

## DYNAMIC COALITION FORMATION AND EQUILIBRIUM POLICY SELECTION\*

Thomas F. COOLEY

*University of Rochester, Rochester, NY 14627, USA*

Bruce D. SMITH

*University of Western Ontario, London, Ont., Canada N6A 5C2  
Rochester Center for Economic Research, Rochester, NY 14627, USA*

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A model of policy formulation is proposed in which the government and the private sector form dynamic coalitions for the purpose of choosing policy actions. However, current players cannot commit their future selves to any particular actions. Defining an equilibrium to be an unblocked sequence of actions, we show that an equilibrium exists and is unique even though, in general, cooperative outcomes are not time-consistent. Our framework has the implication that when private agents and the government have identical preferences, there are no distortions associated with the use of non-lump-sum taxes. Some empirical implications of this result are discussed.

### 1. Introduction

There is now a large literature that analyzes interactions between the government and the private sector in the process of government policy selection. Of particular interest to macroeconomists is the interaction between the government and the rest of the economy in dynamic contexts where the current government cannot commit itself to a future course of action. This situation has been examined from several perspectives. Kydland and Prescott (1977) demonstrated the possibility that government policies derived from the solution of dynamic programming problems can imply future values of optimal policies that will not be thought optimal when the future becomes the present. Subsequent research [e.g., Barro and Gordon (1983a)] raised the possibility that repeated (noncooperative) interactions between the govern-

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ment and the private sector might alleviate Kydland and Prescott's 'time-consistency' problem. And recent research by Chari and Kehoe (1988a, b) and Chari, Kehoe, and Prescott (1988) raises the possibility that strategic interactions among differently dated governments, along with sequentially rational responses by private agents to government policies, can sustain outcomes superior to those obtained under the simplest 'time-consistent' policies.

All of this literature, either implicitly or explicitly, takes a stand on how best to view the nature of government/private-sector interactions. Or, to state the matter somewhat differently, each strand of literature mentioned above provides a model of the joint determination of government policy (and private-sector) actions. The purpose of the present paper is to suggest a model of the process of policy selection that differs from the existing literature in that the government and the private sector are viewed as behaving cooperatively. In order to place this view in perspective, it seems appropriate to begin by reviewing the different stands taken on how the government and the rest of the economy interact in the existing literature on (dynamic) policy selection.

Kydland and Prescott's (1977) analysis of the time-consistency problem adopted the view that the government at each date can choose policies strategically, while the remainder of the economy behaves nonstrategically. In this setting the private sector of the economy can be represented by a (fixed) reaction function and the government can choose policies taking this reaction function as given. An important application of this formulation of 'time consistency' is the Lucas and Stokey (1983) analysis of optimal fiscal policy. For them the private sector is passive, being represented by a mapping of parameters and government policies into a set of competitive equilibria, while the government behaves strategically.<sup>1</sup> Lucas and Stokey also suggest an alternative definition of time consistency; the current policy actions of the current government should be a best response to the strategies chosen by future governments for selecting policies. This definition raises the possibility of strategic interactions among differently dated governments.

Barro and Gordon (1983a) raised the further possibility that the Kydland–Prescott formulation of the interactions between the government and the private sector is too limiting because it neglects the possibility that repeated (noncooperative) interactions between them will permit the private sector (either implicitly or explicitly) to respond strategically to choices of policy actions by the government. This admits the possibility that reputational considerations will sustain equilibria which are superior to simple time-consistent policy choices. It also has the feature that economies can get stuck in bad (e.g., high inflation) equilibria. This literature is reviewed by Rogoff (1987).

<sup>1</sup> Lucas and Stokey use this setting to investigate when optimal taxation and borrowing policies are time-consistent in the sense of Kydland and Prescott.

A third literature, associated with Chari and Kehoe (1988a, b) and Chari, Kehoe, and Prescott (1988), investigates the possibility of strategic interactions among differently dated players in policy games. These papers argue that the requirement that the actions (or strategies) of each dated government be a best response to the strategies chosen by subsequent governments, given that the private sector will respond in a sequentially rational manner to any choice of government policies, can induce the government to follow policies superior to the simple time-consistent policies considered by Kydland and Prescott.

In the present paper we propose an alternative model of the social arrangements used to select policies. Following Barro and Gordon (1983a), we model the private sector as an active player in the process of policy formulation. Also, we follow Chari and Kehoe (1988a, b) and Chari, Kehoe, and Prescott (1988) in allowing for strategic interactions among differently dated governments and differently dated private agents. However, we abandon the notion that the government and the private sector play noncooperatively in a game of policy formulation. Rather, we treat the government (at different dates) and private agents (at different dates) as distinct players in a game of policy formulation. In doing so we create a set of artificial agents, one for the government at each date and one for each private agent at each date. These artificial agents are modelled as meeting outside of time. They are allowed to form coalitions, which consist (potentially) of both government and private-sector players. Members of coalitions coordinate actions, including government policy choices, taking the actions of the complimentary coalition as given. However, players in this game cannot commit their future selves to any particular course of action. An equilibrium of this game consists of a choice of policy actions and private-sector actions that is not blocked by any coalition that can form. However, since players cannot commit their future selves to courses of action, unblocked sets of policies and private-sector choices closely resemble coalition proof equilibria, as defined by Bernheim, Peleg, and Whinston (1987).<sup>2</sup>

Before giving the details of the analysis, it seems appropriate to offer some comments as to why we are motivated to suggest this model of government/private-sector interactions. Assuming that it is desirable to allow the private sector to interact nonpassively with the government, we feel that this approach has some attractive features as a model of social arrangements that are not present in much of the policy games literature reviewed by Rogoff (1987). Specifically, the literature from which many of these policy game models are adapted (duopoly theory) is one where competition is the relevant paradigm. This is a view of government/private-sector interaction that does not fit easily

<sup>2</sup> We view this formulation of the policy-selection process as a close relative of the dynamic-coalitions notion used by Boyd and Prescott (1987) to study the formation of firms and of the notion of an 'equilibrium arrangement' used by Boyd, Prescott, and Smith (1988) to study the same issue.

with some notions of the relationship between a democratic government and the public it represents. This paper suggests an alternative model of policy selection that permits cooperation and coordination between the government and the private sector. Moreover, models where punishment strategies are used to support equilibria often require punishments that current and future players would jointly prefer not to carry out. Or, put differently, we believe that in the event of deviations requiring punishments one should expect to see coalitions forming that modify the punishment to the joint benefit of all subsequent players.<sup>3</sup> Our view of government/private-sector interactions specifically allows for the formation of these coalitions.

We view this description of the policy-selection process as a model of social arrangements, not as a tool for normative selection of policies. Whether it is a useful view of social arrangements depends on its empirical implications and those, in turn, depend on the complementary assumptions one makes about government and private-sector preferences. When the government and private sector have different objective functions, time-consistency problems will arise even though the behavior is cooperative. When objective functions coincide, however, important differences between our cooperative model of social arrangements and the noncooperative view emerge. Section 5 presents an example of an optimal taxation game where the objective functions of the government and the private sector coincide. Taxes that are not lump-sum generally induce distortions that can have serious welfare consequences. However, the unique equilibrium that emerges from the dynamic coalition framework coincides with the equilibrium that would obtain if the government could raise all its revenue via lump-sum taxation. This result has some interesting empirical implications that are discussed in section 6.

## **2. Coalition formation**

In this section we lay out a game in which the government and a representative private agent can form self-sustaining coalitions for the purpose of choosing policies (and other actions). Our notation and physical setting will closely resemble that of Kydland and Prescott (1977). We focus on an environment with a finite time horizon and do not explicitly incorporate uncertainty into the notation, although the latter simplification is inessential. The equilibrium concept that we introduce is very similar to the notion of 'coalition-proof' Nash equilibria introduced by Bernheim, Peleg, and Whinston (1987).

<sup>3</sup>This remark applies both to the policy games literature and to the work of Chari and Kehoe (1988a) on sustainable plans. Specifically, Chari and Kehoe's formulation allows future governments, along with the private sector, to effectively punish deviations by the current government from its equilibrium action. Parenthetically, we would view the formation of these coalitions as involving the kinds of 'speeches' that are often suggested in the literature on refinements of equilibrium concepts. See, e.g., Cho and Kreps (1987).

Let time be indexed by  $t$ ,  $t = 1, \dots, T$ . Let  $\pi = (\pi_1, \pi_2, \dots, \pi_T)$  be a sequence of policy actions for periods  $1, \dots, T$ , and let  $x = (x_1, x_2, \dots, x_T)$  be a sequence of actions of private agents. (Policies and private actions can be viewed as chosen from some compact set, which is suppressed in the discussion.) The government (or policy maker) has the objective function  $S(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$ , which is assumed to be continuous and quasi-concave. The representative private agent has an objective function  $V(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$  with the same properties.

As has been widely noted [see, e.g., Kydland and Prescott (1977, p. 627)], if future policy makers cannot be bound to a decision at  $t = 1$ , then policy makers at each date must be viewed as distinct players. We capture this by adopting the approach that the policy maker at date  $t$  is a separate agent from policymakers at all other dates, as is also the case for the representative private agent at  $t$ . Then policy selection is modelled by allowing all of these agents to form coalitions for the purpose of choosing a sequence of government actions.<sup>4</sup>

We imagine that all (dated) policy makers and private agents meet at the beginning of time to select such a sequence. Some set of actions is proposed. If a coalition of policy makers (and private agents) can form and find a preferred subsequence of policy actions, this initial sequence is blocked. Blocked sequences are never implemented. But, since the payoffs of future players depend on the actions of current players, it is necessary to specify how potential blocking coalitions view the actions of their complements. Here we adopt the natural formulation (natural since current policy makers precede future policy makers in time) that blocking coalitions take the choices of their complements as given. This makes it appropriate to assume, as Kydland and Prescott (1977) do, that private agents take the whole future sequence of government policy actions as given in their decision making.

The sequences of actions that are not blocked at the beginning of time are the cooperative equilibria we are interested in. Lest the notion of all future policy makers and private agents meeting at the beginning of time and forming coalitions seems unnatural, we note that it is common in dynamic economic models to view all agents (including possibly unborn agents) as meeting at the beginning of time to trade in Walrasian auction markets. This permits static competitive equilibrium tools to be employed. Here we allow policy makers and private agents to meet at the beginning of time, to propose sequences of actions, and to form blocking coalitions. This allows us to use standard static core concepts. Thus, ours is simply an analogue (using the notion of coalition formation) to standard approaches to dynamic competitive analysis.

<sup>4</sup>The approach just outlined uses an idea introduced by Myerson (1983), who analyzes the problem of a privately informed principal faced with designing a mechanism for allocating resources. Myerson models the design of the mechanism as the outcome of a process of coalition formation by different possible player types of the same principal.

Formally, then, the *policy maker at t*, denoted agent  $p_t$ , is a policy maker who faces a partial history of choices, which we denote hereafter by  $h_{t-1} = (x_1, \dots, x_{t-1}, \pi_1, \dots, \pi_{t-1})$ , and can choose a sequence  $(\pi_t, \dots, \pi_T)$ . The *private agent at t*, who is also a player in the same cooperative game, is denoted agent  $a_t$ . This agent, facing a given history of choices, can choose a sequence of actions  $(x_t, \dots, x_T)$ .<sup>5</sup>

A *coalition at t* is a subset of players dated  $\tau = t, \dots, T$ . Or, in other words, a coalition at  $t$  is a subset of players  $\{p_\tau\}_{\tau=t}^T \cup \{a_\tau\}_{\tau=t}^T$ . A coalition consisting only of private agents at  $t$ ,  $\{a_\tau\}_{\tau=t}^T$ , obtains the payoff

$$\bar{V}_t \equiv \max_{(x_\tau)_{\tau=t}^T} V(x_t, \dots, x_T, 0, \dots, 0 | h_{t-1}), \quad (1)$$

where  $h_{t-1}$  denotes the inherited history of the game. The interpretation of (1) is as follows: if private agents, and private agents alone, defect from a coalition at date  $t$ , they inherit the past history of the game  $h_{t-1} = (x_1, \dots, x_{t-1}, \pi_1, \dots, \pi_{t-1})$ . Since no policy makers are included in the coalition, the government has been 'shut down' or replaced. We denote 'shutting down' of the government at  $t$  by setting  $\pi_t = \pi_{t+1} = \dots = \pi_T = 0$ . However, private agents from  $t$  on are free to make arbitrary choices  $(x_t, x_{t+1}, \dots, x_T)$ , and hence a coalition of this type obtains the payoff given in (1).<sup>6</sup>

We will say that a set of actions  $(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$  is *blocked* by  $\{a_\tau\}_{\tau=t}^T$  (the coalition of private agents at  $t$ )<sup>7</sup> if there exist values  $(\hat{x}_t, \dots, \hat{x}_T)$  such that

$$V(\hat{x}_t, \dots, \hat{x}_T, 0, \dots, 0 | h_{t-1}) > V(x_1, \dots, x_T, \pi_1, \dots, \pi_T), \quad (2)$$

$$V(\hat{x}_t, \dots, \hat{x}_T, 0, \dots, 0 | h_{t-1}) \geq \bar{V}_s, \quad s = t+1, \dots, T, \quad (3)$$

where  $\bar{V}_s$  is defined by (1).

Eq. (3) merits some discussion. What (3) says is that the set of choices  $(x_1, \dots, x_{t-1}, \hat{x}_t, \dots, \hat{x}_T, \pi_1, \dots, \pi_{t-1}, 0, \dots, 0)$  is not itself blocked by a subset of the coalition  $\{a_\tau\}_{\tau=t}^T$ . That is, it cannot itself be blocked by a coalition of private agents at some date later than  $t$  if  $\{a_\tau\}_{\tau=t}^T$  is to constitute a blocking coalition. Eq. (3) can be viewed as a requirement that a set of actions cannot be blocked by making a threat that will not actually be carried out by

<sup>5</sup>Thus player  $p_t$  chooses a sequence  $(\pi_t, \dots, \pi_T)$ , while player  $p_{t+1}$  chooses  $(\pi_{t+1}, \dots, \pi_T)$ . In equilibrium the choices of each player will be part of an unblocked sequence and hence consistent. We anticipate this and avoid a more complicated notation in which each player chooses  $\pi_t$  and announces a set of future values that may not be implemented.

<sup>6</sup>Alternatively we could think of  $\bar{V}_t$  as being the level of utility that could be attained by a coalition with the next best unblocked government.

<sup>7</sup>Or, as a shorthand, we will often just say is blocked at  $t$ .

subsequent players. Having said this, it will be noted that private agents cannot obtain a payoff exceeding  $\bar{V}_s$  at  $s$ , so that (3) is satisfied trivially. In some sense (3) is an inessential condition, but it is discussed here because an analogous condition plays an important role below.

As a further definition, a set of choices  $(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$  is *blocked by the grand coalition at  $t$*  ( $\{p_\tau\}_{\tau=t}^T \cup \{a_\tau\}_{\tau=t}^T$ ) if there exist values  $(\hat{x}_t, \dots, \hat{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_T)$  such that

$$S(\hat{x}_t, \dots, \hat{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_T | h_{t-1}) > S(x_1, \dots, x_T, \pi_1, \dots, \pi_T), \quad (4)$$

$$V(\hat{x}_t, \dots, \hat{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_T | h_{t-1}) \geq \bar{V}_s, \quad s = t, \dots, T, \quad (5)$$

where  $\bar{V}_s$ ,  $s = t, \dots, T$ , is defined by (1), and such that  $(x_1, \dots, x_{t-1}, \hat{x}_t, \dots, \hat{x}_T, \pi_1, \dots, \pi_{t-1}, \hat{\pi}_t, \dots, \hat{\pi}_T)$  is not itself blocked by a coalition  $\{p_\tau\}_{\tau=s}^T \cup \{a_\tau\}_{\tau=s}^T$ ,  $s \geq t + 1$ .

Some discussion of this definition is in order. Eq. (4) says that for the grand coalition at  $t$  to block the choices  $(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$  there must exist a choice of actions that makes the policy maker strictly better off at  $t$ . Notice that there is no analogous requirement that private agents be made better off. Rather, eq. (5) is an individual rationality condition that asserts a willingness by private agents to join potential blocking coalitions so long as they are not worse off than they would be with no government. This allows the government a wide scope for discretion since it can (jointly with private agents) take any actions that do not result in the government being shut down. In this sense our formulation closely resembles Myerson's (1983), in that most of the interesting play of the game is among policy makers at different dates. Our formulation also closely resembles that of Boyd, Prescott, and Smith (1988). In their set-up, certain agents have a strategic advantage in that they provide something that is not provided by all agents. We could easily allow the government an advantage in providing public services relative to potential alternative governments (as in footnote 6). Then the arguments given by Boyd, Prescott, and Smith (1988) would rationalize the definition of blocking given in (4) and (5).<sup>8</sup> Finally, (5) also requires that the blocking choices  $(\hat{x}_t, \dots, \hat{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_T)$  cannot themselves be blocked by a subset of private agents at a date later than  $t$ .

In addition to (4) and (5) we require that, for  $\{p_\tau\}_{\tau=t}^T \cup \{a_\tau\}_{\tau=t}^T$  to constitute a blocking coalition, there must exist choices  $(\hat{x}_t, \dots, \hat{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_T)$  for this coalition such that  $(x_1, \dots, x_{t-1}, \hat{x}_t, \dots, \hat{x}_T, \pi_1, \dots, \pi_{t-1}, \hat{\pi}_t, \dots, \hat{\pi}_T)$  will not itself be blocked by a coalition that forms at a later date. Thus we do *not*

<sup>8</sup>As an additional comment on the asymmetry between the government and private agents in (4) and (5), one might view this as the analogue of the usual treatment of the government as a Stackelberg leader in noncooperative policy games. Here the government is allowed to select its preferred point in the core.

allow  $(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$  to be blocked by a coalition at  $t$  that threatens to take actions at some date  $s > t$  that will not be carried out at  $s$ .

It is now necessary to write down formally what we mean by this.  $(x_1, \dots, x_{t-1}, \hat{x}_t, \dots, \hat{x}_T, \pi_1, \dots, \pi_{t-1}, \hat{\pi}_t, \dots, \hat{\pi}_T)$  is blocked by a coalition  $\{p_\tau\}_{\tau=s}^T \cup \{a_\tau\}_{\tau=s}^T$  ( $s > t$ ) if any of the following conditions are satisfied.

(i) There exist values  $(\tilde{x}_T, \tilde{\pi}_T)$  such that

$$\begin{aligned} & S(\hat{x}_t, \dots, \hat{x}_{T-1}, \tilde{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_{T-1}, \tilde{\pi}_T | h_{t-1}) \\ & > S(\hat{x}_t, \dots, \hat{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_T | h_{t-1}), \end{aligned} \quad (6)$$

$$V(\hat{x}_t, \dots, \hat{x}_{T-1}, \tilde{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_{T-1}, \tilde{\pi}_T | h_{t-1}) \geq \bar{V}_T.$$

(ii) There exist values  $(\tilde{x}_{T-1}, \tilde{x}_T, \tilde{\pi}_{T-1}, \tilde{\pi}_T)$  such that

$$\begin{aligned} & S(\hat{x}_t, \dots, \hat{x}_{T-2}, \tilde{x}_{T-1}, \tilde{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_{T-2}, \tilde{\pi}_{T-1}, \tilde{\pi}_T | h_{t-1}) \\ & > S(\hat{x}_t, \dots, \hat{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_T | h_{t-1}), \end{aligned} \quad (7)$$

$$V(\hat{x}_t, \dots, \hat{x}_{T-2}, \tilde{x}_{T-1}, \tilde{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_{T-2}, \tilde{\pi}_{T-1}, \tilde{\pi}_T | h_{t-1}) \geq \bar{V}_{T-1},$$

$$V(\hat{x}_t, \dots, \hat{x}_{T-2}, \tilde{x}_{T-1}, \tilde{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_{T-2}, \tilde{\pi}_{T-1}, \tilde{\pi}_T | h_{t-1}) \geq \bar{V}_T,$$

and such that there is no pair  $(x_T^*, \pi_T^*)$  satisfying

$$\begin{aligned} & S(\hat{x}_t, \dots, \hat{x}_{T-2}, \tilde{x}_{T-1}, x_T^*, \hat{\pi}_t, \dots, \hat{\pi}_{T-2}, \tilde{\pi}_{T-1}, \pi_T^* | h_{t-1}) \\ & > S(\hat{x}_t, \dots, \hat{x}_{T-2}, \tilde{x}_{T-1}, \tilde{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_{T-2}, \tilde{\pi}_{T-1}, \tilde{\pi}_T | h_{t-1}), \end{aligned} \quad (8)$$

$$V(\hat{x}_t, \dots, \hat{x}_{T-2}, \tilde{x}_{T-1}, x_T^*, \hat{\pi}_t, \dots, \hat{\pi}_{T-2}, \tilde{\pi}_{T-1}, \pi_T^* | h_{t-1}) \geq \bar{V}_T.$$

Similar conditions are defined for  $s = t + 1, \dots, T - 2$ .

When we say that  $(x_1, \dots, x_{t-1}, \hat{x}_t, \dots, \hat{x}_T, \pi_1, \dots, \pi_{t-1}, \hat{\pi}_t, \dots, \hat{\pi}_T)$  is not blocked at any date later than  $t$ , then condition (i) requires that there be no possible welfare-improving deviation at the terminal date  $T$ . Condition (ii) requires one of two things. Either there is no welfare improving joint deviation  $(\tilde{x}_{T-1}, \tilde{x}_T, \tilde{\pi}_{T-1}, \tilde{\pi}_T)$ , or if there is, a blocking coalition consisting of players  $p_T$  and  $a_T$  will form to block it in turn. Extensions of this notion to periods  $t + 1, \dots, T - 2$  are straightforward. It should be noted that unblocked sequences are very similar to coalition-proof Nash equilibria in the sense of Bernheim, Peleg, and Whinston (1987). This may not be immediately apparent since we require coalitions that form at  $t$  to consist of all agents dated  $t$  or later. However, this does not prevent a player dated  $t$  from undertaking a



unilateral deviation and attempting to form a coalition in which all players dated  $t + 1$  or later follow their initial strategies. The absence of a blocking coalition also implies the absence of a payoff improving deviation by any single player.

### 3. Equilibrium

There is a unique equilibrium to the game laid out above. The equilibrium coincides with the solution to a very simple dynamic programming problem implying that dynamic programming techniques are applicable here, despite the presence of a time-consistency problem which we illustrate below. In this section, we first describe the dynamic programming problem as a prelude to a constructive proof of the existence of equilibrium. To begin, let  $g_T(x_1, \dots, x_{T-1}, \pi_1, \dots, \pi_{T-1})$  and  $f_T(x_1, \dots, x_{T-1}, \pi_1, \dots, \pi_{T-1})$  be the values of  $\pi_T$  and  $x_T$ , respectively, that solve the problem

$$\max_{x_T, \pi_T} S(x_1, \dots, x_{T-1}, x_T, \pi_1, \dots, \pi_{T-1}, \pi_T), \tag{9}$$

subject to

$$V(x_1, \dots, x_{T-1}, x_T, \pi_1, \dots, \pi_{T-1}, \pi_T) \geq \bar{V}_T, \tag{10}$$

where  $\bar{V}_T$  is as defined by (1). Then define recursively, for  $t = 1, \dots, T - 1$ ,  $g_t(x_1, \dots, x_{t-1}, \pi_1, \dots, \pi_{t-1})$  and  $f_t(x_1, \dots, x_{t-1}, \pi_1, \dots, \pi_{t-1})$  to be the values of  $\pi_t$  and  $x_t$  that solve the problem

$$\begin{aligned} \max_{x_t, \pi_t} S[x_1, \dots, x_{t-1}, x_t, f_{t+1}(-), \dots, f_T(-), \\ \pi_1, \dots, \pi_{t-1}, \pi_t, g_{t+1}(-), \dots, g_T(-)], \end{aligned} \tag{11}$$

subject to

$$\begin{aligned} V[x_1, \dots, x_{t-1}, x_t, f_{t+1}(-), \dots, f_T(-), \\ \pi_1, \dots, \pi_{t-1}, \pi_t, g_{t+1}(-), \dots, g_T(-)] \geq \bar{V}_t. \end{aligned} \tag{12}$$

We assume that  $\pi_t = 0$  is a feasible choice for the government at each date, so the constraint set in each of these problems is nonempty. Also, under standard assumptions on  $S(-)$  and  $V(-)$ ,  $g_t(-)$  and  $f_t(-)$  are continuous functions  $\forall t$ , although this is not necessary to the analysis. Thus,  $g_t(-)$  and  $f_t(-)$  give standard dynamic programming solutions to the government's problem of maximizing the value of its objective function subject to a set of individual rationality conditions for private agents.

In the following theorems we state two results. The first is that the choices  $x_t = f_t(x_1, \dots, x_{t-1}, \pi_1, \dots, \pi_{t-1})$ ,  $\pi_t = g_t(x_1, \dots, x_{t-1}, \pi_1, \dots, \pi_{t-1})$ ,  $\forall t$ , are equilibrium choices. The second is that these are the only equilibrium choices.

*Theorem 1.* Suppose the sequence of actions  $(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$  given by  $x_t = f_t(-)$ ,  $\pi_t = g_t(-)$ ,  $\forall t$ , is chosen. Then this set of actions is not blocked.

The proof of Theorem 1 is a straightforward application of backward induction, and hence is left for an appendix. The proof simply involves checking whether any blocking coalition can form. It is apparent that (12) rules out the possibility of any blocking coalition consisting of private agents only, since by definition the payoff received by such a coalition (forming at  $t$ ) is  $\bar{V}_t$ . It is also apparent that  $\{p_T\} \cup \{a_T\}$  is not a blocking coalition, since setting  $x_T = f_T(h_{T-1})$  and  $\pi_T = g_T(h_{T-1})$  gives the (government) utility-maximizing choices of terminal period actions. Thus any blocking coalition must be of the form  $\{p_\tau\}_{\tau=t}^T \cup \{a_\tau\}_{\tau=t}^T$ ,  $t < T$ .

That no such blocking coalition could form can be seen as follows. Suppose such a blocking coalition did form at  $t$  and chose actions  $(\hat{x}_t, \dots, \hat{x}_T, \hat{\pi}_t, \dots, \hat{\pi}_T)$ . There would then be a last date, say  $\bar{k}$ , at which  $x_s \neq f_s(h_{s-1})$  or  $\pi_s \neq g_s(h_{s-1})$  held. But then a coalition  $\{p_\tau\}_{\tau=\bar{k}}^T \cup \{a_\tau\}_{\tau=\bar{k}}^T$  could form and set  $x_\tau = f_\tau(h_{\tau-1})$  and  $\pi_\tau = g_\tau(h_{\tau-1})$ ,  $\forall \tau = \bar{k}, \dots, T$ . By the definition of  $f_\tau(-)$  and  $g_\tau(-)$ , these choices would increase government welfare at date  $\bar{k}$  and would not violate (12). Thus  $(x_1, \dots, x_{t-1}, \hat{x}_t, \dots, \hat{x}_T, \pi_1, \dots, \pi_{t-1}, \hat{\pi}_t, \dots, \hat{\pi}_T)$  would itself be blocked at date  $\bar{k}$ , contradicting the supposition that this blocked  $(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$ . The appendix formalizes this intuition.

As is apparent, the theorem, in addition to establishing the existence of an equilibrium, gives a characterization of this equilibrium as the solution to a simple dynamic programming problem. Further, it is the case that this equilibrium is unique.

*Theorem 2.* Consider a sequence of actions  $(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$ . Suppose that either  $\pi_s \neq g_s(-)$  and/or  $x_s \neq f_s(-)$  for some date  $s$ . Then this choice of actions is blocked.

The method of proof is to fix an arbitrary sequence  $(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$  with either  $x_s \neq f_s(-)$  for some  $s$ , or  $\pi_s \neq g_s(-)$  for some  $s$ . A blocking coalition is then constructed, establishing the theorem.

To begin, let  $\bar{s}$  be the largest date for which  $\pi_s \neq g_s(-)$  or  $x_s \neq f_s(-)$ . Then the coalition  $\{p_\tau\}_{\tau=\bar{s}}^T \cup \{a_\tau\}_{\tau=\bar{s}}^T$  can choose  $\hat{\pi}_{\bar{s}} = g_{\bar{s}}(x_1, \dots, x_{\bar{s}-1}, \pi_1, \dots, \pi_{\bar{s}-1})$ ,  $\hat{x}_{\bar{s}} = f_{\bar{s}}(x_1, \dots, x_{\bar{s}-1}, \pi_1, \dots, \pi_{\bar{s}-1})$ , and can choose  $\hat{\pi}_q = g_q(-)$ ,  $\hat{x}_q = f_q(-)$  for

all  $q \geq \bar{s} + 1$ . Then, by the definitions of  $g_q(-)$  and  $f_q(-)$ ,

$$\begin{aligned} & S(x_1, \dots, x_{\bar{s}-1}, \hat{x}_{\bar{s}}, \dots, \hat{x}_T, \pi_1, \dots, \pi_{\bar{s}-1}, \hat{\pi}_{\bar{s}}, \dots, \hat{\pi}_T) \\ & > S(x_1, \dots, x_T, \pi_1, \dots, \pi_T), \end{aligned} \tag{13}$$

$$V(x_1, \dots, x_{\bar{s}-1}, \hat{x}_{\bar{s}}, \dots, \hat{x}_T, \pi_1, \dots, \pi_{\bar{s}-1}, \hat{\pi}_{\bar{s}}, \dots, \hat{\pi}_T) \geq \bar{V}_{\bar{s}}. \tag{14}$$

Moreover, by the same argument used in the proof of Theorem 1, the actions  $(\hat{x}_{\bar{s}}, \dots, \hat{x}_T, \hat{\pi}_{\bar{s}}, \dots, \hat{\pi}_T)$  are not themselves blocked by a subset of the coalition  $\{p_\tau\}_{\tau=\bar{s}}^T \cup \{a_\tau\}_{\tau=\bar{s}}^T$  (that is, are not themselves blocked at some date later than  $\bar{s}$ ). Hence  $\{p_\tau\}_{\tau=\bar{s}}^T \cup \{a_\tau\}_{\tau=\bar{s}}^T$  satisfies the definition of a blocking coalition, establishing the theorem.

Intuitively, at date  $T$  any actions other than  $f_T(h_{T-1})$  and  $g_T(h_{T-1})$  must be blocked, since these are the optimal actions for any inherited history. Given this, at date  $T - 1$  any action other than  $f_{T-1}(h_{T-2})$  and  $g_{T-1}(h_{T-2})$  [with  $x_T = f_T(-)$  and  $\pi_T = g_T(-)$ ] must be blocked. Backward induction in this manner establishes the uniqueness of the unblocked sequence of actions.

In summary, we have proved that the cooperative game of policy formulation set out above has a unique equilibrium for arbitrary finite horizons. (Whether the same result can be proved in infinite horizon settings is a topic for future research.) Furthermore, the equilibrium of this game can be characterized as the solution of a simple dynamic programming problem.

#### 4. Discussion

The results above have some sharp implications. For instance, if the government objective function coincides with private agents' objective functions, then the equilibrium above is a unique Pareto optimum and is time-consistent. This is easy to see, since in this case the equilibrium of the policy selection process is the solution to the problem

$$\max V(x_1, \dots, x_T, \pi_1, \dots, \pi_T).$$

Clearly  $\max V(x_1, \dots, x_T, \pi_1, \dots, \pi_T) \geq \bar{V}_t, \forall t$  [with  $\bar{V}_t$  defined by (1)], so that the solution to this problem solves the problem (9) and (10) and is an equilibrium. This conclusion raise the question of whether there can be a time-consistency problem in this context. It has already been noted in the literature [see, e.g., Fischer (1980)] than when the government objective function coincides with that of private agents, cooperative behavior<sup>9</sup> eliminates

<sup>9</sup>This is Fischer's usage. His usage of the term cooperative appears to coincide with our equilibrium concept when the objective function of the government coincides with that of the private agents.

time-consistency problems. However, this is not the case when the government's objective function differs from that of private agents. To illustrate this point, we construct an example in which  $(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$  is chosen cooperatively (and with full commitment) at  $t = 1$ . We then show that, in this example, these choices are not time-consistent even though the government and private agents can form arbitrary coalitions.

Our example is a two-period problem with some features that resemble the economy analyzed by Fischer (1980). To make the example more concrete, we will change our notation somewhat. Let  $g_t$  denote government expenditures at  $t$  and let  $c_t$  denote consumption by the representative private agent at  $t$ . At time 1 the representative agent has an endowment of a single good,  $w$ , which can be allocated to consumption, government expenditures, or to an investment. We let  $k_2$  denote the quantity of the good set aside at time 1 to be used in production at  $t = 2$ . Then the technological constraints for this economy are

$$c_1 + g_1 + k_2 \leq w, \quad (15)$$

$$c_2 + g_2 \leq Rk_2. \quad (16)$$

Finally, the objective function of the government is  $S(c_1, c_2, g_1, g_2) = g_2$ , and the objective function of the representative agent is  $V(c_1, c_2, g_1, g_2) = c_1 + \beta \min[c_2, \delta g_2]$ . It is assumed that  $\delta > 0$  and that  $\beta R \geq 1$ . To begin, we analyze the solution to the problem of a government that can commit to a set of choices  $(c_1, c_2, k_2, g_1, g_2)$  at  $t = 1$ . In order to solve this problem, we need to derive  $\bar{V}_1$  as defined by eq. (1). Here, clearly,  $\bar{V}_1 = w$ . Then at  $t = 1$  the government and the representative agent, playing cooperatively, choose  $(c_1, c_2, k_2, g_1, g_2)$  to solve the problem of maximizing  $g_2$  subject to (15), (16), and

$$c_1 + \beta \min[c_2, \delta g_2] \geq w. \quad (17)$$

The solution to this problem will satisfy (17) at equality and will also satisfy  $c_2 \leq \delta g_2$ . To see this, simply notice that if  $c_1, c_2, k_2$ , and  $g_2$  have been chosen so that  $c_2 > \delta g_2$ , then it is possible to raise  $g_2$  and reduce  $c_2$  without violating (16) or (17), and still leave  $c_1$  and  $k_2$  unaltered. Then the solution to the problem above has  $g_1 = 0$ , and from (15)–(17).

$$w - k_2 + \beta \min[Rk_2 - g_2, \delta g_2] = w. \quad (18)$$

Since  $Rk_2 - g_2 \leq \delta g_2$ , (18) may be rewritten as

$$k_2 = \beta(Rk_2 - g_2), \quad (19)$$

or alternatively as

$$g_2 = [(\beta R - 1)/\beta] k_2. \quad (20)$$

Then the government must maximize  $g_2$  subject to (20). Clearly the solution to this problem is to set  $k_2 = w$ , with  $c_1 = 0$  and  $g_2 = (\beta R - 1)w/\beta$ .

If at  $t = 2$  the government and private agents resolve their problem, they will take  $c_1$ ,  $g_1$ , and  $k_2$  as given, and choose  $c_2$  and  $g_2$  to solve the problem of maximizing  $g_2$  subject to (16) and

$$\beta \min[c_2, \delta g_2] \geq \bar{V}_2, \quad (21)$$

with  $\bar{V}_2$  given by (1). Clearly  $\bar{V}_2 = 0$ , so the solution to this problem is to set  $g_2 = Rk_2$  and  $c_2 = 0$ . We see that even though play is cooperative, a time-inconsistency problem can arise.

The equilibrium derived in Theorem 1 for this game sets  $c_2 = g_2 = k_2 = 0$ , and sets  $c_1 = w$ . Again, the 'optimal' solution and the equilibrium for this economy diverge, so the selection of policies via coalition formation does not preclude the occurrence of time-consistency problems.

## 5. Optimal taxation

As indicated previously, under suitable maintained hypotheses, the model of policy determination discussed above is capable of delivering sharp, empirically testable implications. For instance, suppose that the government and the private-sector objective functions coincide, and that the government is charged with choosing sequences of expenditure levels and non-lump-sum taxes. Then the analysis predicts that the unique equilibrium allocation will be the same as if the government could raise all its revenue via lump-sum taxation. We illustrate this result in the context of a two-period model. As will be clear, the analysis could easily be extended to incorporate an arbitrary time horizon.

The notation we employ here is identical to that of section 4, except that now we let agents allocate some labor effort. Let  $\bar{l}$  be the time endowment of the representative agent at each date, and let  $l_t$  denote labor supply at  $t$ . Also, we let  $\tau_t$  denote an  $n$ -vector of date  $t$  tax parameters ( $t = 1, 2$ ). The game is one of optimal taxation, so that the government objective function and private objective functions coincide. This objective function is denoted  $V(c_1, \bar{l} - l_1, c_2, \bar{l} - l_2, g_1, g_2)$ .  $V(\cdot)$  is increasing in each argument, is twice continuously differentiable, and is strictly quasi-concave. The technology of the economy is as follows:

$$c_1 + g_1 + k_2 \leq q(l_1), \quad q' > 0, \quad q'' \leq 0, \quad (22)$$

$$c_2 + g_2 \leq f(l_2, k_2), \quad (23)$$

where  $f$  is increasing in each argument, is twice continuously differentiable, and is concave. In addition, there is an exogenous constraint on how revenue can be raised, so that the government faces a financing constraint at each date:

$$g_1 = R_1(\tau_1, l_1, k_2), \quad (24)$$

$$g_2 = R_2(\tau_2, l_2, k_2). \quad (25)$$

$R_t(-)$ ,  $t = 1, 2$ , is assumed to be continuously differentiable.<sup>10</sup> An example of this setup would be a game where the government chooses a set of proportional taxes on factor incomes.

According to Theorem 1, the unique equilibrium of the cooperative policy game can be determined as follows. At  $t = 2$ , given the inherited choices  $(\bar{c}_1, \bar{l}_1, \bar{g}_1, \bar{k}_2, \bar{\tau}_1)$ , the government chooses  $c_2, l_2, g_2$ , and  $\tau_2$  to solve the problem

$$\max V(\bar{x}_1, \bar{l} - \bar{l}_1, c_2, \bar{l} - l_2, \bar{g}_1, g_2),$$

subject to  $c_2 + g_2 \leq f(l_2, \bar{k}_2)$  and (25).<sup>11</sup> Assuming an interior solution, the first-order conditions for this problem can be manipulated to obtain

$$V_3(-)[f_1(l_2, \bar{k}_2) - \partial R_2/\partial l_2] - V_4(-) + V_6(-)(\partial R_2/\partial l_2) = 0, \quad (26)$$

$$-V_3(-)(\partial R_2/\partial \tau_{2i}) + V_6(-)(\partial R_2/\partial \tau_{2i}) = 0, \quad i = 1, \dots, n, \quad (27)$$

where  $V_j$  and  $f_j$  denote the partial derivatives of these functions with respect to their  $j$ th arguments.

As is apparent from (27), if  $\partial R_2/\partial \tau_{2i} \neq 0$  for some  $i$  in equilibrium, then

$$V_3(-) = V_6(-). \quad (28)$$

Then, using (28) in (26) yields

$$V_3(-) f_1(l_2, \bar{k}_2) = V_4(-). \quad (29)$$

Eqs. (28), (29), and  $c_2 + g_2 = f(l_2, \bar{k}_2)$  determine equilibrium values of  $c_2, l_2$ , and  $g_2$ . Any choice of tax parameters satisfying (25) is then an equilibrium

<sup>10</sup>The omission of  $c_t$  as an argument of the function  $R_t(-)$  amounts to a standard normalization in optimal tax settings.

<sup>11</sup>As above, the constraint  $V(\bar{c}_1, \bar{l} - \bar{l}_1, c_2, \bar{l} - l_2, \bar{g}_1, g_2) \geq \bar{V}_2$  does not bind.

choice. We denote the values  $c_2$ ,  $l_2$ , and  $g_2$  determined in this way as:

$$c_2 \equiv h(c_1, l_1, g_1, k_2, \tau_1), \quad g_2 \equiv m(c_1, l_1, g_1, k_2, \tau_1),$$

$$l_2 \equiv n(c_1, l_1, g_1, k_2, \tau_1).$$

Under our assumptions the above are functions. Also, it should be apparent that these values of  $c_2$ ,  $g_2$ , and  $l_2$  are exactly the values that would be chosen if the government could employ lump-sum taxation at  $t = 2$ .

At  $t = 1$  the government chooses  $c_1$ ,  $l_1$ ,  $k_2$ ,  $g_1$ , and  $\tau_1$  to solve the problem

$$\max V [c_1, \bar{l} - l_1, h(c_1, l_1, g_1, k_2, \tau_1), \bar{l} - n(c_1, l_1, g_1, k_2, \tau_1),$$

$$g_1, m(c_1, l_1, g_1, k_2, \tau_1)],$$

subject to (22) and (24).<sup>12</sup> If we define

$$\tilde{h}(l_1, k_2, \tau_1) \equiv h[q(l_1) - R_1(l_1, k_2, \tau_1) - k_2,$$

$$l_1, R_1(l_1, k_2, \tau_1), k_2, \tau_1],$$

$$\tilde{m}(l_1, k_2, \tau_1) \equiv m[q(l_1) - R_1(l_1, k_2, \tau_1) - k_2,$$

$$l_1, R_1(l_1, k_2, \tau_1), k_2, \tau_1],$$

$$\tilde{n}(l_1, k_2, \tau_1) \equiv n[q(l_1) - R_1(l_1, k_2, \tau_1) - k_2,$$

$$l_1, R_1(l_1, k_2, \tau_1), k_2, \tau_1],$$

then the first-order conditions for this problem (assuming an interior optimum) can be manipulated to obtain

$$V_1(-)[q'(l_1) - \partial R_1/\partial l_1] - V_2(-) + V_3(-)(\partial \tilde{h}/\partial l_1) - V_4(-)(\partial \tilde{n}/\partial l_1) + V_5(-)(\partial R_1/\partial l_1) + V_6(-)(\partial \tilde{m}/\partial l_1) = 0, \quad (30)$$

$$-V_1(-)[1 + \partial R_1/\partial k_2] + V_3(-)(\partial \tilde{h}/\partial k_2) - V_4(-)(\partial \tilde{n}/\partial k_2) + V_5(-)(\partial R_1/\partial k_2) + V_6(-)(\partial \tilde{m}/\partial k_2) = 0, \quad (31)$$

$$-V_1(-)(\partial R_1/\partial \tau_{1i}) + V_3(-)(\partial \tilde{h}/\partial \tau_{1i}) - V_4(-)(\partial \tilde{n}/\partial \tau_{1i}) + V_5(-)(\partial R_1/\partial \tau_{1i}) + V_6(-)(\partial \tilde{m}/\partial \tau_{1i}) = 0, \quad i = 1, \dots, n. \quad (32)$$

<sup>12</sup>Again, the constraint  $V(c_1, \bar{l} - l_1, h(-), \bar{l} - n(-), g_1, m(-)) \geq \bar{V}_1$  is not binding.

We now note some facts about the partial derivatives of the functions  $\tilde{h}(-)$ ,  $\tilde{m}(-)$ , and  $\tilde{n}(-)$ . In particular, since  $c_2 + g_2 \equiv \tilde{h}(-) + \tilde{m}(-) \equiv f(l_2, \bar{k}_2)$ ,

$$\partial \tilde{h} / \partial l_1 + \partial \tilde{m} / \partial l_1 = f_1(-)(\partial \tilde{n} / \partial l_1), \quad (33)$$

$$\partial \tilde{h} / \partial k_2 + \partial \tilde{m} / \partial k_2 = f_1(-)(\partial \tilde{n} / \partial k_2) + f_2(-), \quad (34)$$

$$\partial \tilde{h} / \partial \tau_{1i} + \partial \tilde{m} / \partial \tau_{1i} = f_1(-)(\partial \tilde{n} / \partial \tau_{1i}), \quad i = 1, \dots, n. \quad (35)$$

Now, using (35) in (32) and making note of (28) and (29), we obtain

$$V_1(-) = V_5(-), \quad (36)$$

if  $\partial R_i / \partial \tau_{1i} \neq 0$  for some  $i$ . Further, using (36) and (34) in (31) and making use of (28) and (29), we obtain

$$V_1(-) = V_3(-) f_2(l_2, k_2). \quad (37)$$

Finally, using (36), (33), (28), and (29) in (30) yields

$$V_1(-) q'(l_1) = V_2(-). \quad (38)$$

Conditions (36)–(38) and (22) determine  $c_1$ ,  $l_1$ ,  $g_1$ , and  $k_2$ . The tax parameters  $\tau_{1i}$  can take on any values that satisfy (24) in equilibrium.

This completes the description of equilibrium values in this cooperative game of dynamic optimal taxation. As should be apparent, all equilibrium allocations will be identical to those that would obtain if the government were allowed to employ lump-sum taxation here. Hence, even though the government is formally precluded from the use of lump-sum taxes, there are no distortions from the use of non-lump-sum taxes. In the next section we discuss some empirical findings that we think bear on this implication of the model.

Before doing so, it may be helpful to review the intuition underlying the absence of tax distortions under our model of the policy selection process. In the usual formulation of either static [e.g., Diamond and Mirrlees (1971)] or dynamic [e.g., Lucas and Stokey (1983)] optimal taxation exercises, each private agent views himself as being able to trade with others at (parametric) gross-of-tax prices. In this setting taxes introduce ‘wedges’ either between marginal rates of substitution for different agents or between marginal rates of substitution and marginal rates of transformation.

In the model we propose, on the other hand, trade (and other economic interactions) is accomplished through the formation of coalitions. No agent views himself as being able to trade with other agents in arbitrary amounts at a fixed gross-of-tax price. Thus, any agent who wished to improve upon the



allocation he received would have to form a coalition consisting of himself, other (dated) private agents, and possibly (differently dated) policy makers. But, since all agents have the same objective function and since the initial allocation is Pareto-optimal, it would not be possible to form such a coalition. Thus, no agent (or group of agents) will attempt to deviate from the equilibrium actions described above, despite the absence of commitment and even if the government must move first in setting tax rates each period.<sup>13</sup>

The results just described have, of course, been obtained under the assumption that the government budget is balanced each period [eqs. (24) and (25)]. This assumption is easily dispensed with. To see this suppose that the government is allowed to issue real debt at  $t = 1$ , denoted by  $B$  ( $B < 0$  denotes government lending) and the government can commit itself to repay  $(1 + p)$  per unit borrowed at  $t = 2$ .<sup>14</sup> Then, modifying eqs. (24) and (25) to

$$g_1 = R_1(\tau_1, l_1, k_2) + B, \quad (24')$$

$$g_2 = R_2(\tau_2, l_2, k_2) - (1 + p)B, \quad (25')$$

and adding  $B$  to the list of government choice variables at  $t = 1$  [it is possible either to require  $1 + p$  to equal a market real rate or to allow the government to choose its repayment  $(1 + p)B$ ], it is straightforward to verify that the equilibrium allocation remains unaltered. It is also the case that any choice of  $\tau_1$ ,  $\tau_2$ , and  $B$  that satisfies (24') and (25') continues to constitute an equilibrium. Such a result is, of course, not surprising, since all taxation here is equivalent to lump-sum taxation. Thus our ability to obtain a Ricardian equivalence result, asserting that the timing of taxation is irrelevant, is to be expected. And, of course, the presence of government debt does not alter the time-consistency properties of an equilibrium.<sup>15</sup>

The results of this section may at first glance seem striking and somewhat counter to intuition. However, they simply reflect the fact that, when the objective functions of all players coincide, there is no time-consistency problem. Hence unblocked sequences of actions must be Pareto-optimal so that distortions created by non-lump-sum taxation are internalized. This, of course, leaves open the question of how an unblocked sequence of actions is imple-

<sup>13</sup>It should be apparent that this result does not depend in any way on the economy being dynamic. To see that our results apply in static settings as well, it suffices to observe that, at  $t = 2$ , the economy considered above is a static one.

<sup>14</sup>As in Lucas and Stokey (1983), the government is assumed not to be able to default on this debt.

<sup>15</sup>An interesting question, which would parallel that asked by Lucas and Stokey (1983), is whether it is possible for the government to issue nominally denominated debt in a time-consistent manner. We are not at this point prepared to address such a question, since an interesting model with money would necessarily be an infinite-horizon model. An extension of the analysis to infinite-horizon settings is a topic of future research.

mented in a decentralized fashion. We have no good answer to this question but note that it will arise in the same way in almost any analysis of the core in an economy with public goods or externalities. The potential failure of competitive or Lindahl equilibrium allocations to coincide with core allocations in such contexts is well known [Starrett (1973)].

In view of this remark, it seems to us that whether the model of social arrangements embodied in the core concept has merit for thinking about policy selection compared to noncooperative alternatives depends in part on whether there is any empirical support for its predictions. Under the maintained hypothesis of identical government and private-sector objective functions, our model has some quite sharp empirical implications:<sup>16</sup>

- (a) Distortions associated with non-lump-sum taxation should be small.
- (b) Changes in marginal tax rates, with government expenditure sequences held fixed, should have small effects on equilibrium allocations.
- (c) Ricardian equivalence should obtain.

A rejection of any of these implications would constitute a rejection of the joint hypotheses embodied in our formulation of the policy-selection problem and of coincident objective functions.

Before discussing (in section 6) some potential sources of support for the predictions just described, some comments are in order. First, much of the force of these predictions emerges when they are considered jointly. In particular, it is possible to imagine (a) and (b) not being rejected empirically simply because of almost offsetting income and substitution effects, rather than because our model is 'correct'. Or, alternatively, Ricardian equivalence can obtain locally in a tax-smoothing model, as pointed out by Barro (1979), despite the importance of tax distortions. But a tax-smoothing model, for example, would deliver (a)–(c) together only under very special circumstances. In general, such a model could deliver Ricardian equivalence locally, but would require that there be significant real effects from tax distortions.

Second, if one could derive a model in which tax distortions were important, but in which (a) and (b) were not rejected (say because income and substitution effects were nearly offsetting at observed equilibrium values), prediction (c) would still constitute a means of empirically discriminating among models. For instance, under optimal tax smoothing, Ricardian equivalence will obtain locally, but not for large rearrangements in the timing of taxation. Some large

<sup>16</sup>Naturally, in the absence of some assumptions about the form of the government objective function, the analysis delivers no sharp empirical predictions. This would also be true of any of the policy formulation models discussed in the introduction.

rearrangements in the timing of taxation that appear to be consistent with Ricardian equivalence are discussed in section 6.

Third, suppose (a)–(c) were not rejected in a particular data set, but one believed that tax distortions were important. Ricardian equivalence would obtain (locally) only if tax distortions were being ‘smoothed’ optimally. As discussed by Barro (1979) and emphasized by Bohn (1989), the assumption that tax smoothing was occurring would have empirical implications for the codetermination of marginal (or average) tax rates and other equilibrium quantities. These implications would permit empirical discrimination between this view and ours.<sup>17</sup>

## 6. Concluding comments

There are clearly many ways in which the interactions between the government and the private sector in the policy-selection process can be modelled. The motivation of this paper was to explore the implications of a model in which all agents, including policy makers, can form coalitions and coordinate their choices of actions. In the process of coalition formation, however, no agents were allowed to commit their future selves to any particular course of action. This approach allowed us to take the view that interactions between the government and the private sector have cooperative aspects, while retaining the feature of earlier analyses that time consistency of policy choices is a central issue. We were able to show that simple models with these features are tractable, producing unique equilibria under fairly weak conditions.

Given that it is possible to produce many models of the policy-selection process, it should be hoped that some of these models will give rise to testable implications, allowing empirical discrimination among various models. Under the maintained hypothesis that the objective functions of the government and the private sector coincide, our model has such an implication; there are no distortions from the use of non-lump-sum taxes, and Ricardian equivalence obtains. We would like to conclude by discussing what we regard as some loose empirical support for this view of the policy-selection process.

Suppose one takes the (standard) view that observed tax systems induce distortions. Then there seem to us to be some puzzling empirical claims in various literatures. For instance, Kormendi (1983) and Aschauer (1985) claim to provide evidence supporting a Ricardian equivalence proposition for the postwar U.S. However, except for some local results associated with tax-smoothing models, it is not possible to derive such a proposition if taxation is distorting. Our results, on the other hand, illustrate how Kormendi’s and Aschauer’s findings are possible in an economy with (apparently) distorting

<sup>17</sup>For an argument that the time-series behavior of marginal tax rates is not generally consistent with the implications of tax smoothing, see Seater (1982, pp. 374–375).

taxation. Similarly, Sargent (1982) and Smith (1985a, b, 1987) claim to provide empirical support for models giving rise to Modigliani–Miller theorems for open-market operations. Such theorems can be derived only when nondistorting taxes are available.<sup>18</sup> Thus, the Sargent–Smith claim requires that the apparent use of distorting taxes be illusory.

Furthermore, there are claims in the empirical public finance literature that the marginal excess burden of public funds could be quite low. Browning (1976) estimates the marginal excess burden to be as low as 9 cents per dollar, of which 2 to  $2\frac{1}{2}$  cents represents estimated costs of collection and enforcement. Stuart (1984), using a different methodology, obtains estimates as low as 9 cents on the dollar, while Ballard, Shoven, and Whalley (1985) get estimates as low as 15 cents. While in general the range of estimates obtained is quite large [see, e.g., Browning (1987)] and very sensitive to small variations in parameter values, these results suggest that it is at least possible that distortions due to taxation are small.

Finally, the findings of researchers such as Kydland and Prescott (1982) and Hansen (1985) that competitive models, which are distortion-free, can readily mimic U.S. economic time series, are at least consistent with the idea that economic distortions due to taxation are not important.

While the evidence just cited is not very direct, it suggests that there is some support for the empirical implications of a cooperative view of the problem of policy selection.

## Appendix: Proof of Theorem 1

*Theorem 1.* Suppose the sequence of actions  $(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$  given by  $x_t = f_t(-)$ ,  $\pi_t = g_t(-)$ ,  $\forall t$ , is chosen. Then this set of actions is not blocked.

*Proof.* As noted in the text, it is immediate that there is no blocking coalition consisting of private agents alone. Then we must prove that there is no blocking coalition of the form  $\{p_\tau\}_{\tau=t}^T \cup \{a_\tau\}_{\tau=t}^T$ . The proof is by induction. Consider first period  $T$ . We claim that for any (given) history  $h_{T-1}$  the choices  $x_T^* = f_T(h_{T-1})$  and  $\pi_T^* = g_T(h_{T-1})$  are not blocked at  $T$ . This follows trivially from the definitions of  $f_T(-)$  and  $g_T(-)$ .

To continue with the proof by induction, we now suppose that, for any (given) history  $h_q$ , the values  $x_s^* = f_s(h_{s-1})$  and  $\pi_s^* = g_s(h_{s-1})$ ,  $s = q + 1, \dots, T$ , are such that  $(x_1, \dots, x_q, x_{q+1}^*, \dots, x_T^*, \pi_1, \dots, \pi_q, \pi_{q+1}^*, \dots, \pi_T^*)$  is not

<sup>18</sup>The episodes discussed by Sargent and Smith involve very large open-market operations. In the case of the evidence discussed by Smith, these open-market operations also involved large rearrangements in the timing of taxation. Hence Smith's evidence is consistent with Ricardian equivalence. In addition, these rearrangements were large enough so that they would be difficult to explain by appealing to any local results regarding Ricardian equivalence.

blocked at date  $q + 1$  or later. Then we claim that, if

$$x_q = x_q^* = f_q(h_{q-1}) \quad \text{and} \quad \pi_q = \pi_q^* = g(h_{q-1}),$$

$(x_1, \dots, x_{q-1}, x_q^*, \dots, x_T^*, \pi_1, \dots, \pi_{q-1}, \pi_q^*, \dots, \pi_T^*)$  is also not blocked at date  $q$  or later.

In particular, by induction,  $(x_1, \dots, x_{q-1}, x_q^*, \dots, x_T^*, \pi_1, \dots, \pi_{q-1}, \pi_q^*, \dots, \pi_T^*)$  is not blocked at date  $q + 1$  or later. Then, if it is blocked, it is blocked at date  $q$ . Consequently there exist values  $(\hat{x}_q, \dots, \hat{x}_T, \hat{\pi}_q, \dots, \hat{\pi}_T)$  such that

$$\begin{aligned} & S(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_T, \pi_1, \dots, \pi_{q-1}, \hat{\pi}_q, \dots, \hat{\pi}_T) \\ & > S(x_1, \dots, x_{q-1}, x_q^*, \dots, x_T^*, \pi_1, \dots, \pi_{q-1}, \pi_q^*, \dots, \pi_T^*), \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} & V(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_T, \pi_1, \dots, \pi_{q-1}, \hat{\pi}_q, \dots, \hat{\pi}_T) \geq \bar{V}_k, \\ & k \geq q. \end{aligned} \quad (\text{A.2})$$

Moreover,  $(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_T, \pi_1, \dots, \pi_{q-1}, \hat{\pi}_q, \dots, \hat{\pi}_T)$  cannot itself be blocked at some date later than  $q$ .

Now, for (A.1) and (A.2) to hold, it is clearly the case that  $\hat{x}_k \neq f_k(-)$  and/or  $\hat{\pi}_k \neq g_k(-)$  for some  $k > q$ . Let  $\bar{k}$  be the largest date such that either  $\hat{x}_k \neq f_k(-)$  or  $\hat{\pi}_k \neq g_k(-)$ . We now show that  $(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_T, \pi_1, \dots, \pi_{q-1}, \hat{\pi}_q, \dots, \hat{\pi}_T)$  is blocked at date  $\bar{k}$  by the grand coalition. In particular, set  $\tilde{x}_{\bar{k}} = f_{\bar{k}}(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_{\bar{k}-1}, \pi_1, \dots, \pi_{q-1}, \hat{\pi}_q, \dots, \hat{\pi}_{\bar{k}-1})$ , set  $\tilde{\pi}_{\bar{k}} = g_{\bar{k}}(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_{\bar{k}-1}, \pi_1, \dots, \pi_{q-1}, \hat{\pi}_q, \dots, \hat{\pi}_{\bar{k}-1})$ , set  $\tilde{x}_{k+1} = f_{k+1}(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_{\bar{k}-1}, \tilde{x}_{\bar{k}}, \pi_1, \dots, \pi_{q-1}, \hat{\pi}_q, \dots, \hat{\pi}_{\bar{k}-1}, \tilde{\pi}_{\bar{k}})$ , etc. Then, by the definition of the functions  $f_s(-)$  and  $g_s(-)$ ,

$$\begin{aligned} & S(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_{\bar{k}-1}, \tilde{x}_{\bar{k}}, \dots, \tilde{x}_T, \pi_1, \dots, \pi_{q-1}, \\ & \quad \hat{\pi}_q, \dots, \hat{\pi}_{\bar{k}-1}, \tilde{\pi}_{\bar{k}}, \dots, \tilde{\pi}_T) \\ & > S(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_T, \pi_1, \dots, \pi_{q-1}, \hat{\pi}_q, \dots, \hat{\pi}_T), \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} & V(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_{\bar{k}-1}, \tilde{x}_{\bar{k}}, \dots, \tilde{x}_T, \pi_1, \dots, \pi_{q-1}, \\ & \quad \hat{\pi}_q, \dots, \hat{\pi}_{\bar{k}-1}, \tilde{\pi}_{\bar{k}}, \dots, \tilde{\pi}_T) \geq \bar{V}_k. \end{aligned}$$

Moreover, by induction,  $(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_{\bar{k}-1}, \tilde{x}_{\bar{k}}, \dots, \tilde{x}_T, \pi_1, \dots, \pi_{q-1}, \hat{\pi}_q, \dots, \hat{\pi}_{\bar{k}-1}, \tilde{\pi}_{\bar{k}}, \dots, \tilde{\pi}_T)$  is not blocked at any date later than  $\bar{k}$ . This implies that  $(x_1, \dots, x_{q-1}, \hat{x}_q, \dots, \hat{x}_T, \pi_1, \dots, \pi_{q-1}, \hat{\pi}_q, \dots, \hat{\pi}_T)$  is itself blocked

at  $\bar{k}$ , which contradicts that  $(x_1, \dots, x_{q-1}, x_q^*, \dots, x_T^*, \pi_1, \dots, \pi_{q-1}, \pi_q^*, \dots, \pi_T^*)$  is blocked at  $q$ . Thus, for any history  $h_{q-1}$  the choices  $(x_q^*, \dots, x_T^*, \pi_q^*, \dots, \pi_T^*)$  are not blocked. But then, the choices  $x_t = f_t(-)$ ,  $\pi_t = g_t(-)$ ,  $\forall t$ , are not blocked, establishing the theorem.

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