ^*	Funding liquidity of the asset	$\theta_2 \frac{\theta_2 y_2 - \theta_1 y_1}{\theta_2 - \theta_1}$
k_1	Funding illiquidity of the asset	$\frac{\theta_2 \theta_1 (y_1 - y_2)}{\theta_2 - \theta_1}$
κ	Simplifying symbol such that $\frac{dk_1}{d\theta_2} = -\kappa$	$\frac{\theta_1^2 (y_1 - y_2)}{\theta_2 - \theta_1}$
ι	Simplifying symbol such that $\frac{dk_1}{dy_2} = -\theta_1 \iota$	$\frac{\theta_2}{\theta_2 - \theta_1}$
λ	Value of $p - \rho^*$ in the LC equilibrium	$\frac{4\gamma k_1}{R^2}$
ω	Ratio of λ to k_1	$\frac{R^2}{4\gamma}$
ϕ	Surplus liquidity of the least leveraged firm	$\rho^* - s_{min}$
π	Intermediate term used for simplicity	$r(\theta_2^h y_2^h - p)$
m	Probability of states with non-zero payoff to creditors	$r + (1 - r)q$
s_{max}	Maximum shortfall at which asset is NPV positive	$r\theta_2^h y_2^h + (1 - r)\theta_2^l y_2^l$
\hat{s}	Maximum shortfall that is financed	$\pi + \rho^* + m(p - \rho^*)$
Δs_{max}	Diff. between max. & min. shortfalls for positive NPV projects ($s_{max} - s_{min}$)	$r\theta_2^h y_2^h + (1 - r)\theta_2^l y_2^l - s_{min}$
$\Delta \hat{s}$	Diff. between max. & min. shortfalls for projects that are financed ($\hat{s} - s_{min}$)	$\pi + \phi + m(p - \rho^*)$
\bar{q}	Value of q when the system transitions from FP to PD equilibrium	See Eq. (28)
\bar{q}	Value of q when the system transitions from PD to LC equilibrium	See Eq. (28)
\hat{q}	Value of q when the system transitions from LC to CC equilibrium	See Eq. (28)

Table 3: List of Symbols.

A2. Proof of Lemma (1)

Differentiating k_1 with respect to θ_2 and with respect to y_2 , we get:

$$\frac{dk_1}{d\theta_2} = -\frac{\theta_1^2 (y_1 - y_2)}{\theta_2 - \theta_1} < 0 \quad (A1)$$

$$\frac{dk_1}{dy_2} = -\frac{\theta_2 \theta_1}{\theta_2 - \theta_1} < 0 \quad (A2)$$

Denoting $\kappa = \frac{\theta_1^2 (y_1 - y_2)}{\theta_2 - \theta_1} > 0$ and $\iota = \frac{\theta_2}{\theta_2 - \theta_1} > 0$, we obtain $\frac{d\rho^*}{d\theta_2} = y_2 + \kappa > 0$ and $\frac{d\rho^*}{dy_2} = \theta_2 \iota > 0$.

A3. Proof of Lemma (2)

Given the result in (1), it follows that optimal effort is decreasing in f_r . The expected profits of the lender ef_r is equal to $\frac{1}{\gamma}(R - f_r)f_r$ is quadratic in f_r with a negative coefficient on $(f_r)^2$, implying a concave relationship. The first order condition yields $\frac{1}{\gamma}(R - f_r - f_r) = 0$, i.e., $f_r = \frac{R}{2}$, i.e, the expected profit function is maximized at $f_r = \frac{R}{2}$.

A4. *Proof of Lemma (3)*

When the leverage of a borrower is equal to $\bar{\rho}$, the borrower is indifferent between liquidating δ fraction of the asset to roll over the debt and exercising strategic default. Therefore, we have:

$$\delta p + (1 - \delta)\theta_2 y_2 - \bar{\rho} = (1 - q)(\theta_2 y_2 - \rho^*)$$

Noting that $\delta(\bar{\rho}, p) = \frac{\bar{\rho} - \rho^*}{p - \rho^*}$ and $\theta_2 y_2 - \rho^* = k_1$, we obtain:

$$\begin{aligned} & (\bar{\rho} - \rho^*)p + (p - \bar{\rho})(\rho^* + k_1) - (p - \rho^*)\bar{\rho} = (1 - q)(k_1)(p - \rho^*) \\ \Rightarrow & (p - \bar{\rho})k_1 = (1 - q)(k_1)(p - \rho^*) \\ \Rightarrow & \bar{\rho} = \rho^* + q(p - \rho^*) \quad \text{as } k_1 > 0 \end{aligned} \tag{A3}$$

A5. *Equilibrium Restrictions on face value (f_r), effort aversion parameter (γ) and price p*

Some basic restrictions on the loan face value (f_r), effort aversion parameter (γ) and the financial asset price (p) must be satisfied in equilibrium:

- (i) For non-trivial effort choice, we require $e^* > 0$, i.e., $\frac{1}{\gamma}(R - f_r) > 0$, i.e., $f_r < R$.
- (ii) We require $f_r \leq f_r^m = \frac{R}{2}$, where f_r^m denotes the surplus-liquidity intermediary's profit-maximizing face value. Note that f_r^m can be solved as $\text{argmax}_{f_r} e f_r$ s.t. $e = \frac{1}{\gamma}(R - f_r)$; it follows that $f_r^m = \frac{R}{2}$. Since expected profits are concave in f_r , lenders have no incentive to post a higher face value than f_r^m .
- (iii) $e f_r \geq 1$, otherwise there is no investment in real sector, i.e., $e f_r = \frac{1}{\gamma}(R - f_r)f_r \geq 1$.
- (iv) $\frac{R}{2} - \sqrt{R^2 - 4\gamma} \leq f_r \leq \frac{R}{2}$. The additional restrictions on γ can be derived as follows. Under fair pricing of household loans (i.e., when $e f_r = 1$), the face value f_r is equal to $\frac{R}{2} - \frac{\sqrt{(R^2 - 4\gamma)}}{2}$, which is the lower root of the quadratic equation in f_r . To ensure that $e \leq 1$, we require $f_r \geq 1$, i.e., we require $(R - 2)^2 \geq (R^2 - 4\gamma)$ which implies $\gamma \geq R - 1$. Furthermore, we also require $\gamma \leq \frac{R^2}{4}$; a greater value of γ would result in an imaginary solution for f_r . Combining these restrictions, we require $R - 1 \leq \gamma \leq \frac{R^2}{4}$.
- (v) Combining all the above constraints, we get: $\frac{R}{2} - \sqrt{R^2 - 4\gamma} \leq f_r \leq \frac{R}{2}$.
- (vi) The financial asset price (p) must lie in the interval $(\rho^*, \theta_2 y_2)$.

The last restriction on the price of the financial asset (p) follows because (i) it cannot exceed the expected payoffs on the asset ($\theta_2 y_2$) and (ii) it must be strictly higher than the funding liquidity (ρ^*), otherwise the demand for the asset would be infinite.

A6. Proof of Lemma (4)

Using the results in Lemma (1), namely, $\theta_2 y_2 - \rho^* = k_1$, and Lemma (2), namely, $e = \frac{1}{\gamma}(R - f_r)$, we can re-formulate the optimization problem in (4) - (5) as a Lagrangian optimization problem with μ , η , and ν as Lagrangian parameters. μ is the Lagrangian parameter for the budget constraint, whereas η and ν are the the Lagrangian parameters employed for the non-negativity constraints, $\alpha \geq 0$ and $\beta \geq 0$, respectively.

$$\max_{\alpha > 0, \beta \geq 0} (1 + \alpha)k_1 + \beta e f_r - \mu [\alpha(p - \rho^*) + \beta - (\rho^* - \rho)] - \eta\alpha - \nu\beta \quad (\text{A4})$$

The solution depends on the following first order condition for α , β , μ , η , and ν , respectively.

$$k_1 - \mu(p - \rho^*) - \eta = 0 \quad (\text{A5})$$

$$e f_r - \mu - \nu = 0 \quad (\text{A6})$$

$$\alpha(p - \rho^*) + \beta = (\rho^* - \rho) \quad (\text{A7})$$

$$\alpha = 0 \quad (\text{A8})$$

$$\beta = 0 \quad (\text{A9})$$

Since the secondary market for legacy financial assets must necessarily clear, we impose the condition that $\alpha > 0$, which implies that the Lagrangian parameter $\eta = 0$. It follows from Equation (A5) that

$$\mu = \frac{k_1}{p - \rho^*}. \quad (\text{A10})$$

The real asset market is a primary market and we must account for the possibility of the market being closed ($\beta = 0$) and the market being open ($\beta > 0$); these cases correspond to the Lagrangian parameter, ν , being strictly greater than or equal to 0, respectively. From Equation (A6), we get $\nu = e f_r + \mu$. Thus, after incorporating the result in Equation (A10), we can conclude that when $\nu = 0$,

$$\frac{k_1}{p - \rho^*} = e f_r, \quad (\text{A11})$$

and when $\nu > 0$, we get

$$\frac{k_1}{p - \rho^*} > e f_r. \quad (\text{A12})$$

Note that $\mu > 0$ holds because the budget constraint in (5) is always binding due to non-satiation, i.e., surplus-liquidity intermediaries will always have incentive to deploy their spare liquidity fully in either of the two markets).

A7. Proof of Lemma (5):

We start with the aggregate budget constraint, which equates aggregate supply and demand as shown in Equation (11), restated below:

$$q \int_{\bar{\rho}}^{\rho_{\max}} g(\rho) d\rho + \bar{\beta} \frac{1}{p - \rho^*} = \int_{\rho_{\min}}^{\bar{\rho}} \frac{\rho^* - \rho}{p - \rho^*} g(\rho) d\rho \quad (\text{A13})$$

Integrating the RHS by parts while noting that $G(\rho_{\min}) = 0$, we obtain:

$$q(p - \rho^*) [G(\rho_{\max}) - G(\bar{\rho})] + \bar{\beta} = (\rho^* - \bar{\rho})G(\bar{\rho}) - \int_{\rho_{\min}}^{\bar{\rho}} (-1)G(\rho) d\rho \quad (\text{A14})$$

Substituting for $\bar{\rho}$ from Lemma (3) and rearranging, we obtain:

$$\bar{\beta} = -q(p - \rho^*)G(\rho_{\max}) + \int_{\rho_{\min}}^{\bar{\rho}} G(\rho) d\rho \quad (\text{A15})$$

A8. Proof of Proposition (1)

For parsimony, we characterize the equilibrium in terms of the triplet $(p, \bar{\beta}, f_r)$. In the Fair Pricing (FP) Equilibrium both financial and real assets are fairly priced, i.e., price of an asset is equal to the expected payoff from the asset ($p = E(y_2)$ and $ef_r = 1$) and expected return on investment for surplus-liquidity is 0. This outcome results when the supply of liquidity exceeds the demand for liquidity leading to the satiation of the real asset market even when the price of the financial asset (p) is at its highest possible value of $\theta_2^l y_2$. Consequently, in the FP equilibrium, we have:

$$p = E(y_2) = \theta_2 y_2 \quad (\text{A16})$$

$$ef_r = 1 \Rightarrow \frac{1}{\gamma}(R - f_r)f_r = 1 \Rightarrow f_r = \frac{R}{2} - \frac{\sqrt{R^2 - 4\gamma}}{2} \quad (\text{A17})$$

$$\bar{\beta} = \mathcal{B} \quad (\text{A18})$$

A9. Proof of Proposition (2):

A9.1. Real Asset Price Discrimination Equilibrium (PD)

Conditional on a given θ_2 , the system transitions from the Fair Pricing Equilibrium Region to the Fire Sale Equilibrium Region as q increases and there is too much liquidation of assets at Date 1. In

this situation, the market clearing price (p) falls below the fair value ($\theta_2 y_2$). The real asset market continues to remain fully satiated ($\bar{\beta} = \mathcal{B}$), as in the Fair Pricing region. The price of the financial asset is obtained by substituting for $\bar{\beta} = \mathcal{B}$ in Equation (A15).

The cross-market equilibrium return condition implies that the face value of the real asset loan (f_r) increases to ensure that the returns on both assets are equal. Equation (8) reflects the cross-market equilibrium return condition, yielding:

$$\bar{\beta} > 0 \implies \frac{k_1}{p - \rho^*} = e f_r > \mu > 0. \quad (\text{A19})$$

(A19) can be simplified into a quadratic equation in f_r , after recognizing that $e^* = \frac{1}{\gamma}(R - f_r)$ and $\rho^* = \theta_2 y_2 - k_1$. Note that, in equilibrium, the larger root greater than $\frac{R}{2}$ can be ignored due to constraints expressed in Section (A5), yielding:²²

$$f_r = \frac{R}{2} - \frac{1}{2} \sqrt{R^2 - \frac{4\gamma k_1}{p - \rho^*}} \quad (\text{A20})$$

A9.2. Real Asset Liquidity Crunch Equilibrium (LC)

As q increases in the Price Discrimination region, the face value (f_r) increases in equilibrium (a result that will be shown further down). The maximum value of f_r is equal to $\frac{R}{2}$, as discussed in Section (A5). If the demand for liquidity exceeds supply when f_r is at its highest possible value of $R/2$, supply-demand equilibrium is achieved through the rationing of the real asset market with the aggregate number of real asset loans extended ($\bar{\beta}$) falling below \mathcal{B} . In the LC equilibrium, $f_r = R/2$, $\bar{\beta}$ is given by Equation (A15). p can be obtained as follows from the cross-market equilibrium return condition in Equation (A19) while noting that when $f_r = R/2$, $e f_r = \frac{R^2}{4\gamma}$:

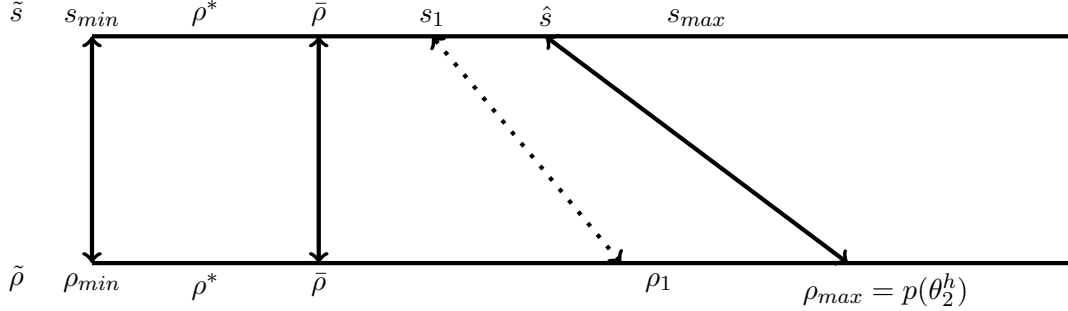
$$\begin{aligned} p - \rho^* &= \frac{k_1}{e f_r} = \frac{4\gamma k_1}{R^2} \\ \Rightarrow p &= \rho^* + \lambda \quad \text{where } \lambda = \frac{4\gamma k_1}{R^2} \end{aligned} \quad (\text{A21})$$

A9.3. Real Asset Credit Crunch Equilibrium (CC)

Note that $\bar{\beta}$ is decreasing in q in the Liquidity Crunch region (a result that will be established further down). Thus, as q increases, $\bar{\beta}$ will decrease and at a sufficiently high value of q , $\bar{\beta}$ will be equal to 0, and the system will transition to the Credit Crunch region. In this case, the equilibrium

²² $f_r^m = R/2$ is the face value of the loan at which the lender's profit is maximized when effort level of the households is endogenously determined. Consequently, it is never in the interest of lenders to charge a face value higher than f_r^m , implying $f_r < f_r^m = R/2$.

Figure 7: Derived Distribution of Debt. The figure below shows a pictorial representation of the mapping between support for $s(\rho)$ and the support for ρ . The full double arrow lines indicate borders around which the ρ function changes and the dotted double arrow lines are specific values of ρ and s used to derive the distribution of ρ given $s \leq \hat{s}$.



price, p , is given by the solution of Equation (12), in which $\bar{\beta}$ is set equal to 0. Furthermore, the cross-market equilibrium return condition is irrelevant. The equilibrium should satisfy (9) and (12) evaluated at $\bar{\beta} = 0$. The equilibrium triplet $(p, \bar{\beta}, f_r)$ will now be reduced to singleton, $p(0)$, because $\bar{\beta}$ and f_r are irrelevant when the real asset market is closed.

A10. Proof of Lemma (6): Derived Distribution of Debt

Figure (7) presents a pictorial representation of the mapping between support for $s(\rho)$ and the support for ρ . To obtain the derived distribution of $\hat{G}(\cdot) = G(\rho|s \leq s_{max})$, we first note that for $\rho_{min} \leq \rho \leq \bar{\rho}$, $\tilde{\rho} = \tilde{s}$ is uniform over $[\rho_{min}, \bar{\rho}]$ because \tilde{s} is uniformly distributed over $[s_{min}, \bar{\rho}]$ with $\rho_{min} = s_{min}$. Then, as shown in the adjoining figure, consider $\rho_1 \in (\bar{\rho}, \rho_{max}]$, where ρ_1 is the face value that finances an investment shortfall of s_1 , and $\rho_{max} = p(\theta_2^h)$. We obtain:

$$\begin{aligned} \hat{G}(\rho_1) &= G(\tilde{\rho} \leq \rho_1 | \tilde{s}(\rho_1) \leq s_{max}) = Prob(\tilde{s}(\rho_1) \leq s_1 | \tilde{s}(\rho_1) \leq s_{max}) \\ &= Prob(\tilde{s}(\rho_1) \leq \bar{\rho} | \tilde{s}(\rho_1) \leq s_{max}) + Prob(\bar{\rho} < \tilde{s}(\rho_1) \leq s_1 | \tilde{s}(\rho_1) \leq s_{max}) \\ &= \frac{\bar{\rho} - s_{min}}{s_{max} - s_{min}} + \frac{s_1 - \bar{\rho}}{s_{max} - s_{min}} = \frac{s_1 - s_{min}}{s_{max} - s_{min}} \end{aligned}$$

Therefore, we have $\hat{G}(\rho)$ specified as follows for $\rho_{min} \leq \rho \leq p$ (where $\bar{p} = \bar{\rho}$):

$$\hat{G}(\rho) = \frac{s(\rho) - s_{min}}{s_{max} - s_{min}} \text{ where } s(\rho) = \begin{cases} \rho, & \text{if } \rho_{min} \leq \rho \leq \bar{\rho} \\ r\rho + (1-r)\bar{p}, & \text{if } \bar{\rho} < \rho \leq p(\theta_2^h) \end{cases} \quad (\text{A22})$$

A11. Model Parameter Space Restrictions

A well defined model parameter space should satisfy the following constraints.²³

$$\theta_2^{min} = (s_{min} + k_1)/y_2^l \quad (A23)$$

$$\Rightarrow \theta_2^{min} = \frac{\theta_1 y_1 + s_{min} + \sqrt{[\theta_1 y_1 + s_{min}]^2 - 4\theta_1 y_2^l s_{min}}}{2y_2^l} \quad (A24)$$

$$(A25)$$

Equation (A23) ensures financial market clearing for any $\theta_2^l \in [\theta_2^{min}, \theta_2^{max}]$ by ensuring that the surplus liquidity in the system is non-negative (i.e., $\phi(\theta_2^{min}) \geq 0$) even for the most severe shock.

A12. Variation of expected Surplus at Date 1 (S_{D1}) with q

First, some notation to simplify the expression for $\Delta \hat{s}$: $\Delta \hat{s} = r\theta_2^h y_2^h + (1-r)\rho^* + (1-r)q(p - \rho^*) - s_{min} = r(\theta_2^h y_2^h - \rho^* - (p - \rho^*)) + (r + (1-r)q)(p - \rho^*) + \rho^* - s_{min} = \pi + m(p - \rho^*) + \phi$ where $\pi = r(r\theta_2^h y_2^h - p)$, $\phi = \rho^* - s_{min}$ and $m = r + (1-r)q$.

In Equation (26), the first term is a constant while both $\bar{\beta}$ and $S_r(\theta_2^l)$ could potentially vary with q . By noting that $e^* = \frac{1}{\gamma}(R - f_r)$, we obtain $S_r(\theta_2^l) = \frac{1}{2\gamma}(R^2 - f_r^2)$. Therefore, we have:

$$\frac{dS_{D1}}{dq} = S_r(\theta_2^l) \frac{d\bar{\beta}}{dq} + \bar{\beta} \frac{dS_r(\theta_2^l)}{dq} = S_r(\theta_2^l) \frac{d\bar{\beta}}{dq} - \bar{\beta} f_r \frac{df_r}{dq} \quad (A26)$$

As both $\bar{\beta}$ and $S_r(\theta_2^l)$ are always positive, using results from Propositions (1) & (19), we obtain:

- (i) FP equilibrium: $\frac{dS_{D1}}{dq} \Big|_{FP} = 0$; as $\frac{d\bar{\beta}}{dq} \Big|_{FP} = 0$ and $\frac{df_r}{dq} \Big|_{FP} = 0$.
- (ii) PD equilibrium: $\frac{dS_{D1}}{dq} \Big|_{PD} = -\bar{\beta} f_r \frac{df_r}{dq} \Big|_{PD} < 0$; as $\frac{d\bar{\beta}}{dq} \Big|_{PD} = 0$ and $\frac{df_r}{dq} \Big|_{PD} > 0$.
- (iii) LC equilibrium: $\frac{dS_{D1}}{dq} \Big|_{LC} = S_r(\theta_2^l) \frac{d\bar{\beta}}{dq} \Big|_{LC} < 0$; as $\frac{d\bar{\beta}}{dq} \Big|_{LC} < 0$ and $\frac{df_r}{dq} \Big|_{LC} = 0$.
- (iv) CC equilibrium: $\frac{dS_{D1}}{dq} \Big|_{CC} = 0$; as $\frac{d\bar{\beta}}{dq} \Big|_{CC} = 0$ and $\frac{df_r}{dq} \Big|_{CC} = 0$.

²³The result in Equation (A24) follows from solving the quadratic equation obtained by substituting $k_1(\theta_2^{min}) = \frac{\theta_2^{min} \theta_1 (y_1 - y_2^l)}{\theta_2^{min} - \theta_1}$ in Equation (A23). The smaller root of the quadratic can be ignored as it does not satisfy the constraint $\theta_2^{min} y_2^l > \theta_1 y_1$.

A13. Variation of expected Surplus at Date 0 (S_{D0}) with q

We differentiate Equation (27) with respect to q , to obtain:

$$\frac{dS_{D0}}{dq} = \frac{[(1-r)\theta_2^l y_2^l + r\theta_2^h y_2^h] - \hat{s} d\hat{s}}{s_{max} - s_{min}} \frac{d\hat{s}}{dq} = \frac{(1-r)[k_1 - q(p - \rho^*)]}{s_{max} - s_{min}} \frac{d\hat{s}}{dq} \quad (A27)$$

As the first term on the RHS in above expression is positive, the sign of $\frac{dS_{D0}}{dq}$ depends only on the sign of $\frac{d\hat{s}}{dq}$. Therefore, using results from Propositions (1) & (19), we obtain

- (i) FP equilibrium: $\frac{dS_{D0}}{dq} \Big|_{FP} > 0$; as $\frac{d\bar{p}}{dq} \Big|_{FP} > 0$.
- (ii) PD equilibrium: $\frac{dS_{D0}}{dq} \Big|_{PD} = 0$; as $\frac{d\bar{p}}{dq} \Big|_{PD} = 0$.
- (iii) LC equilibrium: $\frac{dS_{D0}}{dq} \Big|_{LC} > 0$; as $\frac{d\bar{p}}{dq} \Big|_{LC} > 0$.
- (iv) CC equilibrium: $\frac{dS_{D0}}{dq} \Big|_{CC} = 0$; as $\frac{d\bar{p}}{dq} \Big|_{CC} = 0$.

A14. Proof of Proposition (3): q^{opt} is on the boundary of FP and PD equilibrium

Noting that $S_{Total} = S_{D0} + S_{D1}$, using results from Sub Sections (A12) and (A13) we easily obtain that $\frac{dS_{Total}}{dq} \Big|_{FP} > 0$, $\frac{dS_{Total}}{dq} \Big|_{PD} < 0$ and $\frac{dS_{Total}}{dq} \Big|_{CC} < 0$. In the LC equilibrium, we use $p - \rho^* = \lambda$, $\frac{d\hat{s}}{dq} = (1-r)\lambda$, $\frac{d\bar{\beta}}{dq}$ given by Equation (B13), $f_r = R/2$ and $S_r(\theta_2^l) = \frac{3R^2}{8\gamma} = \frac{3k_1}{2\lambda}$, to obtain:

$$\begin{aligned} \frac{dS_{Total}}{dq} &= \frac{(1-r)[k_1 - q(p - \rho^*)]}{\Delta s_{max}} \frac{d\hat{s}}{dq} + \frac{3(1-r)k_1}{2\lambda} \frac{d\bar{\beta}}{dq} \\ &= \frac{(1-r)(k_1 - q\lambda)}{\Delta s_{max}} (1-r)\lambda - \frac{3(1-r)k_1}{2\lambda} \frac{(\pi + (m - rq)\lambda)\lambda}{\Delta s_{max}} \end{aligned}$$

Notating $\omega = \frac{4\gamma}{R^2} = \frac{\lambda}{k_1}$ and noting that $\pi = r(\theta_2^h y_2^h - p) = r(\theta_2^h y_2^h - \theta_2^l y_2^l) + r(k_1 - \lambda)$, we have:

$$\begin{aligned} \frac{dS_{Total}}{dq} &= -\frac{(1-r)\lambda}{2\Delta s_{max}} \left[\frac{3k_1\pi}{\lambda} + 3(m - rq)k_1 - 2(1-r)(k_1 - q\lambda) \right] \\ &= -\frac{(1-r)k_1}{2\Delta s_{max}} \left[3\pi + 3r\lambda + 3q\lambda - 6rq\lambda - 2\lambda + 2q\omega\lambda + 2r\lambda - 2rq\omega\lambda \right] \\ &= -\frac{(1-r)k_1}{2\Delta s_{max}} \left[(3\pi + 5r\lambda - 2\lambda) + q\lambda(3 + 2\omega - r(6 + 2\omega)) \right] \quad (A28) \end{aligned}$$

We first consider the case where $0 < r \leq \frac{3+2\omega}{6+2\omega}$. The restriction on collateral quality in Proposition (3) can be restated to obtain $r(\theta_2^h y_2^h - \rho^*) \geq k_1 \Rightarrow \pi \geq k_1 - r\lambda$. Therefore, Equation (A28) can be restated to obtain:

$$\frac{dS_{Total}}{dq} \leq -\frac{(1-r)k_1^2}{2\Delta s_{max}} \left[(3 - 2\omega(1-r)) + q\omega(3 + 2\omega - r(6 + 2\omega)) \right] < 0 \quad \forall r \leq \frac{3+2\omega}{6+2\omega} \quad (A29)$$

Next, consider the case where $\frac{3+2\omega}{6+2\omega} \leq r < 1$: In this case, $\frac{dS_{Total}}{dq}$ is increasing in q and therefore, its maximum value is attained at $q = 1$. Therefore, evaluating Equation (A28) at $q = 1$, we obtain the following condition:

$$\frac{dS_{Total}}{dq} \leq -\frac{(1-r)k_1}{2\Delta s_{max}} \left[3\pi + (1-r)(1+2\omega)\lambda \right] < 0 \quad \forall r > \frac{3+2\omega}{6+2\omega} \quad (\text{A30})$$

Combining the results from Equations (A29) and (A30), we have $\frac{dS_{Total}}{dq} < 0$ in the LC equilibrium. As $\frac{dS_{Total}}{dq}$ is strictly increasing in q the FP equilibrium, strictly decreasing in q in the PD and LC equilibria and invariant with q in the CC equilibrium, it follows that S_{Total} is maximized at the boundary between FP and PD equilibrium (i.e., $q^{opt} = \bar{q}$).

For a given set of parameters, we denote the value of q at which the system transitions from FP to PD equilibrium as \bar{q} . \bar{q} can be obtained by solving for q in Equation (18) after setting $\lambda_{PD} = p - \rho^* = k_1$ on the FP-PD boundary. Therefore, we obtain:

$$\begin{aligned} 2\mathcal{B}\Delta s_{max} + 2qk_1 \left(r\theta_2^h y_2^h + (1-r)\rho^* + (1-r)qk_1 - s_{min} \right) - \phi^2 - q^2 k_1^2 - 2q\phi k_1 &= 0 \\ \Rightarrow (1-2r)k_1^2 q^2 + 2rk_1(\theta_2^h y_2^h - \rho^*)q + [2\mathcal{B}\Delta s_{max} - \phi^2] &= 0 \end{aligned} \quad (\text{A31})$$

Solving the above quadratic for q^{opt} , we obtain:²⁴

$$q^{opt} = \frac{-r(\theta_2^h y_2^h - \rho^*) + \sqrt{[r(\theta_2^h y_2^h - \rho^*)]^2 + (1-2r)[\phi^2 - 2\mathcal{B}\Delta s_{max}]}}{(1-2r)k_1} \quad (\text{A32})$$

A15. Proof of Proposition (4)

A15.1. Impact of θ_2^l on q^{opt}

Denoting the FP-PD boundary in the $\theta_2^l - q$ space as $\bar{q}(\theta_2^l)$, we write the boundary as $\lambda_{PD}(\theta_2^l, \bar{q}(\theta_2^l)) = k_1$ and differentiate this expression with respect to θ_2^l to obtain:

$$\begin{aligned} \frac{\partial \lambda_{PD}}{\partial \theta_2^l} + \frac{\partial \lambda_{PD}}{\partial \bar{q}(\theta_2^l)} \frac{d\bar{q}(\theta_2^l)}{d\theta_2^l} &= -\kappa \\ \Rightarrow \frac{d\bar{q}(\theta_2^l)}{d\theta_2^l} &= -\frac{\kappa + \frac{\partial \lambda_{PD}}{\partial \theta_2^l}}{\frac{\partial \lambda_{PD}}{\partial \bar{q}(\theta_2^l)}} > 0 \end{aligned} \quad (\text{A33})$$

For the fraction in the RHS of Equation (A33), the numerator is positive (see Section (A2) and Equation (B25)) while the denominator is negative as $\frac{\partial \lambda_{PD}}{\partial \bar{q}} = \frac{\partial p}{\partial q} \Big|_{PD} < 0$ (see Footnote 19). Thus, the FP-PD boundary is positively sloped in the $\theta_2^l - q$ space implying that q^{opt} decreases with the severity of the economic shock.

²⁴The other root of the quadratic in Equation (A31) can be ignored as for that root we get $q^{opt} < 0$ when $r < 1/2$ and $q^{opt} > 1$ for $r > 1/2$. When $r = 1/2$, Equation (A31) is linear and $q^{opt} = \frac{\phi^2 - 2\mathcal{B}\Delta s_{max}}{(\theta_2^h y_2^h - \theta_2^l y_2^l)k_1}$.

A15.2. Impact of k_1 on q^{opt}

Denoting the PD-FP boundary in the k_1-q space as $\bar{q}(k_1)$, we write the boundary as $\lambda_{PD}(k_1, \bar{q}(k_1)) = k_1$ and differentiate this expression with respect to k_1 to obtain:

$$\begin{aligned} \frac{\partial \lambda_{PD}}{\partial k_1} + \frac{\partial \lambda_{PD}}{\partial \bar{q}(k_1)} \frac{d\bar{q}(k_1)}{dk_1} &= 1 \\ \Rightarrow \frac{d\bar{q}(k_1)}{dk_1} &= \frac{1 - \frac{\partial \lambda_{PD}}{\partial k_1}}{\frac{\partial \lambda_{PD}}{\partial \bar{q}(k_1)}} < 0 \end{aligned} \quad (\text{A34})$$

For the fraction in the RHS of Equation (A34), the numerator is positive (see Equation (B27)) and the denominator is negative (see Footnote (19)). Thus, the PD-FP boundary is negatively sloped in the $k_1 - q$ space implying that q^{opt} is decreasing in collateral quality.

A15.3. Impact of \mathcal{B} on q^{opt}

Denoting the FP-PD boundary in the $\mathcal{B}-q$ space as $\bar{q}(\mathcal{B})$, we write the boundary as $\lambda_{PD}(\mathcal{B}, \bar{q}(\mathcal{B})) = k_1$ and differentiate this expression with respect to \mathcal{B} to obtain:

$$\begin{aligned} \frac{\partial \lambda_{PD}}{\partial \mathcal{B}} + \frac{\partial \lambda_{PD}}{\partial \bar{q}(\mathcal{B})} \frac{d\bar{q}(\mathcal{B})}{d\mathcal{B}} &= 0 \\ \Rightarrow \frac{d\bar{q}(\mathcal{B})}{d\mathcal{B}} &= - \frac{\frac{\partial \lambda_{PD}}{\partial \mathcal{B}}}{\frac{\partial \lambda_{PD}}{\partial \bar{q}(\mathcal{B})}} < 0 \end{aligned} \quad (\text{A35})$$

For the fraction in the RHS of Equation (A35), both the numerator and the denominator are negative (see Equation B28 and Footnote 19)). Thus, the FP-PD boundary is negatively sloped in the $\mathcal{B} - q$ space implying that the optimal q^{opt} is decreasing in the size of the real sector.

A16. Proof of Proposition (5)

To establish Proposition (5), we first evaluate q^{opt} in the presence of binding capital requirements (i.e., $\bar{s} < \hat{s}$) in Section (A16.1), then we evaluate dynamics of q^{opt} in the $\bar{s} - q$ space in Section (A16.2), and finally identify the optimal operating point in the $\bar{s}-q$ that maximizes S_{Total} in Section (A16.2).

A16.1. q^{opt} in the presence of Binding Capital Requirements

Capital requirements are binding when $\bar{p} < \bar{s} \leq \hat{s}$.²⁵ When capital requirements are not binding (i.e., $\bar{s} > \hat{s}$), we note from Footnote (19) that \bar{p} is weakly increasing in q . Consequently, $\hat{s} = r\theta_2^h y_2^h + (1-r)\bar{p}$ is weakly increasing in q and for any given set of system parameters, $\hat{s}(q=0) \leq \hat{s}(q=1)$. Therefore, as capital requirements are imposed it will always become binding at higher values of q before it becomes binding at lower values of q . Thus, two possible cases of binding capital requirements can arise – i) Capital requirements are binding at all q (i.e., $\bar{s} < \hat{s}(q=0) \leq \hat{s}(q=1)$), and ii) Capital requirements are non-binding for $q < q_b$ and binding for $q \geq q_b$ (i.e., $\hat{s}(q=0) < \bar{s} = \hat{s}(q=q_b) \leq \hat{s}(q=1)$).²⁶

To evaluate the impact of q on S_{Total} when \bar{s} is binding, we first establish dynamics of the equilibrium regions when \bar{s} is binding. In the FP equilibrium region, $p = \theta_2^l y_2^l$, $\bar{\beta} = \mathcal{B}$ and $f_r = \frac{R}{2} [1 - \sqrt{1 - \omega}]$.²⁷

In the PD equilibrium region, $p = \rho^* + \lambda_{PD}$, $\bar{\beta} = \mathcal{B}$, and $f_r = \frac{R}{2} \left[1 - \sqrt{1 - \frac{\lambda}{\lambda_{PD}}} \right]$ where λ_{PD} is obtained by solving Equation (A36).²⁸

$$2\mathcal{B}\Delta s_{max} = -2q\lambda_{PD}\Delta\bar{s} + (\phi + q\lambda_{PD})^2 \quad (\text{A36})$$

Differentiating Equation (A36) with respect to q , we obtain:²⁹

$$\begin{aligned} & -2\Delta\bar{s} \left[\lambda_{PD} + q \frac{d\lambda_{PD}}{dq} \right] + 2(\phi + q\lambda_{PD}) \left[\lambda_{PD} + q \frac{d\lambda_{PD}}{dq} \right] = 0 \\ \Rightarrow & [\Delta\bar{s} - \phi - q\lambda_{PD}] \left[\lambda_{PD} + q \frac{d\lambda_{PD}}{dq} \right] = 0 \\ \Rightarrow & \frac{d\lambda_{PD}}{dq} = -\frac{\lambda_{PD}}{q} < 0 \quad \text{as } \bar{s} > \rho^* + q\lambda_{PD} \end{aligned} \quad (\text{A37})$$

By extension, $\frac{dp}{dq} \Big|_{PD} = \frac{d\lambda_{PD}}{dq} < 0$ and $\frac{df_r}{dq} \Big|_{PD} = -\frac{\lambda R}{4\lambda_{PD}^2} \left[1 - \frac{\lambda}{\lambda_{PD}} \right]^{-\frac{1}{2}} \frac{d\lambda_{PD}}{dq} > 0$.³⁰

²⁵ As the objective of capital requirements is to deter liquidation of assets resulting from default, the leverage level ($\rho = \bar{p}$ when $s(\rho) = \bar{p}$) beyond which default becomes viable for firms forms the lower bound for the tightest capital requirement (i.e., $\bar{s} > \bar{p}$).

²⁶ Even when capital controls are binding, \bar{p} is weakly increasing in q as we shall see in Equation (A37) and Footnote (30). Consequently, if capital controls are binding at a given value of q , they never become non-binding as q increases.

²⁷ $\omega = \frac{4\gamma}{R^2}$.

²⁸ Results in Equations (A36 and A38) and Footnote (30) are obtained by solving the fundamental demand-supply relationship for the system in Equation (12) after limiting the maximum leverage in the economy to \bar{s} to obtain $G(\rho_{max}) = G(\rho(\bar{s})) = \frac{\Delta\bar{s}}{\Delta s_{max}}$. Equation (12) gets modified to $q(p - \rho^*)\Delta\bar{s} = \Delta s_{max} \left[\int_{\rho_{min}}^{\bar{\rho}} G(\rho) d\rho - \bar{\beta} \right]$.

²⁹ The final result of Equation (A37) follows from noting that $\Delta\bar{s} - \phi - q\lambda_{PD} = \bar{s} - \bar{p} > 0$ based on the rational lower bound on \bar{s} .

³⁰ Using a similar approach, we can show that when capital requirements are binding, $\frac{d\lambda_{CC}}{dq} = -\frac{\lambda_{CC}}{q} < 0$.

In the LC equilibrium, $p = \rho^* + \lambda$, $\bar{\beta}$ is obtained from Equation (A38) and $f_r = \frac{R}{2}$.

$$2\bar{\beta}\Delta s_{max} = -2q\lambda\Delta\bar{s} + (\phi + q\lambda)^2 \quad (\text{A38})$$

Differentiating Equation (A38) with respect to q , we obtain:

$$\frac{d\bar{\beta}}{dq} = -\frac{\lambda[\Delta\bar{s} - \phi - q\lambda]}{\Delta s_{max}} < 0 \quad (\text{A39})$$

Next, we consider the two components of S_{Total} . Evaluating, S_{D0} when \bar{s} is binding, while using the notations of $\Delta\bar{s} = \bar{s} - s_{min}$ and $\Delta s_{max} = s_{max} - s_{min}$, we obtain:

$$\begin{aligned} S_{D0} &= \int_{s_{min}}^{\bar{s}} E_{\theta_2}[\theta_2 y_2 - s] dH(s) \\ &= \Delta\bar{s} - \frac{1}{2} \frac{(\Delta\bar{s})^2}{\Delta s_{max}} \end{aligned} \quad (\text{A40})$$

As S_{D0} is not a function of q when capital requirements are binding (irrespective of the equilibrium region), S_{D0} remains a constant as q varies from 0 to 1.

Now, evaluating S_{D1} , we have from Equation (26) that $S_{D1} = r\mathcal{B}R + (1-r)\bar{\beta}S_r$. As in the case when \bar{s} was not binding, in the FP equilibrium, both $\bar{\beta} = \mathcal{B}$ and $S_r = \frac{2+\omega+2\sqrt{1-\omega}}{2\omega}$ are invariant in q . Therefore, S_{D1} is invariant in q in the FP equilibrium. In the PD equilibrium, $\bar{\beta} = \mathcal{B}$ is invariant in q , however, $S_r = \frac{R^2 - f_r^2}{2\gamma}$ is a function of q and we have $\frac{dS_r}{dq} = -\frac{f_r}{\gamma} \frac{df_r}{dq} < 0$. Thus, S_{D1} is decreasing in q in the PD equilibrium. In the LC equilibrium, $S_r = \frac{3R^2}{8\gamma}$ is invariant in q , while $\bar{\beta}$ is decreasing in q . Therefore, S_{D1} is decreasing in q in the LC equilibrium. Finally, in the CC equilibrium, as $\bar{\beta} = 0$, S_{D1} is invariant in q .

Combining the above results, we observe that S_{Total} is invariant in q in the FP equilibrium and strictly decreasing in the PD and LC equilibria before it again becomes invariant in q in the CC equilibrium. Thus, when \bar{s} is binding across the entire range of q , S_{Total} is maximized in the FP equilibrium region and $q^{opt} = [0, \bar{q}]$ where \bar{q} is the value of q at which the system transitions from FP equilibrium to PD equilibrium.

On the other hand, when \bar{s} becomes binding at some internal value of $q = q_b$, two cases can occur – a) $q_b \geq \bar{q}$, or b) $q_b < \bar{q}$. We already know from Proposition (3) that when \bar{s} is not binding, $\frac{dS_{Total}}{dq} > 0$ in the FP equilibrium region and $\frac{dS_{Total}}{dq} \leq 0$ in the other three equilibrium regions. Therefore, when $q_b \geq \bar{q}$, capital requirements are non-binding in the FP equilibrium and S_{Total} is maximized at \bar{q} and strictly decreasing thereafter (till it reaches CC equilibrium at \hat{q} , after which S_{Total} is again invariant in q). Consequently, $q^{opt} = \bar{q}$ in this case. When $q_b < \bar{q}$, capital requirements are partially binding in the FP equilibrium. Therefore, S_{Total} increases with q for $q \in [0, q_b]$ and invariant in q

for $q \in (q_b, \bar{q}]$, strictly decreasing in q for $q \in (\bar{q}, \hat{q}]$ and invariant in q for $q \in (\hat{q}, 1]$. Consequently, $q^{opt} = (q_b, \bar{q}]$ in this case. Combining the above results for the two cases, we obtain a general expression for optimal value of q which maximizes S_{Total} as follows: $q^{opt} = [\min(q_b, \bar{q}), \bar{q}]$.³¹

Further, as the cases where \bar{s} is not binding or where \bar{s} is always binding can be seen as subsets of the case where \bar{s} is partially binding, we have in general $q^{opt}(\bar{s}) = [\min(q_b(\bar{s}), \bar{q}(\bar{s})), \bar{q}(\bar{s})]$.

A16.2. Variation of $\bar{q}(\bar{s})$ with \bar{s} in the $q - \bar{s}$ Space

$\bar{q}(\bar{s})$ is given by the locus of points at which $\lambda_{PD}(\bar{q}(\bar{s}), \bar{s}) = k_1$. Differentiating it with respect to \bar{s} , we get:^{32,33}

$$\begin{aligned} \frac{\partial \lambda_{PD}}{\partial \bar{s}} + \frac{\partial \lambda_{PD}}{\partial \bar{q}(\bar{s})} \frac{d\bar{q}(\bar{s})}{d\bar{s}} &= 0 \\ \frac{d\bar{q}(\bar{s})}{d\bar{s}} &= -\frac{\partial \lambda_{PD} / \partial \bar{s}}{\partial \lambda_{PD} / \partial \bar{q}} < 0 \end{aligned} \quad (A41)$$

Note that \bar{q} is a function of \bar{s} and decreasing in \bar{s} when capital controls are binding. Figure (8) displays the variation in $\bar{q}(\bar{s})$ with \bar{s} for the same parameter configuration used in Figure 4. Essentially, as capital controls are tightened (i.e., \bar{s} is reduced), leverage in the economy at Date 0 reduces and consequently, a higher level of bankruptcy exemption (q) is required for the system to go into the fire sale equilibria at Date 1.

A16.3. Optimal $q - \bar{s}$ Combination

To find the optimal combination of q and \bar{s} that maximizes surplus, we take a two-step approach. We first establish optimal level of q for a given \bar{s} and then compare S_{Total} at $q^{opt}(\bar{s})$ across \bar{s} . First, consider a level of \bar{s} such that capital requirements are not binding for the system at any q . Then, based on Proposition (3), $q^{opt} = \bar{q} \in [0, 1]$ and the system is in the FP Equilibrium Region at q^{opt} . Let \bar{s}_0 be the level of \bar{s} such that the controls are just binding at $q = \bar{q}$.

For any $\bar{s} \geq \bar{s}_0$, system dynamics at $q = \bar{q}$ are not affected by the choice of \bar{s} and $q^{opt} = \bar{q}(\bar{s}_0)$.³⁴ Consequently, maximum value of S_{Total} for any $\bar{s} \geq \bar{s}_0$ is given by $S_{Total}(\bar{s}_0, \bar{q}(\bar{s}_0))$. When \bar{s} is

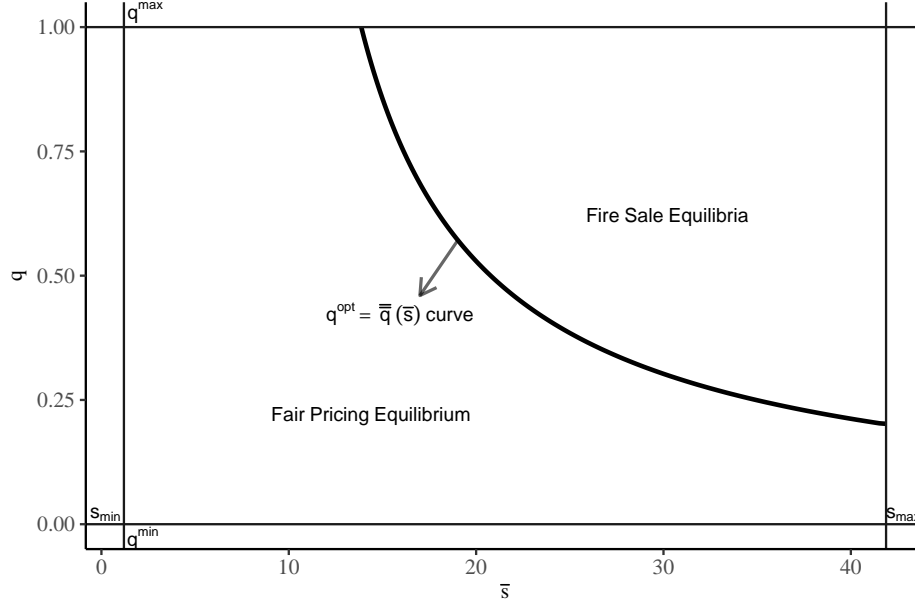
³¹Strictly speaking, $q^{opt} = [\min(\max(0, q_b), \max(0, \bar{q}), 1), \min(\max(0, \bar{q}), 1)]$ as the mathematical solutions for q_b and \bar{q} are not necessarily bound between 0 and 1. However, the simple expression for q^{opt} is sufficient if we replace any negative values by 0 and values exceeding 1 by 1.

³²The final result follows from noting that $\frac{d\lambda_{PD}}{dq} < 0$ and $\frac{d\lambda_{PD}}{d\bar{s}} < 0$.

³³Similarly, we can also show that $\frac{d\bar{q}(\bar{s})}{d\bar{s}} = -\frac{\partial \beta / \partial \bar{s}}{\partial \beta / \partial \bar{q}} < 0$ and $\frac{d\bar{q}(\bar{s})}{dq} = -\frac{\partial \lambda_{CC} / \partial \bar{s}}{\partial \lambda_{CC} / \partial \bar{q}} < 0$.

³⁴Do note that in this case, if $\bar{q} < 1$ (i.e., system transitions into the LC equilibrium at some higher q), capital controls become binding at some value of $q > \bar{q}$ at some $\bar{s} > \bar{s}_0$. However, as established in Section (A16.1), q^{opt} continues to remain at \bar{q} even when \bar{s} is partially binding.

Figure 8: q^{opt} variation with \bar{s} in the presence of capital controls. Optimal bankruptcy exemption parameter ($q^{opt} = \bar{q}(\bar{s})$) curve displayed as \bar{s} varies. Parameter configuration is the same as that used in Figure 4 (i.e. $\theta_1 = 0.02$, $\theta_2^h = 1$, $y_2^l = 15$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$ and $r = 0.6$.)



reduced from this level and capital controls are tightened, based on Equation (A41), $\bar{q}(\bar{s})$ increases. However, from the results of Section (A16.1), we know that $\bar{q}(\bar{s}) \in q^{opt}$ and $S_{Total}(\bar{s}, \bar{q}(\bar{s}))$ is the maximum value of S_{Total} for a given $\bar{s} < \bar{s}_0$. Comparing $S_{Total}(\bar{s}, \bar{q}(\bar{s}))$ with $S_{Total}(\bar{s}_0, \bar{q}(\bar{s}_0))$ when $\bar{s} < \bar{s}_0$, we have $S_{D1}(\bar{s}, \bar{q}(\bar{s})) = S_{D1}(\bar{s}_0, \bar{q}(\bar{s}_0)) = r\mathcal{B}R + (1-r)\mathcal{B}^{\frac{2+\omega+2\sqrt{1-\omega}}{2\omega}}$ as at both points the system is in the FP equilibrium. At the same time, S_{D0} is an increasing function of \bar{s} and invariant in q in the FP equilibrium. Therefore, $S_{D0}(\bar{s}, \bar{q}(\bar{s})) < S_{D0}(\bar{s}_0, \bar{q}(\bar{s}_0))$ as $\bar{s} < \bar{s}_0$ and by extension, $S_{Total}(\bar{s}, \bar{q}(\bar{s})) < S_{Total}(\bar{s}_0, \bar{q}(\bar{s}_0))$. Thus, S_{Total} is maximized at (\bar{s}^{opt}, q^{opt}) such that $\bar{s}^{opt} = \bar{s}_0$ and $q^{opt} = \bar{q}(\bar{s}_0)$.^{35,36}

³⁵Strictly speaking any $\bar{s} > \bar{s}_0$ is also equally optimal and increasing \bar{s} beyond \bar{s}_0 has no impact on q^{opt} .

³⁶There is also a corner case when $\mathcal{B} > \bar{\beta}(q = 0)$ and the system is in either LC or CC equilibria for any q . In such case $q^{opt} = 0$ where the system is in the LC equilibrium at q^{opt} . Further, $\bar{\beta}(q^{opt}) = \frac{\phi^2}{2\Delta s_{max}} < \mathcal{B}$ and the system continues to be in the LC equilibrium for any level of capital controls. Introduction of capital controls only affects the level of liquidation of assets in the economy and has no impact on the surplus liquidity in the system which is given by $\frac{\phi^2}{2\Delta s_{max}}$. At $q = 0$, as there is no liquidation, all surplus liquidity is diverted towards the real asset market and $\bar{\beta}(q = 0) = \frac{\phi^2}{2\Delta s_{max}}$ and it is invariant in \bar{s} . Thus, $q^{opt}(\bar{s}) = 0$ for any level of \bar{s} and we have $S_{D1}(\bar{s}, q^{opt}(\bar{s})) = (1+r)\mathcal{B}\frac{R}{2}$ is invariant in \bar{s} . S_{D0} is an increasing function of \bar{s} when capital controls are binding and invariant in \bar{s} when capital controls are not binding. Therefore, S_{D0} , and by extension S_{Total} , are maximized at the highest possible value of \bar{s} which is binding at q^{opt} . Let \bar{s}_1 be the level of \bar{s} at which capital controls become just binding at $q = 0$. Then we have that S_{Total} is maximized at (\bar{s}^{opt}, q^{opt}) such that $\bar{s}^{opt} = \bar{s}_1$ and $q^{opt} = 0$. Strictly speaking any $\bar{s} > \bar{s}_1$ is also equally optimal and increasing \bar{s} beyond \bar{s}_1 has no impact on q^{opt} .

Appendix B: Internet Appendix

B1. Proof of Existence and Uniqueness of Equilibrium Solution

We can rewrite Equation (A13) that describes the dynamics of the supply and demand for financial assets as follows:³⁷

$$\int_{\rho_{min}}^{\rho^*} \frac{(\rho^* - \rho)}{(p - \rho^*)} g(\rho) d\rho - \frac{\bar{\beta}(p)}{(p - \rho^*)} = \int_{\rho^*}^{\bar{p}} \frac{(\rho - \rho^*)}{(p - \rho^*)} g(\rho) d\rho + \int_{\bar{p}}^{\rho_{max}} qg(\rho) d\rho \quad (B1)$$

$$\text{where } \bar{\beta}(p) = \begin{cases} 0, & \text{if } p < \rho^* + \lambda \\ \bar{\beta}(p) \mid \bar{\beta}(p) \in [0, \mathcal{B}], & \text{if } p = \rho^* + \lambda \\ \mathcal{B}, & \text{if } p > \rho^* + \lambda \end{cases} \quad (B2)$$

The left hand side of Equation (B1) reflects the aggregate demand for financial assets from surplus-liquidity intermediaries, net of their origination of mortgage loans in the real asset market ($\bar{\beta}$). We denote this aggregate demand as $D(p)$. On the other side, the aggregate supply of financial assets by credit-constrained intermediaries in the financial asset market, denoted $S(p)$, is given by the right hand side of Equation (B1). The excess demand, $ED(p) = D(p) - S(p)$, when set equal to 0, yields the financial asset market price (p).

For $p = \rho^*$, $S(p)$ is finite, while $D(p)$ is infinite, and therefore, $ED(p)$ is positive.³⁸ At the other end, for $p > \theta_2^l y_2^l$, $D(p)$ is 0 while $S(p)$ is positive, and therefore, $ED(p)$ is negative.³⁹ Consequently, there always exists at least one solution to $ED(p) = 0$ that corresponds to a price in the range ρ^* to $\theta_2^l y_2^l$. Below, we present a concise expression for excess demand ($ED(p)$), which can also be inferred from Equation (12):

$$ED(p) = \frac{\int_{\rho_{min}}^{\bar{p}} G(\rho) d\rho - q(p - \rho^*)G(\rho_{max}) - \bar{\beta}}{(p - \rho^*)} \quad (B3)$$

If $\frac{d}{dp} [ED(p)] < 0 \forall p \in (\rho^*, \theta_2^l y_2^l)$, it would imply that the solution to $ED(p) = 0$ in the range $(\rho^*, \theta_2^l y_2^l)$ is unique. However, as the denominator of $ED(p)$ in Equation (B3) is always positive for $p \in (\rho^*, \theta_2^l y_2^l)$, it suffices to show that the numerator of $ED(p)$ in Equation (B3) is monotonically

³⁷The restrictions on $\bar{\beta}$ in Equation (B2) arise from the cross-market arbitrage conditions in Lemma (A19). A lower price than $\rho^* + \lambda$ would cause the return from investing in the financial asset market to exceed that from investing in the real asset market, resulting in a market shut down in the real asset market ($\bar{\beta} = 0$). On the other hand if the price is greater than $\rho^* + \lambda$, the return in the real asset market can match any feasible return in the financial asset market and the return in the financial asset market is decreasing in the amount of liquidity supplied to it. Therefore, surplus-liquidity intermediaries exhaust all lending opportunities in the real asset market before supplying to the financial asset market ($\bar{\beta} = \mathcal{B}$).

³⁸At $p = \rho^*$, the cost of acquiring a financial asset is 0. Therefore, even a small number of surplus liquidity firm have the potential to acquire an infinity of financial assets.

³⁹When $p > \theta_2^l y_2^l$, the return on acquiring a financial asset is negative and therefore demand for financial assets is 0.

decreasing in $p \forall p \in (\rho^*, \theta_2^l y_2^l)$ to establish that the excess demand curve intersects the x-axis only once over the interval $(\rho^*, \theta_2 y_2^l)$. We establish this result using $(\hat{G}(\rho))$, the endogenous distribution of leverage that takes into account ex-post dynamics in the economy (see Lemma (6)).⁴⁰ Differentiating the Equation (B3) with respect to p , we get

$$\begin{aligned} \frac{d}{dp} [NUM(ED(p))] &= q\hat{G}(\bar{p}) - q \left[\hat{G}(\rho_{max}) + \frac{(1-r)q(p-\rho^*)}{\Delta s_{max}} \right] - \frac{d\bar{\beta}}{dp} \\ &= -\frac{rq[\theta_2^h y_2^h - \bar{p}]}{\Delta s_{max}} - \frac{(1-r)q^2(p-\rho^*)}{\Delta s_{max}} - \frac{d\bar{\beta}}{dp} < 0 \end{aligned} \quad (B4)$$

Note that the first two terms in Equation (B4) are negative, but the sign of the third term depends on the sign of $\frac{d\bar{\beta}}{dp}$. It can be seen from Equation (B2), $\bar{\beta}$ is a step function of p . Therefore, $\frac{d\bar{\beta}}{dp}$ is 0 for all p not equal to $\rho^* + \lambda$ and is equal to the Dirac Delta function (which is positive) at $p = \rho^* + \lambda$. In short, $\frac{d\bar{\beta}}{dp} \geq 0$.

It follows that $\frac{d}{dp} [NUM(ED(p))] < 0 \forall p \in (\rho^*, \theta_2^l y_2^l)$. Hence the excess demand curve intersects the x-axis only once. This result establishes the existence and uniqueness proof.

B2. Shortfall (s) financed for a given face value (ρ)

Table (4) maps the investment shortfall ($s(\rho)$) that can be financed for a given ρ . Since the payoff potential depends on ρ , the investment shortfall that can be financed changes in specific form over different intervals of ρ , as can be seen in the different rows of Table (4), but is a piece-wise linear function of ρ .

ρ	Default States	Non-default States	Investment Shortfall That is Financed by Debt ($s(\rho)$)
$\rho_{min} \leq \rho \leq \bar{\rho}$	\emptyset	$\Omega_1, \Omega_2, \Omega_3$	ρ
$\bar{\rho} < \rho \leq p(\theta_2^h)$	Ω_2, Ω_3	Ω_1	$r\rho + (1-r)\bar{p}(\theta_2^l)$
$p(\theta_2^h) < \rho$	$\Omega_1, \Omega_2, \Omega_3$	\emptyset	$rp(\theta_2^h) + (1-r)\bar{p}(\theta_2^l)$

Table 4: **Mapping of the Face Value of Liability (ρ).** This table presents mapping between the face value of repo contract ρ and the corresponding investment shortfall, $s(\rho)$, that can be financed at that level of ρ . $s(\rho)$ is equal to the expected ex-ante payoff (at Date 0) that the financiers would receive for face value ρ .

⁴⁰The same result can be obtained when $G(\rho)$ is exogenously specified. In this case, $\frac{d}{dp} [NUM(ED(p))] = -q(G(\rho_{max}) - G(\bar{p})) - \frac{d\bar{\beta}}{dp} < 0$.

B3. Price in the Price Discrimination and Credit Crunch Equilibria

In the Price Discrimination Equilibrium, we solve Equation (B9) and substitute for λ_{PD} in $p|_{PD} = \rho^* + \lambda_{PD}$ to obtain:⁴¹

$$p|_{PD} = \rho^* + \frac{-r(\theta_2^h y_2^h - \rho^*) + \sqrt{r^2(\theta_2^h y_2^h - \rho^*)^2 + (1-2r)(\phi^2 - 2\mathcal{B}\Delta s_{max})}}{(1-2r)q} \quad (\text{B5})$$

Similarly, in the Credit Crunch Equilibrium, we solve Equation (B14) and substitute for λ_{CC} in $p|_{CC} = \rho^* + \lambda_{CC}$ to obtain:

$$p|_{CC} = \rho^* + \frac{-r(\theta_2^h y_2^h - \rho^*) + \sqrt{r^2(\theta_2^h y_2^h - \rho^*)^2 + (1-2r)\phi^2}}{(1-2r)q} \quad (\text{B6})$$

B4. Expression for $\bar{\beta}$ in the Ex Ante Equilibrium

In the ex-ante equilibrium, we use the endogenous distribution of debt obtained in Lemma (6) along with Equation (A15) to solve for $\bar{\beta}$ in the LC equilibrium. Denoting $\hat{s} - s_{min} = \Delta\hat{s}$ and $s_{max} - s_{min} = \Delta s_{max}$ and noting that $p - \rho^* = \lambda$ in the LC equilibrium, we obtain:

$$\bar{\beta} = -q\lambda \frac{\Delta\hat{s}}{\Delta s_{max}} + \int_{\rho_{min}}^{\bar{\rho}} \frac{\rho - s_{min}}{\Delta s_{max}} d\rho$$

Notating $\rho^* - s_{min} = \phi$ and $r(\theta_2^h y_2^h - p) = \pi$ and noting that $\bar{p} = \bar{\rho} = \rho^* + q\lambda$, we get:⁴²

$$\bar{\beta} = -q\lambda \frac{\Delta\hat{s}}{\Delta s_{max}} + \frac{(\phi + q\lambda)^2}{2\Delta s_{max}} \quad (\text{B7})$$

$$= \frac{(\theta_2^l y_2^l - k_1 - s_{min})^2 - q\lambda [2r(\theta_2^h y_2^h - p) + (q + 2r - 2qr)\lambda]}{2(s_{max} - s_{min})} \quad (\text{B8})$$

B5. Proof of Footnote (19):

B5.1. Real Asset Price Discrimination Equilibrium (PD)

Price in the PD region is obtained by using the endogenous distribution of debt from Lemma (6) in Equation (18) and solving for p . Using earlier notations of $\Delta s_{max} = s_{max} - s_{min}$, $\Delta\hat{s} = \hat{s} - s_{min}$, $\phi = \rho^* - s_{min}$ and denoting $p - \rho^* = \lambda_{PD}$, we obtain:

$$\mathcal{B} = -q\lambda_{PD} \frac{\Delta\hat{s}}{\Delta s_{max}} + \int_{\rho_{min}}^{\bar{\rho}} \frac{\rho - s_{min}}{\Delta s_{max}} d\rho$$

⁴¹Note that the other root of the quadratic can be ignored as for that root, $\lambda_{PD} < 0$ when $r < 1/2$ and $\lambda_{PD} > k_1$ when $r > 1/2$. When $r = 1/2$, Equation (B9) is linear in λ_{PD} and can be solved to obtain $\lambda_{PD} = \frac{\phi^2 - 2\mathcal{B}\Delta s_{max}}{(\theta_2^h y_2^h - \rho^*)q}$.

⁴² $\Delta\hat{s} = r\theta_2^h y_2^h + (1-r)\rho^* + (1-r)q(p - \rho^*) - s_{min} = r(\theta_2^h y_2^h - \rho^* - (p - \rho^*)) + (r + (1-r)q)(p - \rho^*) + \rho^* - s_{min} = \pi + m(p - \rho^*) + \phi$ where $\pi = r(\theta_2^h y_2^h - p)$, $\phi = \rho^* - s_{min}$ and $m = r + (1-r)q$. This general result is valid across equilibrium regions. In the PD, LC and CC regions, $p - \rho^*$ is replaced by λ_{PD} , λ and λ_{CC} , respectively.

$$\Rightarrow 2\mathcal{B}\Delta s_{max} = -2q\lambda_{PD}\Delta\hat{s} + (\phi + q\lambda_{PD})^2 \quad (\text{B9})$$

The above quadratic in λ_{PD} can be solved to obtain λ_{PD} which can be used to obtain $p = \rho^* + \lambda_{PD}$. To evaluate the impact of q , we note that $\frac{d\bar{p}}{dq} = \frac{d(q\lambda_{PD})}{dq} = \lambda_{PD} + q\frac{d\lambda_{PD}}{dq}$. Further, $\frac{d\hat{s}}{dq} = (1-r)\frac{d\bar{p}}{dq} = (1-r)\frac{d(q\lambda_{PD})}{dq}$. Differentiating Equation (B9) with respect to q and noting that $\Delta\hat{s} = \pi + \phi + m\lambda_{PD}$, we obtain:

$$\begin{aligned} 0 &= -2 \left[\frac{d\bar{p}}{dq} \Delta\hat{s} + q\lambda_{PD}(1-r)\frac{d\bar{p}}{dq} \right] + 2(\phi + q\lambda_{PD})\frac{d\bar{p}}{dq} \\ \Rightarrow 0 &= -[\pi + (m-rq)] \frac{d\bar{p}}{dq} \\ \Rightarrow \left. \frac{d\bar{p}}{dq} \right|_{PD} &= 0 \quad \text{as } [\pi + (m-rq)] > 0 \end{aligned} \quad (\text{B10})$$

$$\Rightarrow \left. \frac{dp}{dq} \right|_{PD} = \frac{d\lambda_{PD}}{dq} = \frac{1}{q} \left[\left. \frac{d\bar{p}}{dq} \right|_{PD} - \lambda_{PD} \right] = -\frac{\lambda_{PD}}{q} < 0 \quad (\text{B11})$$

f_r in the PD region is a function of p and therefore varies with q . Differentiating Equation (17) with respect to q while noting that $\frac{4\gamma k_1}{R^2} = \lambda$, we obtain:

$$\begin{aligned} \left. \frac{df_r}{dq} \right|_{PD} &= -\frac{1}{4} \left[R^2 - \frac{4\gamma k_1}{p - \rho^*} \right]^{-\frac{1}{2}} (-4\gamma k_1) \left[\frac{-1}{(p - \rho^*)^2} \right] \left[\left. \frac{dp}{dq} \right|_{PD} \right] \\ &= \left[R^2 - \frac{4\gamma k_1}{\lambda_{PD}} \right]^{-\frac{1}{2}} \left[\frac{\gamma k_1}{\lambda_{PD}^2} \right] \left[\frac{\lambda_{PD}}{q} \right] \\ &= \frac{\lambda R}{4q\lambda_{PD}} \left[1 - \frac{\lambda}{\lambda_{PD}} \right]^{-\frac{1}{2}} > 0 \end{aligned} \quad (\text{B12})$$

Finally, as $\bar{\beta} = \mathcal{B}$ in the PD region, $\left. \frac{d\bar{\beta}}{dq} \right|_{PD} = 0$.

B5.2. Real Asset Liquidity Crunch Equilibrium (LC)

In the LC Equilibrium, $p = \rho^* + \lambda$, $\bar{p} = \rho^* + q\lambda$, $f_r = R/2$ and $\bar{\beta}$ is given by Equation (B7). Therefore, $\left. \frac{dp}{dq} \right|_{LC} = 0$, $\left. \frac{d\bar{p}}{dq} \right|_{LC} = \lambda > 0$ and $\left. \frac{df_r}{dq} \right|_{LC} = 0$. We differentiate Equation (B7) with respect to q , noting that $\frac{d\hat{s}}{dq} = (1-r)\lambda$ in the LC equilibrium, to obtain (using the notational simplifications of m , π , ϕ , Δs_{max} and $\Delta\hat{s}$ developed earlier):

$$\begin{aligned} \frac{d\bar{\beta}}{dq} &= -\frac{\lambda [\Delta\hat{s} + (1-r)q\lambda]}{\Delta s_{max}} + \frac{2(\phi + q\lambda)\lambda}{2\Delta s_{max}} \\ \Rightarrow \frac{d\bar{\beta}}{dq} &= -\frac{(\pi + (m-rq)\lambda)\lambda}{\Delta s_{max}} < 0 \end{aligned} \quad (\text{B13})$$

As π , $(m-r)$ and λ are all positive, $\frac{d\bar{\beta}}{dq} < 0$.

B5.3. Real Asset Credit Crunch Equilibrium (CC)

Price in the CC region is obtained by using the endogenous distribution of debt from Lemma (6) in Equation (22) and solving for p . We obtain an expression similar to Equation (B9) with $\mathcal{B} = 0$; a quadratic in λ_{CC} which can be solved to obtain $p = \rho^* + \lambda_{CC}$:

$$0 = -2q\lambda_{CC}\Delta\hat{s} + (\phi + q\lambda_{CC})^2 \quad (\text{B14})$$

Differentiating Equation (B14) with respect to q , we get results similar to Equations (B10) & (B11):

$$0 = [\pi + (m - rq)] \frac{d\bar{p}}{dq} \Big|_{CC} \Rightarrow \frac{d\bar{p}}{dq} \Big|_{CC} = 0 \quad \text{as } [\pi + (m - rq)] > 0 \quad (\text{B15})$$

$$\Rightarrow \frac{dp}{dq} \Big|_{CC} = \frac{d\lambda_{CC}}{dq} = \frac{1}{q} \left[\frac{d\bar{p}}{dq} \Big|_{CC} - \lambda_{CC} \right] = -\frac{\lambda_{CC}}{q} < 0 \quad (\text{B16})$$

Further, as $\bar{\beta} = 0$ and $f_r = R/2$ in the CC equilibrium, it follows that $\frac{d\bar{\beta}}{dq} \Big|_{CC} = 0$ and $\frac{df_r}{dq} \Big|_{CC} = 0$

$$B6. \quad \frac{dp}{d\theta_2^l} \Big|_{LC} > 0 \text{ and } \frac{d\bar{p}}{d\theta_2^l} \Big|_{LC} > 0.$$

In the LC Equilibrium, p is given by $p = \rho^* + \lambda$, and therefore, $\frac{dp}{d\theta_2^l} \Big|_{LC} = y_2^l + (1 - \omega)\kappa > 0$. Further, as $\bar{p} = \rho^* + q\lambda$, we have $\frac{d\bar{p}}{d\theta_2^l} \Big|_{LC} = y_2^l + (1 - q\omega)\kappa > 0$.

$$B7. \quad \frac{dp}{d\theta_2^l} \Big|_{CC} > 0 \text{ and } \frac{d\bar{p}}{d\theta_2^l} \Big|_{CC} > 0.$$

As $\frac{d\hat{s}}{d\theta_2^l} \Big|_{CC} = (1 - r) \left[y_2^l + \kappa + q \frac{d\lambda_{CC}}{d\theta_2^l} \right]$, we rearrange and differentiate Equation (B14) with respect to θ_2^l to obtain:

$$\begin{aligned} 2q \left[\Delta\hat{s} \frac{d\lambda_{CC}}{d\theta_2^l} + (1 - r)\lambda_{CC} \left(y_2^l + \kappa + q \frac{d\lambda_{CC}}{d\theta_2^l} \right) \right] &= 2(\phi + q\lambda_{CC}) \left[y_2^l + \kappa + q \frac{d\lambda_{CC}}{d\theta_2^l} \right] \\ \Rightarrow \frac{d\lambda_{CC}}{d\theta_2^l} &= \frac{(y_2^l + \kappa)(\phi + rq\lambda_{CC})}{q[\pi + (m - rq)\lambda_{CC}]} \end{aligned} \quad (\text{B17})$$

We also have:

$$\frac{dp}{d\theta_2^l} \Big|_{CC} = \frac{d\rho^*}{d\theta_2^l} + \frac{d\lambda_{CC}}{d\theta_2^l} = y_2^l + \kappa + \frac{d\lambda_{CC}}{d\theta_2^l} > \frac{dp}{d\theta_2^l} \Big|_{LC} > 0 \quad (\text{B18})$$

$$\frac{d\bar{p}}{d\theta_2^l} \Big|_{CC} = \frac{d\rho^*}{d\theta_2^l} + q \frac{d\lambda_{CC}}{d\theta_2^l} = y_2^l + \kappa + q \frac{d\lambda_{CC}}{d\theta_2^l} > \frac{d\bar{p}}{d\theta_2^l} \Big|_{LC} > 0 \quad (\text{B19})$$

B8. $\frac{d\bar{\beta}}{d\theta_2^l} > 0$

As $\left. \frac{d\hat{s}}{d\theta_2^l} \right|_{LC} = (1-r)[y_2^l + (1-q\omega)\kappa]$, differentiating Equation (B7) with respect to θ_2^l , yields:

$$2\frac{d\bar{\beta}}{d\theta_2^l}\Delta s_{max} + 2\bar{\beta}(1-r)y_2^l = -2q\Delta\hat{s}(-\omega\kappa) - 2q\lambda(1-r)[y_2^l + (1-q\omega)\kappa] + 2(\phi + q\lambda)[y_2^l + (1-q\omega)\kappa]$$

Rearranging and simplifying, we obtain:⁴³

$$\frac{d\bar{\beta}}{d\theta_2^l} = \frac{[\phi + qr\lambda - (1-r)\bar{\beta}]y_2^l + [\phi + qr\lambda + q\omega(\pi + (m-qr)\lambda)]\kappa}{\Delta s_{max}} > 0 \quad (\text{B20})$$

B9. *Proof:* $\frac{d\bar{\theta}_2^l(q)}{dq} > 0$

$\bar{\theta}_2^l(q)$, the boundary between PD and LC equilibria is defined by the following equation:

$$\bar{\beta}(q, \bar{\theta}_2^l(q)) = \mathcal{B} \quad (\text{B21})$$

Differentiating Equation (B21) with respect to q yields:⁴⁴

$$\begin{aligned} \frac{\partial \bar{\beta}}{\partial q} + \frac{\partial \bar{\beta}}{\partial \bar{\theta}_2^l(q)} \frac{d\bar{\theta}_2^l(q)}{dq} &= 0 \\ \frac{d\bar{\theta}_2^l(q)}{dq} &= -\frac{\partial \bar{\beta}/\partial q}{\partial \bar{\beta}/\partial \bar{\theta}_2^l} > 0 \end{aligned} \quad (\text{B22})$$

B10. *Proof:* $\frac{d\hat{\theta}_2^l(q)}{dq} > 0$

$\hat{\theta}_2^l(q)$, the boundary between the LC and CC regions is defined by the following equation:

$$\lambda_{CC}(q, \hat{\theta}_2^l(q)) = \lambda \quad (\text{B23})$$

Differentiating Equation (B23) with respect to q yields:⁴⁵

$$\begin{aligned} \frac{\partial \lambda_{CC}}{\partial q} + \frac{\partial \lambda_{CC}}{\partial \hat{\theta}_2^l(q)} \frac{d\hat{\theta}_2^l(q)}{dq} &= 0 \\ \frac{d\hat{\theta}_2^l(q)}{dq} &= -\frac{\partial \lambda_{CC}/\partial q}{\partial \lambda_{CC}/\partial \hat{\theta}_2^l} > 0 \end{aligned} \quad (\text{B24})$$

⁴³The result in Equation (B20) follows as $\bar{\beta} \leq \bar{\beta}(q=0) < \phi$.

⁴⁴The final result follows as $\frac{\partial \bar{\beta}}{\partial q} \leq 0$ (see Footnote (19)) and $\frac{\partial \bar{\beta}}{\partial \bar{\theta}_2^l} > 0$ (see Section (B8)).

⁴⁵The final result follows as $\frac{\partial \lambda_{CC}}{\partial q} \leq 0$ (see Footnote (19)) and $\frac{\partial \lambda_{CC}}{\partial \hat{\theta}_2^l} > 0$ (see Section (B7)).

B11. q^{opt} when $\theta_2^h y_2^h - \theta_2^l y_2^l < \left[\frac{8\gamma(1-r)}{3rR^2} - 1 \right] k_1$

When $\theta_2^h y_2^h - \theta_2^l y_2^l < \left[\frac{8\gamma(1-r)}{3rR^2} - 1 \right] k_1$, it can be shown that $\frac{dS_{Total}}{dq} \Big|_{LC} > 0$ for $q < \check{q} = \frac{2\lambda - 5r\lambda - 3\pi}{3\lambda - 6r\lambda + 2(1-r)\omega\lambda}$. Three possible cases arise.

- (i) $\check{q} \leq \bar{q}$: In this case, S_{Total} is always decreasing with q in the LC equilibrium and therefore $q^{opt} = \bar{q}$ (i.e., the border between the FP and PD equilibria).
- (ii) $\bar{q} < q < \hat{q}$: In this case, S_{Total} first increases with q in the LC equilibrium till it reaches a local maxima at $q = \check{q}$, after which it decreases with q . Consequently $q^{opt} = \arg \max_q (S_{Total}(\bar{q}), S_{Total}(\check{q}))$.
- (iii) $\hat{q} \leq \check{q}$: In this case, S_{Total} increases with q across the LC equilibrium, reaching a local maximum value at \hat{q} (i.e., the border of the LC and CC equilibria). Consequently $q^{opt} = \arg \max_q (S_{Total}(\bar{q}), S_{Total}(\hat{q}))$. Note that when $q^{opt} = \hat{q}$, as S_{Total} is invariant with q in the CC equilibrium, $q^{opt} = (\hat{q}, 1)$.

Essentially, when $\theta_2^h y_2^h - \theta_2^l y_2^l < \left[\frac{8\gamma(1-r)}{3rR^2} - 1 \right] k_1$, q^{opt} is one of the following $-\bar{q}, \check{q}, (\hat{q}, 1)$.

B12. *Proof:* $\frac{d\lambda_{PD}}{d\theta_2^l} > 0$

We evaluate the impact of θ_2^l on λ_{PD} . Using results from Section (A2), we have $\frac{d\hat{s}}{d\theta_2^l} = (1-r)\frac{d\bar{p}}{d\theta_2^l} = (1-r)(y_2^l + \kappa + q\frac{d\lambda_{PD}}{d\theta_2^l})$. Differentiating Equation (B9) with respect to θ_2^l to obtain:⁴⁶

$$\begin{aligned} 2\mathcal{B}(1-r)y_2^l &= -2(1-r)q\lambda_{PD}(y_2^l + \kappa + q\frac{d\lambda_{PD}}{d\theta_2^l}) - 2q\Delta\hat{s}\frac{d\lambda_{PD}}{d\theta_2^l} + 2(\phi + q\lambda_{PD})(y_2^l + \kappa + q\frac{d\lambda_{PD}}{d\theta_2^l}) \\ \Rightarrow q[\pi + (m-rq)\lambda_{PD}] \frac{d\lambda_{PD}}{d\theta_2^l} &= (\phi + rq\lambda_{PD})(y_2^l + \kappa) - (1-r)\mathcal{B}y_2^l \\ \Rightarrow \frac{d\lambda_{PD}}{d\theta_2^l} &= \frac{(\phi + rq\lambda_{PD})(y_2^l + \kappa) - (1-r)\mathcal{B}y_2^l}{q[\pi + (m-rq)\lambda_{PD}]} > 0 \end{aligned} \quad (B25)$$

B13. *Proof:* $\frac{\partial\lambda_{PD}}{\partial k_1} < 0$

Collateral quality improves with asset payoff (y_2^l) as k_1 is decreasing in y_2^l . We first evaluate the impact of y_2^l on λ_{PD} . Noting that $\frac{d\hat{s}}{dy_2^l} = (1-r)\frac{d\bar{p}}{dy_2^l} = (1-r)(\theta_2^l\iota + q\frac{d\lambda_{PD}}{dy_2^l})$ from Section (A2), we differentiate Equation (B9) with respect to y_2^l to obtain:⁴⁷

$$\frac{d\lambda_{PD}}{dy_2^l} = \frac{[(\phi + rq\lambda_{PD})\iota - (1-r)\mathcal{B}]\theta_2^l}{q[\pi + (m-rq)\lambda_{PD}]} > 0 \quad (B26)$$

⁴⁶The result in Equation (B25) obtains as the numerator of the fraction in Equation (B25) is positive in the PD region. PD region exists at a given θ_2^l for some q only if $\mathcal{B} < \bar{\beta}(q=0)$ which implies $\mathcal{B} < \phi$.

⁴⁷The result in Equation (B26) follows as $\iota > 1$ and $\phi > \mathcal{B}$.

$$\Rightarrow \frac{\partial \lambda_{PD}}{\partial k_1} = \frac{\frac{d\lambda_{PD}}{dy_2^l}}{\frac{dk_1}{dy_2^l}} = - \frac{\theta_2^l [(\phi + rq\lambda_{PD})\iota - (1-r)\mathcal{B}]}{q\theta_1\iota [\pi + (m-rq)\lambda_{PD}]} < 0 \quad (\text{B27})$$

B14. *Proof:* $\frac{d\lambda_{PD}}{d\mathcal{B}} < 0$

Noting that $\frac{d\hat{s}}{d\mathcal{B}} = (1-r)\frac{d\bar{p}}{d\mathcal{B}} = (1-r)q\frac{d\lambda_{PD}}{d\mathcal{B}}$ while differentiating Equation (B9) with respect to \mathcal{B} , we obtain:

$$\frac{d\lambda_{PD}}{d\mathcal{B}} = - \frac{\Delta s_{max}}{q[\pi + (m-rq)\lambda_{PD}]} < 0 \quad (\text{B28})$$

B15. *Discussion of Proposition (4)*

B15.1. *Impact of θ_2^l on q^{opt}*

As the severity of the economic shock increases, fire-sale effects are triggered at lower levels of q and the optimal q decreases. Figure (4) illustrates this situation. Consider the case of a severe economic shock ($\theta_2^l = \theta_{severe} = 0.30$). In this case, there is an acute shortage of funding liquidity due to the severity of the economic shock. The economy will be in a Liquidity Crunch Equilibrium even at the lowest feasible value of $q = 0$ (which induces the least amount of ex-post liquidation). The solid curve representing the boundary of the Fair Pricing region and the Fire Sale region (depicted by the $\bar{q}(\theta_2^l)$ curve) does not arise in the vertical line drawn at $\theta_2 = 0.30$, i.e., both the Fair Pricing Equilibrium region and the Price Discrimination Equilibrium region vanish for the given level of economic shock. For such a severe economic shock, the economy is always in the Fire Sale Equilibrium region for the entire range of feasible $q \in (0, 1)$. This situation arises because the financial market cannot clear without reducing the supply of loans to the real sector, i.e., the system will always be in the Liquidity Crunch Equilibrium region, and there will some unmet demand in the real sector ($\bar{\beta} < \mathcal{B}$). The system transitions to a Credit Crunch Equilibrium at higher values of q . Interestingly, the ex-ante optimal q^{opt} is equal to 0.

Figure (4) also depicts the situation in which the optimal bankruptcy exemption can be equal to 1. Consider the case of a mild economic shock ($\theta_2^l = \theta_{mild} = 0.75$). In this case, there is sufficient liquidity in the economy that there are no ex-post fire-sale effects. Both the financial asset and the real asset trade at fair value for any level of q . Since there is no negative externality of ex-post liquidation, it is optimal to employ full bankruptcy exemption, which facilitates ex-ante lending that maximizes total surplus in the economy.

B15.2. Impact of k_1 on q^{opt}

In Panel A of Figure 9, we map the equilibria in the system in the (k_1, q) space, which is defined over $k_1 \in [k_{min}, k_{max}]$ and $q \in [0, 1]$. Similar to the analysis behind Figure 4, the $\bar{q}(k_1)$ curve in Panel A of Figure 9 divides the feasible (k_1, q) space into two regions (the Fair Pricing and the Fire Sale Equilibrium region) for any given (k_1, q) combination. Based on Proposition (3), the $\bar{q}(k_1)$ curve represents the q^{opt} for a given k_1 . Note that the curve representing the border of the Fair Pricing and Fire Sale Equilibrium regions is downward sloping in the feasible (k_1, q) space. If collateral quality is sufficiently high, the optimal q can be as high as 1 (see $k_1 = 0.3$ in Panel A of Figure 9). On the other hand, for low quality collateral, the optimal q is 0 (see $k_1 = 1.1$ in Panel A of Figure 9).

B15.3. Impact of \mathcal{B} on q^{opt}

In general, as the size of the real sector \mathcal{B} increases, it is less likely that the real asset market will be fully satiated, but the extent to which the real sector loans are offered depends on the liquidity in the economy. In Panel B of Figure 9, we map the Fair Pricing and the Fire Sale boundary (shown by the $q^{opt}(\mathcal{B} = 0)$ curve) in the (θ_2, q) space for different values of \mathcal{B} . As \mathcal{B} increases, the border of the Fair Pricing Equilibrium and the Fire Sale Equilibrium regions shifts downward (and to the right). This shift causes the optimal q to decrease with \mathcal{B} .

At the extreme, when \mathcal{B} is sufficiently high, even at $q = 0$ when there is no ex-post liquidation, the spare liquidity is insufficient to satisfy the real asset demand. Consequently, the system always lies in the Liquidity Crunch Equilibrium region. This can be seen in Panel B of Figure (9), where for $\theta_2^l = 0.8$ and for $\mathcal{B} = 1.1$, $q^{opt} = 0$. For any higher \mathcal{B} , the optimal q for the given economic shock ($\theta_2^l = 0.8$) will continue to be 0.

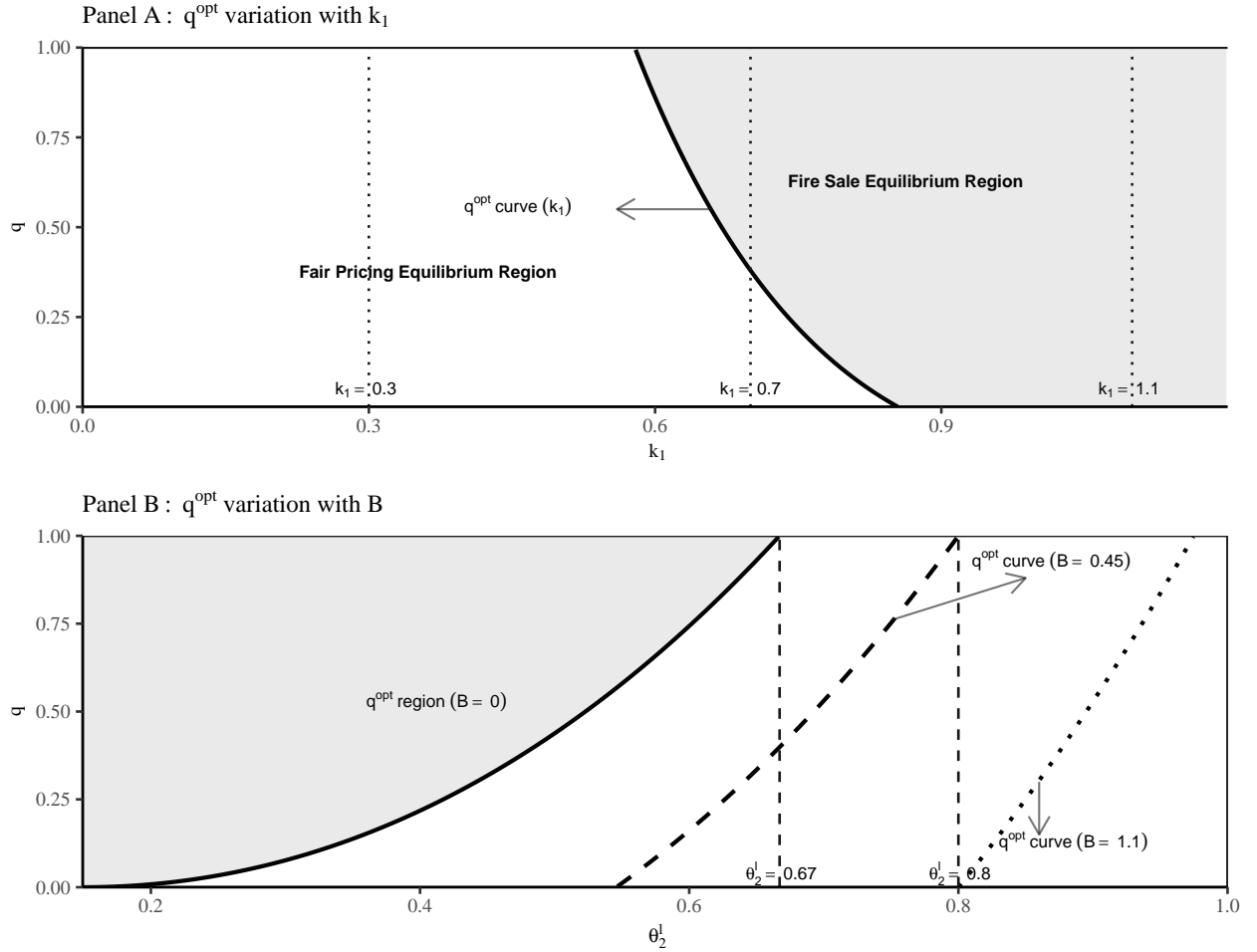
Conversely, as \mathcal{B} decreases, the curve moves toward the northwest of (q, θ_2^l) space. However, this leftward movement is bounded when \mathcal{B} hits 0, i.e., when the real sector is absent. This situation corresponds to the special case of the model examined in Acharya and Viswanathan (2011). with q assumed to be 1. However, in our model, the optimal q could range from an interior value to 1, as can be seen from the $q^{opt}(\mathcal{B} = 0)$ region in Panel B of Figure (9). The specific details can be seen in Appendix A15.3. The combined effect of the economic shock and the size of real sector is discussed in Appendix B16.

In the special case when $\mathcal{B} = 0$, it can be seen from Equation (28) that $\bar{q} > 0$ as $\phi > 0$. However, in this case, the system directly transitions from the Fair Pricing Equilibrium to the

Figure 9: q^{opt} variation with k_1 and \mathcal{B} .

Panel A shows the typical demarcation of the feasible $k_1 - q$ space into the Fair Pricing (FP) and Fire Sale (FS) equilibria. The plot is obtained by evaluating the model for assets varying in their payoffs (y_2^l) leading to variation in their collateral quality (k_1). The solid $q^{opt}(k_1)$ curve represents the boundary between the two equilibrium regions. For a moderate quality asset, indicated by $k_1 = 0.7$, as q is increased from 0, the system transitions from FP equilibrium to FS equilibrium at $q = 0.38$. For a high quality asset indicated by $k_1 = 0.3$, the system remains in FP equilibrium for any q . For a low quality asset, indicated by $k_1 = 1.1$, the system remains in FS equilibrium for any q . $k_1 = 0.30$, $k_1 = 0.7$ and $k_1 = 1.1$ are indicated by the three thin vertical dashed lines. Parameter Configuration: $\theta_2^l = 0.48$, $\theta_1 = 0.02$, $\theta_2^h = 1$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$ and $\mathcal{B} = 0.15$.

Panel B shows the optimal bankruptcy exemption parameter (q^{opt}) curve for three different levels of \mathcal{B} . The solid curve shows q^{opt} for $\mathcal{B} = 0$, the dashed curve shows q^{opt} for $\mathcal{B} = 0.45$ and the dotted curve shows q^{opt} for $\mathcal{B} = 1.1$. The vertical dashed line at $\theta_2^l = 0.67$ indicates the value of θ_2^l at which $q^{opt}(\mathcal{B} = 0) = 1$. The values of \mathcal{B} used to obtain the dashed and dotted q^{opt} curves are chosen such that for $\theta_2^l = 0.8$ (indicated by the second vertical dashed line), we have $q^{opt}(\mathcal{B} = 0.45) = 1$ and $q^{opt}(\mathcal{B} = 1.2) = 0$. Parameter configuration is the same as that used in Figure 4 (i.e. $\theta_1 = 0.02$, $\theta_2^h = 1$, $y_2^l = 15$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$ and $r = 0.6$).



Credit Crunch Equilibrium. Therefore, S_{Total} is invariant in q beyond \bar{q} when $\mathcal{B} = 0$, rendering $q^{opt} = (\bar{q}, 1)$. Further, denoting the value of θ_2^l for which $\bar{q} = 1$ when $\mathcal{B} = 0$ as $\theta_2^{l,B0}$, we obtain $\theta_2^{l,B0} = \frac{s_{min} + (1-r)k_1 + \sqrt{(1-r)^2 k_1^2 + 2rk_1(\theta_2^h y_2^h - s_{min})}}{y_2^l}$.⁴⁸

Now, as \bar{q} is increasing in θ_2^l , for any $\theta_2^l < \theta_2^{l,B0}$, $\bar{q} < 1$ and $q^{opt} = (\bar{q}, 1)$. In addition, as \bar{q} is decreasing in \mathcal{B} , $q^{opt} = \bar{q} < 1$ for any $\mathcal{B} > 0$ for any $\theta_2^l < \theta_2^{l,B0}$.

For $\theta_2^l \geq \theta_2^{l,B0}$, we denote the value of \mathcal{B} at which $\bar{q} = 1$ as \mathcal{B}_1 and the value of \mathcal{B} at which $\bar{q} = 0$ as \mathcal{B}_2 . Again as q^{opt} is decreasing in \mathcal{B} , for a given $\theta_2^l \geq \theta_2^{l,B0}$, we conclude that:

- (i) $q^{opt} = 1$ for $\mathcal{B} \leq \mathcal{B}_1$
- (ii) $0 < q^{opt} < 1$ for $\mathcal{B}_1 < \mathcal{B} < \mathcal{B}_2$
- (iii) $q^{opt} = 0$ for $\mathcal{B} \geq \mathcal{B}_2$

We obtain $\mathcal{B}_1 = \frac{\phi^2 - k_1^2 - 2rk_1(\theta_2^h y_2^h - \theta_2^l y_2^l)}{2\Delta s_{max}}$ and $\mathcal{B}_2 = \frac{\phi^2}{2\Delta s_{max}}$.⁴⁹

B16. The Combined Effect of Economic Shock and Size of Real Sector

The table 5 presents the possible range of q^{opt} for different ranges of θ_2^l and \mathcal{B} .

θ_2^l Range	\mathcal{B} Range	Implication for q^{opt}
$\theta_2^{min} \leq \theta_2^l < \theta_2^{l,B0}$	$\mathcal{B} = 0$	$q^{opt} \in (\bar{q}, 1)$
	$\mathcal{B} > 0$	$0 \leq q^{opt} < 1$
$\theta_2^{l,B0} \leq \theta_2^l \leq \theta_2^{max}$	$0 \leq \mathcal{B} \leq \mathcal{B}_1$	$q^{opt} = 1$
	$\mathcal{B}_1 < \mathcal{B} < \mathcal{B}_2$	$0 < q^{opt} < 1$
	$\mathcal{B}_2 \leq \mathcal{B}$	$q^{opt} = 0$

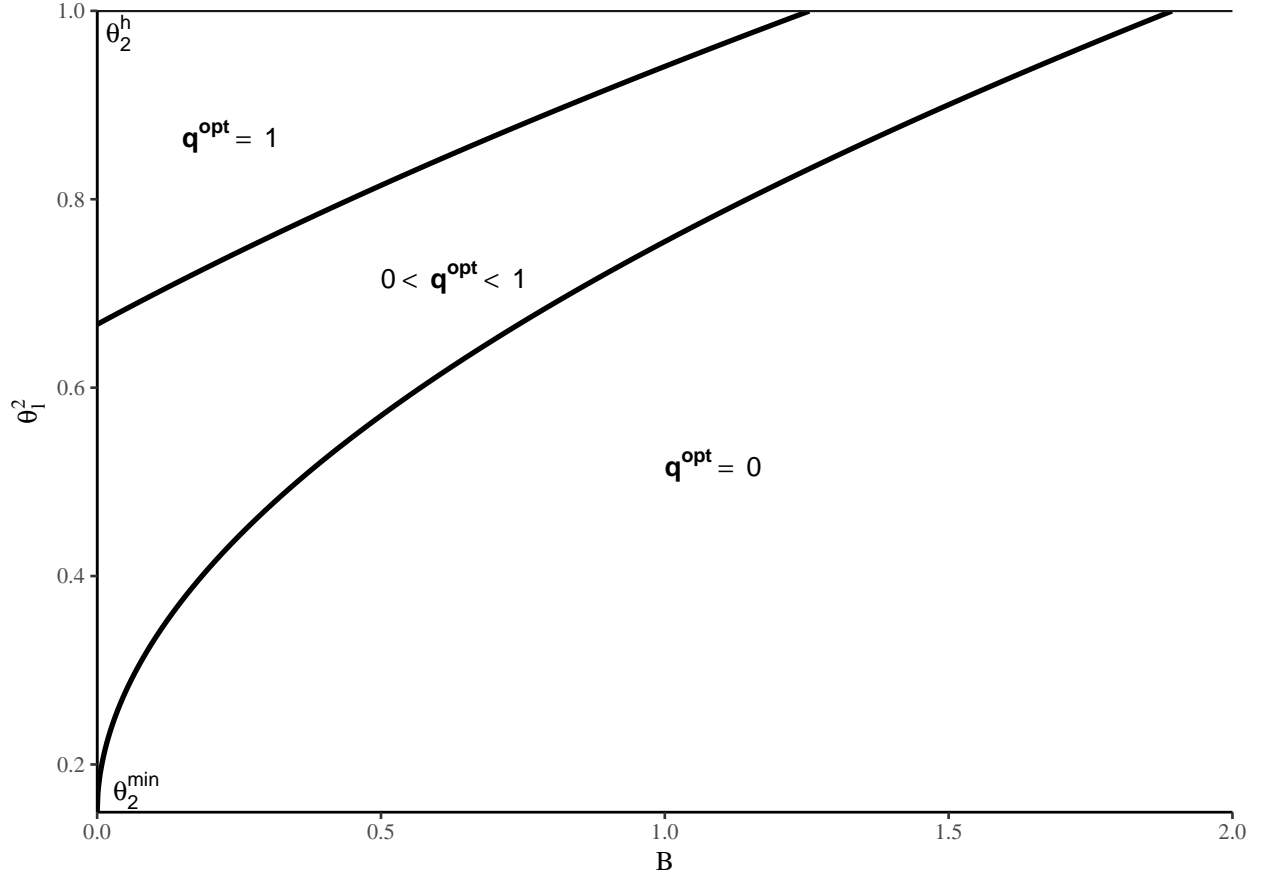
Table 5: **Impact of \mathcal{B} on q^{opt}** . Implication of the size of the real asset market (\mathcal{B}) on the optimal bankruptcy exemption parameter (q^{opt}) for a given level of the economic shock (θ_2^l) is presented.

Figure (10) displays the results of Table 5 in graphical form by presenting the joint impact of the level of the magnitude of the economic shock (θ_2^l) and the size of the real asset market (\mathcal{B}) on q^{opt} . We consider the (\mathcal{B}, θ_2) space and map the three regions of optimal q ($q^{opt} = 0$, an interior q^{opt} , and $q^{opt} = 1$). We see that when the magnitude of the economic shock is mild and the size of the real asset market is small (i.e., top left corner of Fig(10)), $q^{opt} = 1$. As the size of real asset market

⁴⁸To obtain $\theta_2^{l,B0}$, we solve Equation (A31) for θ_2^l after setting $q = 1$ and $\mathcal{B} = 0$.

⁴⁹To obtain \mathcal{B}_1 and \mathcal{B}_2 , we solve Equation (A31) for \mathcal{B} for which $\bar{q} = 1$ and $\bar{q} = 0$, respectively, at a given θ_2^l .

Figure 10: q^{opt} in $\mathcal{B} - \theta_2^l$ space. Demarcation of the $\mathcal{B} - \theta_2^l$ space into regions where $q^{opt} = 0$, $0 < q^{opt} < 1$ and $q^{opt} = 1$. Parameter Configuration used: $\theta_1 = 0.02$, $\theta_2^h = 1$, $y_2^l = 15$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$ and $r = 0.6$.



increases or the severity of the economic shock increases, q^{opt} falls below 1 and moves towards 0 (i.e., bottom right corner of Fig(10)).⁵⁰

⁵⁰Note that in Fig(10), for $\mathcal{B} = 0$ the chart plots the value of \bar{q} , the lower end of the range for q^{opt} as shown in Table 5.

B17. List of Proofs

Main Appendix			
No.	Description of Proof	Appendix Reference	Main Text Reference
1	Funding Liquidity Lemma	Appendix A, SS A2	Lemma (1)
2	Effort Lemma	Appendix A, SS A3	Lemma (2)
3	Optimization Lemma	Appendix A, SS A6	Lemma (4)
4	Market Clearing Lemma	Appendix A, SS A7	Lemma (5)
5	Ex Post: Equilibrium p , f_r and $\bar{\beta}$ in FP Region	Appendix A, SS A8	Proposition (1)
6	Ex Post: Equilibrium p , f_r and $\bar{\beta}$ in FS Region	Appendix A, SS A9	Proposition (2)
7	Ex Ante: Derived Distribution of Debt	Appendix A, SS A10	Section 5, Lemma (6)
8	Ex Ante: Model Restrictions	Appendix A, SS A11	Section 5, SS 5.3
9	Ex Ante: S_{D1} Dynamics	Appendix A, SS A12	Proposition (3)
10	Ex Ante: S_{D0} Dynamics	Appendix A, SS A13	Proposition (3)
11	Ex Ante: q^{opt}	Appendix A, SS A14	Proposition (3)
12	Ex Ante: q^{opt} is increasing in θ_2^l	Appendix A, SS A15.1	Proposition (4)
13	Ex Ante: q^{opt} is decreasing in k_1	Appendix A, SS A15.2	Proposition (4)
14	Ex Ante: q^{opt} is decreasing in \mathcal{B}	Appendix A, SS A15.3	Proposition (4)
15	Optimal $q - \bar{s}$ combination	Appendix A, SS A16	Proposition (5)
Internet Appendix			
No.	Description of Proof	Appendix Reference	Main Text Reference
1	Existence and Uniqueness of Solution	Appendix B, SS B1	Section 5, SS 5.3
2	s for a given ρ	Appendix B, SS B2	Section 5, Lemma (6)
3	Expression for $p _{PD}$ and $p _{CC}$	Appendix B, SS B3	Footnote (19)
4	Ex Ante: Equilibrium $\bar{\beta}$	Appendix B, SS B4	Footnote (19)
5	Ex Ante: PD Region Dynamics	Appendix B, SS B5.1	Footnote (19)
6	Ex Ante: LC Region Dynamics	Appendix B, SS B5.2	Footnote (19)
7	Ex Ante: CC Region Dynamics	Appendix B, SS B5.3	Footnote (19)
8	Ex Ante: $\left. \frac{dp}{d\theta_2^l} \right _{LC} > 0$ and $\left. \frac{d\bar{p}}{d\theta_2^l} \right _{LC} > 0$	Appendix B, SS B6	Footnote (19)
9	Ex Ante: $\left. \frac{dp}{d\theta_2^l} \right _{CC} > 0$ and $\left. \frac{d\bar{p}}{d\theta_2^l} \right _{CC} > 0$	Appendix B, SS B7	Footnote (19)
10	Ex Ante: $\frac{d\bar{\beta}}{d\theta_2^l} > 0$	Appendix B, SS B8	Footnote (19)
11	Ex Ante: $\frac{d\theta_2^l(q)}{dq} > 0$	Appendix B, SS B9	Section 5, SS 5.3
12	Ex Ante: $\frac{d\theta_2^h(q)}{dq} > 0$	Appendix B, SS B10	Section 5, SS 5.3
13	q^{opt} when $\theta_2^h y_2^h - \theta_2^l y_2^l < \left[\frac{8\gamma(1-r)}{3rR^2} - 1 \right] k_1$	Appendix A, SS B11	Proposition (3)
14	Ex Ante: $\frac{d\lambda_{PD}}{d\theta_2^l} > 0$	Appendix B, SS B12	Proposition (4)
15	Ex Ante: $\frac{\partial \lambda_{PD}}{\partial k_1} < 0$	Appendix B, SS B13	Proposition (4)
16	Ex Ante: $\frac{d\lambda_{PD}}{d\mathcal{B}} < 0$	Appendix B, SS B14	Proposition (4)
17	Ex Ante: Discussion of Proposition (4)	Appendix B, SS B15	Proposition (4)
18	Ex Ante: Combined Effect of θ_2^l and \mathcal{B}	Appendix B, SS B16	Proposition (4)

Table 6: List of Proofs in Appendices