Disasters with Unobservable Duration and Frequency: Intensified Responses and Diminished Preparedness*

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Abstract

We study an economy subject to recurrent disasters when the frequency and duration of the disasters are unobservable parameters. Imprecise information about transition intensities increases the probability of the current state effectively lasting forever. In a disaster, uncertainty about duration makes disasters subjectively much worse and can make the welfare value of information extremely high. However, in advance of a disaster, uncertainty about the arrival rate can be welfare-increasing. Agents optimally invest less in mitigation than under full information and pay less for insurance against the next disaster.

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1 Introduction

This paper studies the real effects of parameter uncertainty in a model of repeated disasters. Among the many deeply alarming aspects of the COVID-19 pandemic was the realization of *how little anybody knew* about the right model for what would happen. Two dimensions of structural uncertainty seemed especially salient: uncertainty about the persistence (or duration) of the crisis, and uncertainty about the recurrence (or frequency) of future disasters. Such uncertainty pertains to disasters beyond pandemics. Uncertainty about the duration and frequency of recessions and financial crises is also realistic. Similar considerations apply in the context of climate-related disasters.

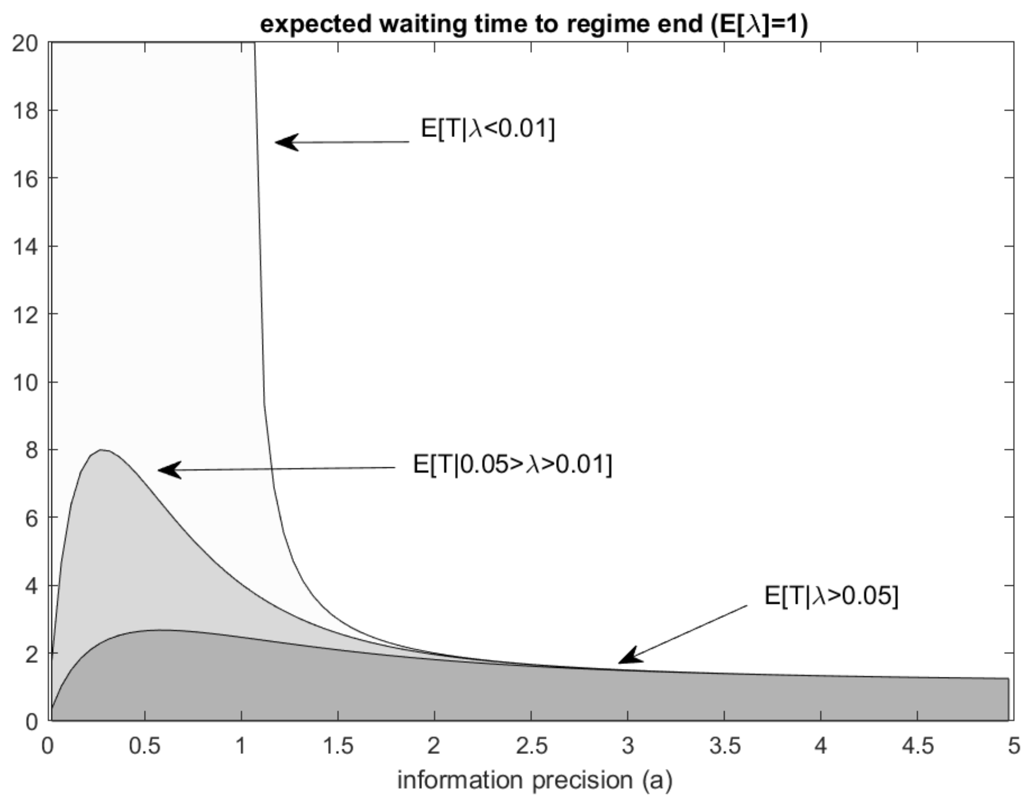
Our model depicts disasters as regimes in which the stock of wealth (potentially including human wealth) is subject to exogenous destruction. The economy transitions stochastically between these episodes and "normal times." Agents update their beliefs by observing the frequencies of transitions, and optimally solve their investment/consumption problem given that information. We derive closed-form expressions for belief dynamics, and we obtain the value function and optimal policies under generalized preferences up to a tractable system of difference/differential equations. We contrast agents' welfare, policies, and incentives in the partial-information setting to the full-information setting.

The main finding is that, when uncertainty about a transition intensity increases, the left tail of the distribution becomes the dominant factor in economic decision-making. That is, even holding the mean belief constant, agents may act as if the transition probability is near zero. In forming expectations and evaluating trade-offs, households rationally place increasing weight on the possibility of the *current* state effectively lasting forever. In a setting where regimes are either "good" or "bad", the result can look like overreaction relative to the full-information benchmark.

The mechanism driving this is that Bayesian updating implies *negative duration dependence*, i.e., that the longer the current state lasts, the longer it is expected to last. This dynamic is quite general: the absence of a transition in a given observation interval shifts the posterior density for the transition intensity towards zero regardless of the form of prior beliefs. Negative duration dependence is equivalent to transition times subjectively exhibiting a decreasing hazard function. Unconditionally, this means that beliefs about future regime durations are described by heavy tailed distributions.

As precision declines, negative duration dependence increases, and the unconditional expected waiting time for the next transition, T, can become unbounded even holding fixed the mean belief about the transition probability. This is depicted graphically in Figure 1 when observers have belief about the transition intensity that is described by a gamma distribution with mean 1 and variance $1/a$. The figure illustrates how, as precision declines, the expected waiting time, $E[T]$, becomes increasingly determined by the possibility that the true value of the intensity is close to zero.

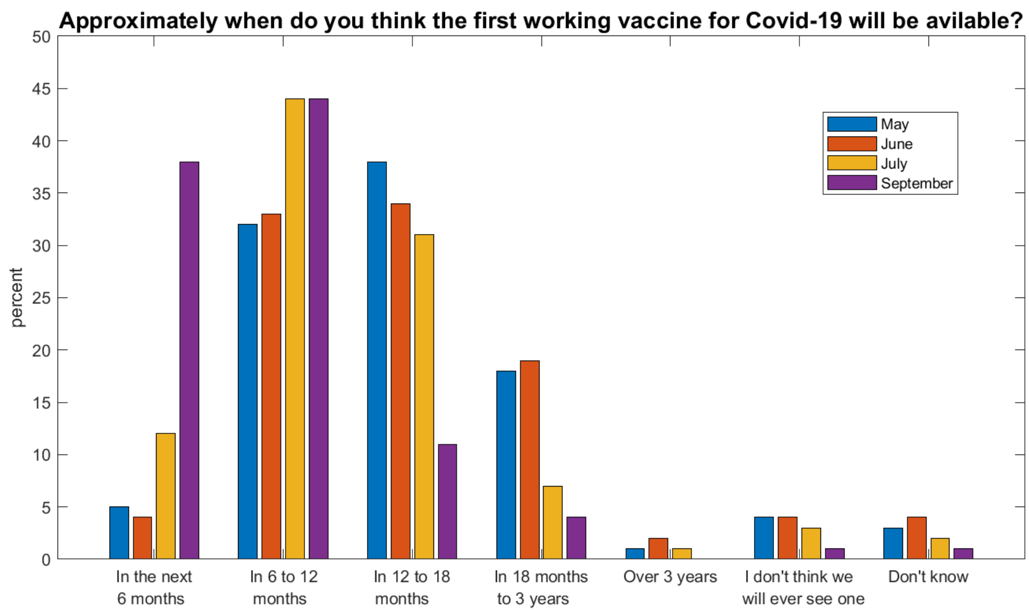
Figure 1: Information Precision and Expected Transition Time



The top line plots the expected waiting time in years for the end of a regime when observers have belief about the intensity per unit time of a switch, λ, that are described by a gamma distribution with mean $E[\lambda] = 1$ and variance $1/a$, where a is the variable on the horizontal axis. The lower lines depict the contribution to this expectation of different components of the belief distribution.

A graphic example of a heavy-tailed waiting time distribution is seen in survey results from 2020 regarding the anticipated arrival time of an effective vaccine against the SARS-CoV-2 virus. The survey, conducted by Deutsche Bank, was sent to 800 global market participants eliciting their forecasts on four dates. As shown in Figure 2, as late as June of that year, and despite much positive clinical trial news, fully 4% of respondents thought the likely time to successful vaccine deployment would be *infinite*. This type of response can be rational under the structural uncertainty that we posit.

Figure 2: Duration Expectations: 2020



Source: Deutsche Bank, dbDIG.

The paper's analysis is based on comparisons of the representative agent's lifetime value function across states. We express these comparisons in terms of welfare costs, meaning the fraction of wealth the agent would be willing to pay to exchange one state for another. We first show that the benefit of ending a disaster is much higher with partial information compared to the full-information benchmark. The welfare gain from reducing the severity of an on-going disaster also increases strongly with parameter uncertainty. These welfare differences map directly to investment incentives. If the economy is augmented to include a mitigation technology, optimal investment in this activity at the onset of a disaster is increasing in parameter uncertainty.[1]

The observation that disasters are subjectively much worse under incomplete infor-

[1]In a similar vein, Barnett et al. (2023) show that uncertainty about infectious parameters within a pandemic leads a central planner with ambiguity averse preferences to impose stricter quarantine measures compared to the full-information benchmark.

mation raises the topic of the value of information. We show that the welfare gain from resolving parameter uncertainty – even without altering the disaster itself – can be as large as or larger than the benefit of ending the disaster. Imprecision acts as an amplification mechanism for perceived risk, leading agents to respond to a disaster with extreme conservatism in their investment/consumption decisions. As with mitigation incentives, our findings have implications for investment in information production. We consider endowing the economy with a technology to increase information precision and show that *marginal* value of information is very high when precision is low. From a policy perspective, reducing uncertainty about the evolution of disasters may be an important mechanism for alleviating their perceived harm.

The information dynamics of the model imply, however, that the welfare and incentive effects reverse prior to a disaster. Beliefs about the arrival rate also exhibit negative duration dependence, which increases when information is imprecise. Repeating the welfare computations in normal times, we show that information about the arrival rate can be welfare-destroying: agents may be subjectively better off with imprecise beliefs. Turning again to incentives, when agents have the option to invest in a mitigation technology prior the onset of a disaster, we show that imprecise information about the disaster frequency induces less mitigation than under full information. For the same reason, agents with less precise information place lower value on disaster insurance.

Taken together, the model describes a belief dynamic across regimes that can lead to seemingly pessimistic behavior in bad times and optimistic behavior in good times. There is some empirical support for this implication. A well-established branch of behavioral economics documents the pattern that economic decision makers tend to ignore the risk of rare adverse events in good times and exaggerate them in bad times. The theory of *diagnostic expectations* has been advanced precisely to account for empirical evidence of this pattern. (See Bordalo et al. (2022) for a recent overview.) Our model presents a rational perspective also potentially consistent with this evidence.

1.1 Related Literature

The paper contributes a new insight to the literature that assesses the welfare costs of disaster risk (see Barro (2009), Pindyck and Wang (2013), Jordà et al. (2020), and Martin and Pindyck (2021)). Acharya et al. (2023) calibrate a version of the model studied here (with full information) to stock market responses to vaccine development news during 2020 in order to estimate the ex ante welfare cost of the pandemic.

A number of papers study learning problems in the context of models with disasters.

Disasters are often parameterized as an exogenous shock process (hitting consumption or the capital stock) whose intensity is unobservable and possibly time-varying (for example, Benzoni et al. (2011) and Wachter and Zhu (2019)). We also have such a shock process, and its intensity varies over time: it is zero in normal times and positive in a disaster regime. However, we assume that agents do know the shock intensity.

In emphasizing uncertainty about persistence, our paper also shares similarities with Gillman et al. (2014) and Ghaderi et al. (2022) in which regimes of differing growth differ in their expected duration. These models assume the regime itself is unobservable. Another related work is Andrei et al. (2019) in which agents do not observe the mean-reversion speed of current consumption shocks and thus face persistence risk. In their model, as in ours, the persistence risk is asymmetric: information about persistence is positive in good times and negative in bad.

Collin-Dufresne et al. (2016) also study a 2-regime rare disaster economy with learning about the switching parameters. They show that, when risk aversion exceeds the inverse of the elasticity of intertemporal substitution, even small amounts of persistence uncertainty can produce large effects on the equity premium and Sharpe ratio. The mechanism they highlight is the increase due to learning in the subjective volatility of consumption growth and marginal utility. While our setting is similar, the real effects we document are driven by the drift of the parameter estimates, not the volatility they induce.[2]

Most of the above papers focus on implications for asset pricing. An exception is Hong et al. (2023) who study implications of time-varying disaster beliefs for willingness to pay for mitigation efforts in the presence of externalities. Our focus too is on welfare effects. We highlight the interaction between unobservable persistence and the current state of the economy in determining the value of information and investment incentives.

2 Model

In this section, we introduce a regime-switching model of disasters under partial information. The goal is to study how the representative agent's value and policy functions vary with information precision.

[2]In contrast, our main findings are *larger* in magnitude when the elasticity of intertemporal substitution is less than the inverse of the coefficient of risk aversion.

2.1 Disaster Dynamics

Following Nakamura et al. (2013), we consider the state of the economy to be either in a "non-disaster" regime or in a "disaster" regime, and denote the state as $s \in \{0,1\}$. Let η denote the probability per unit time (or, intensity) of a disaster arrival, and let λ denote the probability per unit time of a disaster ending.

The model's depiction of the disaster consists of a state-specific stochastic process for the accumulation of wealth. Specifically, let q denote the quantity of productive capital of an individual household (which could be viewed as both physical and human capital). We assume that the stock of q is freely convertible into a flow of consumption goods at rate C per unit time. Then our specification is that q evolves according to the process

$$dq = \mu(s)qdt - Cdt + \sigma(s)qdB_t - \chi(s)qdJ_t \tag{1}$$

where B_t is a standard Brownian Motion and J_t is a Poisson process with intensity $\zeta(s)$. We set $\chi(0) = 0$ and $\chi(1) > 0$ for the disaster state. The Poisson shock captures the risk of an economic loss to the household. While we refer to the occurrence of the state $s = 1$ as the "disaster" (i.e., independent of whether or how many wealth shocks actually occur), somewhat more common in the literature would be to label these dJ shocks themselves as the "disasters", in which case our model maps to a particular specification of time-varying disaster risk, being either "on" or "off" depending on the regime. In Section 3 we will consider augmenting the economy to include real options to mitigate the disaster or acquire information.

2.2 Information Structure

Within a disaster there is likely to be deep uncertainty about *all* the governing parameters. Our focus on the timing parameters is motivated by the experience of COVID-19 in which the likely duration of the pandemic and the frequency of future pandemics were especially urgent questions to resolve. To model this, we assume the switching intensities η and λ are unobservable. While, formally, all disasters have the same parameters, this is not essential. Our main economic conclusions apply as well to the case in which parameter uncertainty re-sets with each new regime.

We will assume that at time zero the agent has beliefs about the two intensity parameters that are described by independent gamma distributions. Each distribution has a pair of non-negative hyperparameters, a^η, b^η and a^λ, b^λ, that are related to the first and second moments via

$$\mathbb{E}[\eta] = \frac{a^\eta}{b^\eta}, \qquad \text{Std}[\eta] = \frac{\sqrt{a^\eta}}{b^\eta}, \tag{2}$$

and likewise for λ. The *relative precision* about η, defined as its mean divided by its standard deviation, is $\sqrt{a^\eta}$.

By Bayes' rule, under this specification, as the agent observes the switches from one regime to the next, her beliefs remain in the gamma class with the hyperparameters updating as follows

$$\begin{aligned} a_t^\eta &= a_0^\eta + N_t^\eta \\ b_t^\eta &= b_0^\eta + t^\eta \end{aligned}$$

where t^η represents the cumulative time spent in state 0 and N_t^η represents the total number of observed switches from 0 to 1. Analogous expressions apply for a^λ and b^λ. Thus, while in $s = 0$, the only information that arrives (on a given day, say) is whether or not we have switched to $s = 1$ on that day. If that has occurred, the counter N^η increments by one and the clock t^η turns off and t^λ turns on. The system is assumed to start in the state $s = 0$ with $N^\eta = N^\lambda = 0$.

The model thus pastes together two linked learning regimes. In each regime, we have a finite dimensional filter in the sense that the two updated parameters fully characterize beliefs about that regime. Further, $\hat{\eta}_t \equiv \mathbb{E}_t[\eta] = a^\eta / b^\eta$, and it remains the case that the agent views this number as the probability per unit time of an instantaneous switch from $s = 0$ to $s = 1$ (again with equivalent expressions for the other regime.)

The gamma-exponential conjugate system is well studied in stochastic process theory (e.g., see Harris and Singpurwalla (1968) and Rubin (1972)). Under these beliefs, the measure for the switching time is a Lomax distribution (Lomax (1954)), whose expectation (in the $s = 0$ regime) is $1/\hat{\eta}$ times $a^\eta / (a^\eta - 1)$. This can be infinite when the relative precision of knowledge of η is low (as illustrated in Figure 1). As we will see, this has important consequences for agents' welfare and optimal behavior

2.3 Preferences

We assume the economy has a unit mass of identical agents (households). Each agent has stochastic differential utility or Epstein-Zin preferences Duffie and Epstein (1992) based on consumption flow rate C, given as

$$\mathbb{J}_t = \mathbb{E}_t \left[\int_t^\infty f(C_{t'}, \mathbb{J}_{t'}) dt' \right] \tag{3}$$

and aggregator

$$f(C, \mathbb{J}) = \frac{\rho}{1-\psi^{-1}} \left[\frac{C^{1-\psi^{-1}} - [(1-\gamma)\mathbb{J}]^{\frac{1}{\theta}}}{[(1-\gamma)\mathbb{J}]^{\frac{1}{\theta}-1}} \right] \tag{4}$$

where ρ is the discount factor, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution (EIS), and $\theta \equiv \frac{1-\gamma}{1-\psi^{-1}}$.The representative agent's problem is to choose optimal consumption $C(s)$ that maximizes the objective function $\mathbb{J}(s)$.[3]

2.4 Solution

Under the model's information setting, the economy is characterized by a six-dimensional state vector consisting of the stock of wealth, q, $a^\eta, b^\eta, a^\lambda, b^\lambda$ and the regime indicator S. However this six-dimensional space can be reduced to three when solving the agent's optimization problem.

Since the switches between states alternate, we can define an integer index M_t to be the total number of switches $N_t^\eta + N_t^\lambda$ and then $N_t^\eta = M_t/2$ when M is even, and $N_t^\lambda = (M_t+1)/2$ when M is odd. Knowing M (along with the priors a_0^η and a_0^λ) is equivalent to knowing a_t^η and a_t^λ. Given these values, specifying the current mean estimates $\hat{\eta}_t$ and $\hat{\lambda}_t$ is equivalent to specifying the remaining hyperparameters b_t^η and b_t^λ.

Within each regime the only changes to the state (apart from q) come through variation in the estimates $\hat{\eta}_t$ and $\hat{\lambda}_t$ which change deterministically with the respective clocks t^η and t^λ. Holding M fixed, the dynamics of $\hat{\eta}_t$ are given by

$$\begin{aligned} d\hat{\eta}_t &= d\frac{a_t^\eta}{b_t^\eta} = a_t^\eta d\frac{1}{b_t^\eta} \\ &= -\frac{a_t^\eta}{(b_t^\eta)^2}\, dt \\ &= -\frac{(\hat{\eta}_t)^2}{a_t^\eta}\, dt. \end{aligned} \tag{5}$$

[3] We recognize the limitations of using a utility specification driven by consumption goods, particularly within a crisis when other considerations (e.g., health) may affect well-being. However, using a familiar formulation ensures that our findings are not driven by non-standard assumptions about utility.

The latter expression says that, until new information arrives, $\hat{\eta}$ decays quadratically and deterministically to zero at a rate that is faster when a^{η} is small. This dynamic defines the negative duration dependence of the system and illustrates its dependence on the degree of information precision.[4]

The agent's Hamilton-Jacobi-Bellman equation links the value functions for states with successively more history. For large M, the estimation errors for both η and λ go to zero:

$$\frac{\mathrm{Std}[\eta]}{\mathbb{E}[\eta]} = \frac{1}{\sqrt{a^{\eta}}} = \frac{1}{\sqrt{a_0^{\eta} + M_t/2}}.$$

Thus the system converges to the full-information solution, which is characterized by two coupled algebraic equations. Appendix (A.2) establishes the following:

Proposition 1. *Let $H(\hat{\eta}, \hat{\lambda}, M)$ denote the solutions to system of coupled first-order differential equations in the appendix. Assuming these are positive, optimal consumption is*

$$C = \rho^{\psi} \ (H)^{-\frac{\psi}{\theta}} q, \tag{6}$$

and the value function of the representative agent is

$$\mathbb{J} \equiv \frac{H(s) q^{1-\gamma}}{1-\gamma}. \tag{7}$$

Note: All proofs appear in the appendix.

The appendix also describes an efficient solution algorithm for the system, and discusses conditions for existence of a unique positive solution.

3 Results

We now turn to numerical analysis to illustrate the model's effects. Our baseline calibration fixes the growth rate $\mu(s)$ and volatility $\sigma(s)$ across regimes to be 0.04 and 0.05. (The values are chosen to approximately capture the growth rate and volatility of aggregate

[4]The ODE in (5) has the exact solution

$$\hat{\eta}_t = \frac{1}{\frac{1}{\hat{\eta}_0} + \frac{t}{a_0^{\eta}}}$$

where t is the time since the regime began.

dividends in non-disaster times.) The disaster shock size is set to $\chi = 0.04$. We fix the disaster shock intensity to be 1.0 in order to interpret χ as the expected loss of wealth per year. We use baseline preference parameters ($\gamma = 4$, $\psi = 1.5$, $\rho = 0.04$) that are broadly consistent with the macro-finance literature. The role of these choices is explored below.

3.1 Information Precision in a Disaster

To start, consider the welfare consequences of parameter uncertainty during a disaster. Since Lucas (1987), a large literature has analyzed the welfare costs of aggregate risks in business cycle models in order to quantify incentives to reduce them. Here, we extend this line of research to encompass the *perceived* risk that stems from parameter unobservability. We address two main questions. First, comparing partial information to full information, how much worse is the disaster compared to the non-disaster state? Second, how much would agents pay to gain information about the unknown parameters?

For any pair of economies or states, $\{i, j\}$, we report the fraction of wealth that the representative agent would be willing to pay for a one-time transition from the worse (j) to the better state (i). The welfare gain is computed as the certainty equivalent change in the representative agent's lifetime value function :

$$1 - \left(\frac{H(j)}{H(i)}\right)^{\frac{1}{1-\gamma}}$$

This definition is standard in the literature.

3.1.1 Welfare Gain from Curtailing a Disaster

To quantify the severity of disasters under our base parameterization, Table 1 reports the welfare gain from ending a disaster, that is, to transitioning from $s = 1$ to $s = 0$ holding everything else fixed. In the context of a pandemic, this could be viewed as the value of a perfectly effective cure or vaccine. Each cell of the table shows this gain for three values of $\hat{\lambda}$ and two values of $\hat{\eta}$. The top panel shows the result when there is no uncertainty about the parameters. In this benchmark case, agents would be willing to pay between roughly 5% and 20% of wealth to return to the normal economic state. The values are intuitively reasonable in the sense that, for $\eta = 0.01$ say, they are not too far from just the expected length of the disaster ($1/\lambda$) times the expected loss of wealth per year, $\chi = 0.04$. Reading across the top panel, the preference parameters do not have large effects on the the full-information values. The bottom panel shows the same computation when agents' current uncertainty about the timing parameters (their posterior standard deviation) is equal to their mean belief about each of them, or their relative precisions are 1.0 for both. This is our baseline case of partial information.[5]

Compared to the top panel, the partial information situation is subjectively much worse. Adding parameter uncertainty greatly increases the resources that the economy would be willing to expend to find a cure or otherwise limit the damage. An analogous computation (omitted for brevity) shows that the welfare benefit from lowering the disaster severity (χ) is also much larger under partial information. To see how these welfare differentials map into investment incentives, suppose now that the economy is endowed with a real option to undertake a lump-sum expenditure, I, to reduce the severity according to $\chi = g(I/q)$ for an arbitrary function $g > 0$ with $g' < 0$. By an argument that we formalize in the online appendix (A.4.1), the sensitivity of the welfare function H to χ effectively pins down the marginal benefit of I. Hence, for standard parameterizations of the mitigation technology, the optimal investment will be strictly greater under partial information than under full information.

3.1.2 Welfare Gain from Resolving Parameter Uncertainty

The results above immediately raise the question of how much agents would be willing to pay to resolve parameter uncertainty, even without curtailing the current disaster. Panel (A) of Table 2 answers this question. For each of the preference configurations considered and for nearly all values of $\hat{\eta}$ and $\hat{\lambda}$, the value of resolving the parameter uncertainty is as large or larger than the value of resolving the ongoing disaster.[6]

It is perhaps not surprising that risk averse agents would be willing to pay to resolve parameter uncertainty. However, as we will see below, this need not always be the case. Moreover, here, it is the *magnitude* of the value that is surprising. The numbers are much larger than typically found in analogous calculations in the literature for other types of risk. In a similar setting, Collin-Dufresne et al. (2016) show that, using a myopic utility benchmark, uncertainty about the persistence of the bad state is an order of magnitude more important than uncertainty about other parameters, e.g., growth rates and volatilities in the two regimes.

Comparing the results in Panel (A) across preference specifications, the value of resolving parameter uncertainty increases with higher risk-aversion (γ), and is lower with

[5]In this case the gamma prior is an exponential distribution. Results are similar for differing initial precisions.

[6]The welfare gain is an understatement in that it excludes any value from, for example, information helping agents' ability to avert future disasters. The model contains no mechanism by which *knowing more about* λ and η allows agents to affect them.

a lower time discount factor (ρ). The γ effect is intuitive: parameter risk increases the subjective volatility of wealth, which agents dislike. Likewise, agents with a longer time horizon (lower subjective discount rate) care strongly about persistence.

The largest effects in Panel (A) come from lowering the elasticity of intertemporal substitution. This is noteworthy because there is a common understanding of Epstein-Zin preferences under which agents with $\psi \leq 1/\gamma$ can be viewed as having a preference for "later resolution of uncertainty," which might suggest that they value information *less* than high EIS agents, whereas here the result is the opposite.[7]

To understand this, note that, with recursive preferences, agents with low EIS cut consumption when the economy enters the disaster state. This is because a low EIS implies strong consumption smoothing motives, and the prospect of lower future wealth motivates a sharp increase in savings. By contrast, a higher EIS implies relatively more concern with the volatility of wealth than consumption smoothing. Agents with a high EIS therefore decrease investment in a disaster. However, the differing consumption responses do not make disasters worse *per se* for agents with low EIS: the top panel of Table 1 shows little effect of the EIS under full information. Instead, it is the extreme *decrease* in consumption as information precision declines that leads to the large welfare losses for these agents. This is again due to the time horizon effect. With low precision of information about λ, there is a chance that the withdrawal of consumption will be effectively permanent.[8]

As with mitigation, there is a direct mapping from the welfare costs of information to investment incentives. The findings above imply that the ability to produce information about the underlying determinants of disaster duration could be an extremely valuable real option. To show this, we can again consider augmenting the economy to have a one-time opportunity to purchase a signal about λ, (e.g., via a "laboratory experiment") in the form of a realization of N transition times of the underlying process for a cost $c(N)$. Note that the outcome will also convey information about the level of λ, which may entail good or bad news. In particular, the worst-case scenario of a very low λ could be revealed, making the purchase of the signal subject to the same risks that are reflected in the partial information value function. It is thus not *a priori* clear that this information-production

[7]See Epstein et al. (2014) for an analysis of the welfare consequences of varying the timing of the resolution of uncertainty.

[8]In Van Nieuwerburgh and Veldkamp (2006) and Kozlowski et al. (2020) learning effects within downturns endogenously cause the downturns to last longer. In our case, the uncertainty-induced investment and consumption distortions do not affect the length of the disaster. However, negative duration dependence implies that the *perceived* duration lengthens the longer the episode goes on.

opportunity will have positive expected value.

In the online appendix (A.4.2), we show that, under a simple linear information cost structure, agents do optimally choose to spend significant fractions of their endowment to buy signals. The expected value from the signal increases rapidly for small increases in precision. From a policy perspective, the implication of the large *marginal* value of information in a disaster is that fundamental research can crucially complement (or perhaps even substitute for) efforts to directly affect the course of the disaster.

3.2 Parameter Uncertainty Prior to a Disaster

The analysis above immediately suggests a corollary: all of the conclusions may be *reversed* prior to a disaster. Low precision of information about the disaster intensity in normal times could cause agents to give increasing weight to *best* case scenarios, namely, that a disaster will never materialize. We now show that, indeed, this can be the case. Moreover, we will see that *both* types of effects – seemingly pessimistic in a disaster and optimistic beforehand – may co-exist.

3.2.1 Value of Information

We start by examining the welfare effect of uncertainty about η when $s = 0$. This effect can be isolated by setting the prior precision for λ to be very high, so that, effectively agents know its value. Panel (B) of Table 2 shows the value of information under these conditions. In the baseline case, the value of information about η indeed can be negative, although the magnitude is not always large. With $\gamma = 2$ the effect can be economically significant: when the point estimate $\hat{\eta}$ is 0.05 the representative agent would be willing to give up to 2.1% of wealth to *not* learn the true disaster frequency.

When information about both λ and η is imprecise, the former typically matters more in the sense that full information is overall welfare improving in both states. Intuitively, the worst-case scenarios still loom large prior to a disaster. However, we can vary the degree to which duration dependence operates in each regime by observing that the percentage drift in the means (which drives the effect) scales with the ratio of the mean to the precision. Thus, when $\hat{\eta}/a_0^{\eta}$ and $\hat{\lambda}/a_0^{\lambda}$ are similar, we obtain similar belief dynamics in the two states. The top panel of Figure 3 illustrates this co-existence of pessimism and optimism in terms of growth rate expectations. Using the parameters in that figure together with $\gamma = 1$, the welfare cost of parameter uncertainty is 3.2% of wealth in the disaster and -3.5% before it. Hence, the incentives to acquire information alternate sign in the two states.

3.2.2 Disaster Mitigation Incentives

We saw above that, when information about disaster duration was imprecise, agents had stronger incentive to end or curtail the disaster. But that logic would now also be expected to flip. When agents place more weight on best-case scenarios, their incentives to invest in mitigation are weaker. To make this explicit, again consider endowing the economy with a one-time real option to expend resources to lower the disaster severity, χ. But now the investment decision is made prior to the onset of a disaster. We argued above that, for any given mitigation technology, the optimal amount invested will scale with the sensitivity of the value function to χ.

The lower panel Figure 3 plots $\log H$ as a function of the disaster severity under full and partial information.[9] When $s = 1$ (right panel) we verify our assertion above that the slope is steeper under partial information. However, with these parameters, when $s = 0$ (left panel) the relation is reversed. Hence, for standard specifications of the mitigation technology, lower precision of information will result in underinvestment or underpreparedness relative to full-information in advance of a disaster.

Continuing the example, while the slopes of the plots for full information and partial information do not appear dramatically different, they still may imply economically important differences for the effect of information precision on mitigation. Consider the mitigation function $\chi(i) = \chi_0 \, e^{-bi}$ where i is the fraction of wealth invested. The left panel of Figure 4 shows optimal investment prior to the disaster when the exercise price of the real option is 1% of wealth, $b = 3$ and $\xi_0 = 0.12$. For low precision, a^{η}, agents will not engage in mitigation at all. However, once a precision threshold is crossed, the option is exercised and investment jumps to over 6% of aggregate wealth resulting in substantial reduction in the disaster severity (right panel). Hence small changes in parameter uncertainty may have important consequences.[10]

3.2.3 Pricing of Disaster Insurance

Another way of capturing preparedness incentives is via willingness to pay for insurance against a disaster. Consider the price of a financial contract which pays 1 upon the arrival of the next disaster. This contract is in net zero supply and does not affect real outcomes. However, its price provides a measure of agents' assessment of the likelihood and timing of a disaster, as well as its consequences in marginal utility terms.

[9] Recall the full value function is negative, so higher values of H are worse.

[10] Imprecision may also exacerbate collective action problems. In this example, incorporating investment externalities can dramatically expand the no-action region.

Proposition 2. *The price, P, in the non-disaster state of the claim which pays 1 upon the arrival of the next disaster, satisfies the equation*

$$-\frac{(\hat{\eta})^2}{a^{\eta}}\frac{\partial P}{\partial \hat{\eta}} + \hat{\eta}\,\frac{H(\hat{\eta},\hat{\lambda},M+1)}{H(\hat{\eta},\hat{\lambda},M)}\,(1-P) - r_0\,P = 0 \tag{8}$$

where r_0 is the riskless rate.[11]

Given the value function solutions, this is a first-order differential equation in $\hat{\eta}$, with boundary condition $P(0) = 0$. Figure 5 plots the solutions for the parameter set we have been considering. Since partial information entails longer expected waiting times, we see that, counterintuitively, the contract is substantially underpriced relative to its full-information value.

Summarizing, this section has shown that information about disaster frequency can be welfare reducing because, with less information, agents rationally believe a disaster may never materialize (the expected waiting time becomes unbounded). This negative value of information may shed light on failure to prepare adequate for disasters and on "don't look up" behavior of seemingly willful ignorance towards their threat.[12]

4 Conclusion

This paper considers the economic effects of uncertainty about state transition probabilities. The main finding is that, as uncertainty increases, the left tail of the distribution becomes the dominant factor in decision-making and welfare, even holding the mean constant. Fully rational agents may act as if the current state will never end. In a setting where regimes are either "good" or "bad", the result can look like overreaction relative to the full-information benchmark, or pessimism in bad times and optimism in good times.

[11] The rate and the pricing kernel are derived in terms of the model primitives in the Appendix.

[12] Models with costly information processing have also been used to explain failure to prepare for disasters. See Maćkowiak and Wiederholt (2018). Aversion to information is explicitly modelled in the preference specification of Andries and Haddad (2020).

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Table 1: Welfare Gain to Ending Disaster

(A) Full Information

Benchmark

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.188	0.090	0.046
	0.05	0.147	0.081	0.0447

$\psi = 0.20$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.205	0.093	0.046
	0.05	0.162	0.085	0.045

$\gamma = 2$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.160	0.079	0.042
	0.05	0.135	0.072	0.04

$\rho = 0.02$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.225	0.100	0.048
	0.05	0.166	0.091	0.047

(B) Partial Information

Benchmark

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.340	0.288	0.235
	0.05	0.246	0.225	0.201

$\psi = 0.20$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.777	0.808	0.827
	0.05	0.646	0.700	0.739

$\gamma = 2$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.292	0.214	0.156
	0.05	0.232	0.186	0.147

$\rho = 0.02$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.519	0.531	0.537
	0.05	0.350	0.378	0.405

The table shows the fraction of wealth the agent would be willing to surrender for a one-time transition out of the disaster state. In Panel (A), agents in the economy know the parameters λ and η. In Panel (B), they have posterior standard deviation equal to their point estimates of these quantities. The benchmark parameters are given in Section 3.

Table 2: The Value of Information

(A) During a Disaster

Benchmark

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.235	0.234	0.220
	0.05	0.182	0.239	0.232

$\psi = 0.20$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.934	0.931	0.921
	0.05	0.922	0.928	0.922

$\gamma = 2$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.178	0.164	0.131
	0.05	0.148	0.169	0.152

$\rho = 0.02$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.641	0.688	0.666
	0.05	0.485	0.619	0.645

(B) Prior to Disaster

Benchmark

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.001	0.001	0.000
	0.05	-0.008	-0.000	0.001

$\psi = 0.20$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.012	0.002	0.001
	0.05	0.079	0.026	0.007

$\gamma = 2$

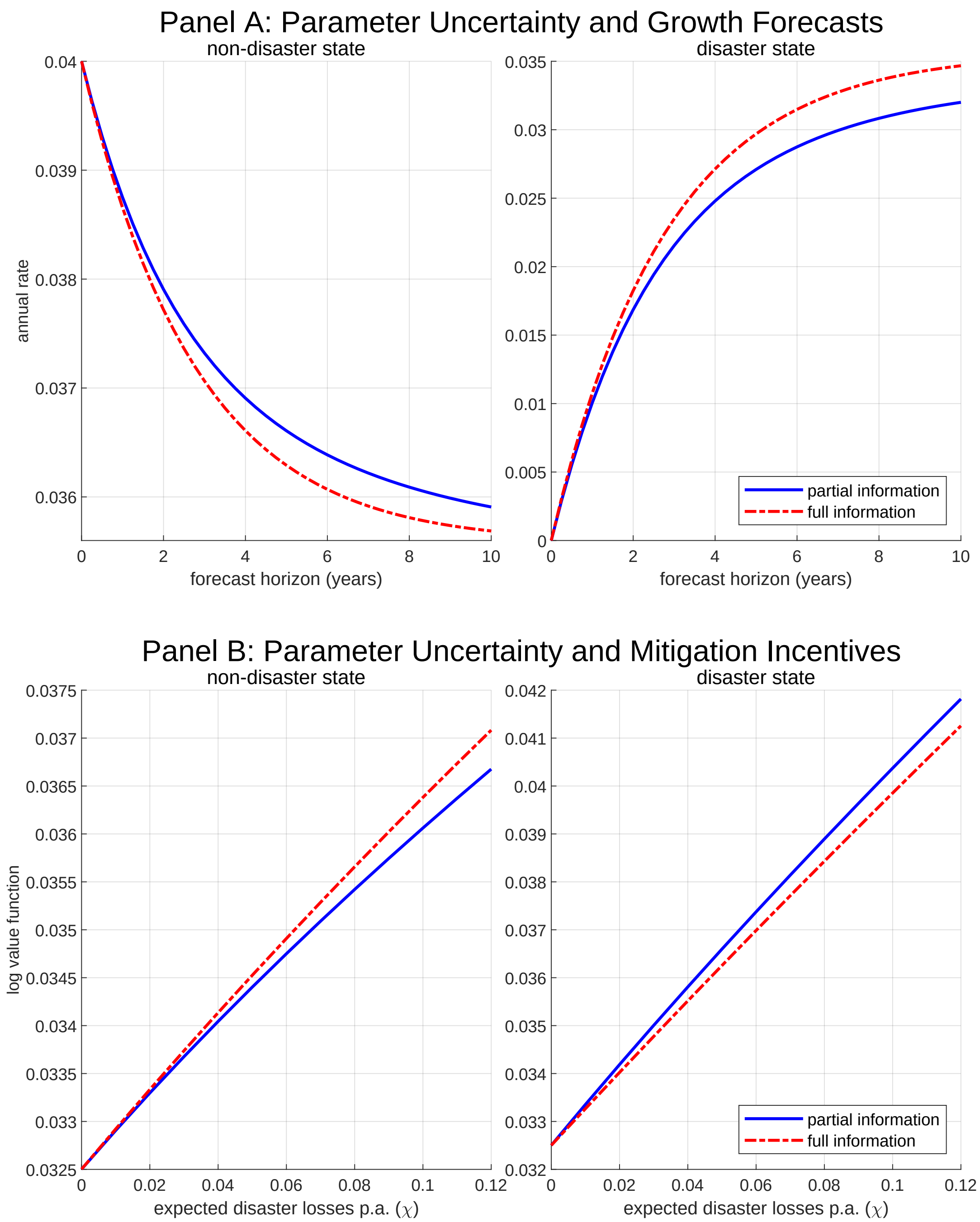
		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	-0.001	-0.000	-0.000
	0.05	-0.021	-0.007	-0.002

$\rho = 0.02$

		$\hat{\lambda}$ 0.2	0.5	1.0
$\hat{\eta}$	0.01	0.050	0.015	0.004
	0.05	0.044	0.047	0.023

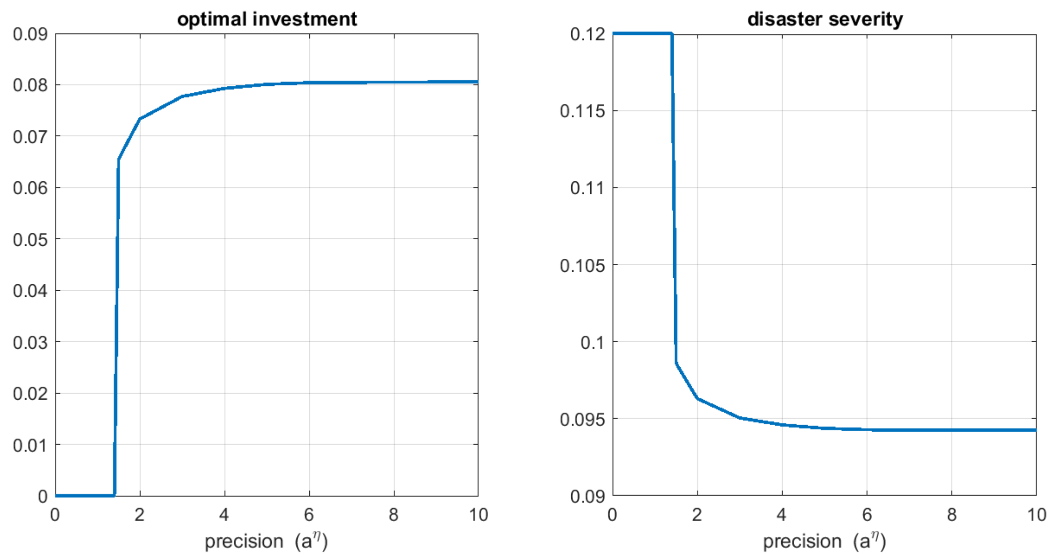
Panel (A) shows the fraction of wealth that the representative agent would be willing to surrender for a transition from partial information to full information (as defined in Table 1) while remaining in the disaster state. Panel (B) shows the fraction of wealth the agent would surrender for a transition from partial information to full information about the disaster intensity η while remaining in the non-disaster state. The agent is assumed to have full information about λ. Benchmark parameters are given in Section 3.

Figure 3: Optimism and Pessimism



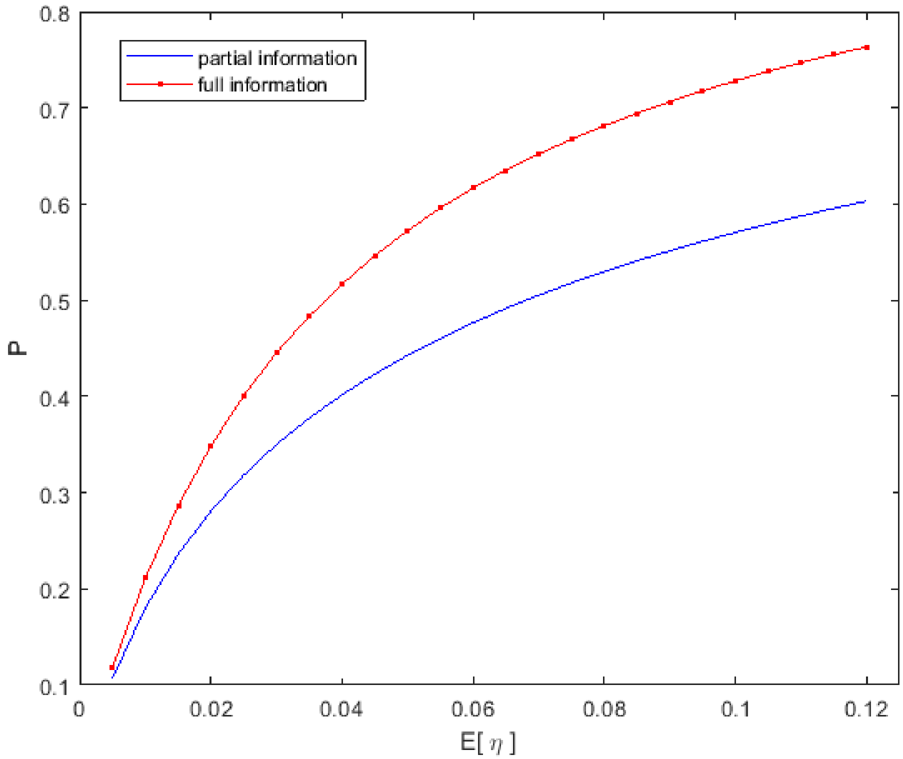
Panel (A) plots subjective expectations for the growth of wealth to different horizons, T. The left panel shows agents' forecasts when in normal times. The right panel shows forecasts during a disaster. The plots take the agent's posterior expected switching intensities for the two states to be $(0.05, 0.20)$ with respective posterior standard deviations of $(0.05, 0.10)$. Panel (B) plots the log value function multiplier, H, as a function of the disaster severity χ also within the disaster (right) and nondisaster (left) states. For each plot, the full-information economy's values are plotted as dotted (red) lines and the partial information ones as solid (blue) lines. Panel (B) uses the benchmark parameters given in the text with $\gamma = 1.01$.

Figure 4: Parameter Uncertainty and Mitigation



The figure plots the optimal investment mitigation (left) and resulting expected annual disaster losses (right) as a function of the precision of information about the arrival intensity, a^η. The economy has a one-time real option to spend a fraction $i + c$ of wealth to lower the disaster severity via $\chi = \chi_0 e^{-bi}$ where $\chi_0 = 0.12, b = 3$ and the fixed costs is $c = 0.01$. Parameter values are the same as in Figure 3.

Figure 5: Disaster Insurance Pricing



The figure plots the price of a contract paying 1 upon the arrival of the next disaster as a function of the mean arrival intensity, $\hat{\eta}$. Other parameter values are the same as in Figure 3.

Online Appendix
Disasters with Unobservable Duration and Frequency: Intensified Responses and Diminished Preparedness

A Proofs and Derivations

A.1 Full Information

To prove Proposition 1, we first treat the case of full-information in which the only state variables are $s \in \{0,1\}$ and q. For ease of notation, define the following combination of preference parameters:

$$e_0 \equiv \frac{\theta}{\psi}\,\rho^{\psi} \quad \text{and} \quad e_1 \equiv -\frac{\psi}{\theta}. \tag{A.1}$$

Also define $\lambda(0) = \eta, \lambda(1) = \lambda$.

Lemma *Denote*

$$g(s) \equiv \theta\,\rho - (1-\gamma)\left(\mu(s) - \frac{1}{2}\gamma\sigma(s)^2\right) - \zeta(s)\ \left([1-\chi(s)]^{1-\gamma} - 1\right) \tag{A.2}$$

for $s \in \{0,1\}$. Let $H(s)$'s denote the solution to the following system of recursive equations:

$$g_0 \equiv g(0) = e_0\,(H(0))^{e_1} + \eta\left[\frac{H(1)}{H(0)} - 1\right] \tag{A.3}$$

$$g_1 \equiv g(1) = e_0\,(H(1))^{e_1} + \lambda\left[\frac{H(0)}{H(1)} - 1\right] \tag{A.4}$$

Assuming the solutions are positive, optimal consumption in state s is

$$C(s) = \rho^{\psi}\ (H(s))^{e_1}\,q, \tag{A.5}$$

and the value function of the representative agent is

$$\mathbb{J}(s) \equiv \frac{H(s)q^{1-\gamma}}{1-\gamma}. \tag{A.6}$$

Proof. Using the evolution of capital stock for the representative agent (1) the Hamilton-

Jacobi-Bellman (HJB) equation for each state s can be written:

$$
\begin{aligned}
0 = \max_C \Big[& f(C, \mathbb{J}(s)) + \mathbb{J}_q(s)(q\mu(s) - C) \\
& + \frac{1}{2}\mathbb{J}_{qq}(s) q^2 \sigma(s)^2 + \zeta(s) \left[\mathbb{J}(s, q(1-\chi(s))) - \mathbb{J}(s,q)\right] \\
& + \lambda(s) \left[\mathbb{J}(s') - \mathbb{J}(s)\right]\Big]
\end{aligned} \tag{A.7}
$$

for $s = \{0,1\}$ and $s' = \{1,0\}$.

Taking the first-order condition with respect to $C(s)$ in (A.7), we obtain

$$
f_c(C, \mathbb{J}(s)) - \mathbb{J}_q(s) = 0. \tag{A.8}
$$

Using $f(C, \mathbb{J})$ from (4) and taking the derivative with respect to C, we obtain

$$
f_c = \frac{\rho C^{-\psi^{-1}}}{[(1-\gamma)\mathbb{J}(s)]^{\frac{1}{\theta}-1}}. \tag{A.9}
$$

Substituting the conjecture $\mathbb{J}(s)$ in equation (7) yields

$$
f_c = \frac{\rho C^{-\psi^{-1}}}{H(s)^{\frac{\gamma-\psi^{-1}}{1-\gamma}} q^{\gamma-\psi^{-1}}}. \tag{A.10}
$$

Then, for state s, we obtain by substituting $\mathbb{J}_q(s) = H(s)q^{-\gamma}$ in (A.8), and simplifying:

$$
C(s) = \frac{H(s)^{-\psi/\theta} q}{\rho^{-\psi}} \tag{A.11}
$$

which agrees with (6) using the definitions of the constants in (A.1).

To verify the conjectured form of the value function, we plug it in to the HJB equation (A.7) and reduce it to the recursive system in the proposition via the following steps:

1. substitute the optimal policy $C(s)$ into the HJB equation (A.7);
2. cancel the terms in q which have the same exponent; and
3. group constant terms not involving Hs and define them to be $g(0)$ for state 0 and $g(1)$ for state 1.

The third step yields the system of recursive equations A.3, A.4. □

Regularity Conditions

The functions $H(s)$ are necessarily bounded by the limiting solutions in which the economy is never in a disaster, H_0^{min}, or is always in a disaster, H_1^{max}. It is straightforward to show that these constants are given by

$$H_0^{min} = \left(\frac{g_0}{e_0}\right)^{1/e_1} \quad \text{and} \quad H_1^{max} = \left(\frac{g_1}{e_0}\right)^{1/e_1}.$$

These quantities are real and positive if g_0, g_1, and e_0 all have the same sign. Given this, it can be shown that a necessary and sufficient condition for existence of a unique solution is that $g_1 < g_0$.

A.2 Proposition 1: HJB System with Parameter Uncertainty

Proof. As noted in the text, the model can be parameterized in terms of the state variables $M, \hat{\eta}, \hat{\lambda}$, and q, where $M = M_t$ is an integer counter that increases on a state switch such that $M_0 = 0$ and even numbered states are the non-disaster epochs and odd numbered states are the disasters. Also, in the non-disaster states, $\hat{\lambda}$ is constant, while $\hat{\eta}$ is constant in disasters. As a consequence, compared with the derivation above for the full-information case, there is now only one additional source of variability in each regime. The dynamics of $\hat{\eta}$ are given in (5) with an analogous expression for and $\hat{\lambda}$. And note that, under the agents' information set, the dynamics of the wealth variable q are identical to the full information dynamics.

As a result, the HBJ equations under partial information are the same as (A.7) above (with state 0 and state 1 being replaced by M and $M+1$) with the addition of a single term on the right side:

$$-\frac{(\hat{\eta})^2}{a^{\eta}} \frac{\partial \mathbb{J}(0)}{\partial \hat{\eta}} \tag{A.12}$$

for $s = 0$, and

$$-\frac{(\hat{\lambda})^2}{a^{\lambda}} \frac{\partial \mathbb{J}(1)}{\partial \hat{\lambda}} \tag{A.13}$$

for $s = 1$. Since, under the agent's information set, the state switches are a point-process with instantaneous intensities $\hat{\eta}$ and $\hat{\lambda}$, these quantities also replace their full information counterparts, η and λ, in multiplying the jump terms in the respective equations.

The next steps in the derivation involving the first order condition for optimal consumption are unchanged from the full-information case. This follows because consumption does not enter into any of the new terms involving the information variables. Replace $\mathbb{J}$ by the conjecture $\frac{q^{1-\gamma}}{1-\gamma}\, H(\hat{\eta},\hat{\lambda},M)$, then a common power of q term is cancelled, and the whole equation is divided by H. These manipulations lead to the above two terms showing up on the right hand side, in a system that is otherwise identical to the full-information system (A.3) and (A.4).

$$g_0 \;=\; e_0 H_M^{e_1} + \hat{\eta}\left(\frac{H_{M+1}}{H_M} - 1\right) - \frac{(\hat{\eta})^2}{a^{\eta} H_M}\,\frac{\partial H_M}{\partial \hat{\eta}} \tag{A.14}$$

$$g_1 \;=\; e_0 H_{M+1}^{e_1} + \hat{\lambda}\left(\frac{H_{M+2}}{H_{M+1}} - 1\right) - \frac{(\hat{\lambda})^2}{a^{\lambda} H_{M+1}}\,\frac{\partial H_{M+1}}{\partial \hat{\lambda}} \tag{A.15}$$

where the constants g_0 and g_1 are as defined in Lemma 1 above.

□

A.2.1 Solution Algorithm

In the full information case, solution of the algebraic system over a grid in the $(\hat{\eta},\hat{\lambda})$ plane is straightforward. The unknown constants $H(s)$ are bounded by the limiting solutions in which the economy is never in a disaster, H_0^{min}, or is always in a disaster, H_1^{max}. The former corresponds to $\eta = 0$ and the latter to $\lambda = 0$.

For the general case, we pick a large even integer M^{max} and assume that the economy has converged to the full information solution with $s = 0$ at M^{max} and $s = 1$ at $M^{max} - 1$. Given these solutions, the HBJ system for $M = M^{max} - 2$ is just a first order ODE, since the jump terms in (A.14)-(A.15) can be explicitly evaluated. For even values of M, the boundary condition at $\hat{\eta} = 0$ is again the full-information solution because the posterior standard deviation $\sqrt{a^{\eta}}\hat{\eta}$ is also zero. (Note that the value of $\hat{\lambda}$ is immaterial if disasters cannot arise.) Likewise, for odd values of M, the boundary condition at $\hat{\lambda} = 0$ is given by the full-information solution. Hence, the first-order ODEs can be explicitly solved in alternating directions. The procedure is then repeated for all lower values of M.

A.3 Pricing Kernel, Riskless Rate and Proposition 2

This section first derives the pricing kernel and riskless rate under partial information. The results are then used to prove Proposition 2 Section 3.2 which describes the pricing equation of insurance against a disaster.

Under stochastic differential utility, the kernel can be represented as

$$\Lambda_t = e^{\int_0^t f_{\mathbb{J}} du} f_C \tag{A.16}$$

where the aggregator function is given in (4). With the form of the value function and the optimal consumption rule from Proposition 1, evaluating the partial derivatives yields (after some rearrangement)

$$\Lambda_t = q^{-\gamma}\; H(\hat{\eta}, \hat{\lambda}, M)\; e^{\int_0^t [c_u\; (\theta-1)-\rho\theta] du} \tag{A.17}$$

where $c = c(\hat{\eta}, \hat{\lambda}, M) \equiv C/q$ is the marginal propensity to consume.

The riskless rate is minus the expected rate of change of $d\Lambda_t/\Lambda_t$ under the agents' information set. Applying Itô's lemma, for even values of M, the expected change is

$$c\;(\theta-1) - \rho\theta\; - \gamma(\mu - c) + \frac{1}{2}\gamma(\gamma+1)\sigma^2$$

$$-\frac{(\hat{\eta})^2}{a^{\eta}}\,\frac{1}{H}\frac{\partial H}{\partial \hat{\eta}}\; + \hat{\eta}\left(\frac{H(M+1)}{H(M)} - 1\right).$$

A key simplification is to observe that, by the HJB equation derived above (see (A.14)), the latter two terms in this expression can be replaced by $g_0 - \frac{\theta}{\psi}c$. This causes all of the terms involving c to exactly cancel. Using the definition of g_0 in (A.2), the remaining terms are just $-\mu + \gamma\sigma^2$. Hence we have shown

$$r_0 = \mu - \gamma\sigma^2.$$

Repeating the above steps for odd values of M and applying the same trick yields

$$r_1 = \mu - \gamma\sigma^2 - \zeta\chi(1-\chi)^{-\gamma}.$$

Turning to the insurance claim, the asset is assumed to make a terminal payout of 1.0 upon the occurrence of the next disaster. Proposition 3 characterizes its price in normal-times prior to that disaster.

Proof. We conjecture that the price, P, of the insurance is not a function of wealth, q. Moreover, when $s = 0$, the state variables a^{η}, a^{λ}, and $\hat{\lambda}$ are all fixed, and $\hat{\eta}$ evolves deterministically according to (5).

By the definition of the pricing kernel, for any claim in the economy, its instantaneous payout per unit time (in this case, zero) times Λ must equal minus the expected change of the product process $P\Lambda$, or

$$\mathcal{L}(\Lambda(q_t, s_t, \hat{\eta}_t)\ P(s_t, \hat{\eta}_t))/\Lambda_t = 0, \tag{A.18}$$

where $\mathcal{L}(X)$ is the drift operator $\mathrm{E}[dX]/dt$ under the agents' information set.

Using Itô's lemma for jumping processes to expand the expected change,

$$-\frac{(\hat{\eta})^2}{a^\eta}\frac{\partial P}{\partial \hat{\eta}} + \mu_\Lambda P + \hat{\eta}\left(\frac{H(M+1)}{H(M)} - P\right) = 0$$

where we have written μ_Λ for the deterministic terms in $d\Lambda_t/\Lambda_t$ and used the fact that $P(M+1) = 1$.

Next, add and subtract $\hat{\eta}(\frac{H(M+1)}{H(M)} - 1)P$ and use the fact that the expected growth rate of the pricing kernel is minus the riskless rate:

$$r_0 = -\mu_\Lambda - \hat{\eta}\left(\frac{H(M+1)}{H(M)} - 1\right)$$

to get (8):

$$-\frac{(\hat{\eta})^2}{a^\eta}\frac{\partial P}{\partial \hat{\eta}} - r_0 P + \hat{\eta}\frac{H(M+1)}{H(M)}(1-P) = 0.$$

□

A.4 Real Options

A.4.1 Mitigation

The text in Section 3 describes endowing the model economy with a one-time real option to invest in a mitigation technology to alter a structural parameter, χ, via $\chi = g(i)$ where I is a lump-sum investment and $i = I/q$. Since the option is a one-shot decision, the post-investment economy is identical to the original model (without the technology) and hence its value function is as derived in the main propositions.

Then, the assertion is that, for two otherwise equal economies E1 and E2, if the sensitivity of the value function, H, to χ is weaker in E1 than in E2, then, if a solution to the real-options problem exists in E2, a solution also exists in E1 with smaller optimal investment.

To see this, view H as a function of χ, and the problem is to choose i to maximize the $H(g(i))(1-i)^{1-\gamma}/(1-\gamma)$ with first order condition $-g'(i)\,\partial \log H(g(i))/\partial\chi = (\gamma - 1)/(1-i)$. Assume $\gamma > 1$. Then the right side (the marginal cost) is an unbounded increasing function of i on $[0,1)$ which is the same for both economies. Call it $RHS(i)$. On the left side (the marginal benefit), the first term is the same for both economies. The hypothesis is that $\partial \log H(\chi)/\partial\chi$ is smaller in E1 than in E2 for all χ implying that the second term is smaller. Hence $LHS1(i) < LHS2(i)$ for all i. Assume $LHS2$ is continuous and declining. Then, if an interior solution, i_2^*, exists, it follows that on $[i_2^*,1)$ we have $LHS1 < LHS2 < RHS$, meaning that there cannot be a solution for E1 in this region. Hence, either there is a solution $i_1^* < i_2^*$ or no interior optimum exists and $i_1^* = 0$ in E1.

A.4.2 Information Production

The top panel of the table below presents the optimal information investment as a fraction of wealth when the economy contains a technology allowing agents to purchase a realization of N transitions of the disaster process, e.g., in a laboratory. The realization increases a^{λ} by N but also alters the mean $\hat{\lambda}$ depending on the (random) time-length of the realization.[1] The table assumes that the information production function is $N = 200\,i$, where $i = I/q$ is the lump-sum investment. The option to make this investment is a one-time occurrence at the on-set of a disaster.

The lower panel reports the welfare gain, in units of wealth, of the investment. The difference between the respective panels can be interpreted as the value-added of the technology.

[1] The time-length is a virtual output. The experiment is assumed to be atemporal. Agents receive the results immediately.

Table A.1: Information Production

(A) Optimal Investment

	Benchmark				$\psi = 0.20$		
		$\hat{\lambda}$				$\hat{\lambda}$	
	0.2	0.5	1.0		0.2	0.5	1.0
$\hat{\eta}$ 0.01	0.015	0.015	0.015	$\hat{\eta}$ 0.01	0.030	0.035	0.040
0.05	0.015	0.015	0.015	0.05	0.035	0.030	0.030
	$\gamma = 2$				$\rho = 0.02$		
		$\hat{\lambda}$				$\hat{\lambda}$	
	0.2	0.5	1.0		0.2	0.5	1.0
$\hat{\eta}$ 0.01	0.015	0.015	0.010	$\hat{\eta}$ 0.01	0.030	0.035	0.035
0.05	0.010	0.015	0.015	0.05	0.025	0.030	0.03015

(B) Welfare Gain

	Benchmark				$\psi = 0.20$		
		$\hat{\lambda}$				$\hat{\lambda}$	
	0.2	0.5	1.0		0.2	0.5	1.0
$\hat{\eta}$ 0.01	0.038	0.046	0.047	$\hat{\eta}$ 0.01	0.314	0.346	0.343
0.05	0.032	0.042	0.042	0.05	0.284	0.320	0.343
	$\gamma = 2$				$\rho = 0.02$		
		$\hat{\lambda}$				$\hat{\lambda}$	
	0.2	0.5	1.0		0.2	0.5	1.0
$\hat{\eta}$ 0.01	0.031	0.032	0.020	$\hat{\eta}$ 0.01	0.114	0.164	0.188
0.05	0.021	0.030	0.026	0.05	0.085	0.130	0.143