When is Less More? Bank Arrangements for Liquidity vs Central Bank Support*

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Abstract

Theory suggests that in the face of fire-sale externalities, banks have incentives to overinvest in order to issue cheap money-like deposit liabilities. The existence of a private market for insurance such as contingent capital can eliminate the overinvestment incentives, leading to efficient outcomes. However, it does not eliminate fire sales. A central bank that can infuse liquidity cheaply may be motivated to intervene in the face of fire sales. If so, it can crowd out the private market and, if liquidity intervention is not priced at higher-than-breakeven rates, induce overinvestment once again. We examine various forms of public intervention to identify the least distortionary ones. Our analysis suggests why private contingent capital dominated in the era preceding central banks and deposit insurance, why it waned subsequently, as well as why banking crises and speculative excesses continue to recur periodically.

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1 Introduction

Over the past century, there has been a significant increase in the scale and scope of central bank interventions in response to banking crises, as well as the expansion of public insurance schemes such as deposit insurance (Bordo and Siklos, 2018; Metrick and Schmelzing, 2021). In particular, since World War II, central bank policy responses to financial crises have become *"close to systematic"* (Ferguson et al., 2023). Puzzlingly, despite the presence of ex-ante public backstops and large-scale public liquidity provision ex post, significant banking crises continue to occur across the world, including in advanced economies with sophisticated regulatory frameworks (for example, the recent banking stress around the failure of Silicon Valley Bank in 2023). Why?

The root of banking crises typically lies in liquidity transformation: banks borrow short to lend or invest long in illiquid assets. Occasionally, accidents happen, and banks fall short of liquidity, forcing them to liquidate assets in a fire sale. Under such circumstances, the availability of additional funds can be helpful to a bank in reducing costly fire sales. What form should that contingent funding take? Should it be in the form of private insurance arrangements, that is, private sector promises to provide the bank transfers in exchange for a pre-committed fee (callable or contingent capital)? Or should it be public liquidity support in the form of central bank bailouts or loans at a high rate (à la Bagehot, 1873) after the need arises? Or should it be pre-positioned liquidity arrangements with the central bank (see Tuckman, 2012; King, 2016; Nelson, 2023; Hanson et al., 2024), and if so, what fee should the central bank charge? We analyze all these mechanisms in this paper and examine how they influence each other.

In Stein (2012), the influential and tractable model we will build our analysis on, a banking sector raises funds from households by issuing a mix of money-like short-term liabilities and long-term bonds, and invests the funds in long-term projects (the banking sector in his model is clubbed together with the project managers for simplicity). Money-like liabilities are cheaper funding than bond funding because holders are willing to pay a liquidity premium for them so long as they stay risk-free. However, at a future date, the projects may be hit by an adverse shock in some state of the world, in which case holders of short-term liabilities will come for their funds. Each (symmetric) bank will have to sell projects in the market to raise funds to repay short-term liability holders, possibly at fire-sale prices. Importantly, the amount of money-like liabilities each bank can issue initially can be constrained by the need to have enough saleable assets at that future date to pay off all the money-like liabilities – a kind of "collateral constraint" on initial money issuance.

The buyers of fire-sold assets in Stein's model are private (in his language, "patient") investors with a limited endowment funds, and the fire-sale discount arises because they will have to buy fire-sold projects instead of investing in their own opportunities, incurring an opportunity cost.

In the Stein model, banks may overinvest in projects in order to alleviate the collateral constraint on financing with cheap money-like liabilities. Each bank does not internalize the fact that while its own collateral constraint is getting loosened, its additional investment and associated fire sales increase the size of the fire-sale discount for all, tightening other banks' collateral constraint. Stein (2012) then examines supervisory and monetary actions that could limit banks' incentive to overinvest in projects and overissue money.

In this paper, we start with a different question. First, is there a private fix – can banks arrange insurance privately to relax the collateral constraint? Second, what can the central bank do to augment private sector efforts, and does it help or hurt? In particular, how does the public provision of (contingent) liquidity affect the private provision of (contingent) capital or liquidity?

We show that there is indeed a simple private solution to overinvestment when money creation is collateral constrained: allow banks to arrange for contingent capital, that is, committed fund inflows from private investors conditional on an adverse shock hitting, for which banks will pay in future normal states. Such an arrangement ensures the collateral constraint never binds because a bank that wants to issue more money-like deposits simply buys more contingent capital insurance. Outcomes are once again socially optimal. Of course, the banking system may issue more money-like liabilities than in the Stein model where such contingent capital is not available. Indeed, under some conditions, it may even fund itself entirely with money-like liabilities, investing in both bonds and projects. In other words, if the premium that households are willing to pay for money-like liabilities is high, banks may take on more liquidity risk than strictly required by project financing in order to fully exhaust the benefit of issuing money-like assets. At any rate, contingent capital inflows ensure that the collateral constraint does not bind and private outcomes are socially optimal.

Such privately arranged contingent funding is not merely a theoretical curiosity. In the past, banks had unlimited liability in a number of countries (Hickson and Turner, 2003; Kenny and Ogren, 2021), most notably during Scotland's free banking era (White, 1995). Another version of contingent liability was where bank shareholders either paid in capital less than the par value of shares and were liable for the remainder on call, or simply had additional liability. For example, most US and Canadian banks operated under double liability until the Great Depression (Macey and Miller, 1992; Turner, 2014; Bodenhorn, 2015; Aldunate et al., 2025), and many UK banks maintained a similar "reserve liability" arrangement until the 1950s, even after they had moved away from unlimited liability. All these are situations where the bank had a call on the capital/wealth of shareholders in distressed times, and the possibility of such capital backing would expand the bank's access to funding and its stability.

More recently, contingent convertible bonds (see Flannery, 2005; Kashyap, Rajan, and Stein, 2008; French et al., 2010; Flannery, 2014) have been proposed to reduce bank debt and enhance equity in distressed times, potentially allowing banks to raise funding. Vallee (2019) and Avdjiev et al. (2020) present empirical evidence that such contingent convertible capital securities can reduce bank fragility.

Of course, contracting frictions can make private contingent funding difficult, thus restoring Stein's overinvestment result. Yet modernity should have improved contractability. The waning of private contingent capital in recent times may have more to do with the scale and scope of central bank interventions than any associated contracting frictions. Indeed, we show that this is theoretically a possibility.

Let us elaborate. What happens if we add an interventionist central bank that can infuse liquidity more cheaply than the private sector (a possibility that became more realistic as countries left the gold standard and final settlement became possible in central bank issued fiat money)? The welfare implications turn on three issues. First, can the central bank commit to a pre-specified intervention policy, or does it react according to the needs of the moment? Second, does the central bank charge the needy bank for its intervention, and under what circumstances does it recover the appropriate charge? Third, is there a cost to the central bank of intervening, and what form does it take?

We focus here on two cases for illustrative purposes. The first is a "bailout" central bank, which seeks only to reduce fire sales ex post by infusing funds into distressed banks, without seeking to recover the funds later from the banks. This will certainly be the case if the central bank has no additional powers of obtaining repayment than the private sector. Anticipating public support on fire sales, banks have an incentive to increase up-front financing through money-like liabilities – in the model, this exactly offsets central bank intervention. Furthermore, if the extent of the bailout is plausibly proportional to the investment (or bank size), banks have the incentive to increase investment since they do not pay a price for the liquidity infusion. The net effect is that realized fire-sale costs do not change since banks take on more illiquidity risks up front to offset central bank intervention. And because banks do not pay the cost of central bank intervention, they overinvest.

Importantly, the anticipation of underpriced central bank intervention could crowd out private insurance arrangements such as contingent capital. In addition, a bailout central bank will induce commercial banks to prefer real investments (banks' asset-side) to contingent capital (banks' liability-side) in supporting deposit (money) issuance. The crowding-out of liability-side private insurance is perhaps reflected in the disappearance of unlimited or even additional shareholder liability, as well as the secular decline in banks' capital ratios in the past 140 years (see, for example, Alessandri and Haldane, 2009).

In the second case, the central bank pre-commits to lend conditional on stress (think of this as deposit insurance or arrangements for banks to pre-position assets with the central bank that they can borrow against in times of liquidity stress). By charging a premium for the liquidity support, the central bank can reduce the bank's incentive to overinvest and potentially restore constrained efficient outcomes. However, a central point in this paper is that shielded from market forces and subject to political pressures, the central bank will invariably charge too low a price for support to market participants (see, e.g., Goodhart, 1995). In particular, we argue the actuarially fair price (the price at which the central bank breaks even on the support) is too low a price to charge for pre-committed liquidity; the right "Bagehot" price is one that reflects the private costs of fire sales even though central bank intervention potentially alleviates fire sales.[1] Yet at this price the central bank will not help boost asset prices relative to what would prevail in the private market, and would end up making money in a systematic way – a red rag for bankers paying the price. Therefore, the actuarially fair price may approximate what is politically possible for the authorities to charge, incentivizing bank overinvestment. Overall, we conclude that increasing anticipated interventions by central banks in stress situations without their adequately charging for public liquidity infusions can help account for the continuing incidence of banking stresses and calls on central bank support.

Worse still is if there is central bank moral hazard – if the central banker's personal costs of intervention are lower than societal costs – for instance, the central banker may worry about the cost of bank failures to their career (Alesina and Tabellini, 1990; Boot and Thakor, 1993). We examine all such possibilities and their effects on welfare.

Thus far, the returns to taking on liquidity risk have been capped by a constant money-financing premium. We now allow the returns to taking liquidity risk to increase without bound. Specifically, in addition to real investments, we examine the effects of central bank intervention on speculative financial investments that are ultimately supported by banks (for example, by funding margin calls via prime brokers). Although not socially beneficial in the model, banks engage in such lending because the profits (carry or prime brokerage fees) earned are privately attractive and increase with leverage. Because the margin calls arrive in the crisis state, they add to calls on bank liquidity. We show a bailout central bank not only continues to distort bank choices towards real investments rather than private liquidity insurance, but also over-intervenes ex post, unlike in the baseline model.

Our paper makes three fundamental contributions. First, the literature on financial fragility (discussed below) typically assumes away private insurance markets, which is at odds with the historical evidence. Second, we suggest an explanation for why private contingent capital disappeared even as contracting likely improved: the crowding out due to the expanding scale and scope of central-bank intervention. Third, higher expectations of inevitably underpriced public insurance in the era of modern banking can amplify liquidity exposures in bank balance-sheet positions and attendant fragility. Hence our framework can account for the evidence that despite intervention, fragility episodes have increased in scale with virtually no decline in incidence.

[1] Bernanke (2008) argues that Bagehot (1873)'s dictum for the central bank to lend freely but at a high price was meant to ration the central bank's holdings of scarce specie, and not to prevent moral hazard. So the dictum would not apply where the central bank controls an elastic currency. Our focus on moral hazard and investment distortions restores the need for the Bagehot dictum, even in a world of fiat money.

1.1 Related Literature

Our analysis starts with Stein (2012), which draws on a large literature on fire-sale externalities.[2] Lorenzoni (2008) shows that two-sided limited commitment between entrepreneurs (borrowers) and consumers (lenders) can induce excessive ex-ante investment, borrowing, and systemic risk via pecuniary externalities in asset prices.[3] Work by Kehoe and Levine (1993) and Rampini and Viswanathan (2010), among others, also endogenizes collateral constraints arising from various forms of limited enforcement. Our paper abstracts from microfounding bank collateral constraints and focuses on the missing market in insurance contracts between banks and other private agents, explaining it as the unintended consequence of central bank interventions.

Closer to our analytical results, Davila and Korinek (2017) show in a model with price-dependent financial constraints that inefficient outcomes arise when insurance markets are incomplete, and therefore advocate that regulators provide contingent financing and stabilize the value of collateral assets. Our analysis clarifies that such contingent financing need not be provided only by a lender of last resort or a public regulator, but also possibly by private agents. In fact, we show private state-contingent insurance claims can get progressively eliminated – a form of endogenous incompleteness – once a lender of last resort comes in, and the associated moral hazard because of the inability to price public insurance appropriately could be welfare-reducing.[4]

From a theoretical perspective, our result that with state-contingent capital, the collateral externality may not lead to inefficiencies in a frictionless world has also been established in different settings in Krishnamurthy (2003), Di Tella (2017), and Asriyan (2020).[5] What is novel in our setting, apart from the specific application, is that with frictions in private insurance markets (as shown in Appendix D), there is over-provision of such state-contingent claims in the private

[2] For an incomplete yet extensive list, see Shleifer and Vishny (1992, 1997), Kiyotaki and Moore (1997), Gromb and Vayanos (2002), Morris and Shin (2004), Allen and Gale (2005), Fostel and Geanakoplos (2008), Brunnermeier and Pedersen (2009), Stein (2009), Geanakoplos (2010), Caballero and Simsek (2013), Moore (2013), Bocola and Lorenzoni (2023). Although addressing different questions, Altavilla, Rostagno, and Schumacher (2025) is a recent contribution that builds on Stein's model augmented with a central bank.

[3] In Lorenzoni (2008), there are different returns to borrower funds intertemporally and across states. Fully state contingent financial contracts would restore equality, but limited commitment inhibits such state contingency. Consequently, borrowers may be better off borrowing less, requiring fewer asset sales in adverse states, and thus effectively transferring resources to those states. Not internalizing the pecuniary externality that depresses asset prices, borrowers do not make such an adjustment.

[4] Jeanne and Korinek (2020) examine in detail the optimal ex-ante versus ex-post central bank policies in a fire-sale model of banking, while Davila and Walther (2022) examine optimal second-best corrective regulations in more general settings with uniform regulation and convex costs. Hachem and Kuncl (2025) explore how regulations could help increase banks' "haircuts" on withdrawals during banking crises. This parallels the private provision of contingent capital in our model. However, depositor haircuts might precipitate runs as depositors anticipate haircuts, so implementation might be difficult (see Diamond and Rajan, 2012).

[5] Krishnamurthy (2003) shows that with adequate state contingency in financial contracts (that is, with insurance added to borrowing), the dependence of borrowing on asset prices can be eliminated, thus eliminating the accelerator effects in Kiyotaki and Moore (1997).

equilibrium, whereas earlier models typically feature an underprovision of private insurance when insurers face collateral constraints (Krishnamurthy, 2003) or face information and trading frictions that lead to illiquidity (Asriyan, 2020). This is because in our model, banks are keen to pocket the "money premium" on deposit liabilities, which can also be manufactured by purchase of contingent capital, but banks do not internalize the externality that makes fire sale worse for everyone else, which leads to excessive money creation through *both* overinvestment in assets and over-purchase of contingent capital insurance.

Diamond and Rajan (2012) and Farhi and Tirole (2012) explore the consequences of uncommitted ex-post interest rate intervention by central banks and illustrate the associated moral hazard which induces risk-taking by banks. While there is a correspondence between liquidity intervention and interest rate intervention, our model further explores how private alternatives are affected by the pricing of public liquidity injections and the central bank's inability to commit to the ex-ante efficient pricing schedule. Our focus is on the optimal mix of asset- versus liability-side choices of banks to support the creation of money, rather than solely on their risk-taking or leverage choices.

Complementary to private contingent capital from outside the banking sector in our representative-bank model, Goodhart and Lastra (2025) focus on the central bank's role as a "leader of last resort," facilitating self-insurance within the banking sector when shocks are idiosyncratic. By contrast, when shocks are instead self-induced through excessive risk-taking, Acharya and Yorulmazer (2007, 2008) and Philippon and Wang (2022) show public support must reward prudent banks rather than expecting them to bail out riskier peers. Acharya, Santos, and Yorulmazer (2010) propose systemic-risk–based deposit insurance premiums to internalize such externalities. In the same spirit, Goodhart and Lastra (2023) advocate charging bank-specific rates and penalties that scale with the duration of central bank liquidity support as a practical tool to deter moral hazard.

2 Setup

We first describe Stein (2012)'s framework. There are three periods: $t = 0, 1, 2$. There are three types of agents: households, banks, and private investors. Traditional entrepreneurs are clubbed with banks to form a composite entity, which both initiates projects and raises finances for them.

2.1 Households

At time 0, the economy has an initial endowment Y. Private investors have initial endowments of W (assumed to be fixed).[6] Households start with disposable wealth $Y - W$ and also are the

[6]Under our baseline parameterization, endogenizing the household's choice of W, the amount of wealth allocated to the private investors, does not change any of the qualitative model results and has negligible effects on the numerical findings. For simplicity of exposition, we therefore treat W as fixed.

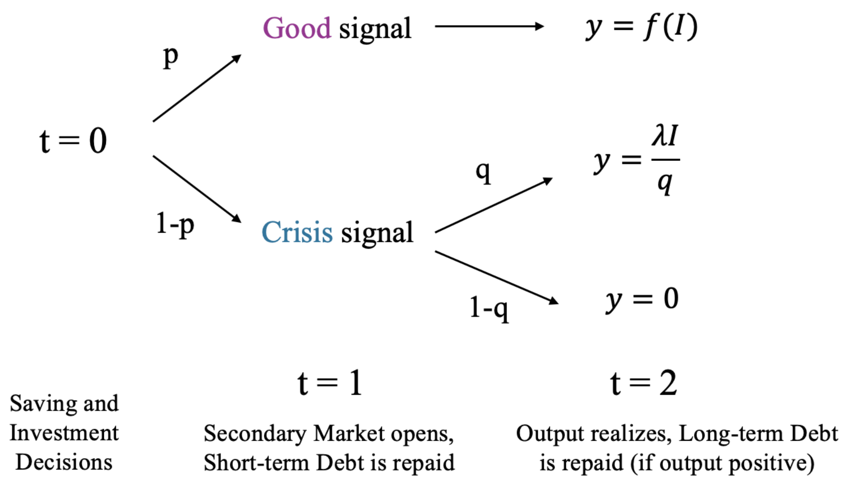

Figure 1: Timing of Events

ultimate owners of the incomes generated by private investors and banks. Households choose between immediate consumption C_0, investment in liquid bank-issued money-like deposits (M), henceforth also called money, or risky bank-issued bonds (B). Households derive additional utility from holding money because it is effectively made riskless. Money pays a gross rate of R^M upon withdrawal at either time 1 or 2. Risky bonds cannot be redeemed at time 1 and pay an expected gross rate R^B at time 2. At time 2, households consume C_2 out of the returns from money and bonds, as well as any income private investors or the banks generate.

The households' utility is given by

$$U = C_0 + \beta\mathbb{E}(C_2) + \gamma M. \tag{1}$$

Given linear preferences, the expected gross return on bonds is $R^B = \frac{1}{\beta} > 1$ and the gross return on money is $R^M = \frac{1}{\beta+\gamma} > 1$, where $\gamma > 0$ is the convenience yield on money ($R^B > R^M > 1$). The money-bond spread is thus assumed to be fixed. Households consume and invest in bank assets.

2.2 Banks

There is a continuum of homogeneous banks with mass 1. For simplicity, we refer to the representative bank or the banking sector as "the bank." The bank raises financing from households by issuing money or bonds, and can invest in real projects. Figure 1 illustrates the timing of events.

On the asset side, the representative bank invests I in real projects at time 0 and gets proceeds y at time 2. At $t = 1$, uncertainty is revealed. With probability p, the economy is in a good state, and the output at $t = 2$ is $f(I) > I$, where f is increasing, concave, and twice differentiable. With probability $1 - p$, the economy is in a crisis state, where the time 2 expected output is $\lambda I \leq I$ and there is a positive probability $1 - q$ that the output is zero.[7]

On the liability side, each bank raises mI in money-like deposits, which can be withdrawn by the households at $t = 1$ or $t = 2$ and pays a gross rate R^M upon withdrawal. Each bank's total money liability is $M = mIR^M$. Following Stein's setup, the convenience yield on money arises only when money is perfectly safe. Hence, observing the crisis state at time 1, the potential for insolvency at time 2 induces depositors to run in order to preserve the safety of their money. To meet depositors' withdrawals amounting to mIR^M, the representative bank sells a fraction $\Delta \in [0, 1]$ of its real assets to raise $\Delta k\lambda I$, where $k \in (0, 1)$ is an endogenous "fire-sale discounted" price, which the bank takes as given. As a result, in the crisis at $t = 1$, depositors are always repaid.

The bank finances real investment I partly with money liability mI. If $m \leq 1$, the bank finances the rest of the investment by issuing $(1 - m)I$ in illiquid bonds that pay a gross rate of R^B at $t = 2$. If $m > 1$, the bank issues more money-like deposits than it needs to invest in projects. For now, we assume it puts the additional $(m - 1)I$ raised in an illiquid storage technology (e.g., bonds issued by other corporate entities) that also pays R^B at $t = 2$. In this case, the bank creates liquid money from illiquid financial investments. Later, we will allow it to make other financial investments.

2.3 Private Investors and Contingent Capital

The bank cannot make credible repayment promises at time 2 after observing the crisis signal – hence it cannot raise financing. This gives it two options to raise funds to repay depositors: sell assets or receive pre-contracted or contingent capital inflows (pre-committed lines of credit will not work because they require repayment). There is a continuum of homogeneous private investors ("the PI") with mass 1 and wealth W. The PI absorbs bank fire sales during crisis periods, make additional real investments at time 1, and new to the Stein framework, provides contingent capital commitments (equivalently, private insurance) in the date-1 crisis state.

2.3.1 Contingent Capital

Specifically, at time 0, the bank can privately contract for the PI to pay an amount $E = \psi I$ as private contingent capital (equivalently, insurance payouts) to the bank if at $t = 1$, the economy is in the crisis state. Naturally, $E \leq W$. In exchange, whenever the economy reaches the good state, the bank pays the PI insurance premium $r^E E$ at date 2. One example is callable or partially paid-up bank equity capital, with the bank able to call up full payment in case of need.

To make money riskless, in the crisis state the bank's promised payment to depositors $M = mIR^M$ must be covered by either fire sale of assets or by private insurance $E = \psi I$ from PIs:

[7]In our setup, the crisis probability $1 - p$ is held fixed throughout for simplicity, including in the presence of central bank interventions introduced in Section 4. Section 7.2 presents empirical evidence broadly consistent with this assumption, as crises remained frequent despite the global expansion of public interventions.

$$\Delta k\lambda I + \psi I = mIR^M \implies \Delta k\lambda I = M - E.$$

With private insurance, only $(M - E)$ of bank assets have to be fire sold. If $\Delta = 1$ (all assets are fire sold), the bank reaches the upper bound on private money creation, which is

$$M^{max} = k\lambda I + E \implies m^{max} = \frac{k\lambda + \psi}{R^M}.$$

Contingent capital thus enhances the maximum amount of money-like deposits that can be issued at date 0 by E $(= \psi I)$.

2.3.2 Late-arriving projects

At $t = 1$, the PI can also invest K in a project that arrives at $t = 1$ to receive output $g(K)$ at $t = 2$, where $g(\cdot)$ is increasing, concave, twice differentiable, and satisfies Inada conditions at 0. In the good state, the PI receives $r^E E$ in insurance premium and can invest its entire W in late-arriving projects. In the crisis state, the PI pays E to the bank, spends $(M - E)$ in fire sales, and invests the rest of its endowment, $W - E - (M - E) = (W - M)$, in late-arriving projects.

3 Baseline Model

We now show that when there are no frictions in contracting private contingent capital, we get the socially optimal outcomes in Stein (2012). We will explore frictions in Section 8.

3.1 Bank's Problem

The representative bank's problem is:

$$\begin{aligned} \max_{m,\psi,I}\ & pf(I) + (1-p)\lambda I - R^B I + \underbrace{mI(R^B - R^M)}_{\text{money spread}} \\ & - p \underbrace{r^E \psi I}_{\text{Cont Cap premium}} + (1-p) \underbrace{\psi I}_{\text{Cont Cap payout}} - (1-p) \underbrace{z[mIR^M - \psi I]}_{\text{loss from fire sales}} \end{aligned} \tag{2}$$

s.t.

$$m \leq m^{max} = \frac{k\lambda + \psi}{R^M}$$

where $z = \dfrac{1-k}{k}$ is the lost return on each dollar of fire-sale proceeds. The Lagrangian is

$$\mathcal{L} = \text{objective} - \eta \left(m - \frac{k\lambda + \psi}{R^M} \right)$$

with Lagrange multiplier $\eta \geq 0$.

The bank's first order conditions (FOCs) then are as follows:

with respect to (w.r.t.) bank's fraction of real investment financed by deposits, m:

$$I[(R^B - R^M) - (1-p)zR^M] = \eta, \tag{3}$$

w.r.t. bank's fraction of real investment covered by private insurance, ψ:

$$pr^E = \frac{\eta}{IR^M} + (1-p)(1+z), \tag{4}$$

and w.r.t. bank's real investment, I (detailed derivations in Appendix A.1):

$$\begin{aligned} pf'(I) + (1-p)\lambda - R^B &= -\left[m(R^B - R^M) - (1-p)z\left(mR^M - \psi\right)\right] + pr^E\psi - (1-p)\psi \\ &= -\frac{\eta}{I}\left[m - \frac{\psi}{R^M}\right]. \end{aligned} \tag{5}$$

The first FOC states that the shadow cost of relaxing the money creation constraint, η, must equal the net marginal benefit of money creation, which is the money premium net of the expected fire-sale loss. The second FOC equalizes the bank's marginal cost of arranging private contingent capital (the expected contingent capital insurance premium charged) with the marginal benefit of private contingent capital, including the expected avoided fire sale costs and the shadow benefit of money creation. The third FOC states that when the money creation constraint is not binding, the marginal benefit from investments is equal to the marginal financing cost R^B. When the constraint is binding, by increasing physical assets I, the bank can issue more uninsured money, which equals to $M - E$ in levels or $m - \frac{\psi}{R^M}$ per dollar of time-0 investment. This relaxes the money creation constraint and generates an additional benefit of investment.

3.2 Private Investor's Problem

The representative private investor's problem is:

$$\max_{M,E} p\left[g(W) + r^E E\right] + (1-p)\left[g(W-M) + \frac{1}{k}(M-E)\right].$$

PI's FOC w.r.t. M, the PI's funds used for fire-sale purchases in the crisis state, is:

$$g'(W-M) = \frac{1}{k} \tag{6}$$

which equalizes the marginal benefit of investing in the g technology with the marginal benefit of obtaining the return on buying fire-sold assets, $\frac{1}{k}$, in the crisis state at $t = 1$.

PI's FOC w.r.t. private liquidity commitment E:

$$pr^E = (1-p)\frac{1}{k} \tag{7}$$

which equalizes the time-0 marginal expected premium on providing private contingent capital with the corresponding expected marginal cost (forgone return of $\frac{1}{k}$ in the bad state). This is the key new additional condition to Stein's framework.

3.3 Private Equilibrium

Next, we define and characterize the private equilibrium.

Definition 1. A private equilibrium is a set of prices and allocations such that taking the prices k, r^E as given, both bank's and PI's FOCs are satisfied, and markets clear. In this case, market clearing implies that $E = \psi I$ and $M = mIR^M$, so that the bank's choices of m, ψ, I are consistent with the PI's choices of M and E.

First note that from agents' FOCs w.r.t. private insurance, (4) and (7),

$$(1-p)\frac{1}{k} = pr^E = \frac{\eta}{IR^M} + (1-p)(1+z).$$

By definition, $1 + z = \frac{1}{k}$, which implies we must have $\eta = 0$: the constraint on money creation is not binding in an interior solution. In the baseline model, with $\eta = 0$, the fire-sale discount k is pinned down by the bank's FOC w.r.t. m (3):

$$\frac{R^B - R^M}{R^M} = (1-p)z = (1-p)(\frac{1-k}{k}), \tag{8}$$

and investment is pinned down by the bank's FOC w.r.t. I (5):

$$pf'(I) + (1-p)\lambda - R^B = 0. \tag{9}$$

The total amount of money liability created is pinned down by the PI's FOC w.r.t. M (6):

$$g'(W - M) = \frac{1}{k},$$

and the total amount of private insurance E is indeterminate as long as it satisfies

$$M - k\lambda I \leq E \leq M, \tag{10}$$

where the first inequality requires that the constraint on money creation is not binding and the second inequality is a natural limit on the amount of private insurance.

Note that in the private equilibrium, there is no solution where the constraint on money creation is binding. If $\eta > 0$, then either the bank or the PI would not be on the respective FOC and E

would be chosen to be as large as possible, which implies $E = M$. But in that case, the constraint on money creation would not be binding, which yields a contradiction. Therefore, we always have $\eta = 0$ in the baseline model. We next show that the private solution is socially optimal.

3.4 Social Planner's Problem

Consider next a benevolent social planner who seeks to maximize the households' utility (1) by choosing the private outcomes of M, E, and I.[8] We can express the planner's problem as follows (detailed steps in Appendix A.2):

$$\max_{m,\psi,I} \; pf(I) + (1-p)\lambda I - R^B I + mI(R^B - R^M) + pg(W) + (1-p)[g(W-M) + M]$$

such that the money creation constraint is met:

$$m \leq m^{max} = \frac{k\lambda + \psi}{R^M},$$

and the planner understands that the private investor's date-1 allocations require that the marginal returns available on late-arriving investment equal returns on assets bought in the fire sale so that

$$g'(W-M) = \frac{1}{k} \implies k = \frac{1}{g'(W - mIR^M)}.$$

The Lagrangian for the planner's problem is:

$$\mathcal{L}^{\mathcal{P}} = \text{planner's objective} - \eta^P \left(m - \frac{k\lambda + \psi}{R^M} \right)$$

with Lagrange multiplier $\eta^P > 0$. Thus, the planner's FOC w.r.t. the fraction of investment financed by deposits, m, is

$$I[(R^B - R^M) - (1-p)zR^M] = \eta^P \left[1 - \frac{g''(W-M)}{(g'(W-M))^2} \lambda I \right] = \eta^P \left(1 + \Omega(M, I)\right) \qquad (11)$$

where the concavity of g implies that $g'' < 0$, and $\Omega(M, I) = -\frac{g''(W-M)}{(g'(W-M))^2}\lambda I > 0$ is a measure of the fire-sale externality associated with increasing the issuance of money, the extent of which depends on the curvature of g. This externality is internalized by the planner in her choice of m if the money-issuance constraint is binding.

The planner's FOC w.r.t. ψ, the fraction of I covered by insurance, is

$$0 = \frac{\eta^P}{IR^M}. \qquad (12)$$

Because there is no social cost of private insurance, the planner can choose as much ψ as

[8] Our notion of efficiency is based on a constrained social planner framework that respects private collateral constraints, following the literature tracing back to Stiglitz (1982) and Geanakoplos and Polemarchakis (1986).

necessary to ensure the money constraint is not binding (i.e., $\eta^P = 0$).

Finally, the planner's FOC w.r.t. investment I (detailed derivations in Appendix A.3) is

$$pf'(I) + (1-p)\lambda - R^B + m(R^B - R^M) + (1-p)\left[-g'(W-M) + 1\right] mR^M$$
$$= -\frac{\eta^P \lambda}{R^M}\left(\frac{g''(W-M)}{(g'(W-M))^2} mR^M\right)$$

which can be rewritten as

$$pf'(I) + (1-p)\lambda - R^B = -m\frac{\eta^P}{I}. \tag{13}$$

After plugging $\eta^P = 0$ into the planner's FOCs and comparing with (8) and (9), it becomes clear that in the baseline model, the planner's choice coincides with the private outcomes.

Proposition 1. The private equilibrium outcome in the baseline model with private contingent capital is efficient.

Let the variables with a $*$ superscript denote the private equilibrium outcome and variables with a P superscript denote the outcomes from solving the planner's problem. Then,

1. The constraint on money creation never binds: $\eta^* = \eta^P = 0$.

2. The private choice of investment and money creation is socially optimal: $I^* = I^P$ and $M^* = M^P$.

3. The amount of private insurance E^* is indeterminate, as long as

$$\max(0, M - k\lambda I) := E^{min} \le E^* \le M,$$

and the same condition applies to E^P.

Consider two features about this efficient private equilibrium. First, the constraint on money creation never binds. In a frictionless market, the contingent capital insurance premium r^E equalizes the PI's marginal cost of forgone fire-sale return $\frac{1}{k}$. From the bank's perspective, this equals the bank's marginal benefit of getting insurance, which includes both savings from avoiding fire sales and the shadow value of relaxing the money constraint. Since the marginal savings from avoiding fire sales equals the PI's marginal cost of forgone fire-sale return, the shadow value of relaxing the money constraint has to be zero. Put differently, in equilibrium the insurance premium adjusts so that enough insurance is bought and the constraint does not bind. Second, in equilibrium, for the marginal dollar in money liability M due at time 1, the bank is completely indifferent between paying depositors through fire sales at a cost of $1 + z$ in the crisis state, or purchasing private insurance at a cost of r^E in the good state. This makes $E \in [E^{min}, M]$ indeterminate.

There are no limits to money financing when accompanied by commensurate contingent capital, so there is no need to increase investment beyond the socially optimal in order to increase money financing. In Stein's model, by contrast, if the money premium $R^B - R^M$ is high, the bank may still want to finance with money but it may not have the ability to sell enough assets to make money riskless in the crisis state ($k\lambda I$ is too low). The binding money issuance constraint then gives the bank additional incentives to invest. Pecuniary externalities then matter because the bank does not take into account its higher investment in depressing fire-sale values for other banks. Hence in his model, banks overinvest if the money premium is high.

From an economic standpoint, Proposition 1 clarifies that the market failure that leads to inefficiency in Stein (2012) is not the fire-sale externality per se, but the missing market for contingent capital provision. This market may be endogenously missing anticipating policy interventions (as we show in Section 4) or due to frictions in committing contingent capital (Section 8).[9]

3.5 Numerical Illustration

Throughout the paper, unless otherwise stated, we illustrate the results with a numerical example with the following parameters and functional specifications: $p = 0.95$, $\lambda = 1$, $W = 140$, $R^B = 1.08$, R^M varies between 1 and 1.075, $f(I) = a\log(I)+I$ where $a = 3.5$, $g(K) = \theta\log(K)$ where $\theta = 140$.

In Figure 2, we plot the equilibrium levels of money liability M, investment I, and the minimum level of private insurance E^{min} for various levels of R^M (that is, various levels of the money-bond spread). In the "low spread" region where R^M is high, the planner's allocations are attainable without any private insurance. However, in the "high spread" region where R^M is low and money issuance is attractive, private contingent capital is procured such that investment and money creation (which increases as the money-bond spread widens) are always at socially optimal level. By contrast, in Stein's model, the money creation constraint binds in the high-spread region and pecuniary externalities operating through the pledgeability constraint would lead to inefficiently high levels of I and M in equilibrium.

3.6 Contingent Capital Over History

Contingent capital in the form of unlimited or contingent liability was ubiquitous in the 18th, 19th, and early 20th century. Even though some forms of contingent liability did not amount to providing insurance at the time of runs but passed on the losses from bank liquidation or closure to shareholders ex post, they expanded the bank's ability to borrow in crisis times because of the

[9]Note that the indeterminacy of E arises only in the frictionless model. Introducing any friction in contracting private contingent capital would lead to a unique level of E, as shown in Section 8. Any additional costs in the insurance market caused by such frictions would lead private agents to choose the minimum feasible level of E and banks would liquidate all available I in the fire-sale market. This motivates our focus in the graphical analysis on $E^{min} = M - k\lambda I$. Conceptually, the frictionless benchmark can be viewed as a limiting case as the costs associated with contracting frictions approach 0.

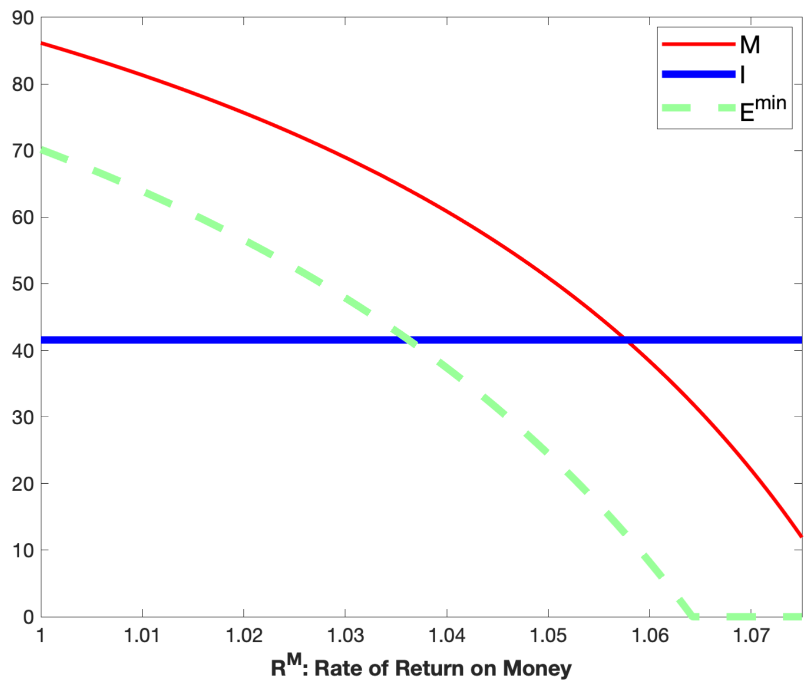

Figure 2: Equilibrium Outcomes in the Baseline Model, for different levels of return on money R^M

perceived additional implicit equity. Consequently they correspond to our model of pre-arranged contingent liquidity or capital.

3.6.1 The Scottish Free Banking Era and Unlimited Liability

The Scottish free banking system from 1716-1845, where most note/deposit-issuing banks had unlimited liability, enjoyed *"remarkable monetary stability"* with no central bank and few legal restrictions (White, 1995), leading to *"considerable agreement that lightly regulated banking was a success in Scotland"* (Briones and Rockoff, 2005). While some attribute part of this success in the 18th century to the presence of large privileged banks acting as quasi-central banks (Cowen and Kroszner, 1989), by 1810 the three chartered banks with limited liability were no larger than other non-chartered banks with unlimited liability and played no supervisory roles. The ensuing period was characterized by a competitive and well-capitalized banking system in which *"none [of the banks] were disproportionately large"* (White 1995, 2014).[10]

By giving the right of note issue only to the Bank of England, the Peel Acts of 1844 and 1845

[10] Cowen and Kroszner (1989) argue that the Bank of England may have acted as an implicit lender of last resort. White (1995) responds that *"in a few cases, the BoE provided loans to Scottish banks, but in other cases (most importantly, during the crises of 1825-6 and 1836-7) it refused to lend."* In fact the BoE explicitly rejected the idea that it had LOLR obligations.

effectively ended the Scottish free banking era. However, many unchartered Scottish banks chose to retain unlimited liability in the 1860s-70s without difficulty in raising capital on a large scale, even after the Companies Act of 1862 allowed them to choose a limited liability structure (White, 1995). While Bagehot was concerned that unlimited liability banks would eventually be owned by impecunious individuals with *"few acres and few shillings,"* making their liabilities effectively limited, Hickson and Turner (2003) show from the archives of an Irish unlimited liability bank that the governing body successfully ensured that ownership remained with the largest and wealthiest shareholders.[11] White (1995) likewise notes that *"all failed [Scottish] banks having more than nine partners were able to pay their liabilities to the public in full."* In other words, the contingent capital contracted with equityholders was large enough to cover the losses in the crisis state.

Unfortunately, the failure in 1878 of the City of Glasgow Bank, the largest unlimited-liability bank ever to collapse at the time with losses exceeding six times its capital, rendered a large number of shareholders insolvent and provoked widespread public backlash, as many uninformed outside shareholders suffered substantial losses from what proved to be a fraudulent and mismanaged institution (Turner, 2014; Goodhart and Lastra, 2020). Amid public concern amplified by exaggerated press narratives, this episode led directly to the 1879 Companies Act, which marked the shift of UK banks from unlimited liability to a form of pre-specified contingent liability (Acheson and Turner, 2008; Turner, 2014; Goodhart and Postel-Vinay, 2024).

3.6.2 Contingent Liability in UK and US Banks

Following the 1879 Companies Act, most UK banks voluntarily continued to operate under contingent liability regimes known variously as "reserve liability" and "uncalled capital" from the 1880s well into the mid-1950s. Turner (2014) explains that *"reserve liability was exactly like uncalled capital except that it was not callable at the discretion of directors. In other words, it could be called on only in the event of a bank collapsing with inadequate funds to pay its depositors."*[12] Grossman and Imai (2013) use data on British banks from 1878 to 1912 and find that banks with more contingent liability took less risk.

A similar contingent liability structure for banks, known as double liability, under which shareholders were liable for an additional amount equal to the bank's paid-in capital, also prevailed

[11] Unlimited liability banks were also established in Ireland from mid-1820s and in Sweden, from 1830 to 1903 (Lakomaa, 2007), Kenny and Ogren (2021) find that such banks took on greater leverage (given the greater implicit equity) and generated higher dividends, return on equity, and maintained strong governance control relative to limited liability banks. Also see Saunders and Wilson (1995) for contingent capital's potential role in financial development.

[12] Button et al. (2015) report that in 1885, the total additional capital that could be called from shareholders of UK banks averaged more than three times their paid-up capital. Turner (2014) also discusses evidence that *"directors of English banks [with contingent liability] were careful in admitting individuals to ownership and they frequently rejected transfers to those deemed unsuitable,"* which likely have contributed to the overall stability of the UK banking system during this period of reserve liability.

in the United States roughly between the Civil War and the Great Depression. Macey and Miller (1992) argue that double liability was *"remarkably effective"* through insolvency assessments, with substantial recovery rates for depositors and other creditors. During that period, American banks subject to stricter liability rules (for equityholders and managers) had less equity and asset volatility, held less risky assets, and were less likely to become distressed during the Great Depression (Esty, 1998; Koudijs, Salisbury, and Sran, 2021; Aldunate et al., 2025). Banks with double liability operated with lower (book) capital ratios and therefore higher leverage, which suggests that bank creditors viewed the contingent liability as a credible guarantee or off-balance-sheet capital (Macey and Miller, 1992; Bodenhorn, 2015). These findings are consistent with our baseline model that suggests the purchase of contingent capital / private insurance can also increase the amount of money-like securities created by a bank.

Despite its apparent success, the wave of bank failures between 1929 and 1933 led to a widespread public perception that double liability had failed as a regulatory system, as damages were often imposed on innocent shareholders with no inside connections to the banks and who had purchased the shares during the economic boom of 1923-1929. Private contractual alternatives could have addressed this problem.[13] However, with the Federal Deposit Insurance Corporation (FDIC) established in 1933, government deposit insurance came to dominate and double liability had been effectively extinguished from the US banking system by 1953. More generally, Turner (2014) notes, *"it is interesting that the United States, Canada, and Great Britain terminated extended liability for banks at roughly the same time,"* between the late 1930s and the 1950s. Below, we suggest an explanation.

4 Central Bank Interventions

In the optimal private solution, at date 1 when the crisis state is realized the private investor diverts funds from investment in later arriving projects toward purchasing assets in the fire-sale market. Seeing this, a socially-minded central bank may be tempted to intervene to infuse liquidity to enhance late investment (the fire sale is simply a transfer). Therefore, we add a central bank that can inject funds $L = \phi I$ (so that ϕ becomes the proportion of total I that is covered by central bank) into the bank to meet depositor withdrawals. Ex post, this reduces fire sales, thus allowing the PI to invest more. Specifically, the PI puts up E in contracted contingent capital but only spends $M - E - L$ on fire sales, and therefore now has $W - M + L$ to invest in the g technology.

[13] For instance, Goodhart and Lastra (2020) propose a two-tiered equity structure, with outsiders subject to limited liability and insiders to a form of contingent / multiple liability. Today, banks issue contingent convertible capital bonds to diversified institutional investors (Flannery, 2005; Kashyap, Rajan, and Stein, 2008), where the bond conversion in crisis times frees up additional borrowing capacity or adds to bank capital. Pre-positioning of private capital prevents the need to search for it in times of trouble.

We assume that central bank funding is not entirely socially costless: it has a deadweight cost $C(L)$ of providing such bailout funding, which can be thought of as a real fiscal or inflation cost – made lower in the transition from the Gold Standard to fiat money. We assume $C(\cdot)$ is (weakly) increasing, convex, and twice differentiable. This cost is borne by households (so it also enters the planner's and the central bank's objectives) but is not recognized by the representative bank and PI.

Regardless of whether the central bank chooses ϕ upfront or conditional on the crisis state, each bank will take ϕ as given, recognizing that L is affected by its investment, I. This form of central bank intervention incentivizes the bank to create money via overinvestment, because it now recognizes that public insurance scales up as it increases I. The key then is how the central bank can offset this incentive by structuring intervention and the associated charge for support appropriately.

Before we go there, we first consider a benevolent planner that can dictate M, E, I and L and recognizes the social cost of intervention $C(L)$. Then, we consider different ways the central bank can intervene. We compare the welfare obtained in these different situations at the end.

4.1 Planner's Problem with Public Liquidity Provision

The planner's problem with contingent public liquidity provision now becomes

$$\max_{m,\psi,\phi,I} \quad pf(I) + (1-p)\lambda I - R^B I + mI(R^B - R^M) + pg(W)$$
$$+ (1-p)\left[g(W - M + L) + M\right] - (1-p)C(L)$$

such that

$$m \leq m^{max} = \frac{k\lambda + \psi + \phi}{R^M}$$

with Lagrange multiplier on the constraint $\eta^P \geq 0$ and the planner knows

$$g'(W - M + L) = \frac{1}{k} \implies k = \frac{1}{g'(W - mIR^M + \phi I)}. \tag{14}$$

From the planner's FOC w.r.t. ψ, we still have that

$$0 = \frac{\eta^P}{IR^M}. \tag{15}$$

Substituting into the FOC w.r.t. m gives

$$I\left[(R^B - R^M) - (1-p)zR^M\right] = \eta^P\left[1 + \Omega(M, I)\right] = 0,$$

which implies the socially optimal fire-sale discount k^P must satisfy

$$\frac{R^B - R^M}{R^M} = (1-p)z = (1-p)(\frac{1}{k} - 1). \tag{16}$$

With $g'(W - M + L) = \frac{1}{k^P}$ and with $\eta^P = 0$, the planner's FOC w.r.t. ϕ can be written as

$$(1 - p)g'(W - M + L) = (1 - p)C'(L). \tag{17}$$

Equations (14) and (17) pin down the socially optimal level of central bank funding L^P and money creation M^P. As before, the socially optimal I^P satisfies

$$pf'(I) + (1 - p)\lambda - R^B = \frac{\eta^P}{I}\left(\frac{\phi}{R^M} - m\right) = 0.$$

While k^P and I^P remain the same as in the planner's problem in Section 3.4, M^P increases by central bank infusion L^P. Cheap central bank liquidity allows for more socially optimal money creation – one way in which the scales are tilted toward central bank intervention in the model.

We now explore a number of different ways the central bank can infuse L, assuming that M, E, I are chosen fully anticipating central bank actions. First, we examine what happens if it intervenes in the crisis state, after seeing fire sales (ex-post lending). Second, we explore the possibility that the bank pre-contracts at date 0 for the central bank's infusion. This is ex-ante commitment, along the lines of several current proposals (King, 2016; Hanson et al., 2024). In either case, the central bank can achieve the social optimal, but only if it charges the bank an appropriate rate. We will see later why that is difficult.

4.2 Central Bank as Ex-post Lender of Last Resort (LOLR)

Suppose a central bank can lend $L = \phi I$ to the bank at date 1, provided it can recover a fixed multiple τ of the LOLR funding from the bank at $t = 2$. It will return the repaid amounts to households, where τ is the gross interest rate the central bank is statutorily required to charge, so $\tau - 1$ is the net return to households from the forced loan. Note that the private sector cannot finance the bank directly since it cannot trust the bank will repay – the only way for the bank to raise assistance from the private sector is by selling assets or by obtaining pre-contracted contingent capital. By allowing for $\tau = 0$, we nest the case where the central bank has no additional powers of recovery than the private sector, and central bank intervention is a pure bailout. When $\tau > 0$, the central bank has greater powers of recovery (for example, through bank taxation) than the private sector as in Holmstrom and Tirole (1998), but it is limited by the bank's net worth in the crisis state.

If the central bank can only intervene ex post in the crisis state at $t = 1$, its objective is

$$\max_{\phi} \; g(W - M + \phi I) - C(\phi I)$$

with FOC w.r.t. ϕ:

$$g'(W - M + L) = C'(L) \tag{18}$$

where the LHS is the expected benefit of funding provision (increasing the investment in the g technology), and the RHS is the cost of providing such funding. This is the same FOC as the planner's FOC w.r.t. ϕ.[14] At time 0, the bank must ensure it meets the date-1 solvency constraint so that it has enough residual assets left to pay the central bank. This implies:

$$\lambda I - (1+z)(M - E - L) \geq \tau L \implies m \leq \frac{k\lambda + \psi + (1-\tau k)\phi}{R^M}, \tag{19}$$

which, for any $\tau > 0$, is tighter than the constraint faced by the bank in the planner's problem: $m \leq \frac{k\lambda+\psi+\phi}{R^M}$. Intuitively, the fact that the bank is required to pay the central bank for liquidity in the risky state rather than in the good state (which would be the case with pre-committed liquidity as we will see) constrains the space for money creation further.

4.2.1 Bank's Problem

Taking ϕ as given, the representative bank's problem is:

$$\max_{m,\psi,I} pf(I) + (1-p)\lambda I - R^B I + \underbrace{mI(R^B - R^M)}_{\text{money-bond spread}} - p \underbrace{r^E \psi I}_{\text{cont cap premium}} + (1-p) \underbrace{\psi I}_{\text{cont cap payout}}$$
$$+ (1-p) \underbrace{\phi I}_{\text{payout from LOLR}} - (1-p) \underbrace{z[mIR^M - \psi I - \phi I]}_{\text{loss from fire sales}} - (1-p) \underbrace{\tau \phi I}_{\text{payment to LOLR}}$$

such that

$$m \leq m^{max} = \frac{k\lambda + \psi + (1-\tau k)\phi}{R^M}$$

with the Lagrange multiplier on the solvency constraint $\eta \geq 0$.

Note that the bank's objective also changes, as in expectation it now has to pay $\tau\phi I$ back at $t = 2$ on the LOLR funding if there is a crisis. The bank's FOCs w.r.t. money creation m and private insurance ψ remain the same as (3) and (4), but in equilibrium with $\eta = 0$, the bank's FOC w.r.t. I now becomes

$$pf'(I) + (1-p)\lambda - R^B = -(1-p)(1+z-\tau)\phi \tag{20}$$

with the additional $-(1-p)(1+z-\tau)\phi$ term reflecting the bank's incentive to overinvest due to the moral hazard created by underpriced central bank liquidity support for any $\tau < 1 + z$.

Meanwhile, the PI takes L as given (correctly perceiving the central bank's choice of ϕ and the bank's choice of I). The equilibrium fire-sale price k is reached only after aggregate money liabilities M increases by the public liquidity L (relative to the equilibrium without a central bank).

[14] The central bank acting ex post at $t = 1$ after the crisis state is realized is indifferent between any level of tax rate τ to charge at $t = 2$, though that tax rate affects the incentives of the bank. The representative bank optimally chooses to accept the LOLR funding in full as long as $\tau \leq 1 + z$.

4.2.2 PI's Problem

Taking L as given, the representative private investor's problem is:

$$\max_{M,E}\; p\left[g(W)+r^{E}E\right]+(1-p)\left[g(W-M+L)+\frac{1}{k}(M-E-L)\right].$$

PI's FOC w.r.t. M, funds for fire-sale purchases and insurance provision in the crisis state, is:

$$g'(W-M+L)=\frac{1}{k} \tag{21}$$

which equalizes the marginal benefit of investing in the g technology with the forgone fire-sale return $\frac{1}{k}$ in the crisis state at $t=1$.

PI's FOC w.r.t. private liquidity commitment E remains the same:

$$pr^{E}=(1-p)\frac{1}{k}. \tag{22}$$

4.2.3 Private Equilibrium with Central Bank Intervention as LOLR

With the usual definitions of equilibrium, it is straightforward to show

Proposition 2. Overinvestment and Underprovision of Private Insurance under an LOLR

Let the superscript LOLR denote equilibrium outcomes with a LOLR central bank charging a rate $\tau \geq 0$ at $t=2$ conditional on the crisis state. Let the superscript P denote the planner's choices, where the planner is also subject to the bank solvency constraint in the crisis state.

1. The level of money creation and LOLR funding is socially optimal. That is, $M^{LOLR}=M^{P}$ and $L^{LOLR}=L^{P}$.

2. However, as long as $\phi^{LOLR}>0$ (and therefore $L^{LOLR}>0$) and $\tau<1+z$), the private equilibrium has an inefficiently high level of investment. That is, $I^{LOLR}>I^{P}$.

3. As a result, the minimum level of private insurance, $E^{min,LOLR}=\max(M^{LOLR}-k^{LOLR}\lambda I^{LOLR}-(1-\tau k)L^{LOLR},0)$, is lower than the socially optimal level if the planner has to respect the same bank solvency constraint (and the planner chooses a non-zero level of E^{min}).

4. There is less overinvestment and greater private contingent capital if the central bank charges more for public liquidity. That is, I^{LOLR} is decreasing in τ and $E^{min,LOLR}$ is increasing in τ.

Proof. All proofs omitted from the main text are provided in Appendix A. □

The prospect of liquidity provision by the central bank liquidity can cause banks to overinvest if the intervention is not priced right, as banks recognize that they can scale up, supported by underpriced liquidity from the LOLR. This also leads to underprovision of private insurance, which is effectively crowded out by LOLR funds.

Furthermore, suppose the social cost of intervention is $C(L) = \frac{1}{2}cL^2$ so that $C'(L) = cL$. We can then show that the private insurance market will be endogenously missing if c, and hence the perceived cost of intervention, is sufficiently low.

Proposition 3. Endogenously Missing Market for Private Insurance

Assume that $C(L) = \frac{1}{2}cL^2$ and in the crisis state, the LOLR central bank's charge for public liquidity τ below fire-sale returns, i.e., $0 \leq \tau < 1 + z = \frac{1}{k}$. If τ is low enough, then there exists a cutoff $\bar{c} \geq 0$ such that for all $c \in (0, \bar{c}]$, $E^{min,LOLR} = 0$.[15]

4.2.4 Discussion of Moral Hazard

Overinvestment due to moral hazard arises because each individual bank perceives that the LOLR central bank covers a fraction ϕ of all its investments. If we instead assume that the representative bank simply takes overall intervention L as given and this LOLR funding does not vary with its individual choice of I, then there is no moral hazard and overinvestment. Which then is more appropriate? The larger the money-funded investment, the higher the required private investor intervention in the fire-sale market if the central bank does not intervene, and the higher the cost of foregone late projects. So it is quite plausible that the ex-post LOLR, motivated by the desire to support real activity, will intervene more if the need arises. Moreover, LOLR interventions in practice are also typically mostly secured by high-quality assets and triggered after observing fire-sales, so it is natural that they scale with the amount of assets likely to be fire-sold.

Note also that the size of the eventual fire-sale discount is determined by the fixed money premium. So central bank intervention does nothing to reduce it. But once its intervention is anticipated and the bank issues more money claims, the fire-sale discount would be much deeper if the central bank did not intervene. This is classic moral hazard.

The central bank can itself be subject to moral hazard, which we will explore in examples: the central bank's own perceived cost of intervention may be $\beta^{CB}C(L)$ with $\beta^{CB} \leq 1$ – it perceives a deadweight cost lower than the true cost because it is either covering up past errors in supervision, or it is fearful of the political reaction if activity falls considerably. This will increase anticipated central bank intervention even further, crowding out private intervention, but also introduce additional costs as the central bank's intervention is excessive even by ex-post considerations.

Finally, the fee charged by the central bank for providing LOLR funds is critical in determining the extent of bank moral hazard. Of course, if the central bank has no additional powers of recovery than the private sector, given it has not contracted support ex ante, it must charge an ex-post rate of

[15] For $\tau = 0$, we can show a clean proof of the statement. For our specifications on f and g for the numerical results, we can also show an explicit upper bound on τ under which this statement will always hold. Otherwise, for the more general LOLR, there could be an implicitly defined upper bound on τ, but it is difficult to derive sufficient conditions where a smaller c always leads to $E^{min} = 0$.

$\tau = 0$. We call this the *bailout central bank*, which we explore below. Alternatively, assuming the central bank has greater powers of recovery than the private sector, in Appendix B.1, we analyze an actuarially fair LOLR (AF-LOLR) – one that will charge a break-even rate $\tau = 1$ at time 2. Finally, and theoretically, the LOLR could eliminate moral hazard by charging $\tau = 1 + z$. Bagehot (1873) anticipated this by urging the central bank to lend freely into a crisis to solvent entities but at a high rate. Our theoretical analysis indicates precisely what that high rate ought to be.

4.2.5 Bailout Central Bank

We plot the outcomes with a bailout central bank ($\tau = 0$) versus the planner's allocations using the same parameters from Section 3. We also specify the social cost of intervention to be $C(L) = \frac{1}{2}cL^2$ with $c = 0.02$ to get a sufficiently high level of bailout funding.

The numerical plots in Figure 3 illustrate Proposition 2. Although the total level of money created M remains efficient, it is now supported in equilibrium by too much investment and too little private insurance than what is socially optimal. Note that for any support from the central bank at a fixed but below-Pigouvian rate, I deviates more from the social optimal as the money premium increases because the subsidy implicit in central bank support, and hence the distortion to investment incentives, increases (see (20)). Moreover, the higher level of investment enables more liquidity to be generated through asset sales, contributing to an endogenous "missing market" for insurance. In Appendix B.1, we show results are qualitatively similar when $\tau = 1$ (that is, the ex-post intervention is actuarially fair), though quantitatively there is less overinvestment distortion.

We also fix a level of money-bond spread by setting $R^M = 1.05$ and examine the effects of varying the cost of bailout funds c in Figure 4. A low cost of c maps to large levels of overinvestment

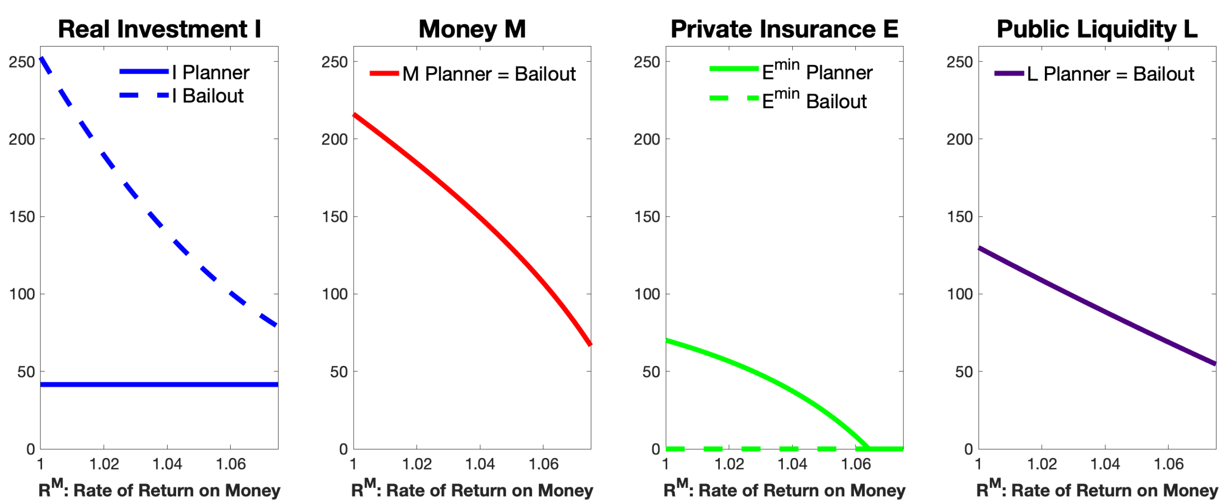

Figure 3: Equilibrium Outcomes with a Bailout Central bank, for different levels of return on money R^M

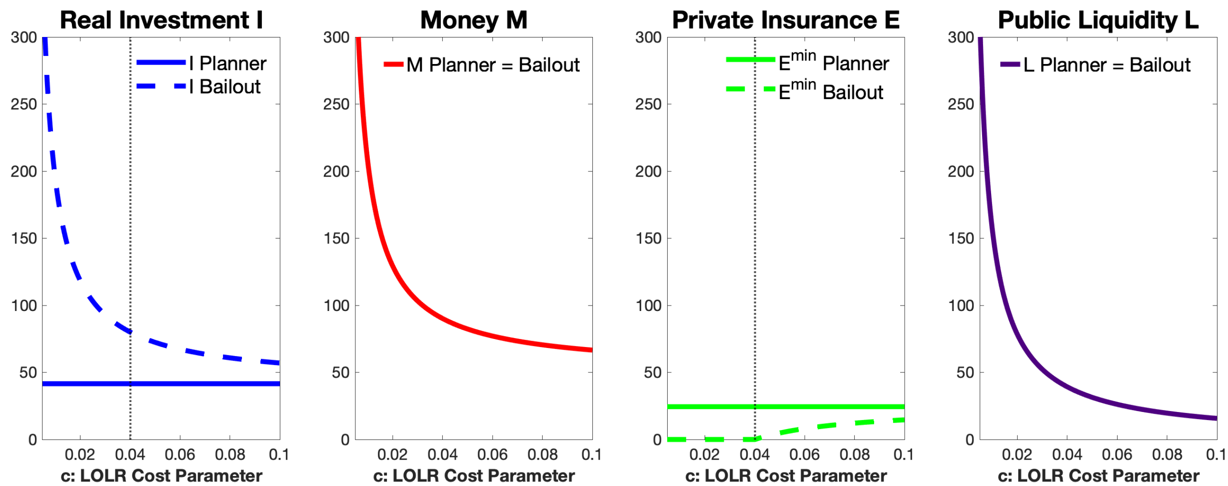

Figure 4: Equilibrium Outcomes with a Bailout Central bank at $R^M = 1.05$, for different levels of bailout cost c

and crowding out of private insurance, as well as high levels of bailout funding L (which in itself is not inefficient absent the moral hazard problem). As indicated by Proposition 3, with a sufficiently low $c < \bar{c}$ (the region to the left of the vertical dotted line in the plots of I and E), private insurance becomes endogenously missing (completely crowded out by public insurance L).

4.3 Provision of Pre-committed Liquidity by Central Bank

An ex-post central bank had limited ability to intervene in the crisis state if $\tau > 0$ because of bank solvency constraints. Now suppose we add to the baseline model a central bank that contracts ex ante (at $t = 0$) to provide a pre-committed level of liquidity in the crisis state at date 1 in return for payment in the good state. This could also be thought of as a form of deposit insurance.[16]

To the extent that banks purchase optimal support ex ante, even an interventionist central bank will not add additional liquidity ex post. Furthermore, since payment takes place in the good state, the central bank is less constrained in the fee it can charge. In the crisis state with probability $1 - p$, the central bank provides $L = \phi I$ to the bank to alleviate the fire sale, but in the good state with probability p, the central bank taxes at a rate τ to collect $\tau L = \tau \phi I$ from the bank. The central bank solves the time-0 planner's problem, but it can only choose ϕ and must respect the privately determined I, M, and E).

[16] When the central bank charges a break-even rate, one can also interpret this as the CB charging p healthy banks to provide the deposit insurance to the $1 - p$ stressed banks. Alternatively, if the time 0 to time 2 episode is repeated many times, the CB breaks even on average.

4.3.1 Actuarially Fair Price for Pre-committed Liquidity

Relative to the bailout central bank, the bank's objective now has an additional term that reflects the tax in the good state for using pre-committed liquidity:

$$\max_{m,\psi,I} pf(I) + (1-p)\lambda I - R^B I + \underbrace{mI(R^B - R^M)}_{\text{money spread}} - p \underbrace{r^E \psi I}_{\text{Cont Cap premium}} - p \underbrace{\tau\phi I}_{\text{CB tax}}$$
$$+ (1-p) \underbrace{\psi I}_{\text{Cont Cap payout}} + (1-p) \underbrace{\phi I}_{\text{CB liquidity}} - (1-p) \underbrace{z[mIR^M - \psi I - \phi I]}_{\text{loss from fire sales}}$$

Once again, the severity of the moral hazard problem depends on τ and to what extent the expected charge helps reduce moral hazard in the bank's FOC w.r.t. investments I. One politically plausible level for τ is the actuarially fair premium, $\tau = \frac{1-p}{p}$, that allows it to break even on loaned funds.

We solve for both central bank actions and bank responses (detailed derivations in Appendix A.6), with the added complication that the central bank sets the level of intervention knowing it will affect private incentives, so it takes all private FOCs as given. We use the following functional specifications for our analytical results (the same ones as used for plotting): $f(I) = a\log(I) + I \implies f'(I) = \frac{a}{I} + 1, g(K) = \theta \log(K) \implies g'(K) = \frac{\theta}{K}, C(L) = \frac{1}{2}cL^2 \implies C'(L) = cL.$

We present the analytical and numerical results for this ex-ante central bank below.

Proposition 4. Pre-committed Liquidity at Actuarially Fair Price (AF-PL)

Let superscript $AFPL$ denote equilibrium outcomes with a central bank that provides pre-committed liquidity at an actuarially fair price. Under our functional specifications,

1. The level of money creation and central bank lending is lower than the socially optimal level (and the ex-post bailout level): $M^{AFPL} < M^P = M^{bailout}$, $L^{AFPL} < L^P = L^{bailout}$.

2. As in the case with a bailout central bank, there is overinvestment and underprovision of contingent capital: $I^{AFPL} > I^P$, $E^{min,AFPL} < E^{min,P}$ (if $E^{\min,P} > 0$).

3. Compared to the bailout bank, moral hazard in investment and crowding-out of contingent capital is less severe: $I^{AFPL} < I^{bailout}$, $E^{min,AFPL} > E^{min,bailout}$.

The dotted line in Figure 5 plots the pre-committed liquidity outcome along with the no central bank case and the bailout case. Note that in the model with an ex-post bailout central bank, M and L are at the same level as the planner's allocations, while E^{min} is too low and I is too high.

In contrast, as seen also from Proposition 4, the AF-PL central bank acts in a constrained, second-best manner: it lowers L and M below the planner's allocations to alleviate moral hazard and to push I and E closer to the efficient levels. So while there is still overinvestment and underprovision of private insurance, it is to a lesser extent than the ex-post case. However, charging

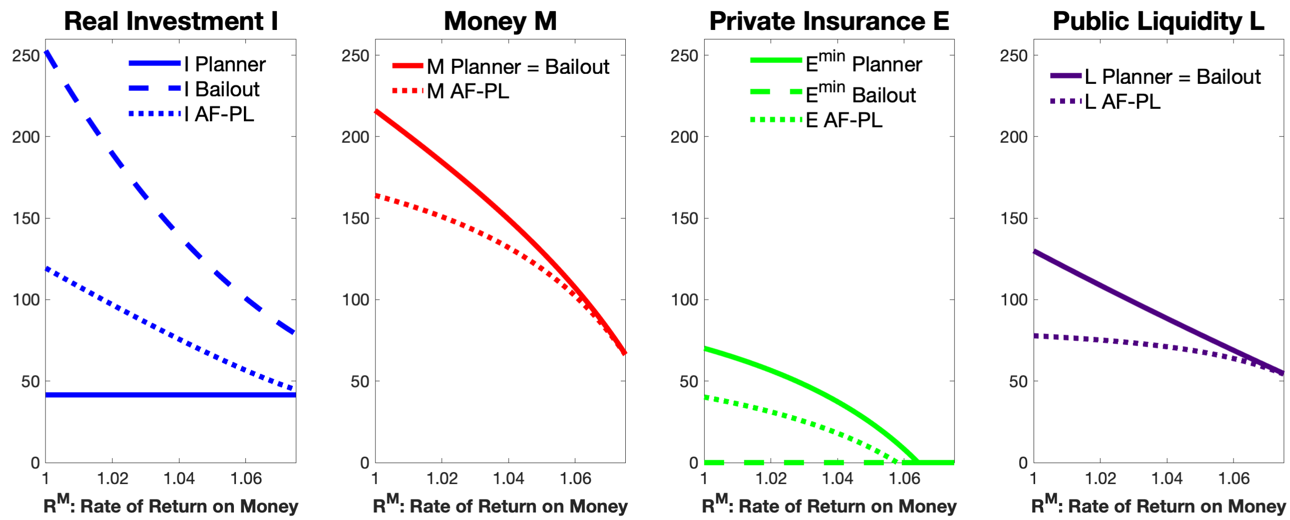

Figure 5: Equilibrium Outcomes for Actuarially Fair Pre-committed Liquidity, for different levels of return on money R^M

only the break-even price for its pre-committed liquidity is not enough to restore efficient outcomes, it has to charge more as we discuss next to implement the planner's choice.

4.3.2 Appropriate Pigouvian Price

Now suppose instead of the actuarially fair tax rate for pre-committed liquidity, the central bank charges $\tau = \frac{1-p}{p}(1+z)$ in the good state. The bank's objective becomes identical to the baseline objective (2) without a central bank. The bank's choice of I now becomes socially optimal and moral hazard is entirely eliminated. The central bank's FOC becomes the planner's condition $g'(W - M + L) = C'(L)$, because the private choice of investment I no longer depends on ϕ as there is no moral hazard. As a result, the central bank that provides pre-committed liquidity and charges $\tau = \frac{1-p}{p}(1+z)$ in the good state, which is above and beyond the actuarially fair premium, is able to achieve the efficient allocations. We will argue shortly why charging such a price is difficult.

4.3.3 Pre-positioned Collateral

If the central bank has the ability to recover more than the private sector in the crisis state, then instead of a capital infusion, the intervention can be a loan, perhaps against assets the private sector would not lend to without massive haircuts in a time of crisis. The pre-positioning of collateral that the central bank can lend against has been suggested by a number of authors (see, for example, King, 2016; Hanson et al., 2024). The repayment of the loan can be spread across good and crisis state based on the bank's ability to pay in those states. All that matters for date 0 incentives is that the expected payment be as in the previous subsection.

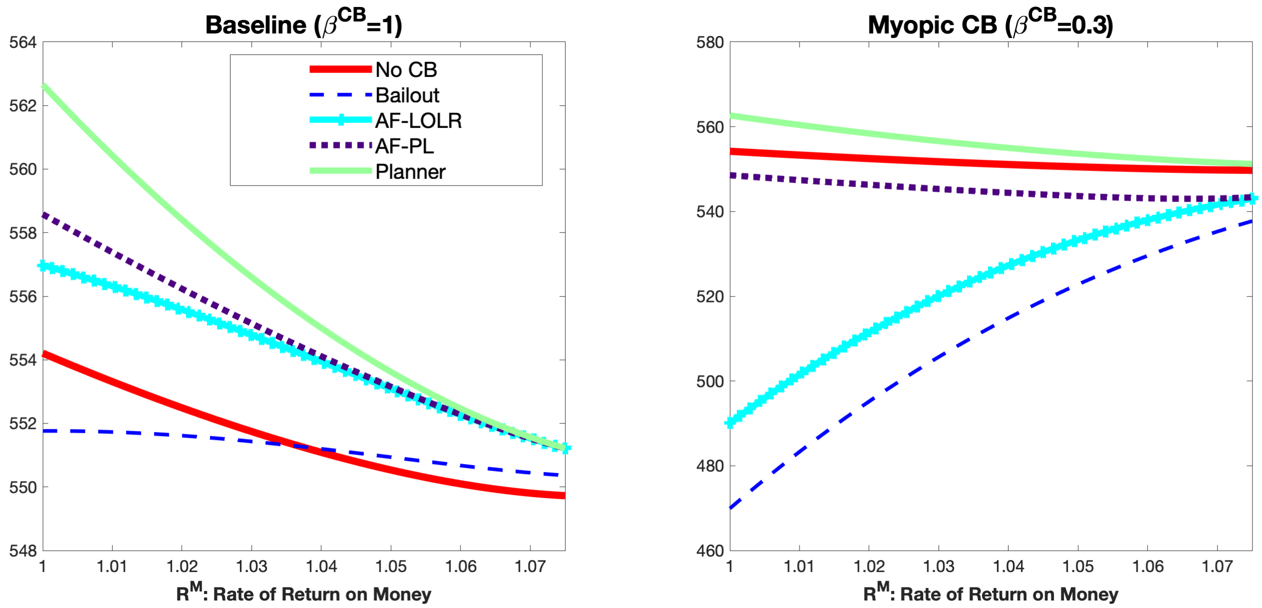

Figure 6: Welfare under Different Types of Interventions, varying the return on money R^M

4.4 Welfare Comparison

Figure 6 plots social welfare for the economy without a central bank, under three types of central bank interventions (bailout, LOLR that breaks even, and actuarially fair pre-committed liquidity), and the social planner's solution where the planner can choose L (which coincides with pre-committed liquidity by central bank at the "right" Pigovian price). The left panel shows results under the baseline parameters for a central bank with no moral hazard ($\beta^{CB} = 1$).

In the low spread region (high R^M) where incentives for excessive money creation are relatively muted, any form of central bank interventions raises welfare, as cheap public support improves outcomes in the crisis state. By contrast, when money-bond spreads are high (low R^M), moral hazard is severe, and introducing a bailout central bank reduces welfare relative to the private equilibrium without a central bank.[17] While pre-committed liquidity delivers higher welfare than bailouts or the ex-post LOLR, all underpriced interventions remain socially suboptimal. When the gains from money creation are high, households benefit from added convenience yield, but moral hazard-induced overinvestment and under-insurance are also most pronounced, making underpriced liquidity support especially distortionary, as shown in the welfare decomposition in Appendix B.3.

The right panel of Figure 6 illustrates the case with central bank moral hazard ($\beta^{CB} = 0.3$), which further distorts incentives toward excessive intervention. Even actuarially fair pre-committed liquidity yields lower welfare than the private equilibrium without a central bank, as commercial

[17] Appendix B.2 shows that with lower c, even an actuarially fair LOLR can yield lower welfare than the no-central bank benchmark.

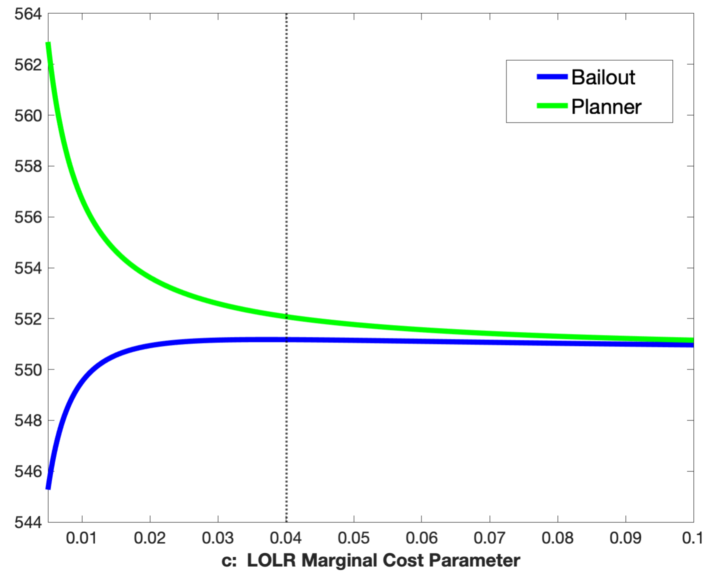

Figure 7: Welfare under Different Types of Interventions at $R^M = 1.05$, varying the central bank funding cost c

bank moral hazard that induces overinvestment becomes aggravated.

Finally, for a fixed level of money spread in the baseline case ($\beta^{CB} = 1$), Figure 7 plots welfare under different levels of c, where the dotted vertical line indicates the level below which private insurance becomes completely crowded out by the central bank. Our point is not that central banking interventions *always* distort outcomes. As shown by the green line, time-1 intervention L is welfare enhancing on its own and rises as its cost (c) falls, reflecting that modern central banks that can create money more elastically because of the shift from the Gold Standard to fiat money have greater potential to improve outcomes. However, the moral hazard distortions induced by central bank support at a fixed but too-low rate also increase as c falls and the quantity of support increases. This makes a central bank with a more elastic balance sheet more distortionary ex ante and potentially more welfare-reducing. So we turn next to why the support is typically underpriced.

5 The Underpricing of Central Bank Support

5.1 The Financial Return on Central Bank Intervention

To fully offset banks' overinvestment incentives, the central bank has to charge $\tau = \frac{1}{k}$ (the Pigouvian/Bagehot price, as also the private fire-sale return) if it charges ex post in the bad state or $\tau = \frac{1-p}{p}(\frac{1}{k})$ if it commits ex ante to charge in the good state. It is straightforward to show that:

Proposition 5. *By charging the Pigouvian price, the central bank will make positive pecuniary profit. Furthermore, under our functional specifications, there exists $\bar{k}$ such that for $k < \bar{k}$, the central bank is profitable even after accounting for indirect intervention cost $C(L)$.*

Note that it is optimal for the central bank to intervene ex post so that $C'(L) = g'(W-M+L) = \frac{1}{k}$. The price it charges ex post is merely a transfer, which allows a range of possibilities even for a socially minded central bank, though the price has ex-ante incentive effects. To the extent that the central bank has no additional powers of recovery, it cannot charge for intervention, and the very presence of the socially minded central bank that does not set up ex-ante structures (see below) will induce moral hazard. To the extent that it has additional powers of recovery but is limited by the solvency constraint, it will still induce moral hazard. It is only when the solvency constraint does not bind and it can credibly announce ex ante, and charge the full Pigouvian price ex post – perhaps spreading the charge across states – that the bank's investment incentives are not distorted. Effectively, though, this will mean charging the private fire-sale price.

The problem, of course, is that it is hard to explain to the public why the central bank should effectively buy assets at the fire-sale price while its average cost of producing liquidity is far lower. If the central bank does this in the midst of stress, it will be accused of price gouging – behaving no better than private sector bottom-fishers who are making profits off the already distressed and weakening them (and some will not be able to pay– the solvency constraint).[18] If the central bank sets up the support scheme ex ante – for example, deposit insurance or pre-positioned collateral – the right price will ensure that, averaged over time, the scheme will make money for the authorities despite the incidence of crises. Invariably, banks will exert political pressure on the authorities for a more actuarially fair scheme, which will then overincentivize investment. Put differently, even when central authorities are sagacious, and charge in anticipation of eventual support, the difficulty of explaining incentive effects to the public (and the political class) will not allow them to charge the right price. Of course, the evidence suggests the authorities rarely charge the right price for the anticipated sequence of stress state interventions, giving a boost to the private sector (see Haddad, Moreira, and Muir, 2025).

In our framework, therefore, the underpriced central bank put is a subsidy to illiquidity risk taking, specifically the risk inherent in financing long-term assets with short-term claims. While the effects are relatively benign when the term spread is small, they become significantly more concerning when the term spread is high, as the subsidy to maturity transformation is then most pronounced. Anticipation of such support amplifies this distortion, and adds to the direct interest-rate effects associated with expansionary central bank policy. Of course, when interest rate policy is focused on slowing the economy, the underpriced central bank put makes its job harder.

[18]Consistent with such behavior, during the 2023 U.S. regional banking crisis, the Fed set up a facility to lend to banks at the full nominal face value of bonds posted as collateral rather than against their depressed market value.

5.2 Underpricing of Public Liquidity as a Source of Franchise Value

In our benchmark model without mispriced public intervention, the constraint is not binding in equilibrium and there are no additional frictions in deposit markets, so the deposit franchise value is zero. In particular, what banks earn as the deposit spread ($R^B - R^M$) is exactly offset by expected fire-sale costs, as shown in (8). By contrast, when public intervention is underpriced (formally, $p\tau < (1-p)(1+z)$), a positive deposit franchise value emerges, as discussed in Appendix B.4. From equation (25), this franchise value is proportional to the size of investment I, scale of intervention ϕ per unit of I, and the degree of underpricing $(1 - p)(1 + z) - p\tau$. These latter two terms also appear as the distortionary wedge in the bank's FOC w.r.t. I. Thus, our stylized model shows that, in addition to traditional sources of bank franchise power (such as market power, switching costs, or deposit stickiness), underpriced government support constitutes an additional and conceptually distinct source of bank franchise value.

Moreover, the franchise value arising from underpriced public liquidity increases with the money-bond premium. This is not because the premium itself contains a bailout component, but because a larger spread expands the scale of money creation M, investment I, and intervention L, thereby magnifying the franchise value generated through underpriced public support.

5.3 Other Policy Tools

In our setup, banks have no incentive to hold cash that earns a gross return below R^M. Appendix B.5 shows that a liquidity-coverage-ratio (LCR)-type regulation raises banks' costs of money creation, which can discipline overinvestment due to moral hazard but also reduces convenience from money. Other policy tools that restore the optimal levels of M and E can improve outcomes with underpriced interventions but cannot restore constrained efficiency either, as private incentives continue to favor I. Consequently, moral hazard persists unless intervention is correctly priced or bank investment incentives are directly altered.

6 Financial Speculation

In the Stein model, the money bond premium is fixed, so it caps the return to money financing. What if investments with greater liquidity dependence have higher returns without cap? If so, the returns from liquidity dependent investment strategies rather than the money bond premium may now determine fire-sale prices, and the extent of distortions from underpriced central bank support. Interestingly, distortions may increase.

6.1 Speculation Technology

Let the real technology $f(\cdot)$ be financed by money (a fraction $m_1 \in [0,1]$) or bonds (a fraction $1 - m_1$). In addition, the bank finances speculation: The bank can invest αI in a leveraged speculative financial technology in addition to the storage technology for excess funds that pays R^B. For each dollar invested in speculation, the bank can enhance leverage l through derivatives strategies. This "search for yield" enhances the total expected return to $1 + ls$, where s is a base spread. In the crisis state, this speculation leads to a margin call and required payment $v(l) > 0$, where $v(\cdot)$ is increasing and convex. So more leverage enhances returns and liquidity exposure.

On the financing side of the αI invested in speculation, a fraction $m_2 \in [0,1]$ is raised via money and the rest is raised via bonds. We impose an additional constraint that $m_2 \geq \bar{m}$, i.e., at least $\bar{m}$ fraction of αI must be money financed. We can attribute this to the bank's duration-hedging motives or regulations (this constraint is not essential but will ensure money financing is not crowded out). So the total liquidity call in the crisis state from speculation becomes $[m_2 + v(l)]\alpha I R^M$.

The total amount of money issued in this economy is $M = m_1 I R^M + m_2 \alpha I R^M = m I R^M$, but the total amount of liquidity demand at time 1 is $(1+\theta)M = M + v(l)\alpha I R^M$ with $\theta = \frac{v(l)\alpha}{m}$, where the last term reflects the additional liquidity demand from margin calls.

We focus on the case where in equilibrium, the speculative technology is lucrative enough so that the gross return $1 + ls$, minus additional expected fire-sale costs due to margin calls pays a net return above R^B and banks do not invest in the simple storage technology. To be precise, $1 + ls - v(l)(1-p)zR^M > R^B$. Notably, this net return on speculation is endogenous: it depends on the leverage choice and fire-sale return, both of which vary with the money-bond spread.

We model the gains from speculation $1 + ls$ and costs associated with the margin calls $v(l)$ as transfers, so the planner's problem remains as in Section 4. For exposition, we add the speculation technology to the benchmark model without and with a bailout central bank.

6.2 Bank's Problem

The bank now chooses $m_1 \in [0,1]$ (fraction of real investment financed by money), $m_2 \in [0,1]$ (fraction of financial speculation financed by money), α (size of financial speculation relative to real investment I), leverage l, along with $\psi = E/I$ and I to solve the following problem:

$$
\max_{m_1 \in [0,1], m_2 \in [0,1], \alpha, l, \psi, I} \underbrace{pf(I) + (1-p)\lambda I - m_1 I R^M - (1-m_1) I R^B}_{\text{real investment}}
$$
$$
+ \underbrace{\alpha I \cdot (1 + ls) - m_2 \alpha I R^M - (1-m_2)\alpha I R^B}_{\text{speculation}}
$$

$$- p \underbrace{r^E \psi I}_{\text{Cont Cap premium}} +(1-p) \underbrace{\psi I}_{\text{Cont Cap payout}} +(1-p) \underbrace{\phi I}_{\text{LOLR payout}}$$

$$- (1-p) \underbrace{z[m_1 I R^M + [m_2 + v(l)]\alpha I R^M - \psi I - \phi I]}_{\text{fire sale losses}}$$

s.t.

$$m_1 + \alpha[m_2 + v(l)] = m \leq \frac{k\lambda + \psi + \phi}{R^M}, \qquad m_2 \geq \bar{m}.$$

6.3 Private Investor's Problem

Taking L and k as given, the PI's problem is essentially unchanged from the baseline, except that the total liquidity provided by the PI to the system is now $(1+\theta)M - L$:

$$\max_{(1+\theta)M,E} p\left[g(W) + r^E E\right] + (1-p)\left[g(W - (1+\theta)M + L) + \frac{1}{k}((1+\theta)M - E - L)\right].$$

The FOCs of the bank and the PI are shown and discussed in detail in Appendix C.1.

6.4 Private Equilibrium

When $1 + ls - v(l)(1-p)zR^M > R^B$ (financial speculation earns a higher net return than the storage technology), the bank only invests in the real technology f and the speculation technology. In equilibrium, to finance speculation, the bank would like to use as much bond financing as possible (to pocket the difference between the net return on speculation and bond financing) and finance the rest using money at a point where the marginal cost of financing via money, which is effectively R^M plus the expected fire-sale cost $(1-p)zR^M$, is equalized with the net return on speculation.

As a result, banks are willing to accept a higher fire-sale cost that equals the difference between net speculation returns and R^M, whereas in the baseline model, the expected fire-sale cost equals the money-bond spread $R^B - R^M$. Therefore, in this model with lucrative speculative investments, real investments f are financed only by bonds at cost R^B (because bond financing is cheaper than the all-in cost of money financing given higher fire-sale costs). In short, we always have $m_1 = 0$, $m_2 = \bar{m}$, so money is created only to finance financial speculation. With more demand for liquidity through margin calls and a lower fire-sale price (though not necessarily more money created), the central bank also intervenes more in the presence of speculation. These results are formalized in the proposition below and fully derived in Appendix C.3 and C.4.

Proposition 6. Equilibrium in the Model with Speculation

Suppose $1 + ls - v(l)(1-p)zR^M > R^B$.

1. The bank finances real investments using bonds only ($m_1 = 0$) *and the bank is on the constraint* ($m_2 = \bar{m}$).

2. Let k^S *and* k^P *be the fire-sale price in the crisis state in the economy with speculation and under the planner's choice, respectively. Then* $k^S < k^P$.

3. With a bailout central bank, there is overinvestment in the real technology and over-intervention by the central bank: $I^S > I^P$, $L^S > L^P$.

4. With or without a central bank, there is higher private and total demand for liquidity in the crisis state in the economy with speculation than under the planner's choice: $(1+\theta)M^S - L^S > M^P - L^P$ *and* $(1+\theta)M^S > M^P$.

In the baseline model with money financing, the fixed money premium pinned down the fire-sale cost. Here, the variable return on the speculative investment increases the equilibrium fire-sale cost above the money premium, and were it not for the constraint requiring minimum money financing, would crowd out the use of money financing entirely. Put differently, the speculative investment, even though it entails margin calls, is a better use of scarce liquidity (because of the higher returns) than money financing with attendant runs. Illiquidity seeking migrates to the highest net-of-liquidity-demand return activity, and this need not be money-financed real investment.

6.5 Numerical Results

We use the same parameters as the baseline setup. We choose $s = 0.01$ as the size of the primitive net "spread" that is magnified by the leveraged speculation trade. For the margin call function, we use $v(l) = 0.002l^2 + 0.001l$. We set the minimum level of money used to fund speculation to be $\bar{m} = 0.5$ (similar results prevail if we use $\bar{m} = 1$). These parameter choices give reasonable levels of speculation returns (net return of 0.1 to 0.13) and leverage (l is around 20-26; the size of the margin call, $v(l)$, is around 0.8 to 1.3), as discussed in detail in Appendix C.4.

With speculation technology, the fire-sale price k is determined jointly with the bank's choice of speculative leverage l, rather than solely by the fixed money-bond spread. Since speculation is partly money-financed, a lower R^M reduces the financing cost of the carry trade, inducing greater money creation and a larger speculative position along the extensive margin (as money financing leverage on the liability side is constant at $m_2 = \bar{m}$). However, this also amplifies fire-sale pressures in the crisis state, so the bank reduces asset-side speculative leverage l along the intensive margin to mitigate convex margin call losses. Our numerical results in Appendix C.4 confirm that in the model with speculation, k becomes less sensitive to movements in R^M, reflecting the endogenous reduction of speculative leverage l as money becomes cheaper and the overall speculation position

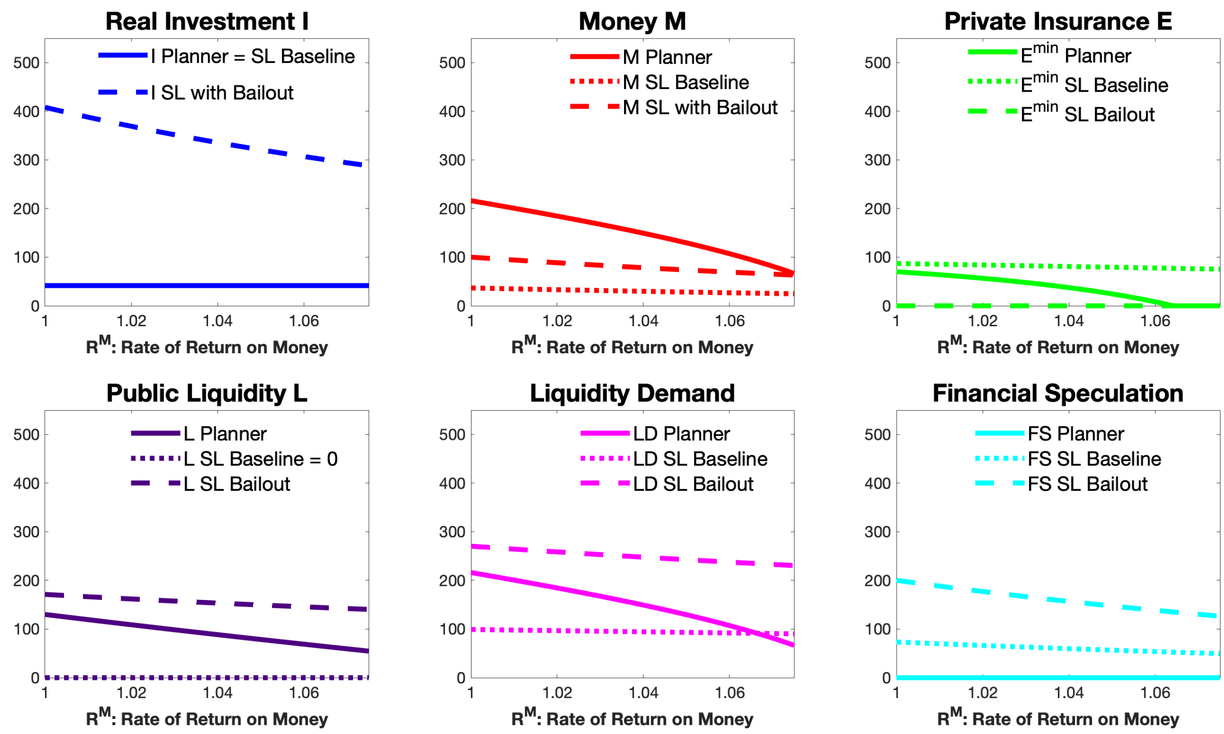

Figure 8: Equilibrium Outcomes with and without Speculative Lending (SL) Opportunities

expands. In short, as R^M decreases, the scale of speculation rises and fire-sale price falls, while the bank optimally chooses lower leverage l per dollar of speculation to partially mitigate fire-sale pressure, yielding a lower per-dollar net return (which remains above R^B).

Figure 8 plots the equilibrium outcomes of the model with speculation and no central bank, with speculation and with a bailout central bank, along with the efficient outcomes from Section 4.1. In the baseline model, with speculation added, there is no distortion to real investment I, though speculation leads to more private liquidity demand $(1 + \theta)M - L$ ($L = 0$ in the baseline with no central banks) that is financed by a higher-than-socially-optimal level of private insurance E.

Once a bailout central bank is added to the model with speculation, the baseline model's results of overinvestment in the real technology and underprovision of liquidity insurance continues to prevail (with an endogenously missing private insurance market). There is also more speculation (as shown on the bottom right panel) after adding a bailout central bank, and due to a more depressed fire-sale price with financial speculation, the level of central bank intervention is also inefficiently high. Furthermore, while the total amount of liquidity demand (dashed pink line, which includes both M and margin calls) is larger than socially optimal (M Planner), fewer money-like deposits are issued (dashed red line) because they compete with margin calls for liquidity in the crisis state.

Figure 9 presents the welfare levels under four scenarios: the planner with contingent public liquidity provision, the benchmark model with a bailout central bank, the speculation model without a bailout central bank, and the speculation model with a bailout central bank.

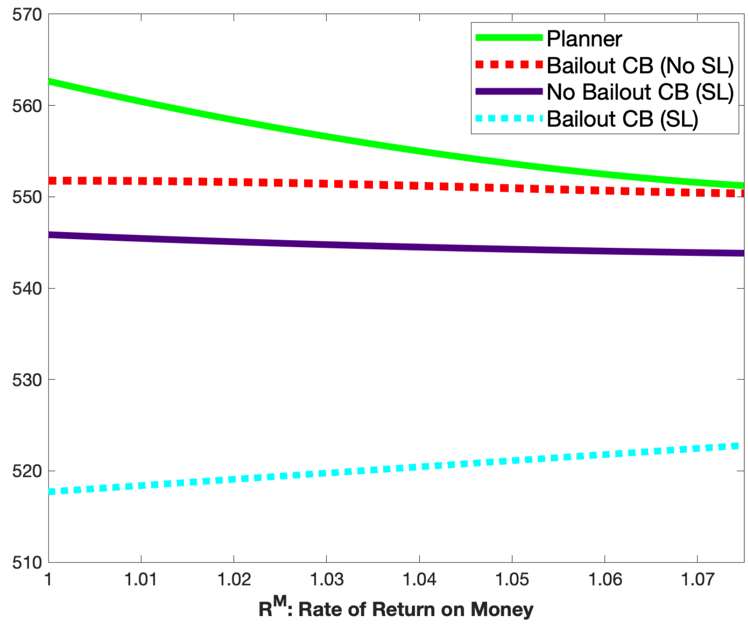

Figure 9: Welfare with vs without Speculative Lending Opportunities

Conceptually, the presence of a bailout central bank and speculative opportunities both lead to distortions that reduce welfare. However, the welfare loss in an economy with speculation and a bailout central bank is not merely the sum of the welfare losses from the baseline model with each friction individually – there is an additional loss from the interactions of the two distortions. As shown in Appendix C.5, much of the interaction effect comes from the fact that the distortion in bank's FOC w.r.t. real investments I is increasing in the amount of fire-sale support from public intervention. As speculation leads to larger fire sales, the overinvestment incentives from the bailout are exacerbated. Another interesting distortion with speculation is that while it creates more liquidity demand at time 1, it creates fewer deposits M, which reduces welfare because of the loss of the household's convenience yield on money.

7 Lender of Last Resort and Financial Speculation

7.1 The Emergence of Public Backstops

Overall, private arrangements of contingent capital appear to have been replaced during the 20th century by public provision of liquidity. Relatedly, there has been a secular decrease in bank equity as a fraction of assets, or equivalently an increase in bank leverage, until capital regulations worldwide raised banks' capital ratio following the Global Financial Crisis (GFC) of 2009. In the United States, Kaufman (1992) and Barth and Miller (2018) document a secular decline in banks' book equity ratios (book capital as a fraction of total book assets), from above 50% in 1840 to below 10% by the late 1940s. In the United Kingdom, Alessandri and Haldane (2009) report a similar fall in bank equity ratios from over 15% in 1880 to below 5% in 1960, while Turner (2014) also shows that the ratio of book capital plus uncalled capital to assets fell even more dramatically, from over 70% in 1885 to below 5% in 1958 as uncalled capital disappeared.

Coincident with the fall in contingent (and normal) capital has been mounting evidence that central banks have become much more willing to intervene in the post World War II era (see, for example, Schularick and Taylor, 2012; Metrick and Schmelzing, 2021), perhaps because democracies have become more sensitive to public pain. This has been accompanied by the increased prevalence of public deposit insurance. Examining 17 major economies over the past four centuries, Ferguson et al. (2023) show that since World War II, central bank policy responses to financial crisis have become "close to systematic," making crises the dominant driver of large central bank balance sheet expansions during the period.[19]

7.2 Continuing Recurrence of Severe Banking Crises

Despite central bank efforts, significant banking crises continue to occur around the world, including in advanced economies. Reinhart and Rogoff (2013) show that, especially since the early 1970s, the incidence of banking crises globally has been at least as frequent, if not more frequent, than in the pre-Great Depression era. In a cross-country study of 14 countries spanning 1870–2008, Schularick and Taylor (2012) also show that while monetary policy responses to financial crises became more aggressive after 1945, the output costs of crises have remained substantial, including to date in the United States (Bouis et al., 2025). In a related vein, Ferguson et al. (2023) find that central bank liquidity support during financial crises has indeed reduced the severity of crises, but also raises the probability of future boom-bust episodes. Such continued crises despite intervention is consistent with central bank induced moral hazard.

[19] Early discussions of the lender of last resort trace back to Thornton (1802) and Bagehot (1873), who emphasized that a LOLR should protect the aggregate money stock rather than individual institutions, lend only to sound institutions at penalty rates with good collateral, and preannounce these conditions in advance of crisis (Humphrey, 1989). In the more modern literature, proponents favor the LOLR's ability to overcome information asymmetries that would lead to insolvency for otherwise sound banks when private monitoring is insufficient, as well as the LOLR's capacity to provide liquidity during episodes of systemic crises (Freixas et al., 2002) or bank runs (a view consistent with the sunspot bank run model of Diamond and Dybvig, 1983). Meanwhile, the moral hazard costs of such a public backstop (Wagster, 2007; Calomiris, 2010; Calomiris and Jaremski, 2019), potential losses to taxpayers, and other potential costs of implementing liquidity provision that reduce a central bank's lending capacity (suggested in Goodhart (1999) and captured by the $C(\cdot)$ function in our model), have also been also well-recognized by policymakers and academics.

7.3 Modern Evidence: Financial Speculation

Finally, our setup of financial speculation that magnifies a small spread via leverage and triggers margin calls in crises mirrors the structure of real-world carry trades, including the Treasury cash-futures basis trade (known as "the basis trade"), where hedge funds take a leveraged long position in cash US Treasuries while simultaneously selling Treasury futures. These extraordinarily leveraged bets allow hedge funds to profit from small spreads (see Kashyap et al., 2025). However, they can also increase systemic stress, as our model suggests. For instance, the US Treasuries market dislocation in March 2020 during the Covid-19 Crisis was largely driven by fire sales from hedge funds that engaged in the basis trade (see, for example, Duffie, 2020; Schrimpf, Shin, and Sushko, 2020; Barth and Kahn, 2021), leading to significant illiquidity in segments of the Treasuries market (He, Nagel, and Song, 2022). While the Federal Reserve's purchase of \$1 trillion in Treasuries helped restore normal functioning of the market (Vissing-Jorgensen, 2021), our model suggests it also increased the expectation of further such intervention, and thus moral hazard.[20]

8 Private Insurance and Limited Commitment

Arguably, we have privileged private arrangements in our analysis by assuming they are frictionless. We can relax this assumption by requiring the PI to escrow the amount pledged for private insurance until the risks of the crisis state dissipate entirely. This introduces an additional cost of providing insurance, over and above the forgone late investment in the crisis state – the expected loss from forgoing late investment even in the normal state. For the bank to wish to buy insurance, there must therefore be an additional marginal benefit to insurance than just preventing fire sales. In the model this comes from alleviating a binding money creation constraint, so we get back the results of the Stein model. In the interests of space, we will only sketch the intuition.

Because their commitment cannot be trusted, the PI must now put E, the funds they promise for contingent capital / private insurance, into a liquid technology in advance and hold it till any possible contingency passes (that is, till date 2).[21] In the good state, instead of investing E into the g technology, the PI gets $h(E)$ from the storage technology and the insurance premium $r^E E$ from the bank at time 2. We assume that $h(\cdot)$ is increasing and weakly concave, and $g'(W - E) > h'(E)$ for all $E > 0$.[22] In the crisis state, the PI's capital payment E is sent to the bank.

[20] See d'Avernas, Petersen, and Vandeweyer (2025) for a complementary analysis of central bank regulations and interventions in the Treasury market in a model with banks and hedge funds.

[21] This requirement can be seen as solving an unmodeled agency problem where insurers can pocket the premium and walk away. Conceptually, one can think of $t = 0$ to $t = 2$ as a fixed period with continuous time, and the liquidity shock at $t = 1$ could arise at any time between the start and the end, so the funds committed to insurance have to be permanently available as liquid collateral. Under this interpretation, the "good" state is the case where the economy reaches $t = 2$ without experiencing any liquidity shock.

[22] Intuitively, the diminishing marginal returns from the liquid storage technology $h(\cdot)$ corresponds to increasing

In Appendix D, we formally describe the private equilibrium and the planner's solution. The key difference from the baseline model in Section 3 is the PI's FOC w.r.t. liquidity commitment E:

$$pr^E = p\left[g'(W-E) - h'(E)\right] + (1-p)\frac{1}{k}. \tag{23}$$

In an interior solution, the time-0 expected marginal benefit of providing private insurance (premium r^E in the good state) is equated to the corresponding marginal cost, which now includes not only the forgone fire-sale return of $\frac{1}{k}$ in the bad state, but also the marginal cost of redirecting funds from the g investment to the liquid technology in the good state.

Therefore, when $E > 0$, the insurance market equates the bank's marginal benefit of receiving insurance to the PI's marginal cost of providing insurance, which implies the bank's shadow cost of money creation equals the PI's incremental costs incurred due to limited commitment:

$$\frac{\eta}{R^M I}+(1-p)\frac{1}{k} = pr^E = p\left[g'(W-E) - h'(E)\right]+(1-p)\frac{1}{k} \implies \frac{\eta}{R^M I} = p\left[g'(W-E) - h'(E)\right].$$

Proposition 7 formalizes that with limited commitment in private insurance markets, Stein (2012)'s baseline results are restored and there is overprovision of private insurance.

Proposition 7. *Let I^*, M^*, E^* denote the private outcomes and I^P, M^P, E^P denote the planner's choices in the model with limited commitment without a central bank.*

1. In the low spread region where the constraint on money creation does not bind ($\eta = \eta^P = 0$), the private outcome is socially optimal. In this case, $I^* = I^P$, $M^* = M^P$, $E^* = E^P = 0$.

2. In the high spread region where the constraint on money creation is binding ($\eta, \eta^P > 0$), there is not only overinvestment and over-issuance of money, but also over-provision of private insurance. That is, $I^* > I^P$, $M^* > M^P$, $E^* > E^P$. Fire sales are also suboptimally severe ($k^* < k^P$).

Remark. As in Stein (2012), in the constrained region, the bank's shadow value of money creation is too high relative to the socially optimal level, as the bank fails to internalize the externality that makes the fire-sale worse for everyone else. Because the bank's shadow value equals the PI's limited commitment costs, there is too much private insurance provided because private agents over-perceive its benefit when the constraint is binding. If there were no fire-sale externality and over-creation of money (that is, $\eta = \eta^P$), then there would be no over-provision of private insurance.

Appendix D shows that our results on central bank intervention continue to hold, and Appendix E shows overinvestment prevails even if the underpriced intervention scales up with M. Also, we show that E decreases as the marginal cost of commitment rises. This is consistent with the view that an increase in contracting frictions, such as the difficulty for shareholders to know that other shareholders would reliably post contingent capital as banks grew larger and the shareholder

marginal costs to overcome the temptation offered by larger amounts of liquid assets. We also assume $h(\cdot)$ has a lower marginal return than $g(\cdot)$ in the good state, so there are always costs to provide insurance due to limited commitment.

base becomes numerous and diverse, resulted in the waning of contingent capital. Yet contingent capital can take other forms that do not require such reliance – for instance, contingent convertible bonds where capital is pre-committed to the bank. More generally, in modern economies with well-developed financial markets, frictions that impede private contingent capital have arguably diminished. Consequently, underpriced central bank intervention may be the more plausible reason for its waning.

9 Conclusion

The incentive for banks to overissue money-like liabilities is an old concern, rendered more salient by recent theoretical modeling. Our work, however, suggests straightforward private remedies. Unfortunately, they do not eliminate fire sales, which may prompt central bank intervention. Limited, pre-committed, and appropriately priced central bank intervention always improves welfare in our model. In contrast, underpriced intervention, which we argue is the norm, is distortionary and crowds out private contingent capital. We do not model dynamics, which may exacerbate distortions. Plausibly, expectations of future central bank intervention grow with intervention. Finally, as the size of the financial sector relative to the central bank increases, and as the fiscal situation of developed countries deteriorates, it is worth asking whether the capacity of the central bank to intervene could eventually be limited, even if it controls an elastic currency. In that case, could private sector risk-taking in mistaken anticipation of sufficient intervention prove particularly costly?

Our model could also find applications in international finance, where countries (substituting for banks in our model) might have the temptation to issue too much short term debt. Fee-based contingent credit lines from multilateral organizations like the IMF could reduce risk-taking, and resemble the contingent capital insurance in our model. Of course, the IMF's willingness to bail out countries without imposing sufficiently stiff conditionality could resemble the bailout in our model. The consequence would be excessive investment and risk taking.[23]

What about regulation and supervision to reduce the gaming of intervention? As the continuing problems in the US financial system (for example, see the Barr (2023) report on the bank failures in March 2023) suggest, and as Barth, Caprio, and Levine (2005) argue, regulation and supervision is no panacea (though see Correia, Luck, and Verner, 2025). All this suggests that finding the right mix of private contracting and public support will be an ongoing topic of debate and research.

[23] We thank Agustin Carstens for suggesting this.

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Appendix

A Key Steps and Proofs

A.1 Bank's FOC wrt I in the Baseline Model in Section 3.1

$$
\begin{aligned}
pf'(I) + (1-p)\lambda - R^B &= -\left[m(R^B - R^m) - (1-p)z[mR^M - \psi]\right] + pr^E\psi - (1-p)\psi \\
&= -\underbrace{m\left[(R^B - R^m) - (1-p)zR^M\right]}_{=\frac{\eta}{I}m} + \underbrace{\psi\left[pr^E - (1-p)(1+z)\right]}_{=\frac{\eta}{R^M I}\psi} \\
&= \frac{\eta}{I}\left[\frac{\psi}{R^m} - m\right],
\end{aligned}
$$

where we use the bank's FOCs (3) and (4) on the second line to arrive at the simplified expression (5).

A.2 Rewriting the Social Planner's Objective in Section 3.4

The social planner maximizes the utility of the representative household, who is endowed with Y units of output at time 0 and has utility function

$$U = C_0 + \beta\mathbb{E}[C_2] + \gamma M,$$

where $C_0 = Y - I - W$ and

$$
\mathbb{E}[C_2] = \begin{cases} f(I) + g(W) & \text{with prob } p \\ \lambda I + g(W - M) + M & \text{with prob } 1 - p \end{cases}.
$$

Therefore,

$$U = Y - I - W + \beta\left[pf(I) + pg(W) + (1-p)\lambda I + (1-p)g(W-M) + (1-p)M\right] + \gamma M.$$

Dropping constants Y and W, and dividing the expression by β (equivalent to multiplying by $R^B = \frac{1}{\beta}$) yields

$$U = -R^B I + pf(I) + pg(W) + (1-p)\lambda I + (1-p)g(W-M) + (1-p)M + \frac{\gamma}{\beta}M,$$

where $\dfrac{\beta+\gamma}{\beta} - 1 = \dfrac{R^B - R^M}{R^M}$. So we can rearrange and get

$$U = pf(I) + (1-p)\lambda I - R^B I + mI(R^B - R^M) + pg(W) + (1-p)[g(W-M) + M],$$

which arrives at the planner's objective in Section 3.4. Note that if $M > I$, so that $M - I$ is invested in a storage technology yielding R^B, then $C_0 = Y - I - W - (M - I)$, while C_2 increases by $R^B(M-I)$ in all states. These two effects offset exactly, so the net contribution of $M - I$ cancels out and the derivation above continues to hold without modification.

A.3 Planner's FOC wrt I in the Baseline Model in Section 3.4

$$\begin{aligned} & pf'(I) + (1-p)\lambda - R^B + m(R^B - R^m) + \\ & +(1-p)\underbrace{\Big[-g'(W-M)+1\Big]}_{=-\frac{1}{k}+1=-z} mR^M = -\frac{\eta^P\lambda}{R^M}\left(\frac{g''(W-M)}{(g'(W-M))^2}mR^M\right) \\ \Longrightarrow\ & pf'(I) + (1-p)\lambda - R^B + \underbrace{\Big[m(R^B - R^m) - (1-p)zmR^M\Big]}_{=m\frac{\eta^P}{I}\left[1-\left(\frac{g''(W-M)}{(g'(W-M))^2}\right)\lambda I\right]} = -\frac{\eta^P\lambda}{R^M}\left(\frac{g''(W-M)}{(g'(W-M))^2}mR^M\right). \end{aligned}$$

This can be further re-written as

$$pf'(I) + (1-p)\lambda - R^B = \eta^P\left(-\frac{g''(W-M)}{(g'(W-M))^2}\lambda m - \frac{m}{I} + \frac{g''(W-M)}{(g'(W-M))^2}\lambda m\right),$$

which leads to the expression in (13):

$$pf'(I) + (1-p)\lambda - R^B = -m\frac{\eta^p}{I}.$$

A.4 Proof of Proposition 2

Part 1. From comparing (8) with (16), note that the expressions for $(1-p)z$ are the same, so the private fire-sale price k is also socially optimal. Then by comparing from (18) and (21) with (14) and (17), we get $M^{LOLR} = M^P$, $L^{LOLR} = L^P$ as they are pinned down by the same FOCs.

Part 2-3. From the bank's FOC w.r.t. I,

$$pf'(I) + (1-p)\lambda - R^B = -(1-p)(1 + z - \tau)\phi,$$

whereas the RHS is 0 in the planner's case. The expressions for z are the same in both cases. Therefore, as long as $\phi > 0$ (some level of LOLR funding) and given that $f''(I) < 0$, we have that

$I^{LOLR} > I^P$. It also follows that $E^{min} = M - k\lambda I - (1 - k\tau)L$ is lower in the private case than what is socially optimal (assuming the social planner is subject to the solvency constraint under the same τ at $t = 2$ in the crisis state), because $M^{LOLR} - L^{LOLR} = M^P - L^P$ but $I^{LOLR} > I^P$.

Part 4. Consider two equilibria with LOLR policies that charge different levels of τ, with $\tau^H > \tau^L$. As shown in Part 1, the levels of M, L, k are at the planner's level regardless of τ. However, consider investment levels I^H, I^L, minimum private insurance $E^{min,H}$, $E^{min,L}$, and fraction of investment covered by public funding ϕ^H and ϕ^L under the equilibrium with an LOLR central bank that charges τ^H and τ^L, respectively. Then, suppose also that $\tau^H \leq 1 + z$,[24]

$$
\begin{aligned}
pf'(I^H) + (1-p)\lambda - R^B &= -(1-p)(1+z-\tau^H)\phi^H, \\
pf'(I^L) + (1-p)\lambda - R^B &= -(1-p)(1+z-\tau^L)\phi^L.
\end{aligned}
$$

First, it cannot be the case that $I^H = I^L$. If so, because $L^H = L^L$, we would have $\phi^H = \phi^L$, but the RHS is different for the two conditions (because $\tau^H > \tau^L$), which would imply $I^H \neq I^L$, arriving at a contradiction.

Second, suppose for a contradiction that $I^H > I^L$. Then because $L^H = L^L$, it must be that $\phi^H < \phi^L$. Then we must have

$$
\begin{aligned}
pf'(I^H) + (1-p)\lambda - R^B &= -(1-p)(1+z-\tau^H)\phi^H \\
&> -(1-p)(1+z-\tau^H)\phi^L \\
&= pf'(I^L) + (1-p)\lambda - R^B.
\end{aligned}
$$

This necessarily implies $f'(I^H) > f'(I^L) \implies I^H < I^L$, a contradiction.

Thus, it must be that $I^H < I^L$, and therefore $E^{min,H} = M - k\lambda I^H - (1 - k\tau^H)L > M - k\lambda I^L - (1 - k\tau^L)L = E^{min,L}$ (where M, L, k are the same for both equilibria).

A.5 Proof of Proposition 3

Part 1 (for $\tau = 0$): First, note that by definition, $E^{min,LOLR} = \max(M^{LOLR} - k\lambda I^{LOLR} - (1 - \tau k)L^{LOLR}, 0) = 0$ if

$$
M^{LOLR} - k\lambda I^{LOLR} - (1-\tau k)L^{LOLR} = M^{LOLR} - L^{LOLR} - k\lambda I^{LOLR} + \tau k L^{LOLR} \leq 0
$$

which can be rewritten as

[24] If the central bank charges beyond the fire-sale savings, banks would simply choose not to receive central bank fundings.

$$k\lambda I^{LOLR} \geq M^{LOLR} - L^{LOLR} + \tau k L^{LOLR}.$$

From the central bank's FOC w.r.t. L (18) and PI's FOC w.r.t. M (21), we have that

$$\frac{1}{k} = g'(W - M^{LOLR} + L^{LOLR}) = C'(L^{LOLR}) = cL^{LOLR} \implies L^{LOLR} = \frac{1}{ck^{LOLR}},$$

and

$$M^{LOLR} - L^{LOLR} = W - (g')^{-1}(\frac{1}{k})$$

where k is a constant that only depends on R^B, R^M, p (as can be seen from bank's FOC w.r.t. m (3) with $\eta = 0$, which remains to be the case with an LOLR central bank added). Since g is monotone increasing and strictly concave, $g' > 0$ and its inverse exists. We have that $M^{LOLR} - L^{LOLR} > 0$ (under the assumption that $g'(W) \geq 1 > \frac{1}{k}$ for any k).

As a result, we get $E^{min,LOLR} = 0$ if and only if

$$k\lambda I^{LOLR} \geq W - (g')^{-1}(\frac{1}{k}) + \frac{\tau}{c} \iff I^{LOLR} \geq \frac{W - (g')^{-1}(\frac{1}{k}) + \frac{\tau}{c}}{k\lambda}$$

So with a bailout central bank where $\tau = 0$, it suffices to show that I is monotone decreasing in c without any bounds, so that the LHS is increasing in c and the RHS is a constant.

From the bank's FOC w.r.t. I, if we multiply I on both sides, we get

$$pf'(I)I + (1-p)\lambda I - R^B I = -(1-p)(1+z-\tau)\phi I.$$

With the functional specification on $C(L)$, we always have that $C'(L) = cL = \dfrac{1}{ck} \implies L = \phi I = \dfrac{1}{ck}$. Therefore, we can rearrange and write

$$(R^B - (1-p)\lambda)I - pf'(I)I = (1-p)(1+z-\tau)\frac{1}{ck}.$$

Note that the LHS is increasing in I and is not bounded above, because $R^B - pf'(I) - (1-p)\lambda > 0$ from the FOC w.r.t. I whenever $\tau < 1 + z$, and $R^B - pf'(I) - (1-p)\lambda$ is increasing in I since $f'' < 0$. Therefore, it is clear that I is a decreasing function of c, and $I \to \infty$ as $c \to 0$ (I grows without bounds as c decreases, which can be easily shown via contradiction).

Part 2 (general case): With $\tau > 0$, there is no guarantee on the existence of a positive $\bar{c}$ and it depends on the specific functional form f as well as the level of τ. For example, using our working assumptions on f and g, we would need $I \geq \dfrac{W - \theta k}{k\lambda} + \frac{\tau}{\lambda} L$ where

$$I = \frac{pa + (1-p)(1+z-\tau)\frac{1}{ck}}{[R^B - (1-p)\lambda - p]}, \qquad L = \frac{1}{ck}.$$

Thus, to make sure we get $E^{min} = 0$ with a small enough c, we would need

$$\frac{pack + (1-p)(1+z)}{[R^B - (1-p)\lambda - p]} - \frac{W - \theta k}{\lambda} c \geq [\frac{1}{\lambda} + \frac{(1-p)}{R^B - (1-p)\lambda - p}]\tau$$

as a sufficient condition, which effectively imposes an upper bound on τ. In a more general setting, it might be possible to get the (possibly implicitly defined) derivative of I w.r.t. $\frac{1}{c}$ from

$$(R^B - (1-p)\lambda)I - pf'(I)I = (1-p)(1+z-\tau)\frac{1}{ck},$$

so we can simply impose that $I'(\frac{1}{c}) > \frac{\tau}{\lambda k}$ which would guarantee $E^{min} = 0$ for a small enough c. This is again an upper bound on τ as long as I is increasing in $\frac{1}{c}$.

A.6 Derivation of the Ex-ante Central Bank's FOC in Section 4.4

The central bank's problem can be written in a way that makes fully transparent how ϕ affects private choices:

$$\begin{aligned}\max_{\phi}\ & pf(I(\phi)) + (1-p)\lambda I(\phi) - R^B I(\phi) + m(\phi)I(\phi)(R^B - R^M) \\ & +(1-p)\left[g\left(W - m(\phi)I(\phi)R^M + \phi I(\phi)\right) + m(\phi)I(\phi)R^M\right] - (1-p)C(\phi I(\phi)).\end{aligned}$$

After some steps (detailed derivations below), the central bank's FOC w.r.t. ϕ can be written as

$$g'(W - M + L) = C'(L) + AL(C'(L) - 1), \tag{24}$$

where $A = \dfrac{(1-p)z}{ap} > 0$ is a constant that is increasing in the bank's marginal benefit from capital infusion (savings from fire sale), and decreasing in the productivity of time-0 investment. The LHS is the marginal social benefit from central bank funding in the crisis state and the RHS is the social cost of central bank funding in the crisis state, now scaled up by an additional term that reflects the severity of moral hazard (from the derivations, we show that it incorporates how a change in ϕ affects I). Note that this FOC takes the same form as the planner's and the bailout / LOLR central bank's, with the additional term $AL(C'(L) - 1)$ which reflects the effects of moral hazard.

A.6.1 Detailed Derivations

The central bank understands the following private decision rules:

1. Bank's FOC w.r.t. m pins down k:

$$(1-p)z = \frac{R^B - R^M}{R^M}.$$

Thus k is invariant to the central bank's actions as the constraint will never be binding in equilibrium.

2. Bank FOC w.r.t. I pins down I:

$$\begin{aligned} pf'(I) + (1-p)\lambda - R^B &= -(1-p)z\phi \\ \implies p[\frac{a}{I} + 1] &= R^B - (1-p)\lambda - (1-p)z\phi \\ \implies \frac{a}{I} &= \frac{1}{p}\left[R^B - (1-p)\lambda - (1-p)z\phi\right] - 1 \\ \implies I &= ap\left[R^B - (1-p)\lambda - p - (1-p)z\phi\right]^{-1}, \end{aligned}$$

where we can clearly observe that I is increasing in ϕ. So with moral hazard, we have $I'(\phi) > 0$, but if there is no moral hazard, $I'(\phi) = 0$. We can also formally derive the expression for $I'(\phi)$:

$$\begin{aligned} I'(\phi) &= -ap\left[R^B - (1-p)\lambda - p - (1-p)z\phi\right]^{-2}(-(1-p)z) \\ &= ap\left[R^B - (1-p)\lambda - p - (1-p)z\phi\right]^{-2}(1-p)z \\ &= I^2\frac{(1-p)z}{ap} \\ &=: AI^2, \end{aligned}$$

where $A = \frac{(1-p)z}{ap} > 0$ is a constant that depends entirely on parameters of the model. It is increasing in the bank's marginal benefit of insurance (savings from fire sale), and decreasing in the productivity of time-0 investment. It only shows up (so that $I'(\phi) \neq 0$) with the moral hazard term present in $-(1-p)z\phi$, otherwise, $I'(\phi) = 0$.

3. PI's FOC w.r.t. M pins down M:

$$g'(W - M + L) = \frac{1}{k} \implies \frac{\theta}{W - M + L} = \frac{1}{k} \implies \theta k = W - M + L.$$

We can think of $m(\phi)$ and $I(\phi)$ as separate functions of ϕ as the LOLR's choice affects the bank's financing and investment decisions, and write

$$\begin{aligned} \theta k &= W - m(\phi)R^M I(\phi) + \phi I(\phi) \\ \implies m(\phi)R^M I(\phi) &= W - \theta k + \phi I(\phi) \\ m(\phi) &= \frac{1}{R^M}\left[\frac{W - \theta k}{I(\phi)} + \phi\right], \end{aligned}$$

so that

$$
\begin{aligned}
m'(\phi) &= \frac{1}{R^M}\left[1 - \frac{W-\theta k}{I^2} I'(\phi)\right] \\
&= \frac{1}{R^M}\left[1 - A(W-\theta k)\right] \\
&= \frac{1}{R^M}\left[1 - A(M-L)\right].
\end{aligned}
$$

So without moral hazard, $m'(\phi) = \frac{1}{R^M} > 0$, but with moral hazard, we can no longer sign $m'(\phi)$ in general but $m'(\phi) < 0$ when $M - L$ (money that is not publicly insured) is large (which is the case with a large W).

For the central bank that charges the actuarially fair τ =1 to break even, its problem can be written in a way that makes clear how ϕ affects private choices:

$$
\begin{aligned}
\max_{\phi} \quad U(\phi) =& \{pf(I(\phi)) + (1-p)\lambda I(\phi) - R^B I(\phi)\} + m(\phi)I(\phi)(R^B - R^M) \\
& (1-p)\left[g\left(W - m(\phi)I(\phi)R^M + \phi I(\phi)\right) + m(\phi)I(\phi)R^M\right] - (1-p)C(\phi I(\phi)).
\end{aligned}
$$

The central bank's FOC w.r.t. ϕ is

$$
\begin{aligned}
0 =& \left[pf'(I) + (1-p)\lambda - R^B\right] I'(\phi) + (R^B - R^M)\left[m'(\phi)I + mI'(\phi)\right] \\
&+ (1-p)g'(W - M + L)\left\{-\left[m'(\phi)I + mI'(\phi)\right]R^M + \phi I'(\phi) + I\right\} \\
&+ (1-p)\left[m'(\phi)I + mI'(\phi)\right]R^M - (1-p)C'(L)\left[\phi I'(\phi) + I(\phi)\right].
\end{aligned}
$$

Substituting $pf'(I) + (1-p)\lambda - R^B = -(1-p)z\phi$, as well as $I'(\phi) = AI^2$ and dividing across by I gives

$$
\begin{aligned}
0 =& \left[-(1-p)z\phi\right] AI + \frac{R^B - R^M}{R^M}\left[m'(\phi) + mAI\right]R^M \\
&+ (1-p)g'(W - M + L)\left\{-\left[m'(\phi) + mAI\right]R^M + \phi AI + 1\right\} \\
&+ (1-p)\left[m'(\phi) + mAI\right]R^M - (1-p)C'(L)\left[\phi AI + 1\right].
\end{aligned}
$$

Now substitute that $L = \phi I$, $M = mIR^M$ everywhere to obtain

$$
\begin{aligned}
(1-p)C'(L)\left[AL + 1\right] =& -(1-p)zAL + \frac{R^B - R^M}{R^M}\left[m'(\phi)R^M + AM\right] \\
&+ (1-p)g'(W - M + L)\left\{-\left[m'(\phi)R^M + AM\right] + AL + 1\right\} \\
&+ (1-p)\left[m'(\phi)R^M + AM\right].
\end{aligned}
$$

Recognizing that $m'(\phi)R^M = 1 - A(M-L) \implies m'(\phi)R^M + AM = 1 + AL$:

$$(1-p)C'(L)(1+AL) = -(1-p)zAL + \frac{R^B - R^M}{R^M}(1+AL)$$
$$+ (1-p)g'(W-M+L)\left[-(1+AL) + AL + 1\right] + (1-p)(1+AL).$$

Substituting $\frac{R^B-R^M}{R^M} = (1-p)z$, we can simplify as follows:

$$-(1-p)zAL + (1-p)z(1+AL) + (1-p)(1+AL) = (1-p)C'(L)(1+AL).$$

Using $g'(W-M+L) = \dfrac{1}{k}$, this can be written as Equation (24):

$$(1-p)(1+z) = (1-p)\left[C'(L) + AL(C'(L)-1)\right].$$

A.7 Proof of Proposition 4

Part 1. Consider equation (24):

$$\frac{1}{k} = g'(W-M+L) = C'(L) + AL(C'(L)-1).$$

and denote the RHS as $s(L) = C'(L) + AL(C'(L)-1) = cL + AL(cL-1)$.

First, note that the socially optimal L^P satisfies

$$\frac{1}{k} = 1 + z = C'(L^P) = cL^P \implies L^P = \frac{1}{ck}.$$

Second, note that $RHS(L)$ is increasing for any $L \geq L^P$ because $cL^P - 1 = \frac{1}{k} - 1 = 1 + z - 1 = z > 0$, so that all parts of $s(\cdot)$ are increasing in L. The same observation can be made by looking at $s'(L) = c + 2AcL - A > 0$ for any $L \geq \frac{1}{ck}$.

Third, suppose for a contradiction that the ex-ante central bank chooses the level of pre-commited liquidity to be some $L^{AFPL} \geq L^P$. Then because s is increasing for any $L \geq L^P$,

$$s(L^{AFPL}) \geq s(L^P) = cL^P + AL^P(cL^P - 1) > cL^P = \frac{1}{k},$$

because $AL^P(cL^P-1) > 0$ with $L^P = \frac{1}{ck}$. This implies

$$C'(L^{AFPL}) + AL^{AFPL}(C'(L^{AFPL}) - 1) = s(L^{AFPL}) > \frac{1}{k}.$$

which implies L^{AFPL} does not satisfy the CB's FOC (24). This is a contradiction, as the CB's problem must have an interior solution since $g(\cdot)$ is concave and $C(\cdot)$ is convex.

Therefore, we must have that $L^{AFPL} < L^P = L^{bailout}$. Because $(M-L)$ is pinned down by the same FOC $g'(W-M+L) = \frac{1}{k}$ and k is the same across all baseline cases, we also have $M^{AFPL} < M^P = M^{bailout}$.

Part 2. From the bank's FOC w.r.t. I,

$$pf'(I^{AFPL}) + (1-p)\lambda - R^B = -(1-p)z\phi^{AFPL} < 0.$$

whereas the planner's choice satisfies

$$pf'(I^P) + (1-p)\lambda - R^B = 0,$$

with the RHS being 0 in the planner's case. The expressions for $(1-p)z$ are the same in all cases. Therefore, as long as $\phi^{AFPL} > 0$ (some level of bailout funding, which is always the case in our specification), we have that $I^{AFPL} > I^P$. It also follows that $E^{min} = M - k\lambda I - L$ is lower with precommitted liquidity than what is socially optimal, because $M^{AFPL} - L^{AFPL} = M^P - L^P$ but $I^{LOLR} > I^P$.

Part 3. Suppose, for a contradiction, $I^{AFPL} \geq I^{bailout}$. Then because $L^{AFPL} < L^{bailout}$, it must be that $\phi^{AFPL} < \phi^{bailout}$. From the bank's FOC w.r.t. I:

$$pf'(I^{AFPL}) + (1-p)\lambda - R^B = -(1-p)z\phi^{DI},$$

and

$$pf'(I^{bailout}) + (1-p)\lambda - R^B = -(1-p)(1+z)\phi^{bailout}.$$

To have $I^{AFPL} \geq I^{bailout}$, we must have

$$-(1-p)z\phi^{AFPL} < -(1-p)(1+z)\phi^{bailout},$$

but

$$\phi^{AFPL} < \phi^{bailout} \implies -(1-p)z\phi^{AFPL} > -(1-p)z\phi^{bailout} > -(1-p)(1+z)\phi^{bailout},$$

which yields a contradiction.

Thus, it must be that $I^{AFPL} < I^{bailout}$, and therefore $E^{min,AFPL} > E^{min,bailout}$.

A.8 Proof of Proposition 5

While the central bank's objective is to maximize social welfare, its pecuniary profit from intervention is

$$p\tau L - (1-p)L,$$

and its profit net of social intervention costs is

$$p\tau L - (1-p)L - (1-p)C(L).$$

When the central bank charges the actuarially-fair price $\tau = \frac{1-p}{p}$ (if it can charge in the good state), the pecuniary profit is trivially 0, and the profit net of intervention costs equals $-(1-p)C(L)<0$.

By constrast, when the central bank charges the Pigouvian price $\tau = \frac{1-p}{p}(1+z) = \frac{1-p}{p}\frac{1}{k}$ to restore constrained efficient outcomes, then as long as $k < 1$ (i.e., fire-sale losses are present, which always holds with $R^M < R^B$), the central bank's pecuniary profit is

$$(1-p)(\frac{1}{k}-1)L > 0,$$

and its profit net of intervention costs is

$$(1-p)(\frac{1}{k}-1)L-(1-p)C(L),$$

which is positive if k is sufficiently small (the fire-sale losses are sufficiently large).

Under our specification that $C(L) = \frac{c}{2}L^2$, since the equilibrium requires $cL = C'(L) = g'(W-M+L) = \frac{1}{k}$, the profit net of intervention costs becomes

$$(1-p)(\frac{1}{k}L-L-\frac{c}{2}L^2) = (1-p)L(\frac{1}{k}-1-\frac{1}{2k}) = (1-p)(\frac{1}{2k}-1)L > 0 \iff k < \bar{k} = 0.5$$

So when fire-sale losses are sufficiently severe, the central bank earns positive gains even accounting for its indirect intervention costs.

A.9 Proof of Proposition 6

Suppose the primitive parameters are such that in equilibrium in the economy with speculation, $1+ls-v(l)(1-p)zR^M > R^B$. The detailed equilibrium conditions for the model with speculation are derived in Appendix C.

1. $m_1 = 0$ and $m_2 = \bar{m}$ are shown in Section C.3.

2. To see that $k^S < k^P$, note that from Section C.3, $(R^B - R^M) < (1-p)z^S R^M$ but in the baseline model $(R^B - R^M) = (1-p)z^P R^M$. Therefore, $z^S > z^P$ and therefore $k^S < k^P$ as $z = \frac{1}{k} - 1$.

3. With a bailout central bank, since we always have $\frac{1}{k} = C'(L)$, a lower level of k implies a higher level of L, as $C(\cdot)$ is increasing and convex so $C'(\cdot)$ is increasing in L. Moreover, in both the benchmark and this extension, when a bail-out central bank is added, I is pinned down by:
$$pf'(I)+(1-p)\lambda-R^B = -(1-p)(1+z)\phi,$$

Note that we must have a larger I under the speculation extension. For a contradiction, suppose that I were smaller in the extended model with speculators, then because L is now higher, it must be that ϕ is higher, which from this FOC implies I must be higher, yielding a contradiction.

4. In the model with speculation, with $m_2 = \bar{m}$, we have that $\theta = \frac{v(l)\alpha}{m} = \frac{v(l)\alpha}{\bar{m}\alpha} = \frac{v(l)}{\bar{m}}$, and

$$g'(W - (1 + \frac{v(l)}{\bar{m}})M^S + L^S) = \frac{1}{k^S},$$

where $L = 0$ maps to the case without a central bank. In contrast, in the baseline model, $g'(W - M^P + L^P) = \frac{1}{k^P} < \frac{1}{k^S}$, therefore since $g'(\cdot)$ is decreasing, we must have that the total private liquidity demand at time 1 in the crisis state is higher with speculation: $(1 + \frac{v(l)}{\bar{m}})M^S - L^S > M^P - L^P$. Since $L^S > L^P$, it is also the case that the total liquidity demand is also higher: $(1 + \frac{v(l)}{\bar{m}})M^S > M^P$. The amount of money liability created, however, could be lower than the efficient level, as $(\frac{v(l)}{\bar{m}})M$ is used to satisfy margin calls.

A.10 Proof of Proposition 7

In the low-spread region with $\eta = \eta^P = 0$, the private FOCs coincide with the planner's FOCs.

Furthermore, suppose $E > 0$, from either the planner's FOC w.r.t. ψ or the private agents' FOC w.r.t. ψ and E, we must have

$$(1 - p)\frac{1}{k} = pr^E = p\left[g'(W - E) - h'(E)\right] + (1 - p)\frac{1}{k} \implies g'(W - E) - h'(E) = 0.$$

With $E > 0$, the LHS $g'(W - E) - h'(E)$ must be positive. Therefore, if we have $g'(W - E) > h'(E)$, then it must be that $E = 0$ when $\eta = \eta^P = 0$.

Now we prove the part with $\eta > 0$.

Proof for E:

If $\eta > 0$, note that the private FOCs w.r.t. private insurance can be combined and written as

$$p(g'(W - E^*) - h'(E^*)) = \frac{\eta^*}{I^* R^M}.$$

Similarly, the socially optimal E^P must satisfy

$$p\left[g'(W - E^P) - h'(E^P)\right] = \frac{\eta^P}{I^P R^M}.$$

Suppose, for a contradiction, that $E^* \leq E^P$, then because the LHS is increasing in E, we must have

$$\frac{\eta^P}{I^P} \geq \frac{\eta^*}{I^*}.$$

Note that the private FOC w.r.t. m gives

$$\left[(R^B - R^M) - (1-p)z^* R^M\right] = \frac{\eta^*}{I^*},$$

whereas the planner's FOC w.r.t. m gives

$$\left[(R^B - R^M) - (1-p)z^P R^M\right] = \frac{\eta^P}{I^P}[1 + \Omega(M^P, I^P)] > \frac{\eta^P}{I^P} \geq \frac{\eta^*}{I^*}.$$

So it must be that $z^P < z^*$ which implies $k^P > k^*$. Then we have

$$g'(W - M^*) = \frac{1}{k^*} > \frac{1}{k^P} = g'(W - M^P) \implies M^* > M^P.$$

The planner's FOC w.r.t. I is

$$pf'(I^P) + (1-p)\lambda - R^B = -\frac{\eta^P}{I^P}\left(m^P - \frac{\psi^P}{R^M}\right)$$

whereas the bank's FOC w.r.t I can be written as

$$pf'(I^*) + (1-p)\lambda - R^B = -\frac{\eta^*}{I^*}\left[m^* - \frac{\psi^*}{R^m}\right]$$

Note that in the constrained region, we must have that $m^P - \frac{\psi^P}{R^M}, m^* - \frac{\psi^*}{R^m} > 0$. There are two cases, both leading to a contradiction.

Case 1: $I^P \geq I^*$, then in the constrained region $k^P \lambda I^P + E^P = M^P < M^* = k^* \lambda I^* + E^*$ contradicts with $k^P > k^*$, $E^P \geq E^*$ and $I^P \geq I^*$.

Case 2: $I^P < I^*$. Then from the two FOCs w.r.t. I, the following must hold:

$$m^P - \frac{\psi^P}{R^M} < m^* - \frac{\psi^*}{R^m} \implies \frac{k^P \lambda}{R^M} < \frac{k\lambda}{R^M} \implies k^P < k,$$

which yields a contradiction with $k^P > k$.

Therefore, we must have $E^* > E^P$ and hence $\frac{\eta^P}{I^P} < \frac{\eta^*}{I^*}$.

Proof for M: Suppose $M^P \geq M^*$. Then we must have

$$M^* \leq M^P \implies \frac{1}{k^*} = g'(W - M^*) \leq g'(W - M^P) = \frac{1}{k^P} \implies k^P \leq k^*.$$

We have already established $E^P < E^*$,so in the binding region, we must have that

$$k^P \lambda I^P = M^P - E^P \geq M^* - E^* = k^* \lambda I^* \implies k^P \lambda I^P \geq k^* \lambda I^*.$$

Since $k^P < k^*$,this necessarily implies that $I^P > I^*$.

The planner's FOC w.r.t. I is

$$pf'(I^P) + (1-p)\lambda - R^B = -\frac{\eta^P}{I^P}\left(m^P - \frac{\psi^P}{R^M}\right),$$

whereas the bank's FOC w.r.t I can be written as

$$pf'(I^*) + (1-p)\lambda - R^B = -\frac{\eta^*}{I^*}\left[m^* - \frac{\psi^*}{R^m}\right].$$

Since $I^P > I^*$and $\frac{\eta^P}{I^P} < \frac{\eta^*}{I^*}$, we must have $m^P - \frac{\psi^P}{R^M} > m^* - \frac{\psi^*}{R^m} \implies \frac{k^P\lambda}{R^M} > \frac{k^*\lambda}{R^M} \implies k^P > k^*$, yielding a contradiction with $k^P < k^*$ from earlier.

Therefore, we must have $M^* > M^P$ and $k^P > k$.

Proof for I: We have already established that $E^P < E^* \implies \frac{\eta^P}{I^P} < \frac{\eta^*}{I^*}$, $M^P < M^* \implies k^P > k^* \implies m^P - \frac{\psi^P}{R^M} > m^* - \frac{\psi^*}{R^m}$ in the binding region. Then $I^P < I^*$ following the same reasoning as in Stein (2012).

B Additional Results in the Baseline Model with a Central Bank

B.1 Actuarially Fair Ex-post LOLR

Consider here an LOLR central bank that is perceived to charge a break-even rate $\tau = 1$ at time 2 conditional on the crisis state. As shown in Figure 10, such an ex-post actuarially-fair LOLR (AF-LOLR) alleviates moral hazard relative to the bailout central bank in Figure 3, as there is less severe overinvestment and underprovision of private insurance, but it still cannot attain the planner's solution.[25] To counteract the moral hazard effect of LOLR, the LOLR has to charge a higher than actuarially fair rate on public liquidity provision. This may not be feasible if the high rate is not politically feasible (banks claim the central bank is gouging them) or if it is not collectible because the central bank has limited additional powers of recovery than the private sector.

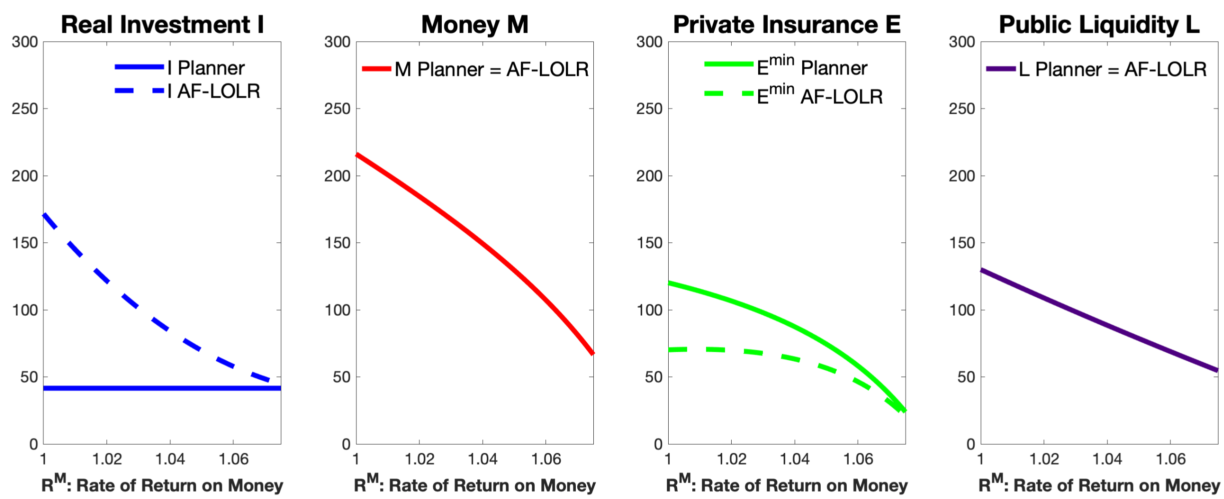

Figure 10: Equilibrium Outcomes with an Actuarially Fair LOLR central bank, for different levels of return on money R^M

B.2 Welfare in the Benchmark Model under a Lower c

Using the baseline parameters but lowering c from 0.02 to 0.002, we obtain a case in Figure 10 where, even without political myopia, the lower cost of intervention intensifies moral hazard make an actuarially fair LOLR welfare-reducing when money-bond spreads are high.

[25]To ensure comparability, the planner's solutions plotted with the AF-LOLR also respects the solvency constraint for the repayment of taxes by the banks.

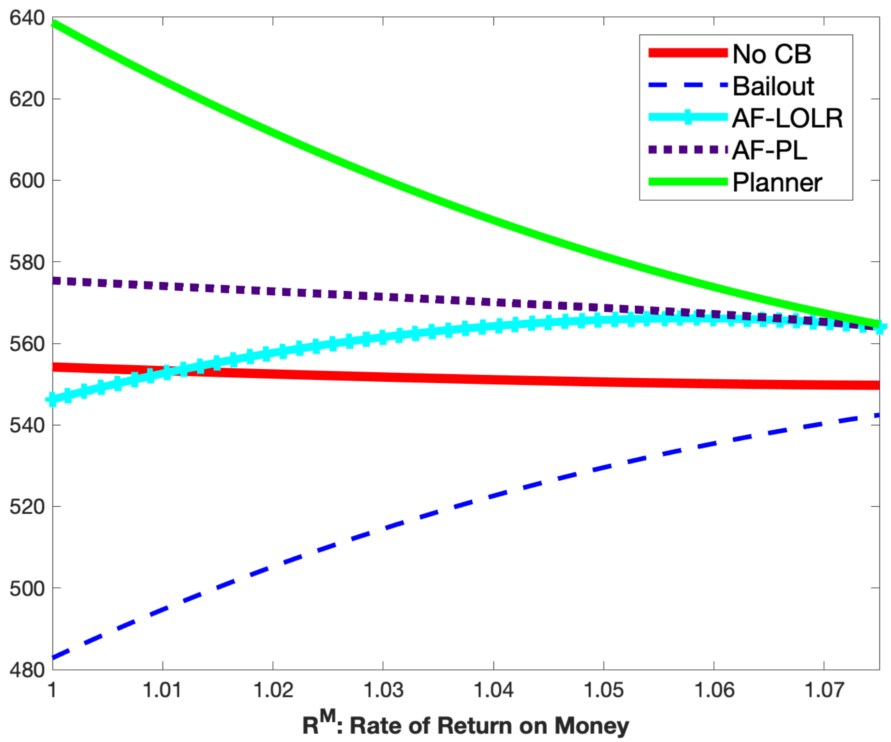

Figure 11: Welfare under Different Types of Interventions, varying the return on money R^M, with $c = 0.002$

B.3 Welfare Decomposition

Social welfare can be decomposed into four parts (ignoring some constants):

$$\underbrace{pf(I) + (1-p)\lambda I - R^B I}_{\text{Real Investment}} + \underbrace{mI(R^B - R^M)}_{\text{Money Premium}} + \underbrace{(1-p)M}_{\text{Money Payout in Crisis}}$$
$$+ \underbrace{[pg(W) + (1-p)g(W - M + L) - R^B W]}_{\text{PI Investment}} \underbrace{-(1-p)C(L)}_{\text{LOLR Cost}}$$

Figure 12 illustrates the welfare decomposition for our results in the baseline model without a central bank, with a planner that also chooses L, and a bailout central bankOnce a central bank is introduced and there is no moral hazard distortion (as in the Planner case), because the central bank has the power to create money elastically, both the money premium and the money payout in the crisis increases (by L), which improves welfare at the cost of the LOLR cost $C(L)$ in the crisis state. However, under a bailout central bank (or the underpriced actuarially fair LOLR), moral hazard leads to overinvestment which has the potential to lower overall welfare relative to the no central bank case.

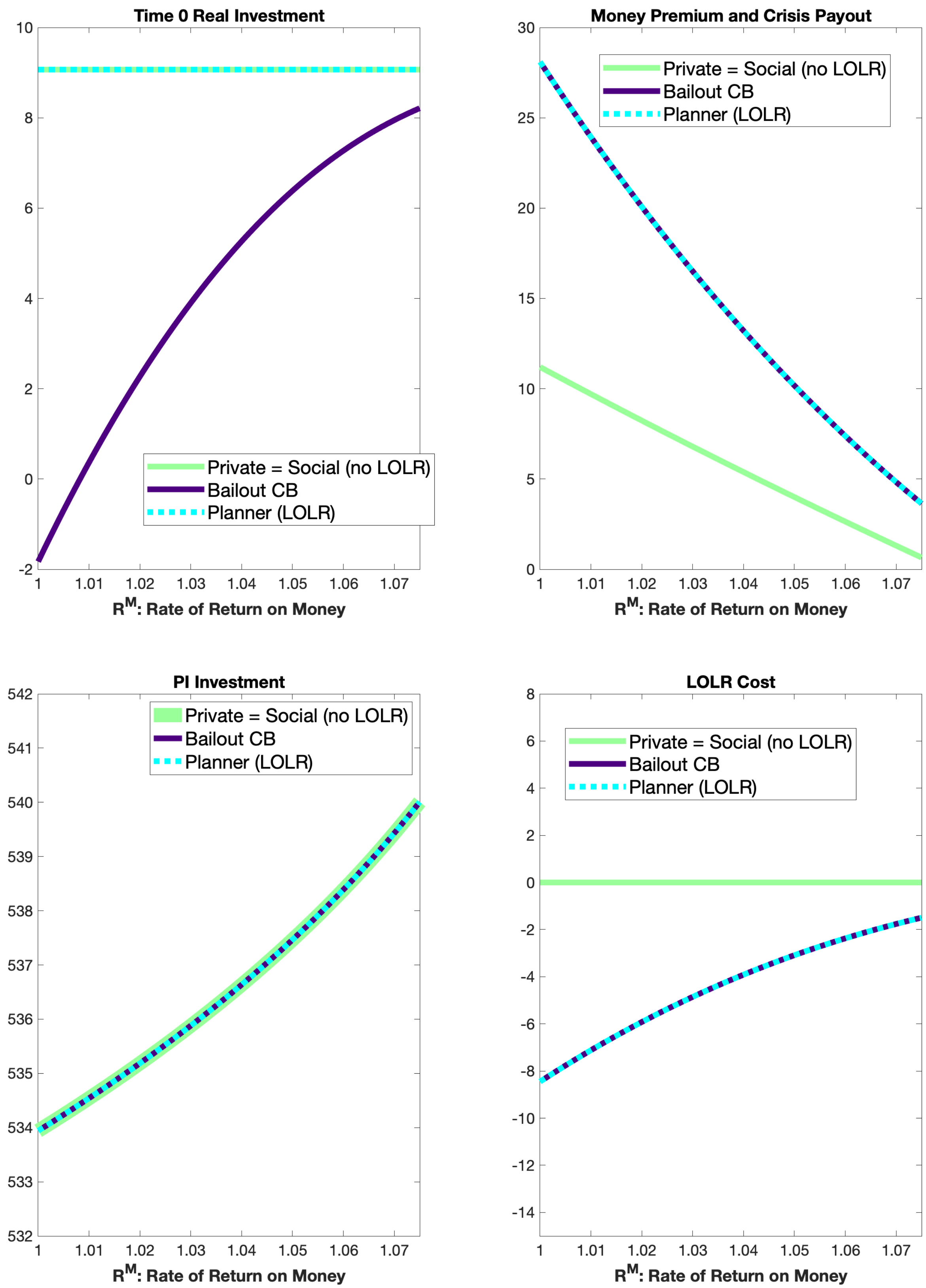

Figure 12: Welfare Decomposition

B.4 Bank Franchise Value Decomposition

Consider the bank's profit function in the benchmark model with a central bank:

$$\max_{m,\psi,I} pf(I) + (1-p)\lambda I - R^B I + \underbrace{mI(R^B - R^M)}_{\text{money spread}} - p \underbrace{r^E \psi I}_{\text{Cont Cap premium}} - p \underbrace{\tau\phi I}_{\text{CB tax}}$$
$$+ (1-p) \underbrace{\psi I}_{\text{Cont Cap payout}} + (1-p) \underbrace{\phi I}_{\text{CB liquidity}} - (1-p) \underbrace{z[mIR^M - \psi I - \phi I]}_{\text{loss from fire sales}}$$

In equilibrium, the money spread $mI(R^B - R^M)$ cancels out with losses from fire-sales $(1-p)zmIR^M$, fire-sale payout of contingent $(1-p)(1+z)\psi I$ cancels out with private insurance premium $pr^E\psi I$, so the bank's profit is

$$\max_{m,\psi,I} pf(I) + (1-p)\lambda I - R^B I - p \underbrace{\tau\phi I}_{\text{CB tax}} + (1-p) \underbrace{\phi I}_{\text{CB liquidity}} + (1-p) \underbrace{z[\phi I]}_{\text{fire sale loss saving}} \tag{25}$$

If there were no underpriced intervention, the last three terms also cancel out. However, if intervention were underpriced, $L = \phi I$ of the total money creation M leads to a positive franchise value as the bank pays less in the good state than it benefits from public intervention in the bad state. The franchise value is $[(1-p)(1+z) - p\tau]L = [(1-p)(1+z) - p\tau]\phi I$, which is underpricing of intervention times size of intervention.

This same underpricing term also appears as the wedge in the bank's first-order condition with respect to I:

$$pf'(I) + (1-p)\lambda - R^B = -\left[(1-p)(1+z) - p\tau\right]\phi,$$

showing that underpriced public liquidity distorts the bank's marginal investment condition by exactly the amount of the franchise value per unit of investment.

B.5 Liquidity Requirement and Other Policy Tools

In our setup, banks have no incentive to hold cash that earns a gross return of 1, or any gross return less than R^M.[26] Suppose instead that a liquidity-coverage-ratio (LCR)-type regulation requires

[26]This is without loss of generality, since a bank always has the option to change the mix of its short-term versus long-term debt, which has a similar buffering benefit and is more cost-effective. There are two ways to reduce asset sales in the bad state by \$1: either borrow an extra dollar of long-term debt at time 0 and have the proceeds on hand as cash that pays \$1 or the return on the liquid technology, which has net cost $R^B - 1$ (or R^B minus the return on the liquid technology), or borrow an extra dollar of long-term debt at time 0 so as to reduce the amount of short-term borrowing by \$1, with net cost $R^B - R^M$. Hence, the latter approach is strictly preferred, and banks endogenously choose not to engage in simply having cash on hand, as long as cash earns less than R^M. We consider the case where cash earns 1

banks to hold a fraction $0 < \omega < 1$ of total deposits as cash that is always available to meet depositor demand. This requirement is equivalent to forcing the bank to pay $R^{M\prime} = R^M + (R^M - 1)\frac{\omega}{1-\omega} > R^M$ on the money M that is used to finance long-term investment, while simultaneously creating $M' = \omega(M + M')$ of deposits that it simply hold as cash (which does not finance any investment and thus carries no run risk). [27]

As shown in Appendix B.5, such a liquidity requirement reduces the private benefit of issuing money to finance long-term investment by imposing an additional cost. With a bailout central bank, this mitigates overinvestment and improve fire-sales outcomes. However, it also lowers the effective money-bond spread and reduces the convenience yield generated by money M that finances long-term investment (a lower money-bond spread corresponds to lower constrained efficient welfare, as shown in Figure 6). Moreover, although money held as cash M' also generates a convenience yield, holding cash purely for its convenience lowers households' utility relative to consuming at time 0 (since $R^M > 1 \iff \beta + \gamma < 1$). M' could also be viewed as banks' deposits at other banks (e.g. under reserve requirements), which does not earn convenience yield for households and makes it even more undesirable from a welfare standpoint. Thus, while a liquidity requirement can improve outcomes relative to those under a bailout central bank, it does not restore the constrained-efficient allocation.

One could also consider the "cap-and-trade" approach of Stein (2012) (executed through open market operations and a reserve requirement) that allows regulators to implement the constrained-efficient level of M, or impose a requirement on banks' contingent capital to implement the planner's choice of E. However, when public liquidity is underpriced, these tools — while potentially welfare-improving — cannot restore the constrained-efficient outcome. Even if M and E are fixed so that efficient levels of asset-side investment I and liability-side arrangements E can support optimal money creation of M, private incentives continue to tilt strongly towards increasing I. Moral hazard therefore cannot be fully eliminated without either correcting the underpricing of public intervention or directly altering banks' investment incentives.

B.5.1 Details on Liquidity Requirement

With our parameter restriction $\beta + \gamma < 1$ which implies $R^M > 1$, consider a liquidity-coverage-ratio-type regulation that requires banks to hold a fraction ω of total deposits M as cash. Effectively, the bank pays $R^{M\prime} = R^M + (R^M - 1)\frac{\omega}{1-\omega} > R^M$ on M, the deposits that are used to finance long-term investment (real investment or storage technology), while simultaneously creating an additional

for exposition below.

[27] For example, if $R^M = 1.02$ (a 2% net interest rate) and banks are required to hold $\omega = 20\%$ of deposits as cash, the requirement is equivalent to the bank paying $R^{M\prime} = 1.025$ (a 2.5% net interest rate) on M while creating an additional deposit of $M' = \omega(M + M') = 0.25M$ backed entirely by cash.

stock of deposits $M' = \omega(M + M') = \frac{\omega}{1-\omega}M$ that it simply hold as cash.

First, note that the additional $M' > 0$ is welfare reducing, holding all other choices equal. Starting from the primitive utility function, with M', the primitive welfare of the household become

$$U = C_0 + \beta\mathbb{E}[C_2] + \gamma M + \gamma M',$$

where $C_0 = Y - I - W - M'$ and

$$C_2 = \begin{cases} f(I) + g(W) + M' & \text{with prob } p \\ \lambda I + g(W - M) + M + M' & \text{with prob } 1 - p \end{cases}.$$

Therefore,

$$U = Y - I - W - M' + \beta\left[pf(I) + pg(W) + (1 - p)\lambda I + (1 - p)g(W - M) + (1 - p)M + M'\right] + \gamma M + \gamma M'.$$

Dropping constants Y and W, and dividing the expression by β (equivalent to multiplying by $R^B = \frac{1}{\beta}$) yields

$$U = -R^B I + pf(I) + pg(W) + (1-p)\lambda I + (1-p)g(W-M) + (1-p)M + \frac{\gamma}{\beta}M + M' + \frac{(\gamma - 1)}{\beta}M',$$

where $\dfrac{\beta + \gamma}{\beta} - 1 = \dfrac{R^B - R^M}{R^M}$. So we can rearrange and get

$$U = pf(I) + (1-p)\lambda I - R^B I + mI(R^B - R^M) + pg(W) + (1-p)[g(W-M) + M] + (\frac{R^B}{R^M} - R^B)M'.$$

So this requirement simply adds an additional term with M' with a coefficient $\frac{R^B}{R^M} - R^B < 0$. Thus, all else equal, any amount of forced cash holdings M' decreases welfare, even if M' were to earn the convenience yield on money which may not necessarily be the case.

Second, consider the bank's problem when facing a higher effective rate on deposits $R^{M\prime} > R^M$. Since the bank has no incentive to hold any cash (it incurs a fixed amount of loss, $R^M - 1$, per unit of deposit held as cash), it will hold exactly the amount required by regulation, $M' = \omega(M + M')$, so its choice of M' is always pinned down by M. The bank's problem and equilibrium conditions therefore remains unchanged except that R^M is now replaced by $R^{M\prime}$. With a higher $R^{M\prime}$, we have

$$\frac{R^B - R^{M\prime}}{R^{M\prime}} = (1 - p)z = (1 - p)(\frac{1 - k}{k}),$$

which implies that the fire-sale loss z falls and the fire-sale price k rises. Then $g'(W - M + L) = \frac{1}{k} = C'(L)$ implies that intervention L and money used to finance long-term investment, M both falls. Lastly, from the bank's FOC w.r.t. I,

$$pf'(I) + (1-p)\lambda - R^B = -(1-p)(1+z-\tau)\phi,$$

we can show that I must fall. Suppose instead I were higher, then the LHS must be lower. Since z has fallen, it must be that ϕ is also higher, which implies $L = \phi I$ must rise, which yields a contradiction with the earlier result that L falls.

Therefore, with a higher R^M, investment choice I is lower – there is less overinvestment. Whether this reduction in overinvestment improves welfare, however, depends on the extent of underpricing in public intervention and on how much money convenience yield is lost through the regulatory requirement.

C Model with Speculative Technology

C.1 Bank's Problem

$$
\begin{aligned}
\max_{m_1\in[0,1], m_2\in[0,1], \alpha, l, \psi, I} \quad & \underbrace{pf(I) + (1-p)\lambda I - m_1 IR^M - (1-m_1)IR^B}_{\text{real investment}} \\
& + \underbrace{\alpha I \cdot (1+ls) - m_2 \alpha IR^M - (1-m_2)\alpha IR^B}_{\text{speculation}} \\
& - p \underbrace{r^E \psi I}_{\text{insurance premium}} + (1-p) \underbrace{\psi I}_{\text{insurance payout}} + (1-p) \underbrace{\phi I}_{\text{LOLR payout}} \\
& - (1-p)\underbrace{z[m_1 IR^M + [m_2 + v(l)]\alpha IR^M - \psi I - \phi I]}_{\text{fire sale}}
\end{aligned}
$$

s.t.

$$m_1 + \alpha[m_2 + v(l)] = m + \alpha v(l) \leq \frac{k\lambda + \psi + \phi}{R^M}$$

$$m_2 \geq \bar{m}$$

where $z = \dfrac{1-k}{k}$ is the net loss on fire-sold assets. The Lagrangian is

$$\mathcal{L} = \text{objective} - \eta \left(m_1 + \alpha[m_2 + v(l)] - \frac{k\lambda + \psi + \phi}{R^M} \right) + \eta_2(m_2 - \bar{m})$$

with Lagrange multipliers $\eta, \eta_2 \geq 0$.

The bank's first order conditions (FOCs) then are as follows:

w.r.t. bank's fraction of real investment financed by deposits, $m_1 \in [0,1]$:

$$I[(R^B - R^M) - (1-p)zR^M] = \eta,$$

where if LHS<RHS, we run into the corner solution that $m_1 = 0$ and the real investments are entirely bond financed.

w.r.t. the bank's choice of leverage on the speculative trade, l:

$$s - v'(l)(1-p)zR^M = \frac{\eta}{I} v'(l),$$

so for each dollar invested, the marginal benefit of increasing leverage, s, is equal to the additional fire-sale costs due to the margin calls and increased shadow costs (if the constraint is binding) caused by the increase in leverage.

w.r.t. α, bank's speculative technology as a fraction of real investment:

$$I[(1 + lS - m_2 R^M - (1 - m_2)R^B - (1 - p)z(m_2 + v(l))R^M] = \eta(m_2 + v(l)),$$

which can be written as

$$1 + lS - R^B + m_2 R^B - m_2 R^M - m_2(1 - p)zR^M - v(l)(1 - p)zR^M = \frac{\eta}{I}(m_2 + v(l))$$

w.r.t. the fraction of speculative technology financed by money, m_2:

$$I[(\alpha R^B - \alpha R^M) - (1 - p)z\alpha R^M] + \eta_2 = \eta\alpha,$$

w.r.t. bank's fraction of real investment covered by private insurance, ψ:

$$pr^E = \frac{\eta}{IR^M} + (1 - p)(1 + z),$$

and w.r.t. bank's real investment, I:

$$\begin{aligned}
&\underbrace{pf'(I) + (1 - p)\lambda - m_1 R^M - (1 - m_1)R^B}_{\text{real investment}} \\
&+ \underbrace{\alpha \cdot (1 + ls) - m_2\alpha R^M - (1 - m_2)\alpha R^B}_{\text{speculation}} \\
-p \underbrace{r^E\psi}_{\text{insurance premium}} &+(1 - p) \underbrace{\psi}_{\text{insurance payout}} +(1 - p) \underbrace{\phi}_{\text{LOLR payout}} \\
&-(1 - p) \underbrace{z[m_1 R^M + [m_2 + v(l)]\alpha R^M - \psi - \phi]}_{\text{fire sale}} = 0
\end{aligned}$$

which implies

$$\begin{aligned}
&pf'(I) + (1 - p)\lambda - R^B \\
= &- \left[m_1(R^B - R^M) + \alpha[1 + ls - m_2 R^M - (1 - m_2)\alpha R^B] - (1 - p)zR^M\,[(m_1 + (m_2 + v(l))\alpha)]\right] \\
&+ pr^E\psi - (1 - p)\psi - (1 - p)z\psi - (1 - p)(1 + z)\phi.
\end{aligned}$$

At the bank's optimal solution, $m_1 = 0$, substituting the early FOCs into the FOC w.r.t. I, we recover

$$pf'(I) + (1 - p)\lambda - R^B = -\frac{\eta}{I}\left[\alpha(m_2 + v(l)) - \frac{\psi}{R^M}\right] - (1 - p)(1 + z)\phi.$$

C.2 Private Investor's Problem

Taking L and k as given, the PI's problem is essentially unchanged from the baseline, except that the total liquidity provided by the PI to the system is now $(1+\theta)M - L$[28]:

$$\max_{(1+\theta)M,E} p\left[g(W) + r^E E\right] + (1-p)\left[g(W - (1+\theta)M + L) + \frac{1}{k}((1+\theta)M - E - L)\right].$$

PI's FOC w.r.t. $(1+\theta)M$, the PI's funds used for fire-sale purchases and private insurance in the crisis state is:

$$g'(W - (1+\theta)M + L) = \frac{1}{k}$$

which equalizes the marginal benefit of investing in the g technology with the forgone return on fire sales $\frac{1}{k}$ in the crisis state at $t = 1$.

PI's FOC w.r.t. private liquidity commitment E:

$$pr^E = (1-p)\frac{1}{k}.$$

C.3 Private Equilibrium

To solve for the private equilibrium, we first note that from agents' FOC w.r.t. private insurance,

$$(1-p)\frac{1}{k} = pr^E = \frac{\eta}{IR^M} + (1-p)(1+z).$$

By definition, $1 + z = \frac{1}{k}$, which implies we must have $\eta = 0$: the constraint on money creation is not binding in an interior solution.

For a given level of fire-sale price k, the levels of investment and central bank's intervention are pinned down by the bank's FOC w.r.t. I and the central bank's FOC w.r.t. ϕ (in the baseline model with out a central bank, $\phi = 0$):

$$pf'(I) + (1-p)\lambda - R^B = -(1-p)(1+z)\phi. \tag{26}$$

$$\frac{1}{k} = C'(L) = C'(\phi I) \tag{27}$$

In the baseline model, with $\eta = 0$, given a level of k and leverage l, the bank's FOC w.r.t. α and m_1 are fairly similar:

$$1 + ls - R^B + m_2 R^B - m_2 R^M - m_2(1-p)zR^M - v(l)(1-p)zR^M = \frac{\eta}{I}(m_2 + v(l)) = 0,$$

[28] With $m_1 = 0$ and $m_2 = \bar{m}$ in equilibrium as shown in C.3, $m = \bar{m}\alpha$ and $\theta = \frac{v(l)}{\bar{m}}$

$$[(R^B - R^M) - (1-p)zR^M] = \frac{\eta}{I} = 0,$$

which implies

$$\frac{1}{m_2}(1 + ls - R^B) + R^B - R^M = [1 + \frac{v(l)}{m_2}](1-p)zR^M,$$

$$(R^B - R^M) = (1-p)zR^M.$$

Note that as long as $(1 + ls - R^B) > v(l)(1-p)zR^M$, it must be that the first FOC holds but LHS<RHS for the second FOC. Therefore, $m_1 = 0$ and $m = m_2\alpha$: money is only created to finance speculation, while the real investment is entirely financed by illiquid bonds.

Moreover, when $m_1 = 0$, from the bank's FOC w.r.t. m_2:

$$I[(\alpha R^B - \alpha R^M) - (1-p)z\alpha R^M] + \eta_2 = \eta\alpha = 0,$$

it must be that $\eta_2 = -\alpha i[(R^B - R^M) - (1-p)zR^M] > 0$ which implies $m_2 = \bar{m}$ if $\alpha > 0$. That is, when where is speculation ($\alpha > 0$), the funding constraint on the speculative technology is binding, as the banks creates as little money as possible to be able to earn more than R^B on the asset side and finance at the cost of R^B.

To pin down k and l, note that the bank's FOC w.r.t. l implies that

$$s - v'(l)(1-p)zR^M = \frac{\eta}{I}v'(l) = 0 \implies \frac{s}{v'(l)} = (1-p)zR^M. \tag{28}$$

Substituting into the bank's FOC w.r.t. α at $m_2 = \bar{m}$ yields:

$$\frac{1}{\bar{m}}(1 + ls - R^B) + R^B - R^M = [1 + \frac{v(l)}{\bar{m}}](1-p)zR^M = [1 + \frac{v(l)}{\bar{m}}]\frac{s}{v'(l)}. \tag{29}$$

Moreover, the PI's FOC w.r.t. M pins down $M = \bar{m}\alpha IR^M$ (since $m_1 = 0$, $m_2 = \bar{m}$) which effectively pins down α:

$$g'(W - (1 + \frac{v(l)}{\bar{m}})M + L) = \frac{1}{k}, \tag{30}$$

Therefore, assuming $(1 + ls - R^B) > v(l)(1-p)zR^M$ which in equilibrium amounts to $(1 + ls - R^B) > v(l)\frac{S}{v'(l)}$, we will always have $m_1 = 0$ and $m_2 = \bar{m}$. We have that l is pinned down by (29), k is pinned down by (28), L is pinned down by (27), M is pinned down by (30), and I is pinned down by (26).

Lastly, the total amount of private insurance E is indeterminate as long as it satisfies

$$(1 + \frac{v(l)}{\bar{m}})M - k\lambda I - L \leq E \leq (1 + \frac{v(l)}{\bar{m}})M - L, \tag{31}$$

where the first inequality requires that the constraint on money creation is not binding and the

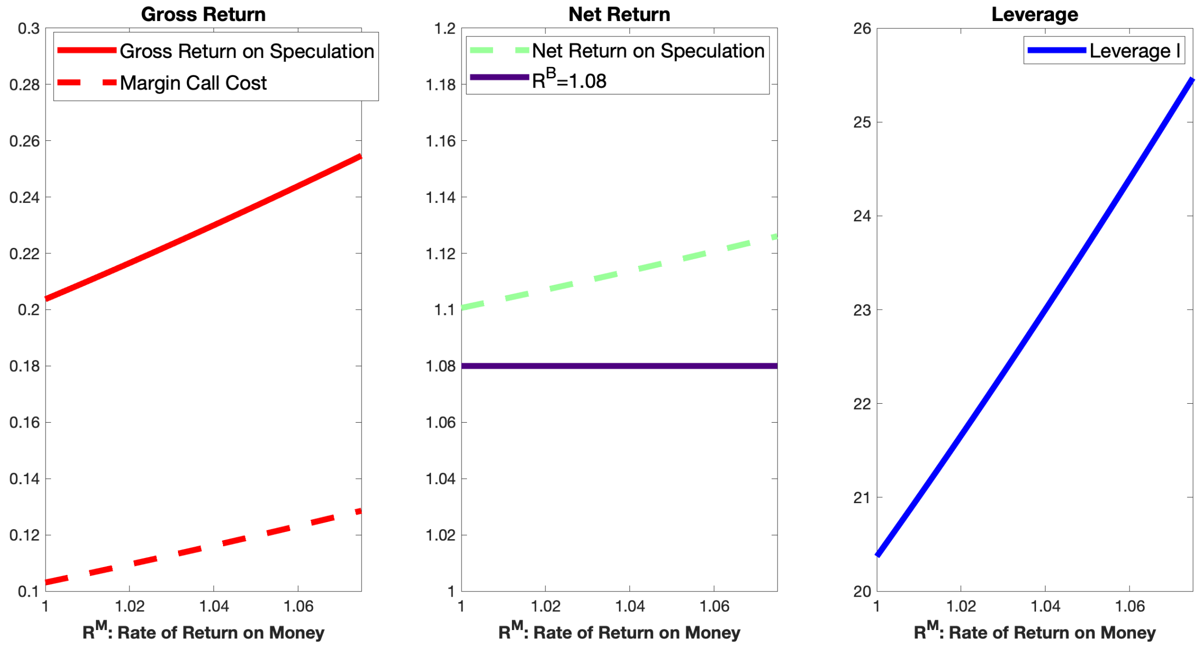

Figure 13: Returns and Leverage Choice from Speculation

second inequality is a natural limit on the amount of private insurance.

C.4 Leverage Choice and Speculation Returns

Figure 13 illustrates the bank's leverage choice and the associated return from speculation under our benchmark specification. As R^M decreases, money financing becomes more lucrative, and the total amount of speculation (part of which must be funded via money) rises accordingly However, the increase in the total amount of speculation and money creation would result in a decline in fire-sale price all else equal, so the bank optimally choose to employ less leverage per dollar of speculative investment. Lower leverage reduces the gross return on speculation but also lowers the fire-sale costs by reducing margin-call liquidity demand. Consequently, the per dollar net return on speculation is lower as leverage falls, yet the total amount invested in speculation rises, as does money creation and the magnitude of margin calls in the crisis state. As shown in Figure 14, introducing speculation lowers fire-sale prices relative to the benchmark model. Moreover, the equilibrium fire-sale price k becomes less sensitive to movements in R^M, reflecting the endogenous reduction of leverage as money becomes cheaper and the speculative scale expands.

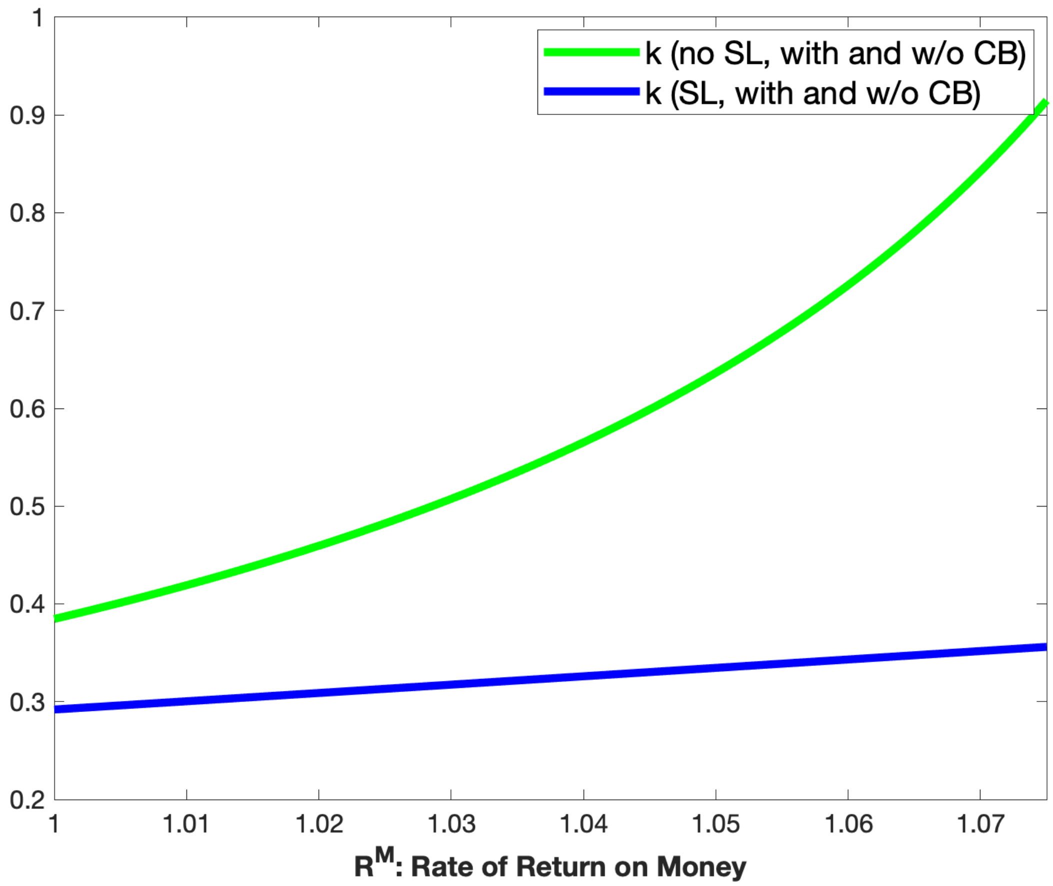

Figure 14: Fire-sale Price k

C.5 Welfare Decomposition

With speculation, the social welfare for the allocations are computed as

$$\underbrace{pf(I)+(1-p)\lambda I-R^B I}_{\text{Real Investment}}+\underbrace{mI(R^B-R^M)}_{\text{Money Premium}}+\underbrace{(1-p)M}_{\text{Money Payout in Crisis}}$$
$$+\underbrace{\left[pg(W)+(1-p)g(W-(1+\theta)M+L)-R^B W\right]}_{\text{PI Investment}}\underbrace{-(1-p)C(L)}_{\text{LOLR Cost}}$$

where $[m_2+v(l)]\alpha IR^M := (1+\theta)M$ represents the total liquidity demand at time 1, to meet both money liabilities and margin calls.

In the baseline model, the welfare loss only comes from real investment (overinvestment and therefore underprovision of private insurance to create an optimal amount of money), as the level of money and LOLR intervention is efficient relative to the planner that can choose L. With speculation added and without a central bank, there is no distortion on time 0 investment, though the lack of L leads to too little money created relative to the planner's choice. Moreover, with speculation and margin calls, less "good money" is created so the society earns less from the premium on demandable money liabilities. After adding a central bank to the speculation model, we can see that

1. Moral hazard becomes more severe than without speculation so there is too much time 0 real investment I.

2. Money premium is extremely low because of speculation (slightly counterbalanced by the central bank),

3. There is too much central bank intervention L, more than the case without speculation.

4. Note that

$$g'(W - (1 + \theta)M + L) = \frac{1}{k} = C'(L)$$

whereas the socially optimal solution satisfies

$$g'(W - M^{social} + L^{social}) = \frac{1}{k^{social}} = C'(L^{social})$$

With $k^{social} > k$, we have that too much central bank intervention $L > L^{social}$, too much liquidity demand $(1 + \theta)M > M^{social}$ but with a large margin call θ, we have that $M < M^{social}$ in our numerical illustrations. Moreover, $M - L > M^{social} - L^{social}$ so there is less invested in the g technology at time 1 in the crisis state relative to the constrained-efficient level, as shown in the welfare decomposition below.

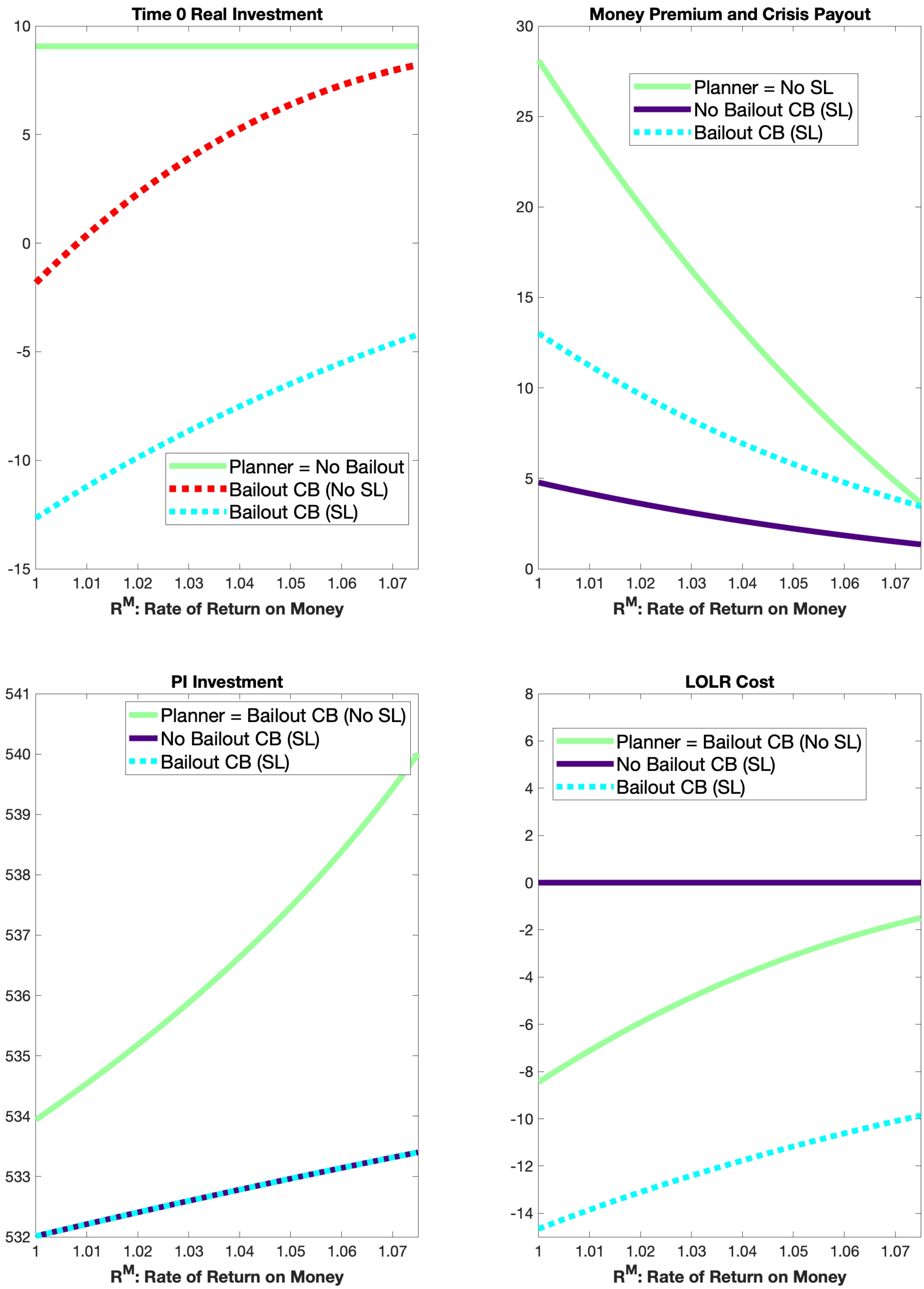

Figure 15: Welfare Decomposition

Conceptually, we can think of having a bailout central bank and having speculative opportunities as both distortionary and reduces welfare. However, the welfare loss from having a bailout central bank in an economy with speculation is not merely the sum of the welfare losses from the two individually, there is an additional loss from the interactions of the two distortions (as seen from the gap between the green line and the blue line). Our interpretation is that the interaction effect mainly comes from the distortions of moral hazard and socially wasteful speculation on real investments.

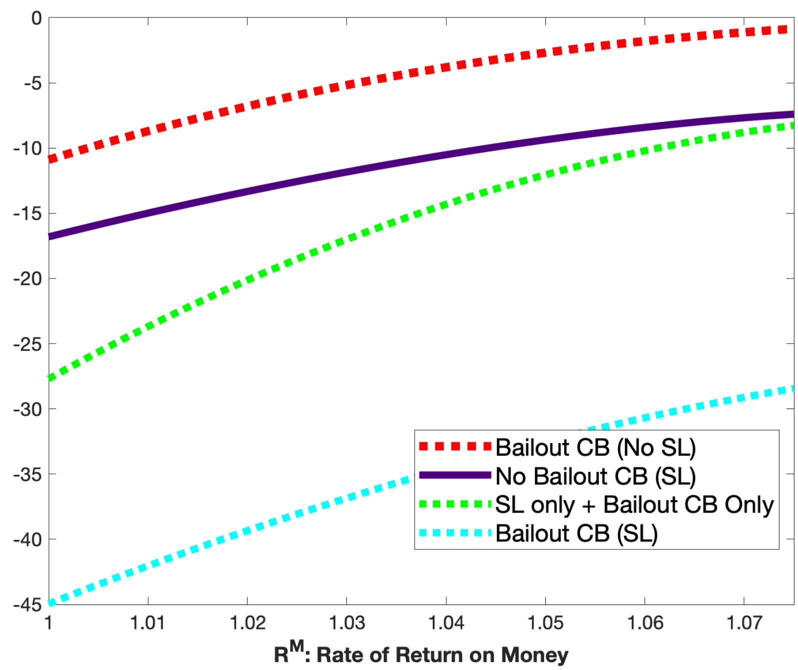

Figure 16: Welfare Loss Relative to Planner's Choice

D Model with Frictions in the Insurance Market

D.1 Private Equilibrium

The bank's problem and FOCs, as well as the equilibrium definition and market clearing conditions, remain the same as in Section 3. However, the private investor's problem becomes

$$\max_{M,E}\; p\left[g(W-E)+h(E)+r^E E\right]+(1-p)\left[g(W-M)+\frac{1}{k}(M-E)\right].$$

PI's FOC w.r.t. M remains (6), but PI's FOC w.r.t. private liquidity commitment E is now:

$$pr^E = p\left[g'(W-E)-h'(E)\right]+(1-p)\frac{1}{k},$$

so that the time-0 expected marginal benefit of providing private insurance (premium r^E in the good state) is equated to the corresponding marginal cost (forgone return of $\frac{1}{k}$ in the bad state and the forgone g investment in the good state when funds are placed in the liquid technology).

From the bank's FOC w.r.t. private insurance (4), and the PI's FOC w.r.t. E above, we can see that when $E > 0$ (interior solution), the insurance market equates the bank's marginal benefit of receiving insurance to the PI's marginal cost of providing insurance, which implies the bank's shadow cost of money creation equals the PI's incremental costs incurred due to limited commitment:

$$\frac{\eta}{R^M I}+(1-p)\frac{1}{k} = pr^E = p\left[g'(W-E)-h'(E)\right]+(1-p)\frac{1}{k} \implies \frac{\eta}{R^M I} = p\left[g'(W-E)-h'(E)\right].$$

The other private equilibrium conditions remain the same as in Section 3, except we do not necessarily have $\eta = 0$ in all cases. In particular,

$$(R^B - R^M) - (1-p)zR^M = \frac{\eta}{I},$$

and investment is pinned down by the bank's FOC w.r.t. I:

$$pf'(I)+(1-p)\lambda - R^B = -\frac{\eta}{I}\left[m-\frac{\psi}{R^M}\right]$$

The total amount of money liability created is pinned down by the PI's FOC w.r.t. M (6):

$$g'(W-M) = \frac{1}{k},$$

Lastly, the money creation constraint $M \leq k\lambda I + E$ must be satisfied, and it must be binding if $\eta > 0$. If $E = 0$, the FOCs w.r.t. E hold with inequality instead.

D.2 Planner's Solution

The planner's objective becomes

$$U = pf(I) + (1-p)\lambda I - R^B I + mI(R^B - R^M)$$
$$+ p\left[g(W-E) + h(E)\right] + (1-p)\left[g(W-M) + M\right].$$

As a result, the planner's FOC w.r.t. ψ and I takes the following forms

$$p\left[g'(W-E) - h'(E)\right] = \frac{\eta^P}{IR^M}, \tag{32}$$

$$pf'(I) + (1-p)\lambda - R^B = -\frac{\eta^P}{I}(m - \frac{\psi}{R^M}), \tag{33}$$

whereas the planner's FOC w.r.t. m still have the same expressions as (11).

D.3 Additional Results

D.3.1 Costs of Limited Commitment

Furthermore, we can prove analytically that higher costs due to limited commitment (e.g., when the marginal return on the liquid technology falls) lead to lower levels of private insurance and a more binding money creation constraint. Proposition 8 shows a comparative statics with $h_1'(\cdot) < h_2'(\cdot)$, so that the first liquid storage technology yields a lower marginal return. which increases the opportunity cost of holding liquid E for the PIs.

Proposition 8. *Consider two economies with limited commitment, $i = 1, 2$ with h_1, h_2 increasing and weakly concave, $h_1'(\cdot) < h_2'(\cdot)$ over the domain of E, and all other parameters and functional forms are identical. Then the equilibrium satisfies $E_1 \leq E_2$. Furthermore, if the constraint is binding, it must be that $E_1 < E_2$ and $\frac{\eta_1}{I_1} > \frac{\eta_2}{I_2}$.*

Proof. See Appendix D.6. □

D.3.2 Bailout Central Bank

To illustrate the effects of public support when private insurance is subject to frictions, we introduce a bailout central bank into the model with limited commitment, using the same setup as in Section 4.2 with $\tau = 0$. Proposition 9 shows that a lower ex-post intervention cost for the central bank reduces private contingent capital, and when this cost is sufficiently low (i.e., when the central bank is more powerful ex post), private insurance is always under-provided.

Proposition 9. *Suppose $C(L) = \frac{c}{2}L^2$ so that $C'(L) = cL$.*

1. *Consider two economies with limited commitment and a bailout central bank, $i = 1, 2$, that differ only in the parameter c with $c_1 < c_2$. Then $E_1 \leq E_2$. Moreover, if the constraint is binding in the equilibrium under c_2,, it must be that $E_1 < E_2$.*

2. *Furthermore, starting from an equilibrium in an economy with limited commitment with $\eta^P > 0$, if c is sufficiently small, the bailout central bank crowds out contingent capital relative to the planner's allocation: $E^* < E^P$.*

Proof. See Appendix D.7. □

The numerical results below parallel the findings from Proposition 7 and 9: there is over-provision of capital insurance in the economy without a central bank, and under-provision of private insurance (an endogenously missing market with $E = 0$) once a bailout central bank is added. Therefore, under the same mechanism that is more cleanly illustrated in Section 4 with a frictionless insurance market, the presence of a bailout central bank continues to crowd out private insurance and remains potentially welfare-reducing.

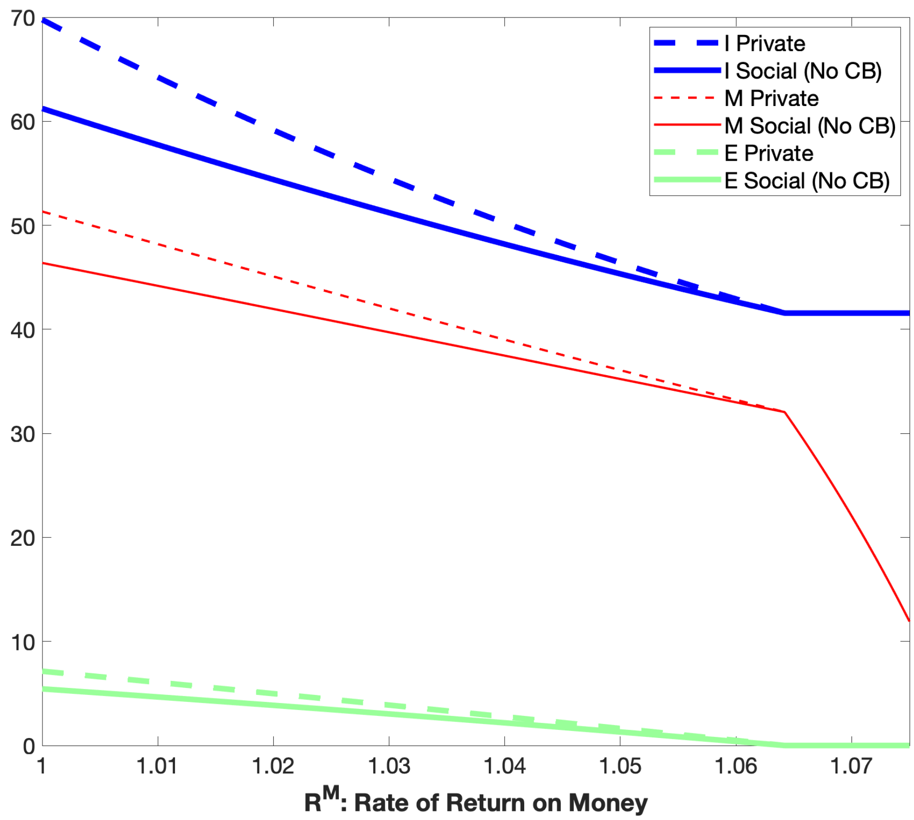

Figure 17: Equilibrium Outcomes in the Model with Limited Commitment, for different levels of return on money R^M

D.4 Baseline Model with Limited Commitment

Using the same parameters as before and assuming that the liquidity technology is a cash technology ($h(E) = E, h'(E) = 1$), the numerical results from Figure 17 confirm the findings from Proposition 7, where the variables with "Social" subscripts denote the allocations from a planner that does not have access to public liquidity L. Notably, there is also over-provision of capital insurance both in aggregate ($E > E^P$) and in terms of the fraction of bank size I insured ($\psi > \psi^P$) where $\psi = \frac{E}{I}$. Importantly, this result will be reversed after adding a bailout central bank.

D.5 Adding a Bailout Central Bank

Figure 18 compares the level of equilibrium allocations under a bailout central bank relative to the planner's allocations. There is substantial overinvestment due to moral hazard engendered by the presence of a bailout central bank, which leads to over-creation of money (in the high spread region) despite underprovision of private insurance (an endogenously missing market where $E = 0$). We emphasize here that there is underprovision of private insurance after a bailout central bank is introduced, which reverses the over-provision result from the private equilibrium in the presence of limited commitment in the private insurance market. Under the same mechanism that is more cleanly illustrated in Section 4 with a frictionless insurance market, the presence of LOLR crowds out private insurance.

Under this parameterization, the level of bailout funds, L, is very close to L from the planner's solution. This can be understood by looking at two opposing forces that affect $L = \phi I$. On the one hand, due to moral hazard, there is substantial overinvestment that raises L in the private

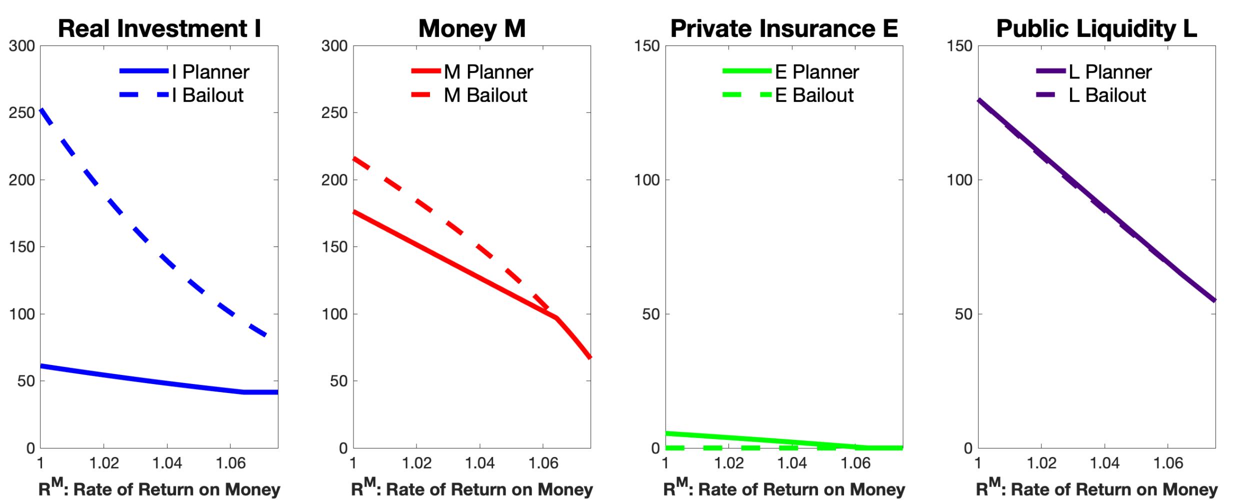

Figure 18: Equilibrium Outcomes with Limited Commitment in the Private Insurance Market and a Bailout Central Bank, for different levels of return on money R^M

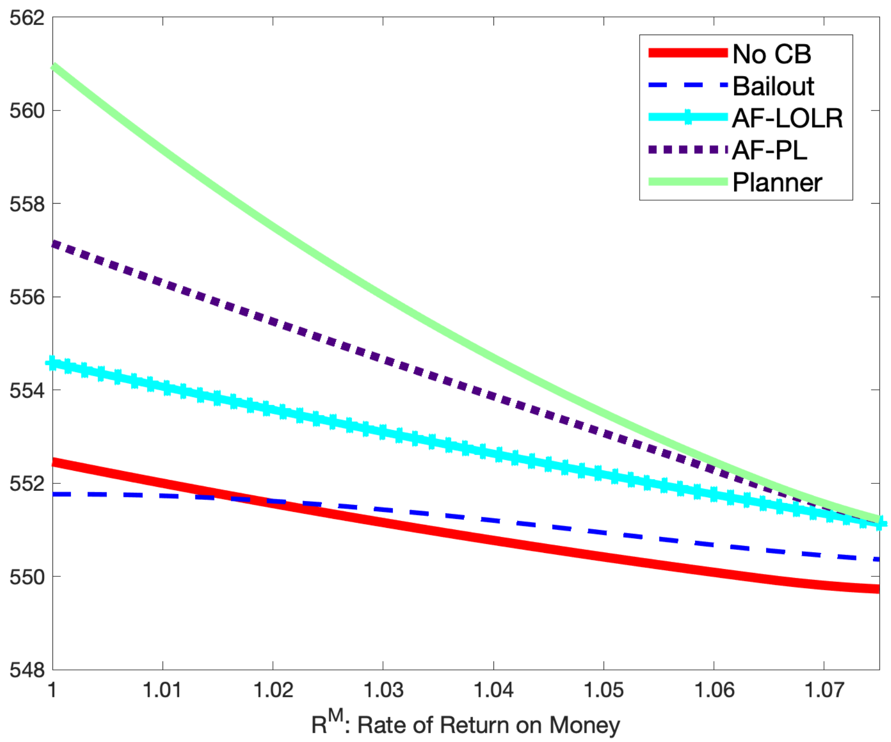

Figure 19: Welfare with Limited Commitment of Private Insurance

equilibrium relative to the planner's allocation. On the other hand, the bailout central bank only acts ex post, so it does not recognize that ex ante, a higher level of ϕ, the fraction of investment covered by public liquidity, also relaxes the money creation constraint of the bank in the high spread region where the constraint is binding. As a result, it under-supplies ϕ relative to the planner's choice. Under our baseline parameters, the two opposing forces happen to almost cancel each other out.[29] Regardless, as shown in Figure 19, in the model with limited commitment, adding a bailout central bank still crowds out private insurance which remains potentially welfare-reducing.

Finally, the welfare results from adding the actuarially fair ex-post LOLR or pre-committed liquidity to the model with limited commitment remain qualitatively similar as the results from the baseline model. The core mechanism in Section 4 that the provision of public liquidity crowds out private liquidity arrangements and leads to overinvestment unless it is judiciously priced remains significant even in the presence of a binding constraint on money creation.

[29]This seems to hold only under when we use $p = 0.95$. It is not a general result in the model with limited commitment that L is also very close to the planner's level.

D.6 Proof of Proposition 8

Proof. Note that since h_1, h_2 are increasing and weakly concave, h_1', h_2' are positive and weakly decreasing.

Case 1: $\eta_1 = \eta_2 = 0$,

$$p[g'(W - E_1) - h_1'(E_1)] = 0 = p[g'(W - E_2) - h_2'(E_2)].$$

If $E_1 > E_2$, $p[g'(W - E_1) - h_2'(E_1)] \geq p[g'(W - E_2) - h_2'(E_2)] = 0 = p[g'(W - E_1) - h_1'(E_1)]$, contradicting with $h_1'(E_1) < h_2'(E_1)$.

Case 2: $\eta_2 = 0$ but $\eta_1 > 0$. Then $p[g'(W - E_2) - h_2'(E_2)] = 0 < p[g'(W - E_1) - h_1'(E_1)]$. Suppose $E_2 < E_1$, then since $\frac{\eta_2}{I_2} < \frac{\eta_1}{I_1}$, it must be that $k_2 < k_1$,which implies $M_2 > M_1$. Since $k_2\lambda I_2 + E_2 > M_2 > M_1 = k_1\lambda I_1 + E_1$, but $k_2 < k_1$ and $E_2 < E_1$,we must have that $I_2 > I_1$. The FOCs w.r.t. I (shown below) then imply that $-\frac{\eta_1}{I_1 R^M}(k_1\lambda) > -\frac{\eta_2}{I_2}(m_2 - \frac{\psi_2}{R^M}) = 0$,contradicting with $0 = \frac{\eta_2}{I_2} < \frac{\eta_1}{I_1}$ and $k_1 > 0$ (which holds under the assumptions on $g(\cdot)$).

$$pf'(I_1) + (1-p)\lambda - R^B = -\frac{\eta_1}{I_1}(m_1 - \frac{\psi_1}{R^M}) = -\frac{\eta_1}{I_1 R^M}(k_1\lambda).$$

$$pf'(I_2) + (1-p)\lambda - R^B = -\frac{\eta_2}{I_2}(m_2 - \frac{\psi_2}{R^M}) = 0.$$

Case 3: $\eta_2 > 0$ but $\eta_1 = 0$. Such case cannot arise in equilibrium. Otherwise, we must have $g'(W - E_2) - h_1'(E_2) > g'(W - E_2) - h_2'(E_2) > g'(W - E_1) - h_1'(E_1) = 0$, which implies $E_2 > E_1$. Furthermore, $\frac{\eta_2}{I_2} > \frac{\eta_1}{I_1}$ implies that $k_2 > k_1$ and therefore $M_2 < M_1$. Then since $\eta_2 > 0$ but $\eta_1 = 0$, we have that $k_2\lambda I_2 + E_2 = M_2 < M_1 < k_1\lambda I_1 + E_1$,so it must be that $I_2 < I_1$. The FOCs below then imply that $-\frac{\eta_2}{I_2 R^M}(k_2\lambda) > -\frac{\eta_1}{I_1}(m_1 - \frac{\psi_1}{R^M}) = 0$,contradicting with $\frac{\eta_2}{I_2} > \frac{\eta_1}{I_1} = 0$ and $k_2 > 0$.

$$pf'(I_1) + (1-p)\lambda - R^B = -\frac{\eta_1}{I_1}(m_1 - \frac{\psi_1}{R^M}) = 0.$$

$$pf'(I_2) + (1-p)\lambda - R^B = -\frac{\eta_2}{I_2}(m_2 - \frac{\psi_2}{R^M}) = -\frac{\eta_2}{I_2 R^M}(k_2\lambda).$$

Case 4: Now consider the case where $\eta_1, \eta_2 > 0$. Suppose, for a contradiction, that $E_2 \leq E_1$. Then $\frac{\eta_2}{I_2} = p[g'(W - E_2) - h_2'(E_2)] \leq p[g'(W - E_1) - h_2'(E_1)] < p[g'(W - E_1) - h_1'(E_1)] = \frac{\eta_1}{I_1}$. Then it must be that $k_2 < k_1$, which implies $M_2 > M_1$. Furthermore, in the constrained region, $k\lambda I + E = M \implies \frac{k\lambda + \psi}{R^M} = m$, so

$$pf'(I_i) + (1-p)\lambda - R^B = -\frac{\eta_i}{I_i}(m_i - \frac{\psi_i}{R^M}) = -\frac{\eta_i}{I_i R^M}(k_i\lambda), \quad i = 1, 2.$$

Since $k_2 < k_1$ and $\frac{\eta_2}{I_2} < \frac{\eta_1}{I_1}$,

$$pf'(I_2) + (1-p)\lambda - R^B = -\frac{\eta_2}{I_2 R^M}(k_2\lambda) > -\frac{\eta_1}{I_1 R^M}(k_1\lambda) = pf'(I_1) + (1-p)\lambda - R^B$$

which implies $I_2 < I_1$. But then we have $M_2 = k_2\lambda I_2 + E_2 < k_1\lambda I_1 + E_1 = M_1$, a contradiction.

Now suppose $E_2 > E_1$ but $\frac{\eta_2}{I_2} \geq \frac{\eta_1}{I_1}$. Then it must be that $k_2 \geq k_1$ and therefore $M_1 \geq M_2$. Following the same logic above, we would have $I_2 \geq I_1$, so $M_2 = k_2\lambda I_2 + E_2 > k_1\lambda I_1 + E_1 = M_1$, a contradiction. □

D.7 Proof of Proposition 9

Part 1: We first consider the case where $\eta_1, \eta_2 > 0$. Suppose, for a contradiction, $\eta_1, \eta_2 > 0$ and $E_1 \geq E_2$. Then since $p\left[g'(W-E) - h'(E)\right] = \frac{\eta}{IR^M}$,where the LHS is increasing in E, it must be that $\frac{\eta_1}{I_1} \geq \frac{\eta_2}{I_2}$. Because $[\frac{R^B-R^M}{R^M} - (1-p)z_I] = [\frac{R^B-R^M}{R^M} - (1-p)(\frac{1}{k_i} - 1)] = \frac{\eta_i}{I_i R^M}$,this implies that $k_1 \geq k_2$, and as a result, $k_1\lambda I_1 + E_1 = M_1 - L_1 \leq M_2 - L_2 = k_2\lambda I_2 + E_2$, which implies $k_1 I_1 \leq k_2 I_2$ and thus $I_1 \leq I_2$. From the bank's FOC w.r.t. I, we have

$$pf'(I_i) + (1-p)\lambda - R^B = -\frac{\eta_i}{I_i}(m_i - \frac{\psi_i}{R^M}) - (1-p)(1+z_i)\phi_i = -\frac{\eta_i}{I_i R^M}(k_i\lambda + \phi_i) - (1-p)\frac{1}{k_i}\phi_i, \quad i = 1, 2.$$

$I_1 \leq I_2$ implies $RHS_1 \geq RHS_2$, so it must be that

$$\frac{\eta_1}{I_1 R^M}(k_1\lambda + \phi_1) + (1-p)\frac{1}{k_1}\phi_1 \leq \frac{\eta_2}{I_2 R^M}(k_2\lambda + \phi_2) + (1-p)\frac{1}{k_2}\phi_2$$

Substituting $\frac{\eta_i}{I_i R^M} = \frac{R^B-R^M}{R^M} - (1-p)(\frac{1}{k_i} - 1) = D - (1-p)\frac{1}{k_i}$ where $D = \frac{R^B-R^M}{R^M} + 1 - p > 0$, we can write this condition as

$$D(k_1\lambda + \phi_1) - (1-p)\lambda - (1-p)\frac{1}{k_1}\phi_1 + (1-p)\frac{1}{k_1}\phi_1 = D(k_1\lambda + \phi_1) - (1-p)\lambda \leq D(k_2\lambda + \phi_2) - (1-p)\lambda$$

which implies it must be that $\phi_1 \leq \phi_2$ (and thus $L_1 \leq L_2$).

Therefore, from $L_i = \phi_i I_i = \frac{1}{c_i k_i} \implies \frac{1}{c_i} = \phi_i k_i I_i$,we have that

$$\frac{1}{c_1} = k_1 I_1 \phi_1 \leq k_2 I_2 \phi_2 = \frac{1}{c_2} \implies c_1 \geq c_2,$$

a contradiction.

If $\eta_1 = 0$ and $\eta_2 > 0$, then since $p\left[g'(W-E) - h'(E)\right] = \frac{\eta}{IR^M}$,where the LHS is increasing in E, it must be that $E_1 < E_2$.

When c_2 is such that both equilibria is unconstrained, the proof in part 2 below shows that $k\lambda I$ is sufficient to cover $M - L$ and therefore the choice of E would be 0 (or more generally, pinned down by the PI's FOC w.r.t. E, which does not differ between $c_1 < c_2$). Therefore, we would have $E_1 = E_2$.

<u>Part 2</u>. With a bailout central bank, the bank's FOC w.r.t. investment becomes

$$pf'(I) + (1-p)\lambda - R^B = \frac{\eta}{I}(\frac{\psi}{R^m} - m) - (1-p)(1+z)\phi$$

Since f is concave, $f'(\cdot)$ is monotone decreasing, so when $\eta = 0$, we have that $I'(\phi) > 0$, which implies that ϕ and I move in the same direction as L. Then consider an unconstrained equilibrium such that $\eta^* = 0$ and therefore k^*follows from $R^B - R^M = (1-p)z^* R^M$. From the central bank's FOC w.r.t. ϕ, we always have

$$C'(L^*) = cL^* = \frac{1}{k^*} = g'(W - M^* + L^*) \implies L^* = \frac{1}{ck^*}, M^* - L^* = W - g'^{-1}(\frac{1}{k^*})$$

so k^* and $M^* - L^*$ are constants given the model parameters and therefore $L^*$is decreasing in c^* and unbounded, which implies that I^* and ϕ^* are also decreasing in c and unbounded. If c is sufficiently low, which produces a sufficiently large L^* and therefore I^* under k^*,then $\eta = 0$ always holds as long as $k^*\lambda I^* > M^* - L^*$. Thus, we have an unconstrained equilibrium under the bailout central bank. In contrast, the planner has $I'(\phi) = 0$ as its choice of I is independent of ϕ so the planner's choice could still be constrained under the same c.

E Private Insurance and Public Liquidity Support as a Fraction of M

Now suppose both private and public insurance are on the amount of money created (payments to depositors), M, rather than on the banks' level of assets. The contingent capital purchased by banks from PIs are in proportion with M.

To make money riskless, in the crisis state, the bank's promised payment to depositors $M = mIR^M$ must be covered by either fire sale of assets or by the private insurance $E = \psi M$ from private investors:

$$\Delta k\lambda I + \psi M = M \implies \Delta k\lambda I = (1-\psi)M = (1-\psi)mIR^M.$$

With private insurance, only $(M - E)$ of bank assets have to be fire sold. If Δ =1 (all assets are fire sold), the bank reaches the upper bound on private money creation, which is

$$M^{max} = \frac{k\lambda I}{1-\psi} \implies m^{max} = \frac{1}{1-\psi}\frac{k\lambda}{R^M}.$$

We assume that the central bank's liquidity injection takes the form $L = \phi M$, so that ϕ becomes the proportion of total M that is insured / covered by central bank. In this case, the money creation constraint simplifies to

$$m \leq \frac{1}{1-\psi-\phi}\frac{k\lambda}{R^M}.$$

The central bank then chooses ϕ, but each bank takes ϕ as given yet recognizes that L is affected by its money creation, M, which can possibly lead to a moral hazard problem if the incentives of money creation M are not appropriately corrected. The presence of this form of central bank intervention could lead to excessive money creation as banks recognize that public insurance also scales up as they increase M.

E.1 Benchmark Model with $L = \phi M$

E.1.1 Private Investor's Problem

Taking L as given, the representative private investor's problem is:

$$\max_{M,E}\ p\left[g(W) + r^E E\right] + (1-p)\left[g(W - M + L) + \frac{1}{k}(M - E - L)\right].$$

PI's FOCs w.r.t. M, the PI's funds used for fire-sale purchases and private insurance in the crisis state, is:

$$g'(W - M + L) = \frac{1}{k}$$

which equalizes the marginal benefit of investing in the g technology with the forgone return on fire sales $\frac{1}{k}$ in the crisis state at $t = 1$.

PI's FOC w.r.t. private liquidity commitment E remains the same:

$$pr^E = (1-p)\frac{1}{k}.$$

The PI's FOCs are largely unchanged.

E.1.2 Bank's Problem

With an LOLR, the additional break-even constraint can be expressed as

$$\lambda I - (1+z)(M-E-L) - \tau L \geq 0 \implies (1-\psi-\phi(1-\tau k))m \leq \frac{k\lambda}{R^M}$$

which, for any τ >0, is tighter than the previous constraint faced by a planner: $[1-\psi-\phi]m \leq \frac{k\lambda}{R^M}$.

Taking ϕ as given, the representative bank's problem is:

$$\max_{m,\psi,I} pf(I) + (1-p)\lambda I - R^B I + \underbrace{mI(R^B - R^M)}_{\text{money spread}} - p \underbrace{r^E\psi M}_{\text{insurance premium}}$$
$$+ (1-p)\underbrace{\psi M}_{\text{insurance payout}} + (1-p)\underbrace{\phi M}_{\text{LOLR payout}} - (1-p)\underbrace{z[mIR^M - \psi M - \phi M]}_{\text{fire sale}} - (1-p)\underbrace{\tau\phi M}_{\text{payment to LOLR}}$$

such that

$$[1-\psi-\phi(1-\tau k)]m \leq [1-\psi-\phi(1-\tau k)]m^{max} = \frac{k\lambda}{R^M}$$

with Lagrange multiplier on the break-even constraint $\eta \geq 0$.

Note that the bank's objective also changes, as in expectation it now has to pay back at $t = 2$ $\tau\phi M$ on the LOLR funding if there is a crisis. But as long as $\tau < 1 + z$, the bank will receive the benefit of avoiding fire-sale costs, so there is moral hazard.

with respect to (w.r.t.) bank's fraction of real investment financed by deposits, m:

$$I[(R^B - R^M) - (1-p)zR^M - pr^E\psi R^M + (1-p)(1+z)\psi R^M + (1-p)(1+z-\tau)\phi R^M] = \eta[1-\psi-\phi],$$

w.r.t. bank's fraction of money covered by private insurance, ψ:

$$pr^E = \frac{\eta}{IR^M} + (1-p)(1+z),$$

w.r.t. I (substituting in $\eta = 0$ as the constraint still does not bind):

$$
\begin{aligned}
& pf'(I) + (1-p)\lambda - R^B \\
= \quad & -m[(R^B - R^M) - (1-p)zR^M - pr^E\psi R^M + (1-p)(1+z)\psi R^M(1-p)(1+z-\tau)\phi R^M] \\
& = -\tfrac{\eta}{I} m[1 - \psi - \phi] = 0
\end{aligned}
$$

In equilibrium with $\eta = 0$, the FOC w.r.t. M implies that

$$
(R^B - R^M) + (1-p)(1+z-\tau)\phi R^M = (1-p)zR^M.
$$

The first FOC shows that with central bank intervention, the private marginal benefit of money creation increases due to underpricing of the intervention: as long as $\tau < 1 + z$, we would have a larger z relative to the benchmark case where $R^B - R^M = (1-p)zR^M$. Then since $g'(W - M + L) = \frac{1}{k} = C'(L)$, we have that both private money $M - L$ and public support L will be higher than the outcomes in the private equilibrium. There is further incentive to over-create money as z becomes higher and thus k becomes lower, so total money M would also be higher relative to both the private equilibrium and the $L = \phi I$ case.

However, in this $L = \phi M$ specification, investment is undistorted when the constraint isn't binding, as there is no additional wedge in the bank's FOC w.r.t. I. The moral hazard effect operates purely on the liability side. If we introduce private frictions in the private insurance market so that the constraint binds, distortions reappear: the bank has incentives to overinvest not only because of the fire-sale externality in incomplete markets, but also because underpriced central bank interventions amplifies the gains from overinvestment.

Our benchmark highlights a distinct mechanism: even holding $M - L$ fixed (both money and intervention at the efficient levels), the mix of money creation via I versus E becomes distorted due to moral hazard on bank size or investment. While our benchmark illustrates distortions on quantities (composition of M), this extension illustrates how intervention could possibly distorts prices and the level of money creation.

E.2 Limited Commitment Model with $L = \phi M$

E.2.1 Private Investor's Problem with Limited Commitment

The PI's problem is:

$$
\max_{M,E} \; p\left[g(W-E) + h(E) + r^E E\right] + (1-p)\left[g(W-M+L) + \frac{1}{k}(M-E-L)\right].
$$

PI's FOC w.r.t. M:

$$
g'(W - M + L) = \frac{1}{k}
$$

PI's FOC w.r.t. private liquidity commitment E is now:

$$pr^E = p\left[g'(W-E) - h'(E)\right] + (1-p)\frac{1}{k}.$$

E.2.2 Private Equilibrium with a Bailout Central Bank

The bank's problem remains the same except that the constraint could be binding. A bailout central bank's FOC w.r.t. ϕ continues to be $g'(W-M+L) = C'(L)$. As in the limited commitment model in the main text, the private agents' FOC w.r.t. private insurance implies that the Lagrangian multiplier on the constraint is equal to the PI's commitment costs: $\frac{\eta}{IR^M} = p\left[g'(W-E) - h'(E)\right]$.

E.2.3 Planner's Problem

The planner's problem is:

$$\begin{aligned}\max_{I,m,\psi,\phi} U =& pf(I) + (1-p)\lambda I - R^B I + mI(R^B - R^M) \\ &+ p\left[g(W-E) + h(E)\right] + (1-p)\left[g(W-M+L) + M\right] - (1-p)C(L).\end{aligned}$$

s.t.

$$[1-\psi-\phi]m \leq= \frac{k\lambda}{R^M}$$

As a result, the planner's FOC w.r.t. m, ϕ,ψ and I takes the following forms, taking as given $g'(W-M+L) = \frac{1}{k}$.

FOC w.r.t. ϕ:

$$(1-p)g'(W-M+L) + \frac{\eta^P}{IR^M}[1+\Omega(M,I,L)] = (1-p)C'(L)$$

where $\Omega(M,I,L) = -\frac{g''(W-M+L)}{(g'(W-M+L))^2}\lambda I$.

FOC w.r.t. ψ:

$$p\left[g'(W-E) - h'(E)\right] = \frac{\eta^P}{IR^M}.$$

FOC w.r.t. m (after simplification):

$$\frac{R^B - R^M}{R^M} - (1-p)z = \frac{\eta^P}{IR^M}[1+\Omega(M,I,L)].$$

FOC w.r.t. I (after simplification):

$$pf'(I) + (1-p)\lambda - R^B = -\frac{\eta^P}{I}(m[1-\phi-\psi]).$$

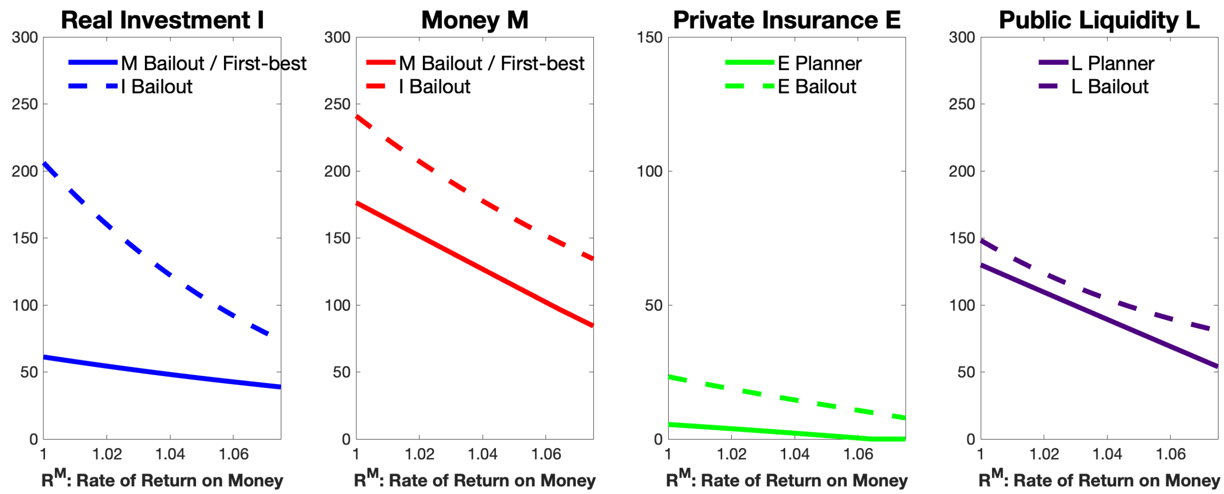

Figure 20: Equilibrium Outcomes with Limited Commitment in the Private Insurance Market and a Bailout Central Bank that intervenes with $L = \phi M$, for different levels of return on money R^M

E.2.4 Numerical Illustration

As shown in Figure 20, when the constraint becomes binding in the limited commitment model, the bank has incentives to exploit the underpriced intervention by over-creating money. This can occur through both excessive investment and excessive purchases of private insurance, as in the benchmark with $L = \phi M$. As in the benchmark model with $L = \phi M$, the bank is incentivized not only to overinvest but also to over-create money and over-purchase insurance, anticipating intervention. As a consequence, the central bank over-intervenes in equilibrium, which further depresses the fire-sale price.

Furthermore, in this extension, we also find that the presence of a bailout central bank amplifies overinvestment as the money-bond spread increases. This effect is not merely a manifestation of the standard Stein mechanism where overinvestment arises because a binding constraint distorts value in incomplete markets, but is further magnified by the underpriced intervention. Indeed, we verify that a lower ex-post intervention cost c leads to more severe overinvestment, more excessive money creation and intervention. Thus, the qualitative conclusions from the baseline model with $L = \phi I$ carry over: higher spreads induce excessive investment even in the presence of frictions.