Corporate resiliency and the choice between financial and operational hedging[*]

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Abstract

We investigate how firms manage financial default risk (on debt) and operational default risk (on delivery obligations). Financially constrained firms reduce operational hedging through inventory and supply chain in favor of cash holdings. Our model predicts that firms' markup increases with financial default risk as they cut operational hedging costs. Empirical analysis confirms this prediction and shows that the markup-credit risk relationship strengthens during adverse aggregate shocks, particularly for firms exposed to lending disruptions. Market power alone cannot explain this relationship, which reflects firms' strategic adjustments in operational hedging practices.

KEYWORDS: FINANCIAL DEFAULT, OPERATIONAL DEFAULT, RESILIENCE, LIQUIDITY, RISK MANAGEMENT, INVENTORY, SUPPLY CHAINS

JEL: G31, G32, G33

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1. Introduction

This paper examines how firms manage the dual risks of financial and operational default. Firms face contractual obligations on two fronts: financial debt contracts and operational contracts to deliver goods and services to customers. When adverse economic shocks occur, firms can face financial default on their debt obligations and operational default on their delivery commitment to customers as was the case during the Covid-19 pandemic. Given constraints on internal cash flows and limited access to the capital market, firms must optimize their resource allocation to mitigate these two distinct default risks. We develop a theoretical framework that analyzes this fundamental tradeoff in resource allocation between financial and operational hedging, and provide empirical evidence supporting our model's predictions. In particular, we find that greater financial risk predicts lower spending on operational hedging, particularly when firms become financially constrained.

Our paper thus studies the tension between operational resiliency and financial efficiency, viz., the tradeoff that the firm faces between allocating liquid resources to operational hedging and to the prevention of financial distress. The firm's need to optimally balance these two hedging demands—financial hedging and operational hedging—provides a novel explanation for heterogeneity in operational resilience across firms. In particular, our analysis demonstrates why highly leveraged, financially constrained firms may rationally choose lower levels of operational hedging.

While financial default has been extensively studied, research on operational default remains relatively limited. The Covid-19 pandemic and its aftermath have highlighted the critical importance of corporate operational resilience to shocks that disrupt supply chains, deplete inventory, and impair firms' ability to meet their delivery obligations. Firms employ various operational hedging strategies to mitigate such negative shocks, including maintaining excess inventory, diversifying and geographically relocating supplier networks, and developing backup production capacity. These operational hedging methods are, however, financially costly. Firms are nevertheless willing to endure some higher production costs in

order to mitigate the risk of operational disruption and failure to deliver on their obligations to customers. Such a failure not only impairs the firms' immediate cash flow but they also impose a penalty on their reputation and franchise value.

Evidence from a global survey by the Institute for Supply Management illustrates the widespread impact of operational disruptions during the Covid-19 pandemic. By May 2020, 97% of the organizations reported actual or anticipated negative impacts from coronavirus-induced supply chain disruptions.[1] Consequently, U.S. manufacturing was operating at 74% of normal capacity, with Europe at 64%. While North American firms reportedly maintained sufficient inventory for current operations, confidence in inventory adequacy declined significantly: to 64% in the U.S., 49% in Mexico and 55% in Canada. Similar concerns emerged in Asia, with firms in Japan and Korea expressing lack of confidence that they would have sufficient inventory for Q4 of 2020; and almost half of the firms maintained above-normal inventory levels. This operational stress prompted a strategic shift, with 29% of organizations initiating or planning to re-shore or near-shore some or most operations.[2] However, this trend toward increased operational resilience faced resistance by some firms. Several corporate executives and investors contended that such operational hedging strategies compromised financial efficiency.[3]

Our theoretical framework models a competitive (price-taking) levered firm facing dual risks. The first risk arises from a shock to cash flows generated by assets in place, which can create liquidity shortfalls leading to financial default and elimination of equity value. The second risk stems from a shock that disrupts the firm's operation, which in turn reduces output and income. The shocks are potentially correlated, for example, driven by common macroeconomic factors. Both financial and operational defaults impose losses on firm value. The firm must optimally allocate its cash inflows between two purposes: building a buffer

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[1] https://www.prnewswire.com/news-releases/covid-19-survey-round-3-supply-chain-disruptions-continue-globally-301096403.html. See also "Businesses are proving quite resilient to the pandemic", The Economist, May 16th 2020, and "From 'just in time' to 'just in case'", Financial Times, May 4, 2020.

[2] "Reshoring" and "nearshoring" are the processes of bringing the manufacturing operations to the firm's country or a country nearby, respectively.

[3] "Will coronavirus pandemic finally kill off global supply chains?" Financial Times, May 27, 2020. https://www.ft.com/content/4ee0817a-809f-11ea-b0fb-13524ae1056b

against the financial default risk, and developing operational resilience to ensure fulfillment of its customer contracts.

Our model captures a key timing mismatch: while operational hedging increases future cash flows by reducing the risk of operational disruption, which benefits the firm, the occurrence of financial default can terminate the firm's operations *before* this benefit materializes. That is, there is a potential maturity mismatch between operational and financial risk. In principle, the firm can mitigate this maturity mismatch by borrowing against the enhanced long-term cash flows that operational hedging provides. Doing so can potentially alleviate the need of substituting between financial hedging and operational hedging. However, this is feasible only if the firm can credibly pledge future cash flow to outside investors. This intuition gives rise to our key prediction: when pledgeability is limited, financial and operating hedging become substitutes. In other words, a financially constrained and leveraged firm must decide between using cash to mitigate the risk of financial default or to mitigate the risk of operational default.

Our model generates a clear empirical prediction: for firms with significant credit risk and limited access to external capital, operational hedging decreases as the firm's credit spread (which reflects financial default risk) increases. Since operational hedging raises the firm's production costs and lowers the profit margin, higher credit risk, which lowers operational hedging, raises the profit margin or the "operational spread". This results in a positive relationship between credit spread and operational spread.[4]

We empirically test our model's main prediction: the operational spread, measured by markup ((Sales − cost of goods sold)/sales), increases with the firm's credit risk, which we measure using Altman's Z-score (Altman, 1968, 2013a). A higher negative Z-score indicates greater credit risk, making it a suitable proxy for near-term financial default risk as captured in our model. Our tests control for firm characteristics that include three measures of market power, which is usually associated with markup.[5] The results support our hypothesis. The

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[4]Notably, in our model, the effect of credit risk on operational hedging stems primarily from lack of funds to spend on operational hedging, which is an investment in operational resiliency, rather than from debt overhang, which reduces the incentives to invest, as in Myers (1977).

[5]We control for market power by (i) an indicator variable for the firm being among the top 4 sellers in

estimated effect of -(Z-score) on markup is positive and it is statistically and economically significant: an increase of one standard deviation in the firm's -(Z-score) raises the firm's markup by 7% relative to the sample average. Our analysis thus introduces a new determinant of the firm's markup, while the market power variables are not found to predict higher markup. We also find that higher credit risk significantly lowers the cost of goods sold after controlling for the firm's characteristics and sales, supporting the model's prediction by which credit risk affects markup through our proposed operational hedging mechanism.

An important prediction of our model is that financial constraints amplify the positive relationship between markup and credit risk. We test this prediction by examining how the positive markup-credit risk relationship changes during two types of financing shocks. First, we study NBER-designated recessions, periods when external financing generally becomes constrained. We find that the positive effect of credit risk on markup strengthens significantly during recessions. Importantly, this result is driven by a parallel strengthening of the negative effect of credit risk on cost of goods sold during recessions, suggesting that the markup-credit risk relationship is driven at least partly by costs. Second, we employ shocks to firms' credit supply during the subprime crisis, following Chodorow-Reich (2014)'s study of the negative impact on firms of the exogenous shocks to their relationship banks, following the collapse of Lehman Brothers in September 2008, which increased financial constraints on firms. We find that for firms that were more exposed to the exogenous credit-supply shocks, credit risk had both a significantly more positive effect on markup and a significantly more negative effect on cost of goods sold. This test helps alleviate concerns about the endogeneity of credit risk as we study here the effect of the pre-crisis credit risk on the post-crisis outcomes for financially-constrained firms.[6]

We attend to an alternative explanation for the positive effect of the firm's credit risk on its markup that is based on market power. Chevalier and Scharfstein (1994) and Gilchrist et al. (2017) propose that liquidity-constrained firms with market power can raise their prod-

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the industry, (ii) the share of the firm's sales in the industry sales, and (iii) the Industry*time periods fixed effects, which implicitly captures the industry's concentration.

[6]Some other studies also examine the impact of the subprime mortgage crisis on firms' other real decisions (Giroud and Mueller, 2016).

uct prices (or keep prices higher in downturns) in order to boost their short-run profitability and meet immediate liquidity needs, even if it hurts their market share and long-term profitability. Notably, all our regressions include control variables that proxy for the firm's market power as well as industry-year-quarter fixed effects which capture industry-wide changes in product prices and in concentration ratios.

We test explicitly the market power explanation of the positive relationship between markup and financial default risk. We estimate our models on the determinants of markup separately for firms which are among the top 5% industry sellers and for the remaining firms. By the market power hypothesis, the markup-credit risk relationship is positive only for firms with higher market power, which enables them to set their prices. However, we find that this is not the case. The positive markup-credit spread relationship is insignificant for the top 5% industry sellers while it is positive and significant for the rest of the firms, which are more competitive. This finding also holds in the contexts of NBER recessions and the 2008 shocks to lending banks. Even in periods of acute liquidity shortages, the positive markup-credit risk relationship is not associated with greater market power. Yet, the prevalence of this relationship for competitive firms lends support to our model.

Finally, we test an *ex-post* prediction of our model: when credit risk is high enough, avoiding financial default becomes the dominant consideration affecting firm value because the firm is likely to default financially before any operational default occurs, making operational hedging irrelevant for protecting firm value. We test this prediction indirectly using stock returns during the Covid era (2020–2021). By our model, pre-Covid operational hedging choices should matter less for firms that enter Covid with an already-high credit risk. The results support the predictions of our theory: investing in operational hedging helps firms preserve their franchise value following adverse operational shocks, but only if their credit risk is relatively low.

In summary, our novelty is in proposing that firms need to hedge not only against defaults on their financial contracts (their debt obligations) but also against defaults on their operational contracts (their commitments to deliver products to customers). Because both

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forms of hedging impose demands on the firms' limited resources, financially constrained firms face a tradeoff allocating those resources to protect against these dual default risks.

1.1 Related literature

Our paper is related to studies of the real effects of financing frictions (see Stein (2003) for a review) which show that financing frictions can affect investment decisions and employment (Lemmon and Roberts, 2010; Duchin, Ozbas, and Sensoy, 2010; Almeida et al., 2012; Giroud and Mueller, 2016, among others). The literature also studies the effect of financial constraints and financial distress on financial policies such as cash, credit lines, and risk management (e.g., Almeida, Campello, and Weisbach, 2004; Sufi, 2009; Bolton, Chen, and Wang, 2011; Acharya, Davydenko, and Strebulaev, 2012).

In particular, our paper relates closely to Rampini and Viswanathan (2010). They show that more financially distressed firms may reduce risk management to save liquidity for current investment. However, our paper differs from Rampini and Viswanathan (2010) in three important ways. First, in Rampini and Viswanathan (2010), debt is fully collateralized in all states, which makes debt riskless. Thus, their model is silent regarding the relationship between a firm's credit risk and risk management. In contrast, in our model debt is risky because of uncertainty in cash flow and maturity mismatches between the firm's cash flow and debt obligations. Second, we introduce the notion of operational risk — default risk on supplier contract — that rationalizes a firm's incentive to engage in operational hedging. This notion allows us to study the relationship between credit risk and a firm's operational hedging policy. Third, one key model implication in Rampini and Viswanathan (2010) is that a firm with lower net worth does not conserve any liquidity, because its return on investment is so high that it exceeds the return on liquidity hoarding. In our paper, an incentive to conserve liquidity arises for firms with lower net worth due to the presence of risky debt. This latter pattern is documented in Acharya, Davydenko, and Strebulaev (2012); however, they do not analyze the interaction of credit and operational risk, which we study both theoretically and empirically.

Our paper also relates to Froot, Scharfstein, and Stein (1993), who propose a theory for the rationale for corporate hedging. In Froot, Scharfstein, and Stein (1993), hedging against cash shortfalls helps the firm mitigate the risk of not being able to finance valuable investment opportunities. In a more recent paper, Gamba and Triantis (2014) study firms' risk management policies through holding liquid assets (cash equivalent), purchasing financial derivatives, and maintaining operational flexibility. They demonstrate that the strongest motivation for hedging is to avoid financial distress. They show in the model that the three risk management tools are more of complements than substitutes, and cash holding is the most effective out of these three risk management mechanisms. We highlight instead that avoidance of financial default can make financial hedging and operational hedging substitutes. In our model, operational hedging is not a means to avoid financing shortfall, but it is rather the other way around: Hedging against a shortfall of cash that presents a financial default risk reduces the resources allocated to operational hedging for firms facing financial constraints or having low pledgeability of cash flows.[7] Recently, Hu, Varas, and Ying (2021) theoretically show that long-term debt has the benefit of risk management — long-term creditors share the loss of the firm value during the economic downturn. Consistently, we show that a firm's overall credit risk imposes a higher pressure for the firm to give up more operational hedging, in order to conserve more cash to withstand the imminent financial default risk.

Finally, our paper adds to the emerging literature of risk management in production networks. Kulchania and Thomas (2017) find that firms hold more cash to mitigate the consequences of supply chain disruption led by deregulation of trucking industry. Recently, Grigoris, Hu, and Segal (2022) empirically and theoretically study the relationship between trade credit extension to customers and risk premia. Specifically, firms that offer more trade credit earn lower risk premia. Finally, Ersahin, Giannetti, and Huang (2024) exploit the incidence of natural disasters to study how production networks adapt to idiosyncratic shocks, finding that trade credit extension keeps supply chains stable except when suppliers

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[7]See Bianco and Gamba (2019) for a recent theoretical contribution focusing on the risk management role of inventory. They focus on an all-equity firm so do not analyze the effect of credit risk on operational hedging as we do.

are financially constrained. By offering more trade credit to customer firms, a supplier firm hedges against its customer firms' default risk, and therefore lowers the cost of searching for new customers. Our novelty lies with the fact that we allow firms to default on *both* debt contracts and contracts with their customers. This extension gives rise to the competition between financial and operational hedging for the limited liquidity resources of the firm.

2.   The model

2.1   Model setup

This section develops a model of a competitive (price-taking) levered firm's optimal operational hedging policy when facing two types of costly default: financial default (on debt service) and operational default (on customer contract). We build on the financial hedging framework of Acharya, Davydenko, and Strebulaev (2012) by incorporating operational hedging, where financial hedging takes the form of cash savings to prevent default on debt that matures before customer contract settlement dates.

The model considers a single-levered firm with existing debt F in a three-period economy $(t = 0, 1, 2)$, where the debt matures at $t = 1$. The firm owns assets that generate cash flow x_t at $t = 0$ and $t = 1$, with x_2 representing the firm's franchise value. The firm also has a customer contract that requires delivery of I units of goods at unit price p at $t = 2$, where the firm acts as a price-taker.

The model incorporates a random shock u that simultaneously impacts both the firm's period 1 cash flow and its production capacity for customer contract fulfillment, making it a systematic shock. The shock u is realized at $t = 1$. Specifically, the firm's cash flow at $t = 1$ is given by $x_1 = \bar{x}_1 + u$, and its production capacity decreases from I to $(1 - \delta(u))I$, where $\delta(u)$ is a decreasing and convex function in u with continuous and finite first- and second-order derivatives.

The shock u follows a probability distribution with density function $g(u)$ on support $[0, \infty)$, cumulative distribution function $G(u)$ and hazard function $h(u) = \frac{g(u)}{1-G(u)}$. For

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analytical tractability, we assume that u follows an exponential distribution with parameter α, such that $g(u) = \alpha e^{-\alpha u}$ and $G(u) = 1 - e^{-\alpha u}$. This assumption yields a constant hazard function $h(u) = \alpha$.[8] The complete timeline of the model is illustrated in Figure 1, which we further elaborate upon next.

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$p[(1 - \delta(u))I + i] + x_2$ is realized.

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If $[1 - \delta(u)]I + i < I$,

operational default occurs.

$p[(1 - \delta(u))I + i] + (1 - \lambda)x_2$

is realized.

t ——————————————————————— 1

0

Debt level F is maturing in $t = 1$.   cash flow $x_1 = \bar{x}_1 + u$ is realized.

A contract for output I is deliverable in $t = 2$.   Debt F matures.

Cash flow x_0 is realized.

Operational hedging amount i is chosen.   If $x_0 + \bar{x}_1 + u < F + K(I + i)$,

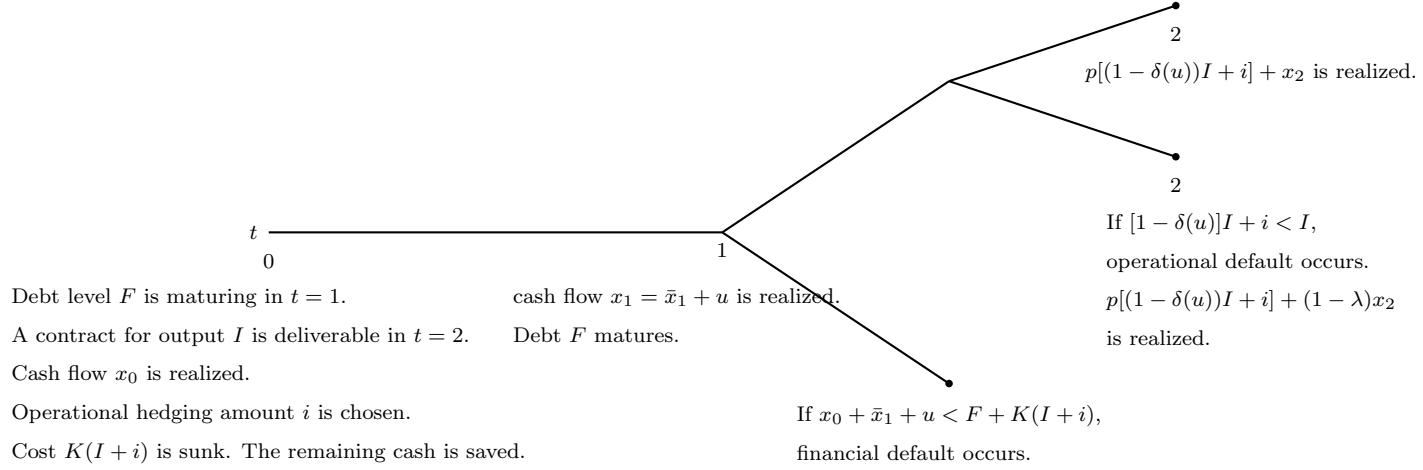Cost $K(I + i)$ is sunk. The remaining cash is saved.   financial default occurs.

Figure 1: The timeline of the model

At date $t = 0$, the assets in place generate a positive cash flow $x_0 > 0$. The firm begins production of I units of goods for delivery at $t = 2$ and can choose to hedge operational risk through a marginal investment i. This investment results in total deliverable goods of $(1 - \delta(u))I + i$, where i can also be interpreted as inventory, and/or spare production capacity.[9] The combined cost of the production and operational hedging is represented by an increasing and convex cost function $K(I + i)$ with continuous and finite first- and second-order derivatives.

Initially, we assume that market frictions prevent the firm from accessing external financing at $t = 0$ and $t = 1$. Consequently, the firm's date-0 disposable cash consists solely of internal cash flow, resulting in cash reserve of $c = x_0 - K(I + i)$. The legacy debt F, payable at $t = 1$, cannot be renegotiated due to high bargaining costs (for example, when held by

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[8]Exponential distribution is a special case of Gamma distribution, which has been widely used to model the jump size distribution of uncertainty shocks in finance (e.g., Johnson, 2021).

[9]In our model the firm is operationally inflexible in the sense that its production amount is confined by the size of the customer contract. We do so to focus on the firm's operational hedging decisions, rather than its investment/disinvestment decisions.

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dispersed bondholders prone to coordination problems). This debt must be serviced from internal funds, $c + x_1$, and failure to fully repay the debt at $t = 1$ results in financial default and liquidation. In this case, the firm loses both its future cash flow from contract delivery, $p[(1 - \delta(u))I + i]$, and its franchise value, x_2. Given that period-1 cash flow x_1 is random, full debt repayment cannot be guaranteed. Furthermore, even without financial default, failing to deliver the contracted I units of goods constitutes operational default, resulting in a partial loss $\lambda \in (0, 1)$ of the franchise value, x_2–for instance, through reputation damage when customers switch to alternative suppliers.

2.2   Discussion

The qualitative results of our model remain robust to variations in its exact specification, provided four key assumptions are maintained: First, default must involve deadweight costs to shareholders. While our model assumes complete loss of future cash flows upon default, the results extend straightforwardly to cases of partial loss. Second, the debt must mature before the customer contract settlement date. This timing creates a fundamental maturity mismatch between debt obligations and customer contract fulfillment. Third, the firm cannot raise significant external financing against future income from customer contract settlement at date-2. If firms could pledge a large portion of settlement income as collateral, current and future cash holdings would become substitutes, eliminating the need for precautionary cash savings. This would remove the tension between financial and operational hedging. While our base model completely prohibits external financing, we later extend it to allow borrowing up to a fraction τ of contract settlement cash flows at $t = 2$. Our main results persist as long as τ remains sufficiently small, reflecting limited pledgeability–a condition likely universal in practice. Fourth, the $t = 1$ shock must affect both date-1 cash flow and the firm's ability to fulfill its customer contract. Though we model this through a single shock affecting both assets in place and production capacity, the framework could accommodate multiple correlated shocks.

2.3 Optimal hedging policies

The firm's optimal hedging policy depends on how the likelihood of financial default compares to that of operational default, which requires us to analyze the shock levels that trigger each type of default. We begin by deriving these threshold values.

At date 1 the firm's available cash for debt service consists of its cash reserve $(x_0 - K(I+i))$ plus interim-period asset cash flow $(x_1 = \bar{x}_1 + u)$, totaling $x_0 - K(I+i) + x_1$. The "financial default boundary" u_F represents the minimum shock level needed to avoid default by enabling full repayment of F: $u_F = F + K(I+i) - x_0 - \bar{x}_1 = \bar{F} + K(I+i)$, where $\bar{F} = F - x_0 - \bar{x}_1$ represents the net debt (debt minus predictable cash flows at date 0 and 1). This financial default boundary u_F increases with both net debt ($\bar{F}$) and operational hedging level (i). When the realized shock u falls between 0 and u_F, the firm defaults on its debt and equity holders receive nothing.

We also consider the possibility of default on the firm's contract with its customer. At date-2, the firm can deliver $(1 - \delta(u))I + i$ units of goods. The firm defaults on its contract with the customer if this amount falls below the production commitment I. The "operational default boundary" u_O represents the minimum shock level that enables full delivery of the contracted amount and avoids operational default: $(1 - \delta(u_O))I + i = I$, or equivalently, $u_O = \delta^{-1}\left(\frac{i}{I}\right)$.

Since the loss function δ decreases in u, its inverse δ^{-1} decreases in i. Consequently, the operational default boundary u_O decreases with the operational hedging level i chosen at date-0, meaning that operational hedging reduces the operational default risk. When the realized shock u falls between 0 and u_O, the firm defaults on its contract with the customer, causing equity holders to lose a portion λ of the franchise value x_2.

The operational default boundary u_O affects equity value function only when it exceeds the financial default boundary u_F. This creates a key challenge in solving the model since both u_F and u_O are endogenously determined by the firm's hedging policy. We solve for the firm's optimal operational hedging policy, with details provided in Appendix I.B.[10]

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[10]It is straightforward to consider hedging being undertaken by a manager who maximizes equity value

Our main result, formalized in **??** below, captures the fundamental tradeoff the firm faces between financial and operational hedging. As a firm's interim financial leverage increases– that is, as net debt $\bar{F}$ maturing at date-1 rises–the motive for financial hedging dominates operational hedging concerns. The firm responds by reducing investment in operational hedging to conserve cash, thereby better protecting against financial default risk. This leads to lower optimal operational hedging levels, denoted generically as i^{**} to encompass all cases.

Proposition 2.1. *The firm's optimal operational hedging policy i^{**} decreases in net debt $\bar{F}$.*

(Proof is in Appendix IC.IV.)

2.4 Operational spread (Markup) and credit spread

An important corollary is that a firm's credit spread is typically positively related with its operational spread or the markup. To see this, we define the credit spread as the ratio of the face value (F) to the market value (L) of the debt minus 1, where the market value of the debt is:

$$L = F - \int_0^{u_F} (u_F - u)\, g(u) du \ . \tag{2.1}$$

The integral term represents the expected loss in value in bankruptcy. The operational spread is defined as the markup, $p - K'(I + i)$. By Proposition 2.1, when x_0 and x_1 are held constant, the optimal operational hedging policy i^{**} decreases with the debt level F. This implies a positive correlation between the credit spread and the operational spread, provided that the market price of debt L/F decreases with F. Intuitively, as the face value of debt F increases, credit spread widens through a *direct* effect: As face value F increases, the promised debt payment becomes larger, making default more likely and widening the credit spread. However, there is an *indirect effect* of F on credit spread through operational hedging: As face value F increases, the firm reduces operational hedging to conserve liquidity for debt payments, countervailing the *direct* effect on the credit spread. Suppose that $K(I +$

net of personal costs arising from firm's bankruptcy (see, for example, Gilson (1989)).

$i) = \kappa N(I+i)$, where $\kappa > 0$ and $N(I+i)$ is a differentiable function that is increasing and convex in $(I+i)$, then we obtain that:

Proposition 2.2. *For κ sufficiently small, the credit spread and the operational spread are positively correlated along the equilibrium path as F varies.*

(Proof is in Appendix IC.V.) When κ is small, the direct effect dominates, maintaining the standard relationship where credit spreads increase with the face value of debt.[11] We verify Proposition 2.2 numerically in Section 2.6.

2.5   Model extensions

2.5.1   The effect of partial pledgeability

Our base case model in Section 2 assumes the firm has no access to external financing. We extend the model to consider partial pledgeability ("PP") of cash flow from the customer contract settlement, using subscript PP to denote quantities in this extension. The results remain qualitatively identical to the base case where the firm cannot pledge any date-2 cash flows to creditors.

At $t = 1$ we allow the firm to use a fraction τ of its proceeds from date-2 customer contract settlement $(\tau p[(1 - \delta(u))I + i])$ as collateral for new financing, where $0 \leq \tau \leq 1$. When $\tau = 0$ we recover the base case with extreme financing frictions, while $\tau = 1$ represents frictionless access to external capital backed by future cash flows. In practice, τ measures the firm's ease of access to cash flow financing.

The key difference from the base case lies in the determination of the financial default boundary. At $t = 1$, raising new financing is value-neutral conditional on survival. Without loss of generality, we can assume that the firm raises an amount equal to the cash shortfall at the financial default boundary, $\tau p[(1 - \delta(u_{F,PP}))I + i]$.[12] The total cash available for debt service at date 1 becomes $x_0 - K(I+i) + x_1 + u + \tau p[(1 - \delta(u_{F,PP}))I + i]$, comprising three

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[11]In Appendix IC.V, we explicitly derive the upper bound of κ.

[12]This amount can always be raised for $u \in [u_{F,PP}, \infty]$, since $\delta(u)$ decreases in u by assumption, making the pledgeable income $\tau p[(1 - \delta(u))I + i]$ increasing in u.

components: the cash reserve $x_0 - K(I + i)$, the random cash flow $x_1 = \bar{x}_1 + u$, and the newly borrowed amount $\tau p[(1 - \delta(u_{F,PP}))I + i]$.

The operational default boundary u_O remains unchanged from the base case, while the financial default boundary takes a new form:

$$u_{F,PP} = \bar{F} + K(I + i) - \tau p[(1 - \delta(u_{F,PP}))I + i] \,. \tag{2.2}$$

For sufficiently low values of τ, the optimal hedging policy maintains the same structure as in the baseline case. Specifically, the optimal operational hedging (i_{PP}^{**}) decreases as the inherited net debt level ($\bar{F}$) increases.

Proposition 2.3. *If $\tau < \bar{\tau}$, the firm's optimal operational hedging policy i^{**} decreases in $\bar{F}$.*

(Proof is outlined in Appendix IC.VI). When $\tau = 0$, we recover the zero-pledgeability case from our base model in Section 2. Since all quantities vary continuous with τ, both Proposition IC.1 and Proposition 2.3 hold when τ is sufficiently low ($\tau \in [0, \bar{\tau}]$). Moreover, as τ increases, the F-region where debt levels do not influence the operational hedging policy expands.

2.5.2 Hedging along the supply chain

We can extend our model to consider the firm that hedges operational default risk through supplier diversification rather than excess inventory or spare production capacity. Let the production function be $K = K(I, n)$, where $n \geq \underline{n}$ represents the measure of suppliers enlisted in the firm's production process, and $\underline{n}$ is the minimal measure of suppliers needed for production.[13]

We assume that a more diversified supply chain (larger n) increases costs, implying positive first- and second-order partial derivatives of K with respect to n: $K_n(I, n) > 0$ and $K_{nn}(I, n) > 0$. We assume that the production loss function $\delta(u, n)$ now depends on

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[13]We assume that n is the measure rather than a count of suppliers to maintain consistency with the first-order conditions in our baseline model.

both the production shock u and the measure of suppliers n. Consistent with our baseline model, $\delta(u, n)$ is decreasing and convex in both arguments with continuous and finite first- and second-order derivatives, $\delta_u(u, n) < 0$, $\delta_n(u, n) < 0$, $\delta_{uu}(u, n) > 0$ and $\delta_{nn}(u, n) > 0$. In addition, we also assume a negative cross-partial derivative $\delta(u, n)$, $\delta_{un}(u, n) < 0$.

In this framework, the operational default threshold u_O satisfies $\delta(u_O, n) = 0$, implying $\frac{\partial u_O}{\partial n} = -\frac{\delta_n(u_O, n)}{\delta_u(u_O, n)} < 0$. The second-order derivative of u_O with respect to n remains positive, as in the baseline case. Under these conditions, our previous results extend naturally: operational hedging through supply chain diversification (n) decreases with the firm's credit risk.

2.6   Numerical illustration

In this section, we present comparative statics to illustrate two key relationships implied by our model solutions: (1) the correlation between optimal hedging policy i^{**} and debt F maturing at date-1, and (2) the relationship between credit spread and operational spread.[14]

For our analysis, we examine the generalized model from Section 2.5.1 that allows for partial pledgeability ($\tau \in [0, 1]$). We make the following parametric assumptions for numerical illustration: cash flow shock (u) follows an exponential distribution with rate parameter $\alpha = 0.05$, giving a probability density function $g(u) = 0.05e^{-0.05u}$. The production loss function is assumed to be $\delta(u) = e^{-u}$. We assume a quadratic production cost function $K(I + i) = \kappa(I + i)^2$ with $\kappa = 0.1$.[15]

Figure 1 presents the relationship between optimal operational hedging i^{**} and debt levels F under three different pledgeability scenarios—low ($\tau = 0$), intermediate ($\tau = 0.4$) and ($\tau = 0.8$)—depicted by blue, red, and yellow lines, respectively. The results reveal several systematic patterns in the optimal hedging behavior. First, when the debt level F is sufficiently low, i^{**} remains constant across all pledgeability scenarios, as the debt can

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[14]While our model treats the debt level (F) as exogenous, one could extend it to solve for optimal capital structure by incorporating tax benefits of debt, though this is beyond our current focus.

[15]Additionally, we set the following parameter values: I (Contractual delivery amount) $= 3$, λ (Proportional cost of operational default) $= 0.5$, p (Unit price) $= 1.2$, x_0 (Cash flow at date-0) $= 5$, $\bar{x}_1$ (Certain cash flow at date-1) $= 5$, and x_2 (Franchise value at date-2) $= 10$.

be repaid at date-1 with certainty regardless of production shock realizations. Second, as F increases beyond this region, i^{**} demonstrates a negative correlation with the debt level maturing at date-1. Third, this negative relationship exhibits a greater magnitude (steeper slope) and persists over a broader range of debt levels F when the pledgeability τ is lower. These findings indicate that the optimal operational hedging policy decreases monotonically with the amount of interim-maturing debt, with this effect being particularly pronounced when the firms faces constraints in accessing external funding (low pledgeability τ).

[INSERT Figure 1.]

Figure 2 examines the relationship between credit spread and operational spread (measured by markup $p - K'(I + i)$) in equilibrium.[16] The analysis reveals a positive correlation between these spreads along the equilibrium path of optimal hedging policies across different debt levels F. This correlation becomes more pronounced as pledgeability τ decreases. These findings support a key prediction of our model: a firm with higher credit risk optimally reduces operational hedging to a greater extent, thereby preserving cash at date-0 to hedge against financial default risk. This reduction in operational hedging manifests as a higher markup in equilibrium, establishing a systematic link between credit risk and operational hedging decisions.

[INSERT Figure 2.]

3. Empirical analysis

Our model produces two hypotheses on the link between operational hedging and credit risk. First, greater credit risk or probability of default lowers operational hedging, indicated by an increase in the price-to-unit cost difference, or the firm's markup. Second, the positive relationship between markup and credit risk is stronger for firms that are financially constrained, indicated in our model as having a lower pledgeability of future cash flows.

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[16]Note that in the partial pledgeability setting ($\tau > 0$), the market value of debt $L = F - \int_0^{u_F} [u_F - u - \tau p(\delta(u_F) - \delta(u))I] g(u) du$.

We measure operational hedging by $Markup = (Sales - CGS)/Sales$, where CGS is the cost of goods sold. Increased spending on operational hedging lowers Markup. Operational hedging is measured by a higher level of inventory. Hoarding inventory indicates the firm's propensity to engage in operational hedging, and by greater breadth of supply chains, which includes expanding and diversifying the number of suppliers with whom the firm works.[17]

We begin with an example illustrating the interaction between the firm's credit risk and its operational hedging policies. Vail Resorts, Inc., a mountain resort company included in our sample, was heavily indebted before the subprime mortgage crisis and the Great Recession. In its 2008 and 2009 annual reports, management expressed concerns regarding the company's highly levered capital structure. Item 1A, Risk Factors, says "Our indebtedness could adversely affect our financial health and prevent us from fulfilling our obligations." To make things worse, its lenders (U.S. Bank and Wells Fargo) experienced a total of 2.7% drop in loan provision during the financial crisis. Correspondingly, Vail held 5.6% less inventory (on average, scaled by its sales) during the recession period compared with the periods before that, and it also terminated the strategic alliance program with Ricoh Co., Ltd. a Japanese company that was Vail's office equipment supplier and stopped being a significant customer with General Mills, a consumer food company (Source: FactSet Revere database). In the meantime, its markup increased by 10.7%. In this case, a high credit risk in a period of tight financing appears to have lowered Vale's inventory and forced the company to reduce its contracting with its suppliers, which led to lower per-unit costs.

Our empirical analysis is structured as follows. We begin by validating that our proposed measures of operational hedging—inventory and supply chain breadth—are consistent with our model mechanics. In the model, operational hedging mitigates the effects of shocks to firms' output, or sales for price taker firms. We test whether the decline in sales higher inventory and greater supply chain breadth mitigate the sales decline during economic shocks measured by the NBER-designated recessions. And, we test whether Markup, our measure of the operational spread, declines with inventory and supply chain breadth.

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[17]Operational hedging may encompass other measures; inventory and supply chain diversification may be the most salient and easy to measure.

We then test the two major predictions of the model. First, we test whether Markup increases in the firm's credit risk measured by the negative value of Altman's (1968) Z-score, which indicates the likelihood of default and positively affects credit spreads. We also test if and on CGS declines in credit risk. Second and most importantly, we use financing shocks to test whether lower pledgeability or tighter financial constraints strengthen the positive relationship between $Markup$ and credit risk. The first test examines the relationship between $Markup$ and $-(Z - score)$ during NBER recession periods, when capital markets are depressed and financing is scarce. The second test employs the shocks to lenders during the subprime mortgage crisis of 2008, which curtailed their ability to provide credit to their relationship borrowers, following Chodorow-Reich (2014). We test whether the Markup (and CGS) of exposed firms—those whose lenders were more strongly hit by the crises—exhibited a stronger relationship with their $-(Z - score)$ measured before the crisis. We conclude by testing whether the effect of financing distress on Markups operates through market power, which enables firms to raise prices. By our proposed mechanism, $Markup$ rises in response to financial distress because firms lower their operational hedging costs.

3.1 Data and empirical definitions

We employ quarterly data from 1971 to April 2020, a span of 197 quarters, from Compustat. We exclude firms in the financial industries (SIC codes 6000-6999) and utility industries (SIC codes 4900-4949), and firm-quarters involved in major mergers (Compustat footnote code AB). We include firm-quarter observations with market capitalization greater than $10 million and quarterly sales more than $1 million at the beginning of the quarter, inflation adjusted to the end of 2019. Our sample includes 18, 338 firms with an average asset value of $2.7 billion dollars (inflation adjusted to the end of 2019). Altogether we have 573, 041 firm-quarters.

3.1.1 Variable definitions

Operational spread is measured by Markup, defined as sales ($SALEQ$) minus cost of goods sold ($COGSQ$) divided by sales. Markup, which is the price-unit cost spread, proxies for our model's spread between price and the marginal cost of production of the output quantity. Our second dependent variable is CGS, defined as the cost of goods sold ($COGSQ$) scaled by assets (ATQ), which increases with operational hedging.

Our key explanatory variable proxies for the firm's ability to pay off its debt liabilities, viz., the negative of Altman's (1968) Z-score.[18] The model includes variables that control for the firm's investment and its debt capacity: Total assets in logarithms, which account for the firm's size; Tobin's Q, which accounts for the firms growth opportunities, calculated as the sum of common shares outstanding ($CHOQ$) multiplied by the stock price at the close of the fiscal quarter ($PRCCQ$), preferred stock value ($PSTKQ$) plus dividends on preferred stock ($DVPQ$), and liabilities (LTQ), scaled by total assets (e.g., Covas and Den Haan, 2011); and three variables that affect the debt capacity, (iii) cash holdings ($CHEQ$), (iv) cash flow ($IBQ + DPQ$) and (v) tangible assets ($PPENTQ$), all scaled by total assets. In models with CGS/Assets as the dependent variable we add to the control variables contemporaneous sales-to-assets ratio because cost of goods sold is partly and mechanically related to sales. We also control for market power, which affects the firm's markup (Lerner, 1934) and inventory behavior (Amihud and Medenelson, 1989), using two variables: a dummy variable that equals one if the firm ranks among the top four sellers in the industry in a given quarter and zero otherwise, and the firm's Sales/Industry sales. Throughout, we use Fama and French's 48 industries.

Operational hedging is indicated by inventory and by supply chain breadth. Inventory ($INVTQ$) scaled by sales proxies for the excess production indicated by i in the theoretical model.[19] The supply chain hedging variable is created using information from the Factset

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[18]Das, Hanouna, and Sarin (2009) finds that corporate bond yields spreads are decreasing in the Z-score. Since $EBIT$ is not available in Compustat quarterly data, we use $OIBDP$ instead in our calculation of the Z-score (Chen et al., 2017).

[19]The 2020 Covid-19 pandemic highlighted the importance of inventory—which in many cases was impossible to replenish at reasonable cost or in a timely manner—and of supply chain diversification to circumvent

Revere relationship database on firms' suppliers.[20] It contains a comprehensive relationship-level data between firms, starting from April 2003. An observation in this database is the relationship between two firms with information about the identities of the related parties, the start and end date of the relationship, the type of the relationship (e.g., competitor, supplier, customer, partner, etc.), and the firms' geographic origins. We aggregate the relationship-level data to firm-quarter level and calculate three measures of supply chain hedging for each firm in each quarter: (i.) ln(1+number of suppliers); (ii.) ln(1+number of supplier regions), where supplier regions are country and state/province combination; (iii.) ln(1+number of out-of-region suppliers), that is, suppliers that are not from the firm's region. We merge the supply-chain data with our main sample, yielding a total of $151,985$ firm-quarter observations covering $6,204$ firms from mid-2003 to the first quarter of 2020. The median firm has 4 suppliers from 3 regions in a given quarter, out of which 3 suppliers are not from the firm's region. The supply chain hedging index, SCH, is the first principal component from the principal component analysis (PCA) of the three individual measures over the whole panel. This first principal component explains 97% of the sample variance. The three measures (i)-(iii) have very similar weights being, respectively, 0.575, 0.580 and 0.578. A higher value of SCH indicates greater supply chain breadth and a more intensive hedging along the supply chain.

Table 1 presents summary statistics of the variables in our study. All continuous variables in our analysis are winsorized at the 1% and 99% tails.

[INSERT Table 1.]

3.2 Hedging operational risk through supply chain and inventory

Operational hedging in our model—indicated by i—can be interpreted as either building up extra inventory (Section 2) or a more stable supply chain (Section 2.5.2). More generally,

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shutdowns of some manufacturing facilities.

[20]Factset Revere has much better coverage of supply chain information than the Compustat segment data and used by some studies about supply chain (e.g., Ding et al., 2020).

it reflects spending on slack and excess means that enable firms to produce the requisite output in case of operational stress. Greater i increases production expenditures while enabling firms to deliver on their contracts and to have higher sales in times of a negative economic shocks that depress output. We thus examine first the effect of our measures of operational hedging—inventory holding and supply chain hedging—on firms' sales during periods of recessions, using the NBER designation.

For each recession period, we estimate a separate cross-sectional regression with the dependent variable being $\Delta(Sales/Assets)$, the change in the average level of firm sales (scaled by total assets) between the recession quarters and the eight-quarter period before the recession. Because a recession may have warning signs which affect the firms' operational hedging before its onset we use the inventory and supply chain hedging data that ends four quarters before the onset of each recession. The control variables are fixed as of the latest quarter before the onset of the recession. In these regressions, we exclude firm-quarters with zero inventory. The model includes industry fixed effects and standard errors are clustered at industry level.

[INSERT Table 2.]

Table 2 presents the results. Higher levels of inventory and supply chain hedging before the recession mitigates the decline in sales during the recession compared with the average sales during the eight pre-recession quarters. Naturally, sales declined during the recessions,[21] but less so for firms with higher inventory and supply chain breadth before the recession. The coefficients of the pre-recessions Inventory/Sales and SCH are all positive and significant, averaging 0.02 across the six recessions. In Table 1, the mean Inventory/Sales ratio is 0.6. Thus, a firm with 0.1 increase in this ratio had a 0.02 lower decline in its Sales/Asset during recessions (the average Sales/Asset is 0.33). For SCH we have data only for the recession of 2007Q4—2009Q2. There too, the coefficient is positive and significant. Overall, we find

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[21]The average sales-assets ratio is 0.012 lower during the recessions, compared with the previous eight-quarter periods. The average decline in sales-assets ratio ranges from -0.023 to 0.007, across the six recessions in our sample. Apart from the first recession (1973Q4—1975Q1), all recessions witness an average decline in sales-assets ratio.

that firms with higher levels of operational hedging suffer less severe disruptions in output deliveries when recession shocks hit.

3.3   Markup, CGS and operational hedging

In our model, the operation spread declines when the firm increases spending on operational hedging. We now test whether Markup, our empirical measure of the operational spread, declines in inventory and supply chain breadth, our empirical measures of operational hedging. We estimate the following model using quarterly data:

$$Markup_{j,t} = \beta_1 * \text{Inv/Sales}_{j,t-1} + \beta_2 * \text{SCH}_{j,t-1} + \text{Control variables}_{j,t-1}$$

$$+ \text{Firm FE} + \text{Industry} * \text{Year-Qtr FE} \tag{3.1}$$

By our model, we expect $\beta_1 < 0$ and $\beta_2 < 0$. We also estimate the model with the dependent variable being $CGS/Assets_{j,t}$. Here we expect $\beta_1 > 0$ and $\beta_2 > 0$.

[INSERT Table 3.]

In Table 3 we find that that Markup and CGS are both affected by the two variables that indicate operational hedging. Higher values of inventory and supply chain hedging, which raise the firm's unit cost, significantly lower Markup and raise CGS. To illustrate the economic significance of the estimated effect, the estimation in column (1) imply that one standard deviation increase in SCH lowers markup by 0.01 and one standard deviation increase in Inventory/sales lowers markup by 0.04. These values are sizable relative to the mean Markup, which is 0.317. After controlling for firms' market power variables and industry-quarter fixed effects (Column (2)), the estimated effects of SCH is 0.007 while that of Inventory/Sales remains the same. Notably, the coefficients of the two market power variables included in Column (2), the top 4 industry seller dummy variable and the firm's Sales/total industry sales ratio, are 0.0056 and -0.57 with respective standard errors of 0.0040 and 0.13. Thus, there is no evidence that market power drives up the markup. Overall, the

results suggest that Markup and CGS reflect in part the effects of the firm's operational hedging on its performance.

3.4   Baseline results

The main hypothesis of our model that firms closer to financial distress reduce spending on operational hedging, resulting in a higher operation spread, which we proxy by Markup, and in lower costs which we measure by CGS/assets. We estimate the following model:

$$Y_{j,t} = \beta_1*-(\text{Z-score})_{j,t-1} + \text{Control variables}_{j,t-1} + \text{Firm FE} + \text{Industry}*\text{Year-Qtr FE} \quad (3.2)$$

$Y_{j,t}$ is either $Markup_{j,t}$ or $CGS/Assets_{j,t}$ and -(Z-score), which is lagged, increases in the firm's credit risk, which implies a higher default spread. By our hypothesis, -(Z-score) has a positive effect on Markup and a negative effect on CGS/Assets. The model includes the control variables used earlier as well as the firm and industry-quarter fixed effects; standard errors are clustered by firm and by year-quarter.

[INSERT Table 4.]

Table 4 presents our baseline results. As predicted in Proposition 2.3, the operational spread measured by Markup is positively affected by the firm's -(Z-score). A higher likelihood of financial default and a greater need for liquidity to hedge financial risk makes firms reduce spending on operational hedging. Then unit cost declines and Markup increases. The economic meaning of the estimated effects is seen in that by column (1), an increase of one standard deviation in -(Z-score) raises the firm's markup by 7% relative to its average markup value, or by 5% relative to its average after controlling for market power and industry-quarter fixed effects. In columns (3) and (4) we find that an increase of one standard deviation in -(Z-score) lowers the CGS/Assets by 2% relative to its average CGS/Assets value, with almost a similar effect after controlling for market power variables and industry-quarter fixed effects. We note again that the market power variables do not have a positive effect on

Markup.[22] Our findings are thus consistent with our key model prediction that the need to avoid financial default induces firms to shift funds away from operation hedging in order to support financial resilience.

3.5 Effect of financial constraint

Our model predicts that a lower pledgeability of the firm's future cash flows—a lower τ in our model—leads to a stronger positive markup-credit risk relationship. A firm facing lower pledgeability is financially constrained. The research question is whether financial constraint makes the markup-credit risk relationship stronger. Because financial constraints is endogenously related to the firm's performance we employ in our tests two exogenously imposed shocks to financial constraints: economic recessions and the 2008 financial crisis. In the first test we examine the effect of a systematic increase in financial constraint, while in the second test we examine the effect of idiosyncratic increase in financial constraints on the firm's markup-credit risk relationship. By our model's prediction, these shocks increase the positive effect of credit risk on Markup and its negative effect on CGS because firms must shift liquidity from operational hedging to hedge against financial default.

3.5.1 Recession periods

Market liquidity is scarce during economic recessions, making it harder for firms to raise capital upon demand if they need to service their financial obligations. We test whether during recessions there is a stronger effect of credit risk on markups, as our model predicts would be the case when pledgeability is limited, meaning that forms are more financially constrained. Our test augments the baseline estimation in Table 4 of the effect of -(Z-score) on Markup by adding an interaction term -(Z-score)×Recession, where Recession is a dummy variable that equals one during the NBER-designated recession quarters and zero otherwise. The values of -(Z-score) and of the firm control variables are fixed for the duration of the

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[22]The coefficients of the two market power variables included in Column (2), the top 4 industry seller dummy variable and the firm's Sales/total industry sales ratio, are -0.0019 and -0.28 with standard errors of 0.0047 and 0.078, respectively.

recession periods at their respective values as of the most recent quarter before the starts of the recession because their values may themselves be affected by the recession.[23]

The results, presented in Table 5, suggest that firms with higher credit risk—higher -(Z-score)—before the onset of a recession reduce their operational hedging, reflected in a greater increase in their Markup (Panel A) and a greater decline in their CGS (Panel B). This results is reflected in the interaction terms -(Z-score)×Recession having positive and significant coefficients in the Markup regressions, columns (1)-(2), and negative and significant coefficients in the CGS regressions, columns (3)-(4). These results support our model's prediction that faced with financial constraint, firms are more aggressive in shifting liquidity from operational hedging to financial hedging, thus lowering their unit cost and raising their markup.

[INSERT Table 5.]

3.5.2 Credit supply shocks in 2008

The second test of the effects of financing constraints on the markup-credit risk relationship employs the firm-specific exposure of firms to the credit shock due to the 2008 financial crisis. During this crisis, especially starting in the fall of 2008, a number of banks could no longer extend credit to firms with which they had lending relationship beforehand. We test whether for firms whose lenders were adversely affected by the 2008 crisis, called "exposed firms", there was a stronger effect -(Z-score)—a positive effect on Markup and a negative effect on CGS/Assets. By our model, exposed firms with higher credit risk should have allocated more resources when becoming financially constrained in order to avoid financial default while reducing spending on operational hedging, thus lowering their cost and raising their Markup.

We employ three measures of the adverse impact of the 2008 crisis on lenders' ability to provide credit, proposed by Chodorow-Reich (2014).[24] (i) %Loans reduction, the number

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[23]See the recommendation, for instance, in Roberts and Whited (2013) on the issue of studying the effects of shocks on the dependent variables.

[24]We thank Chodorow-Reich for sharing his data with us.

of loans that the lenders of a firm extended to all firms (excluding the firm in question), in the nine-month period from October 2008 to June 2009, relative to the average of 18-month period of October 2005 to June 2006 and October 2006 to June 2007. (ii) Lehman exposure, the exposure to Lehman Brothers through the fraction of a bank's syndication portfolio where Lehman Brothers had a lead role. (iii) ABX exposure, the extent of banks' exposure to toxic mortgage-backed securities, calculated using the correlation between banks' daily stock return and the return on the ABX AAA 2006-H1 index.

The relationship between our sample firms and bank lenders is calculated using data from the LPC-Dealscan database. For each firm and each of the three measures, we calculate a weighted average of the measure over all members of the last pre-crisis loan syndicate of the firm, where the weight is the lender's share in the firm's last pre-crisis loan syndicate. The detailed construction of the three variables at the firm level is in Chodorow-Reich (2014). We construct the three variables such that a larger value implies greater exposure to the financial crisis through the firm's lenders. For this analysis, we restrict our sample firms to the 2,429 firms in Chodorow-Reich (2014) database.

We use the following regression specification:

$$
\begin{aligned}
Y_{j,k,t} = {} & \alpha + \beta_1 * -(Z-score)_{j,2007} * Lender\ exposure_{j,t} + \beta_2 * Lender\ exposure_{j,t} \\
& + \sum_m \beta_{3,m} * Controls\ variable_{m,j,t-1} \\
& + \sum_m \beta_{4,m} * Controls\ variable_{m,j,t-1} * Lender\ exposure_{j,t} + \theta_j + \eta_{k,t} + \epsilon_{j,t}\ , \quad (3.3)
\end{aligned}
$$

where $Y_{j,k,t}$ denotes either Markup or CGS/Assets for firm j in industry k in quarter t. The values of Y before and after the crisis are over the pre-crisis and post-crisis two-year periods, July 2006 to June 2008 and January 2009 to December 2010, respectively. Notably, -(Z-score)$_{j,2007}$ is fixed before the crisis as of the end of 2007. Lender exposure equals zero for the pre-crisis period and equals its actual respective values for the post-crisis period. The control variables are the same as in the baseline regression (Table 4), being fixed at the end of year 2007 for the post-crisis period. The model includes firm fixed effects and

Industry-quarter fixed effects and standard errors are clustered at firm levels.

[INSERT Table 6.]

Table 6 presents the results. Consistent with our proposition, the coefficient β_1 is positive and significant for the Markup model (even-numbered columns) and negative and significant for the CGS/Assets model (odd-numbered columns). These results are consistent for all three Lender exposure variables. Markup increased for firms with higher credit risk whose lenders were more adversely affected by the financial crisis. These firms reduced spending on operational hedging, as evident from the lowering of CGS/Assets, shifting resources to avoid financial default. Gauging the economic significance of the joint impacts of the firm's credit risk and its exposure to financial crisis using Column (1) as an example, we find that a one unit higher value of the firm's -(Z-score) and a reduction of the number of loans by its lender to other borrowers by 10% during the financial crisis led to a wider Markup by 0.009. This constitutes 2.5% of the average Markup in the pre-crisis period. By Column (3), one unit higher value of -(Z-score) and a 10% exposure of lender to Lehman led to Markup to widen by 0.016.

Next, we study the dynamic effects of the interaction term $\beta_1 * \text{-(Z-score)}_{j,2007} * \text{Lender exposure}_{j,t}$ before and after the crisis. We replace the Lender Exposure variable in equation (3.3) with an interaction terms $\beta_n * \text{-(Z-score)}_{j,2007} * (\text{Lender exposure}, D_n)$ where the dummy variable D_n equals one for the indicated quarter n and zero otherwise. The indicated n equals $-4, \ldots, -1, 1, \ldots, 4, 5+$. This numbering applies to the last four quarters in the pre-crisis period, Q3-Q4 of 2007 and Q1-Q2 of 2008, then for the four post-crisis quarters, Q1-Q4 of 2009, with $D_{5+} = 1$ for the quarters Q1-Q4 of 2010. We expect the coefficients β_n to be insignificant in the pre-crisis period, $n = -4, \ldots, -1$, and to be significantly positive for the post-crisis period, $n = 1, \ldots, 4$.

Table 7 presents the results. In all columns, the coefficients β_n are mostly significant after the crisis starting from $n = 2$ while being insignificant before the crisis. At the bottom of each column we present F-tests of the joint significance of all the coefficients β_n, conducted

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separately for the four pre-crisis quarters and the four post-crisis quarters. In all tests, the F-value shows strong statistical significance of the coefficients β_n for the post-crisis four quarters while it shows insignificance for the pre-crisis four quarters. Figure 3 illustrates the point estimates, as well as the 95% confidence intervals of the coefficients on the product of -(Z-score) and alternative measures of lender exposure for the periods of four quarters before and after the financial crisis.

Table 7 presents the results. In all columns, the coefficients β_n are mostly significant after the crisis starting from $n = 2$ while being insignificant before the crisis. At the bottom of each column we present F-tests of the joint significance of all the coefficients β_n, conducted separately for the four quarters before the crisis and the four quarters after it. In all tests, the F-value shows strong statistical significance of the coefficients β_n for the post-crisis four quarters while it shows insignificance of the coefficients for the pre-crisis four quarters. Figure 3 illustrates the point estimates, as well as the 95% confidence intervals of the coefficients on the product of -(Z-score) and alternative measures of lender exposure for the periods of four quarters before and after the financial crisis.

[INSERT Table 7.]

Overall, the results show that the tension between operational hedging spending and the needs to avoid financial default is stronger when the firm is hit by a negative shock to its ability to raise capital. Then, it foregoes spending on operational hedging activities and diverts cash to service its financial needs, causing its markup to rise.

3.6   Market power and the markup-credit spread relationship

We now present and test an alternative hypothesis on the markup-credit risk behavior due to Chevalier and Scharfstein (1994) and Gilchrist et al. (2017). In our model, firms are price takers and change in markup reflect changes in marginal cost due to operational hedging. A higher credit risk induces the firm to lower its cost, resulting in a wider markup. Chevalier and Scharfstein (1994) and Gilchrist et al. (2017) propose that firms with market power that

are subject to financial liquidity constraint and higher credit risk may raise their product prices in order to increase short-term cash flow, which would alleviate their liquidity needs. The cost of doing that is forgoing the benefits of higher market share and long run profit. This behavior is more feasible if the customer base is sticky. By this analysis, the positive effect of credit risk and financial constraint on markup is stronger for firms with market power. Naturally, the two explanations for the positive markup-credit risk relationship are not mutually exclusive. It can well be that both motives play a role: In response to financial distress, firms with market power raise their markup by both raising prices and reducing operation hedging and costs, and competitive price-taking firms raise their markup by reducing operational hedging and costs. The question is whether firms without market power also exhibit a positive markup-credit spread relationship, especially when becoming financially constrained.

We test the effect of market power on the markup-credit risk relationship by re-estimating our markup models separately for firms that rank among the top 5% of firms in their industry in terms of sales scaled by industry sales and for the remaining firms. The top 5% firms are viewed as having a greater market power than the remaining firms, which are more likely to be price takers. We call these two groups HMP and LMP for high and low market power, respectively. The models of Chevalier and Scharfstein (1994) and Gilchrist et al. (2017) explain the positive markup-credit risk relationship only for the HMP firms but not for the LMP firms.

We first replicate the markup test in Table 4 on the effect of -(Z-score) on Markup. In Table 8, Panel A, we find that for HMP firms the coefficient of -(Z-score) is practically zero (t-statistic = 0.09) while this coefficient for LMP firms is positive and highly significant. It follows that higher markup due to higher financial risk occurs among LMP firms while being absent among HMP firms.

In Panel B of Table 8, where we replicate the markup tests of Table 5, we find that for HMP firms higher credit risk has a positive but insignificant effect on Markup during economic recessions, while having practically a zero effect otherwise. For LMP firms, Markup

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rises in -(Z-score) and further rises during recessions, when firms are subject to financial constraint, as predicted by our model.

In Panel C of Table 8, we replicate the analysis of Table 6 as it pertains to markup. We test whether firms that were exposed to lenders that were shocked during the 2008 financial crisis were more likely to raise Markup if their financial risk was higher. Specifically, we test the reaction of Markup to the interaction variable -(Z-score)×Lender exposure, where -(Z-score) is set at the pre-crisis level for the duration of the crisis. We find that LMP firms that were exposed to shocked lenders significantly raised their Markup because this exposure increased these firms' financial constraints, inducing them to cut on on operational hedging costs. For HMP firms we find that the coefficient of -(Z-score)×Lender exposure is insignificant for exposure measured by %loan reduction and by ABX exposure. For the Lehman exposure measure, the coefficient of -(Z-score)×Lender exposure is positive and marginally significant in the range of 5% and 10% significance levels (t-statistic = 1.95).

In conclusion, we find that higher financial risk and financial constraint raise markups significantly for firms with low market power which are usually price takers which cannot raise their markup by raising prices. By our model, they raise their markup by reducing their unit cost through a reduction in operational hedging, which enables them to shift liquid resources to hedge against financial risk. For firms with market power, we do not find a significant relationship between credit risk and markup, although in times of economic shocks they show a weakly positive markup-credit risk relationship.

3.7 Operational hedging and value change during Covid-19

Our model suggests that operational hedging does not affect the firm's expected franchise value when credit risk is high (Appendix IB.II). This is because the lower probability that the firm will survive the Period 1 shock, which may lead to default, lowers the probability of it realizing the benefit from delivering the contracted output in Period 2 and its future franchise value thereafter, denoted x_2. With a sufficiently high credit risk, operational default risk is dominated by financial default risk. Then, variations in operational hedging have little or

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no effect on the firm's value.

We test this prediction indirectly using the firms value change during the Covid era as the Covid shock can be viewed as a very low realization of the shock variable u in our theory, which creates both financial and operational default risks. By our analysis, pre-Covid operational hedging choices matter less for the value of firms that entered Covid with pre-existing high credit risk. In our test, we estimate a cross-section regression of the firm stock return during 2020—2021 on our two measures of operational hedging, SCH (supply chain hedging) and Inventory/Sales ratio, both at the end of 2019. The control variables are Book/Market ratio and Size (in logarithm), which are known to affect stock returns across firms, using the end-of-2019 values. The model also includes the percent change in sales during 2019 to control for mechanical changes of Inventory/Sales ratio in 2019 due to sale changes. And we include industry fixed effects. Finally, we split our sample into two halves by the sample median of -(Z-score) and estimate regressions separately for each group, which enables to distinguish between results for high and low credit risk firms.

Table 9 presents the results. Entering the Covid crisis with a higher SCH, which means a better supply chain hedging due to diversified chain of suppliers, contributed significantly to having a higher stock return during the crisis, while the effect of Inventory/Sales ratio is insignificant. However, it is only for firms with low credit risk—those with below-median -(Z-score)—that operational hedging significantly contributed to value. The coefficients of both SCH and Inventory/sales are positive and significant. For firms with higher credit risk—those with above-median -(Z-score)—there is an insignificant effect of both SCH and Inventory/Sales on value. The results thus suggest that in times of supply chains disruptions and depletion of inventories, having invested in operational hedging is value-increasing as long as the firm is not threatened by imminent financial default. These results are consistent with the intermediate predictions of our theory (Appendix IB.II).

[INSERT Table 9.]

4. Conclusion

This paper examines how corporations balance financial efficiency and operational resiliency. We develop a model of a competitive (pricing-taking) firm that must allocate liquidity resources between two forms of hedging: financial hedging through cash reserves to reduce the risk of financial default, and operational hedging through investment in inventory and supply chain network that reduces the risk of operational default such as a failure to deliver on obligations to customers. Our analysis shows that this tradeoff is especially pronounced when the firm faces external financing constraints, leading to a positive relationship between operational spread (markup) and credit risk.

We present empirical evidence supporting our model predictions. We first establish that markup effectively captures a firm's operational hedging activities, as measured by inventory holdings and supply chain hedging. We then demonstrate a robust positive relationship between the firm's markup and its credit risk. This relationship strengthens when the firm faces heightened incentives to preserve liquidity for averting financial default. Specifically, the markup-credit risk relationship intensifies during economic recessions and became particularly pronounced following the subprime financial crisis, especially for firms whose lenders experienced greater crisis exposure. Overall, our empirical findings strongly support our key premise that the tension between financial and operational hedging becomes more acute when the firm encounters greater external financing constraints.

We conclude by identifying promising directions for future research. From a theoretical perspective, extending our partial equilibrium framework to a general equilibrium production network model would offer valuable insights. In such a model, product pricing, credit risk and operational hedging decisions would emerge as equilibrium outcomes of the interconnected system, with each firm's operational hedging choices influencing the operational risks faced by its upstream and downstream network partners. This approach would enable analysis of network externalities in operational hedging, particularly the potential underinvestment in operational resiliency that may arise from credit risk spillovers across connected firms. From

an empirical perspective, several important questions await investigation. These include developing a more granular understanding of various operational hedging strategies, evaluating their comparative effectiveness, and precisely measuring their impact on product prices. Addressing these questions will require more comprehensive data on firms' operational hedging practices.

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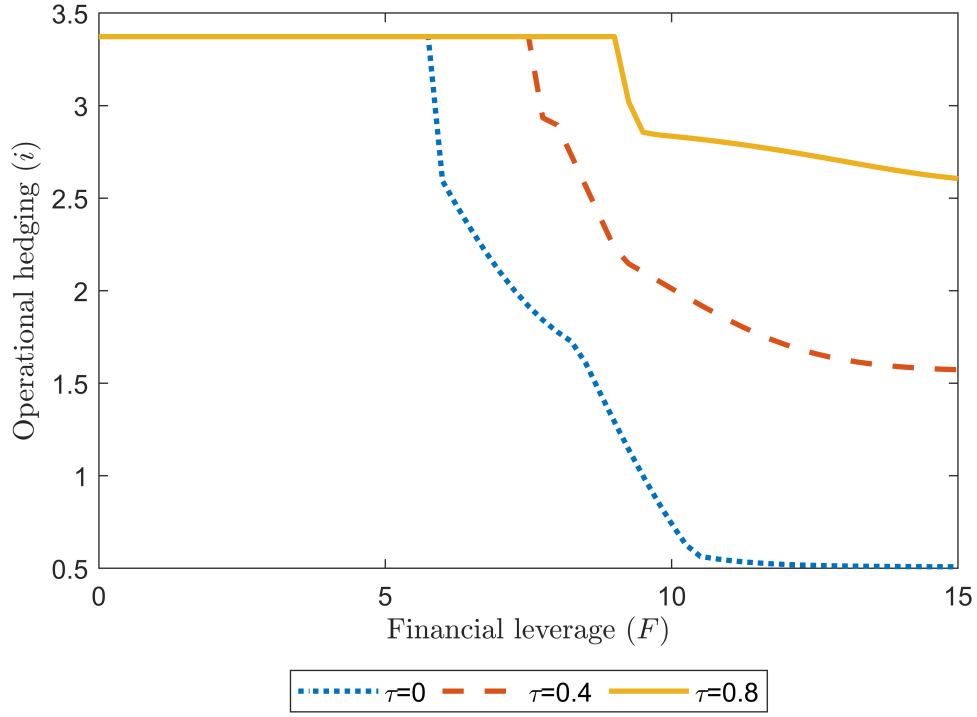
Figure 1: **Firm's optimal hedging policy** i^{**} **and debt level** F

Optimal hedging policy i^{**} given debt level F for $\tau = 0$, $\tau = 0.4$ and $\tau = 0.8$, where τ is a measure of the extent of the need for pledgeability, which proxies financial constraint.

Figure 2: **Credit spread and operational spread**

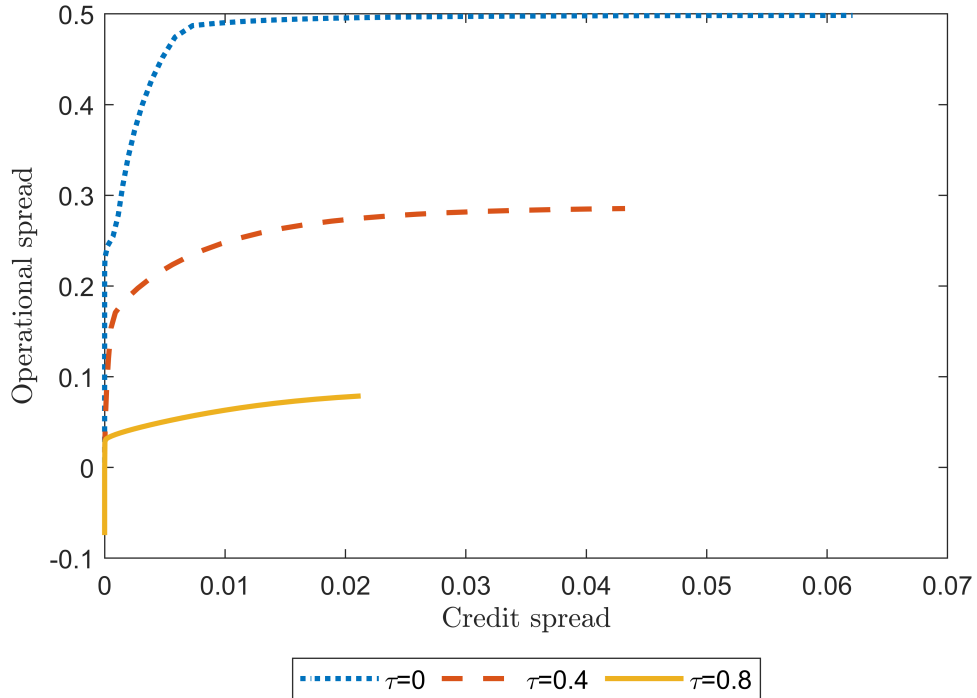The credit spread and operational spread under the optimal hedging policy i^{**} given debt level F for $\tau = 0$, $\tau = 0.4$ and $\tau = 0.8$.

Figure 3A: Markup – Coefficient on -(Z-score)×LE: % # Loans reduction

Markup: % # Loan reductions
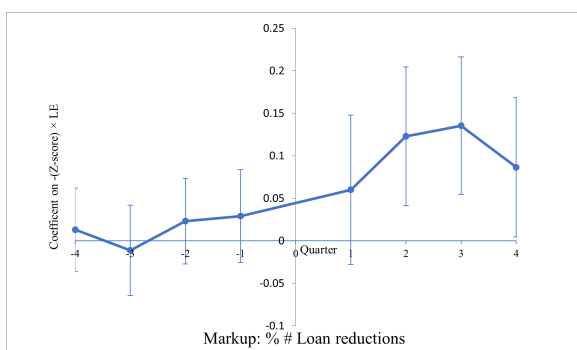
Figure 3D: CGS – Coefficient on -(Z-score)×LE: % # Loans reduction

CGS: % # Loan reductions
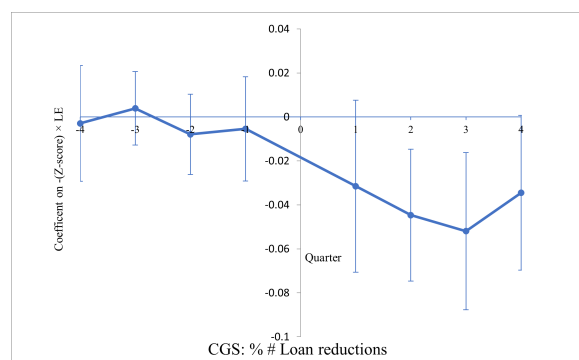
Figure 3B: Markup – Coefficient on -(Z-score)×LE: Lehman exposure

Markup: Lehman exposure
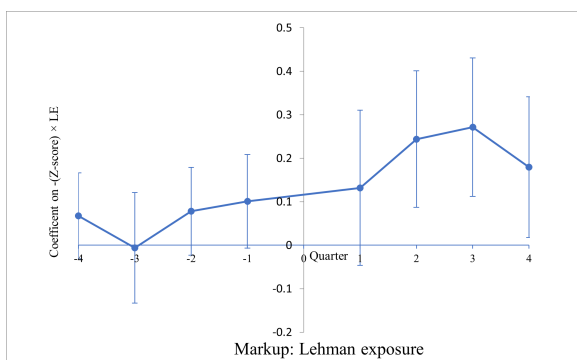
Figure 3E: CGS – Coefficient on -(Z-score)×LE: Lehman exposure

CGS: Lehman exposure
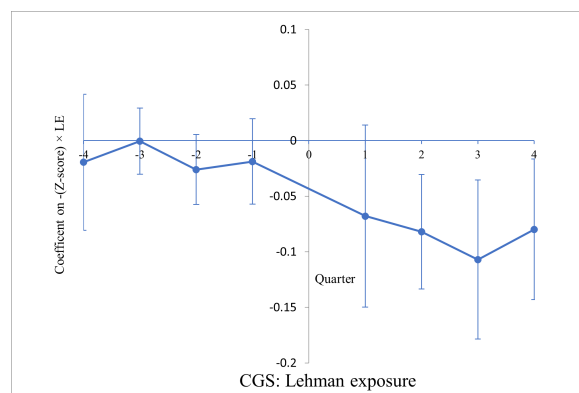
Figure 3C: Markup – Coefficient on -(Z-score)×LE: ABX exposure

Markup: ABX exposure
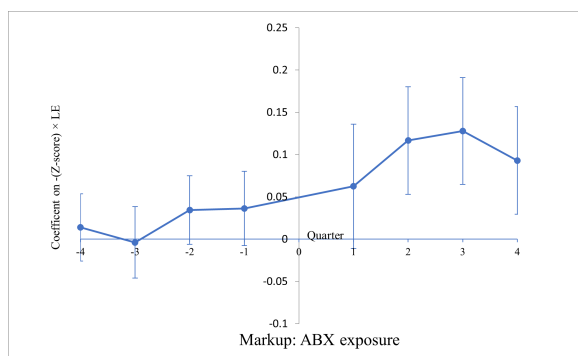
Figure 3F: CGS – Coefficient on -(Z-score)×LE: ABX exposure

CGS: ABX exposure
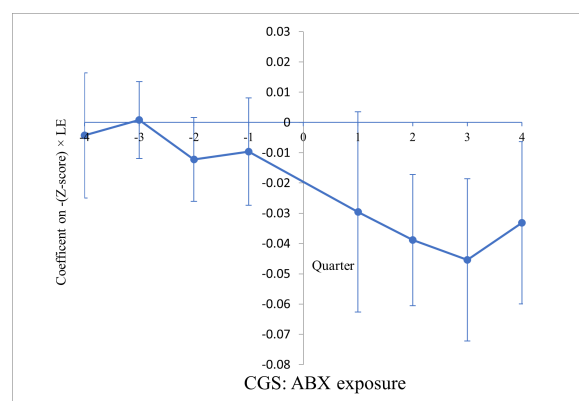
Figure 3: **Markup, CGS and credit risk: Dynamic effects of exposure to the financial crisis**

This figure plots the the point estimates of the coefficients on -(Z-score)*LE in the markup and CGS regressions, as of Table 7, and their 95% confidence intervals.

Table 1: **Summary statistics** — Compustat 1973-2020

Summary statistics of the variables in our sample from 1971 to April 2020. The data are quarterly from Compustat; The variable names are in parentheses. Markup = (Sales($SALEQ$) − cost of goods sold($COGSQ$))/Sales. CGS/assets = CGS($COGSQ$)/total assets(ATQ). Z-score, a measured credit risk, is calculated from quarterly data following Altman (1968). Tobin's Q is the firm's market value (the sum of common shares outstanding($CHOQ$) multiplied by stock price at the close of the fiscal quarter($PRCCQ$), preferred stock value($PSTKQ$), dividends on preferred stock($DVPQ$) and liabilities(LTQ)), divided by its total assets, following Covas and Den Haan (2011). Cash holdings ($CHEQ$), Cash flow ($= IBQ + DPQ$) and Tangible assets ($PPENTQ$) are divided by total assets. Market power is measured by two variables, all employing Fama and French's 48 industries: a dummy variable for the top 4 industry seller, $= 1$ if the firm's sales are among the top four sellers in the industry (0 otherwise); and firm's Sales/Industry sales. The operational hedging variables include Inventory-to-sales ratio ($INVQ$)/Sales (limited to strictly positive values), and Supply chain hedging index, composed from a principal component analysis (PCA) that employs three measures: (i) ln(1+number of suppliers), (ii) ln(1+number of supplier regions), (iii) ln(1+number of suppliers not from the firm's region). Data are quarterly (source: Factset), covering 6,204 firms from mid-2003 to the first quarter of 2020. SCH (supply chain hedging) is the first principal component score that equals $0.575 \times$ (i) $+ 0.580 \times$ (ii) $+ 0.578 \times$ (iii), where (i)—(iii) indicate the above three measures.

We require that the lagged firm capitalization is at least \$10 million and quarterly sales are at least \$1 million (inflation adjusted to the end of 2019). All continuous variables are winsorized at both the 1st and 99th percentiles.

VARIABLES	N	Mean	S.D.	P25	P50	P75
Markup: (sales-cogs)/sales	572,345	0.317	0.428	0.208	0.338	0.508
CGS/assets	569,049	0.209	0.188	0.079	0.162	0.277
-(Z-score)	573,041	-3.542	5.872	-3.995	-2.082	-1.081
Tobin's Q	573,041	1.981	1.597	1.073	1.446	2.211
Cash holdings	573,041	0.164	0.197	0.024	0.082	0.232
Cash flow	573,041	0.010	0.056	0.005	0.021	0.035
Asset tangibility	573,041	0.303	0.243	0.104	0.235	0.448
Top 4 industry seller	573,041	0.039	0.193	0.000	0.000	0.000
Sales/industry sales	573,041	0.009	0.026	0.000	0.001	0.005
Total assets	573,041	2,738.859	8,390.609	79.178	299.321	1,338.695
Inventory/sales	465,600	0.592	0.520	0.224	0.489	0.793
Supply chain hedging (SCH)	116,430	-0.010	1.697	-1.334	-0.381	0.956

Table 2: **The effect of operational hedging on changes in sales during NBER recessions**

Cross-sectional regressions of changes in the Sales/Assets ratio during recessions compared with the pre-recession on the level of firms' operational hedging variables, Inventory/Sales and SCH. The dependent variable is Δ(Sales/Assets), the difference between average sales/assets during the recession quarters and average sales/assets over eight quarters before the recession. The recession quarters are as designated by the NBER. The main explanatory variables are Inventory/Sales ratio and SCH measured by the supply chain hedging PCA index, fixed at four quarters before the onset of recession (or earlier). All regressions include control variables: Tobin's Q, natural logarithm of total assets, cash holdings, cash flow, and asset tangibility. All the control variables are fixed as of the last quarter before the onset of each recession. We include Fama-French 48 industry fixed effects and cluster the standard errors at industry level. $*$, $**$, $***$ denote significance below 10%, 5%, and 1% levels, respectively.

VARIABLES	Δ sales/assets					
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Inventory-sales ratio						
Recession period	1973Q4	1979Q2	1981Q2	1989Q4	2001Q1	2007Q4
	—	—	—	—	—	—
	1975Q1	1980Q2	1982Q2	1991Q1	2001Q3	2009Q2
Inventory/sales	0.037**	0.016**	0.013*	0.016***	0.021***	0.011**
Standard error	0.015	0.008	0.007	0.004	0.004	0.005

Panel B: Supply chain hedging PCA, for the recession of 2007Q4 to 2009Q2	
	Supply chain hedging (SCH)
SCH	0.002**
Standard error	0.001
Control variables	Yes
FF-48 industry fixed effects	Yes

Table 3: **Markup, CGS and operational hedging**

Estimation of the relationship between Markup (columns (1) and (2)), CGS/Assets (columns (3) and (4)) and two measures of operational hedging: SCH (supply chain hedging), the first principal component of three supply chain diversification measures, and Inventory/Sales ratio. The control variables include Tobin's Q, ln(Total assets), Cash holdings, Cash flow, and Tangible assets. In CGS regressions (columns (3) and (4)) we also control for contemporaneous Sales/assets. The variables are defined in Table 1. In even-numbered columns, we also control for market power variables: a dummy variable for the top 4 industry seller and the firm's Sales/total industry sales. All explanatory variables are lagged by one quarter. The regressions include firm and Fama-French 48 Industry×year-quarter fixed effects. Standard errors are clustered at firm and year-quarter levels. *, **, * * * denote significance below 10%, 5%, and 1% levels, respectively.

VARIABLES	*Markup*		*CGS/assets*	
	(1)	(2)	(3)	(4)
SCH	-0.0076***	-0.0042**	0.00088***	0.00063**
	(0.0021)	(0.0018)	(0.00030)	(0.00029)
Inventory/sales	-0.076***	-0.076***	0.0061***	0.0067***
	(0.014)	(0.014)	(0.0013)	(0.0013)
Market power variables	No	Yes	No	Yes
Other Control variables	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes
Industry×Year-quarter fixed effects	No	Yes	No	Yes
Observations	93,853	92,762	93,772	92,681
R-squared	0.698	0.718	0.975	0.977

Table 4: **Markup, CGS and credit risk**

Regressions of Markup and CGS/Asset on credit risk, measured by Altman's (1968) -(Z-score). The dependent variables are the quarterly Markup (columns (1) and (2)) and CGS/Assets (columns (3) and (4)). The control variables include Tobin's Q, Ln(total assets), Cash holdings, Cash flow, and Tangible assets. In the CGS models we also control for contemporaneous Sales/Assets. In even-numbered columns, we also control for market power variables: a dummy variable for the top 4 industry seller, and Sales/Total industry sales. All explanatory variables are lagged by one quarter. The regressions include firm and Fama-French 48 Industry×year-quarter fixed effects. Standard errors are clustered at firm and year-quarter levels. $*$, $**$, $***$ denote significance below 10%, 5%, and 1% levels, respectively.

VARIABLES	Markup		CGS/assets	
	(1)	(2)	(3)	(4)
-(Z-score)	0.0037***	0.0029***	-0.00058***	-0.00054***
	(0.00057)	(0.00053)	(0.000080)	(0.000079)
Market power variables	No	Yes	No	Yes
Other Control variables	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes
Industry×Year-quarter fixed effects	No	Yes	No	Yes
Observations	571,388	564,418	568,015	561,177
R-squared	0.614	0.634	0.949	0.951

Table 5: **Markup, CGS and credit risk: NBER recessions**

Regressions of Markup or CGS/Assets on -(Z-score) that interacts with a Recession dummy variable which equals one for the following quarters: 1973Q4–1975Q1, 1979Q2–1980Q2, 1981Q2–1982Q2, 1989Q4–1991Q1, 2001Q1–2001Q3 and 2007Q4–2009Q2. We exclude the Covid-related recession during 2020Q1. For each recession, the values of -(Z-score) and the control variables during the recession periods are fixed as of the last quarter before the onset of the recession. The control variables are as in Table 4, and we control for contemporaneous Sales/Assets in the CGS regressions. The variable definitions are in Table 1. The regressions include firm and Fama-French 48 Industry×year-quarter fixed effects. Standard errors are clustered by firm and year-quarter levels. $*$, $**$, $***$ denote significance below 10%, 5%, and 1% levels, respectively.

VARIABLES	*Markup*		*CGS/assets*	
	(1)	(2)	(3)	(4)
-(Z-score)×Recession	0.0019**	0.0016***	-0.00023**	-0.00025**
	(0.00075)	(0.00051)	(0.00011)	(0.00010)
-(Z-score)	0.0035***	0.0028***	-0.00057***	-0.00053***
	(0.00056)	(0.00052)	(0.000077)	(0.000077)
Market power variables	No	Yes	No	Yes
Other Control variables	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes
Industry×Year-quarter fixed effects	No	Yes	No	Yes
Observations	563,120	554,348	560,343	551,691
R-squared	0.616	0.636	0.948	0.950

Table 6: **Markup, CGS and credit risk: Exposure to the financial crisis**

Regressions of Markup and CGS on firms' -(Z-score) that interacts with the extent of exposures to the 2008 financial crisis. The sample include the $2,429$ firms in Chodorow-Reich (2014). The two-year periods before and after the crisis are July 2006 to June 2008, and January 2009 to December 2010, respectively. The three measures for crisis exposure are %Loans reduction, Lehman exposure and ABX exposure, using Chodorow-Reich's (2014) variables. The variable Lender exposure equals zero for the two-year period before the crisis, and equals its actual respective value for the two-year period after the crisis. The values of -(Z-score) are as of the end of 2007. The firm-level control variables (including market power variables) are as in Table 4, fixed at the end of 2007 for the entire post-crisis period. In CGS regressions we also control for the contemporaneous Sales/Assets. The specification is as in the model

$$Y_{j,k,t} = \alpha + \beta_1 * -(Z - score)_{j,2007} * Lender\ exposure_{j,t} + \beta_2 * Lender\ exposure_{j,t}$$

$$+ \sum_m \beta_{3,m} * Control\ variable_{m,j,t-1}$$

$$+ \sum_m \beta_{4,m} * Controls\ variables_{m,j,t-1} * Lender\ exposure_{j,t} + \theta_j + \eta_{k,t} + \epsilon_{j,t}\ ,$$

The variables are defined in Table 1. The regressions include Market power variables, Market power variables×lender exposure, other Control variables, other Control variables×lender exposure, firm and Fama-French 48 Industry×year-quarter fixed effects. Standard errors are clustered by firm. $*$, $**$, $***$ denote significance below 10%, 5%, and 1% levels, respectively.

VARIABLES	% # Loans reduction		Lehman exposure		ABX exposure	
	Markup	*CGS/assets*	*Markup*	*CGS/assets*	*Markup*	*CGS/assets*
	(1)	(2)	(3)	(4)	(5)	(6)
-(Z-score)×lender exposure	0.086**	-0.030***	0.160**	-0.058***	0.084***	-0.027***
	(0.034)	(0.011)	(0.072)	(0.021)	(0.027)	(0.008)
Lender exposure	-0.699	0.017	-0.969	-0.149	-0.902**	0.019
	(0.455)	(0.157)	(0.689)	(0.221)	(0.410)	(0.129)
Market power variables			Yes			
Market power variables×lender exposure			Yes			
Other Control variables			Yes			
Other Control variables×lender exposure			Yes			
Firm fixed effects			Yes			
Industry×Year-quarter fixed effects			Yes			
Observations	20,621	20,613	20,621	20,613	20,621	20,613
R-squared	0.905	0.987	0.905	0.987	0.906	0.987

Table 7: **Markup, CGS and credit risk:**
Dynamic effects of exposure to the financial crisis

Regressions of Markup and CGS/Assets on firms' -(Z-score) that interacts with the extent of lender exposures to the 2008 Financial Crisis. The estimation is of model (3.3), replacing the Lender Exposure variable with an interaction terms $\beta_n \times -(Z-score)_{j,2007} \times (LE, D_n)$. LE is lender exposure, measured by three variables. D_n is a dummy variable that equals one for the indicated quarter n and zero otherwise, where n equals $-1, -2, -3, -4, +1, +2, +3, +4, +5+, +5+$ capturing quarters +5 to + 8. This numbering applies to the last four quarters in the pre-crisis period, Q3-Q4/2007 and Q1-Q2/2008, and for the four post-crisis quarters, Q1-Q4/2009, with $D_{5+} = 1$ for the quarters Q1-Q4/2010. The lender exposure (LE) measures are due to Chodorow-Reich (2014); see details in Table 6. The values of -(Z-score) and the control variables are as of the end of 2007. The last two rows show the results from F-test for joint significance of the coefficients of the interaction terms between -(Z-score) and the size of LE for quarters D_n. The regressions include firm and Fama-French 48 Industry×year-quarter fixed effects. Standard errors are clustered by firm. $*, **, ***$ denote significance below 10%, 5%, and 1% levels, respectively.

VARIABLES	% # Loans reduction		Lehman exposure		ABX exposure	
	Markup	*CGS/assets*	*Markup*	*CGS/assets*	*Markup*	*CGS/assets*
$-(Z-score) \times LE, D_{-4}$	0.013	-0.003	0.068	-0.019	0.014	-0.004
	(0.025)	(0.013)	(0.050)	(0.031)	(0.020)	(0.011)
$-(Z-score) \times LE, D_{-3}$	-0.011	0.004	-0.006	-0.00041	-0.004	0.001
	(0.027)	(0.009)	(0.065)	(0.015)	(0.022)	(0.006)
$-(Z-score) \times LE, D_{-2}$	0.023	-0.008	0.078	-0.026	0.034*	-0.012*
	(0.026)	(0.009)	(0.051)	(0.016)	(0.021)	(0.007)
$-(Z-score) \times LE, D_{-1}$	0.029	-0.005	0.101*	-0.019	0.036	-0.010
	(0.028)	(0.012)	(0.055)	(0.020)	(0.022)	(0.009)
$-(Z-score) \times LE, D_1$	0.060	-0.032	0.132	-0.068	0.062*	-0.030*
	(0.045)	(0.020)	(0.091)	(0.042)	(0.037)	(0.017)
$-(Z-score) \times LE, D_2$	0.123***	-0.045***	0.244***	-0.082***	0.117***	-0.039***
	(0.042)	(0.015)	(0.080)	(0.026)	(0.032)	(0.011)
$-(Z-score) \times LE, D_3$	0.135***	-0.052***	0.272***	-0.107***	0.128***	-0.045***
	(0.041)	(0.018)	(0.081)	(0.036)	(0.032)	(0.014)
$-(Z-score) \times LE, D_4$	0.086**	-0.035*	0.180**	-0.080**	0.093***	-0.033**
	(0.042)	(0.018)	(0.082)	(0.032)	(0.032)	(0.014)
$-(Z-score) \times LE, D_{+5+}$	0.083*	-0.021	0.170*	-0.046*	0.087**	-0.022**
	(0.043)	(0.014)	(0.091)	(0.025)	(0.034)	(0.010)
Lender exposure, D_n				Yes		
Control variables				Yes		
Control variables×Lender exposure				Yes		
Firm fixed effects				Yes		
Industry×year-quarter fixed effects				Yes		
Observations	19,914	19,906	19,914	19,906	19,914	19,906
R-squared	0.903	0.987	0.903	0.987	0.904	0.987
F-statistic for $n = +1$ to $+4$	3.83***	2.88**	3.79***	3.08**	4.62***	3.57***
F-statistic for $n = -1$ to -4	0.63	0.74	1.29	1.00	1.21	1.57

Table 8: **Markup and credit risk: Separate estimations by market power**

Panel A is a replication of the Markup test of Table 4: regressions of Markup on -(Z-score). Panel B is a replication of the Markup test in Table 5: regressions of Markup and CGS on -(Z-score) that interacts with indicators of NBER recession periods. Panel C is a replication of the Markup tests in Table 6: regressions of Markup on -(Z-score) that interacts with the exposures to the 2008 Financial Crisis. Firms are sorted in each quarter into two groups by their sales scaled by the total industry sales: Those that rank among the top 5% of firms (HMP) in the industry and the remaining firms (LMP). We then estimate our models separately for each group based on the sorting that is done in the lagged quarter. The dependent variable is Markup. All models include control variables, firm and Industry×year-quarter fixed effects.

	Panel A: Baseline		Panel B: NBER recessions	
	HMP-Top 5%	LMP	HMP-Top 5%	LMP
VARIABLES	Markup			
	(1)	(2)	(3)	(4)
-(Z-score)	0.00013	0.0029***	-0.00030	0.0028***
	(0.0015)	(0.00053)	(0.0015)	(0.00052)
-(Z-score)×Recession			0.0024	0.0016***
			(0.0018)	(0.00051)
Control variables		Yes		
Firm fixed effects		Yes		
Industry×year-quarter fixed effects		Yes		
Observations	23,681	539,460	23,282	529,811
R-squared	0.903	0.633	0.901	0.635

Panel C: 2008 Financial Crisis

	% # Loans reduction		Lehman exposure		ABX exposure	
	HMP-Top 5%	LMP	HMP-Top 5%	LMP	HMP-Top 5%	LMP
VARIABLES	*Markup*					
	(1)	(2)	(3)	(4)	(5)	(6)
-(Z-score)×lender exposure	0.219	0.086**	0.723*	0.164**	0.316	0.085***
	(0.211)	(0.034)	(0.370)	(0.072)	(0.239)	(0.028)
Lender exposure	2.394	-0.663	0.682	-1.015	1.495	-0.903**
	(2.908)	(0.505)	(4.093)	(0.774)	(2.509)	(0.449)
Control variables			Yes			
Firm fixed effects			Yes			
Industry×year-quarter fixed effects			Yes			
Observations	829	19,605	829	19,605	829	19,605
R-squared	0.982	0.903	0.982	0.903	0.982	0.903

Table 9: **Operational hedging and stock return during Covid period**

Cross-section regressions of firms' stock returns over the two-year period 2020-2021 on measures of operational hedging at the end of 2019: SCH that measures the supply chain diversification, and Inventory/Sales ratio, both defined in Table 1. The control variables are Book/market ratio and equity capitalization (both in logarithm), denoted ln(B/M) and ln(Size), and the % changes in sales in 2019. The regressions include Fama-French 48 Industry fixed effects. High and low -(Z-score) are defined as -(Z-score) being above and below the sample median, respectively. $*$, $**$, $***$ denote significance below 10%, 5%, and 1% levels, respectively.

| | 2020 — 2021 stock return | | |
| | Full sample | High -(Z-score) | Low -(Z-score) |
	(1)	(2)	(3)
Ln(inventory/sales)	0.015	-0.020	0.051*
	(0.030)	(0.054)	(0.029)
SCH	0.064***	0.021	0.049**
	(0.023)	(0.035)	(0.022)
Ln(B/M)	-0.227***	-0.161	-0.296***
	(0.075)	(0.184)	(0.074)
Ln(size)	-0.135***	-0.171***	-0.052*
	(0.034)	(0.057)	(0.026)
% changes in sales, 2019	-0.190	-0.359	0.132
	(0.218)	(0.354)	(0.254)
Industry fixed effects		Yes	
Observations	1,664	795	737
R-squared	0.070	0.096	0.161

Internet Appendix

I.A.   Complete table of Table 4

Table IA.1: **Markup, CGS and credit risk — Complete table**

This table reports the complete table of Table 4.

VARIABLES	Markup		CGS/assets	
	(1)	(2)	(3)	(4)
-(Z-score)	0.0037***	0.0029***	-0.00058***	-0.00054***
	(0.00057)	(0.00053)	(0.000080)	(0.000079)
Tobin's Q	0.021***	0.019***	-0.0048***	-0.0048***
	(0.0020)	(0.0019)	(0.00036)	(0.00035)
Ln assets	0.0073***	0.0058**	0.0035***	0.0036***
	(0.0028)	(0.0026)	(0.00049)	(0.00054)
Cash holdings	-0.070***	-0.065***	0.0010	0.0012
	(0.015)	(0.015)	(0.0022)	(0.0022)
Cash flow	0.91***	0.85***	-0.19***	-0.18***
	(0.044)	(0.038)	(0.0072)	(0.0069)
Asset tangibility	-0.035**	-0.0061	-0.015***	-0.015***
	(0.014)	(0.014)	(0.0029)	(0.0029)
Top 4 industry seller		-0.00019		0.00018
		(0.0047)		(0.0019)
Sales/industry sales		-0.28***		0.071***
		(0.078)		(0.021)
Sales/AT			0.75***	0.75***
			(0.0054)	(0.0054)
Market power variables	No	Yes	No	Yes
Other Control variables	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes
Industry × Year-quarter fixed effects	No	Yes	No	Yes
Observations	571,388	564,418	568,015	561,177
R-squared	0.614	0.634	0.949	0.951

I.B. Model Solutions Under Different Default Risk Scenarios

We solve the model by considering three different scenarios based on the relative magnitudes of financial default boundary (u_F) and operational default boundary (u_O). First, we analyze the benchmark case where net debt is zero ($\bar{F} = 0$), then examine the case where financial default risk dominates ($u_F \geq u_O$), and finally study the case where operational default risk dominates ($u_F < u_O$). This systematic approach allows us to fully characterize the optimal hedging policies under all possible scenarios.

IB.I Benchmark: Optimal hedging policy $\bar{i}$ when $F = 0$

For net debt level $\bar{F}$ close to zero, the financial default boundary remains below the operational default boundary when the firm chooses the first-best hedging level $\bar{i}$ that ignores credit risk, provided the following technical assumption holds:[25]

Assumption IB.1.
$$K(I + \bar{i}) < x_0 \ , \tag{IB.1}$$

where $\bar{i}$ is the solution of equation (IC.2).

This assumption guarantees that the firm's date-0 cash flow is sufficient to implement the first-best operational hedging level $\bar{i}$ when only operational default risk is present.

We now analyze cases where both financial and operational defaults are possible. In these scenarios, the firm faces a fundamental tradeoff between allocating resources to prevent financial default and maintaining operational hedging. For our analysis, we maintain the following technical assumption:

Assumption IB.2. *The commitment to production I is sufficiently high and $\frac{K'(I+\bar{i})}{I}$ is sufficiently low that the inequalities $p - K'(I) > (pI + x_2)\alpha K'(I)$ and* (IC.15)< 0 *hold simultaneously.*

This assumption ensures the existence of unique and positive interior optimal hedging levels i^* and $\hat{i}^*$ in regions where $u_F \geq u_O$ and $u_F < u_O$, respectively.

The economic interpretation of these conditions is straightforward. The requirement that I be sufficiently high means that the firm's customer contract is substantial enough to warrant operational hedging consideration. Following standard investment literature, we

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[25]We provide a formal proof in Appendix IC.I.

specify the production cost function as quadratic: $K(I + i) = \kappa(I + i)^2$, where $\kappa > 0$. The condition that $\frac{K'(I+\bar{i})}{I}$ be sufficiently low is satisfied by choosing a sufficiently small κ, ensuring that production costs scale reasonably with quantity. This provides the firm flexibility in operational hedging decisions even at high production levels.

IB.II   Optimal hedging policy i^* when $u_F \geq u_O$

When the firm's inherited debt level is sufficiently high that the financial default boundary exceeds the operational default boundary for all cahedging levels $i \in [0, \bar{i}]$, then the firm would have already declared financial default at date-1 for the shock values that would trigger the operational default. Thus, operational default boundary does not enter the equity value function in this case. In Appendix IC.II, we show that in this case, a unique optimal operational hedging policy exists and the optimal hedging policy decreases with the firm's inherited debt level.

IB.III   Optimal hedging policy $\hat{i}^*$ when $u_F < u_O$

We now examine a more interesting case where the firm's inherited debt level is sufficiently low such that the financial default boundary remains below the operational default boundary for $i \in [0, \bar{i}]$. In this case, the operational default boundary enters the equity value function. The equity value $\hat{E}$ equals E (given in (IC.4)) minus the expected cost proportional to the date-2 franchise value, λx_2:

$$\hat{E} = E - \int_{u_F}^{u_O} \lambda x_2 g(u) du \ , \tag{IB.2}$$

When equity holders choose the optimal level of operational hedging i to maximize $\hat{E}$, it yields the following first-order condition:

$$p - K'(I + i) = [V(u_F, i) - \lambda x_2] h(u_F) K'(I + i) + \frac{\lambda x_2 g(u_O)}{[1 - G(u_F)] I \delta'(u_O)} \ , \tag{IB.3}$$

where $V(u_F, i) \equiv p\left[(1 - \delta(u_F))I + i\right] + x_2$ is the firm's date-2 cash flow and franchise value at the financial default boundary.

Let $\hat{i}^*$ denote the firm's hedging policy that satisfies (IB.3). The left-hand side, $p - K'(I + i)$, represents the marginal profit or loss from additional operational hedging. Note that this term can be negative if the optimal $\hat{i}^*$ is high enough that the marginal production cost $K'(I + i)$ exceeds the price p.

A marginal increase in i has opposing effects on expected losses from operational and financial default. It increases the expected cost of financial default by raising the financial default boundary u_F,[26] while decreasing the expected cost of operational default by reducing the operational default boundary u_O, as captured by the last term in (IB.3). Thus, the first-order condition (IB.3) indicates that the firm chooses $\hat{i}^*$ to equate its "markup" with the net effect of these two forces–the marginal increase in expected financial default cost and the marginal decrease in expected operational default cost. Furthermore, we prove that when $u_F < u_O$ for $i \in [0, \bar{i}]$, higher inherited net debt leads to lower optimal operational hedging:

We prove in Appendix IC.III that the first-order condition (IB.3) yields a unique and positive interior solution $\hat{i}^*$ that maximizes E subject to $D(i, \bar{F}) > 0$ for $i \in [0, \bar{i}]$ under Assumption IB.2. By comparing the first-order conditions (IC.2), (IC.5) and (IB.3), we can establish that $\bar{i} > \hat{i}^* > i^*$. Intuitively, when the firm's inherited net debt $\bar{F}$ is sufficiently low, the operational default boundary u_O exceeds the financial default boundary u_F. In this case, operational default risk becomes the primary concern for equity holders, leading to higher investment in operational hedging. The following lemma, proved in Appendix IC.III, formalizes the above relationship.

Lemma IB.1. $\bar{i} > \hat{i}^* > i^*$ *if Assumption IB.2 holds.*

Furthermore, we prove in Appendix IC.III that when $u_F < u_O$ for $i \in [0, \bar{i}]$, higher inherited net debt leads to lower optimal operational hedging when I is sufficiently high:

Lemma IB.2. *When $\bar{F}$ is such that $0 < u_F < u_O$ for $i \in [0, \bar{i}]$, the optimal operational hedging policy $\hat{i}^*$, if exists, decreases in the firm's net debt level $\bar{F}$.*

IB.IV   Optimal operational hedging policy and net debt $\bar{F}$

We now formally characterize the relationship between the firm's optimal operational hedging policy and its inherited net debt level $\bar{F}$.

Define $\bar{F}_{fb}$ as the maximum net debt level that allows the firm to repay its debt at date-1 when choosing the optimal hedging policy $\bar{i}$ that maximizes the unlevered firm value, as derived in Appendix IB.I. Mathematically, $\bar{F}_{fb} + K(I + \bar{i}) = 0$. When $\bar{F} \leq \bar{F}_{fb}$, short-term debt is riskless and the firm chooses the optimal hedging policy as if there were no debt.

Recall that $D = u_F - u_O$ represents the difference between financial and operational default boundaries. Let $D^*(\bar{F}) \equiv D(i^*(\bar{F}), \bar{F})$ and $\hat{D}^*(\bar{F}) \equiv D(\hat{i}^*(\bar{F}), \bar{F})$ denote these differences when the firm chooses hedging policies i^* and $\hat{i}^*$, respectively. There exist unique

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[26]Notice that the loss conditional on a financial default is reduced by λx_2 because the firm has already lost λx_2 due to operational default when it declares financial default.

thresholds $\bar{F}_0$ and $\bar{F}_1$ such that such that $\hat{D}^*(\bar{F}_0) = 0$ and $D^*(\bar{F}_1) = 0$. When $\bar{F} < \bar{F}_0$, we have $\hat{D}^* < 0$ and when $\bar{F} > \bar{F}_0$, we have $\hat{D}^* > 0$. Similarly, $D^* < 0$ when $\bar{F} < \bar{F}_1$, and $D^* > 0$ when $\bar{F} > \bar{F}_1$. Proposition IB.1 summarizes this relationship between the firm's optimal operational hedging policy and its net debt level maturing at date-1:

Proposition IB.1. *If Lemma IC.3 holds, then*

    I. *If $0 \leq \bar{F} \leq \bar{F}_{fb}$, the firm's optimal operational hedging policy is $\bar{i}$.*

    II. *If $\bar{F}_{fb} < \bar{F} \leq \bar{F}_0$, the firm's optimal operational hedging policy is $\hat{i}^*$.*

    III. *If $\bar{F}_0 < \bar{F} < \bar{F}_1$, the firm's optimal operational hedging policy is $\tilde{i}$ such that $u_F = u_O$.*

    IV. *If $\bar{F} \geq \bar{F}_1$, the firm's optimal operational hedging policy is i^*.*

I.C.   Detailed proofs of Appendix I.B

IC.I   Proofs of the benchmark case in which $F = 0$

Consider first a benchmark case when the debt level $F = 0$. In this case, financial default is irrelevant: $u_F = 0$. The firm will choose the hedging policy $\bar{i}$ that maximizes the unlevered date-0 equity value:

$$\bar{E} = \int_0^\infty \left[x_0 - K(I+i) + \bar{x}_1 + u + p\left[(1 - \delta(u))I + i\right] + x_2 \right] g(u) du - \int_0^{u_O} \lambda x_2 g(u) du \ . \ \text{(IC.1)}$$

The last term of Equation (IC.1) reflects the proportional loss of franchise value in case of operational default. The first-order condition is

$$\frac{\partial \bar{E}}{\partial i} = p - K'(I+i) - \lambda x_2 \frac{g(u_O)}{I\delta'(u_O)} = 0$$

$$p - K'(I+i) = \lambda x_2 \frac{g(u_O)}{I\delta'(u_O)} \ , \tag{IC.2}$$

where $u_O = \delta^{-1}\left(\frac{i}{I}\right)$. Define $\bar{i}$ the solution for the first-order condition (IC.2). In what follows, we show that $\bar{i}$ is also the unique optimal hedging level that maximizes the equity value (IC.1), under Assumption IB.1.

    Since $D(i, \bar{F})$ is continuous in $\bar{F}$, u_F is always smaller than u_O for $i \in [0, \bar{i}]$ when $\bar{F}$ is sufficiently small.

The second-order derivative of $\bar{E}$ with respect to i is:

$$\frac{\partial^2 \bar{E}}{\partial i^2} = -K''(I+i) - \frac{\lambda x_2}{I^2} \frac{g'(u_O) - g(u_O)\frac{\delta''(u_O)}{\delta'(u_O)}}{[\delta'(u_O)]^2} < 0 \qquad (IC.3)$$

Since $\delta(u)$ is decreasing and convex in u, $\frac{\partial^2 \bar{E}}{\partial i^2}$ is always negative if the production commitment I is sufficiently high. In other words, the objective function $\bar{E}$ is concave in i. Thus, $\bar{i}$ is the unique optimal solution that maximizes the equity value (IC.1).

IC.II Proofs of optimal hedging policy when $u_F \geq u_O$

The total payoff to equity holders is the sum of cash flows from assets in place and the payoff from the contractual fulfillment to customers, less the production cost, the operational hedging cost and the debt repayment, provided that the firm does not default on its debt in the interim. The market value of equity is therefore given as:

$$E = \int_{u_F}^{\infty} \left[u - u_F + p\left[(1 - \delta(u))I + i\right] + x_2 \right] g(u) du , \qquad (IC.4)$$

where u_F is given in (2.3). $(u - u_F)$ is the amount of cash left in the firm after debt F is repaid, and $p\left[(1 - \delta(u))I + i\right] + x_2$ is the firm's period-2 cash flow and franchise value, conditional on the firm not defaulting in the interim.

Equity holders choose the level of operational hedging i to maximize equity value E in (IC.4), which yields the following first-order condition:

$$p - K'(I+i) = V(u_F, i)h(u_F)K'(I+i) , \qquad (IC.5)$$

where $V(u_F, i) \equiv p\left[(1 - \delta(u_F))I + i\right] + x_2$ is the firm's date-2 cash flow and franchise value at the financial default boundary. Define i^* as the firm's hedging policy that satisfies (IC.5). On the one hand, a marginal increase in operational hedging yields a marginal profit equal to its markup $p - K'(I+i)$. On the other hand, a marginal increases in operational hedging also increases the expected cost of financial default, which is the product of three terms on the right-hand side of Equation (IC.5): the first term is the loss of date-2 cash flow and franchise value if financial default occurs; the second term is the hazard rate of a financial default; and, the last term is the marginal effect of additional operational hedging on the financial default boundary u_F. The first-order condition says that the firm chooses the hedging policy i^* such that the markup is equal to the marginal increase of the expected financial default

cost.

Comparing the first-order conditions (IC.2) and (IC.5), it is straightforward that $\bar{i} > i^*$.[27] Next, we show that the first-order condition (IC.5) admits a unique and positive interior solution i^* that maximizes E subject to $D(i, \bar{F}) > 0$ for $i \in [0, \bar{i}]$ under Assumption IB.2.

Lemma IC.1. *If Assumption IB.2 holds and $\bar{F}$ is sufficiently large, then the first-order condition* (IC.6) *guarantees a unique and positive interior solution i^* that maximizes E subject to $D(i, \bar{F}) > 0$.*

Proof. Since u is exponentially distributed on $[0, \infty)$ with $g(u) = \alpha e^{-\alpha u}$ and $h(u) = \alpha$, the first-order condition (IC.5) simplifies to

$$p - K'(I + i) = V(u_F, i)\alpha K'(I + i) . \tag{IC.6}$$

Define i^* is the firm's optimal hedging policy that satisfies (IC.6).

First, we show that i^* that satisfies the first-order condition (IC.5) is the unique optimal solution for the maximization problem when $u_F > u_O$. Define $S = p - K'(I + i) - V(u_F, i)h(u_F)K'(I + i)$. Taking the derivative of S with respect to i:

$$\frac{\partial S}{\partial i} = -\left[\begin{array}{c} K''(I + i) + \frac{\partial V(u_F, i)}{\partial i}h(u_F)K'(I + i) \\ +V(u_F, i)\frac{\partial h(u_F)}{\partial u_F}\frac{\partial u_F}{\partial i}K'(I + i) + V(u_F, i)h(u_F)\frac{\partial^2 u_F}{\partial i^2} \end{array}\right] \tag{IC.7}$$

$$\frac{\partial V(u_F, i)}{\partial i} = p[1 - \delta'(u_F)IK'(I + i)] > 0 \tag{IC.8}$$

and

$$\frac{\partial^2 u_F}{\partial i^2} = K''(I + i) > 0 \tag{IC.9}$$

Using these quantities,

$$\frac{\partial S}{\partial i} = -\left[\begin{array}{c} K''(I + i) + p[1 - \delta'(u_F)IK'(I + i)]h(u_F)K'(I + i) \\ +V(u_F, i)\frac{\partial h(u_F)}{\partial u_F}[K'(I + i)]^2 + V(u_F, i)h(u_F)K''(I + i) \end{array}\right] \tag{IC.10}$$

$\frac{\partial S}{\partial i}$ is smaller than zero. Thus, the second-order condition for maximization $[1 - G(u_F)]\frac{\partial S}{\partial i}$ at $i = i^*$ is smaller than zero. By the first-order condition (IC.5), $S = 0$ if $i = i^*$. Since $\frac{\partial S}{\partial i} < 0$, we have $S > 0$ if $i < i^*$ and $S < 0$ if $i > i^*$. Since $\frac{\partial}{\partial i}E = [1 - G(u_F)]S$, E increases

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[27]We prove this claim formally in Appendix IC.III.

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in i for $i < i^*$ and decreases in i for $i > i^*$. Therefore i^* is the unique optimal solution to the maximization problem.

Now we prove that Assumption IB.2 is sufficient condition that guarantees a positive interior solution i^* and $D(i^*, \bar{F}) > 0$ when $\bar{F}$ is sufficiently large. Denote $\underline{i}$ such that $p - K'(I + \underline{i}) = (p(I + \underline{i}) + x_2)\alpha K'(I + \underline{i})$. Notice that $\underline{i}$ must be strictly greater than zero. This is because the left hand-side of the above equation decreases with i, the right hand-side increases with i, and left hand-side is strictly greater than the right hand-side when $i = 0$ by Assumption IB.2, since $K(I + i)$ is convex in i. For any $\bar{F} > 0$, the right hand-side of the first-order condition (IC.6) when $i = \underline{i}$ is $V(u_F, \underline{i})\alpha K'(I + \underline{i})$, which is smaller than $(p(I + \underline{i}) + x_2)\alpha K'(I + \underline{i}) = p - K'(I + \underline{i})$. The left hand-side of the first-order condition (IC.6) decreases with i. The right hand-side of the first-order condition (IC.6) increases with i. This is because u_F increases with i and $\delta(u)$ decreases with u. Consequently, $(1 - \delta(u_F))$ increases with i. $K'(I+i)$ increases with i because the convexity of K in i. So the optimal i^* that satisfies the first-order condition (IC.6) must be strict greater than $\underline{i}$. Denote $\bar{F}_M$ such that $D(\underline{i}, \bar{F}_M) = 0$. Then for any $\bar{F} \geq \bar{F}_M$, we must have $D(i^*(\bar{F}), \bar{F}) > D(\underline{i}, \bar{F}) > 0$. This is because $D(\bar{F}, i)$ increases in $\bar{F}$ and i, and $i^*(\bar{F}) > \underline{i}$. Thus, we have proved that for $\bar{F} > \bar{F}_M$, the first-order condition (IC.6) admits a positive interior solution i^* and the financial default boundary u_F is greater than the operational default boundary u_O when the firm chooses the optimal hedging policy i^*. Since we have proved that the first-order condition (IC.6) is also the sufficient condition for the solution of the constrained maximization problem subject to $D(i, \bar{F}) > 0$, we have proved Lemma IC.1. Q.E.D.

Lemma IC.2 states that the optimal optimal operational hedging policy decreases in the firm's net debt level $\bar{F}$ in this case:

Lemma IC.2. *When $\bar{F}$ is sufficiently high such that $u_F > u_O$ for $i \in [0, \bar{i}]$, the optimal operational hedging policy i^*, if exists, decreases in the firm's net debt level $\bar{F}$.*

Proof. Notice that the optimal hedging policy i^* and the associated financial default boundary u_F are all functions of $\bar{F}$. The firm's optimal operational hedging policy i^* decreases in $\bar{F}$. Define $M(i^*(\bar{F}), \bar{F}) \equiv E(i^*(\bar{F}), \bar{F})$ the value function under optimal hedging policy i^*. By the first-order condition, $\frac{\partial M}{\partial i^*} = 0$. Differentiating both sides with respect to $\bar{F}$:

$$\frac{\partial^2 M}{\partial i^{*2}} \frac{\partial i^*}{\partial \bar{F}} + \frac{\partial M}{\partial i^* \partial \bar{F}} = 0 \tag{IC.11}$$

From equation (IC.11) we get $\frac{\partial i^*}{\partial \bar{F}} = -\frac{\partial^2 M}{\partial i^* \partial \bar{F}} / \frac{\partial^2 M}{\partial i^{*2}}$. Since $\frac{\partial^2 M}{\partial i^{*2}} < 0$ by the second-order

condition, the sign of $\frac{\partial i^*}{\partial \bar{F}}$ is the same as the sign of $\frac{\partial M}{\partial i^* \partial \bar{F}}$.

$$
\begin{aligned}
\frac{\partial^2 M}{\partial i^* \partial \bar{F}} &= [1 - G(u_F)] \left[pI\delta'(u_F) \frac{\partial u_F}{\partial \bar{F}} h(u_F) K'(I + i^*) - V(u_F, i^*) \frac{\partial h(u_F)}{\partial u_F} \frac{\partial u_F}{\partial \bar{F}} K'(I + i) \right] \\
&= [1 - G(u_F)] \left[pI\delta'(u_F) h(u_F) K'(I + i^*) - V(u_F, i^*) \frac{\partial h(u_F)}{\partial u_F} K'(I + i) \right]
\end{aligned}
$$
(IC.12)

Since u follows a exponential distribution, $\frac{\partial h(u_F)}{\partial u_F} = 0$. Thus, Equation (IC.12) is smaller than zero. Therefore, $\frac{\partial i^*}{\partial \bar{F}} < 0$. Q.E.D.

IC.III Proofs of optimal hedging policy when $u_F < u_O$

We begin this subsection by proving the following lemma:

Lemma IC.3. *If Assumption IB.2 holds, then $\hat{i}^*$ that satisfies* (IB.3) *uniquely maximizes* $\hat{E}$.

Proof. First, we show that $\hat{i}^*$ that satisfies the first-order condition (IB.3) is the unique optimal solution for the maximization problem. Define $\hat{S} = p - K'(I + i) - [V(u_F, i) - \lambda x_2] h(u_F) K'(I + i) - \frac{\lambda x_2 g(u_O)}{1 - G(u_F)} \frac{\partial u_O}{\partial i}$. Taking the derivative of $\hat{S}$ with respect to i:

$$
\frac{\partial \hat{S}}{\partial i} = - \left[\begin{array}{c} K''(I + i) + \frac{\partial V(u_F, i)}{\partial i} h(u_F) K'(I + i) + [V(u_F, i) - \lambda x_2] \frac{\partial h(u_F)}{\partial u_F} \frac{\partial u_F}{\partial i} K'(I + i) \\ + [V(u_F, i) - \lambda x_2] h(u_F) \frac{\partial^2 u_F}{\partial i^2} + \lambda x_2 \frac{\partial}{\partial i} \left[\frac{g(u_O)}{[1 - G(u_F)] I \delta'(u_O)} \right] \end{array} \right]
$$
(IC.13)

$$
\frac{\partial}{\partial i} \left[\frac{g(u_O)}{[1 - G(u_F)] I \delta'(u_O)} \right] = \left[\frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{[1 - G(u_F)][\delta'(u_O)]^2 I} + \frac{g(u_F)K'(I + i)g(u_O)}{[1 - G(u_F)]^2} \right] \frac{1}{I \delta'(u_O)}
$$
(IC.14)

The absolute value of (IC.14) is small if Assumption IB.2 holds. Using quantities (IC.8), (IC.9) and (IC.14), $\frac{\partial \hat{S}}{\partial i}$ is

$$
\frac{\partial \hat{S}}{\partial i} = - \left[\begin{array}{c} K''(I + i) + p[1 - \delta'(u_F) I K'(I + i)] h(u_F) K'(I + i) \\ + [V(u_F, i) - \lambda x_2] h(u_F) K''(I + i) \\ + \lambda x_2 \left[\frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{[1 - G(u_F)][\delta'(u_O)]^2 I} + \frac{g(u_F)K'(I+i)g(u_O)}{[1 - G(u_F)]^2} \right] \frac{1}{I \delta'(u_O)} \end{array} \right]
$$
(IC.15)

$\frac{\partial \hat{S}}{\partial i}$ is always smaller than zero, thus, the second-order condition for maximization $[1 - G(u_F)] \frac{\partial \hat{S}}{\partial i}$ at $i = \hat{i}^*$ is smaller than zero. By the first-order condition (IB.3), $\hat{S} = 0$ if $i = \hat{i}^*$. Since $\frac{\partial \hat{S}}{\partial i} < 0$, we have $\hat{S} > 0$ if $i < \hat{i}^*$ and $\hat{S} < 0$ if $i > \hat{i}^*$. Since $\frac{\partial}{\partial i} \hat{E} = [1 - G(u_F)] \hat{S}$, $\hat{E}$

increases in i for $i < \hat{i}^*$ and decreases in i for $i > \hat{i}^*$. Therefore, $\hat{i}^*$ is the unique optimal solution to the maximization problem. Q.E.D.

Now we prove Lemma IB.1. i^* satisfies the first-order condition (IC.5):

$$p - K'(I + i^*) = V(u_F, i^*)h(u_F)K'(I + i^*)$$

$$> V(u_F, i^*)h(u_F)K'(I + i^*) - \lambda x_2 h(u_F)K'(I + i^*) + \frac{\lambda x_2 g(u_O)}{[1 - G(u_F)]I\delta'(u_O)}$$
(IC.16)

The inequality holds because $\lambda x_2 h(u_F)K'(I+i^*) > 0$ and $\frac{\lambda x_2 g(u_O)}{[1-G(u_F)]I\delta'(u_O)} < 0$. Now taking the derivative of both sides of the first-order condition in the case $u_O > u_F$, (IB.3), with respect to i. The derivative of the left-hand side is $-K''(I + i)$. The derivative of the right-hand side is

$$p[1 - \delta'(u_F)IK'(I + i)]h(u_F)K'(I + i) + [V(u_F, i) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}[K'(I + i)]^2$$

$$+ [V(u_F, i) - \lambda x_2]h(u_F)K''(I + i)$$

$$+ \lambda x_2 \left[\frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{[1 - G(u_F)][\delta'(u_O)]^2 I} + \frac{g(u_F)K'(I + i)g(u_O)}{[1 - G(u_F)]^2} \right] \frac{1}{I\delta'(u_O)}$$
(IC.17)

The quantity (IC.17) is always greater than $-K''(I + i)$ by Assumption IB.2. Thus the left-hand side of Equation (IB.3) decreases in i and the right-hand side of Equation (IB.3) increases in i. Since $\hat{i}^*$ satisfies the first-order condition in $u_O > u_F$ case, (IB.3). We must have $\hat{i}^* > i^*$. Meanwhile, $\bar{i}$ satisfies the first-order condition (IC.2):

$$p - K'(I + i^*) = \lambda x_2 \frac{g(u_O)}{I\delta'(u_O)}$$

$$< V(u_F, i^*)h(u_F)K'(I + i^*) - \lambda x_2 h(u_F)K'(I + i^*) + \frac{\lambda x_2 g(u_O)}{[1 - G(u_F)]I\delta'(u_O)}$$
(IC.18)

In a similar way, we can prove that $\bar{i} > \hat{i}^*$.

In what follows, we prove Lemma IB.2: the firm's optimal operational hedging policy $\hat{i}^*$ decreases in $\bar{F}$. Define $\hat{M}(\hat{i}^*(\bar{F}), \bar{F}) \equiv E(\hat{i}^*(\bar{F}), \bar{F})$ the value function under the optimal hedging policy $\hat{i}^*$. Similar to the case $u_F > u_O$, $\frac{\partial \hat{i}^*}{\partial \bar{F}} = -\frac{\partial^2 \hat{M}}{\partial \hat{i}^* \partial \bar{F}} / \frac{\partial^2 \hat{M}}{\partial \hat{i}^{*2}}$. Since $\frac{\partial^2 \hat{M}}{\partial \hat{i}^{*2}} < 0$ by the

second-order condition, the sign of $\frac{\partial \hat{i}^*}{\partial \bar{F}}$ is the same as the sign of $\frac{\partial \hat{M}}{\partial \hat{i}^* \partial \bar{F}}$.

$$
\begin{aligned}
\frac{\partial^2 \hat{M}}{\partial \hat{i}^* \partial \bar{F}} &= [1 - G(u_F)] \left[\begin{array}{c} pI\delta'(u_F)\frac{\partial u_F}{\partial \bar{F}}h(u_F)K'(I+\hat{i}^*) - [V(u_F,\hat{i}^*) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}\frac{\partial u_F}{\partial \bar{F}}K'(I+i) \\ - \frac{\lambda x_2}{I}\frac{g(u_O)g(u_F)}{[1-G(u_F)]^2\delta'(u_O)}\frac{\partial u_F}{\partial \bar{F}} \end{array} \right] \\
&= [1 - G(u_F)]\frac{\partial u_F}{\partial \bar{F}} \left[pI\delta'(u_F)h(u_F)K'(I+i^*) - \frac{\lambda x_2}{I}\frac{g(u_O)g(u_F)}{[1-G(u_F)]^2\delta'(u_O)} \right]
\end{aligned}
$$
(IC.19)

Since u follows an exponential distribution, $\frac{\partial h(u_F)}{\partial u_F} = 0$. Thus, Equation (IC.19) is always smaller than zero if the production commitment I is sufficiently high. Therefore, $\frac{\partial \hat{i}^*}{\partial \bar{F}} < 0$ if the production commitment I is sufficiently high.

IC.IV   Proofs of Proposition IB.1 and Proposition 2.1

We now prove Proposition IB.1 and Proposition 2.1. First, let us recall the key definitions and thresholds:

The threshold $\bar{F}_{fb}$ represents maximum net debt level that allows the firm to repay its debt at date-1 when choosing the optimal unlevered hedging policy $\bar{i}$, satisfying $\bar{F}_{fb} + K(I+\bar{i}) = 0$. The difference between financial and operational default boundaries is denoted by $D = u_F - u_O$. When the firm chooses hedging policies i^* and $\hat{i}^*$, these differences are given by $D^*(\bar{F}) \equiv D(i^*(\bar{F}), \bar{F})$ and $\hat{D}^*(\bar{F}) \equiv D(\hat{i}^*(\bar{F}), \bar{F})$, respectively. The thresholds $\bar{F}_0$ and $\bar{F}_1$ are defined by $\hat{D}^*(\bar{F}_0) = 0$ and $D^*(\bar{F}_1) = 0$.

This subsection will show that these thresholds $\bar{F}_0$ and $\bar{F}_1$ exist uniquely and satisfy $\bar{F}_0 < \bar{F}_1$. Moreover, we will prove that $\hat{D}^* < 0$ when $\bar{F} < \bar{F}_0$ and $\hat{D}^* > 0$ when $\bar{F} > \bar{F}_0$. Similarly, we will demonstrate that $D^* < 0$ when $\bar{F} < \bar{F}_1$; and $D^* > 0$ when $\bar{F} > \bar{F}_1$.

First, we note that when the inherited net short-term debt level is sufficiently low ($\bar{F} \leq \bar{F}_{fb}$), the optimal hedging policy $\bar{i}$ maximizes equity value as if the firm were unlevered. When $\bar{F} > \bar{F}_{fb}$, the firm must choose an optimal hedging policy i that balances the tradeoff between financial and operational default risks, which we elaborate on below.

Notice that $D(i, \bar{F})$ is continuously differentiable in both i and $\bar{F}$ with partial derivatives:

$$
\frac{\partial D}{\partial i} = K'(I+i) - \frac{1}{I\delta'(u_O)} , \tag{IC.20a}
$$

$$
\frac{\partial D}{\partial \bar{F}} = 1 . \tag{IC.20b}
$$

Notice that $\frac{\partial D}{\partial i} > 0$ because $K'(I+i) > 0$ and $\delta'(u) < 0$ by assumption. The following

lemma is for technical purpose. It facilitates our proof that both $D^*(\bar{F}) = 0$ and $\hat{D}^*(\bar{F}) = 0$ have unique solutions, which we denote as $\bar{F}_0$ and $\bar{F}_1$, respectively.

Lemma IC.4.

$$\frac{dD^*}{d\bar{F}} > 0 \text{ if } u_F(i^*) \geq u_O(i^*) \tag{IC.21a}$$

$$\frac{d\hat{D}^*}{d\bar{F}} > 0 \text{ if } u_F(\hat{i}^*) \geq u_O(\hat{i}^*) \tag{IC.21b}$$

Proof. First we prove the following inequality:

$$\frac{dD^*}{d\bar{F}} = \frac{\partial D^*}{\partial \bar{F}} + \frac{\partial D^*}{\partial i^*}\frac{\partial i^*}{\partial \bar{F}} > 0 \tag{IC.22}$$

Using Equations (IC.20a) and (IC.20b) Inequality (IC.22) is equivalent to

$$\left[K'(I + i^*) - \frac{1}{I\delta'(u_O)}\right]\left(-\frac{\partial i^*}{\partial \bar{F}}\right) < 1 \tag{IC.23}$$

From Appendix IC.II, $\frac{\partial i^*}{\partial F} = -\frac{\partial^2 M}{\partial i^* \partial F}/\frac{\partial^2 M}{\partial i^{*2}}$. $\frac{\partial^2 M}{\partial i^* \partial F}$ is given by Equation (IC.12). $\frac{\partial^2 M}{\partial i^{*2}}$ is given by $[1 - G(u_F)]\frac{\partial S}{\partial i^*}$ where $\frac{\partial S}{\partial i^*}$ is given by Equation (IC.10) at $i = i^*$. Thus, Inequality (IC.23) is equivalent to

$$\frac{V(u_F, i^*)\frac{\partial h(u_F)}{\partial u_F}K'(I + i^*) - pI\delta'(u_F)h(u_F)K'(I + i^*)}{\begin{bmatrix} K''(I + i^*) + p[1 - \delta'(u_F)IK'(I + i^*)]h(u_F)K'(I + i^*) \\ +V(u_F, i^*)\frac{\partial h(u_F)}{\partial u_F}[K'(I + i^*)]^2 + V(u_F, i^*)h(u_F)K''(I + i^*) \end{bmatrix}}\frac{1 - I\delta'(u_O)K'(I + i^*)}{-I\delta'(u_O)} < 1 \tag{IC.24}$$

Algebraic simplification shows that the above inequality is equivalent to

$$V(u_F, i^*)\frac{\partial h(u_F)}{\partial u_F}K'(I + i^*) + pI[\delta'(u_O) - \delta'(u_F)]h(u_F)K'(I + i^*)$$

$$< [1 + V(u_F, i^*)h(u_F)]K''(I + i^*)[-I\delta'(u_O)] \tag{IC.25}$$

Since u follows a exponential distribution, $\frac{\partial h(u)}{\partial u} = 0$ and the first term of the left-hand side of Inequality (IC.25) is equal to zero. the second term on the left-hand side is (weakly) smaller than zero if $u_F \geq u_O$ because $\delta(u)$ is convex in u. Therefore the left-hand side of Inequality (IC.25) is (weakly) smaller than zero. The right-hand side of Inequality (IC.25) is strictly

greater than zero. Therefore, Inequality (IC.25) holds and $\frac{dD^*}{d\bar{F}} > 0$.

Now we prove the following inequality:

$$\frac{d\hat{D}^*}{d\bar{F}} = \frac{\partial \hat{D}^*}{\partial \bar{F}} + \frac{\partial \hat{D}^*}{\partial \hat{i}^*}\frac{\partial \hat{i}^*}{\partial \bar{F}} > 0 \qquad (IC.26)$$

Inequality (IC.26) is equivalent to

$$\left[K'(I + \hat{i}^*) - \frac{1}{I\delta'(u_O)} \right]\left(-\frac{\partial \hat{i}^*}{\partial \bar{F}} \right) < 1 \qquad (IC.27)$$

From Appendix IC.III, $\frac{\partial \hat{i}^*}{\partial F} = -\frac{\partial^2 \hat{M}}{\partial \hat{i}^* \partial \bar{F}}\Big/\frac{\partial^2 \hat{M}}{\partial \hat{i}^{*2}}$. $\frac{\partial^2 \hat{M}}{\partial \hat{i}^* \partial \bar{F}}$ is given by Equation (IC.19). $\frac{\partial^2 \hat{M}}{\partial \hat{i}^{*2}}$ is given by $[1 - G(u_F)]\frac{\partial \hat{S}}{\partial \hat{i}^*}$ where $\frac{\partial \hat{S}}{\partial \hat{i}^*}$ is given by Equation (IC.15) at $i = \hat{i}^*$. Thus, Inequality (IC.27) is equivalent to

$$\frac{\left[\begin{array}{c} [V(u_F, \hat{i}^*) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}K'(I + \hat{i}^*) \\ -pI\delta'(u_F)h(u_F)K'(I + \hat{i}^*) \\ +\frac{\lambda x_2}{I}\frac{g(u_O)g(u_F)}{[1-G(u_F)]^2\delta'(u_O)} \end{array} \right]}{\left[\begin{array}{c} K''(I + \hat{i}^*) + p[1 - \delta'(u_F)IK'(I + \hat{i}^*)]h(u_F)K'(I + \hat{i}^*) \\ +[V(u_F, \hat{i}^*) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}[K'(I + \hat{i}^*)]^2 \\ +[V(u_F, \hat{i}^*) - \lambda x_2]h(u_F)K''(I + \hat{i}^*) \\ +\lambda x_2\left[\frac{g'(u_O)\delta'(u_O)-g(u_O)\delta''(u_O)}{[1-G(u_F)][\delta'(u_O)]^2I} + \frac{g(u_F)K'(I+\hat{i}^*)g(u_O)}{[1-G(u_F)]^2} \right]\frac{1}{I\delta'(u_O)} \end{array} \right]}\frac{1 - I\delta'(u_O)K'(I + \hat{i}^*)}{-I\delta'(u_O)} < 1 \qquad (IC.28)$$

Algebraic simplification shows that the above inequality is equivalent to

$$[V(u_F, \hat{i}^*) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}K'(I + \hat{i}^*) + pI\left[\delta'(u_O) - \delta'(u_F)\right]h(u_F)K'(I + \hat{i}^*) + \frac{\lambda x_2}{I}\frac{g(u_O)g(u_F)}{[1 - G(u_F)]^2\delta'(u_O)}$$

$$< \left[1 + [V(u_F, \hat{i}^*) - \lambda x_2]h(u_F) \right]K''(I + \hat{i}^*)\left[-I\delta'(u_O) \right] - \lambda x_2\frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{[1 - G(u_F)][\delta'(u_O)]^2I} \qquad (IC.29)$$

Since u follows a exponential distribution, $\frac{\partial h(u)}{\partial u} = 0$ and the first term of the left-hand side of Inequality (IC.29) is equal to zero. the second term on the left-hand side is (weakly) smaller than zero if $u_F \geq u_O$ because $\delta(u)$ is convex in u. The first term of the right-hand side of Inequality (IC.29) is strictly greater than zero. Therefore, to show that Inequality

(IC.29) holds, we need to show that:

$$\lambda x_2 \frac{1}{I} \frac{g(u_O)g(u_F)}{[1-G(u_F)]^2\delta'(u_O)} < -\lambda x_2 \frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{[1-G(u_F)][\delta'(u_O)]^2 I} \qquad \text{(IC.30)}$$

Or, equivalently,

$$\lambda x_2 \frac{1}{I} \frac{g(u_O)g(u_F)}{[1-G(u_F)]^2\delta'(u_O)} + \lambda x_2 \frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{[1-G(u_F)][\delta'(u_O)]^2 I} < 0$$
$$\Leftrightarrow \frac{\lambda x_2}{I[1-G(u_F)]\delta'(u_O)} \left[\frac{g(u_O)g(u_F)}{[1-G(u_F)]} + \frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{\delta'(u_O)} \right] < 0$$
$$\Leftrightarrow \frac{g(u_O)g(u_F)}{[1-G(u_F)]} + \frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{\delta'(u_O)} > 0 \qquad \text{(IC.31)}$$

Since $g(u) = \alpha \exp(-\alpha u)$, $\alpha g(u) = -g'(u)$, and $\frac{g(u_F)}{[1-G(u_F)]} = \alpha$, the inequality (IC.31) is equivalent to

$$\frac{\delta''(u_O)}{\delta'(u_O)} < 0 \qquad \text{(IC.32)}$$

which always holds since $\delta(u)$ decreases and convex in u by assumption. Therefore, $\frac{d\hat{D}^*}{d\bar{F}} > 0$.

Q.E.D.

We turn to the formal proof of Proposition IB.1 and Proposition 2.1. First, i^* and $\hat{i}^*$ are continuously differentiable in $\bar{F}$ and $D(i, \bar{F})$ is continuously differentiable in both i and f. It follows that $D^*(\bar{F})$ and $\hat{D}^*(\bar{F})$ are continuously differentiable, thus continuous in $\bar{F}$.

Secondly, from Appendix IB.II and Appendix IB.III, we know that u_F is greater than u_O, i.e., $D^*, \hat{D}^* > 0$ when $\bar{F}$ is sufficiently high, i.e., $\bar{F} \geq \bar{F}_M$. To see this, from Lemma IC.1,$D^* > 0$ if $\bar{F} \geq \bar{F}_M$. From Lemma IB.1, for a given $\bar{F}$, $\hat{i}^* > i^*$. Since $D(i, \bar{F})$ increases in i, $\hat{D}^* > 0$ when $\bar{F} \geq \bar{F}_M$. On the other hand, if $F = 0$, $u_F = 0$, which is always lower than u_O. Since $D^*(\bar{F})$ and $\hat{D}^*(\bar{F})$ are continuous in $\bar{F}$, $D^*, \hat{D}^* < 0$ for all $\bar{F}$ that is sufficiently low. Again by the continuity of $D^*(\bar{F})$ and $\hat{D}^*(\bar{F})$ in $\bar{F}$, there must exist $\bar{F}_0$ and $\bar{F}_1$ such that $\hat{D}^*(\bar{F}_0) = 0$ and $D^*(\bar{F}_1) = 0$. By Lemma IC.4, $\frac{d\hat{D}^*}{d\bar{F}} > 0$ whenever $\hat{D}^* \geq 0$ and $\frac{dD^*}{d\bar{F}} > 0$ whenever $D^* \geq 0$. It follows that $\bar{F}_0$ and $\bar{F}_1$ are unique. Moreover, $\hat{D}^* < 0$ for all $\bar{F} < \bar{F}_0$ and $\hat{D}^* > 0$ for all $\bar{F} > \bar{F}_0$. Similarly, $D^* < 0$ for all $\bar{F} < \bar{F}_1$ and $D^* > 0$ for all $\bar{F} > \bar{F}_1$.

From Lemma IB.1, $\hat{i}^* > i^*$ for any given $\bar{F}$. At $\bar{F} = \bar{F}_1$, $D^*(\bar{F}_1) = 0$. Since $\frac{\partial D}{\partial i} > 0$, we must have $\hat{D}^*(\bar{F}_1) = D(\hat{i}^*(\bar{F}_1), \bar{F}_1) > 0$. Thus, $\bar{F}_1 > \bar{F}_0$.

To conserve space, we omit the argument $\bar{F}$ in i^*, $\tilde{i}$ and $\hat{i}^*$. If $\bar{F} \leq \bar{F}_0$, then $D^* < 0$ and $\hat{D}^* \leq 0$. Thus, maximizing the equity value subject to $u_F \leq u_O$ will yield the optimal operational hedging policy $\hat{i}^*$. Meanwhile, maximizing the equity value subject to $u_F \geq u_O$

will yield a corner solution $\tilde{i} > i^*$, in which $\tilde{i}$ is such that $D(\tilde{i}, \bar{F}) = 0$. Indeed, for a given $\bar{F}$ in this region, the feasible set of i for the maximization problem of the equity value subject to $u_F \geq u_O$, if not empty, is $i \geq \tilde{i} > i^*$. From Appendix IC.II, the equity value E decreases in i for $i > i^*$. Since $\tilde{i}$ is also feasible for the maximization problem of the equity value subject to $u_F \leq u_O$ and $\hat{E} = E$ when $i = \tilde{i}$, $\tilde{i}$ must yield a lower expected payoff for the shareholders, compared with $\hat{i}^*$. Thus, the optimal operational hedging policy is $\hat{i}^*$.

If $\bar{F}_0 < \bar{F} < \bar{F}_1$, then $D^* < 0$ and $\hat{D}^* > 0$. Thus, maximizing the equity value subject to $u_F \leq u_O$ or subject to $u_F \geq u_O$ will yield the same corner solution $\tilde{i}$, in which $\tilde{i}$ is such that $D(\tilde{i}, \bar{F}) = 0$. This is because, for a given $\bar{F}$ in this region, the feasible set of i for the maximization problem of the equity value subject to $u_F \geq u_O$ is $i \geq \tilde{i} > i^*$, and from Appendix IC.II, equity value E decreases in i for $i > i^*$. Meanwhile, the feasible set of i for the maximization problem of the equity value subject to $u_F \leq u_O$ is $i \leq \tilde{i} < \hat{i}^*$ and from Appendix IC.III, $\hat{E}$ increases in i for $i < \hat{i}^*$. Thus, the optimal operational hedging policy is $\tilde{i}$.

If $\bar{F} \geq \bar{F}_1$, then $D^* \geq 0$ and $\hat{D}^* > 0$. Thus, maximizing the equity value subject to $u_F \geq u_O$ will yield the optimal operational hedging policy i^*. Meanwhile, maximizing the equity value subject to $u_F < u_O$ will yield a corner solution $\tilde{i} < \hat{i}^*$. Indeed, for a given $\bar{F}$ in this region, the feasible set of i for the maximization problem of the equity value subject to $u_F \leq u_O$, if not empty, is $i \leq \tilde{i} < \hat{i}^*$ and from Appendix IC.III, $\hat{E}$ increases in i for $i < \hat{i}^*$. Since $\tilde{i}$ is also feasible for the maximization problem of the equity value subject to $u_F \geq u_O$ and $\hat{E} = E$ when $i = \tilde{i}$, $\tilde{i}$ must yield a lower expected payoff for the shareholders, compared with i^*. Thus, the optimal operational hedging policy is i^*.

Now we prove Proposition 2.1. From Proposition IB.1 and Lemma IC.2, when $\bar{F} > \bar{F}_1$, $i^{**} = i^*$ and thus decreases in $\bar{F}$. Similarly, from Proposition IB.1 and Lemma IB.2, when $\bar{F} < \bar{F}_0$, $i^{**} = \hat{i}^*$ and thus decreases in $\bar{F}$. Moreover, $\frac{\partial \tilde{i}}{\partial \bar{F}} = -\frac{\partial D}{\partial \bar{F}} / \frac{\partial D}{\partial \tilde{i}}$. Since both partial derivatives on the right-hand side are positive from Inequalities (IC.20a) and (IC.20b), $\frac{\partial \tilde{i}}{\partial \bar{F}} < 0$. When $\bar{F}_0 < \bar{F} < \bar{F}_1$, $i^{**} = \tilde{i}$ and thus decreases in $\bar{F}$. Lastly, at $\bar{F} = \bar{F}_1$, since $D^* = 0$, $i^* = \tilde{i}$, so $i^{**} = i^* = \tilde{i}$ at $\bar{F} = \bar{F}_1$ and thus is continuous in $\bar{F}$ at $\bar{F} = \bar{F}_1$. Similarly, at $\bar{F} = \bar{F}_0$, since $\hat{D}^* = 0$, $\hat{i}^* = \tilde{i}$, so $i^{**} = \hat{i}^* = \tilde{i}$ at $\bar{F} = \bar{F}_0$ and thus is continuous in $\bar{F}$ at $\bar{F} = \bar{F}_0$. Therefore, i^{**} decreases in $\bar{F}$.

IC.V   Proof of Proposition 2.2

Along the equilibrium path, operational spread increases in debt level F. we prove that the credit spread, defined in Section 2.4, also increases in F along the equilibrium path, thereby establishing the positive operational spread-credit spread relationship. Note that the latter

is equivalent to the elasticity of the market value of debt L with respect to the face value of debt F smaller than 1. The derivative of L with respect to F is

$$\frac{dL}{dF} = 1 - G(u_F) - K'(I + i^{**})\frac{\partial i^{**}}{\partial F}G(u_F) . \tag{IC.33}$$

To prove (IC.33) $< \frac{L}{F}$, we need to show that

$$-K'(I + i^{**})\frac{\partial i^{**}}{\partial F}G(u_F) \leq \int_0^{u_F} ug(u)du = \frac{1}{\alpha}\left[1 - (1 + u_F)e^{-\alpha u_F}\right]$$

$$\Leftrightarrow -\alpha K'(I + i^{**})\frac{\partial i^{**}}{\partial F} \leq 1 - \frac{u_F e^{-\alpha u_F}}{G(u_F)} , \tag{IC.34}$$

where we have used $g(u) = \alpha e^{-\alpha u}$ and $G(u_F) = 1 - e^{-\alpha u_F}$. Suppose $K(I + i) = \kappa N(I + i)$, then inequality (IC.34) holds when $\kappa \leq -\frac{1}{\alpha N'(I+i^{**})\frac{\partial i^{**}}{\partial F}}\left(1 - \frac{u_F e^{-\alpha u_F}}{G(u_F)}\right)$.

IC.VI Proof of Proposition 2.3

The value of equity when $u_{F,PP} > u_O$ can be written as

$$E_{PP} = \int_{u_{F,PP}}^{\infty} \left[(u - u_{F,PP}) - \tau p[(1 - \delta(u_{F,PP}))I + i] + p[(1 - \delta(u))I + i] + x_2\right]g(u)du . \tag{IC.35}$$

The value of equity when $u_{F,PP} < u_O$ is $E_{PP} - \int_{u_F}^{u_O} \lambda x_2 g(u)du$.

The partial pledgeability case can be solved in a comparable manner as the zero pledgeability case. We define $\hat{i}^*_{PP}$ as the optimal hedging policy that maximizes the equity value when $u_{F,PP} < u_O$; $\tilde{i}_{PP}$ as the optimal hedging policy that equalizes the operational and financial default boundaries $u_O(\tilde{i}_{PP}) = u_{F,PP}(\tilde{i}_{PP}, \bar{F})$; and, i^*_{PP} as the optimal hedging policy that maximizes the equity value when $u_{F,PP} > u_O$. Specifically, i^*_{PP} and $\hat{i}^*_{PP}$ are given respectively by the following first-order conditions:

$$p - K'(I + i^*_{PP}) = V(u_{F,PP}, i^*_{PP}) h(u_{F,PP}) \frac{[K'(I + i^*_{PP}) - \tau p]}{[1 - \tau p\delta'(u_{F,PP})I]} , \tag{IC.36}$$

$$p - K'(I + \hat{i}^*_{PP}) = \left[V(u_{F,PP}, \hat{i}^*_{PP}) - \lambda x_2\right] h(u_{F,PP}) \frac{[K'(I + \hat{i}^*_{PP}) - \tau p]}{[1 - \tau p\delta'(u_{F,PP})I]}$$

$$+ \frac{\lambda x_2 g(u_O)}{[1 - G(u_{F,PP})]I\delta'(u_O)} . \tag{IC.37}$$

Define $\bar{F}_{fb,PP}$ to be such that

$$\bar{F}_{fb,PP} + K(I + \bar{i}_{PP}) = \tau * p * \bar{i}_{PP} \ . \tag{IC.38}$$

In other words, $\bar{F}_{fb,PP}$ is the maximal net debt level such that the firm is able to pay back the debt at date-1 even if the production shock u is severe enough to obliterate the entire production capacity I. $\bar{F}_{0,PP}$ and $\bar{F}_{1,PP}$ are defined analogously to the respective thresholds in Proposition IB.1: $\bar{F}_{0,PP}$ is such that $u_{F,PP}(\hat{i}^*_{PP}, \bar{F}_{0,PP}) = u_O(\hat{i}^*_{PP})$; $\bar{F}_{1,PP}$ is such that $u_{F,PP}(i^*_{PP}, \bar{F}_{1,PP}) = u_O(i^*_{PP})$. The following proposition characterizes the firm's optimal hedging policy as a function of $\bar{F}$ when the pledgeability is imperfect, i.e., $\tau < \bar{\tau} < 1$:[28]

Proposition IC.1. *There exists $\bar{\tau} < 1$ such that if $\tau < \bar{\tau}$, then*

    I. If $0 \leq \bar{F} \leq \bar{F}_{fb,PP}$, the firm's optimal operational hedging policy is $\bar{i}$.

    II. If $\bar{F}_{fb,PP} < \bar{F} \leq \bar{F}_{0,PP}$, the firm's optimal operational hedging policy is $\hat{i}^*_{PP}$.

    III. If $\bar{F}_{0,PP} < \bar{F} < \bar{F}_{1.PP}$, the firm's optimal operational hedging policy is $\tilde{i}_{PP}$ such that $u_{F,PP} = u_O$.

    IV. If $\bar{F} \geq \bar{F}_{1,PP}$, the firm's optimal operational hedging policy is i^*_{PP}.

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[28]The proofs of Proposition IC.1 and Proposition 2.3 are similar to the base case although the algebra is much more involved. The proofs are available upon request.