Sovereign Debt and Economic Growth when Government is Myopic and Self-interested[†]

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Abstract

We examine how a sovereign's ability to borrow abroad affects the country's growth and steady state consumption, assuming that the government is both myopic and self-interested. Surprisingly, government myopia can increase a country's access to external borrowing. In turn, access to borrowing can extend the government's effective horizon as the government's ability to borrow hinges on it convincing creditors they will be repaid, which gives it a stake in incentivizing private production and savings despite its self-interest. In a high-saving country, the lengthening of the government's effective horizon can incentivize it to tax less, resulting in a "growth boost", with higher steady-state household consumption than if it could not borrow. However, in a country that saves little, the government may engage in more repressive policies to enhance its debt capacity and spending. This could lead to a "growth trap" where household steady-state consumption is lower than if the government had no access to external borrowing. We discuss the effectiveness of alternative debt policies, including multilateral institutions declaring the sovereign's debt odious, debt relief, and debt ceilings.

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1 Introduction

Is the ability to borrow in international markets good for a country, especially a developing one? Many theories of international borrowing emphasize the better risk-sharing a country can achieve. In case of an economic or natural calamity, it can borrow to smooth consumption. It can also draw on international savings to finance domestic growth (see, for example, Kletzer and Wright [2000]). Yet it is hard empirically to see a positive correlation between a developing country's use of foreign financing and good outcomes such as stronger economic growth (see Aizenman, Pinto and Radziwill [2004], Prasad et al. [2006], and Gourinchas and Jeanne [2013]). What might explain the divergence between theory and evidence?

One limitation with many existing models is that they tend to assume that the government of the country in question maximizes the utility of its citizenry over the long run. Yet an important reality in many developing countries is that their governments often are myopic (have short horizons) and are self-interested (spend wastefully in ways that do not benefit citizens). Whether poverty adversely affects governance, or whether poor governance entrenches poverty is unclear.

A second limitation is related to the first. Once the government is assumed to maximize the welfare of its citizenry, often the best thing it can do is to default on its foreign debt (see, for example, Bulow and Rogoff [1989*a*], Bulow and Rogoff [1989*b*], and Tomz [2012]). To explain the existence of sovereign debt, researchers then have to appeal to a variety of mechanisms that enforce sovereign repayment such as a government's concern for its reputation or the possibility of punishment strategies by creditors. While theoretically interesting, there is little empirical evidence for these mechanisms (in particular, see Eichengreen [1987], Özler [1993], Flandreau and Zumer [2004], Sandleris et al. [2004], Arellano [2008], Panizza, Sturzenegger and Zettelmeyer [2009]).

We examine the desirability of sovereign debt in a model that addresses these limitations. We consider a country with a representative household each period – the household is a composite of households and the productive private sector, and we will use these terms interchangeably. The other agents in the model are the government and international investors.

The household has an initial endowment (smaller if a developing country) that it can either consume, save by buying government bonds, or invest in private enterprise. It maximizes the sum of its consumption this period and the discounted endowment left behind for the next generation, a proxy for the future stream of its descendants' consumption.

The country's government rules only for one period, and thus has a short horizon. It is assumed to spend in ways that do not enhance citizen welfare.[1] The government maximizes

[1]These include, for instance, wasteful populist spending (such as election propaganda), white elephant

the resources it can raise for spending, which consist of the sum of the taxes it levies on private sector real output and the amount it can raise through debt issuance (net of repayment of past debt). Importantly, the government does not tax household savings in government debt, leaving the burden of taxation on easily-identifiable real private investment.

Government debt is short-term and issued to both domestic households and foreign investors.[2] Successor governments inherit the obligation to repay sovereign debt, though they can default. If the government defaults on past debt, it pays the default cost (we elaborate shortly) and cannot issue new debt for the rest of the period. International lenders do not care about the quality of government spending, but will lend only if they expect to get their money back with interest. Therefore, given the model has no uncertainty, there will be no over-lending and no default in equilibrium. This allows us to highlight the central tradeoffs.

Following a recent set of papers, we assume the government cannot default selectively on foreign debt holders. This would be true if it issued bearer bonds or if foreign debt holders could sell out to domestic holders as default became more likely. [3]

We assume the default costs rise in the size of sovereign bonds held by domestic households, due for example to the political price the government pays for hurting influential citizens. So the government does not default on sovereign debt for two reasons. First, it will incur the default cost immediately. Second, it has a short horizon, so it does not trade off the cost of default against the present value of the outstanding debt, but instead only against the net debt repayments it has to make in its period in power. This implies that a sizeable amount of debt stock can be supported with modest costs of default.

Our central focus is on how access to sovereign borrowing, and foreign borrowing in particular, affects the government's tax policy and thus steady state outcomes. Absent government borrowing (what we term "debt autarky"), the myopic government will set the tax rate on private output at the level that trades off the disincentivizing effect of a higher tax rate on private investment against its direct positive impact on government revenues (the "Laffer curve" maximizing level). However, the myopic government's access to borrowing alters the tax it wants to impose on the household sector today, for the tax alters how much it can borrow today. First, a higher tax pushes more of the household's current endowment into financial savings (which are not taxed). This raises the default costs on government borrowing, raises the borrowing amount the government can credibly repay, and hence its ability to borrow. However, a higher

projects (such as presidential palaces or gigantic power plants that are not economic to run), or plain theft (luxury flats in Miami or London or Cayman Island bank accounts).

[2] All our results are robust to allowing for longer-maturity debt.

[3] See Broner, Martin and Ventura [2010], Bolton and Jeanne [2011], Acharya and Rajan [2013], Gennaioli, Martin and Rossi [2014], Acharya, Drechsler and Schnabl [2014], Broner and Ventura [2016], Andrade and Chhaochharia [2018], and Farhi and Tirole [2018], for modeling and applications of this assumption.

tax rate today can also lower the private sector's endowment next period, lowering successor governments' resources to repay debt. This hurts the current government's ability to borrow.

The ability to borrow therefore gives even a myopic government a stake in the country's future, beyond the horizon it is in power. Furthermore, depending on parameters and the relative size of the effects just described, access to government borrowing can worsen or improve steady state outcomes – even for a myopic, self-interested government, access to foreign borrowing is not an unmitigated blessing or curse, it depends.

More specifically, for a country with a high propensity to save among domestic households, access to foreign borrowing can effectively increase the government's horizon and reduce its oppressive taxation. Intuitively, the government's debt capacity is not increased by raising taxes and forcing more savings into its domestic bonds, but instead it is increased by reducing taxes and increasing the ability and willingness of future governments to repay. The lower taxation induced by access to borrowing enhances steady-state consumption relative to debt autarky, *i.e.*, there is a "growth boost".

Conversely, for a poor country with low starting endowment and a low propensity to save among the citizenry, the government may set higher-than-autarky tax rates. This could push the country into a lower consumption "growth trap", precisely because each myopic government represses in order to enhance its debt issuance, in the process leaving the next period government also with a low-endowment economy that is heavily indebted so that the repression gets entrenched *ad infinitum*. For the citizens of such countries, the government's access to borrowing is truly odious. A high enough initial endowment, however, mitigates government repression, allowing the country to escape the trap. Interestingly, while governments in our model are all myopic and self-interested, poor country governments are intrinsically more repressive because their environment incentivizes them to be so.

We also examine outcomes when a government only has access to domestic debt and compare them to ones when it has access to foreign borrowing also. Growth traps can emerge in both situations but over a wider parameter range without access to foreign debt, and growth boosts appear only with access to foreign debt.

The existing literature has sometimes assumed government myopia, but rarely self-interest also at the same time. Our stark assumptions, though not implausible, are quite distant from the usual (and equally stark) assumption that the government has a long term and public interest perspective. Our assumptions do allow sovereign borrowing to be justified with relatively small default costs. Moreover, the model has interesting implications. For instance, net debt service is more important in determining defaults than the stock of debt, suggesting more defaults when global interest rates rise. Also, a moderate restructuring of the time profile of debt payments may be enough to get a government to be compliant with debt payments, large scale debt write downs may be unnecessary.

The model helps shed light on other issues. For instance, a number of papers (see Aizenman, Pinto and Radziwill [2004], Prasad et al. [2006], and Gourinchas and Jeanne [2013]) have documented a puzzling weak or negative correlation between a developing country's growth and its reliance on foreign borrowing. Our model offers a potential explanation for this phenomenon, which is in the spirit of discussion in Aguiar and Amador [2011] and Gourinchas and Jeanne [2013], that there can be an *endogenous* selection of which countries rely more on foreign borrowing, rather than some direct adverse effect of foreign borrowing on country growth and development.

Separately, a literature on "odious" debt (see Buchheit, Gulati and Thompson [2006], Jayachandran and Kremer [2006] and Sander [2009]) takes the view that allowing access to external debt gives a self-interested government more resources to waste or steal, with the repayment eventually extracted by international lenders from the citizens. Therefore, some commentators advocate declaring debt issued by such governments odious and limiting the enforcement of such debt in international courts. While we have little to say on brutal governments that hurt their citizenry or invade neighbors, we do emphasize the possibility that access to borrowing will affect even the myopic self-interested government's incentives and behavior, sometimes favorably. External debt need not be odious even if the government is.

We discuss the effect of policies such as debt relief and debt ceilings on the welfare of the citizenry. Typically, modest debt relief in our model will do little for a country's citizens even if it is in a growth trap. The current government will simply use the expanded space to borrow, and spend the amounts raised quickly. It will soon be back to pre-relief levels of debt – experience suggests this was not an idle concern with the debt relief measures undertaken in developing countries in the late 1990s and early 2000s. In contrast to the ineffectiveness of debt relief on its own, debt relief can be very effective in enhancing a country's growth when coupled with debt ceilings that limit borrowing by the government (either through a constitutional debt ceiling or informal limits agreed to by all creditors). Of course, for countries where access to debt boosts growth, binding debt ceilings will hurt country welfare.

Finally, we examine the effects of shocks. Despite the fact that government defaults are costly by design in our model, we observe that countries in a growth trap can at times benefit from default caused by unanticipated shocks. Because growth is suppressed by the government's repressive policies intended to boost borrowing, a significant one-period growth spurt can arise from the economy entering debt autarky post default (see Levy-Yeyati and Panizza [2011] for empirical evidence on sovereign default and subsequent growth). In some cases, the spurt can be such that the economy escapes the growth trap.

Our paper builds on Acharya and Rajan [2013], who present a two-period (three-date) model of sovereign debt with a myopic wasteful government. Their model does not permit them to examine long-run or steady-state equilbria, nor do they address the choice between consumption, investment, and savings by the household sector. Our model enables us to examine dynamics and steady states, wherein lie the key results of our paper; for instance, that governments can have an incentive to lower taxes to boost growth is specific to our dynamic analysis. Our paper is also related to Basu [2009], Bolton and Jeanne [2011], and Gennaioli, Martin and Rossi [2014], who also tie the costs of sovereign default to the amount of debt held by domestic banks. They examine the trade-offs between more credible sovereign borrowing (when domestic banks hold more sovereign bonds) against the greater costs when the sovereign defaults. A version of this trade-off is also in our model, but our fundamental assumption – of myopic self-interested governments – is different from these papers and our focus is on how access to sovereign borrowing can alter long-run growth.

On this last point, our paper is related to Aguiar, Amador and Gopinath [2009] and Aguiar and Amador [2011] who also examine theoretically the relation between (foreign) sovereign borrowing and long-run growth. Their models vary the extent of government myopia in the presence of limited commitment and show that sufficiently high myopia can result in an inefficient steady-state outcome or in slow convergence to the steady state. Relatedly, Aguiar, Amador and Fourakis [2020] calibrate a range of related models to quantify the welfare costs of access to sovereign debt. In contrast, we consider a myopic but wasteful government throughout, and examine the effect of obtaining access to foreign debt, domestic debt, or being shut out from borrowing.

The rest of the paper is as follows. In Section 2, we discuss the baseline model and the main Bellman equation capturing the model dynamics. In Section 3, we present an in-depth analysis of steady states and explain how a growth trap or growth boost arises, as well as discuss its policy implications. In Section 4, we consider model robustness and extensions. In Section 5, we analyze the effectiveness of debt ceilings and debt relief. In Section 6, we discuss the impact of unanticipated shocks to the economy in the steady state and derive further policy implications. Finally, we offer concluding remarks and possible future extensions in Section 7.

2 Baseline Model

We consider an overlapping generations model with a country and the rest of the world. The country is a small open economy with two agents, the private sector and the government. Time is discrete and the horizon is infinite. A period represents the life of the government.

The private sector is a representative household and firm, combining both consumption and private production. It maximizes the sum of the log of current period consumption c_i and the log of next period endowment e_{i+1} (which is left for the next generation) times a parameter ρ, where $\rho \in (0, \frac{1}{r})$ captures the household's propensity to save/leave bequests, where $r > 0$ is the world interest rate. At the beginning of the period i, the household inherits an endowment e_i, consisting of the previous period after-tax household production as well as the gross returns from financial savings, which it allocates to consumption c_i, financial savings s_i, and physical investment k_i so as to maximize utility. Physical investment produces $f(k_i)$ at the end of the period, where $f' > 0$ and $f'' < 0$.

The government in our model is incumbent for only one period. It is myopic in that its sole objective is to maximize its spending over the period, and self-interested in that spending does not directly augment the economy's endowment or private consumption. The assumption on myopia is in the spirit of Alesina and Tabellini [1990] that politicians discount the future at a greater rate than does the citizenry. The self-interested spending could be on itself (high government salaries or corruption), on grandiose white elephant projects, or on political propaganda.[4] We could also include populist spending that is visible but does not enhance consumer utiliy much (for example, circuses). The government finances the spending by imposing a tax on the private sector, as well as issuing debt which is sold to both domestic and foreign investors. The government can tax the production at a rate t_i , with proceeds $t_i f(k_i)$; the net proceeds for the household from production is therefore $(1-t_i)f(k_i)$.

The government can borrow by issuing debt which we assume is short-term, *i.e.*, it matures next period, and pays the required world interest rate of r. Nothing hinges on the short-term nature of debt as we show by allowing the issuance of long-term debt in section 4.1. Foreign investors invest in the country's sovereign debt as well as its private sector's debt. We assume the government cannot default selectively on foreign debt holders, which would be true if it issued bearer bonds or if foreign debt holders could sell out to domestic holders as default became more likely. All we really need, however, is that a default on external sovereign debt spills over to domestic debt. This is hardwired in the model by assuming the two forms of debt are indistinguishable, but there are a variety of other sources of spillover that could be invoked. For instance, in Sandleris [2010], even public defaults on only foreign-held debt lead to domestic output losses because they send a negative signal about the state of the economy.[5]

[4] Scholl [2017] and Chatterjee and Eyigungor [2019] also consider private benefits to myopic governments in models that feature uncertainty and political turnover (spending can affect election outcomes) to derive dynamics leading up to a sovereign debt crisis. In contrast, our model has no uncertainty and therefore no default in equilibrium. Our focus is on how long-run endowment is affected by access to debt, domestic and foreign.

[5] There is other evidence consistent with such spillovers. Borensztein and Panizza [2009] show that public defaults are associated with banking crises; Brutti [2011] finds more financially dependent sectors tend to grow relatively less after sovereign default; De Paoli, Hoggarth and Saporta [2009] show that sovereign default is associated with substantial output costs for the domestic economy; Arteta and Hale [2008] use firm-level data to show that syndicated lending by foreign banks to domestic firms declines after default; Ağca and Celasun [2012] also use firm-level data to show the corporate borrowing costs increase after default.

While the household can save abroad, we assume that it has a mild home bias so financial savings, if s_i is positive, are invested in domestic government bonds at the rate r (rather than internationally) whenever the government borrows. In other words, when the government borrows internationally, the supply of bonds is equal to or greater than the domestic savings. We focus throughout on this case as we are interested in understanding how a myopic self-interested government's access to foreign borrowing affects economic growth relative to no foreign borrowing. Note also that if s_i is negative, the household borrows from abroad.

We also assume the private household's financial savings into government debt are not taxed (equivalently, savings in government debt are taxed at a lower rate than household investment in real assets). This is a key assumption. Consider three justifications. First, fixed hard assets are easier to tax than fungible financial savings. Since financial savings are more mobile and also easily converted to concealable assets like gold, the government typically keeps taxes on financial savings relatively low. Second, we have in mind here both actual taxes as well as the implicit taxes the government collects through corruption, which usually fall more heavily on business enterprise. Third and most important, needy governments tend to direct flows toward themselves through *financial repression*. For instance, capital controls are deployed to ensure that domestic savings do not leave the economy, financial institutions like banks are required to allocate a significant part of their assets to government debt, and tax breaks are provided to domestic investors for the earnings on government bond holdings, potentially crowding out the private sector's access to finance (effectively a tax).[6] For simplicity, we do not model any of these effects, assuming they are fully captured by the tax falling only on real investment. It should be kept in mind, though, that real repression (high taxes on private sector real investment) and financial repression (guiding financial savings into government instruments) are two instruments – possibly employed together in practice – for the government to achieve the same objective at the expense of the private sector.[7]

If the government defaults, the economy's infrastructure incurs direct damage – for instance, banks holding government debt are "run" upon, the payment system freezes, and repo markets collateralized by government debt are disrupted. To ensure the private sector produces this period (and can be taxed) the government has to commit a part of its spending to a mopping-up cost of default which we model as $C + zD^{Dom}$, where $C > 0$ captures a fixed cost of default, D^{dom} is the face value of government debt held by the domestic residents at the time of de-

[6] Gennaioli, Martin and Rossi [2018] find that there is a negative and statistically significant correlation between a bank's holding of domestic government bonds and its loans-to-assets, especially in developing countries.

[7] Reinhart, Kirkegaard and Sbrancia [2011], Reinhart [2012], Reinhart and Sbrancia [2015], and Chari, Dovis and Kehoe [2020] look at financial repression as a way to ease the repayment burden for a country. Roubini and Sala-i Martin [1992] model financial repression as a way to raise "easy" resources for the public budget when tax evasion by the private sector is high.

fault, and $z > 0$ is a default cost parameter which measures the domestic financial sector's use of sovereign debt in transactions (for example, its value as safe assets in collateralizing transactions or its presence in bank portfolios). Parameter z could also be thought of as a measure of the financial sector's sophistication or development.[8] In addition to incurring the default cost, the government is excluded post default from debt markets for the rest of its term – this could be thought of as the time debt is being renegotiated (Panizza, Sturzenegger and Zettelmeyer [2009] find this to be about 4 years in defaults after 1991, typically the term of an elected government). The defaulting government thus experiences "debt autarky" with no access to the sovereign debt market. We assume that investors – both domestic and foreign – are fully rational and are therefore willing to lend to the government only to the extent that the debt will be fully repaid in the next period.

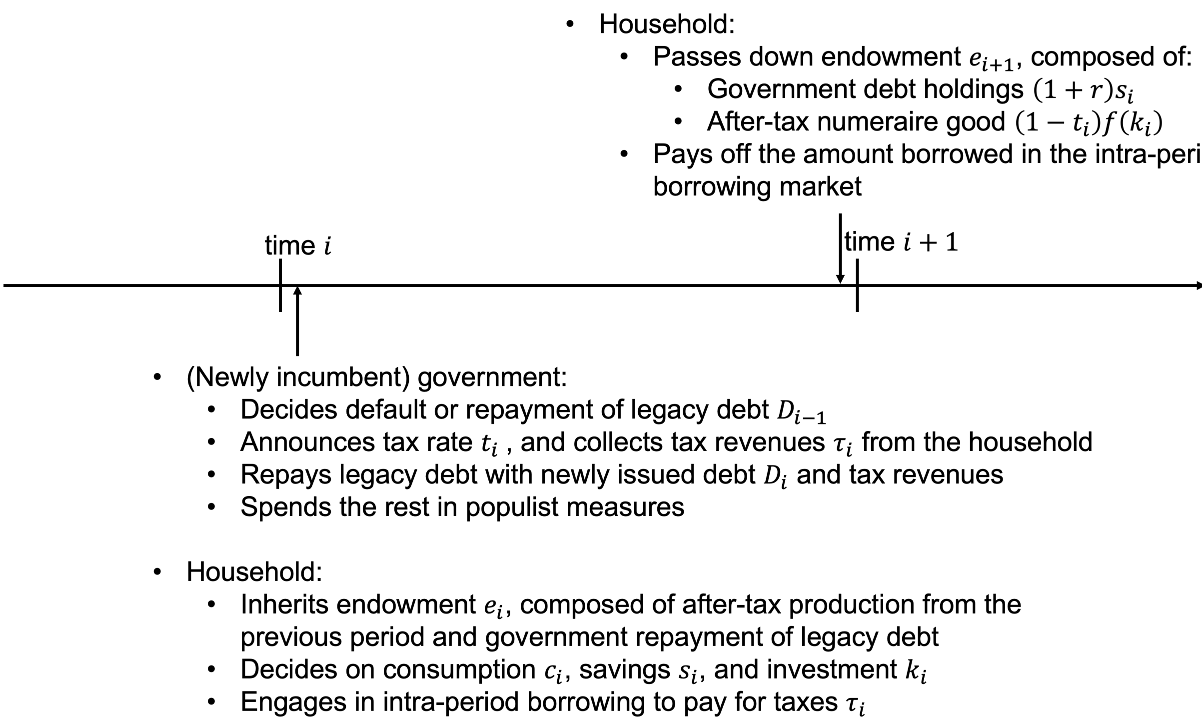

Figure 1: Timeline of the model.

To keep matters simple, we assume the government makes all decisions and takes all actions at the beginning of the period. The government decides whether to repay past debt and what tax rate to set. It uses both the proceeds of new debt issued as well as taxation to repay old debt. Since the household receives taxable income from productive investment only at the end

[8] Because household savings s can be negative in our model when initial endowment is low but productivity of capital is high, we need a high enough C to ensure that the default cost itself never becomes negative.

of the period, we assume it borrows from the international market within the period to pay taxes in advance (and this borrowing is repaid out of production revenues before the period ends). We assume only debt held between periods accrues interest. These assumptions save us from keeping separate track of old sovereign debt paid from tax revenues and old sovereign debt paid from borrowing. It changes nothing material in the model. The timeline of the model is shown in Fig. 1.

2.1 Household problem

Start with the household's problem in period i. The representative household receives an endowment e_i from the past generation, and takes the tax rate t_i as given. Its problem can be summarized as the following constrained optimization:

$$\max_{c_i,e_{i+1},k_i,s_i} \quad \ln c_i + \rho \ln e_{i+1} \tag{2.1}$$

$$s.t. \quad c_i + s_i + k_i \leq e_i, \text{ and} \tag{2.2}$$

$$e_{i+1} \leq (1+r)s_i + (1-t_i)f(k_i). \tag{2.3}$$

Setting λ and μ as the Lagrangian multipliers for constraints in (2.2) and (2.3), respectively, the first order conditions (FOC's) for our four choice variables yields:

$$c_i: \quad 0 = \frac{1}{c_i} + \lambda; \tag{2.4}$$

$$s_i: \quad 0 = \lambda - (1+r)\mu; \tag{2.5}$$

$$k_i: \quad 0 = \lambda - (1-t_i)f'(k_i)\mu; \text{ and} \tag{2.6}$$

$$e_{i+1}: \quad 0 = \frac{\rho}{e_{i+1}} + \mu. \tag{2.7}$$

It is easily seen (see Lemma C.1 in the appendix) that FOC's (2.4) - (2.7) lead to the following set of decision functions for the households:

$$k_i = f'^{-1}\left(\frac{1+r}{1-t_i}\right), \tag{2.8}$$

$$c_i = \kappa_0[(1+r)(e_i - k_i) + (1-t_i)f(k_i)], \tag{2.9}$$

$$e_{i+1} = \kappa_1[(1+r)(e_i - k_i) + (1-t_i)f(k_i)], \text{ and} \tag{2.10}$$

$$s_i = \kappa_1(e_i - k_i) - \kappa_0(1-t_i)f(k_i); \text{ where} \tag{2.11}$$

$$\kappa_0 := \frac{1}{(1+\rho)(1+r)}; \text{ and } \kappa_1 := \frac{\rho}{1+\rho}.$$

Remark 2.1. We discuss some properties of the solutions (2.8) - (2.11):

1. The household's physical investment is a function of the exogenous interest rate and the government-set tax rate only (see (2.8)). So the total amount of tax collected by the government is $tf(k(t))$, a function of t. We denote this function as $\tau(t)$.
2. Note from (2.9) and (2.10) that $\forall i$, $c_i = \frac{1}{\rho(1+r)} e_{i+1}$. This implies that there is a one-to-one relationship between the level of endowment and consumption in the model.
3. Note from (2.10) that the next-period endowment depends on the current-period endowment linearly with a coefficient $\kappa_1(1+r)$. In order to rule out exploding economies, we impose a condition that $\kappa_1(1+r) < 1 \Leftrightarrow \rho < 1/r$.
4. Note in (2.11) that household financial savings are increasing in the tax rate t (because investment is decreasing in the tax rate from (2.8)).[9]

2.2 Government problem: debt autarky

Let us turn now to the government's problem. The government decides whether to service legacy debt, sets the tax rate, and issues the maximum new debt consistent with these decisions, while expecting the household to react according to (2.8)–(2.11). The benchmark case is one where the government cannot issue any debt (so the household's financial savings are invested abroad). Since this government can only spend what it raises from tax, it will simply choose a tax rate that maximizes tax revenues $\tau(t)$. Let $**$ denote this benchmark "debt autarky" case:

$$\begin{aligned}
t^{**} &:= \text{benchmark tax rate} = \operatorname*{argmax}_t \tau(t), \\
k^{**} &:= \text{benchmark investment} = k(t^{**}), \text{ and} \\
\tau^{**} &:= \text{benchmark tax revenue} = \tau(t^{**}) = t^{**} f(k^{**}).
\end{aligned}$$

For instance, in the case of a power production function $f(k) = Ak^\gamma$, $t^{**} = 1-\gamma$.

2.3 Optimization problem of myopic government with debt

Consider now the government's problem when it can borrow. We will denote the face value of debt borrowed in period i as D_i. The government has legacy debt payment $(1+r)D_{i-1}$ due, of

[9] Under the log-utility assumption for households, investment declines and savings increase with the tax rate t; in other words, real and financial repression map one-for-one in this case. With a more general utility function for households, the impact of the tax rate on savings would depend on the elasticity of inter-temporal substitution. In this case, the government may have to employ financial repression explicitly (forced savings), in addition to economic repression (taxation), to channel savings to its bonds.

which $(1+r)D_{i-1}^{Dom}$ is to domestic investors. Suppose for now that the government finds default suboptimal and decides to pay back the legacy debt. It finances its spending by issuing debt D_i and collecting taxes from the private sector at rate t_i. Suppose that the next government's "spendable", the maximum resource that it can raise through taxation and borrowing, is S_{i+1}. Debt issuance D_i today is then constrained by the next-period government's *ability to pay*:

$$D_i(1+r) \leq S_{i+1}. \tag{2.12}$$

Consider now the next-period government's *willingness to pay*. In the event that the next-period government defaults, its tax revenues are at the autarky level τ^{**}. It follows that in order for the next-period government to be willing to pay, the amount it can spend if it doesn't default should be more than τ^{**} minus the spending to clean up the post-default financial disruption:

$$\begin{aligned}
&\underbrace{S_{i+1} - D_i(1+r)}_{\text{net spending on no default}} \geq \underbrace{\tau^{**}}_{\text{revenues in autarky}} - \underbrace{(C + zD_i^{Dom}(1+r))}_{\text{spending to clean up default}} && (2.13)\\
\Rightarrow &D_i(1+r) \leq S_{i+1} + zD_i^{Dom}(1+r) + C - \tau^{**}\\
\Rightarrow &D_i(1+r) \leq S_{i+1} + zs_i(1+r) + C - \tau^{**}. && \text{(in equlibrium)}
\end{aligned}$$

Since both the ability-to-pay constraint as well as the willingness-to-pay constraint must be met, the effective constraint on current-period debt is

$$\begin{aligned}
&D_i(1+r) \leq \min\{S_{i+1}, S_{i+1} + zs_i(1+r) + C - \tau^{**}\}\\
\Rightarrow &D_i(1+r) \leq S_{i+1} - \max\{0, \tau^{**} - C - zs_i(1+r)\}. && (2.14)
\end{aligned}$$

It can be seen that $\tau^{**} - C - zs_i(1+r) = 0$ traces the threshold between willingness-to-pay and ability-to-pay constraint; when $\tau^{**} - C - zs_i(1+r)$ is positive, the willingness-to-pay constraint is binding, whereas when it is negative, the ability-to-pay constraint is binding.

Notice also from (2.11) that s_i increases linearly in e_i. This implies that for sufficiently high endowments, $\tau^{**} - C - zs_i(1+r) < 0$, implying that the ability-to-pay constraint is binding. Conversely, for sufficiently low levels of endowment, the willingness-to-pay constraint is binding. Two interesting elements of our formulation can thus already be seen: First, the ability to issue debt effectively elongates the myopic government's horizon, and second, the government's myopia can make debt more easily sustainable.

2.3.1 How debt elongates the government's horizon

Constraint (2.14) highlights the double-edged nature of sovereign debt that is at the heart of our model. On the one hand, if the willingness-to-pay constraint is binding, D_i increases in financial savingss_i, which incentivizes the myopic government to repress real private investment with higher taxation in order to boost financial savings in government debt. On the other hand, when focusing on the next-period government's available resources to pay debt (whether the ability-to-pay constraint is binding or not), it turns out that D_i increases in S_{i+1}, which increases in e_{i+1}. From this perspective, the current-period government has an incentive to increase next-period endowment by lowering taxation and boosting real investment. As we show in the following sections, these differing incentives mean the government can under-tax or over-tax relative to our benchmark case, which is the debt autarky optimum ($\operatorname{argmax}_t tf(k(t))$). What it will do depends on which of the two incentives is stronger. If S_{i+1} is more sensitive to current-period taxation than the penalty term $\max\{0, \tau^{**} - C - zs_i(1+r)\}$, then the myopic government will choose a lower-than-benchmark tax rate, otherwise it will choose a higher-than-benchmark tax rate. Furthermore, the current-period government sees

$$\text{spending} = S_i - \text{legacy debt} = \max_t \left[D_i + \tau(t) - D_{i-1}(1+r)\right], \tag{2.15}$$

and the debt capacity D_i implicitly depends on the tax rate also via its dependence on S_{i+1} and/or s_i. Therefore, the problem is inherently infite-horizon, even though the myopic government only optimizes a one-period problem. This is why debt is potentially a horizon-lengthening device.[10]

2.3.2 How the government's short horizon affects debt sustainability

Conversely, let us rewrite the willingness-to-pay condition (2.13) after substituting $S_{i+1} = D_{i+1} + \tau(t_{i+1})$. We get

$$\underbrace{\left(C + zD_i^{Dom}(1+r)\right)}_{\text{spending to clean up default}} \geq \underbrace{D_i(1+r) - D_{i+1}}_{\text{benefit from avoiding net debt service}} + \underbrace{\tau^{**} - \tau(t_{i+1})}_{\text{benefit from increased tax revenues}} \tag{2.16}$$

Essentially the government's short horizon means that even though it can default on the entire stock of debt that is built up, the benefit it sees is only the avoided debt service over its short horizon (with debt in steady state so that $D_i = D_{i+1}$, this amounts to just the interest on

[10]Note that other long-term assets, even those not issued by the government, can help elongate its horizon, albeit more obliquely. For instance, if the government charges capital gains taxes, it has an incentive to care about the current value of equity, which depends on growth outcomes beyond the government's horizon.

debt) and the increase in tax revenues when default eliminates any restraint on taxation. Put differently, the default costs do not need to be high enough to exceed the benefits of not paying the outstanding stock of debt. The latter would require default costs to be implausibly high (see, for example, the discussion in Panizza, Sturzenegger and Zettelmeyer [2009]). Instead, for a short-horizon government to continue servicing its debt, the cost of default only needs to outweigh the flow benefits of default over a single period (see also Bulow and Rogoff [1989*b*]).

2.4 Recursive formulation of the government's problem

Let us formulate the government's problem recursively. Note that a myopic government i takes e_i, D^{Dom}_{i-1}, and D_{i-1} as given, and maximizes (2.15). This implies that the natural set of state variables is $(e_i, D^{Dom}_{i-1}, D_{i-1})$; however, since legacy debt D_{i-1} enters (2.15) only additively, the maximization problem is independent of D_{i-1}. Moreover, D^{Dom}_{i-1} only governs the government's decision to default or not. Therefore, conditional on the government finding default suboptimal, the only state variable is economy's endowment e_i. Furthermore, since a myopic government will always maximize $D_i + \tau(t)$, we can replace D_i with the expression in (2.14). Note that since the maximum is derived from the no-default condition for the next government, there will be no government defaults in our model on the equilibrium path. Therefore, we have:

Lemma 2.1. *(Main Bellman equation) The government's value function is*

$$S(e) = \max_t \quad \left[\frac{1}{1+r}\big[S(e') - \max\{0, \tau^{**} - C - zs(1+r)\}\big] + \tau(t)\right] \tag{2.17}$$

$$s.t. \quad e' = \kappa_1\big[(1+r)(e-k(t)) + (1-t)f(k(t))\big], \tag{2.18}$$

$$s = \kappa_1(e-k(t)) - \kappa_0(1-t)f(k(t)), \textit{ and} \tag{2.19}$$

$$k(t) = f'^{-1}\Big(\frac{1+r}{1-t}\Big). \tag{2.20}$$

The value function $S(e)$, as well as the policy function $t(e)$, i.e., the taxation rule conditional on the myopic government finding default suboptimal, constitute the complete solution for (2.17), which is sufficient for the no-default equilibrium path.

The decision rule encompassing (off-equilibrium) default can be obtained by revisiting the two constraints, (2.12) and (2.13); for given endowment e, legacy domestic debt D^{Dom}_{-1} (the face value of which is $(1+r)D^{Dom}_{-1}$), and legacy total debt D_{-1} (the face value of which is $(1+r)D_{-1}$),

1. *If $S(e) - (1+r)D_{-1} < 0$, the government cannot pay back the legacy debt and defaults. Upon default, it enters autarky and charges the autarkic tax rate t^{**}.*
2. *If $S(e) - (1+r)D_{-1} < \tau^{**} - C - z(1+r)D^{Dom}_{-1}$, the government potentially can pay back the*

legacy debt, but finds defaulting more advantageous. In other words it defaults strategically, enters autarky, and charges the autarkic tax rate t^{**}.

3. *If neither of the above two conditions apply, then the government pays back the legacy debt, charges tax $t(e)$ and issues $D(e) := S(e) - \tau(t(e))$ amount of debt. Government spending is $S(e) - (1+r)D_{-1}$.*

Finally, note that the debt issuance $D(e)$ can be further decomposed into $D^{Dom} = s(e, t(e))$ and $D^{For} = D(e) - s(e, t(e))$. Formally characterizing the solution of the Bellman equation requires a set of regularity conditions set out in Definition 2.1, imposed mainly to ensure convexity and single-crossing properties of the derived functions. Any power production function of the form $f(k) = Ak^{\gamma}$ automatically meets regularity conditions A and B below, and therefore will be used in all our numerical exercises throughout (as in Fig. 2). All proofs are in appendix C.

Definition 2.1. We assume that the following regularity conditions are met:

A. (Convexity of investment in t) $k(t)$ is decreasing and convex in t, from which it follows that private profit $\pi(t)$ is also decreasing and convex in t.

B. (Single-crossing properties) $\frac{k'(t)}{\pi'(t)}$ is decreasing in t, and $\frac{\tau'(t)}{\pi'(t)}$ is strictly increasing in t.

C. (Minimal government feasibility in autarky) $\tau^{**} > C$.

We can then derive the following result concerning the value and the policy functions:

Proposition 2.2. *There is a unique bounded and weakly monotonic value function $S(e)$, and a corresponding policy function $t(e)$, that solve (2.17),with the following properties:*

1. *$S(e)$ is weakly concave, and $S'(e) \to 0$ as $e \to \infty$.*
2. *$\exists \hat{e}^1 \leq \hat{e}^2$ such that for $e < \hat{e}^1$, only the willingness-to-pay constraint binds; for $e > \hat{e}^2$, only the ability-to-pay constraint binds; and, for $e \in [\hat{e}^1, \hat{e}^2]$, both constraints bind.*
3. *$t(e)$ is continuous, (weakly) increasing in the region $e \in [0, \hat{e}^1]$, (weakly) decreasing in the region $[\hat{e}^1, \hat{e}^2]$, and (weakly) increasing in the region $[\hat{e}^2, \infty)$. Also, $t(e) \to t^{**}$ as $e \to \infty$.*

We provide numerical examples that solve the problem of a given period's government for different endowments and help understand the proposition.

2.5 Numerical Examples

Example I. Fig. 2 shows a solution from the model specialized to $f = 3k^{.65}$, $r = 10\%$, $z = 4$, $\rho = 2.3$, and $C = 1$. The solution in this case possesses the following properties:

(i) There exists a low-e region (see Fig. 2, regions annotated “WTP”) where only the willingness-to-pay constraint is binding. In this region, the future government’s ability to pay exceeds its willingness to pay. The government gains debt capacity by pushing default costs up, that is, with high repressive taxes that channel incremental household endowments entirely into savings in government bonds.

(ii) There exists a middle-e region (see Fig. 2, regions annotated “WTP & ATP”) where the optimal solution for the government is to “slide” between the two constraints, *i.e.*, setting $\tau^{**} - C - z(1+r)s = 0$. In this region, the tax rate policy $t(e)$ is always strictly decreasing in e (see Fig. 2(b)). Essentially, the government channels incremental endowment into household investment (see Fig. 2(d)) by lowering taxes, which increases the household’s future endowment and the future government’s ability to pay. Marginal household productivity is high enough that the current government’s borrowing capacity increases more than the foregone taxes. Household financial savings (see Fig. 2(c)) are constant so the incremental borrowing is all foreign. The limit of this process is reached when household productivity falls enough at high enough investment that incremental reductions in the tax rate do not incentivize enough production and borrowing capacity to offset the loss in tax revenues. The limiting lower bound for the tax rate turns out to be the autarkic tax rate.

(iii) There exists a high-e region (see Fig. 2, regions annotated “ATP”) where only the ability-to-pay constraint is binding. Large-endowment economies have so much domestic savings that strategic default is ruled out. However, when the willingness-to-pay constraint is not binding, the size of the government’s future surplus and its ability to borrow today does not vary with the endowment (see Fig. 2(e)). The reason is interesting. In this region, government debt capacity rises by less than the loss of tax revenues when taxes are lowered below the autarkic rate. So the government fixes taxes at the autarkic rate, which does not vary with endowment. Consequently, household investment is commensurately fixed, and all incremental endowment goes into household financial savings, which crowds out government foreign borrowing but does not add to overall government borrowing. In sum, a myopic government with a wealthy household sector taxes as if it has no access to debt.

[Fig. 2 about here]

Example II. It turns out that the tax rates in WTP and WTP & ATP region need not always be higher than the autarkic tax rate. Figure 3 shows the solution properties for a different case, which arises for instance for parameters $f = 3k^{.65}$, $r = 1\%$, $z = 1.1$, $\rho = 3.1$, and $C = 1$, where default costs are lower and the household propensity to save is higher compared to the previous example. In Fig. 3(b), we see that the government charges a tax rate lower than the autarkic rate $(1-\gamma = 0.35)$. This is because boosting private sector growth is in the myopic government's incentive, as doing so increases its debt capacity by increasing the next government's willingness to pay. In particular, as the household savings rate is high, future savings can be boosted effectively by raising future endowments, i.e., by promoting growth today.[11] As can be seen in Fig. 3(e), the amount of debt that a government can borrow is a sharply increasing function of endowment, until the willingness-to-pay constraint eases off and the ability-to-pay constraint kicks in. When this happens, the government starts charging tax rates closer to the autarkic tax rates, because its tax policies have little effect on the amount of debt it can borrow.

[Fig. 3 about here]

It turns out that these differing cases – whether governments choose higher or lower than autarkic tax rates in the willingness-to-pay region – can lead to differing steady states for the economy, which we study next.

3 Steady States and their Properties

Consider a planner whose utility is the discounted sum of each generation's utility. Let this utility be denoted as $U(\{c_i\}_{i=0}^{\infty}, \{e_i\}_{i=0}^{\infty}; \beta)$, where β denotes the planner's discount rate. It follows that, for such a planner with arbitarily long horizon ($\beta \to 1$), the ordering of steady states governs the ordering of the planner's utility. Let us now characterize steady states and the path towards them. We first need some definitions regarding the growth path:

Definition 3.1. Given the solution program $t(e)$ from the Bellman equation (2.17) and the private sector reaction function (2.18)–(2.20), we define

- An endowment path $\{e_i\}_{i=0}^{\infty}$ as $e_{i+1} := e_+(e_i, t(e_i))$ starting at e_0. In addition, we define $e_\infty(e_0)$ as the limit (if it exists) of this endowment path: $e_\infty(e_0) := \lim_{i\to\infty} e_i$.
- Steady state (e^{ss}, t^{ss}) as a pair satisfying[12]

$$t^{ss} = t(e^{ss}), \text{ and} \tag{3.1}$$

$$e^{ss} = e \text{ such that } e = e_+(e, t^{ss}). \tag{3.2}$$

- As discussed earlier, consumption at the steady state $c^{ss} = \frac{1}{\rho(1+r)}e^{ss}$.

[11]Interestingly, in the boost case, actions by the international community to improve enforcement of external sovereign debt may eliminate the willingness-to-pay constraint, and make the country worse off.

[12]In addition, a no-saddle-point condition is imposed as follows: $\exists\ \epsilon > 0$ such that for all $e \in (e^{ss} - \epsilon, e^{ss} + \epsilon)$, $e_\infty(e) = e^{ss}$. This excludes the measure-zero set of fixed-point endowments on which a small shock can push the endowment path away from the fixed point in the long run.

From Proposition 2.2, it must be the case that e^{ss} is in (i) the willingness-to-pay constraint region; or, (ii) the ability-to-pay constraint region; or, (iii) the "sliding" region. We derive the necessary conditions for the steady state should one or more exist in each of the three regions.

Suppose first that e^{ss} exists in the willingness-to-pay constraint region (region (i)). We note that using the envelope condition ($\frac{dS}{dt}\cdot\frac{dt}{de} = 0$) as well as the definition $e^{ss} = e_+(e^{ss}, t^{ss})$, we can get the exact $\frac{dS}{de}$ at this point:

$$\begin{aligned}\frac{dS}{de} &= \kappa_1 \frac{dS}{de} + z\kappa_1 \\ \Rightarrow \frac{dS}{de} &= z\frac{\kappa_1}{1-\kappa_1} = \rho z.\end{aligned} \tag{3.3}$$

In words, when the willingness-to-pay constraint binds, an increase in current endowment increases the current government's spendable, both by increasing future endowment, which increases future spendable and current borrowing capacity, as well as increasing current household financial savings (which increases the government's ability to borrow directly). Also, the optimal t should satisfy the FOC:

$$\frac{1}{1+r}\left[\frac{de_+}{dt}\frac{dS}{de} + z(1+r)\frac{ds}{dt}\right] + \tau' = 0. \tag{3.4}$$

Plugging (3.3) into (3.4), we get the following characteristic equation:

$$\frac{de_+}{dt}\underbrace{\frac{dS}{de}}_{=\rho z} + z(1+r)\frac{ds}{dt} + (1+r)\tau' = 0. \tag{3.5}$$

The first term in (3.5) is negative because greater taxation shrinks the amount the household allocates to productive investment, reducing production and growth, the household's future endowment, and hence what the future government can spend. The second term is positive because greater taxation increases the amount devoted to domestic financial savings (because of repression), and hence enhances the government's willingness to pay and its ability to borrow. The third term is the effect of taxation directly on tax revenues.

It is straightforward to see that (3.5) is independent of e. Therefore, it follows that if such a steady state were to exist, the tax rate t^{ss} can be completely characterized from the model primitives, which we define as t^W. Then, the corresponding endowment e^{ss} can be derived simply by solving $e^W = e_+(e^W, t^W)$. We denote this as **steady state W**. So it is possible that the optimal tax rate, t^W, can be greater than the autarkic tax rate t^{**} if the government's incentive to repress dominates its incentive to foster growth, or smaller if the reverse is true. We offer an in-depth discussion of this in Proposition 3.1.

Next, suppose that e^{ss} exists in region (ii), the ability-to-pay constraint region. The corresponding envelope condition and the FOC yield respectively

$$\frac{dS}{de} = \kappa_1 \frac{dS}{de} \Rightarrow \frac{dS}{de} = 0 \tag{3.6}$$

In the ability-to-pay region, therefore, taking taxation as constant, an increase in household endowment has no effect on the government's ability to spend. Incremental household endowment simply goes into consumption and household financial savings (because household investment is fully determined by the tax rate). Financial savings do not change the government's ability to borrow in this region.

$$\frac{de_+}{dt} \underbrace{\frac{dS}{de}}_{=0} + (1+r)\tau' = 0. \tag{3.7}$$

Following the same logic as for case (i), it follows that, if such a steady state were to exist, the tax rate t^{ss} must be equal to $t^A = \text{argmax}_t\, \tau = t^{**}$. Again, e^{ss} in this region can be derived by solving $e^A = e_+(e^A, t^{**})$. Note that the steady-state taxation will be set at the debt autarky level, even though the government will be borrowing. We denote this as **steady state A**, which achieves the same endowment as the benchmark autarky case.

Finally, suppose that e^{ss} exists in region (iii). Since it is sliding between the constraints, and because it is a steady state, the following must be simultaneously met:

$$e = e_+(e,t), \text{ and} \tag{3.8}$$

$$0 = \tau^{**} - C - z(1+r)s(e,t). \tag{3.9}$$

We refer to the solution (e^S, t^S) for (3.8)-(3.9) as **steady state S**.

In Appendix B, we formally characterize the three steady states A, W, and S, and argue why the limit of any endowment path must be one of them. We also discuss the conditions under which each of the steady states can exist. Importantly, when multiple steady states exist, the limit of an endowment path depends on the initial endowment; in particular, endowment paths starting from lower endowments converge to a lower steady state than those starting from higher endowments. This is the core reason why growth traps exist in our model. More surprisingly, there can also be growth boosts as we will see shortly.

We now turn to the central result of the paper, *i.e.*, whether access to international borrowing helps or hurts a country when its government is myopic and self-interested. We use the notation

$\{e_n^{**}\}_{n=0}^{\infty}$ where $e_{n+1}^{**} = e_+(e_n^{**}, t^{**})$ and the corresponding steady state as e_{∞}^{**}.[13]

Proposition 3.1. *Access to sovereign borrowing can lead the government to set steady-state taxation at levels that are below or above the benchmark. Steady-state endowments and consumption vary correspondingly. Specifically :*

- *Suppose that $t^{**} < t^W$. Then, $e_{\infty}(e_0)$ is in general not independent of e_0, and $e_{\infty}(e_0) \leq e_{\infty}^{**}$ always. In particular, for a set of parameters of strictly positive measure, $\exists \bar{\bar{e}}$ such that*
  - *$\forall e_0 < \bar{\bar{e}}$, $e_{\infty}(e_0) < e_{\infty}^{**}$ **(Growth Trap)**, and*
  - *$\forall e_0 \geq \bar{\bar{e}}$, $e_{\infty}(e_0) = e_{\infty}^{**}$ **(Benchmark)**.*
- *Suppose instead that $t^{**} \geq t^W$. Then, $e_{\infty}(e_0)$ is independent of e_0 and $e_{\infty}(e_0) \geq e_{\infty}^{**}$ always. Depending on the parameter set,*
  - *e_{∞} is either equal to e_{∞}^{**} **(Benchmark)**, or*
  - *e_{∞} is strictly greater than e_{∞}^{**} **(Growth Boost)**.*

In Appendix B Lemma B.2, we also characterize equilibrium quantities of government debt and its composition as well as of government spending in these steady states.

In order to graphically illustrate these growth dynamics for a myopic and self-interested government that can borrow internationally, we show in Fig. 4 the simulated endowment paths for three different sets of parameters. In Fig. 4(a), both steady states A and W exist. Therefore, the long-run or steady-state endowments depend on the initial endowment. Indeed, it can be observed that economies starting at sufficiently low endowments may never escape the lower endowment steady state. The willingness-to-pay constraint will always be binding, with high repressive taxation. This leads the economy to a growth trap. In fact, the growth in endowment can be negative as seen in Fig. 4(a) for some starting endowments, so that economies end up poorer because of government repression at the trap steady state. However, if the economy were to start at a higher endowment, then the willingness-to-pay constraint is never binding, and the economy converges to the "better" steady state. Put differently, government behavior can be more rapacious in poor economies, precisely because households have so little, and not because of any cultural propensity to be rapacious.

In the case of Fig. 4(b), only steady state A exists and there is no growth trap. Therefore, all economies eventually converge to the benchmark steady state. Obviously, poorer economies take longer to reach there.

Finally, in Fig. 4(c), only steady state W exists, but in this case the willingness-to-pay constraint incentivizes government to keep taxes low. This allows it to enhance future private

[13] We exclude measure zero events as even a small perturbation would remove the possibility of their existence.

endowments, the future government's spendable, and thus its own borrowing today, more than it can raise its borrowing by raising taxes and forcing more financial savings. The equilibrium tax rate is smaller than that of the benchmark case ($t^W < t^{**}$). Borrowing acts as a growth boost, and all economies converge to a better-than-benchmark equilibrium, no matter what endowment they start with. While not shown in the figure, steady state S behaves similarly to this case of a growth boost. Note that when the "growth boost" steady state exists, it is the unique steady state. In contrast, when the "growth trap" steady state exists, it occurs only for low initial endowments, and at sufficiently high initial endowments, the benchmark steady state exists; in other words, there are multiple steady states based on the level of initial endowment.

[Fig. 4 about here]

Two parameters, the household propensity to save, ρ, and the default cost parameter, z, are critical in determining the nature of steady state(s) that arise, as hinted in Examples I and II above. Start first with the propensity to save. Growth traps exist only for economies with low propensities to save and at low endowments. Here is why. As mentioned before, the government in the willingness-to-pay region trades off the incentive to boost growth against the repression incentive. The boost incentive is greater for the governments of higher-saving economies because the growth of private endowments is more sensitive to taxation. The government in this case opts to boost growth, purely in the interest of increasing its debt capacity, by increasing the amount which the next government is willing to pay back. Through generations of governments, the growth boost persists, and depending on the household savings parameter, the economy may or may not grow out of the willingness-to-pay constraint; when it does not, the government charging lower-than-benchmark tax rates and the growth boost become permanent features.

Conversely, the repression incentive is larger for governments of economies that save little, since more domestic financial savings are necessary for the government to borrow internationally.[14] When these economies start out at low endowments, repression by successive governments ensures endowments never grow large enough to escape the willingness-to-pay region and a trap results. We formalize the preceding arguments in Appendix Section B.1, which leads to the following results:

Proposition 3.2. *A necessary and sufficient condition for $t^{**} < t^W$, which is a necessary condition for the growth trap to exist, is an upper bound on the propensity to save ρ:*

$$t^{**} < t^W \Leftrightarrow \rho < \frac{1}{t^{**}}. \tag{3.10}$$

[14]An interesting question is what happens in a country where households have the possibility of capital flight. In that case, it may be that the country behaves as if the propensity to save is low. Of course, the higher consequent taxation may prompt more capital flight. This is worth exploring in future research.

Proposition 3.3. *A sufficient condition for the economy to converge to the benchmark steady state is a lower bound on the propensity to save ρ:*

$$\rho \in \left(\bar{\rho}, \frac{1}{r}\right), \ \ \textit{where } \bar{\rho} < \frac{1}{r}. \tag{3.11}$$

The intuition is that with a high propensity to save, household endowments grow quickly, enabling the economy to escape from the willingness-to-pay region to the ability-to-pay region swiftly, and in turn, leading to convergence to the benchmark case. It is in the interim range of values of propensity to save ρ that the possibility of a growth boost arises.

This is where the second parameter, default cost parameter z, gains importance. Recall z reflects the importance of government bonds to the domestic financial sector, and is a measure of the sophistication or development of the country's financial system. Whether the steady state is strictly boosted by access to foreign borrowing depends on whether the default cost parameter z is sufficiently small. Here is why: Note that the growth boost in our model occurs only when the economy's steady state remains in the willingness-to-pay region, which is when $\tau^{**} - C - zs(1+r) \geq 0$. Therefore, when z is low, $\tau^{**} - C - zs(1+r)$ stays positive and the willingness-to-pay constraint can remain binding for a longer duration; conversely, when z is high, the willingness-to-pay region is small and the steady state moves quickly to the benchmark steady state which is in the ability-to-pay region.

These results on how the savings parameter ρ and the default cost parameter z affect the nature of the steady state (growth trap, benchmark or growth boost) are illustrated in Fig. 5, where we plot different steady state equilibria (Figure 5 (a)) and steady state endowments (Figure 5 (b) and 5(c)) for different parameter values for ρ and z. In sum, this suggests that developing countries with low financial sophistication z and moderately-high propensities to save ρ will tend to benefit most from access to foreign borrowing, as measured by reaching higher steady-state endowments, even though their governments are myopic and self-interested.

[Fig. 5 about here]

Let us set these results in relation to the literature. Aguiar and Amador [2011], for example, study a neoclassical growth model with sovereign debt. Due to political frictions, the present-day government places a much higher weight on current household consumption relative to that in future, but nevertheless has the same discount rate as households; this leads to an anticipation of government default when debt is high along with possible expropriation via high taxes on capital, and therefore ex-ante under-investment in capital. This slows down the economy's rate of convergence to the efficient steady state, though does not alter the eventual steady state. In Aguiar, Amador and Gopinath [2009], the government's discount rate is higher than that of households. With this change in the government's objective, the economy is always trapped at levels of capital investment below the efficient one if the government discount rate is high enough. In these papers, even though the government cares about the welfare of the citizenry, sufficient myopia induces it to have a greater propensity to default on debt, causing debt to be a greater overhang on capital investment.

In contrast to these papers, the government in our model is not just myopic, it does not care about the citizenry's consumption. So debt not only effectively extends the government's horizon, it also gives the government a reason to care about the future citizenry (because of the taxable output or financial savings they generate). Because of these attributes, government borrowing in our model can lead to better long-run outcomes than the autarky steady state.

3.1 Implications for sovereign debt

A large literature on sovereign debt that we cannot do justice to attempts to explain (with only moderate success) why countries repay their foreign debt.[15] Recent papers that rely on the inability of the sovereign to discriminate between debt holders of different nationalities (see Broner and Ventura [2016], Gennaioli, Martin and Rossi [2014], Guembel and Sussman [2009]), or the sovereign's inability to prevent foreigners from trading debt to domestic institutions if a selective default is announced (see Broner, Martin and Ventura [2010] or Broner and Ventura [2016]), improve our understanding. The difficulty in discriminating between domestic and foreign holders then allows researchers to focus on what the costs of defaulting on domestic holders might be. This is a question to which researchers have more plausible answers. These include the cost of setting off panics in, or decapitalizing, the domestic banking system as in Gennaioli, Martin and Rossi [2014], the loss in activity if banks have a harder time finding safe collateral with which to transact (see Bolton and Jeanne [2011]), or the risks to re-election of antagonizng powerful domestic investors.

Yet, if the size of foreign debt were large would these costs not be dwarfed? Our assumption of government myopia helps us address this – the perceived benefits of default may not be large for a myopic government. Indeed, as (2.16) suggests, all the myopic government cares about are the flow benefits of default, which may be significantly smaller than that associated with wiping out the stock of debt. This is why a fair amount of external debt can be sustained even if the default costs z are modest. Indeed, while Acharya and Rajan [2013] also assume a myopic government, because their analysis is in a two-period setting, they require $z > 1$ for external

[15] See Eaton and Gersovitz [1981], Grossman and Van Huyck [1988], Bulow and Rogoff [1989*a*], Bulow and Rogoff [1989*b*], Fernandez and Rosenthal [1990], Eaton and Fernandez [1995], Cole and Kehoe [1998], Guembel and Sussman [2009], Reinhart and Rogoff [2010], Amador [2012], and Tomz [2012], and the surveys by Aguiar and Amador [2011] and Panizza, Sturzenegger and Zettelmeyer [2009].

debt to be feasible. Our framework does not require such high default costs because the per-period net debt service in a multi-period model is much smaller, so the benefits of default are proportionately smaller.

Our framework has other implications. In traditional models that focus on default benefits being proportional to the stock of debt, default should be more likely when interest rates are low (and discounted debt stock values high). In our model, default is more likely if debt service costs unexpectedly rise, that is, in periods of rising rates.

Government myopia in our dynamic framework can also explain why a modest reprofiling of debt after a default can be enough to make the debt creditworthy. Default in our model occurs when the flow benefits of default exceed the cost. A successor government that can renegotiate the stock of defaulted debt down to a level that future governments will pay, and create some additional room for it to issue new debt to fund its own spending, will be perfectly happy to renegotiate the debt to this level and regain good standing; this incumbent government does not bear the cost of the future debt repayment, while it benefits from regaining access to debt markets. This could explain both why negotiated haircuts on defaulted debt can be modest (Aguiar and Amador [2014] find the median country exits restructuring carrying a 5 percent higher debt-to-GDP load than at the time of default) and why creditors are happy lending again – the reprofiling makes the new debt sustainable given the modest benefits of default.

Finally, because the costs of default in our model are one-off, while the benefits of default are flows each period, a government that has a longer horizon may have a greater incentive to default because it cumulates the benefits over multiple periods. This is in contrast to Hatchondo, Martinez and Sapriza [2009], where a lower level of debt is sustainable when a myopic government is in power than when a patient government is in power. The reason for the difference in our results is simple – the costs of default in their model come in the future, so the impatient myopic government discounts them more. In contrast, the costs of default in our model are experienced by the government that triggers default, while most of the debt repayment is beyond the government's horizon, so myopic governments have greater incentives to repay debt.

3.2 Weak or negative correlation between foreign finance and growth

A number of studies (see Aizenman, Pinto and Radziwill [2004], Prasad et al. [2006], and Gourinchas and Jeanne [2013]) have explored whether countries that borrow more internationally do better – this literature focuses on the *intensive* margin (while the "odious" debt literature focuses on the *extensive* margin). The surprising finding is of a weak positive or even significant negative correlation between developing country growth and its use of foreign borrowing, within the set of countries that all have the ability to borrow internationally.[16]

Our model can shed light on this pattern that Gourinchas and Jeanne [2013] term "the allocation puzzle". To see this, suppose the differential reliance on foreign borrowing across countries arises due to differences across countries in the citizen's propensity to save (ρ), keeping the nature of the government the same (myopic and self-interested). Let us focus on the willingness-to-pay region or the sufficiently low endowment region which typically represents developing countries and emerging markets.

Then, our results on growth traps and growth boosts (Propositions 3.1–3.3) imply that in developing countries, a higher propensity to save (high ρ) means the country will avoid growth traps, potentially even experiencing a growth boost. This will drive the steady-state endowment up, and the extent of foreign borrowing relative to the endowment down; conversely, a lower propensity to save (low ρ) is associated with repression and growth traps, which drive the steady-state endowment down and the extent of foreign borrowing up. To the extent that the steady-state endowment proxies for measures of well-being such as consumption and growth, our model can generate the negative relationship between foreign borrowing and the measures documented in the literature.[17]

Our model clarifies the broader point that *ceteris* is not *paribus* across countries, so the relationship between foreign borrowing and economic growth may be confounded by the endogenous selection of which countries rely more on foreign borrowing. It is not that foreign financing is necessarily bad for developing country growth, but that the very characteristics that lead some countries to have more foreign financing, viz., low endowments and low propensities to save, typically also lead to greater repression by their governments.

Relatedly, in Aguiar and Amador [2011] countries with more fractured politics (and thus with more short-term incumbent governments) tend to spend more, and have higher outstanding net foreign liabilities, which leads them to grow slower because of an effective debt overhang. In their model, external sovereign debt has a direct adverse effect. In our model, it is coincidental with repressive regimes.

Gourinchas and Jeanne [2013] conclude that the finding that high productivity countries receive lower external capital flows is not driven by investment wedges (lower returns on cap-

[16] In particular, Prasad et al. [2006] find that over the period 1970-2004, there is no positive correlation for nonindustrial countries between current account balances and growth, or equivalently, that developing countires that have relied more on foreign finance have not grown faster in the long run, and have typically grown more slowly. They conclude this runs counter to the predictions of standard theoretical models. Similarly, Aizenman, Pinto and Radziwill [2004] construct a self-financing ratio for countries in the 1990s and find that countries with higher ratios grew faster than countries with lower ratios.

[17] This intuition can be sketched analytically and the negative relationship verified numerically for low financial development z (details are in appendix D). Interestingly, the relationship for countries with more sophisticated financial systems (higher z) may be different, an implication for which there is some evidence (see, for example, Prasad et al. [2006]).

ital discouraging capital flows) but savings wedges (high productivity countries having greater realized savings). Our paper offers a further elaboration of this argument. Greater realized savings may be because of the country's greater intrinsic propensity to save. This, in turn, reduces the distortionary tax the government imposes on capital investment (a lower investment wedge), and leads to a convergence to a higher steady state output.

3.3 Odious debt

Should countries with odious governments have access to external debt or not? Sack [1927] (see also Buchheit, Gulati and Thompson [2006], Jayachandran and Kremer [2006] and Sander [2009]) suggests that debt should be deemed *odious* and not transferable to successor regimes if (a) it was incurred without the consent of the people (b) it was not for their benefit and (c) the lender knew or should have known about the lack of consent and benefit. Our myopic self-interested government does not ask the household how much it should borrow, nor is the borrowed amount used for the benefit of the household. Lenders are perfectly happy lending since they get repaid in equilibrium. So the sovereign debt in our model meets these conditions of being odious.

The value of declaring as odious any future issue of debt that meets the above criteria is that it prevents wasteful new spending, and the accumulation of debt that successor governments will have to repay. It can disincentivize odious governments from coming to power by reducing the size of the prize from doing so (see Jayachandran and Kremer [2006]). It can also make it harder for such regimes to stay in power by reducing the resources they have to spend.

Our model does not speak to the process by which the odious government comes to power, but certainly suggests that the ability to borrow can mitigate repressive behavior. The key to the change in its behavior on gaining access to debt is the nature of the country's environment – for instance, the propensity to save of households (ρ), the size of their endowment (e_0), or the centrality of government debt to the private sector's functioning (as captured in the default cost parameter z). Governments may choose growth-enhancing policies relative to the autarky benchmark in order to boost their successor government's willingness to repay, and thereby, increase borrowing today; this dynamic enables the economy to experience a growth boost in the form of a steady-state endowment that is above the autarkic one. An odious regime, therefore, does not always imply that access to borrowing has odious consequences.[18] The need to borrow could place limits on how odious a regime can get.

[18] A related but different point is made in Janus [2012]: a limitation on debt issuance makes it less worthwhile for the odious government to stay in power, giving it more incentive to be rapacious say in taxation or additional borrowing, even if that raises the risk it is turfed out. In our model, the government cannot change its limited term in office, so all the improvement in incentives comes from the direct horizon-lengthening effects of debt.

Of course, we also show the converse possibility: access to borrowing can lead the government to repress its country into a poverty trap (Kharas and Kohli [2011]), especially if the country is poor (small endowments) and has a low propensity to save. The more general point is that international engagement, whether through trade or capital flows, can worsen or restrain bad behavior. The precise circumstances matter.

Even if access to international borrowing leads to a growth trap in our model, because the country does not start with a blank slate, a declaration that the new debt issued by the government is odious and unenforceable is not necessarily beneficial to its citizens. Such a declaration will immediately trigger default (since the government cannot borrow to repay legacy debt), which may be costlier to the country's citizens than keeping access open. It may be better, as we will see in Section 5, for the country to be eased into a better equilibrium through a combination of debt relief and debt ceilings.[19]

Before we conclude this section, we must point out that governments in our model are myopic and self-interested or corrupt but not brutal. Some commentators (see, for example, Bolton and Skeel [2007]) have in mind regimes that freely imprison, maim, and murder their citizens (or those of neighboring countries) when they use the term "odious". Of course, in such situations, we will also have to model the negative utility to citizens and neighbors from the government spending more on truncheons, rifles and flame-throwers, which may far outweigh the effects of lower taxes. We have little to say about such regimes.

4 Robustness and Extensions

Let us now examine the robustness of the basic model and some extensions.

4.1 Longer debt maturity

In our model, all government debt is short-term, maturing in the next period. We show in Online Appendix E.1 that this assumption is immaterial to the main results of our paper: Long-term debt is identical to short-term debt in its effects if the government can always buy back and reissue the bonds (given there is no default in equilibrium the price of debt remains unaffected).

[19] Stepping outside the model, the odious debt declaration, while benefiting from being simple, may also have unintended consequences. One of them is for a country that does not currently have an odious government. The increased possibility that one of its successors could be deemed "odious" could reduce its prospects for rolling over debt, and thus close the market for new debt issuance today. This too could precipitate costly default, as well as reduce the probability of a non-odious regime staying in power. Since few countries can guarantee the quality of successor governments, the unintended consequences of easing the process by which debt can be declared "odious" could be quite substantial.

Intuitively, what matters regardless of the maturity of the debt is the net debt service. It follows therefore that the endowment/tax rate paths are identical under debt of any maturity.[20]

4.2 Domestic debt only

Let us turn to a different question. How would the household fare if the government could not borrow internationally, but could borrow from the domestic household? To focus only on the effect of domestic versus foreign borrowing, we assume domestic debt continues to be issued at the world interest rate. It turns out that removing access to foreign debt does not necessarily improve the long-run consumption of the household, for the government now faces a different incentive to repress.

The household's problem is the same as earlier. However, the government's problem set up in section 2.3 changes. The government is still constrained by the next government's ability to pay. However, if z is sufficiently high ($z > 1$ suffices) the government does not face the willingness-to-pay constraint anymore – the government never finds it optimal to default as all of its debt is held domestically. If the face value of the legacy debt is D, the gain from defaulting is D, whereas the loss from defaulting is the default cost $C + zD$. As $C > 0$, if $z > 1$ the loss is always greater than the gains, implying that a strategic default is never optimal.[21]

We then have the following new Bellman equation to solve for the domestic-debt only case:

$$S(e) = \max_t \left[\min \left\{ \frac{1}{1+r} S(e'), s \right\} + \tau(t) \right] \tag{4.1}$$

where endowment e', savings s and capital $k(t)$ follow equations (2.18)-(2.20) as before. Note that the debt the government can raise today in (4.1) is the minimum of the present value of the future surplus and current savings, because with no large foreign sector to absorb its debt issue, the government can only sell what its citizens demand. This then implies the government in choosing taxes faces traditional incentives for financial repression – that is, to direct domestic savings towards its own debt. We then have

1. If $S(e^{ss}) < (1+r)s(e^{ss}, t^{ss})$ (ability-to-pay constrained), and $t^{ss} = t^{**}$.
2. If $S(e^{ss}) > (1+r)s(e^{ss}, t^{ss})$ (savings-constrained), and $t^{ss} > t^{**}$ that sets $s_t(e^{ss}, t^{ss}) + \tau'(t^{ss}) = 0$ (independent of e^{ss}).

[20] There is a growing body of literature now that analyzes long-term sovereign borrowing under a variety of assumptions on government ability to trade and ability to commit. We cannot do justice to this literature (see, for example, the recent paper by DeMarzo, He and Tourre [2021] and references therein). Interestingly, sovereign access to debt when the government is myopic results in a welfare loss for the more patient citizenry in some of this literature (DeMarzo, He and Tourre [2021] in particular), whereas this is not always the case in our model.

[21] The condition $z > 1$ is only a sufficient condition to ensure no default. In practice, a weaker condition would suffice since all that needs to be offset are the flow benefits of default, as discussed earlier.

It turns out that there are two possible steady states in equilibrium. When the savings constraint binds in equilibrium, the myopic government faces a direct incentive to financially repress; it wants to funnel private endowments into savings by increasing taxation on the real production, the proceeds of which it uses for its wasteful programs. When it does not bind, we get the autarkic level of taxation.

In the earlier case with foreign borrowing, domestic debt helped enhance the cost of default. Here, it supplies the entire borrowing needs of the government. So the steady state outcomes could be quite different. For instance, we can already see from the discussion above that we never get a growth boost when the government is restricted only to domestic borrowing. We now compare steady state outcomes with and without access to foreign debt more systematically: First, we note that under the same conditions, a government with access to domestic debt only is more likely to plunge the economy into a growth trap than one with foreign debt access. Fig. 6(a)-(b) illustrate an example where a growth trap exists for the government with access to domestic debt only, but does not for the government with access also to foreign debt, under the same parameter configurations. We state these observations more formally below.

Lemma 4.1. *A growth trap exists when the government cannot access foreign debt whenever it exists when the government can access foreign debt. Conversely, the growth trap may not exist when the government can access foreign debt even if it exists when the government cannot. In addition, there is no growth boost when the government cannot access foreign debt.*

Second, we compare the severity of growth traps in the two cases. The degree of financial sophistication z is an important factor governing the level of steady state consumption and endowment reached with access to foreign borrowing. Specifically, when z is low, access to foreign debt ameliorates the growth trap as in Fig. 6(c). It raises the steady state consumption and endowment relative to the steady state with the government having access only to domestic debt. The opposite is true when z is high (see Fig. 6(d)).

Lemma 4.2. *Suppose that the parameter configuration admits growth traps under both cases, with and without access to foreign debt. Then, for sufficiently high financial sophistication z, the growth trap is worse in the case with access to foreign debt. For z sufficiently close to 1, the growth trap is worse in the case without access to foreign debt.*

[Fig. 6 about here]

The intuition is straightforward: with foreign debt access, the incentive to financially repress comes from the incentive to increase the default cost of the next government, which increases debt capacity today. Because the default cost is proportional to z, the financial repression incentive is amplified by z. In contrast, for a government without access to foreign debt, this parameter is irrelevant (once above a threshold) because the government does not default strategically. This is why z governs the relative severity of growth traps in the two cases.

4.3 Productive government investment

We relax in Online Appendix E.2 the assumption that the self-interested government simply spends on current wasteful projects. Suppose that a public investment made in the beginning of the current period (when the government undertakes other spending) yields return at the beginning of the next period. This captures the notion that public projects, such as a state-owned steel plant or a toll road or climate change mitigation, are long-term in nature. Since the return is generated only next period, the myopic current government does not enjoy the future cash flow *per se*. However, non-zero investment may still be in its incentive if it increases its debt capacity. We show formally that this is the case if the next-period government is in the ability-to-pay region, but not necessarily if it is in the willingness-to-pay region. The implication is that governments of a country with low endowment (developing countries), which are likely to be in the willingness-to-pay region, cannot take advantage of public investment opportunities, not because they are less capable or more corrupt than rich-country governments, but because once again their circumstances give less of an incentive to do so.

5 Policy Instruments

Could poor countries escape growth traps? What policies would lenders have to follow? We now discuss the effectiveness of policies such as debt relief and debt ceilings from the perspective of the citizenry.

5.1 Debt relief

Consider first debt relief, that is, forgiveness of a certain amount of the face value of debt. We do not have a traditional Myers-style debt overhang problem in our framework, whereby the fear of the government raising taxes to service its debt causes the private sector to underinvest – indeed, the government in our model taxes heavily in order to maximize its spending, which already causes underinvestment relative to a low tax or no tax regime. Debt relief alone is inconsequential in our model. It simply allows the current-period government to increase spending by the amount of the relief (also see Aguiar and Amador [2011]).

Lemma 5.1. *In an equilibrium path, any debt relief in a period is transfered one-to-one to government spending in that period. The ensuing tax rates and endowment paths remain unchanged.*

This is not very far from reality. Of the 36 countries that received significant official debt relief under the Highly Indebted Poor Country (HIPC) Initiative and Multilateral Debt Relief Initiative (MDRI) in the early 2000s, 15 were either back in debt distress or had a high risk of debt distress by 2019. Another 13 had a moderate risk of debt distress.[22] Even the remaining did not all have a low risk of debt distress – some simply did not produce the data to compute debt sustainability.

5.2 Debt ceiling

What about capping the government's ability to borrow with a constitutional debt ceiling (as, for example, in Germany) or through a common understanding imposed by external lenders (as, for instance, in the call for multilateral agencies like the IMF to monitor and limit debt buildup in poor countries). For instance, Alfaro and Kanczuk [2017] model a government with present-biased preferences, and argue this leads to an over-accumulation of debt. They find that a rule placing a ceiling on the amount of debt that can be borrowed performs much better than a rule limiting maximum deficits, and virtually approximates the optimal rule.

In our framework, despite government myopia, borrowing can boost steady-state endowments. So if planners want to maximize the country's steady-state endowment, debt ceilings are appropriate when borrowing leads to a growth trap but not when it boosts growth.

To see this, suppose that debt ceiling takes the general form $\{\bar{D}_i\}_{i=0}^{\infty}$ where each government i faces the debt ceiling $\bar{D}_i$. Let us denote the current government's spendable surplus as $S(e;\bar{D}_0,\bar{D}_1,\ldots)$. We have:

Proposition 5.2. *$S(e;\bar{D}_0,\bar{D}_1,\ldots)$ is weakly decreasing in all debt ceilings, $\bar{D}_i$, current ($i=0$) and future ($i>0$). It follows that lowering the debt ceiling – whether for the government itself or future governments – weakly decreases the current government's ability to spend.*

We now consider a special form of debt ceiling where $D_i = \bar{D}\ \forall i$ (flat debt ceiling). Define $e_\infty(e_0;\bar{D})$ as the limit of the endowment sequence under debt ceiling $\bar{D}$. We first prove that

Proposition 5.3. *(Optimal debt ceiling). Suppose that $t^{**} < t^W$ (corresponding to the trap case). Then, in general $e_\infty(e_0) \leq e_\infty(e_0;\bar{D})$. In particular, there exists a threshold debt ceiling $\bar{\bar{D}} = D^W$ such that for all $\bar{D} < \bar{\bar{D}}$, $e_\infty(e_0;\bar{D}) = e_\infty^{**}$ for all e_0, completely removing the trap.*

[22] See https://www.worldbank.org/en/topic/debt-sustainability#2 for a list of countries and the risk of debt distress prepared by the World Bank.

Suppose instead that $t^{**} > t^W$ (corresponding to the boost case). Then, in general $e_\infty(e_0) \geq e_\infty(e_0; \bar{D})$. Similarly, $\exists \bar{\bar{D}}$ such that for all $\bar{D} < \bar{\bar{D}}$, $e_\infty(e_0; \bar{D}) = e^{**}_\infty$ for all e_0.

Fig. 7 offers an illustration. In Fig. 7(a), the debt ceiling is placed on the parameter case where the growth trap exists ($t^{**} < t^W$). The debt ceiling generally reduces the tax rate for most values of endowment. In Fig. 7(b), the debt ceiling is placed on the parameter case where the growth boost exists ($t^{**} > t^W$), and it increases the tax rate everywhere.

[Fig. 7 about here]

5.3 Debt relief coupled with debt ceiling

We saw earlier that debt relief alone had little effect over the long run. However, when coupled with a debt ceiling, debt relief can be beneficial in moving a country to a better equilibrium. Suppose, that the debt ceiling was not initially in place and the economy is in a growth trap. Only a debt ceiling below the steady-state level of debt will have effect, but imposing it will cause the country to default, thus causing it to incur the deadweight costs. Therefore, if default is a dominated option, any attempt to impose a debt ceiling should first be preceded by debt relief so as to avoid immediate default.[23]

Formally, let the debt amount be reduced by fraction ξ. A one-time debt restructuring scheme then can be summarized by a pair $(\xi, \bar{D})$. We analyze how various restructuring schemes $(\xi, \bar{D})$ can affect the utilities of different interested parties.

We first take the perspective of external creditors. Assuming debt has to be reduced, they would want to minimize ξ given $\bar{D}$, such that relief is enough to prevent default. Intuitively, ξ required to prevent default is a decreasing function of the debt ceiling $\bar{D}$, as a lower ceiling constrains the government's resources more. By Proposition 5.3, lowering $\bar{D}$ eventually gets the economy out of the trap. It follows, then, that finding an efficient scheme can be reduced to finding the threshold debt ceiling $\bar{\bar{D}}$ at or below which the economy escapes the trap. The threshold $\bar{\bar{D}}$ is lower than the debt issued in steady state W, as anything weakly higher is not going to change the current and subsequent government's behavior. We formalize this below.

Proposition 5.4. *Consider an economy in a growth-trap steady state with endowment e^W, debt D^W, and tax policy $t^W > t^{**}$. For any debt ceiling $\bar{D}$, debt relief ξ prevents government default if and only if*

$$\xi \geq \xi^{min}(\bar{D}) := 1 - \frac{S(e^W; \bar{D}) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{(1+r)D^W}.$$

[23] Note that default followed by debt autarky can ameliorate repressive taxation and potentially help the economy move from a growth trap to a higher steady state, as we will see later in Section 6.1. Here, we focus on the case where default is not welfare-improving in the long run.

Since $S(e^W;\bar{D})$ is increasing and continuous in $\bar{D}$, $\xi^{min}(\bar{D})$ is decreasing and continuous in $\bar{D}$.

A debt restructuring scheme that minimizes ξ while ensuring no default as well as no growth trap $(e_\infty = e^{**})$ can be characterized as choosing the debt ceiling $\bar{\bar{D}}$ that is arbitarily smaller than the steady-state level of debt

$$\bar{\bar{D}} := D^W = \frac{\tau^W - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{r},$$

and choosing a ξ arbitarily close to 0. At this debt ceiling, the tax rate is initially arbitarily close to t^W as well.

Fig. 8(a) illustrates the patterns exhibited by $\xi^{min}(\bar{D})$ and $e_\infty(\bar{D})$. Note first a sharp discontinuity of $e_\infty(\bar{D})$; for $\bar{D}$ higher than the steady-state level D^W, the trap is unchanged. For $\bar{D}$ slightly lower than D^W, the trap is suddenly removed. However, $\xi^{min}(\bar{D})$ is continuous in $\bar{D}$, and need only be vanishingly small, with the debt ceiling dislodging the country from the trap steady state, and the ensuing endowment dynamics taking it to the ability-to-pay region.

[Fig. 8 about here]

How about household interests? While any debt ceiling below D^W ensures long run convergence to the ability-to-pay steady state, lower debt ceilings induce faster convergence to the long-run endowment. Fig. 8(b) illustrates this point. At a debt ceiling just below the threshold (99.95% of D^W), it takes about 100 periods for the economy to reach the benchmark steady state, whereas a lower debt ceiling (80% in the figure) achieves it in 40 periods. Intuitively, governments do not start charging the autarkic tax rate right away; if the debt ceiling is just below D^W, they will set the tax rate just below t^W and it declines only slowly to the autarkic tax rate. Convergence is faster when the debt ceiling is set lower and debt relief is set accordingly higher, as can be seen in Fig 8(c). Formalizing the preceding argument:

Proposition 5.5. *Suppose that the economy is trapped at endowment e^W. Suppose now that a permanent debt ceiling $\bar{D}$ is placed at $t = 0$, along with adequate levels of debt relief such that the debt ceiling does not trigger default. Let $\{t_i^{\bar{D}}\} := \{t_0^{\bar{D}}, t_1^{\bar{D}}, \ldots\}$ denote the collection of tax rates that the governments in periods $i = 0, 1, 2, \ldots$ charge, and similarly, let $\{e_i^{\bar{D}}\} := \{e_0^{\bar{D}}, e_1^{\bar{D}}, \ldots\}$ be the corresponding endowments. Then, for two debt ceilings $\bar{D}^1 < \bar{D}^2$, $t_i^{\bar{D}^1} \leq t_i^{\bar{D}^2}$ holds for all $i \in \mathbb{Z}_+$. This immediately implies that $e_i^{\bar{D}^1} \geq e_i^{\bar{D}^2}$ for all i as well.*

Propositions 5.4 and 5.5 show that there is an understandable conflict of interest between creditors and the domestic private sector on the extent of government debt haircuts. While creditors would prefer the minimum debt relief that allows the country to escape the growth trap, the domestic private sector would prefer higher levels of debt relief for faster convergence to the steady state. Of course, debt renegotiation will result from bargaining, taking these and other factors into account. It should also be noted that debt ceilings are inherently time-inconsistent. While suitable debt relief combined with a debt ceiling is in the present government's incentive, it is not in the future governments' incentive; future governments benefit, if possible, from removing or relaxing the debt ceilings and increasing their spending by borrowing more. And future creditors have an incentive to lend. Finally, the knife-edged nature of debt ceilings and debt relief (no effect above a threshold ceiling, large effects below so minor debt relief is enough) are largely driven by the fact that in the model there is no uncertainty and all parameters are exactly known. In the presence of various forms of uncertainties, the optimal debt relief would likely be higher.

6 Unexpected Shocks

Let us now examine the effects of unexpected shocks to model parameters on model outcomes. We focus on the case where the model exhibits both steady states A (autarky benchmark) and W (trap). We assume the model economy has stayed at either of the steady states for a long enough time, i.e., endowment, taxes, and debt issuances are as in Lemma B.2. We then consider an unexpected shock – at the beginning of the period – to the current endowment e; a permanent shock to the propensity to save ρ; a permanent shock to private sector productivity ϕ which level-shifts the production function $f(k) \to \phi \times f(k)$; and a permanent shock to the interest rate r, and analyze the effects on (i) the current government's decision to default, and (ii) the steady states. We first consider the impact of a "small" shock, i.e., (theoretically) an infinitesimal perturbation of one parameter at the base case of parameters for which the steady states are considered, and next, the impact of a "large" shock.

6.1 Small shocks in the trap steady state

We formally characterize the impact of small shocks in the trap steady state in the Online Appendix F. Perhaps the most intriguing result is that government spendable in the growth-trap steady state W is negatively related to the productivity parameter. This is driven by two forces: (i) An increase in productivity induces a decrease in financial savings by the private sector; in steady state W, this drives down the government debt capacity. (ii) An increase in productivity also increases tax revenue in case of default, which weakens the government's commitment to not default, thereby further reducing the debt capacity. Lower debt capacity will in turn trigger default if the government had previously maximized borrowing.

Consider next a small endowment shock. In the growth-trap steady state W, even a small negative endowment shock causes default. However, somewhat counter-intuitively the shock may be beneficial in the long run: Because the next-period government is in autarky, it charges the autarkic tax rate, which is lower than the original repressive tax rate; as a result, the economy gets a large push to growth in the following period. In some cases, this boost in growth can be large enough to eventually get the economy out of the growth trap it was originally in. Note that this is a different feature of our model relative to other models such as Aguiar, Amador and Gopinath [2009] wherein economies stuck in an inefficient steady state only revert to that same state following endowment shocks. In our model, different steady states are possible based on the country's endowment, so shocks can change the eventual steady state.[24]

[Fig. 9 about here]

Panel (a) of Fig. 9 illustrates this result. The economy, initially in steady state W, is given a small shock (5% of original endowment) in period 10, causing a sovereign default in the next period. However, in the following period the government charges the autarkic tax rate which boosts growth significantly. This boost is large enough to counter the effects of the initial contraction so that in the long run the economy converges to the higher steady state A. Interestingly, an economy initially in steady state A is impervious to small endowment shocks. In panel (b), such an economy is given a small (5%) shock to original endowment. This does not trigger government default. In this sense, government debt of this economy is "safe". The economy goes through a minor contraction but bounces back to its original path.

To summarize, policy intervention might be unnecessary in response to small unexpected shocks, even when such shocks lead to sovereign defaults. However, we show next that this is not necessarily the case while considering the impact of large shocks.

6.2 Large shocks in the trap steady state

Large shocks such as natural calamities or wars or pandemics can lead to significantly different implications compared to small shocks. To show this, we focus on shocks to endowment: In panel (c) of Fig. 9, the economy, initially in the growth-trap steady state W, experiences a large adverse shock (loss of 50% of original endowment). In this case too, the government defaults; however, unlike the case of a small shock (panel (a)), the economy is unable to recover from

[24] Levy-Yeyati and Panizza [2011] use quarterly data to study the evolution of GDP growth around twenty-three default episodes that took place between 1982 and 2003 and find that defaults tend to follow output contractions. This is perhaps not so surprising. What is more interesting is that using quarterly data, they find that defaults tend to be associated with the beginning of the recovery. Whether it is just a coincidence that the default is associated with the business-cycle trough, or whether, as in our model, it could result in pro-growth policies, is a matter for further research.

the initial shock in spite of the short-term boost to growth. It converges back to the growth-trap steady state W. In fact, panel (d) shows that a large shock can cause even the government of the economy initially in steady state A to default, unlike the case of a small shock (panel (b)). With a large shock, the economy is pushed into a growth trap and the endowment only converges to the lower steady state W.

Consider then the impact of a large unexpected endowment shock such as a pandemic on a developing country with a myopic self-interested government. The pandemic clearly reduces production, taxes, future endowments, and the government's ability to service debt, possibly pushing the country into a growth trap. Furthermore, the nature of the shock is such that the government must undertake socially useful healthcare expenditures and also boost fiscal transfers to boost household endowments. Our model suggests that an efficient mechanism to help the developing economy recover well from such a shock could be "targeted debt relief," *i.e.*, a combination of (i) debt relief to avoid the default costs which can be a significant shock to government resources; and, (ii) continued access to debt markets, with the utilization of proceeds from debt issuance monitored (perhaps by a multilateral agency) for specific deployment toward containing the pandemic and its economic fallout. Within the context of our model, even a myopic self-interested government will have some interest in containing the pandemic and helping households survive – the fruits of that spending will be reaped within their horizon. However, they have little interest in spending that has benefits outside that horizon, so they will underspend relative to the socially desirable level, and access to borrowing will not help them spend better. Therefore, some amount of monitoring of the targeted relief is warranted.

6.3 Shocks in the boost steady state

In contrast to the model parameter configuration where a growth trap exists, a parameter configuration which admits a boost state does not suffer long-run consequences of small or large shocks. Mathematically, this arises because there is only one steady state in such a case; at any starting endowment, the economy always converges to the unique (boost) steady state.

7 Conclusion

The key takeaway from our paper is that sovereign debt is a double-edged sword when governments are myopic and self-interested. When the economy is poor and has a low propensity to save, access to debt can lead to a growth trap where the economy's steady state is worse than under debt autarky as successive governments adopt repressive policies to channel domestic savings to government bonds. In other cases, however, access to debt can extend the horizon of a myopic self-interested government, resulting in steady states that are the same as or even better than autarky. When debt induces a growth trap, policy instruments such as debt ceilings can be effective, provided there is adequate commitment to enforce them. Small endowment shocks can release an economy from a growth trap; however, large adverse shocks can push an economy that is not in a trap into one. Some of these interesting implications of our model are worthy of further empirical investigation.

An interesting extension would be to endow the otherwise myopic and self-interested government with some regard for the current-period consumption of citizens, as might be the case for economies with stronger institutions governing government behavior. Another extension could be to model the differences between economic and financial repression, examine their relative benefits from the standpoint of a myopic self-interested government, and understand their impact on debt and economic growth. Finally, in the presence of uncertainty, a myopic government would have to choose between issuing large quantities of risky debt, or smaller quantities of riskless debt, with differing implications for the lengthening of horizon and equilibrium costs of default. There is clearly scope for more research analyzing such tradeoffs.

References

Acharya, Viral V, and Raghuram G Rajan. 2013. "Sovereign debt, government myopia, and the financial sector." *Review of Financial Studies*, 26(6): 1526–1560.

Acharya, Viral V, Itamar Drechsler, and Philipp Schnabl. 2014. "A pyrrhic victory? Bank bailouts and sovereign credit risk." *Journal of Finance*, 69(6): 2689–2739.

Ağca, Şenay, and Oya Celasun. 2012. "Sovereign debt and corporate borrowing costs in emerging markets." *Journal of International Economics*, 88(1): 198–208.

Aguiar, Mark, and Manuel Amador. 2011. "Growth in the shadow of expropriation." *The Quarterly Journal of Economics*, 126(2): 651–697.

Aguiar, Mark, and Manuel Amador. 2014. "Sovereign debt." In *Handbook of international economics*. Vol. 4, 647–687. Elsevier.

Aguiar, Mark, Manuel Amador, and Gita Gopinath. 2009. "Investment cycles and sovereign debt overhang." *The Review of Economic Studies*, 76(1): 1–31.

Aguiar, Mark, Manuel Amador, and Stelios Fourakis. 2020. "On the welfare losses from external sovereign borrowing." *IMF Economic Review*, 68(1): 163–194.

Aizenman, Joshua, Brian Pinto, and Artur Radziwill. 2004. "Sources for financing domestic capital – Is foreign saving a viable option for developing countries?" National Bureau of Economic Research Working Paper 10624.

Alesina, Alberto, and Guido Tabellini. 1990. "A positive theory of fiscal deficits and government debt." *The Review of Economic Studies*, 57(3): 403–414.

Alfaro, Laura, and Fabio Kanczuk. 2017. "Fiscal rules and sovereign default." National Bureau of Economic Research.

Amador, Manuel. 2012. "Sovereign debt and the tragedy of the commons." *Stanford University Working Paper*.

Andrade, Sandro C, and Vidhi Chhaochharia. 2018. "The costs of sovereign default: Evidence from the stock market." *Review of Financial Studies*, 31(5): 1707–1751.

Arellano, Cristina. 2008. "Default risk and income fluctuations in emerging economies." *American Economic Review*, 98(3): 690–712.

Arteta, Carlos, and Galina Hale. 2008. "Sovereign debt crises and credit to the private sector." *Journal of international Economics*, 74(1): 53–69.

Basu, Suman. 2009. "Sovereign debt and domestic economic fragility." *Manuscript, Massachusetts Institute of Technology*.

Bolton, Patrick, and David Skeel. 2007. "Odious debts or odious regimes?" *Law and Contemporary Problems*, 70(4): 83–107.

Bolton, Patrick, and Olivier Jeanne. 2011. "Sovereign default risk and bank fragility in financially integrated economies." *IMF Economic Review*, 59(2): 162–194.

Borensztein, Eduardo, and Ugo Panizza. 2009. "The costs of sovereign default." *IMF Staff Papers*, 56(4): 683–741.

Broner, Fernando, Alberto Martin, and Jaume Ventura. 2010. "Sovereign risk and secondary markets." *American Economic Review*, 100(4): 1523–55.

Broner, Fernando, and Jaume Ventura. 2016. "Rethinking the effects of financial globalization." *The Quarterly Journal of Economics*, 131(3): 1497–1542.

Brutti, Filippo. 2011. "Sovereign defaults and liquidity crises." *Journal of International Economics*, 84(1): 65–72.

Buchheit, Lee C, G Mitu Gulati, and Robert B Thompson. 2006. "The dilemma of odious debts." *Duke Law Journal*, 56: 1201.

Bulow, Jeremy, and Kenneth Rogoff. 1989*a*. "A constant recontracting model of sovereign debt." *Journal of Political Economy*, 97(1): 155–178.

Bulow, Jeremy, and Kenneth Rogoff. 1989*b*. "Sovereign debt: Is to forgive to forget?" *American Economic Review*, 79(1): 43.

Chari, VV, Alessandro Dovis, and Patrick J Kehoe. 2020. "On the optimality of financial repression." *Journal of Political Economy*, 128(2): 710–739.

Chatterjee, Satyajit, and Burcu Eyigungor. 2019. "Endogenous political turnover and fluctuations in sovereign default risk." *Journal of International Economics*, 117: 37–50.

Cole, Harold L, and Patrick J Kehoe. 1998. "Models of sovereign debt: Partial versus general reputations." *International Economic Review*, 55–70.

DeMarzo, Peter M, Zhiguo He, and Fabrice Tourre. 2021. "Sovereign debt ratchets and welfare destruction." *NBER Working Paper*.

De Paoli, Bianca, Glenn Hoggarth, and Victoria Saporta. 2009. "Output costs of sovereign crises: some empirical estimates."

Eaton, Jonathan, and Mark Gersovitz. 1981. "Debt with potential repudiation: Theoretical and empirical analysis." *Review of Economic Studies*, 48(2): 289–309.

Eaton, Jonathan, and Raquel Fernandez. 1995. "Sovereign debt." *Handbook of International Economics*, 3(C): 2031–2077.

Eichengreen, Barry. 1987. "Til debt do us part: the US capital market and foreign lending, 1920-1955." *NBER Working Paper*.

Farhi, Emmanuel, and Jean Tirole. 2018. "Deadly embrace: Sovereign and financial balance sheets doom loops." *Review of Economic Studies*, 85(3): 1781–1823.

Fernandez, Raquel, and Robert W Rosenthal. 1990. "Strategic models of sovereign-debt renegotiations." *Review of Economic Studies*, 57(3): 331–349.

Flandreau, Marc, and Frederic Zumer. 2004. "The making of global finance 1880-1913." *Development Centre of the Organisation for Economic Co-operation and Development*.

Gennaioli, Nicola, Alberto Martin, and Stefano Rossi. 2014. "Sovereign default, domestic banks, and financial institutions." *Journal of Finance*, 69(2): 819–866.

Gennaioli, Nicola, Alberto Martin, and Stefano Rossi. 2018. "Banks, government bonds, and default: What do the data say?" *Journal of Monetary Economics*, 98: 98–113.

Gourinchas, Pierre-Olivier, and Olivier Jeanne. 2013. "Capital flows to developing countries: The allocation puzzle." *Review of Economic Studies*, 80(4): 1484–1515.

Grossman, Herschel I, and John B Van Huyck. 1988. "Sovereign debt as a contingent claim: excusable Default, repudiation, and reputation." *The American Economic Review*, 78(5): 1088.

Guembel, Alexander, and Oren Sussman. 2009. "Sovereign debt without default penalties." *Review of Economic Studies*, 76(4): 1297–1320.

Hatchondo, Juan Carlos, Leonardo Martinez, and Horacio Sapriza. 2009. "Heterogeneous borrowers in quantitative models of sovereign default." *International Economic Review*, 50(4): 1129–1151.

Janus, Thorsten. 2012. "Odious debt in an imperfect world." *Review of Development Economics*, 16(2): 305–317.

Jayachandran, Seema, and Michael Kremer. 2006. "Odious debt." *American Economic Review*, 96(1): 82–92.

Kharas, Homi, and Harinder Kohli. 2011. "What is the middle income trap, why do countries fall into it, and how can it be avoided?" *Global Journal of Emerging Market Economies*, 3(3): 281–289.

Kletzer, Kenneth M, and Brian D Wright. 2000. "Sovereign debt as intertemporal barter." *American Economic Review*, 90(3): 621–639.

Levy-Yeyati, Eduardo, and Ugo Panizza. 2011. "The elusive costs of sovereign defaults." *Journal of Development Economics*, 94(1): 95–105.

Özler, Şule. 1993. "Have commercial banks ignored history?" *American Economic Review*, 83(3): 608–620.

Panizza, Ugo, Federico Sturzenegger, and Jeromin Zettelmeyer. 2009. "The economics and law of sovereign debt and default." *Journal of Economic Literature*, 47(3): 651–98.

Prasad, Eswar, Raghuram G Rajan, Arvind Subramanian, et al. 2006. "Patterns of international capital flows and their implications for economic development." In *Proceedings of the 2006 Jackson Hole Symposium, Federal Reserve Bank of Kansas City*. 119–158.

Reinhart, Carmen M. 2012. "The return of financial repression." *CEPR Discussion Paper No. DP8947*.

Reinhart, Carmen M, and Kenneth S Rogoff. 2010. "Growth in a time of debt." *American Economic Review*, 100(2): 573–78.

Reinhart, Carmen M, and M Belen Sbrancia. 2015. "The liquidation of government debt." *Economic Policy*, 30(82): 291–333.

Reinhart, Carmen M, Jacob F Kirkegaard, and M Belen Sbrancia. 2011. "Financial repression redux." *Finance and Development*, 22–26.

Roubini, Nouriel, and Xavier Sala-i Martin. 1992. "A growth model of inflation, tax evasion, and financial repression." National Bureau of Economic Research.

Sack, Alexander Nahum. 1927. *Les effets des transformations des Etats sur leurs dettes publiques et autres obligations financieres : traite juridique et financier.* Recueil Sirey.

Sander, Frederico Gil. 2009. "Odious Debt as a Principal-Agent Problem." *Debt Relief and Beyond: Lessons Learned and Challenges Ahead*, 229.

Sandleris, Guido. 2010. "Sovereign defaults, domestic credit market institutions and credit to the private sector." *Mimeo Universidad Torcuato Di Tella*.

Sandleris, Guido, Gaston Gelos, Ratna Sahay, and Guido Sandleris. 2004. *Sovereign borrowing by developing countries.* Washington, DC: International Monetary Fund.

Scholl, Almuth. 2017. "The dynamics of sovereign default risk and political turnover." *Journal of International Economics*, 108: 37–53.

Tomz, Michael. 2012. *Reputation and international cooperation: sovereign debt across three centuries.* Princeton, NJ: Princeton University Press.

A Figures

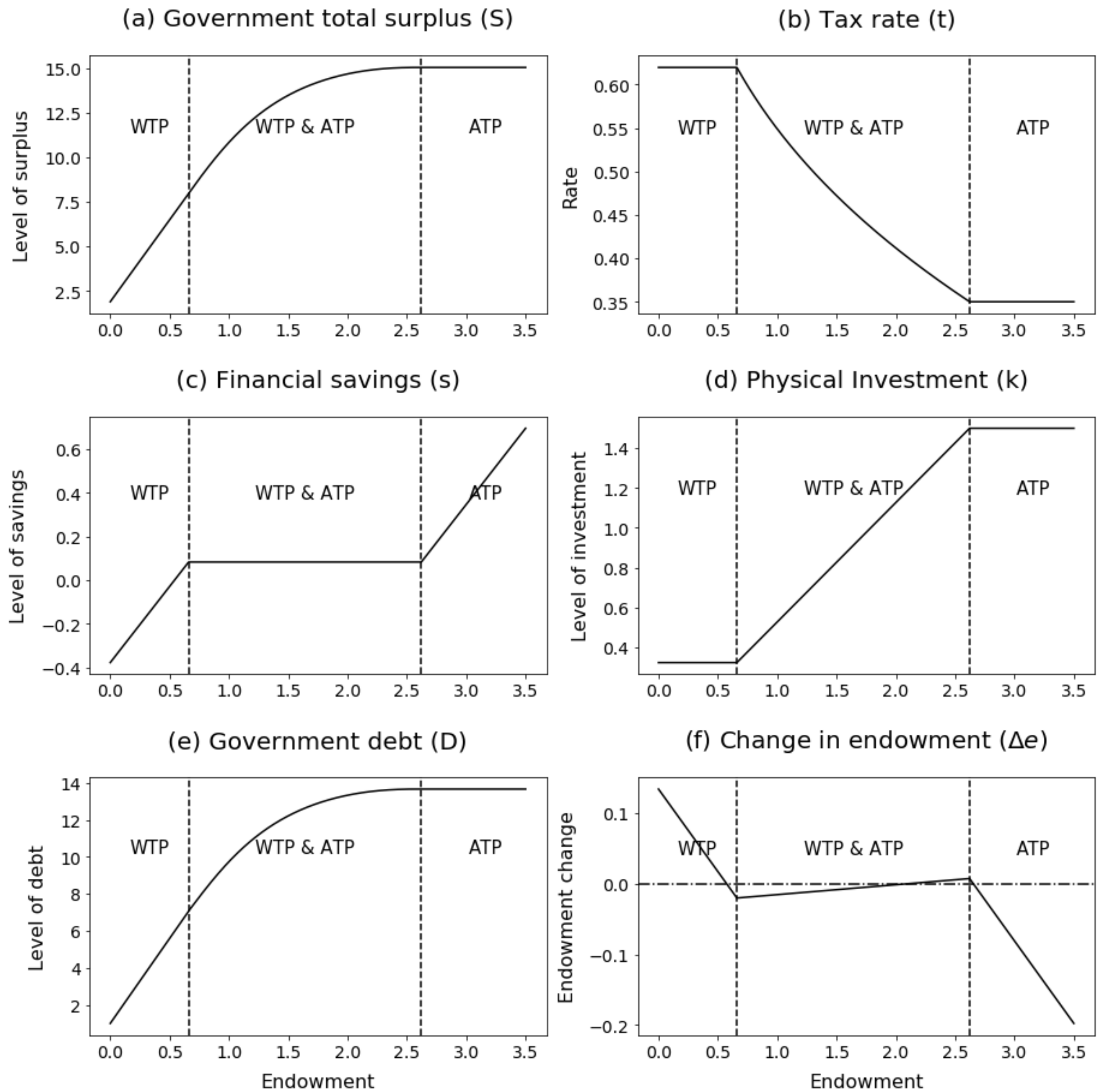

Figure 2: Solution from the baseline model, with parameters $f = 3k^{.65}$, $r = 10\%$, $z = 4$, $\rho = 2.3$ and $C = 1.0$. "WTP" stands for willingness-to-pay region; "ATP" for the ability-to-pay region; and "WTP & ATP" for the sliding region where both willingness-to-pay and ability-to-pay constraints bind.

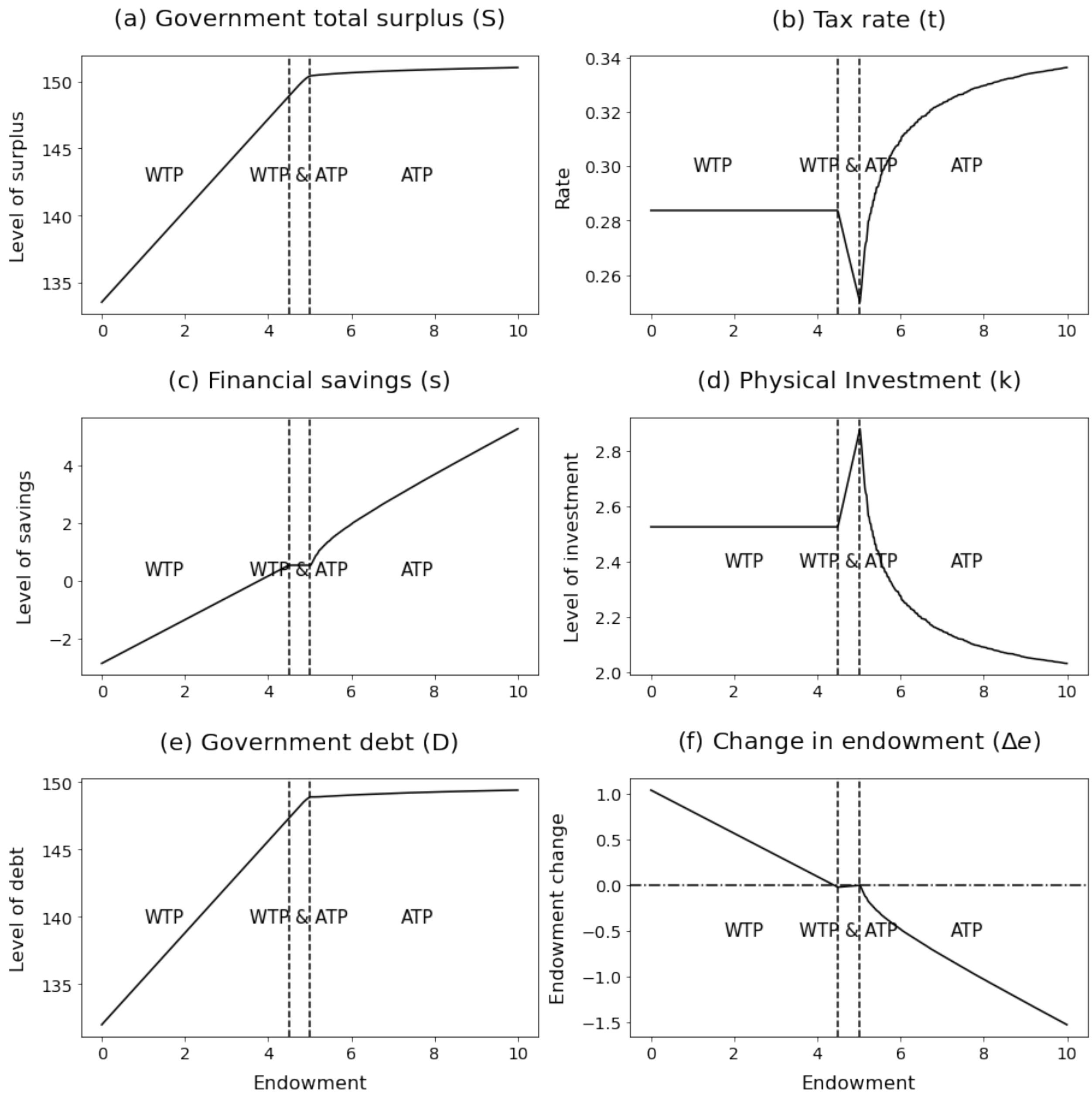

Figure 3: Solution from the baseline model, with parameters $f = 3k^{.65}$, $r = 1\%$, $z = 1.1$, $\rho = 3.1$ and $C = 1.0$. "WTP" stands for willingness-to-pay region; "ATP" for the ability-to-pay region; and "WTP & ATP" for the sliding region where both willingness-to-pay and ability-to-pay constraints bind.

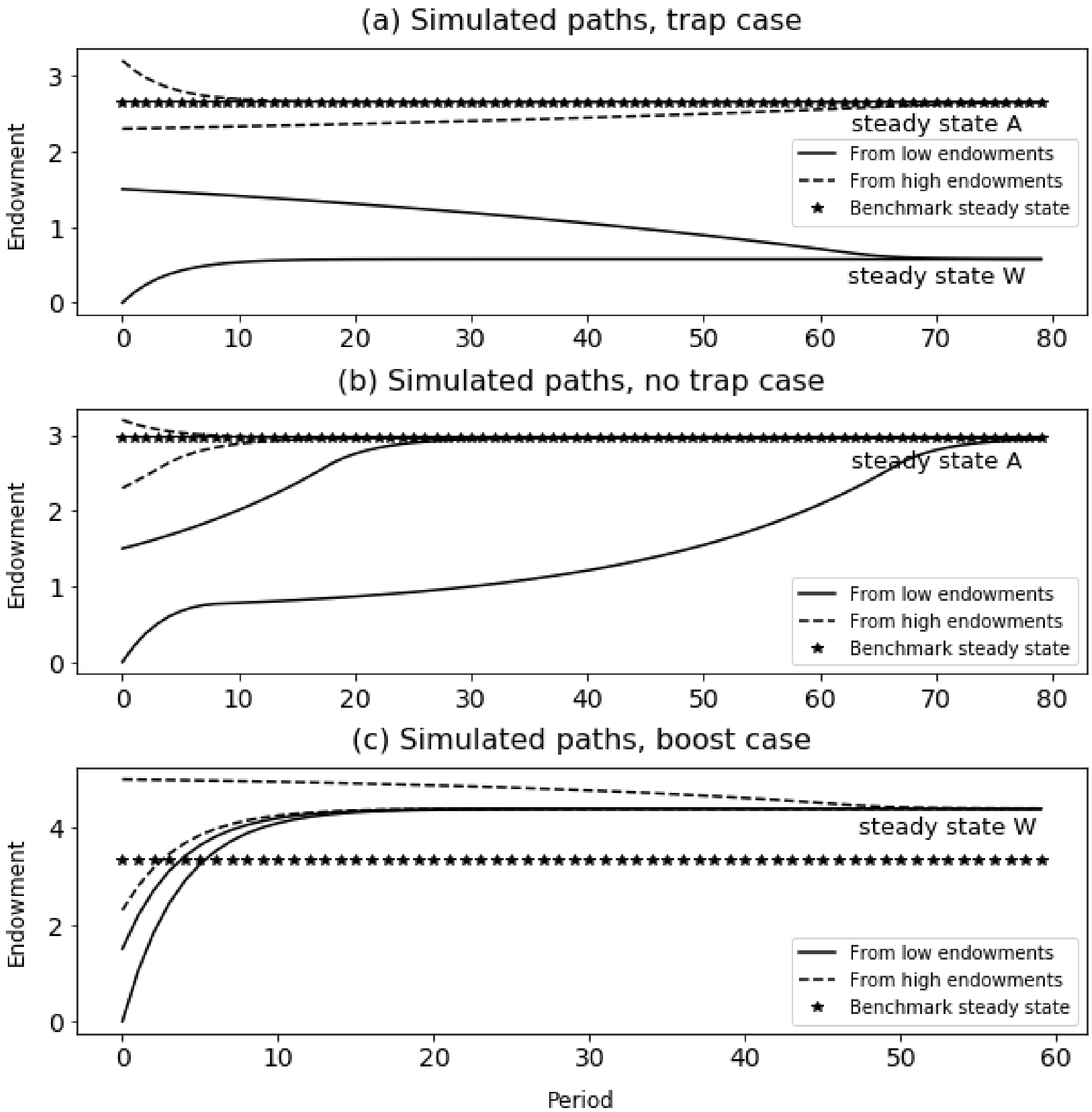

Figure 4: Simulated endowment paths for three different parameter sets. The model in panel (a) exhibits two steady states, W and A. Endowment paths starting from low endowments (solid lines) converge to steady state W (lower), whereas those starting from high endowments (dashed lines) converge to steady state A (higher). The model in panel (b) exhibits only one steady state (steady state A). All endowment paths converge to the same endowment regardless of the starting endowment. The model in panel (c) exhibits only steady state W. Contrary to other parameter configurations, steady state W in this case is at a higher endowment level than the benchmark autarky case. All endowment paths converge to the same endowment regardless of the starting endowment. Parameters used: $f = 3k^{.65}$, $C = 1$, (a) $r = 10\%$, $\rho = 2.3$, and $z = 4$. (b) $r = 10\%$, $\rho = 2.5$, and $z = 4$. (c) $r = 1\%$, $\rho = 3.1$, and $z = 1.1$.

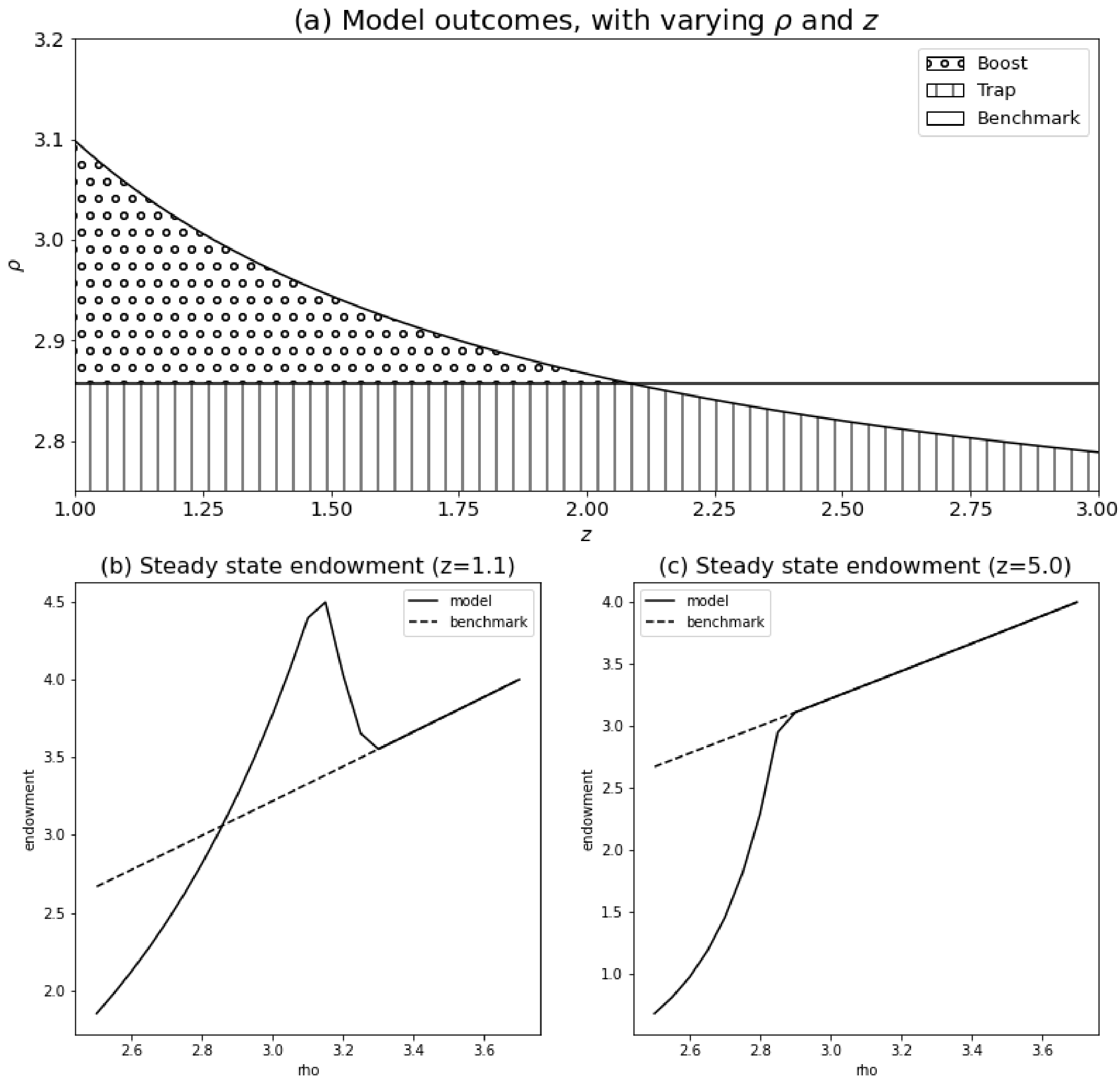

Figure 5: (Top) Model outcomes in terms of steady states. The straight horizontal line is at $\rho = \frac{1}{t^{**}}$, markedly separating the boost and trap cases. (Bottom) Steady steady outcomes, at low ($z = 1.1$) and high ($z = 5.0$), with varying ρ. Parameters used: ρ and z are varied, and $f = 3k^{.65}$, $r = 3\%$, and $C = 1.0$.

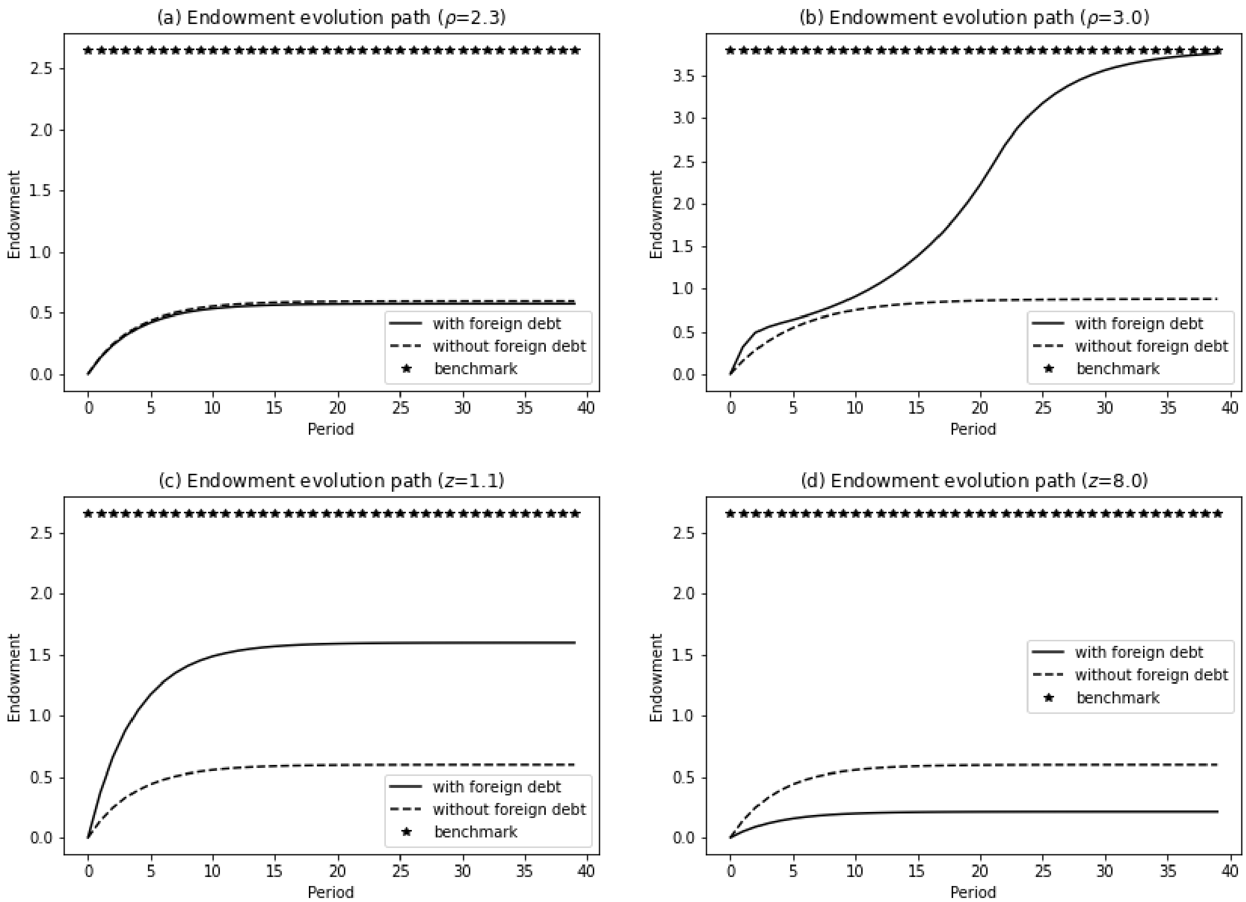

Figure 6: Growth traps, with and without access to foreign debt. (a) Growth trap exists in both cases. (b) Growth trap exists only in the case without foreign debt access. (c) Growth trap is worse in the case without foreign debt access. (d) Growth trap is worse in the case with foreign debt access. Parameters: (top) $f = 3k^{.65}$, $r = 10\%$, $z = 4$, and $C = 1.0$. (bottom) $f = 3k^{.65}$, $r = 10\%$, $\rho = 2.3$, and $C = 1.0$.

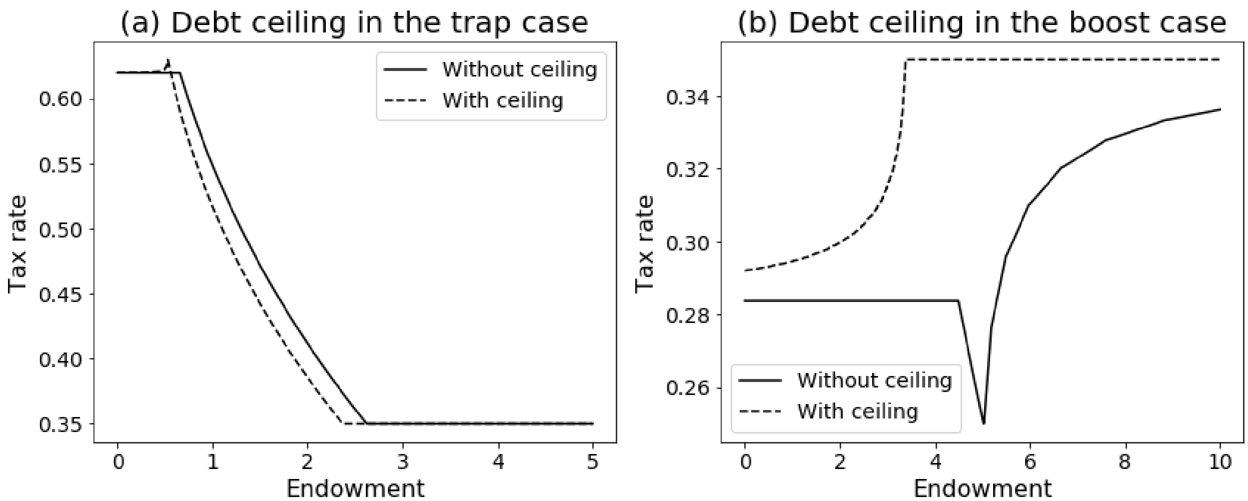

Figure 7: Tax policy of a myopic government facing a debt ceiling equal to 95% of the debt amount taken at steady state W, D^W. In panel (a), the debt ceiling is placed on a model which originally exhibited a growth trap. It can be seen that the debt ceiling lowers the tax rate for the most part. In panel (b), the debt ceiling is placed on a model which originally exhibited a growth boost. In this case, the debt ceiling raises the tax rate uniformly. Parameters used: (a) $f = 3k^{.65}$, $r = 10\%$, $z = 4$, $\rho = 2.3$ and $C = 1.0$. (b) $f = 3k^{.65}$, $r = 1\%$, $z = 1.1$, $\rho = 3.1$ and $C = 1.0$

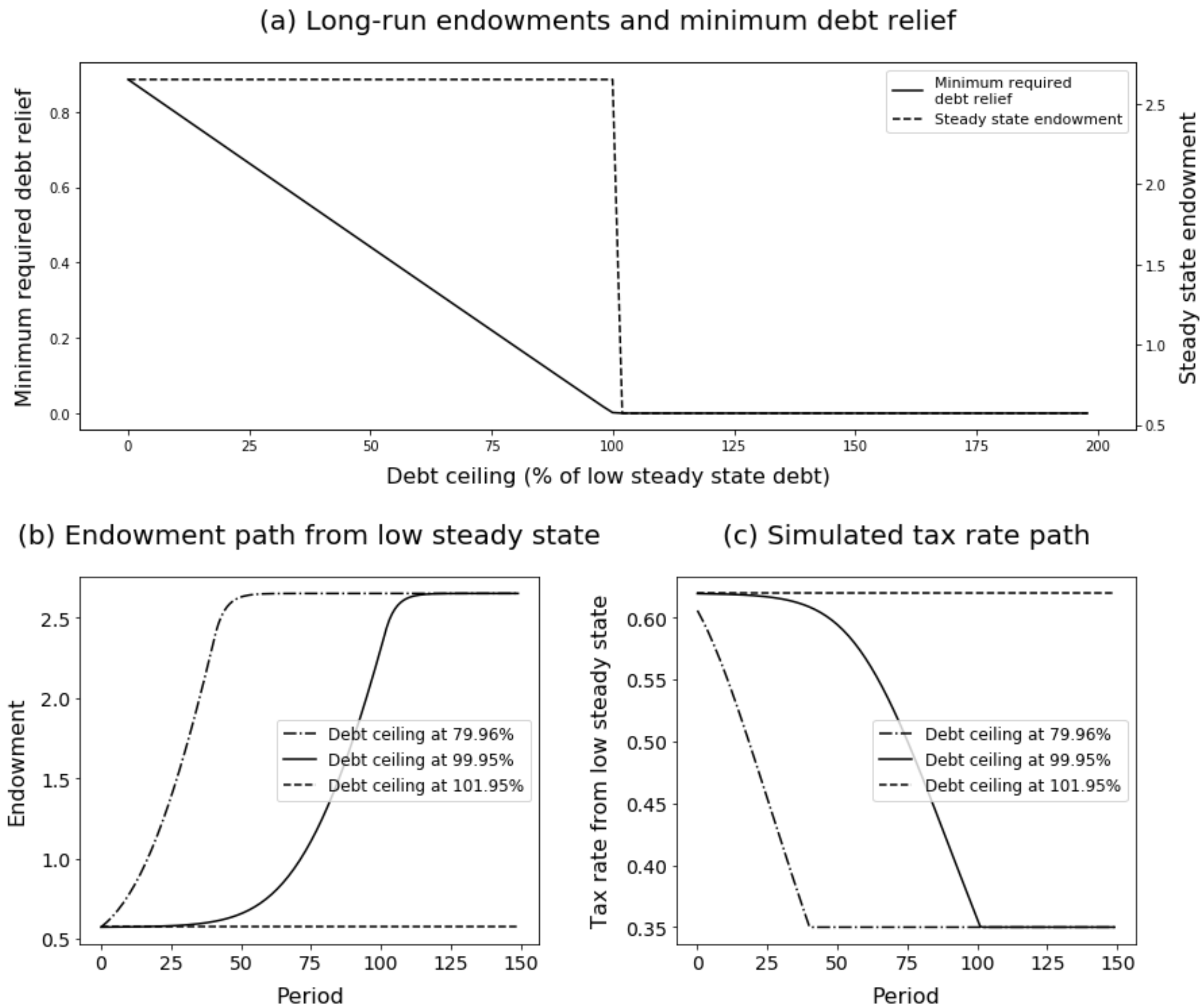

Figure 8: (a) Minimum required relief (left scale) and steady-state endowment (right scale), as functions of debt ceiling. Simulated endowment (b) and tax rate (c) paths after different levels of debt ceilings are placed on a trapped economy. In all figures, The debt ceilings are expressed as % of the level of debt in steady state W, D^W. Parameters used: $f = 3k^{.65}$, $r = 10\%$, $z = 4$, $\rho = 2.3$ and $C = 1.0$.

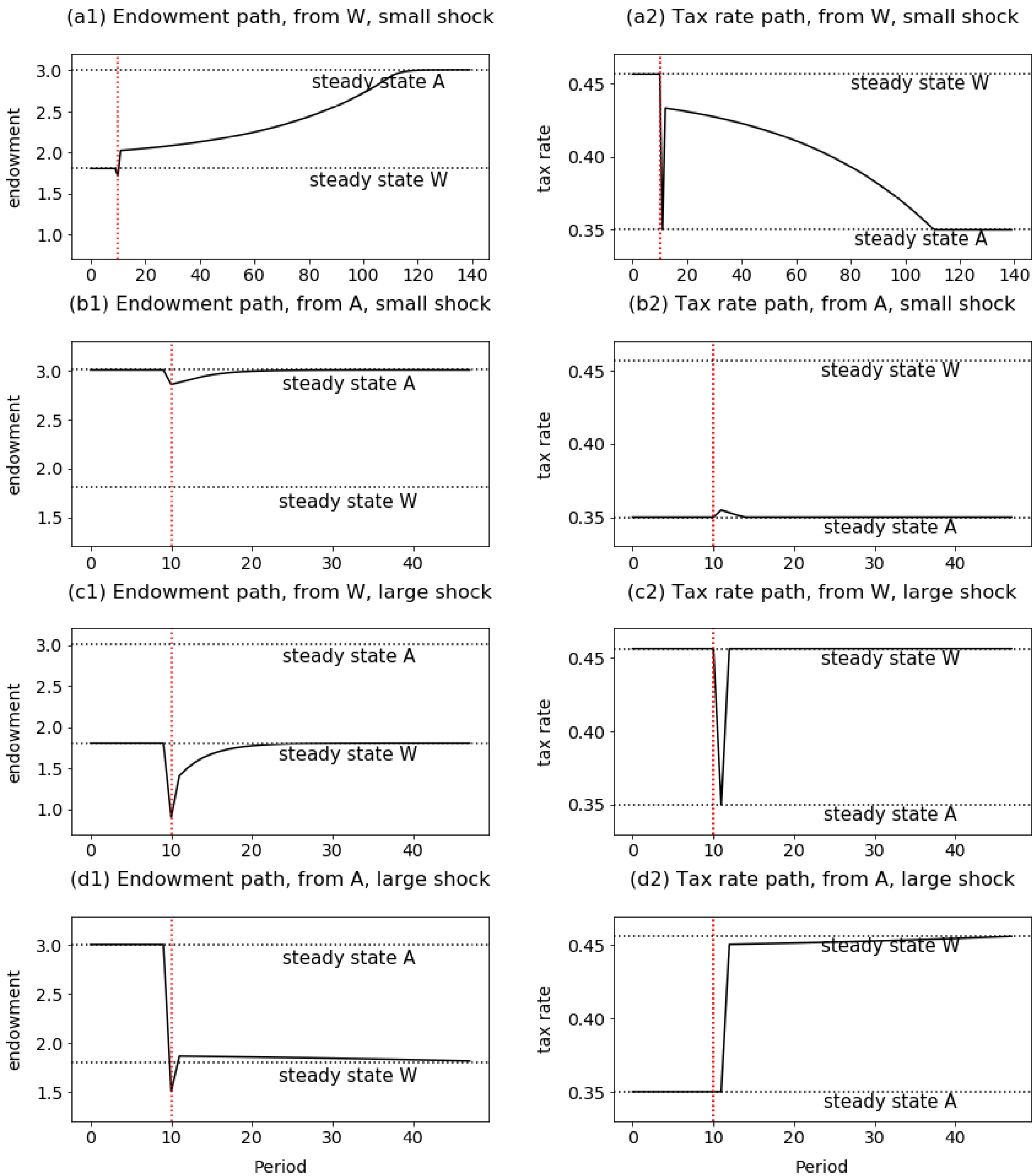

Figure 9: Short- and long-run results of small (5% of original) and large (50% of original) negative endowment shock , for economies in steady states W (growth trap) and A (autarky). The shock is experienced shortly before the end of period 10. Panels (a) and (c) pertain to economies initially in steady state W, whereas panels (b) and (d) pertain to those initially in steady state A. All economies except the one initially in steady state A and experiencing a small shock (panel (b)) go through a default in period 10. The following parameters are used: $f = 3k^{.65}$, $r = 4\%$, $z = 4.24$, $\rho = 2.72$ and $C = 1.0$.

B Characterization of the Steady States

In Section 3, we have stated that the steady state has to fall in one of (i) ability-to-pay region, (ii) willingness-to-pay region, and (iii) sliding region. We then derived necessary conditions for a steady state in each of the three regions:

$$e^A = e_+(e^A, t^A);\text{ and } \tau'(t^A) = 0. \qquad \text{(steady state A)}$$

$$e^W = e_+(e^W, t^W);\text{ and } \frac{\rho z}{1+r}\frac{de_+}{dt}(e^W, t^W) + z\frac{ds}{dt}(e^W, t^W) + \tau'(t^W) = 0. \qquad \text{(steady state W)}$$

$$e^S = e_+(e^S, t^S);\text{ and } \tau^{**} - C - z(1+r)s(e^S, t^S). \qquad \text{(steady state S)}$$

In addition, for steady states A and W, the other necessary condition is that they indeed fall under the correct regions. That is,

$$\text{Steady state A exists only if } \tau^{**} - C - zs(e^A, t^{**}) \leq 0, \text{ and} \tag{B.1}$$

$$\text{Steady state W exists only if } \tau^{**} - C - zs(e^W, t^W) > 0. \tag{B.2}$$

We show in Lemma C.6, via an application of the contraction-mapping theorem, that conditions in (B.1) and (B.2) are not only necessary, but also sufficient for the existence of each of the steady states, respectively.

Finally, we prove in Lemma B.1 that because any endowment path $\{e_i\}_{i=0}^{\infty}$ (see Definition 3.1) is a monotonic sequence, it must have a limit. Moreover, the limit must be one of the steady states characterized above. In Lemma B.1 as well as Appendix C, we make use of the intermediate function e^{sat}:

Definition B.1. Define the following function:

$$\begin{aligned} &e^{sat}(t) := e \quad s.t. \quad e_+(e,t) = e \\ \Rightarrow &e^{sat}(t) = \frac{(1-t)f(k(t)) - (1+r)k(t)}{1/\kappa_1 - (1+r)}\,. \end{aligned} \tag{B.3}$$

In intuitive terms, $e^{sat}(t)$ is the point towards which the economy "saturates" under the given t: $\left[\lim e_n = e_+(e_+(\cdots(e_+(e,t),\cdots),t),t) = e^{sat}(t)\right]$. It also follows that for a given t, at $e > e^{sat}(t)$ the economy is "contracting" $(e_+(e,t) < e)$, and at $e < e^{sat}(t)$, the economy is "growing" $(e_+(e,t) > e)$.

Summarizing all arguments above, we have the following formal result:

Lemma B.1. *Any endowment path $\{e_i\}_{i=0}^{\infty}$ is a monotone sequence (increasing or decreasing) and has a limit. It follows that $e_\infty(e_0)$ is always well-defined. Furthermore, $e_\infty(e_0)$ is always one of* *three possible steady states:*

- ***(steady state A)** Steady state is in the ability-to-pay constraint region $(\hat{e}^2, \infty)$, and $e^{ss} = e^A := e^{sat}(t^{**})$.*
- ***(steady state W)** Steady state is in the willingness-to-pay constraint region $[0, \hat{e}^1)$, and $e^{ss} = e^W := e^{sat}(t^W)$ where*

$$t^W = t \quad \text{such that} \quad \rho z \frac{de_+}{dt} + z(1+r)\frac{ds}{dt} + (1+r)\tau' = 0.$$

- ***(steady state S)** The sliding region is a singleton set ($\hat{e}^1 = \hat{e}^2$), and the steady state is in this set. In this case, the pair (e^S, t^S) simultaneously solve*

$$\begin{aligned} e &= e_+(e, t), \text{ and} \\ 0 &= \tau^{**} - C - z(1+r)s(e, t). \end{aligned}$$

In general, in the case where there are multiple steady states in the model, $e_\infty(e_0)$ is not independent of e_0. In particular, $e_\infty(e_1) \leq e_\infty(e_2)$ if $e_1 < e_2$.

The proof is in the appendix. Notably, steady state S exists only when the sliding region is a singleton set; this is because when it is of positive measure, the steady state within the region is bound to be a saddle point.

Finally, in Lemma C.7 we discuss how six different parameter cases yield distinct combinations of the above three steady states, which provide the basis for Proposition 3.1.

B.1 Savings parameter and growth traps

We showed in Proposition 3.1 that $t^W > t^{**}$ is a necessary condition for a growth trap to exists for lower endowments. In this section, we analyze the government incentives in the willingness-to-pay region to show how ρ emerges as a critical parameter.

First, suppose that the economy is in the willingness-to-pay region. Government's optimal tax rate is chosen as the following:

$$t^W := \underset{t}{\text{argmax}} \quad \left[\frac{1}{1+r}\left[S(e') - \tau^{**} + C + zs(1+r)\right] + \tau(t)\right].$$

Note that $e' = \kappa_1\left[\pi(t) + (1+r)e\right]$. Differentiating, and collecting all terms except the last, we get

$$\underbrace{\frac{dS}{de}}_{=\rho z}\frac{\rho}{1+r}\pi'(t)-z\big[\rho k'(t)+\frac{1}{1+r}\frac{d}{dt}(1-t)f(k(t))\big].$$

Whether t^W is lower or higher than $t^{**}=\operatorname{argmax}_t tf(k(t))$ depends on whether this expression, evaluated at $t=t^{**}$, is positive or not. The two conflicting incentives for the myopic government follow:

$$\underbrace{\underbrace{\frac{dS}{de}}_{=\rho z}\frac{\rho}{1+r}\pi'(t)}_{\text{Incentive to lower taxes to boost growth to increase next-period government's spendable}} - \underbrace{z\big[\rho k'(t)+\frac{1}{1+r}\frac{d}{dt}(1-t)f(k(t))\big]}_{\text{Incentive to repress investment with higher taxes to increase next-period government's willingness-to-pay}}.$$

In the equation above, we observe that (i) z enters linearly in both terms, so that when determining the sign of the expression, z is irrelevant; (ii) ρ enters as a quadratic term in the first term ($+$ incentive to grow), and as a linear term in the second term ($-$ incentive to grow). This is because the savings parameter ρ influences both the marginal sensitivity of the future endowment to current tax rate ($\frac{de_+}{dt}$) and the marginal sensitivity of next period government's repayment capacity to endowment ($\frac{dS}{de}$). For high enough ρ, the first term dominates and the myopic government chooses an even lower tax rate than benchmark. For low enough ρ, the second term dominates and the opposite occurs. In the proof of Proposition 3.2, we show that the threshold savings parameter is equal to $\frac{1}{t^{**}}$.

B.2 Equilibrium quantities at the steady states

Below, we characterize the steady states that occur in the ability-to-pay (A) and the willingness-to-pay (W) regions by providing expressions for the equilibrium quantities of debt and its composition as well as of government spending.

Lemma B.2. *Suppose that the model parameters admit two steady states, depending on the starting endowment e_0. Consider a steady state where all subsequent governments choose the same policies (t,D) with none defaulting. Then, equilibrium quantities chosen at the two steady states can be derived as the following, where* PV *stands for the "present value of":*

Steady state A. *In the ability-to-pay region steady state, the tax rate is t^{**} and the corresponding endowment is $e^A=e^{sat}(t^{**})$. The debt D^A, its domestic and foreign components, and government spending are:*

- $D^A = \frac{\tau^{**}}{r} = PV(\text{future period tax revenue})$,
- $D^{Dom} = s(e^A, t^{**})$,
- $D^{For} = \frac{\tau^{**}}{r} - s(e^A, t^{**})$, *and*
- *Government spending* $= 0$.

Steady state W. *In the willingness-to-pay region steady state W, the tax rate is chosen at* $t^W > t^{**}$ *and the corresponding endowment is* $e^W = e^{sat}(t^W) < e^{**}$. *The debt* D^W*, its domestic and foreign components, and government spending are:*

- $D^W = \frac{\tau(t^W) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{r} = NPV(\text{future period tax revenue - spending})$,
- $D^{Dom} = s(e^W, t^W)$,
- $D^{For} = \frac{\tau(t^W) - [\tau^{**} - C - z(1+r)s(e^W, t^W)]}{r} - s(e^W, t^W)$, *and*
- *Government spending* $= \tau^{**} - C - z(1+r)s(e^W, t^W)$.

Interestingly, in the ability-to-pay region, the borrowing by the previous government leaves the current government with no room to spend. In contrast, the government in the willingness-to pay-region can spend $\tau^{**} - C - z(1+r)s(e^W, t^W)$. In steady state, all future governments will act in the exact same way, collecting taxes $\tau(t^W)$ and spending $\tau^{**} - C - z(1+r)s(e^W, t^W)$. It follows that the debt capacity of the government in this steady state equals to the present value of tax revenues, net of spending.

Online Appendix To

Sovereign Debt and Economic Growth when Government is Myopic and Self-interested

C Online Appendix: Proofs and Mathematical Analysis

Lemma C.1. *Household's optimization problem in (2.1) - (2.3) is associated with the following first order conditions with respect to the four choice variabless:*

$$c_i: \quad 0 = \frac{1}{c_i} + \lambda; \tag{C.1}$$

$$s_i: \quad 0 = \lambda - (1+r)\mu; \tag{C.2}$$

$$k_i: \quad 0 = \lambda - (1-t_i)f'(k_i)\mu; \text{ and} \tag{C.3}$$

$$e_{i+1}: \quad 0 = \frac{\rho}{e_{i+1}} + \mu. \tag{C.4}$$

The system of FOC's (C.1) - (C.4) is solved by the following set of decision functions:

$$\begin{aligned} k_i &= f'^{-1}\Big(\frac{1+r}{1-t_i}\Big), \\ c_i &= \kappa_0[(1+r)(e_i - k_i) + (1-t_i)f(k_i)], \\ e_{i+1} &= \kappa_1[(1+r)(e_i - k_i) + (1-t_i)f(k_i)], \text{ and} \\ s_i &= \kappa_1(e_i - k_i) - \kappa_0(1-t_i)f(k_i); \text{ where} \\ \kappa_0 &:= \frac{1}{(1+\rho)(1+r)}; \text{ and } \kappa_1 := \frac{\rho}{1+\rho}. \end{aligned} \tag{C.5}$$

Proof: *Combining (C.2) and (C.3), we get the investment decision as a function of tax rate t_i only:*

$$k_i = f'^{-1}\Big(\frac{1+r}{1-t_i}\Big). \tag{C.6}$$

Combining (C.1), (C.2), and (C.4), we obtain the following marginal condition between the next-period endowment e_{i+1} and the current-period consumption c_i:

$$\frac{1}{c_i} - (1+r)\frac{\rho}{e_{i+1}} \Rightarrow e_{i+1} = \rho(1+r)c_i. \tag{C.7}$$

Given our four equations (two each from resource constraints and FOC's), we solve for the four *unknowns. c_i can be solved by adding (2.3) to $(1+r)\times$ (2.2) and plugging in (C.7):*

$$
\begin{aligned}
&(1+r)c_i+\cancel{(1+r)s_i}+(1+r)k_i+\underbrace{e_{i+1}}_{=\rho(1+r)c_i}=(1+r)e_i+\cancel{(1+r)s_i}+(1-t_i)f(k_i)\\
\Rightarrow&(1+r)(1+\rho)c_i=(1+r)(e_i-k_i)+(1-t_i)f(k_i)\\
\Rightarrow&c_i=\underbrace{\frac{1}{(1+\rho)(1+r)}}_{=\kappa_0}[(1+r)(e_i-k_i)+(1-t_i)f(k_i)].
\end{aligned}
$$

and k_i is determined in (C.6). Similarly, we can derive conditions for e_{i+1} and s_i:

$$
\begin{aligned}
e_{i+1}&=\kappa_1[(1+r)(e_i-k_i)+(1-t_i)f(k_i)],\ \textit{and}\\
s_i&=\kappa_1(e_i-k_i)-\kappa_0(1-t_i)f(k_i).
\end{aligned}
$$

■

In the following proofs, we make use of the following shorthand functions,

$$e_+(e,t):=\kappa_1[(1+r)(e-k(t))+(1-t)f(k(t))]; \tag{C.8}$$

$$s(e,t):=\kappa_1(e-k(t))-\kappa_0(1-t)f(k(t)); \tag{C.9}$$

$$\pi(e,t):=(1-t)f(k(t))-(1+r)k(t), \tag{C.10}$$

for the next-period endowment, financial savings, and private profit from investment respectively, based on the current-period endowment e and tax rate t.

Proof of Proposition 2.2: It suffices to show that the mapping T implied by the Bellman equation preserves monotonicity and concavity. In what follows, we denote $F:\mathbb{R}_+\to\mathbb{R}$ as a generic weakly increasing and concave function. In addition, we let e_1 and e_2 denote generic real values of endowments where $e_1<e_2$, and t_1, t_2 the respective optimal tax rates.

Monotonicity. Observe first that both $e_+(e,t)$ and $s(e,t)$, defined respectively in (C.8) and (C.9), are increasing in e. Next, note that

$$
\begin{aligned}
TF(e_2)&=\max_t\frac{1}{1+r}[F(e_+(e_2,t))-\max\{0,\tau^{**}-C-z(1+r)s(e_2,t)\}]+\tau(t)\\
&\geq\frac{1}{1+r}[F(e_+(e_2,t_1))-\max\{0,\tau^{**}-C-z(1+r)s(e_2,t_1)\}]+\tau(t_1)\\
&\geq\frac{1}{1+r}[F(e_+(e_1,t_1))-\max\{0,\tau^{**}-C-z(1+r)s(e_1,t_1)\}]+\tau(t_1)\\
&=TF(e_1).
\end{aligned}
$$

This proves the preservation of monotonicity under the mapping T. ■

(i) Concavity. Take some (e_1,t_1), (e_2,t_2) and $\alpha\in(0,1)$. Let

$$\begin{aligned} &e_\alpha := (1-\alpha)e_1+\alpha e_2;\\ &t_\alpha : e_+(e_\alpha,t_\alpha) = (1-\alpha)e_+(e_1,t_1)+\alpha e_+(e_2,t_2). \end{aligned}$$

It is immediate that such a t_α always exists. We prove the following lemma first:

Lemma C.2. *For (e_1,t_1), (e_2,t_2), and (e_α,t_α) defined as above,*

$$\begin{aligned} \tau(t_\alpha) &\geq (1-\alpha)\tau(t_1)+\alpha\tau(t_2);\\ s(e_\alpha,t_\alpha) &\geq (1-\alpha)s(e_1,t_1)+\alpha s(e_2,t_2). \end{aligned}$$

Proof: From the definition of t_α, denoting $k_\alpha := k(t_\alpha)$, $f_\alpha := f(k(t_\alpha))$, $s_\alpha := s(e_\alpha,t_\alpha)$, and $\pi_\alpha := \pi(t_\alpha)$, and recognizing that by definition $e_\alpha = (1-\alpha)e_1+\alpha e_2$, it follows that

$$\begin{aligned} &e_+(e_\alpha,t_\alpha) = (1-\alpha)e_+(e_1,t_1)+\alpha e_+(e_2,t_2)\\ \Rightarrow &(1-t_\alpha)f_\alpha-(1+r)k_\alpha = (1-\alpha)[(1-t_1)f_1-(1+r)k_1]+\alpha[(1-t_2)f_2-(1+r)k_2]\\ \Rightarrow &\pi(t_\alpha) = (1-\alpha)\pi(t_1)+\alpha\pi(t_2), \end{aligned}$$

where π is defined in (C.10). From Lemma 1.1 in the Online Appendix, assumptions stated in Definition 2.1 imply that

$$k(t_\alpha) \leq (1-\alpha)k(t_1)+\alpha k(t_2); \tag{C.11}$$

$$\tau(t_\alpha) \geq (1-\alpha)\tau(t_1)+\alpha\tau(t_2). \tag{C.12}$$

In addition, from the definition of π in (C.10), we also have that

$$\begin{aligned} &\pi_\alpha = (1-\alpha)\pi_1+\alpha\pi_2\\ \Rightarrow &(1-t_\alpha)f_\alpha-(1+r)k_\alpha = (1-\alpha)(1-t_1)f_1+\alpha(1-t_2)f_2-(1+r)((1-\alpha)k_1+\alpha k_2)\\ \Rightarrow &(1-t_\alpha)f_\alpha = (1-\alpha)(1-t_1)f_1+\alpha(1-t_2)f_2-\underbrace{(1+r)((1-\alpha)k_1+\alpha k_2-k_\alpha)}_{\geq 0}\\ \Rightarrow &(1-t_\alpha)f_\alpha \leq (1-\alpha)(1-t_1)f_1+\alpha(1-t_2)f_2, \end{aligned}$$

which leads to

$$s_\alpha = \kappa_1(e_\alpha-k_\alpha)-\kappa_0(1-t_\alpha)f_\alpha$$

$$\geq (1-\alpha)s_1 + \alpha s_2. \qquad \blacksquare$$

To show that concavity is preserved under T, we need to show that

$$TF(e_\alpha) \geq (1-\alpha)TF(e_1) + \alpha TF(e_2).$$

First, by the definition of t_α and the concavity of F,

$$\begin{aligned} & e_+(e_\alpha, t_\alpha) = (1-\alpha)e_+(e_1, t_1) + \alpha e_+(e_2, t_2) && (\because \text{ Construction of } t_\alpha) \\ \Rightarrow & F(e_+(e_\alpha, t_\alpha)) \geq (1-\alpha)F(e_+(e_1, t_1)) + \alpha F(e_+(e_2, t_2)). && \text{(C.13)} \end{aligned}$$

Second, since $\max(x, y) + \max(a, b) \geq \max(x + a, x + b)$, we have

$$\begin{aligned} & (1-\alpha)\max\{0, \tau^{**} - Cz(1+r)s_1\} + \alpha \max\{0, \tau^{**} - Cz(1+r)s_2\} \\ \geq & \max\{0, \tau^{**} - C - z(1+r)[(1-\alpha)s_1 + \alpha s_2]\} \\ \geq & \max\{0, \tau^{**} - C - z(1+r)s_\alpha\}. && \text{(C.14)} \end{aligned}$$

Then,

$$\begin{aligned} TF(e_\alpha) = & \max_t \frac{1}{1+r}[F(e_+(e_\alpha, t)) - \max\{0, \tau^{**} - C - z(1+r)s(e_\alpha, t)\}] + \tau(t) \\ \geq & \frac{1}{1+r}[F(e_+(e_\alpha, t_\alpha)) - \max\{0, \tau^{**} - C - z(1+r)s(e_\alpha, t_\alpha)\}] + \tau(t_\alpha) \\ \geq & (1-\alpha)TF(e_1) + \alpha TF(e_2), \end{aligned}$$

where the last step comes from the combination of (C.13), (C.14), and (C.12). ■

(ii) (Binding constraints). We prove the following logically equivalent statement: let $e_1 < e_2$. If at e_1 the ability-to-pay constraint is binding, that so it must at e_2 also. If instead at e_2 the willingness-to-pay binds, then so it must at e_1 also.

Proof: First let us set forth the associated first-order conditions (FOC's). If at e the ability-to-pay constraint is binding, then the following FOC is satisfied:

$$\begin{aligned} & \underbrace{\frac{de_+}{dt}}_{=\pi'(t)} \frac{dS}{de} + (1+r)\tau'(t) = 0 \\ \Rightarrow & \underbrace{\frac{dS}{de} + (1+r)\frac{\tau'(t)}{\pi'(t)}}_{:=FOC^{ability}(e,t)} = 0. \end{aligned}$$

If instead at e the willingness constraint is binding, then the following FOC is satisfied:

$$\begin{aligned}
&\underbrace{\frac{de_+}{dt}}_{=\pi'(t)}\frac{dS}{de} + z(1+r)\frac{ds}{dt} + (1+r)\tau'(t) = 0 \\
\Rightarrow &\underbrace{\frac{dS}{de} + z(1+r)\frac{s'(t)}{\pi'(t)} + (1+r)\frac{\tau'(t)}{\pi'(t)}}_{:=FOC^{willingness}(e,t)} = 0.
\end{aligned}$$

Since $s' > 0$ and $\pi' < 0$, it follows that $FOC^{willingness}(e,t) < FOC^{ability}(e,t)$ always.

If both are binding, then it must be that $\tau^{**} - C - z(1+r)s = 0$ and

$$\begin{aligned}
FOC^{ability}(e,t) &> 0, \text{ and} \\
FOC^{willingness}(e,t) &< 0.
\end{aligned}$$

as increasing t by dt would enter the region where only the ability-to-pay constraint is binding ($\tau^{**} - C - z(1+r)s < 0$) and increase the objective function by $\pi' FOC^{ability}(e,t)dt$. Since $\pi' < 0$ and $dt > 0$, $FOC^{ability}$ must be greater than 0 for this not to be a perturbation that increases the objective function. Similar argument applies in the opposite direction ($dt < 0$) for $FOC^{willingness}$.

We then prove the following lemma:

Lemma C.3. *Both $FOC^{ability}(e,t)$ and $FOC^{willingness}(e,t)$ are (weakly) decreasing in e and (strictly) increasing in t.*

Proof: For $FOC^{ability}(e,t)$, observe that $e_+(e,t)$ is increasing in e and decreasing in t. Combined with the fact that S is concave, it follows that dS/de is decreasing in e and increasing in t. From the assumptions stated in Definition 2.1, $\frac{\tau'}{\pi'}$ is increasing in t. This proves the properties for $FOC^{ability}(e,t)$.

For $FOC^{willingness}(e,t)$, it only remains to be proved that $\frac{s'}{\pi'}$ is increasing in t as the function is independent of e. Notice that since $\pi = (1-t)f - (1+r)k$ and $s = \kappa_1(e-k) - \kappa_0(1-t)f$,

$$\begin{aligned}
\frac{s'}{\pi'} &= \frac{-\kappa_1 k' - \kappa_0(\pi' + (1+r)k')}{\pi'} \\
&= -[\kappa_1 + \kappa_0(1+r)]\frac{k'}{\pi'} - \kappa_0.
\end{aligned}$$

Since $\frac{k'}{\pi'}$ is assumed to be decreasing in t in Definition 2.1, this proves the properties for $FOC^{willingness}(e,t)$. ■

Now, consider the first case where at e_1 the ability-to-pay constraint is binding and suppose per contra that at e_2 the ability-to-pay constraint is non-binding. This implies that

$$\tau^{**} - C - z(1+r)s(e_1, t_1) \leq 0, \text{ and}$$
$$\tau^{**} - C - z(1+r)s(e_2, t_2) > 0.$$

Observe that since s is increasing in both e and t,

$$\begin{aligned}
&\tau^{**} - C - z(1+r)s(e_2, t_2) > 0 \geq \tau^{**} - C - z(1+r)s(e_1, t_1) \\
\Rightarrow & z(1+r)s(e_2, t_2) < z(1+r)s(e_1, t_1) \\
\Rightarrow & z(1+r)s(e_1, t_2) < z(1+r)s(e_1, t_1) \\
\Rightarrow & t_1 > t_2.
\end{aligned}$$

At e_1, the FOC should be met, which implies that $FOC^{ability}(e_1, t_1) = 0$ and accordingly $FOC^{willingness}(e_1, t_1) < 0$. At e_2, $FOC^{willingness}(e_2, t_2) = 0$ and accordingly $FOC^{ability}(e_2, t_2) > 0$. Comparing $FOC^{ability}$ evaluated at different parameters,

$$FOC^{ability}(e_2, t_2) > 0 = FOC^{ability}(e_1, t_1) > FOC^{ability}(e_2, t_1) \Rightarrow t_2 > t_1,$$

leading to a contradiction. The proof of the second case is a mirror image. ■

(iii) (Continuity). By the theorem of the maximum, we only have to prove that for each e, there is a unique t that maximizes the objective function. First observe that, since $s(e, t)$ is concave in t, the penalty function $-\max\{0, \cdot\}$ is concave in t. Next, Let e be an arbitrary number and consider $t_1 < t_2$ and suppose per contra that t_1 and t_2 both achieve the maximum. Consider an arbitrary $\alpha \in (0, 1)$ and pick t_α as in Lemma C.2. By the stated lemma and the fact that S is concave, we know respectively that

$$\tau(t_\alpha) \geq (1-\alpha)\tau(t_1) + \alpha\tau(t_2), \text{ and}$$
$$S(e'(t_\alpha, e)) \geq (1-\alpha)S(e'(t_1, e)) + \alpha S(e'(t_2, e)).$$

Since this holds true for any arbitrary α, by picking t_α we should achieve a larger objective function. The claim is then proved by contradiction. ■

$t(e)$ **increasing in** $[0, \bar{e}^1]$**:** Suppose not, and suppose that $e_1 < e_2$ and $t_1 > t_2$. This creates the following contradiction:

$$0 = FOC^{willingness}(t_1, e_1) \geq FOC^{willingness}(t_1, e_2) > FOC^{willingness}(t_2, e_2) = 0.$$

$t(e)$ **decreasing in** $[\bar{e}^1, \bar{e}^2]$**:** In this region, the optimal t is such that $\tau^{**} - C - z(1+r)s = 0$. The proof follows from the fact that s is increasing in both e and t.

$t(e)$ **increasing in** $[\bar{e}^2, \infty]$**:** Suppose not, and suppose that $e_1 < e_2$ and $t_1 > t_2$. This creates the following contradiction:

$$0 = FOC^{ability}(t_1, e_1) \geq FOC^{ability}(t_1, e_2) > FOC^{ability}(t_2, e_2) = 0.$$

(iv) (Asymptotics). We first prove that $S(e)$ is bounded. First observe that, since $\max\{0, \tau^{**} - C - zs(1+r) \geq 0$, $S(e)$ is bounded from above by an alternative value function $\tilde{S}(e)$

$$\tilde{S}(e) := \max_t \frac{1}{1+r}\tilde{S}(e') + \tau(t)$$

for which the solution is simply $\tilde{S} = \frac{\tau^{**}}{r}$. Therefore, we conclude that $S(e) \leq \frac{\tau^{**}}{r}$ $\forall e$. Combined with the fact that $S(e)$ is weakly increasing and concave in e, we have that $S'(e) \to 0$ as $e \to \infty$. Note, then, at sufficiently high e, the optimal $t = \text{argmax}_t \frac{1}{1+r} S(e') + \tau(t) = t^{**}$. ■

Proof of Lemma B.1: In order to prove this lemma, we prove Lemmas C.4 - C.6 first.

Lemma C.4. *Any endowment path* $\{e_i\}_{i=0}^{\infty}$ *is a monotone sequence (increasing or decreasing). This immediately implies that any growth path has a limit, and it must be a fixed point of the policy function* $h(e) := e_+(e, t(e))$.

Proof: It suffices to prove that $h(e)$ is a monotonic increasing function, because $e_i < e_{i+1} = h(e_i)$ would imply that $e_{i+2} = h(e_{i+1}) > h(e_i) = e_{i+1}$, which leads by induction that $e_{j+1} > e_j$ for $\forall j \geq i$. We have proved in Proposition 2.2 that there are three regions to consider: $[0, \hat{e}^1]$, $[\hat{e}^1, \hat{e}^2]$, and $[\hat{e}^2, \infty]$. We prove piecewise monotonicity in each of these regions, which suffices for overall monotonicity given the continuity of $t(e)$ proved in Proposition 2.2. Recall from (C.8) that $e_+(e, t)$ is increasing in e and decreasing in t.

- (Region 1) Take $e_1 < e_2$, $e_1, e_2 \in [0, \hat{e}^1]$ and suppose per contra $h(e_1) > h(e_2)$. This must imply that $t_1 < t_2$. Note that $FOC^{willingness}$ must be met at both points and recall that both $\frac{s'}{\pi'}$ and $\frac{\tau'}{\pi'}$ are strictly increasing in t (Lemma C.3). This leads to

$$\begin{aligned} 0 &= \frac{dS}{de}\Big|_{h(e_1)} + z(1+r)\frac{s'(t_1)}{\pi'(t_1)} + (1+r)\frac{\tau'(t_1)}{\pi'(t_1)} \\ &< \frac{dS}{de}\Big|_{h(e_1)} + z(1+r)\frac{s'(t_2)}{\pi'(t_2)} + (1+r)\frac{\tau'(t_2)}{\pi'(t_2)} && (\because t_1 < t_2) \\ &\leq \frac{dS}{de}\Big|_{h(e_2)} + z(1+r)\frac{s'(t_2)}{\pi'(t_2)} + (1+r)\frac{\tau'(t_2)}{\pi'(t_2)} && (\because h(e_1) > h(e_2) \text{ and concavity of } S) \end{aligned}$$

$$= 0.$$

which is a contradiction.

- (Region 2) Take $e_1 < e_2$, $e_1, e_2 \in [\hat{e}^1, \hat{e}^2]$. We have proved in Proposition 2.2 that $t_1 > t_2$ in this region. Therefore $h(e_1) < h(e_2)$ immediately follows.
- (Region 3) This part is similar to region 1. ■

Lemma C.4 allows us limit the analysis of only the fixed points of the policy function $h(e)$. Essentially, these are steady states defined in Definition 3.1 plus the saddle fixed points. Saddle fixed points are limiting endowments of a measure zero starting endowment - only if it starts at that exact point - and therefore we exclude them from our analysis.

Next we characterize all possible steady states. Recall that $e^{sat}(t)$ is defined in (B.3). In addition, we define an additional auxiliary function $e^{abil}(t)$ in (C.15):

Definition C.1. Define the following function:

$$\begin{aligned} e^{abil}(t) &:= e \quad s.t. \quad \tau^{**} - C - z(1+r)s(e,t) = 0 \\ \Rightarrow e^{abil}(t) &= k(t) + \frac{(1-t)f(k(t))}{\rho(1+r)} + \frac{\tau^{**} - C}{z\kappa_1(1+r)}; \text{ and,} \end{aligned} \tag{C.15}$$

In intuitive terms, for any given t, $e^{abil}(t)$ is the boundary endowment at which both constraints are binding ($\tau^{**} - C - z(1+r)s(e,t) = 0$).

Lemma C.5. *e^{ss} must satisfy one of the following:*

- *(Steady state W) $e^{ss} \in [0, \hat{e}^1)$ and is characterized by*

$$\begin{aligned} &t^W := t \quad \textit{such that} \quad \rho \frac{de_+}{dt} z + z(1+r)\frac{ds}{dt} + (1+r)\tau' = 0; \\ &e^{ss} = e^{sat}(t^W). \end{aligned} \tag{C.16}$$

- *(Steady state A) $e^{ss} \in (\hat{e}^2, \infty)$ and is characterized by $e^{ss} = e^{sat}(t^{**})$.*
- *(Steady state S) $e^{ss} = \hat{e}^1 = \hat{e}^2$, and is characterized by t^{ss} such that $e^{ss} = e^{abil}(t^{ss}) = e^{sat}(t^{ss})$.*

Proof: It is straightforward to see that e^{ss} must belong in one of the three regions $[0, \hat{e}^1)$, $[\hat{e}^1, \hat{e}^2]$, $(\hat{e}^2, \infty)$. We first prove that in the interior in the region ($[0, \hat{e}^1)$) and region ($(\hat{e}^2, \infty)$), the fixed points must take the aforementioned form. Suppose that $e^{ss} \in [0, \hat{e}^1)$. Then, in the neighborhood of e^{ss}, the Bellman equation is

$$S = \max_t \frac{1}{1+r}\Big[S(e') - \tau^{**} + C + zs(1+r)\Big] + \tau(t).$$

From the envelope condition, we get that $\frac{dS}{de} = \rho z$. Then, the optimal t can be derived by solving the following isolated equation:

$$\rho \frac{de_+}{dt} z + z(1+r)\frac{ds}{dt} + (1+r)\tau' = 0. \tag{C.17}$$

Finally, since e^{ss} must be a fixed point, it follows that $e^{ss} = e^{sat}(t^W)$ where t^W is the solution to (C.17). The steady-state endowment in the region $\big((\hat{e}^2, \infty)\big)$ can be obtained similarly.

Next, we prove that if $\hat{e}^1 < \hat{e}^2$, then e^{ss} cannot belong to the middle region $([\hat{e}^1, \hat{e}^2])$. We prove that in order for a fixed point $t^{ss} : e^{abil}(t^{ss}) - e^{sat}(t^{ss}) = 0$ to be a stable point, $\frac{d}{dt}e^{abil}(t) - \frac{d}{dt}e^{sat}(t)$ must be non-positive at t^{ss}. Suppose per contra that $\frac{d}{dt}e^{abil}(t) - \frac{d}{dt}e^{sat}(t) > 0$. Note that in a small neighborhood of e^{ss}, the two functions can be approximated as

$$e^{abil}(t) = e^{ss} + \frac{d}{dt}e^{abil}(t)(t - t^{ss}) \Rightarrow {e_{abil}}^{-1}(e) = t^{ss} + \left(\frac{d}{dt}e^{abil}(t)\right)^{-1}(e - e^{ss});$$
$$e^{sat}(t) = e^{ss} + \frac{d}{dt}e^{sat}(t)(t - t^{ss}) \Rightarrow {e_{sat}}^{-1}(e) = t^{ss} + \left(\frac{d}{dt}e^{sat}(t)\right)^{-1}(e - e^{ss}).$$

Note that in this neighborhood $e < e^{ss} \Rightarrow e^{-1}_{abil}(e) > e^{-1}_{sat}(e)$.

Suppose now WLOG[25] that in the left neighborhood of e^{ss}, the optimal policy is sliding between the two constraints, *i.e.*, $t(e) = {e_{abil}}^{-1}(e)$. Consider e in this neighborhood $e \in (e^{ss} - \epsilon, e^{ss})$ and consider $e_+(e, t(e))$. By definition of e^{sat}, $e_+(e,t) < e$ if and only if $t > e^{-1}_{sat}(e)$. Therefore, it follows that $e_+(e, t(e)) = e_+(e, {e_{abil}}^{-1}(e)) < e$. Since this applies to all elements of the left neighborhood of e^{ss}, combined with the fact from Lemma C.4 $h(e)$ is a monotonic increasing function, that endowment paths are it follows that e can never converge to e^{ss}. Therefore, $e^{ss} \notin [\hat{e}^1, \hat{e}^2]$ if $\hat{e}^1 < \hat{e}^2$.

We next prove that the derivative condition $\frac{d}{dt}e^{abil}(t) - \frac{d}{dt}e^{sat}(t) \leq 0$ is impossible. Recall that $e^{abil}(t) - e^{sat}(t) = \psi_1\pi(t) + \psi_2 k(t) + \psi_3$ where ψ_2 and ψ_3 are positive. By the definition of t^{ss},

$$\psi_1\pi(t^{ss}) + \psi_2 k(t^{ss}) + D = 0 \Rightarrow \psi_1\pi(t^{ss}) + \psi_2 k(t^{ss}) < 0 \Rightarrow \psi_1 < -\psi_2\frac{k(t^{ss})}{\pi(t^{ss})}, \text{ so that}$$

[25] without loss of generality

$$
\begin{aligned}
\frac{d}{dt}e^{abil}(t^{ss}) - \frac{d}{dt}e^{sat}(t^{ss}) &= \psi_1\pi'(t^{ss}) + \psi_2 k'(t^{ss}) \\
&> -\psi_2\frac{k(t^{ss})}{\pi(t^{ss})}\pi'(t^{ss}) + \psi_2 k'(t^{ss}). \quad (\because \pi' < 0)
\end{aligned}
$$

Note that

$$
\begin{aligned}
-\psi_2\frac{k(t^{ss})}{\pi(t^{ss})}\pi'(t^{ss}) + \psi_2 k'(t^{ss}) \geq 0 &\Leftrightarrow -\frac{\pi'(t^{ss})}{\pi(t^{ss})} + \frac{k'(t^{ss})}{k(t^{ss})} \geq 0 \qquad (\because \psi_2, k > 0) \\
&\Leftrightarrow -\frac{d}{dt}\log(\pi(t^{ss})) + \frac{d}{dt}\log(k(t^{ss})) \geq 0 \\
&\Leftrightarrow \frac{d}{dt}\log\left(\frac{k(t^{ss})}{\pi(t^{ss})}\right) \geq 0 \\
&\Leftrightarrow \frac{d}{dt}\frac{k(t^{ss})}{\pi(t^{ss})} \geq 0 \\
&\Leftarrow \frac{k(t)}{\pi(t)} \text{ is weakly increasing.}
\end{aligned}
$$

Therefore, the assumption that $\frac{k(t)}{\pi(t)}$ is weakly increasing (it is constant for power production function) is a sufficient condition for any fixed point in $[\hat{e}^1, \hat{e}^2]$ not to be a stable fixed point. ■

Lemma C.6. *The following facts are true:*

1. *Steady state W* $\left(e^{ss} \in [0, \hat{e}^1)\right)$ *exists if and only if* $e^{abil}(t^W) \geq e^{sat}(t^W)$.
2. *Steady state A* $\left(e^{ss} \in (\hat{e}^2, \infty)\right)$ *exists if and only if* $e^{abil}(t^{**}) \leq e^{sat}(t^{**})$.
3. *If either of conditions A and B are met, then* $\hat{e}^1 < \hat{e}^2$ *almost always, implying that steady state S cannot exist.*
4. *If neither of conditions A and B are met, then* $\hat{e}^1 = \hat{e}^2$ *and the only steady state is steady state S:* $e^{ss} = \hat{e}^1 = \hat{e}^2$.

Proof: The proof follows four steps A-D below.

1. The "only if" part is proved in Lemma C.5. To show the "if" part, recall the Bellman equation

$$
\begin{aligned}
S(e) = \max_t \quad & \left[\frac{1}{1+r}\left[S(e') - \max\{0, \tau^{**} - C - zs(1+r)\}\right] + \tau(t)\right] \qquad \text{(C.18)} \\
s.t. \quad & e' = \kappa_1\left[(1+r)(e - k(t)) + (1-t)f(k(t))\right],
\end{aligned}
$$

$$s = \kappa_1(e - k(t)) - \kappa_0(1-t)f(k(t)), \text{ and}$$
$$k(t) = f'^{-1}\Big(\frac{1+r}{1-t}\Big).$$

Now conjecture that $S(e) = \alpha + \beta e$ and $t(e) = t^W$ $\forall e \le e^{abil}(t^W)$. It can be verified that the conjecture is correct if

$$\alpha = \frac{1+r}{r} - r(\tau^{**} - C), \text{ and}$$
$$\beta = \rho z.$$

owing to the fact that $e'(e, t^W) < e^{abil}(t^W)$ if $e < e^{abil}(t^W)$ and thus the ability-to-pay constraint is never binding in this region.

2. Similar to 1., we can verify a conjectured partial solution $S(e) = \frac{1+r}{r}\tau^{**}$ and $t(e) = t^{**}$ $\forall e \ge e^{abil}(t^{**})$, owing to the fact that $e'(e, t^{**}) > e^{abil}(t^{**})$ if $e < e^{abil}(t^{**})$ and thus the willingness-to-pay constraint is never binding in this region.

3. Suppose per contra that steady state A exists, and that $\hat{e}^1 = \hat{e}^2$. Note that steady state W cannot exist as it would directly violate the continuity of $t(e)$ proved in Proposition 2.2. Now suppose that it does not, and consider an endowment e arbitarily lower than $\hat{e}^1$. Because steady state W does not exist, the next-period endowment must be over $\hat{e}^2$, at which point the spendables function S is a constant value. Note that this would imply the optimal tax rate t to be the solution of:

$$\begin{aligned} t &= \operatorname*{argmax}_t \Big[\frac{1}{1+r}\big[S(e') - \tau^{**} + C + zs(1+r)\}\big] + \tau(t)\Big] && (\because e < \hat{e}^1)) \\ &= \operatorname*{argmax}_t \big[zs(e,t) + \tau(t)\big] && (\because S(e') \text{ is constant}) \end{aligned}$$

which is almost surely different from $t^{**} := \operatorname{argmax} \tau(t)$. This violates the continuity of $t(e)$. The proof of the case where steady state W exists is a mirror image. ■

4. This immediately follows from Lemma C.5. ■

Proof of Proposition 3.1: The following corollary of Lemma C.6 is a sufficient condition for the proposition:

Lemma C.7. *We analyze six different parameter cases, which span all possible cases due to the fact that $e^{abil}(1) > e^{sat}(1)$ always, and the single-crossing properties implied by the assumptions in Definition 2.1. [Refer to Figs. 1–4 of the Online Appendix for the solution characteristics for each of the six cases.]*

- ***Case A.*** *$t^{**} < t^W$, and*

  - ***A1. (Benchmark)*** *$e^{sat}(t) \geq e^{abil}(t)$ for both t^{**} and t^W: Regardless of the starting endowment e_0, the economy converges to e^{**}_∞ $\big(\forall e_0,\ e_\infty(e_0) = e^{**}_\infty\big)$.*
  - ***A2. (Trap)*** *$e^{sat}(t) \leq e^{abil}(t)$ for both t^{**} and t^W: Regardless of e_0, the economy converges to the same point **lower** than the benchmark limit $\big(\forall e_0,\ e_\infty(e_0) = e^{sat}(t^W) < e^{**}_\infty\big)$.*
  - ***A3. (Trap or Benchmark)*** *$e^{sat}(t^{**}) > e^{abil}(t^{**})$ and $e^{sat}(t^W) < e^{abil}(t^W)$: There is a unique crossing point for the two functions e^{sat} and e^{abil}, say $\bar{\bar{e}}_A$. Then,*

$$e_\infty(e_0) = \begin{cases} e^{sat}(t^W) & \text{if } e_0 < \bar{\bar{e}}_A; \text{ and} \\ e^{**}_\infty & \text{if } e_0 \geq \bar{\bar{e}}_A. \end{cases}$$

- ***Case B.*** *$t^{**} \geq t^W$, and*

  - ***B1. (Benchmark)*** *$e^{sat}(t) \geq e^{abil}(t)$ for both t^{**} and t^W: Regardless of e_0, the economy converges to e^{**}_∞ $\big(\forall e_0,\ e_\infty(e_0) = e^{**}_\infty\big)$.*
  - ***B2. (Boost)*** *$e^{sat}(t) \leq e^{abil}(t)$ for both t^{**} and t^W: Regardless of e_0, the economy converges to the same point **higher** than the benchmark limit $\big(\forall e_0,\ e_\infty(e_0) = e^{sat}(t^W) > e^{**}_\infty\big)$.*
  - ***B3. (Boost)*** *$e^{sat}(t^{**}) < e^{abil}(t^{**})$ and $e^{sat}(t^W) > e^{abil}(t^W)$: There is a unique crossing point for the two functions e^{sat} and e^{abil}, say $\bar{\bar{e}}_B$. Then, regardless of e_0, the economy converges to $\bar{\bar{e}}_B$ which is **higher** than the benchmark limit $\big(\forall e_0,\ e_\infty(e_0) = \bar{\bar{e}}_B > e^{**}_\infty\big)$. Also, it is only at this singleton point that both constraints are binding.*

Proof of Proposition 3.2: Note that t^W maximizes

$$t^W = \operatorname*{argmax}_t \rho z \frac{\kappa_1}{1+r}\pi(t) - z[\kappa_1 k(t) + \kappa_0(1-t) f(k(t))] + \tau(t). \tag{C.19}$$

Note that

$$\rho z \frac{\kappa_1}{1+r}\pi(t) - z[\kappa_1 k(t) + \kappa_0(1-t) f(k(t))] + \tau(t) = \tau(t) - z(\frac{1-\rho}{1+r}\pi(t) + k(t))$$

Since by assumption π and k are convex, and τ is concave, expression in (C.19) is concave. This implies that $t^W > t^{**}$ if and only if the FOC at t^{**} is positive. This translates to

$$\frac{\rho z \kappa_1}{1+r}\pi'(t^{**}) - z\kappa_1 k'(t^{**}) + z\kappa_0 f(k(t^{**})) - z\kappa_0(1+r)k'(t^{**}) + \tau'(t^{**}) > 0, \tag{C.20}$$

It is sufficient to derive conditions for (C.20) to hold. Using $\pi'(t) = -f(k)$ as well as

$$\begin{aligned}
&\left(tf(k(t))\right)'|_{t^{**}} = 0\\
\Rightarrow & f(k(t^{**})) + tf'(k)k'(t^{**}) = 0\\
\Rightarrow & f(k(t^{**})) = -t\frac{1+r}{1-t}k'(t^{**}),
\end{aligned}$$

we can simplify the expression in (C.20) as the following:

$$\begin{aligned}
&\frac{\rho z\kappa_1}{1+r}\pi'(t^{**}) - z\kappa_1 k'(t^{**}) + z\kappa_0 f(k(t^{**})) - z\kappa_0(1+r)k'(t^{**}) + \tau'(t^{**}) > 0\\
\Rightarrow & z\kappa_0\Big[\rho^2 t^{**}\frac{1+r}{1-t^{**}} - \rho(1+r) - t^{**}\frac{1+r}{1-t^{**}} - (1+r)\Big]k'(t^{**}) > 0\\
\Rightarrow & z\kappa_0\frac{1+r}{1-t^{**}}\big[t^{**}\rho^2 - (1-t^{**})\rho - 1\big] < 0.
\end{aligned}$$

The characteristic quadratic equation has two roots:

$$\frac{(1-t^{**}) \pm \sqrt{((1-t^{**})^2 + 4t^{**})}}{2t^{**}} = \left\{\frac{1}{t^{**}}, -1\right\}.$$

Since $\rho > 0$, the second root is economically irrelevant and therefore we get that

$$t^W > t^{**} \Leftrightarrow \rho < \frac{1}{t^{**}}. \qquad \blacksquare$$

Proof of Proposition 3.3: First, we prove that $t^{**} < 1$. Recall that $t^{**} = \text{argmax}_t\, \tau(t)$ and $\tau(t) \geq 0$. Since $\tau(1) = 0$ always, it cannot be the case that $1 = \text{argmax}_t\, \tau(t)$. Therefore, $t^{**} < 1$. Further, t^{**} does not vary with ρ.

Next, we prove that for any $t < 1$, $\exists\hat{\rho}$ such that $e^{abil}(t) < e^{sat}(t)$. Recall that

$$\begin{aligned}
e^{abil}(t) &= k(t) + \frac{(1-t)f(k(t))}{\rho(1+r)} + \frac{\tau^{**} - C}{z\left(\frac{\rho}{1+\rho}\right)(1+r)}, \text{ and}\\
e^{sat}(t) &= \frac{(1-t)f(k(t)) - (1+r)k(t)}{\frac{1}{\rho} - r}.
\end{aligned}$$

Note that for $t < 1$, $(1-t)f(k(t)) - (1+r)k(t) > 0$, and that keeping all else equal, $e^{sat}(t)$ is monotonically increasing in ρ, reaching infinity as $\rho \to \frac{1}{r}$, whereas e^{abil} is monotonically decreasing in ρ. It follows that for any given $t < 1$, there must exist a threshold $\hat{\rho}(t) < \frac{1}{r}$ such that $e^{sat}(t) > e^{abil}(t)$.

Finally, it suffices to consider the case where $\rho > \frac{1}{t^{**}}$, under which case $t^W < t^{**}$. Notice that due to the single-crossing properties of e^{abil} and e^{sat}, $e^{sat}(t^{**}) > e^{abil}(t^{**}) \Rightarrow e^{sat}(t^W) > e^{abil}(t^W)$ in this case. Given that t^{**} does not vary with ρ, it follows that for $\rho > \bar{\rho} = \hat{\rho}(t^{**})$, $e^{sat}(t) > e^{abil}(t)$ for both t^{**} and t^W. From Lemma C.7, this implies that model outcomes are either A1 or B1, where endowments always converge to the benchmark steady state. ■

Proof of Proposition 5.2: The formal problem is stated for the general case in Lemma C.8:

Lemma C.8. *Conditional on not defaulting, government's actions are independent of past government debt ceilings and legacy debt. Suppose that the debt ceiling that the government in period i faces is $\bar{D}_i$, $\forall i \in \mathbb{Z}_+$. Then, the current government's problem can be summarized as solving the following Bellman equation:*

$$S(e;\bar{D}_0,\bar{D}_1,\ldots) = \max_t \left[\min\Big[\frac{1}{1+r}(S(e';\bar{D}_1,\bar{D}_2,\ldots) - \max\{0,\tau^{**}-C-zs(1+r)\}),\bar{D}_0\Big] + \tau(t)\right] \tag{C.21}$$

$$\begin{aligned} s.t.\quad & e' = \kappa_1\big[(1+r)(e-k(t)) + (1-t)f(k(t))\big], \\ & s = \kappa_1(e-k(t)) - \kappa_0(1-t)f(k(t)), \text{ and} \\ & k(t) = f'^{-1}\Big(\frac{1+r}{1-t}\Big). \end{aligned}$$

Then, similarly to Lemma 2.1, the decision rule encompassing default for government i which has inherited an economy with endowment e_i, legacy debt D_{i-1}, and legacy domestic debt D_{i-1}^{Dom} can be characterized as the following. For the sake of brevity, we use the notation $S_i(\cdot) := S(\cdot\,;\bar{D}_i,\bar{D}_{i+1},\ldots)$ and $t_i(\cdot) := t(\cdot\,;\bar{D}_i,\bar{D}_{i+1},\ldots)$.

1. *If $S_i(e_i) - (1+r)D_{i-1} < 0$, the government cannot pay back the legacy debt and defaults. Upon default, it enters autarky and charges autarkic tax rate t^{**}.*
2. *If $S_i(e_i) - (1+r)D_{i-1} < \tau^{**} - C - z(1+r)D_{i-1}^{Dom}$, the government potentially can pay back the legacy debt, but finds defaulting more advantageous. In other words it strategically defaults, enters autarky, and charges the autarkic tax rate t^{**}.*
3. *If neither of the above two conditions apply, then the government pays back the legacy debt, charges tax $t_i(e_i)$ and issues $S_i(e_i) - \tau(t_i(e_i))$ amount of debt. Total spending of the government is $S_i(e_i) - (1+r)D_{i-1}$.*

The flat debt ceiling case corresponds to setting $D_i = \bar{D}\ \forall i$. Let us first prove that the mapping $T(\bar{D})$:

$$F \to T(\bar{D})F = \max_t \frac{1}{1+r}\min\Big[F(e') - \max\{0,\tau^{**}-C-zs(1+r)\},\bar{D}\Big] + \tau(t),$$

is monotonic:

$$F \le G \quad \forall e \Rightarrow TF \le TG \quad \forall e; \text{ and} \tag{C.22}$$
$$\bar{D}^1 \le \bar{D}^2 \Rightarrow T(\bar{D}^1)F \le T(\bar{D}^2)F \quad \forall e. \tag{C.23}$$

In the interest of brevity, let us define:

$$T^t(\bar{D})F := \frac{1}{1+r}\min\Big[F(e') - \max\{0, \tau^{**} - C - zs(1+r)\}, \bar{D}\Big] + \tau(t),$$

so that $T(\bar{D}) = \max_t T^t(\bar{D})$. Note that fixing t, T^t is a monotonic transformation: $F \ge G \Rightarrow T^tF \ge T^tG$, $\bar{D}^1 \le \bar{D}^2 \Rightarrow T(\bar{D}^1)F \le T(\bar{D}^2)F$. Next, we prove (C.22) and (C.23).

Proof of (C.22). Suppose per contra that for some e, $TF > TG$. Let the associated tax rates be t_F and t_G. This leads to the following contradiction:

$$\begin{aligned}
T^{t_F}F(e) &> T^{t_G}G(e) && \text{(by assumption)}\\
&\ge T^{t_F}G(e) && (\because \text{ optimality of } t_G)\\
&\ge T^{t_F}F(e). && (\text{monotonicity of } T^t)
\end{aligned}$$

Proof of (C.23). Similarly, suppose per contra that $T(\bar{D}^1)F > T(\bar{D}^2)F$ for some e. Let the associated tax rates be t_1 and t_1. This leads to the following contradiction:

$$\begin{aligned}
T^{t_1}(\bar{D}^1)F(e) &> T^{t_2}(\bar{D}^2)F(e) && \text{(by assumption)}\\
&\ge T^{t_1}(\bar{D}^2)F(e) && (\because \text{ optimality of } t_G)\\
&\ge T^{t_1}(\bar{D}^1)F(e). && (\text{monotonicity of } T^t)
\end{aligned}$$

Now consider two generic value functions $S^1 := S(\cdot; \bar{D}_1, \ldots, \bar{D}_n^1, \ldots)$ and $S^2 := S(\cdot; \bar{D}_1, \ldots, \bar{D}_n^2, \ldots)$ where the debt ceiling is different for only one period $i = n$, and suppose WLOG that $\bar{D}_n^1 < \bar{D}_n^2$. Note that

$$\begin{aligned}
S^1 &= \Big(\prod_{i=1}^{n-1} T(\bar{D}_i)\Big) T(\bar{D}_n^1) S^{n+1}, \text{ and}\\
S^2 &= \Big(\prod_{i=1}^{n-1} T(\bar{D}_i)\Big) T(\bar{D}_n^2) S^{n+1};
\end{aligned}$$

where $S^{n+1} := S(\cdot; \bar{D}_{n+1}, \bar{D}_{n+2}, \ldots)$. Note that from (C.23),

$$S_n^1 := T(\bar{D}_n^1)S^{n+1} \le T(\bar{D}_n^2)S^{n+1} =: S_n^2.$$

Then, by successive application of (C.22) for $i = 1, \ldots, n-1$, we derive that $S^1 \leq S^2$. ■

Proof of Proposition 5.3: First note that in this special case the Bellman equation takes the following form:

$$\begin{aligned} S(e;\bar{D}) = \max_t \quad & \Big[\frac{1}{1+r}\min\big[S(e';\bar{D}) - \max\{0, \tau^{**} - C - zs(1+r)\}, \bar{D}\big] + \tau(t)\Big] \qquad \text{(C.24)} \\ s.t. \quad & e' = \kappa_1\big[(1+r)(e-k(t)) + (1-t)f(k(t))\big], \\ & s = \kappa_1(e-k(t)) - \kappa_0(1-t)f(k(t)), \text{ and} \\ & k(t) = f'^{-1}\Big(\frac{1+r}{1-t}\Big). \end{aligned}$$

It follows similarly to Lemma C.6 that there are only two possible steady states, A and W, which must satisfy conditions specified in Lemma C.5. What remains to be proved is that the necessary and sufficient condition for the willingness-to-pay region steady state W to exist is that $\bar{D} \geq \bar{\bar{D}}$ for some $\bar{\bar{D}}$.

Let us conjecture that $\bar{\bar{D}} = D^W$ defined in Lemma B.2, and suppose first that $\bar{D} > D^W$. Note that in steady state W, the current and all future governments on the equilibrium path take on the debt of amount D^W which is below the debt ceiling. Using this logic, we can verify that a conjectured partial solution $S(e;\bar{D}) = S(e)\ \forall e \leq \hat{e}^1$ solves the Bellman equation in (C.24), similarly to Lemma C.6. By the uniqueness of the solution, this proves that $\bar{D} > D^W$ does not alter the behavior of the model economy for $e < \hat{e}^1$.

Now suppose instead that d$\bar{D} < D^W$. We know that if the steady state were to exist, the tax rate must satify (C.16), and that $e^{ss} = e^{sat}(t^W)$. We then verify the impossibility of the existence by observing the fact that at (e^{ss}, t^W), the optimality condition is violated because of the debt ceiling binding.

It can be seen that once the debt ceiling starts binding, the marginal sensitivity of the first term (min$\{\cdot, \bar{D}$) to the tax rate is zero. Therefore, the government's choice of tax rate in this case would be t^{**}. Therefore, if steady state W is removed, the only steady state that can survive is $e^A = e^{sat}(t^{**})$. ■

One way to see this intuitively is to analyze the marginal incentives for a myopic government in the short run. Recall the original Bellman equation and suppose for simplicity that e is in the willingness-to-pay region:

$$t(e) = \operatorname*{argmax}_t \frac{1}{1+r}\Big[S(e') - \tau^{**} + C + z(1+r)s\Big] + \tau(t).$$

Recall that the myopic governments' optimal taxation was chosen by trading off the incentive to boost ($\frac{de'}{dt}\frac{dS}{de} < 0$) and to repress ($\frac{ds}{dt} > 0$). We consider two cases:

- The debt ceiling is imposed only on the current government. In this case, the problem is changed to

$$t(e) = \underset{t}{\operatorname{argmax}} \frac{1}{1+r}\Big[\min\{S(e') - \tau^{**} + C + z(1+r)s, \bar{D}\}\Big] + \tau(t).$$

  If $\bar{D}$ is low enough so that $S(e') - \tau^{**} + C + z(1+r)s$ is greater than or equal to $\bar{D}$, then the government's marginal incentives to both boost or repress disappear. Therefore, the government would simply choose $t = t^{**}$ that maximizes $\tau(t)$.

- The debt ceiling is imposed on all future governments but not on the current government. In this case, the problem is changed to

$$t(e) = \underset{t}{\operatorname{argmax}} \frac{1}{1+r}\Big[S(e';\bar{D}) - \tau^{**} + C + z(1+r)s\Big] + \tau(t).$$

  The incentive to repress remains unchanged; however, because $S(e')$ is constrained by $\bar{D}$ in some states of the world, the incentive to boost is lower. Therefore, the government engages in even higher repression than without debt ceiling.

Given that a flat ceiling is a combination of the debt ceiling now and a debt ceiling starting tomorrow for ever, it follows that a debt ceiling either moves the tax rate to the benchmark tax rate t^{**}, or induces the government to repress even more. It follows that if $t^{**} < t^W$, then the debt ceiling could improve the steady state by achieving the benchmark steady state instead. On the other hand if $t^{**} > t^W$, then the debt ceiling always hurts when it is binding.

Proof of Proposition 5.4: In a steady state, the government defaults if and only if the new government spendings under the debt restructuring scheme, $\big(S(e^W;\bar{D}) - (1+r)(1-\lambda)D^W_{-1}\big)$, is lower than the original spending $\big(\tau^{**} - C - z(1+r)s(e^W,t^W)\big)$, the expression for which is derived in Lemma B.2. Observe that

$$\begin{aligned}
&S(e^W;\bar{D}) - (1+r)(1-\lambda)D^W_{-1} \geq \tau^{**} - C - z(1+r)s(e^W,t^W) \\
\Rightarrow &(1-\lambda) \leq \frac{S(e^W;\bar{D}) - [\tau^{**} - C - z(1+r)s(e^W,t^W)]}{(1+r)D^W_{-1}} \\
\Rightarrow &\lambda \geq 1 - \frac{S(e^W;\bar{D}) - [\tau^{**} - C - z(1+r)s(e^W,t^W)]}{(1+r)D^W_{-1}}.
\end{aligned}$$

■

Proof of Proposition 5.5: First observe that for all endowment paths starting from the trap endowment, the debt ceiling is binding. Therefore, there are only three possible choices of tax rate: choose tax rate such that either (i) $S(e';\bar{D}) - \tau^{**} + C + zs(1+r) = \bar{D}$, (ii) $S(e';\bar{D}) = \bar{D}$ or (iii) $S(e';\bar{D}) - \tau^{**} + C + zs(1+r) > \bar{D}$ and $\tau'(t) = 0$.

We show that in all possible cases, $t(e;\bar{D})$ is weakly decreasing in $\bar{D}$, having e fixed. Observe that using the envelope theorem – given that the debt ceiling is binding – yields $\frac{\partial S(e;\bar{D})}{\partial \bar{D}} < 1$. Using this, and supposing $\bar{D}^1 < \bar{D}^2$, we assess the property in each case:

[(i)]

1. $S(e';\bar{D}) - \tau^{**} + C + zs(1+r) - \bar{D} = 0$. Note that the LHS is decreasing in $\bar{D}$, and therefore t has to increase the LHS to counteract. The LHS is decreasing in t implying that t should be decreasing as $\bar{D}$ is decreasing.
2. $S(e';\bar{D}) - \bar{D} = 0$. This case is similar to case (i) above.
3. $S(e';\bar{D}) - \tau^{**} + C + zs(1+r) > \bar{D}$ and $\tau'(t) = 0$. In this case $t = t^{**}$ and therefore the stated condition that $t(e;\bar{D})$ is weakly decreasing condition in $\bar{D}$ is preserved. ■

Proof of Proposition F.1: The partial derivatives of S and e in the two steady states were proved in Lemma C.6. For the savings parameter ρ, notice first that an application of envelope theorem on the Bellman equation in (2.17) yields, in steady state W:

$$\frac{\partial S}{\partial \rho} = \frac{1}{1+r}\Big[\frac{\partial S}{\partial \rho} - \frac{\partial \tau^{**}}{\partial \rho} + z\frac{\partial s}{\partial \rho}(1+r)\}\Big] + \frac{\partial \tau(t)}{\partial \rho}$$
$$\Rightarrow \frac{\partial S}{\partial \rho} = \frac{1+r}{r} z \frac{\partial s}{\partial \rho}(1+r) > 0 \qquad (\because \tfrac{\partial \tau}{\partial \rho} = 0)$$

It follows similarly that at steady state A, $\frac{\partial S}{\partial \rho} = 0$. For the productivity parameter ϕ, an application of envelope theorem yields, in steady state W:

$$\frac{\partial S}{\partial \phi} = \frac{1}{1+r}\Big[\frac{\partial S}{\partial \phi} - \frac{\partial \tau^{**}}{\partial \phi} + z\frac{\partial s}{\partial \phi}(1+r)\}\Big] + \frac{\partial \tau(t)}{\partial \phi}$$
$$\Rightarrow \frac{r}{1+r}\frac{\partial S}{\partial \phi} = z\frac{\partial s}{\partial \phi}(1+r) - \frac{1}{1+r}\frac{\partial \tau^{**}}{\partial \phi} + \frac{\partial \tau(t)}{\partial \phi}$$
$$\Rightarrow \frac{r}{1+r}\frac{\partial S}{\partial \phi} = \underbrace{z\frac{\partial s}{\partial \phi}(1+r)}_{<0} - \left[\frac{1}{1+r}\tau^{**} - \tau(t)\right] \qquad (\because \tfrac{\partial \tau(t)}{\partial \phi} = \tau(t))$$

Now notice that since $\tau^{**} = \max_s \tau(s) \geq \tau(t)$, the second term $\left[\frac{1}{1+r}\tau^{**} - \tau(t)\right] > 0$ for sufficiently low r. The partial derivative in steady state A $\frac{\partial S}{\partial \phi} > 0$ follows similarly. ■

D Weak or Negative Correlation between Foreign Finance and Growth (section 3.2)

We analyze below the channels driving the complex relationship between the steady-state endowment, e^W, and the foreign debt, D^{For}, normalized by endowment. The intuition is as follows.

From Lemma B.2, we can decompose $\frac{D^{For}}{e^W}$ as the following:

$$\frac{D^{For}}{e^W} = \underbrace{\frac{\tau(t^W)/r}{e^W}}_{\sum \text{tax revenues}} - \underbrace{\frac{(\tau^{**} - C - z(1+r)s(e^W, t^W))/r}{e^W}}_{\text{willingness-to-pay wedge}} - \underbrace{\frac{s(e^W, t^W)}{e^W}}_{\text{domestic debt}} . \tag{D.1}$$

As ρ increases, the steady-state endowment is higher mechanically as households prefer endowment over consumption, but the repressive tax rate t^W decreases (see Fig. 10(a) and (b)). As a result, the first term on the right hand side in (**??**), which is proportional to tax revenues and inversely proportional to endowment, is decreasing.

[Fig. 10 about here]

However, rearranging slightly, the other terms on the right hand side are increasing in ρ. Since e^W increases with ρ, $-\frac{(\tau^{**}-C)}{e^W}$ is increasing in ρ. Furthermore, $\frac{s(e^W,t^W)}{e^W}$ is multiplied by a positive coefficient for z sufficiently high (note that for z close to or greater than one, $z\frac{(1+r)}{r} - 1 > 0$). This term is increasing in ρ since savings increase at a faster rate than the endowment as ρ increases.

When z is low, the first term in (**??**) can dominate and $\frac{D^{For}}{e^W}$ may be decreasing in ρ, as shown in Fig. 10(e), whereas e^W is increasing in ρ regardless of z (Fig. 10(c) and (d)). This gives rise to a *negative* relation between the foreign debt to endowment ratio and the steady-state endowment.

In contrast, when z is high, the term containing $\frac{s(e^W,t^W)}{e^W}$ dominates the decrease in repression so that the foreign debt normalized by endowment is increasing in ρ, giving rise to a *positive* relation between the foreign debt to endowment ratio and steady-state endowment.

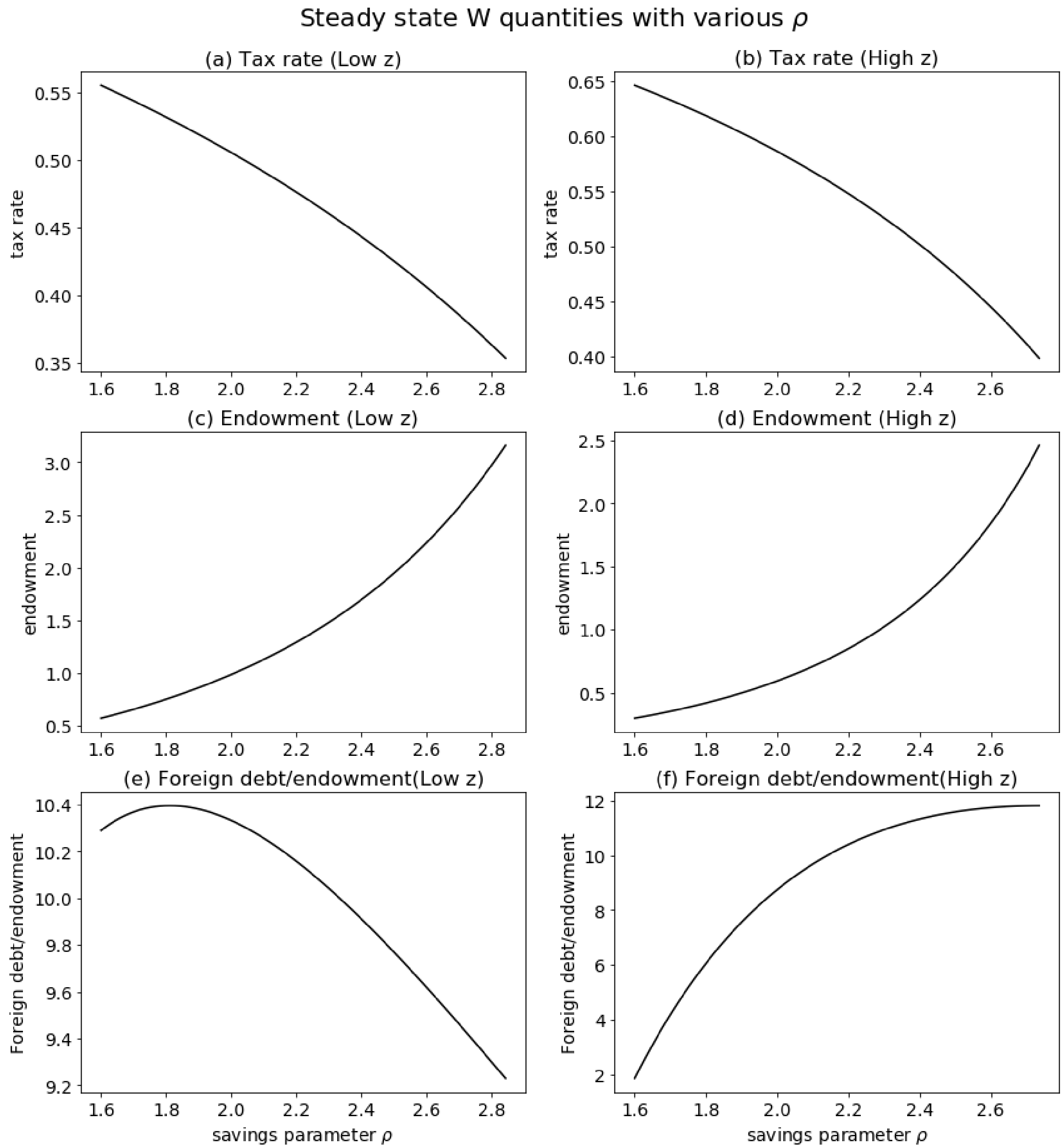

Figure 10: Comparative statics on ρ – households' propensity to save – to tax rates, endowments, and foreign debt normalized by endowment, in the willingness-to-pay steady state. The following parameters are used: $f = 3k^{.65}$, $r = 10\%$, $C = 1.0$, low $z = 1.1$, high $z = 2$.

E Robustness

E.1 Longer debt maturity

In our model, all government debt is short-term, maturing in the next period. In this section, we show that this assumption is immaterial to the main results of our paper. Intuitively, what matters regardless of the maturity of the debt is the net debt service. So long as the government can issue or buy back debt up to its riskless debt capacity, the net debt service will remain the same, regardless of the maturity of the debt issued.

To see this, assume that the government issues perpetual bonds which it can buy back or sell at the market price as warranted. Let us define D_i as the stock of perpetual debt that government i owes to the public at the end of period i. Now consider the next-period government's constraints. The government should be *able* to pay back the interest on debt stock:

$$rD_i \le S_{i+1}$$

The government should also be *willing* to repay:

$$\begin{aligned}
&\underbrace{S_{i+1} - rD_i}_{\text{net spending on no default}} \quad \ge \quad \underbrace{\tau^{**}}_{\text{revenues in autarky}} \quad - \underbrace{\left(C + zD_i^{Dom}(1+r)\right)}_{\text{spending to clean up default}} \\
\Rightarrow & rD_i \le S_{i+1} + zs_i(1+r) + C - \tau^{**}
\end{aligned}$$

Note that a household wanting to save s_i amount of wealth will hold that amount of domestic perpetuities, and sell it off at the next period. Combining the expressions above, assuming the myopic government maximizes borrowing,

$$rD_i = S_{i+1} - \max\{0, \tau^{**} - C - zs_i(1+r)\}$$

Now, the spendable amount S_{i+1} is:

$$S_{i+1} = \underbrace{(D_{i+1} - D_i)}_{\text{additional debt raised}} + \underbrace{\tau(t_{i+1})}_{\text{tax collected}}$$

Let us define $S_i' := S_i + D_{i-1} = D_i + \tau(t_i)$ and recursively formulate this problem on S'. Plugging this into the above equation, we get

$$\begin{aligned}
& rD_i = S_{i+1}' - D_i - \max\{0, \tau^{**} - C - zs_i(1+r)\} \\
\Rightarrow & (1+r)D_i = S_{i+1}' - \max\{0, \tau^{**} - C - zs_i(1+r)\}
\end{aligned}$$

$$\Rightarrow S_i' = \frac{1}{1+r}\Big[S_{i+1}' - \max\{0, \tau^{**} - C - zs_i(1+r)\}\Big] + \tau(t_i)$$

Therefore, we have the exact same form of recursion as in our base model. Long-term debt is identical to short-term debt as the government can always buy back and re-issue the bonds and given there is no default in equilibrium the price of debt remains unaffected. It follows that the endowment/tax rate paths are identical under this setup. Put differently, nothing hinges on the assumption that the government issues only short-term debt, it can issue debt of any maturity.

E.2 Productive government investment

We have assumed in the baseline model that the self-interested government simply spends on current wasteful projects. What if it has access to a productive technology which yields a cash flow of $g(I)$ for the government in the next period, in return for today's investment I? This is best thought of as investment in a state-owned steel plant or a toll road or climate change mitigation. We assume that the investment is made in the beginning of the current period, when the government undertakes other spending, and the return of the investment is at the beginning of the next period – this captures the notion that public projects are typically long term. We assume that the government technology g satisfies Inada conditions, *i.e.*, $g'(0) \to \infty$, $g' > 0$, $g'' < 0$.

Since $g(I)$ is generated only in the next period, the myopic current government does not enjoy the future cash flow *per se*. However, non-zero investment may still be in the government's incentive if it increases its debt capacity. Interestingly, the government will invest if it is in the ability-to-pay region, but not necessarily if it is in the willingness-to-pay region.

To see this, suppose for simplicity that the next period government's total surplus is S, not including the revenue from the long term investment, and the option to invest in technology g is only available to the current government. Note that the next-period government's ability-to-pay constraint, with respect to the current government's debt issuance D and investment I is now :

$$D(1+r) \leq S + g(I) \Rightarrow D \leq \frac{1}{1+r}\left(S + g(I)\right). \tag{E.1}$$

Clearly, if the next-period government is constrained by the ability to pay, an investment in government technology I increases the debt capacity of the current government by $\frac{1}{1+r}g(I)$. In contrast, the next-period government's willingness-to-pay constraint is:

$$S + g(I) - D(1+r) \geq \tau^{**} - \text{default cost} + g(I) \tag{E.2}$$

$$\Rightarrow D \leq \frac{1}{(1+r)}\left[S - \tau^{**} + \text{default cost}\right]. \tag{E.3}$$

Interestingly, if the next-period government is constrained by the willingness to pay, investment does not help the current government's debt capacity at all. Although the incremental cash flow $g(I)$ increases the net spending by the future government in case it honors the legacy debt, it also increases its net spending in the default state by exactly the same amount. The two effects offset each other in determining the willingness to pay constraint, so that the debt capacity is left unchanged by long-term investments.

We illustrate this in Fig. 11. The corresponding formal result is summarized in the following Lemma:

Lemma E.1. *The government's problem, with access to a technology that for investment I generates cash flow $g(I)$ acrruing to the next-period government, is characterized by the following Bellman equation:*

$$S(e) = \max_{t,I} \quad \left[\frac{1}{1+r}\left[S(e') + \min\{g(I), C + zs(1+r) - \tau^{**}\}\right] + \tau(t)\right] - I.$$

The optimal investment function $I(e)$ has the following property: $\exists \bar{e}^1_{gcf} < \bar{e}^2_{gcf}$ such that $\forall e < \bar{e}^1_{gcf}$, $I(e) = 0$, and $\forall e > \bar{e}^2_{gcf}$, $I(e) = I^{**} := \text{argmax}_i \left[\frac{1}{1+r} g(i) - i\right]$. In other words, governments in economies with low endowments may not see any value in spending productively, even if the technology exists.

Proof: First, note that since $g(I)$ is concave, the optimal I is always smaller or equal to I^{**}. We then consider the two limits of the endowment.

Consider $e \to 0$. For sufficiently small e, $C + zs(1+r) - \tau^{**} < 0$, implying that $\min\{g(I), C + zs(1+r) - \tau^{**}\} = C + zs(1+r) - \tau^{**}$ $\forall I \geq 0$. In this case, the dependence of the objective function on I only comes from the $-I$ term. Therefore, the maximum is achieved at $I = 0$, regardless of other values.

Then consider $e \to \infty$. For sufficiently large e, $C + zs(1+r) - \tau^{**} > g(I^{**})$, implying that $\min\{g(I), C + zs(1+r) - \tau^{**}\} = g(I)$ $\forall I \in [0, I^{**}]$. In this case, the optimization problem is separable for I, *i.e.*, $I(e) = \text{argmax}_i \left[\frac{1}{1+r} g(i) - i\right] = I^{**}$.

In the interim region, the optimal I is such that it slides between the two constraints, i.e., $g(I) = C + zs(1+r) - \tau^{**}$. ■

As we have noted earlier, countries with low endowments (developing countries) are likely to be in the willingness-to-pay region. The government of the developing country cannot take advantage of public investment opportunities, not because it is less capable or more corrupt than a rich-country government, but because the willingness-to-pay constraint binds more strongly. Effectively, public investment does nothing to alleviate this constraint, so the government sees no value in such investments. Once again, developing country governments according to this result are not intrinsically unwilling to make public investments, their circumstances give them less incentive to do so.[26]

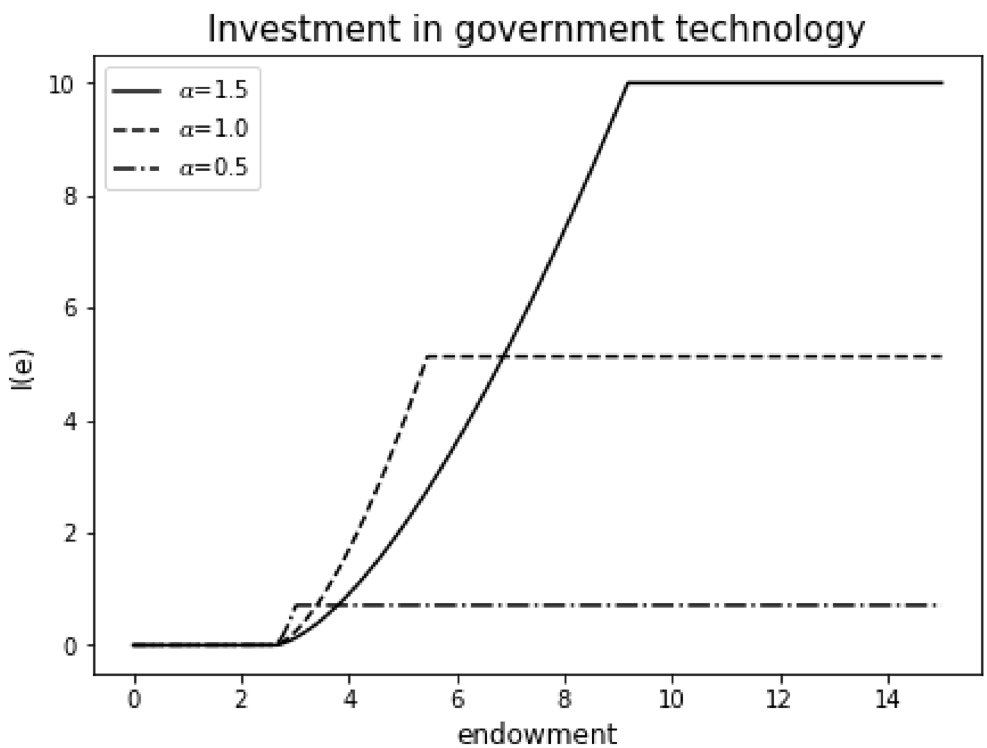

Figure 11: Numerical solution for the extension with government technology. α is the varied parameter, where $g(\cdot) = \alpha \times f(\cdot)$. All other parameters are the same as in Fig. 2; $f = 3k^{.65}$, $r = 10\%$, $z = 4$, $\rho = 2.3$ and $C = 1.0$.

F Small Shocks in the Trap Steady State (Section 6.1)

Proposition F.1. *Consider the government's spendables function $S(e; \rho, \phi, r)$ where ρ, ϕ, and r are savings parameter, productivity parameter, and interest rate, respectively. Partial derivatives of the spendables function with respect to e, ρ, ϕ, and r (for sufficiently low r), at steady state A and growth-trap steady state W, are as follows:*

$$\left.\frac{\partial S}{\partial e}\right|_{e^W} > 0, \quad \left.\frac{\partial S}{\partial \rho}\right|_{e^W} > 0, \quad \left.\frac{\partial S}{\partial \phi}\right|_{e^W} < 0, \quad \left.\frac{\partial S}{\partial r}\right|_{e^W} < 0; \ \textit{and}$$

[26] Note also that if the developing country made "seizable" investment abroad, which could be appropriated by the foreign lender in case of default, it would alleviate even the willingness-to-pay constraint, since the country would retain the fruits of the investment only if it serviced its debt. Somewhat perversely, this gives the poor developing country an incentive to make productive investments abroad rather than at home – foreign exchange reserves could be thought of as such an investment.

$$\left.\frac{\partial S}{\partial e}\right|_{e^A} = 0, \quad \left.\frac{\partial S}{\partial \rho}\right|_{e^A} = 0, \quad \left.\frac{\partial S}{\partial \phi}\right|_{e^A} > 0, \quad \left.\frac{\partial S}{\partial r}\right|_{e^A} < 0.$$

At steady states, a shock triggers default if and only if it decreases current spendables S. It follows then that

1. *In steady state W, a negative shock to endowment e, a negative shock to savings ρ, and a positive shock to productivity ϕ, all trigger default.*
2. *In steady state A, a negative shock to productivity ϕ triggers default.*
3. *A positive shock to interest rate r triggers default in both steady states.*
4. *Endowments in both steady states are positively related to savings ρ and productivity ϕ:*

$$\frac{\partial e^W}{\partial \rho}, \quad \frac{\partial e^W}{\partial \phi} > 0; \textit{ and}$$
$$\frac{\partial e^A}{\partial \rho}, \quad \frac{\partial e^A}{\partial \phi} > 0.$$