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ABSTRACT

We study the welfare implications of uncertainty in business cycle models. In the modern business cycle literature, multiplicative real shocks to production and/or preferences play an important role as the impulses that produce aggregate fluctuations. Introducing shocks in this way has the implication that fluctuating economies may enjoy higher welfare than their steady state counterparts. This occurs because purposeful agents make use of uncertainty in their favor. The result holds for a range of reasonable parameter values and in various models considered in the business cycle literature. One notable implication is that the welfare cost estimates which have been obtained in the literature using only consumption series may be biased and possibly seriously.

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1. Introduction

Robert Lucas (1987) obtained an upper bound estimate of the welfare gain from eliminating consumption risk by replacing postwar U.S. consumption with a consumption series without fluctuations. He assumed a representative agent with a constant relative risk aversion (CRRA) utility function. His estimates of the welfare cost of consumption fluctuations are very small, no more than 0.008 percent of aggregate consumption assuming logarithmic preferences. The fact that these estimates were so small stimulated interest in the issue of whether other features of the economy would significantly increase the estimated magnitude of the cost of aggregate fluctuations. Imrohroglu (1989) and Krusell and Smith (1999) introduced incomplete markets and uninsurable individual risk and found higher welfare costs. Cho et al. (1997) calculated the welfare cost of business cycle fluctuations in a model with nominal wage contracts. In their model, the welfare loss derives entirely from labor supply risk and the costs are higher than those found by Lucas. Obstfeld (1994) and Dolmas (1998) introduced non-expected utility type preferences and found much larger welfare costs associated with business cycles.

Recently, Alvarez and Jermann (2004) have obtained much larger estimates for the welfare cost of consumption fluctuations, using asset prices: their estimates range between 0.08 percent and 0.49 percent of lifetime consumption. Barro (2006)

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introduced disaster risk including war and obtained welfare costs of around 20 percent of GDP. Even with the usual economic fluctuations, he obtained costs of around 1.5 percent of GDP. Barlevy (2004) looked into the effect of uncertainty on growth and its consequences on welfare. Barlevy's estimates are about two orders of magnitude greater than Lucas' estimates.¹

This paper considers the welfare consequences of the shocks that generate business cycles. We argue that the technology shock in the real business cycle literature is not always detrimental to economic welfare. Since there are no distortions in prototypical real business cycle models like Kydland and Prescott (1982), Long and Plosser (1983), and Hansen (1985), aggregate fluctuations in these models still result in Pareto optimal allocations. It may seem natural to think that these fluctuating economies obtain lower welfare than their steady state counterparts, because the latter does not suffer from any uncertainty while the former does. We argue that this is not always correct. That is, economies with business cycle fluctuations may enjoy higher welfare than their steady state counterparts.

We understand that the last statement sounds counterintuitive. But, if we think of the way productivity shocks enter real business cycle models, the result is quite natural. The key to understanding how welfare could increase with uncertainty is to realize that the shocks to production are multiplicative and productive inputs like labor are variable. If there is a favorable productivity shock, output increases one-for-one, given the inputs. In addition, firms may employ more inputs with an increase in productivity so output can increase further. In other words, an increase in productivity will raise output more than proportionally and thus the reduced form (equilibrium) production function is convex with respect to the shock. Accordingly, introducing uncertainty through multiplicative productivity shocks raises average output.

The conventional way of thinking about the welfare costs of business cycles is this. Imagine that consumers are risk averse and offer these consumers two possible consumption streams, one which is constant and the other which has the same mean but fluctuates around the mean. Risk averse consumers would always prefer the smooth consumption stream and would require some additional average consumption to be indifferent between the two. This is the logic of the Lucas experiment and it is uncontroversial. This effect is the **fluctuations effect** of the uncertainty and it is always detrimental to welfare. But, suppose that consumers can take advantage of the uncertainty by working harder and investing more when productivity is high. In that case, the mean values of equilibrium output and consumption change with the uncertainty because agents try to make use of the uncertainty in their favor. We call this the **mean effect** of the business cycle uncertainty. If the mean decreases with uncertainty, economic welfare worsens and the uncertainty unequivocally lowers welfare. However, if the mean increases, and if the mean effect dominates the fluctuations effect, welfare increases with uncertainty. To correctly measure the welfare cost of business cycles, we have to know something about the size of the two effects. That, in turn, depends on how risk averse the agents are and how the uncertainty enters the model economy. Note, however, that the conventional approach is to look only at the fluctuations effect and that alone will always lead one to conclude that business cycle uncertainty reduces economic welfare.

We emphasize that, for uncertainty to increase the economic welfare, it has to be multiplicative to the choices which can be adjusted in response to it. That is, the mean effect is positive in the case of multiplicative shocks and so there is a possibility of welfare increasing with the shocks. In the case of additive shocks, the mean effect is negative in most of the cases of which we are aware and, thus, there is no possibility of welfare increasing with them. Multiplicative shocks include technology shocks, which are used extensively in the literature, preference shocks, seasonal shocks, investment specific shocks, shocks to income tax rates etc. Examples of shocks that are usually additive encompass monetary shocks, government expenditure shocks etc. In sum, economic welfare may increase with uncertainty because purposeful agents can make use of shocks in their favor, which is possible when the shocks are multiplicative to endogenous choices.

We examine the welfare costs of uncertainty in dynamic general equilibrium models where shocks are a source of fluctuations. In each of the cases considered in the paper we contrast two Pareto optimal economies; one subject to uncertainty and hence fluctuating, and the other one at its steady state. We then see which economy obtains higher utility. We show many cases where the economy with uncertainty has higher utility than the counterpart steady state economy. This result is robust to the range of reasonable parameter values typically considered in the literature.

The next section presents two examples, which illustrate the issue. Section 3 presents the welfare analysis in a prototypical real business cycle model and looks for the range of parameter values that yield welfare gains under business cycle uncertainty. We consider closed and open economies to highlight the effects of adjusting capital and labor separately. Section 4 discusses several related issues. Section 5 concludes.

2. Examples

The welfare cost of uncertainty depends on whether economic agents have some means to make use of the uncertainty in their favor. The first example shows that if there are no such means, uncertainty is certainly detrimental to economic welfare as in Lucas (1987). But the second example shows that if the agents have some endogenous choices that allow them to make use of the uncertainty in their favor, an increase of economic welfare with uncertainty is possible.

¹ This paper has been circulated since 1999. Since then there have been many important contributions in the cost of business cycle literature. In particular, the cost of individual risk together with the cost of aggregate risk have been studied in depth. Contributions include Heathcote et al. (2008, 2009), Storesletten et al. (2001), De Santis (2007), Krebs (2003, 2007) etc. These contributions are discussed in Section 4.

2.1. An endowment economy

Consider the following endowment economy, which is basically identical to the one considered by Lucas (1987). The representative agent maximizes the following lifetime utility.

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} c_t^{1-\sigma} \right\}, \tag{1}$$

where E_0 is the expectations operator conditional on the initial period information Ω_0 , c_t is the period t consumption, $0 < \beta < 1$ is the utility discounting factor, and $\sigma > 0$ is the relative risk aversion parameter. The agent faces the following constraint for consumption in each period:

$$c_t \leq e_t, \tag{2}$$

where e_t is the endowment in period t . Assume that e_t follows an *i.i.d.* process.

$$\ln(e_t) = \varepsilon_t, \tag{3}$$

where $\varepsilon_t \sim i.i.d. N(-\gamma_\varepsilon^2/2, \gamma_\varepsilon^2)$. That is, e_t follows an *i.i.d.* log-normal process.² If we assume (3), we have $E(e_t) = 1$ and $Var(e_t) = \exp(\gamma_\varepsilon^2) - 1$. Hence a change in the variance of ε_t is a mean-preserving spread of the endowment shocks.³

If we use the endowment process (3) in the lifetime utility (1), we have the lifetime utility:

$$U_0 = \frac{1}{(1-\sigma)(1-\beta)} \cdot \exp\left(-\frac{\sigma(1-\sigma)\gamma_\varepsilon^2}{2}\right). \tag{4}$$

Now it is straightforward to take the derivative:

$$\frac{\partial U_0}{\partial \gamma_\varepsilon^2} = -\frac{\sigma}{2(1-\beta)} \cdot \exp\left(-\frac{\sigma(1-\sigma)\gamma_\varepsilon^2}{2}\right) < 0. \tag{5}$$

Economic welfare decreases unequivocally with the uncertainty.⁴ Note that the agent cannot alter anything in this setup in response to uncertainty and so is not able to make use of it. The next example is one where an agent can alter the labor input.

2.2. An economy with endogenous labor

This example has endogenous labor so the agent can choose to supply labor in the way that makes use of multiplicative productivity shocks. Consider the following real business cycle model. The representative agent is assumed to have the preferences:

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} (c_t - \alpha n_t)^{1-\sigma} \right\}, \tag{6}$$

² Assuming more realistic process for e_t does not change the result qualitatively. The key is that the agent does not have any means to make use of the uncertainty.

³ To see this, suppose a random variable X has a log-normal distribution as:

$$\ln(X) \sim N(\mu, \gamma^2).$$

Then the mean and variance of X can be obtained as:

$$E(X) = \exp\left(\mu + \frac{\gamma^2}{2}\right), \quad Var(X) = \exp(2\mu + \gamma^2)[\exp(\gamma^2) - 1].$$

Hence the mean of X changes whenever the value of γ^2 changes. To have a distribution of X whose mean does not depend on γ^2 , we have to change the distribution of $\ln(X)$ as:

$$\ln(X) \sim N\left(\Gamma - \frac{\gamma^2}{2}, \gamma^2\right),$$

where Γ is a constant. Then we can have the mean and variance of X as follows:

$$E(X) = \exp(\Gamma), \quad Var(X) = \exp(2\Gamma)[\exp(\gamma^2) - 1],$$

and hence the mean of X does not depend on γ^2 . Now a change in γ^2 means a mean preserving spread of the random variable X .

⁴ As Lucas (1987) shows, a deterministic trend in endowment (and hence in consumption) does not change the result.

where E_0 is the expectations operator conditional on the initial period information Ω_0 , c_t is the period t consumption and n_t represents hours of work in the period. β is the utility discounting factor, and σ and α are preference parameters. We assume that $0 < \beta < 1$, $\sigma > 0$ and $\alpha > 0$. Because this preference specification abstracts from wealth effects, it keeps the problem simple. It has been used by many authors including [Greenwood et al. \(1988\)](#) and [Hercowitz and Samson \(1992\)](#). We assume that output is produced according to the production function:

$$y_t = A_t k_t^\theta n_t^{1-\theta}, \quad (7)$$

where y_t , A_t , k_t denote output, productivity shock, capital stock in period t and $0 < \theta < 1$. The capital stock obeys the law of motion:

$$k_{t+1} = (1 - \delta)k_t + i_t, \quad (8)$$

where i_t denotes investment, δ the depreciation rate and k_0 the initial capital stock, which is given. To have a closed-form solution, we consider the case when there is full depreciation, i.e. δ is equal to one.⁵ We also assume that the productivity shock follows an *i.i.d.* log-normal process. Specifically, assume:

$$\ln(A_t) \sim N\left(-\frac{\tau^2}{2}, \tau^2\right). \quad (9)$$

That is, as in the previous example, A_t follows an *i.i.d.* log-normal process. Now the mean and variance of A_t can be obtained as in the previous example and we write them here for later reference:

$$E(A_t) = 1, \quad \text{Var}(A_t) = \exp(\tau^2) - 1. \quad (10)$$

Hence a change in τ^2 implies a mean preserving spread of A_t .

Following [Rothschild and Stiglitz \(1970, 1971\)](#), if we define an increase in uncertainty by a mean preserving spread of the distribution, the critical value for the uncertainty to increase utility can be obtained as⁶:

$$\frac{1 - \sigma}{\theta} = 1 \iff \theta + \sigma = 1. \quad (11)$$

If (11) holds, the lifetime utility does not depend on τ^2 and hence the mean preserving spread of the distribution of the shock does not affect the expected lifetime utility. In other words, the fluctuations and the mean effect are balanced. However, if $\theta + \sigma > 1$, the lifetime utility function is concave in the shock and hence the fluctuations effect dominates the mean effect. In this case, the conventional wisdom holds, i.e. the uncertainty reduces welfare. On the other hand, if $\theta + \sigma < 1$, the lifetime utility function is convex in the shock and the mean effect dominates the fluctuations effect. That is, welfare increases with uncertainty. This result confirms that the effect of uncertainty on the economy depends critically on the parameters determining the elasticity of labor demand, i.e. θ , and risk aversion, i.e. σ .⁷ In other words, if the elasticity of labor demand is large and/or if the degree of risk aversion is small, the possibility that uncertainty increases economic welfare is higher.

3. Productivity shocks and welfare

The examples just presented make it clear that the welfare of an economy can increase with the introduction of uncertainty, even if the parameter values required for this to happen make it seem unlikely. The key to the result is the endogenous choice, i.e. labor choice in the previous examples, which can be made by the agents to make use of the uncertainty in their favor. There are many endogenous choices for agents facing uncertainty. In this section we consider capital and labor. First, we consider an economy having only capital as a variable input. Then we add labor together with capital. Finally, we will open the economy and let capital move across countries as in [Backus et al. \(1992, 1995\)](#).

Since the seminal papers of [Kydland and Prescott \(1982\)](#) and [King et al. \(1988a, 1988b\)](#), linear approximation methods have been a workhorse to approximate the solution to non-linear, dynamic, stochastic, general equilibrium models. If shocks driving aggregate fluctuations are “small” enough, first-order approximations are found to be remarkably accurate to characterize the local existence and determinacy of equilibrium solutions and to generate the second moments of associated endogenous variables. The indisputable consensus, however, is that first-order approximation techniques are inadequate for

⁵ Thus, as shown in [Appendix A](#), the resource constraint in this model economy reads as

$$c_t + k_{t+1} = y_t,$$

for every $t \geq 0$.

⁶ See [Appendix A](#) for derivation.

⁷ The elasticity of labor demand is $1/\theta$. Note also that the elasticity of labor supply matters in more general cases which will be studied later.

welfare comparisons across alternative stochastic environments.⁸ It is generally accepted that a correct second-order approximation of the equilibrium welfare function requires a second-order approximation to policy functions. Therefore, to obtain accurate welfare evaluations, we employ second-order perturbation methods developed by Schmitt-Grohe and Uribe (2004, 2007) for the numerical solution to the relevant business cycle models discussed in this study.⁹

3.1. Closed economies

Capital and labor are both variable factors of production and they also enter the production technology in a multiplicative way. Therefore, it seems necessary to analyze the welfare effect of being able to adjust capital and/or labor in response to productivity shocks. The example in the previous section illustrates the case when the welfare gain results from adjusting labor. Here we consider the welfare effects from adjusting capital with the labor supply fixed. The corresponding recursive representation of the representative agent's problem can be written as:

$$\begin{aligned}
 V(A_t, k_t) = \max & \left\{ \frac{1}{1-\sigma} c_t^{1-\sigma} + \beta E_t[V(A_{t+1}, k_{t+1})] \right\} \\
 \text{s.t.} & \quad (1) \ c_t + i_t = A_t k_t^\theta \\
 & \quad (2) \ k_{t+1} = (1-\delta)k_t + i_t \\
 & \quad (3) \ \ln(A_t) = \rho \ln(A_{t-1}) + \varepsilon_t \\
 & \quad (4) \ c_t \geq 0, k_0 \text{ is given,}
 \end{aligned} \tag{12}$$

where i_t denotes investment and the productivity shock innovation, ε_t , has an *i.i.d.* normal distribution $N(-\frac{\tau_\varepsilon^2}{2(1+\rho)}, \tau_\varepsilon^2)$.¹⁰ The parameter values are set as: $\beta = 0.99$, $\theta = 0.36$, $\delta = 0.025$, $\rho = 0.95$. We allow σ to vary to see the effect of risk aversion. We have also simulated the cases of τ_ε between 0.003 and 0.019.¹¹ The welfare effects derived from endogenous capital adjustment in the model (12) are summarized in Fig. 1.¹² The welfare gain or loss depends critically on the risk

⁸ Kim and Kim (2003) find that in a simple open economy, the welfare evaluations based on a first order approximation to the equilibrium policy functions could be erroneous but the appropriate second-order approximations can bring correct welfare rankings.

⁹ We have used their computing programs posted in their homepages.

¹⁰ If we characterize the mean and variance of the shock innovation ε_t in this way, the unconditional mean and variance of A_t is as follows. First, if we represent the technology shock process with an MA process, we have:

$$A_t = \exp \left\{ \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-j} \right\}.$$

For a normal random variable X , we can have the following.

$$E[\exp(aX)] = \exp \left\{ aE(X) + \frac{a^2 \text{Var}(X)}{2} \right\},$$

where a is an arbitrary parameter. Hence the first two moments of A_t can be obtained as:

$$\begin{aligned}
 E(A_t) &= \prod_{j=0}^{\infty} \exp \left[\rho^j E(\varepsilon_t) + \frac{\rho^{2j} \text{Var}(\varepsilon_t)}{2} \right] = \exp \left\{ \sum_{j=0}^{\infty} \left[\rho^j E(\varepsilon_t) + \frac{\rho^{2j} \text{Var}(\varepsilon_t)}{2} \right] \right\}, \\
 E(A_t^2) &= \prod_{j=0}^{\infty} \exp [2\rho^j E(\varepsilon_t) + 2\rho^{2j} \text{Var}(\varepsilon_t)] = \exp \left\{ \sum_{j=0}^{\infty} [2\rho^j E(\varepsilon_t) + 2\rho^{2j} \text{Var}(\varepsilon_t)] \right\}.
 \end{aligned}$$

Now if we substitute the mean and the variance of ε_t in the above expression, we can verify that the mean of A_t is 1. In addition, since

$$\text{Var}(A_t) = E(A_t^2) - [E(A_t)]^2,$$

we can write the variance of A_t as:

$$\text{Var}(A_t) = \sum_{j=0}^{\infty} [\exp(\rho^{2j} \tau_\varepsilon^2) - 1].$$

A change in the variance of ε_t , τ_ε^2 , is a mean preserving spread of the random variable A_t in the sense of Rothschild and Stiglitz (1970, 1971).

¹¹ Numerous authors including Hansen (1985) have used 0.007 for τ_ε .

¹² Following Lucas (1987), our welfare measure is defined to be the percentage change in consumption, uniform across all dates and values of the shocks, required to leave the representative consumer indifferent between consumption instability due to A_t and a perfectly smooth consumption path; it is denoted by:

$$\frac{\Delta c}{\bar{y}} \times 100,$$

where \bar{y} is the steady state output.

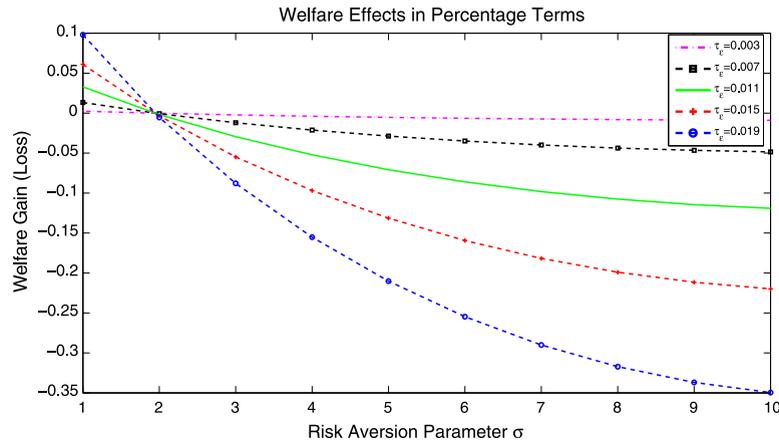


Fig. 1. Variable capital and fixed labor.

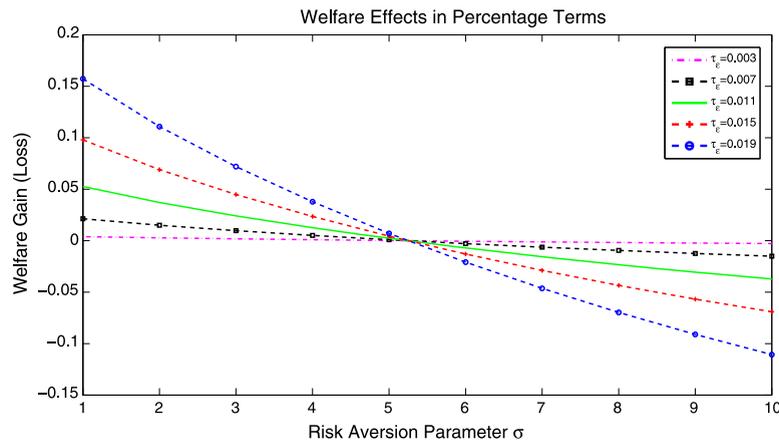


Fig. 2. Variable capital and variable labor.

aversion parameter σ . The critical value for σ at which the effect of uncertainty changes from beneficial to detrimental is less than 2, but purposeful agents in the model economy can still make use of uncertainty in their favor by adjusting capital via investment. The implied welfare measure has the maximal upper bound of about 0.1% of output.

Fig. 2 presents the welfare gain (loss) from being able to adjust both capital and labor jointly. Note that the recursive representation of the benchmark business cycle model can be written as:

$$\begin{aligned}
 V(A_t, k_t) = \max & \left\{ \left(\frac{1}{1-\sigma} \right) [c_t^\alpha (1-n_t)^{1-\alpha}]^{1-\sigma} + \beta E_t[V(A_{t+1}, k_{t+1})] \right\} \\
 \text{s.t.} & \quad (1) \ c_t + i_t = A_t k_t^\theta n_t^{1-\theta} \\
 & \quad (2) \ k_{t+1} = (1-\delta)k_t + i_t \\
 & \quad (3) \ \ln(A_t) = \rho \ln(A_{t-1}) + \varepsilon_t \\
 & \quad (4) \ c_t, i_t \geq 0, 0 \leq n_t \leq 1, k_0 \text{ is given.}
 \end{aligned} \tag{13}$$

The parameter values are set as: $\beta = 0.99$, $\alpha = 0.35$, $\theta = 0.36$, $\delta = 0.025$, $\rho = 0.95$. Relative to the case with variable capital only (Fig. 1), the welfare gains are obviously larger. The critical value for σ is greater than 5, which is more than doubled, and the relative size of welfare gains is almost doubled as well. The welfare gain now has a maximal upper bound of 0.16% of output. Note that the critical value for σ to imply a welfare gain with uncertainty is much higher than in the simple examples in the previous section.¹³ This model economy differs in two important ways from the simple examples. First, the shock is now persistent. Second, there is realistic capital accumulation so intertemporal substitution is more meaningful. If there is a favorable realization of the shock, they will work longer, produce more and save more. This means higher production efficiency because of the convexity of the reduced form production function with respect to the shock. Since

¹³ Since $\theta = 0.36$, the critical value in the example in Section 2 is $\sigma = 0.64$.

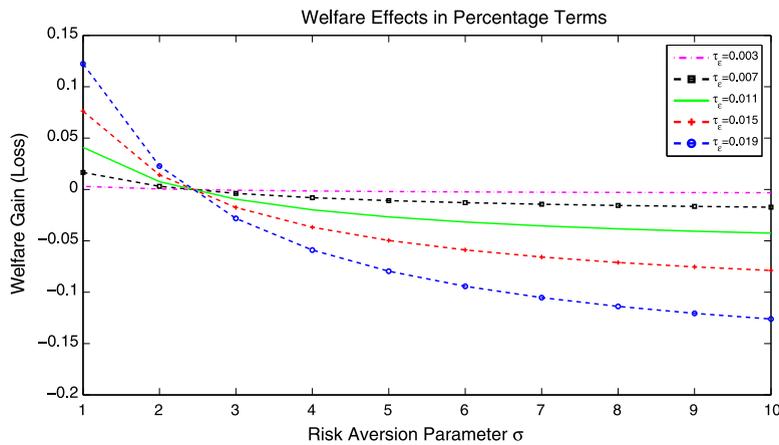


Fig. 3. Separable preferences (risk aversion).

most of the models in the real business cycle literature have assumed a value of σ around 2, uncertainty is beneficial in those models.¹⁴ In sum, given any value of σ which is less than 5, increasing uncertainty raises the welfare of the economy.

To separate the welfare effect of risk aversion and the intertemporal substitution elasticity of labor supply, we introduce the following separable preferences:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - B \frac{n_t^{1+\gamma}}{1+\gamma} \right) \right\}$$

where $\frac{1}{\gamma}$ represents the Frisch elasticity of labor supply.¹⁵ With $\gamma = 1$, Fig. 3 represents the welfare effects of risk aversion σ . Given the size of the uncertainty,¹⁶ τ_ϵ , the welfare gain decreases with the value of σ . The critical value for σ at which the effect of uncertainty turns from beneficial to detrimental is about 2.5. It takes place at the value where the curves intersect in the figure. Models with preferences that are separable between consumption and leisure usually assume (at least in the real business cycle literature) that preferences are logarithmic in consumption ($\sigma = 1$), where uncertainty is beneficial.

With $\sigma = 1$, Fig. 4 shows the welfare effect of the intertemporal substitution elasticity of labor supply. The welfare gain decreases with the value of γ . However, given the logarithmic utility of consumption, the welfare gain from labor supply uncertainty is always positive over a plausible range of the value of γ . This can be explained as follows. First of all, increasing the value of γ makes the agent more risk averse to labor supply uncertainty. However, increasing the value of γ makes the economy fluctuate less. Fig. 4 shows that the latter effect dominates the former one.

3.2. Comparison with Lucas (1987)

It is illuminating to compare our welfare result with that of Lucas (1987). One notable feature of Lucas' welfare analysis is that the standard deviation of log consumption about trend is calibrated to be about 0.013; by contrast, the benchmark parametrization of the size of TFP shock τ_ϵ is 0.00712 in all business cycle models studied in our welfare analysis, which leads to unambiguously smaller consumption volatility than Lucas' calculation. Thus, for the comparison with Lucas, we increase the size of TFP shock τ_ϵ to 0.021 in the model with Cobb–Douglas preferences so that the consumption volatility σ_c is equal to 0.013 as in Lucas (1987).¹⁷

¹⁴ See, for example, Prescott (1986), Stockman and Tezar (1995), and Backus et al. (1992).

¹⁵ We set the parameter values except those of the preference as in the previous economy. The values of the preference parameters are set in the following way. First of all, we assume that $B = 3$, which implies the hours of work are about one third of endowment of time when $\sigma = 1$ and $\gamma = 1$. The values of the other two preference parameters are allowed to vary to see the impact of the changes on the welfare cost of the business cycle. When the value of the risk aversion parameter σ varies, that of γ is assumed to be 1. This value implies the elasticity of intertemporal substitution to be 1. Although the estimates of the elasticity for women and youth are much higher than 1 according to the micro studies of labor supply, this value is a bit higher than the estimates for men obtained in the literature (see Pencavel, 1986; Killingsworth and Heckman, 1986). However, recent aggregate labor studies like Alogoskoufis (1987) and Cho et al. (1998) obtained the estimate of the elasticity higher than 1. In fact, the intertemporal elasticity of labor supply is assumed to be much higher than 1 in most real business cycle models. On the other hand, when the value of the elasticity of labor supply parameter is allowed to vary, we assume the value of σ to be 1. This value implies the logarithmic preferences and it has been used in the literature numerous times (for example, see Burnside and Eichenbaum, 1996).

¹⁶ Recall that τ_ϵ is assumed to take a value between 0.003 and 0.019, which includes almost all the values used in the real business cycle literature.

¹⁷ To facilitate further comparison, the welfare measure now will be presented as

$$\frac{\Delta c}{\bar{c}} \times 100$$

where \bar{c} is the steady state consumption.

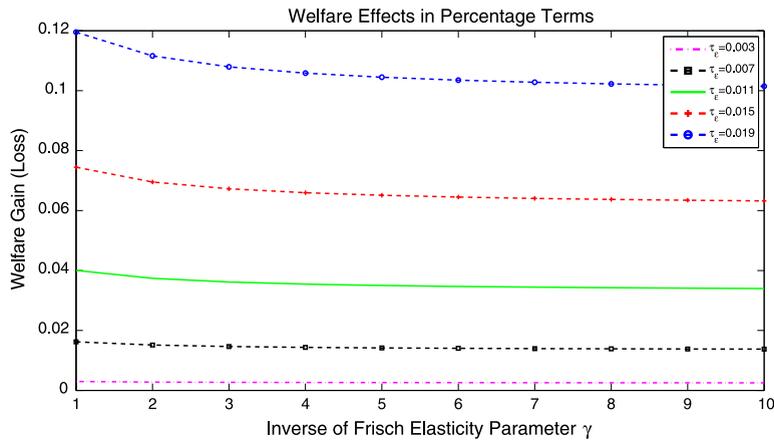


Fig. 4. Separable preferences (labor supply elasticity).

Table 1
Cost of consumption instability: comparison.

Model	Risk aversion σ		
	1	5	10
Lucas (1987) ^a	-0.008%	-0.042%	-0.084%
Baseline ^b	0.258%	0.0213%	-0.175%
Mean effect	0.590%	0.798%	1.266%

^a Adapted from Table 2 in Lucas (1987).

^b RBC model with Cobb–Douglas preferences.

As would be expected, given the same variability of consumption as in Lucas ($\sigma_c = 0.013$), economic fluctuations can be beneficial when agents can take advantage of fluctuations in productivity by choosing savings and labor endogenously (see Table 1). When such endogenous choices are introduced, however, a high risk aversion (e.g. $\sigma = 10$) can also dramatically increase the welfare cost of business cycles relative to the case without endogenous choices, since agents with higher risk aversion want smoother consumption streams and indeed achieve them by adjusting capital and labor endogenously.

In this context, it is also possible to make a distinction between the fluctuations and mean effect of business cycle uncertainty. Again, we increase the size of TFP shock τ_ϵ to 0.021 in the baseline model with Cobb–Douglas preferences so that the consumption volatility σ_c is equal to 0.013. Without any endogenous choices, risk-averse consumers would always suffer from welfare costs: these costs originate from the fluctuations effect. However, once we allow for endogenous adjustment of labor and capital, the mean effect of the uncertainty is unequivocally operative.¹⁸ In a low-risk aversion world (i.e. $\sigma = 1$), the mean effect of the business cycle uncertainty is 0.590%, which will surpass the overall welfare gains of 0.258%. As conjectured, both hours flexibility and capital accumulation can reduce the welfare cost of business fluctuations. Without the mean effect, *ceteris paribus*, the fluctuations effect, which is unambiguously detrimental to welfare, is approximately estimated to be -0.332%.

As a low risk-aversion world evolves to a high risk-aversion world, the mean effect is striking. By contrast, in a high risk aversion world (i.e. $\sigma = 10$), consumers have a stronger incentive to smooth consumption: they invest more due to precautionary savings and work harder due to labor flexibility. The overall effect is that the mean effect is dominated by the fluctuations effect as in Lucas (1987). However, the corresponding mean effect is now a huge number: 1.266% of the steady state consumption. This finding is not inconsistent with our general intuition: consumers can take advantage of the uncertainty by working harder and investing more when productivity is high.

¹⁸ Mimicking Floden (2001) and Heathcote et al. (2008), the mean effect is defined as the value for ω^{mean} that solves the following equation:

$$u((1 + \omega^{\text{mean}})\bar{c}, 1 - \bar{n}) = u(E(c(A_t, k_t)), 1 - E(n(A_t, k_t)))$$

where $u(c, 1 - n)$ represents Cobb–Douglas preferences. Here $c(A_t, k_t)$ and $n(A_t, k_t)$ are the policy functions of consumption and labor, respectively. Since Cobb–Douglas preferences satisfy the homogeneity property such that $u(xc, 1 - n) = g(x)u(c, 1 - n)$ for any x and some g , Proposition 1 in Floden (2001) is immediately applicable and thus our total welfare effect can be decomposed into two components, the mean effect ω^{mean} and the fluctuations effect $\omega^{\text{fluctuations}}$. In other words, the total welfare effect $\frac{\Delta c}{\bar{c}}$ can be expressed as

$$\frac{\Delta c}{\bar{c}} \simeq \omega^{\text{mean}} + \omega^{\text{fluctuations}},$$

up to second-order terms.

3.3. Open economies

In a closed production economy, we have demonstrated that the welfare effects of being able to adjust capital in response to productivity shocks could be beneficial, independently of being able to adjust labor. Nevertheless, adjusting capital over the business cycle is easier when the economy is open. Capital can be imported freely from abroad (Backus et al., 1992, 1995). In order to capture the welfare effects of importing and increasing capital in sync with positive productivity shocks, we consider a streamlined version of the model of Backus et al. (1992, 1995) in which inventory accumulation, leisure durability and the time-to-build structure of capital accumulation have been eliminated.¹⁹

In this economy, each country's representative agent has the following lifetime utility:

$$U_j = E_0 \sum_{t=0}^{\infty} \beta^t U(c_{jt}, 1 - n_{jt})$$

where

$$U(c_{jt}, 1 - n_{jt}) = \frac{[(c_{jt})^\alpha (1 - n_{jt})^{1-\alpha}]^{1-\sigma}}{1 - \sigma} \quad (j = h, f)$$

and h and f denote home and foreign country. Output in country j is

$$y_{jt} = A_{jt} (k_{jt})^\theta (n_{jt})^{1-\theta}.$$

The world resource constraint for the single good which the two countries produce is

$$c_{ht} + c_{ft} + i_{ht} + i_{ft} = y_{ht} + y_{ft}.$$

The capital accumulation technology is given by:

$$k_{jt+1} = (1 - \delta)k_{jt} + x_{jt}.$$

For simplicity's sake, we assume that $\text{corr}(\log A_{ht}, \log A_{ft}) = 0$ and the shock processes are

$$\begin{pmatrix} \log A_{ht+1} \\ \log A_{ft+1} \end{pmatrix} = \begin{pmatrix} \rho & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} \log A_{ht} \\ \log A_{ft} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{ht+1} \\ \varepsilon_{ft+1} \end{pmatrix}.$$

The recursive representation is:

$$\begin{aligned} V(k_{ht}, k_{ft}, A_{ht}, A_{ft}) = \max & \quad \chi U(c_{ht}, 1 - n_{ht}) + (1 - \chi)U(c_{ft}, 1 - n_{ft}) \\ & \quad + \beta E_t[V(k_{ht+1}, k_{ft+1}, A_{ht+1}, A_{ft+1})] \\ \text{s.t.} & \quad (1) \quad c_{ht} + c_{ft} + i_{ht} + i_{ft} = y_{ht} + y_{ft} \\ & \quad (2) \quad y_{ht} = A_{ht} (k_{ht})^\theta (n_{ht})^{1-\theta} \\ & \quad (3) \quad y_{ft} = A_{ft} (k_{ft})^\theta (n_{ft})^{1-\theta} \\ & \quad (4) \quad k_{ht+1} = (1 - \delta)k_{ht} + i_{ht} \\ & \quad (5) \quad k_{ft+1} = (1 - \delta)k_{ft} + i_{ft} \\ & \quad (7) \quad \log A_{ht+1} = \rho \log A_{ht} + \varepsilon_{ht+1} \\ & \quad (8) \quad \log A_{ft+1} = \rho \log A_{ft} + \varepsilon_{ft+1}, \end{aligned} \tag{14}$$

where χ is the corresponding Negishi–Mantel weight.

It will suffice to experiment with the case where productivity shocks occur only in the home country. In response to productivity shocks A_{ht} , the representative agent in the country h will be able to adjust capital by investing within the country h and also importing capital from the foreign country f .²⁰ Note that in our theoretical economy a *symmetric* competitive equilibrium with complete contingent claims markets is implemented via the optimum problem (14) in which we maximize $\chi u_h + (1 - \chi)u_f$ subject to the technology and the world resource constraint.²¹ In this competitive equilibrium consumers

¹⁹ Klein and Ventura (2009) studied the welfare effect of removing barriers to labor mobility across countries. They found large welfare gain from removing the barriers.

²⁰ In the simplified version of the Backus–Kehoe–Kydland model, the welfare measure will be defined as follows:

$$\frac{\Delta c^h}{\bar{y}_h} \times 100$$

where Δc^h is the home country's gains (costs) in terms of consumption goods and \bar{y}_h is the steady state output in home country.

²¹ What is meant by “symmetric” is that the Negishi–Mantel weights are equal, i.e. $\chi = \frac{1}{2}$.

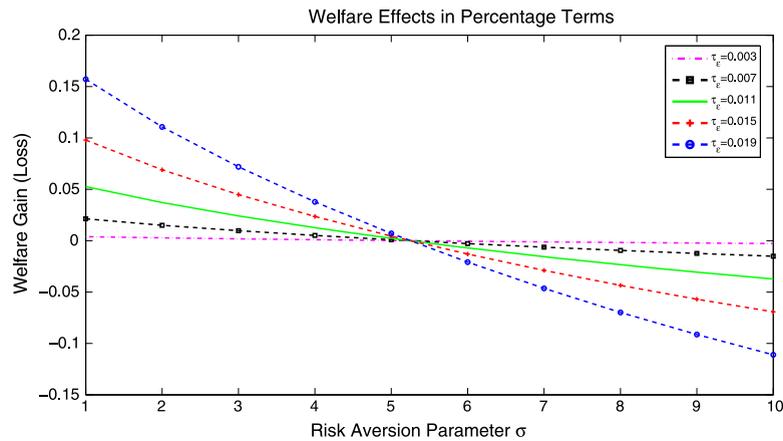


Fig. 5. Variable capital cum variable labor in open economy (autarchy).

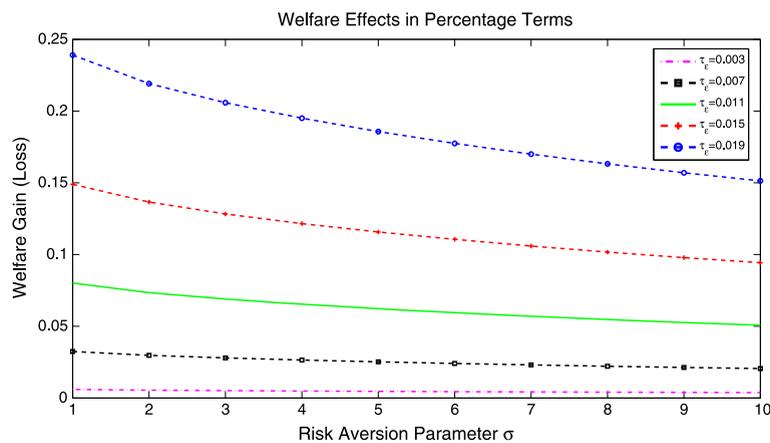


Fig. 6. Variable capital cum variable labor in open economy (risk-sharing).

can trade complete contingent claims to diversify country-specific risk and thus the marginal utility of consumption is equated across countries for each state of nature. In other words, agents in the model have the ability to shift perfectly substitutable goods costlessly and to trade them in complete markets for state-contingent claims. When the home country only experiences a rise in productivity, for instance, resources are shifted without any frictions to the more productive location, the home country.

To highlight the welfare effects of freely mobile capital in the open economy, we first consider an extreme *autarchy* equilibrium, in which we eliminate from our two-country model all trade in goods and assets across two countries but allow for productivity shocks only in the home country. In this autarchy equilibrium, our two-country model should be identical to our benchmark closed economy with Cobb–Douglas preferences when productivity shocks in the home country have the same magnitude as their closed economy counterparts. As a result, welfare gains from business cycles should be the same. Indeed, as would be anticipated, the two-country model under autarchy (Fig. 5) displays the exactly same welfare effects of business cycles as the closed economy counterpart (Fig. 2). Thus, our guess is that when there are no impediments to importing capital from abroad, welfare gains from business cycles should be larger than in the closed economy (interchangeably, the autarchy economy). Fig. 6 presents the welfare effects of the simplified version of the Backus–Kehoe–Kydland model, where both capital and labor are varying. In the model, business cycles are always beneficial: across the “reasonable” range for risk aversion, σ , between 1 and 10, business cycles can always be exploited by agents in their favor, resulting in welfare gains. As would be expected, the relative extent of welfare gains is much larger and more persistent than in the closed economy counterparts (the implied welfare measure has a maximal upper bound of 0.24%).

4. Discussion

The magnitude of welfare changes with uncertainty depends on the means by which the agents can make use of the uncertainty in their favor. The means in the previous sections are labor and capital. There can be many other means which can be used by the agent in an uncertain economic environment. One notable means that we can think of is cyclical factor utilization (for example see [Bils and Cho, 1994](#); [Greenwood et al., 1988](#)). If the intensity of the use of a production input

can be adjusted procyclically, the welfare gain can be larger. For example, consider an agent with the same preferences as in the second example in Section 2, (6), but with the following production function:

$$y_t = A_t (n_t^\psi k_t)^\theta n_t^{1-\theta}, \quad 0 < \psi < 1, \quad (15)$$

where n_t^ψ is a rate of capital utilization as a function of labor input.²² If the agent has this additional margin of adjustment, the equilibrium output can be obtained as:

$$y_t = \left(\frac{1 - \theta(1 - \psi)}{\alpha} \right)^{[1 - \theta(1 - \psi)] / [\theta(1 - \psi)]} A_t^{1/[\theta(1 - \psi)]} k_t^{1/(1 - \psi)}. \quad (16)$$

Note that without cyclical factor utilization, the equilibrium output in this economy is written as

$$y_t = \left(\frac{1 - \theta}{\alpha} \right)^{[1 - \theta]/\theta} A_t^{1/\theta} k_t. \quad (17)$$

The equilibrium production function (16) exhibits more curvature in the shock A_t than its benchmark counterpart (17), which means that the mean effect can be much larger with the factor utilization. Procyclical factor utilization can also take place along labor effort margin, which has a similar effect.

Another margin that can be used by the agents is home production (for example, see [Benhabib et al., 1991](#); [Greenwood and Hercowitz, 1991](#); [Greenwood et al., 1995](#)). First of all, if the total working hours at home and in the marketplace stay stable and only the composition changes in response to shocks to market production, the welfare loss due to hours and consumption fluctuations will be minimized and hence the possibility of welfare increasing with business fluctuations will rise. Furthermore, shocks to home production can be interpreted as multiplicative shocks to preferences. As shown in [Appendix A](#), multiplicative preference shocks can increase welfare. In sum, home production can be a means of buffering the fluctuations effect of shocks to market production and, at the same time, shocks to home production themselves may increase the welfare.

[Gomes et al. \(2001\)](#) introduced job search in a real business cycle model. They also found a welfare gain from business cycle fluctuations. In their model, the welfare gain results from two factors. Although they do not mention it in the paper, the first factor is the mean effect due to the multiplicative productivity shock. The second factor is the option value of job search which depends positively on the size of the uncertainty. Since their main concern is with the welfare gain due to job search, it seems desirable to disentangle the welfare effect of the two factors.

In an important contribution, [Heathcote et al. \(2008\)](#) introduced idiosyncratic wage risk in an environment with endogenous labor supply and partial insurance.²³ The wage risk has a permanent component, which cannot be insured, and a temporary component, which can be perfectly insured. They derived explicit analytical solutions for equilibrium allocations and studied three welfare effects of rising wage dispersion, completing markets and eliminating risk. In particular, they found that the rise of labor market risk in the U.S. labor market over the past 30 years incurs a welfare loss of 7.5% of lifetime consumption due to larger uninsurable fluctuations in individual consumption and hours. However, they also found that a welfare gain of 5% of lifetime consumption is associated with the rise in the labor market risk. In sum, they found that the welfare cost of the rise of labor market risk in the U.S. over the past 30 years is 2.5% of lifetime consumption.²⁴

There is without doubt a marked difference between the aggregate cost of business cycles and the cost to individuals. Heterogeneity, especially employment status, matters in terms of individual specific costs of the business cycle.²⁵ This point was well established by [Imrohoroglu \(1989\)](#). Of course, the issue of what factors determine the employment status is important, but we beg the question. But, even in a world with heterogeneity the issue raised in this paper is important. Suppose an economy consists of two groups of agents, employed and unemployed, and that productivity shocks drive the economy. The agents who are employed in the market can effectively make use of the shocks in their favor, but the unemployed agents cannot. That is, the employment status of an agent decides whether he/she has the means of making use of the business cycle uncertainty in their favor or not. Unemployed agents experience only the fluctuations effect so they bear the brunt of the business cycle. [Krusell and Smith \(1999\)](#)²⁶ document this result in an economy where agents face idiosyncratic risk. This line of research is clearly important and the distribution of the cost of business cycles across the agents may be more important than the average cost of the business cycle fluctuations.

²² Equilibrium output in the case of cyclical factor utilization can be obtained in the same manner as in the second example in Section 2.

²³ [Storesletten et al. \(2001\)](#) introduced individual specific shocks together with aggregate productivity shock in an overlapping generations environment. The key to their model is the countercyclical heteroskedasticity of the individual specific shock distribution. They found much larger welfare cost than those [Lucas \(1987\)](#) had found.

²⁴ [De Santis \(2007\)](#) introduced individual risk and incomplete consumption insurance in a similar environment. However, the risk is not on wage but on consumption and hence the possibility of increasing welfare due to a rise in risk does not arise. He also obtained large welfare cost of business cycles. [Ellison and Sargent \(2012\)](#) extended De Santis model by introducing robustness concern over model specification.

²⁵ For surveys of models with heterogeneity, see [Rios-Rull \(1995\)](#), [Krusell and Smith \(2006\)](#) and [Heathcote et al. \(2009\)](#).

²⁶ See also [Krebs \(2003, 2007\)](#).

5. Conclusion

This paper considers the welfare effect of uncertainty in business cycle models. We showed that when the uncertainties are multiplicative, as usually assumed in the real business cycle literature, welfare may be higher in an economy with aggregate fluctuations than in the counterpart economy without uncertainty. This finding holds true over the range of parameter values that have been used many times in the literature.

The findings in this paper may have some implications for stabilization policies. If the shocks initiating the business cycles are additive, then there is no possibility of fluctuations being beneficial and hence stabilizing the fluctuations is welfare improving.²⁷ However, if the shocks are multiplicative as in the real business cycle literature and as in the case of some seasonal fluctuations or some preference shocks, stabilizing the fluctuations may not be welfare improving. Policies that respond to shocks have to take account of the source of shocks and often will have the implication that the optimal policy will cause the economy to fluctuate more. This, for example, is the nature of the optimal and time-consistent monetary policy in Cooley and Quadrini (2000).

The results in this paper suggest that analyzing the welfare cost of business cycles by looking at the time series data of a few specific macroeconomic variables may lead to the wrong conclusion. Any business cycle uncertainty has two effects, the fluctuations and the mean effect. The method of obtaining the cost of the business cycle by comparing the utility of the actual consumption series to the utility of the mean of the actual series ignores the mean effect of business cycle uncertainty so the estimate of the welfare cost can be correct only when the mean effect happens to be zero.

The notion of “making hay while the sun shines” is well enshrined as a principle of the business cycle. Rational economic agents would respond to favorable shocks and it is this that produces what we have called the “mean effect”. If the mean effect is positive enough to dominate the fluctuations effect as in many real business cycle models, the business cycle is welfare improving.

Appendix A

A.1. Derivation of (11)

The resource constraint for the economy is:

$$c_t + k_{t+1} = A_t k_t^\theta n_t^{1-\theta}. \quad (\text{A.1})$$

Now the problem facing the representative agent is to maximize the lifetime utility, (6), subject to the resource constraint, (A.1).

The first order conditions for the utility maximization are:

$$\alpha = (1 - \theta) A_t k_t^\theta n_t^{-\theta}, \quad (\text{A.2})$$

$$(c_t - \alpha \cdot n_t)^{-\sigma} = \beta \theta E_t \{ A_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta} \cdot (c_{t+1} - \alpha \cdot n_{t+1})^{-\sigma} \}. \quad (\text{A.3})$$

Solving (A.2) in terms of the working hours, we have,

$$n_t = b A_t^{1/\theta} k_t, \quad (\text{A.4})$$

where $b = [(1 - \theta)/\alpha]^{1/\theta}$. Using (A.4) in the production function, (7), we get the reduced form production function in terms of the real shock and the capital stock.

$$y_t = d A_t^{1/\theta} k_t, \quad (\text{A.5})$$

where $d = b^{1-\theta}$. The reduced form (equilibrium) solution for output is convex in the shock since $0 < \theta < 1$, and hence production smoothing is not optimal when there are shocks to the technology. In addition, increasing uncertainty raises the expected output. This is what we refer to as the mean effect of the uncertainty.

To solve for consumption and saving (investment, capital accumulation), we first guess for the solution as follows.

$$c_t = \lambda y_t, \quad k_{t+1} = (1 - \lambda) y_t. \quad (\text{A.6})$$

Using (A.1) in (A.3), we can obtain the following.

$$\lambda = 1 - \{ \beta \theta d^{1-\sigma} E [A_t^{(1-\sigma)/\theta}] \}^{1/\sigma}, \quad (\text{A.7})$$

where $d = b^{1-\theta}$. The coefficient λ is a function of the size of the uncertainty. If we use the fact that A_t follows an *i.i.d.*

²⁷ Of course, the policy should not involve any distortions. If the policy involves welfare costs, it is not clear whether the stabilization is welfare improving.

log-normal process, we can solve for λ as follows²⁸:

$$\lambda = 1 - \left\{ \beta \theta d^{1-\sigma} \exp \left\{ \left(\frac{1-\sigma}{\theta} - 1 \right) \left(\frac{1-\sigma}{\theta} \right) \frac{\tau^2}{2} \right\} \right\}^{1/\sigma}. \tag{A.8}$$

Hence the fraction of consumption out of output decreases with τ^2 and that of investment increases with it if $\sigma > 1$ or $\sigma + \theta < 1$. That is, precautionary savings depends on the value of the preference and production parameters. If $\theta + \sigma = 1$, the size of the uncertainty does not affect the savings rate $1 - \lambda$, i.e. capital accumulation.²⁹ However, if $\theta + \sigma < 1$, increasing uncertainty implies more precautionary savings and vice versa.

Sandmo (1970) showed that when there are changes in the degree of income uncertainty, the response of precautionary savings depends on the sign of the third derivative of the utility function. In our case, the type of uncertainty is different from that studied by Sandmo. In our case production parameters matters together with preference parameters.

To express the lifetime utility in terms of the underlying parameters including productivity variance, define the period utility function as:

$$u_t = \frac{1}{1-\sigma} \cdot (c_t - \alpha \cdot n_t)^{1-\sigma}. \tag{A.9}$$

Then using the analytical solution, (A.7), we have:

$$u_t = \frac{(\lambda d - \alpha b)^{1-\sigma}}{1-\sigma} \cdot [(1-\lambda)d]^{(1-\sigma)t} A_t^{(1-\sigma)/\theta} A_{t-1}^{(1-\sigma)/\theta} \dots A_0^{(1-\sigma)/\theta} k_0^{1-\sigma}. \tag{A.10}$$

If we assume the expected lifetime utility is finite,³⁰ given the initial capital stock, it can be obtained as:

$$\begin{aligned} EU &= \frac{(\lambda d - \alpha b)^{1-\sigma} k_0^{1-\sigma}}{1-\sigma} \cdot \sum_{t=0}^{\infty} \{ [\beta [(1-\lambda)d]^{(1-\sigma)}]^t [E(A_t^{(1-\sigma)/\theta})]^{t+1} \} \\ &= \frac{(\lambda d - \alpha b)^{1-\sigma} k_0^{1-\sigma} E(A_t^{(1-\sigma)/\theta})}{(1-\sigma) \{ 1 - \beta [(1-\lambda)d]^{(1-\sigma)} [E(A_t^{(1-\sigma)/\theta})] \}}, \end{aligned} \tag{A.11}$$

where we used the assumption that the shock follows an *i.i.d.* process. Now the effect of an increase in the uncertainty is not so straightforward but the critical value for the parameters to imply increasing utility with uncertainty can be obtained in the following way. First, we have the following from (A.7):

$$E(A_t^{(1-\sigma)/\theta}) = \frac{(1-\lambda)^\sigma}{\beta \theta d^{1-\sigma}}. \tag{A.12}$$

²⁸ Note the following. If we let

$$Z_t = A_t^{(1-\sigma)/\theta},$$

we have

$$\begin{aligned} \frac{\partial Z_t}{\partial A_t} &= \left(\frac{1-\sigma}{\theta} \right) A_t^{[(1-\sigma)/\theta]-1} \\ \frac{\partial^2 Z_t}{\partial A_t^2} &= \left(\frac{1-\sigma}{\theta} \right) \left(\frac{1-\sigma}{\theta} - 1 \right) A_t^{[(1-\sigma)/\theta]-2}. \end{aligned}$$

Hence if $\sigma > 1$, we can conclude that Z_t is convex with respect to A_t . This means that the expected value of Z_t is increasing with τ^2 .

²⁹ If capital stock grows in the economy, working hours also grow and hence the constraint on the total available hours will be violated. Using (A.5) in (A.6), we have the following.

$$\frac{k_{t+1}}{k_t} = (1-\lambda)dA_t^{1/\theta}.$$

To guarantee that the economy is fluctuating around the steady state, we need to assume that:

$$(1-\lambda)dE[A_t^{1/\theta}] = d\{\beta \theta d^{1-\sigma} E[A_t^{(1-\sigma)/\theta}]\}^{1/\sigma} \cdot E[A_t^{1/\theta}] = 1.$$

This condition keeps capital stock from growing or from shrinking in the long run. See King et al. (1988a, 1988b) for more detailed discussion. We do not have growth in the example so explosive growth is not an issue.

³⁰ The condition is the following:

$$\beta [(1-\lambda)d]^{1-\sigma} E(A_t^{(1-\sigma)/\theta}) < 1.$$

If we substitute this expression in (A.11), we obtain the following expression for the lifetime expected utility³¹:

$$\begin{aligned}
 EU &= \frac{k_0^{1-\sigma} (\lambda d - \alpha b)^{1-\sigma} (1 - \lambda)^\sigma}{\beta(1 - \sigma)d^{1-\sigma} [\lambda - (1 - \theta)]} \\
 &= \frac{k_0^{1-\sigma} [\lambda - (1 - \theta)]^{-\sigma} (1 - \lambda)^\sigma}{\beta(1 - \sigma)}.
 \end{aligned}
 \tag{A.13}$$

Hence the expected utility depends on λ and σ . First, consider the case that $0 < \sigma < 1$. If λ goes up in this case, utility in (A.13) will decrease and vice versa. However, as was mentioned previously in a footnote, λ is related to the variance of the shock τ^2 inversely when $\theta + \sigma < 1$, which means that utility is increasing with τ^2 . If $\theta + \sigma = 1$, λ does not respond to the changes in the variance, τ^2 , and thus the fluctuations and the mean effect are balanced, which means that business cycle uncertainty does not affect expected lifetime utility. But if $\theta + \sigma > 1$, λ is increasing with τ^2 , which means that utility is decreasing with τ^2 . Now consider the case that $\sigma > 1$. If λ goes up in this case, utility in (A.13) will increase and vice versa. In addition, we can show from (A.8) that λ goes down as the variance of the shock τ^2 increases³² and thus utility will always decrease with the uncertainty. The same is true in the case that $\sigma = 1$.

Using the assumption that A_t follows an *i.i.d.* log-normal distribution as (9), the unconditional mean of the lifetime utility can be obtained as:

$$EU = \frac{(d - \alpha b)^{1-\sigma}}{(1 - \sigma)(1 - \beta)} \cdot \exp\left\{\left(\frac{1 - \sigma}{\theta} - 1\right)\left(\frac{1 - \sigma}{\theta}\right)\frac{\tau^2}{2}\right\}.$$

Hence if we define the degree of uncertainty with the variance of normal distribution, τ^2 , we have the same condition for welfare increase with the uncertainty.

A.2. Preference shocks and welfare

The key to the results in the text is that the variable factors of production respond positively to the multiplicative productivity shock. With this construct, we show that purposeful agents in the economy make use of uncertainty in their favor. This sort of response to uncertainty is not restricted to multiplicative technology shocks. In the case of (multiplicative) preference shocks, we can also show that welfare increases with uncertainty.³³

Following Long and Plosser (1983), consider the simplest real business cycle model with log-linear preferences, Cobb-Douglas production function, and 100 percent capital depreciation:

$$\begin{aligned}
 V(B_t, k_t) &= \max \left\{ \log(c_t) + B_t \log(1 - n_t) + \beta E_t[V(B_{t+1}, k_{t+1})] \right\} \\
 \text{s.t. } (1) \quad &c_t + k_{t+1} = k_t^\theta n_t^{1-\theta} \\
 (2) \quad &k_{t+1} = (1 - \delta)k_t + i_t \\
 (3) \quad &\ln(B_t) \sim \text{i.i.d. } N\left(\log(B) - \frac{\tau^2}{2}, \tau^2\right) \\
 (4) \quad &c_t, k_{t+1} \geq 0, 0 \leq n_t \leq 1, k_0 \text{ is given,}
 \end{aligned}
 \tag{A.14}$$

³¹ Note the following in the derivation.

$$\frac{\alpha b}{d} = \frac{\alpha b}{b^{1-\theta}} = \alpha b^\theta = \left\{ \left(\frac{1 - \theta}{\alpha}\right)^\theta \right\}^{1/\theta} \alpha = 1 - \theta.$$

³² Note the following in (A.8). If we let

$$Z_t = A_t^{(1-\sigma)/\theta},$$

we have

$$\begin{aligned}
 \frac{\partial Z_t}{\partial A_t} &= z \left(\frac{1 - \sigma}{\theta}\right) A_t^{\{[(1-\sigma)/\theta]-1\}} \\
 \frac{\partial^2 Z_t}{\partial A_t^2} &= \left(\frac{1 - \sigma}{\theta}\right) \left(\frac{1 - \sigma}{\theta} - 1\right) A_t^{\{[(1-\sigma)/\theta]-2\}}.
 \end{aligned}$$

Hence if $\sigma > 1$, we can conclude that Z_t is convex with respect to A_t . This means that the expected value of Z_t is increasing with τ^2 .

³³ Chang et al. (2013) confirm that the effects of imperfect aggregation manifest themselves through the presence of preference shocks (the so-called labor market wedge) in the representative-agent model; the preference shocks may reflect a specification error rather than a fundamental driving force behind business cycles. Thus welfare cost calculation based on a parameterized utility function with preference shocks may be misleading. Nevertheless, this "reduced-form" example provides some useful intuition for an alternative welfare-improving channel.

where B is constant and $\delta = 1$. Here, B_t is the preference shock with constant mean and the only source of aggregate fluctuations in the model economy.

The equilibrium solutions for the above problem (A.14) can be obtained analytically as follows:

$$c_t = (1 - \beta\theta)k_t^\theta n_t^{1-\theta} \quad (\text{A.15})$$

$$k_{t+1} = \beta\theta k_t^\theta n_t^{1-\theta} \quad (\text{A.16})$$

$$n_t = \frac{1 - \theta}{(1 - \theta) + (1 - \beta\theta)B_t}. \quad (\text{A.17})$$

Note that $V(B_t, k_t)$ represents the lifetime utility from period t on and can be obtained analytically using the equilibrium solutions (A.15), (A.16), and (A.17). Hence we can take the first and the second derivative of $V(B_t, k_t)$ with respect to the preference shock B_t , leading to³⁴:

$$\frac{\partial V}{\partial B_t} = \log \left[\frac{(1 - \beta\theta)B_t}{(1 - \theta) + (1 - \beta\theta)B_t} \right] < 0 \quad (\text{A.18})$$

$$\frac{\partial^2 V}{\partial B_t^2} = \frac{(1 - \theta)}{[(1 - \theta) + (1 - \beta\theta)B_t]B_t} > 0. \quad (\text{A.19})$$

Thus, V is decreasing in the shock B_t but at a decreasing rate. That is, V is convex in B_t so introducing uncertainty in this way increases welfare. Multiplicative preference shocks may not be detrimental to welfare just as technology shocks.

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³⁴ See the previous version of the paper for derivation of the lifetime utility. It can be supplied on request.

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