Optimal Monetary Policy in a Phillips-Curve World[1]

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Abstract

In this paper we study the optimal monetary policy in a model that integrates the modern theory of unemployment with a liquidity model of monetary transmission. Two policy environments are considered: period by period optimization (time consistency) and full commitment (Ramsey allocation). When the economy is subject to aggregate productivity shocks, the optimal monetary policy is pro-cyclical. In addition to these cyclical properties, we also characterize the long-term properties of the optimal policy and we show that the time-consistent policy may induce lower inflation than the commitment policy. This is in contrast to many studies arguing that higher inflation may result from the lack of policy commitment.

“*Despite its disrepute within important academic and policymaking circles, the Phillips Curve persists in U.S. data. Simple econometric procedures detect it.*”

Thomas Sargent, 1998

Introduction

On September 29, 1998 the Federal Open Market Committee met to determine the course of monetary policy. All of the economic indicators available at the time suggested that the economy was in remarkably good shape; unemployment was very low, the inflation rate was close to zero, and the economy was growing steadily although, it was thought, not as fast as in the first quarter of the year. Surprisingly, given the available data, the FOMC decided to ease monetary policy by lowering the Federal Funds rate 25 basis points. The one concrete reason cited in support of this action was that credit seemed to be getting tight, as evidenced by the fact that commercial borrowers were experiencing increasing difficulty obtaining financing.[1] In their words “...an easing policy action at this point could provide added insurance against the risk of a further worsening in financial conditions and a related curtailment in the availability of credit to many borrowers.” In the period between the September meeting and the November meeting the Fed took steps to lower the Federal Funds rate again. At the November meeting they faced much the same economic background, strong growth, low inflation, and very low unemployment, and yet they decided to ease rates again. In the fourth quarter of 1998 real GDP grew at a rate of 6.1 percent. Viewed in terms of the conventional wisdom about monetary policy rules, this sequence of decisions seems somewhat surprising. In this paper we argue that the Fed was

[1]This shortage of liquidity was thought to be due in part to the Russian bond default and the crisis faced by Long Term Capital Management.

doing exactly what the optimal and time-consistent monetary policy would recommend.

The search for formal rules that describe how policy has been conducted or should be conducted has been an important area of research. Thoughtful economists like John Taylor (1993, 1998) Lars Svensson (1997b, 1997a) and Clarida, Gali, & Gertler (1999), have proposed formal rules for monetary policy that are grounded in careful empirical research. The Taylor rule, for example, is one in which monetary policy responds to inflation or expected inflation and the gap between actual and potential output. If output is below potential output, the implication is that monetary policy should ease. If output is above potential output monetary policy should tighten. The conventional interpretation of this rule in the face of the facts that prevailed in the autumn of 1998 would hold that inflation and expected inflation were low (essentially zero) and that output was at or below the potential level.

The notion that monetary policy should respond to the output gap is based on the view that inflationary monetary policies have expansionary effects on the real activity, at least in the short-run. This view derives from a robust empirical feature of post-war U.S. data, the positive correlation between inflation and employment (or output), as documented by Sargent (1998). We refer to this empirical observation as the Phillips curve relation. The view that inflationary monetary policies have expansionary effects on the economy, is also supported by recent empirical studies, which find significant liquidity effects of monetary policy shocks.[2] The idea that the objective of the monetary authority should be to smooth employment (or output) by expanding the stock of money during periods of low employment and reducing the stock of money during periods of high employment follows

[2]See, for example, Christiano, Eichenbaum, & Evans (1996), Hamilton (1997), Leeper, Sims, & Zha (1996).

from these observations.

If the optimal policy was to smooth employment and if the monetary authority acts optimally, we should observe a negative correlation between monetary aggregates and employment, at least for those aggregates that are under the control of the monetary authority. Data from the post-war period, however, show that employment is positively correlated with all monetary aggregates, including the monetary base.[3] There are important issues of causality here but the simple correlations suggest that post-war U.S. monetary policy has been pro-cyclical rather than counter-cyclical. If so it may have reinforced business fluctuations rather than smoothed them.

Our goal in this paper is to characterize the optimal monetary policy in a general equilibrium model where there is a direct link between monetary policy and employment. The real side of the economy is characterized by a search and matching framework with equilibrium unemployment. In this framework we introduce a monetary sector in which changes in the supply of money change the supply of loanable funds, which in turn affect the nominal interest rate. These liquidity effects cause changes in real activity by lowering the cost of working capital for firms. In this way the model captures the "cost channel" of monetary transmission that Barth & Ramey (1999) find significant for the propagation of monetary shocks in the economy.

In this framework we study the optimal monetary policy under two policy environments. In the first environment we assume that monetary policy interventions are decided on a period-by-period basis, and the monetary authority cannot credibly commit to long-run plans. This implies that the type of policies we analyze are time-consistent. In studying the time-consistent policies, we restrict the analysis to policies that are Markov-stationary,

[3]See Cooley & Hansen (1995) for a documentation of the main monetary facts in the U.S. economy.

that is, policy rules that only depend on the current (physical) states of the economy. In the second policy environment, instead, we assume that the monetary authority is able to commit to long-term plans (Ramsey allocation).

There are two main findings. The first finding concerns the cyclical properties of the optimal policy while the second concerns the long-term properties. Regarding the cyclical properties, we show that in both environments—with and without commitment—the optimal policy is pro-cyclical when business cycle fluctuations are driven by technology shocks. More specifically, the optimal policy increases the stock of money when employment and output are high and reduces the stock of money when employment and output are low. In this sense, the observed properties of the post-war data as discussed above are not inconsistent with the optimal policy of a benevolent policy maker. In addition, the optimal growth rate of money displays some persistence in the sense of being serially correlated, which is also a feature of the post-war history of the U.S. economy. To generate this positive correlation, the search framework in the labor market plays a crucial role. Without this framework, the current growth rate of money would be negatively correlated with the previous rate.

This result can be explained as follows: an increase in employment and output in the economy that results from productivity shocks increases the nominal interest rate. The increase in the interest rate generates inefficiencies that a benevolent policy maker would like to eliminate. Because changes in the stock of money have liquidity effects, the way to prevent an interest rate increase is to expand the stock of money when the shock is high. Interpreted correctly, such a policy is consistent with the Taylor rule. If the economy experiences a positive technology shock, then output will be below the potential level associated with that shock unless more liquidity is supplied to the economy. Therefore, the stock of money has to increase when output increases. Now, because the search and matching frictions in the labor market generate a persistent response of employment to shocks (hump-shape response) and output grows for more than one period, then the optimal stock of money also grows for more than one period. In the absence of matching frictions, output will grow only in the first period and then return to the steady state. In this case the first order serial correlation of the optimal growth rate of money would be negative: it would be above the steady state only in the first period and below the steady state afterwards.

The second important finding is that there are important differences between the time-consistent policy and the optimal policy under commitment. Again, the matching framework that characterizes the real sector of the model plays a crucial role in this . We show that when the worker's share of the matching surplus is small, the time-consistent policy is less inflationary than the optimal policy with commitment. This result contrasts with earlier studies of optimal monetary policy, such as Barro & Gordon (1983).

The intuition for this result is simple. In this economy there are two possible sources of inefficiency. The first inefficiency derives from the cost of financing the current production plan. This cost is determined by the nominal interest rate. On this dimension, a Friedman rule of a zero nominal interest rate would be optimal. The second source of inefficiency derives from the matching frictions in the labor market. If the worker's share of the surplus is smaller than a certain value, the high profitability of a match for the firm induces an excessive creation of vacancies. The policy maker can reduce the profitability of a match by increasing the inflation rate. However, the decision to create new vacancies is not affected by either the current inflation rate or by the next period inflation rate. What affects the return on a new vacancy is the change in prices two periods from now. The policy maker is able to *credibly* affect the inflation rate two periods from now, only if it can commit. Otherwise, after the new vacancies have been created, it no longer has the incentive to inflate. The lack of commitment, then, implies that the time-consistent policy is given by a simple Friedman rule of a zero nominal interest rate while the optimal policy with commitment will set nominal interest rates that are larger than zero. In the long run, higher interest rates are associated with higher inflation rates (Fisher rule).

The properties of the optimal and time-consistent policy have also been studied by Peter Ireland (1996), but in a different environment in which monetary policy interventions affect the real sector of the economy through the rigidity of nominal prices. In Ireland's model the optimal monetary policy is also pro-cyclical when business fluctuations are driven by technology shocks. However, the Ireland's model does not generate the positive serial correlation in the growth rate of money as our model does. In addition, his results do not extend to our long-term results for which policy commitment can affect the properties of the optimal policy as described above.

A study of the differences between time-consistent policies and optimal policies with commitment in models with sticky prices and liquidity effects is also conducted by Albanesi, Chari, & Christiano (1999). In contrast to our paper, they do not find important long-term differences in the environment with and without commitment. We reach a different conclusion because of the more complex dynamics introduced by the labor market in our economy (searching and matching frictions).

The plan of the paper is as follows. In section 1 we describe the model and in section 2 we define the optimal policy in the two policy environments: absence of commitment and full commitment. Section 3 characterizes the analytical properties of the optimal policy and section 4 examines the quantitative properties of the time-consistent policy. Finally, section 5 concludes.

1 The economy

We describe here a monetary economy that is specifically designed to generate the liquidity effect of monetary interventions, that is a reduction in the nominal lending rate after a monetary expansion. The reduction in the cost of borrowing, in turn, leads to an expansion in the real sector of the economy. By designing the economy so that inflationary policies have expansionary effects, we capture the main idea behind the Phillips curve relation, that is, the idea that in the short run there is a trade-off between inflation and the real activity (a Phillips curve world), and this trade-off can be used for the design of monetary policy. However, rather than taking this relation as given, we derive it from a fully specified model. The advantage of this modeling strategy is that it allows us to define the objective of the policy maker as maximizing the welfare of the agents in the economy (benevolent policy maker), rather than defining it as an arbitrary objective over inflation and output or unemployment.

1.1 The monetary authority and the intermediation sector

The total amount of households' nominally denominated assets is denoted by M. We interpret M as a broad monetary aggregate and will refer to M as money. Part of these assets are used for transactions and the remaining quantity is held in the form of bank deposits. The funds collected by banks are then used to make loans to firms.

The monetary authority controls the quantity of money in the economy by making transfers to the households. Monetary transfers are in the form of bank deposits. Denoting by g the growth rate of money, the monetary transfers are equal to gM.

Given the interpretation of M as a broad monetary aggregate, it would be more realistic to assume that the monetary authority controls M only imperfectly. One way to formalize this idea is to assume that $g = \hat{g} + \varepsilon$, where $\hat{g}$ is perfectly controlled by the monetary authority and ε is some monetary shock that is observed after the choice of $\hat{g}$. In the first part of the paper, we keep the assumption that this shock is zero. Then in section 4.2 we will consider this second source of uncertainty. As we will see, the properties of the optimal policy are not affected by the presence of the monetary shock.

For monetary interventions to have a liquidity effect, that is a fall in the nominal interest rate after a monetary expansion, some form of rigidity has to be imposed in the households' ability to readjust their stock of deposits. We assume that the households choose the stock of nominal deposits at the end of each period, after all transactions have taken place, and they must wait until the end of the next period before being able to change their portfolio. Thus, the household cannot immediately readjust its portfolio of deposits after the observation of a new shock or after the monetary authority has changed its monetary tools. Denote by D the pre-transfers households' deposits. Because the monetary transfers are in the form of bank deposits and households cannot readjust their stock of deposits, the funds available to banks to make loans are $D + gM$. Therefore, an increase in the growth rate of money increases the stock of loanable funds, which in turn induces a fall in the nominal interest rate. This is a very standard assumption in the class of "limited participation" models as in Lucas (1990), Fuerst (1992) and Christiano & Eichenbaum (1995).

1.2 Households

There is a continuum of households of total measure 1 that maximize the expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t (c_t - \chi_t a) \tag{1}$$

where c_t is consumption of market produced goods, a is the disutility from working and χ_t is an indicator function taking the value of one if the agent is employed and zero if unemployed.

Households own three types of assets: transaction funds (cash), nominal deposits and firms' shares. Denoting by m the pre-transfers nominally denominated assets and by d the quantity of these assets kept in the form of deposits, the household's transaction funds are $m - d$. In each period, agents are subject to the following cash-in-advance and budget constraints:

$$P(c + i) \leq m - d \tag{2}$$

$$P(c + i) + m' = m + gM + (d + gM)R + \chi Pw + P\pi n \tag{3}$$

The variable P is the nominal price, i is the household's investment in the shares of new firms, n identifies the number of firms' shares that the household owns and π the dividends paid by these firms. The wage received by an employed worker is denoted by w and it is paid at the end of the period. The determination of the wage will be specified below. The after-transfer stock of deposits is $d + gM$ (remember that transfers are in the form of bank deposits). These deposits earn the nominal interest rate R.

1.3 Production

The production sector is characterized by a search-matching framework similar to the labor-search model of Pissarides (1988) and the exogenous separation version of Mortensen & Pissarides (1994). The production technology displays constant returns-to-scale with respect to the number of employees. Without loss of generality, it is convenient to assume that there is a single firm for each worker. The search for a worker involves a fixed cost κ

and the probability of finding a worker depends on the matching technology $\mu V^{\alpha}(1-N)^{1-\alpha}$, where V is the number of vacancies (number of firms searching for a worker), $1-N$ is the number of searching workers. The parameter α is restricted to assume values between zero and one. The probability that a searching firm finds a worker is denoted by q and it is equal to $\mu V^{\alpha}(1-N)^{1-\alpha}/V$, while the probability that an unemployed worker finds a job is denoted by h and is equal to $\mu V^{\alpha}(1-N)^{1-\alpha}/(1-N)$. Job separation is exogenous and occurs with probability λ. There is no cost to searching for an unemployed worker. Employed workers cannot search for a new job.

If the search process is successful, the firm operates the technology $y = Ax^{\nu}$, where A is the aggregate level of technology, x is an intermediate input. Output goods and intermediate goods are perfect substitutes, and therefore, the relative price of these two goods is 1. The aggregate technology level A is equal to e^{z} where z is an aggregate technology shock that follows a first order Markov process with transition density function $\Gamma(z, z')$.

Transactions for the purchase of the intermediate good require liquid funds. Firms get this cash by borrowing from a financial intermediary at the nominal interest rate R. By assuming that the intermediate input has to be paid in advance, the model captures the "cost channel" of monetary transmission which has been found to be empirically relevant for the propagation of monetary shocks in the economy.[4] See Barth & Ramey (1999).

The contract signed between the firm and the worker specifies the wage w so that the

[4]In alternative, we could assume that working hours are flexible and the intermediate input is replaced by the number of working hours. The properties of this alternative model would be equivalent to the current model if we assume that the part of the worker's payment that compensates the disutility from working, has to be paid in advance.

worker gets a share η of the surplus generated by the match. The assumption of a constant sharing fraction of the surplus is standard in this class of models and it is motivated theoretically by assuming Nash bargaining between the firm and the worker, with η the bargaining power of the worker relative to the firm. As we will see, this parameter plays a crucial role in characterizing the properties of the optimal policy. The wage, $w(\mathbf{s})$, depends on the states of the economy $\mathbf{s}$, as specified below.

1.3.1 Firms

Firms post vacancies and implement optimal production plans to maximize the welfare of their shareholders. Denote by $J(\mathbf{s})$ the value of a match for the firm measured in terms of current consumption. This is given by:

$$J(\mathbf{s}) = \tilde{\pi}(\mathbf{s}) + \beta(1-\lambda)EJ(\mathbf{s}') \tag{4}$$

For notational convenience, we have redefined the function $\tilde{\pi}(\mathbf{s}) = E[\beta P(\mathbf{s})\pi(\mathbf{s})/P(\mathbf{s}')]$, where $\pi(\mathbf{s})$ are the dividends paid by the firm to the shareholders at the end of the period. The function expresses the current value for the shareholder of the dividend paid by the firm. Because dividends are paid at the end of the period, the shareholder needs to wait until the next period to transform monetary assets into consumption. This implies that the current value in terms of consumption of one unit of money received at the end of the period is $\beta P(\mathbf{s})/P(\mathbf{s}')$.

The dividends paid to the shareholders are equal to the output produced by the firm minus the cost for the intermediate input, $x(1+R)$, and the labor cost, w:

$$\pi = Ax^{\nu} - x(1+R) - w. \tag{5}$$

Notice that the cost for the intermediate input also includes the interest paid on the loan used to finance the payment of the input.

Given $J(\mathbf{s})$, the value of a vacancy is denoted by $Q(\mathbf{s})$ and is defined as:

$$Q(\mathbf{s}) = -\kappa + q(\mathbf{s})\beta EJ(\mathbf{s}') + (1 - q(\mathbf{s}))\beta EQ(\mathbf{s}') \tag{6}$$

Because the value of a vacancy must be zero in equilibrium, that is, $Q(\mathbf{s}) = 0$, equation (6) becomes:

$$\kappa = q(\mathbf{s})\beta EJ(\mathbf{s}') \tag{7}$$

Equation (7) is the arbitrage condition for the posting of new vacancies, and accordingly, for the creation of new jobs. It simply says that, in equilibrium, the cost of posting a vacancy, κ, is equal to the discounted expected return from posting the vacancy.

Consider now the value of a match for a worker. Define $W(\mathbf{s}, \varphi)$ and $U(\mathbf{s})$ to be, respectively, the value of a match and the value of being unemployed in terms of current consumption. They are defined as:

$$W(\mathbf{s}) = \tilde{w}(\mathbf{s}) - a + (1 - \lambda)\beta EW(\mathbf{s}') + \beta\lambda EU(\mathbf{s}') \tag{8}$$

$$U(\mathbf{s}) = h(\mathbf{s})\beta EW(\mathbf{s}') + (1 - h(\mathbf{s}))\beta EU(\mathbf{s}') \tag{9}$$

where $\tilde{w} = E[\beta P(\mathbf{s})w(\mathbf{s})/P(\mathbf{s}')]$. As with dividends, the wage $w(\mathbf{s})$ is multiplied by the term $E\beta P(\mathbf{s})/P(\mathbf{s}')$ because wages are paid at the end of the period. Adding equations (4) and (8), and subtracting equation (9), gives the total surplus generated by the match $S(\mathbf{s})$. The surplus is shared between the worker and the firm according to the fixed proportion η, that is, $W(\mathbf{s}) - U(\mathbf{s}) = \eta S(\mathbf{s})$ and $J(\mathbf{s}) = (1 - \eta)S(\mathbf{s})$. Using this sharing rule and equation

(7), the surplus of the match can be written as:

$$S(\mathbf{s}) = \tilde{\pi}(\mathbf{s}) + \tilde{w}(\mathbf{s}) - a + \frac{(1-\lambda-\eta h(\mathbf{s}))\kappa}{(1-\eta)q(\mathbf{s})} \tag{10}$$

Moreover, by equating $W(\mathbf{s}) - U(\mathbf{s})$ to $\eta S(\mathbf{s})$, and using (5), we derive the wage $w(\mathbf{s})$ which is equal to:

$$w(\mathbf{s}) = \eta(Ax^{\nu} - x(1+R)) + \frac{(1-\eta)a}{E\left(\frac{\beta P(\mathbf{s})}{P(\mathbf{s}')}\right)} + \frac{\eta h(\mathbf{s})\kappa}{q(\mathbf{s})E\left(\frac{\beta P(\mathbf{s})}{P(\mathbf{s}')}\right)} \tag{11}$$

The wage $w(\mathbf{s})$ as well as the surplus generated by the match depend on the intermediate input x. Because the firm and the worker are splitting the surplus, the optimal input x maximizes this surplus. Based on this, we have:

Proposition 1.1 *The optimal input x is given by:*

$$x = \left(\frac{\nu A}{1+R}\right)^{\frac{1}{1-\nu}}$$

Proof 1.1 *By differentiating the surplus in equation (10) after substituting $\pi(s) + w(s) = Ax^{\nu} - x(1+R)$, we get the result.*

According to proposition 1.1, the intermediate input, and therefore, firm's output, is decreasing in the nominal interest rate R. This is because the interest rate increases the marginal cost of the intermediate input. Therefore, one way in which monetary policy interventions have an impact on the real sector of the economy, is by affecting the equilibrium interest rate.

Using equations (7) and (4) we derive:

$$\frac{\kappa}{q(\mathbf{s})} = \beta\tilde{\pi}(\mathbf{s}') + \beta E\left(\frac{(1-\lambda)\kappa}{q(\mathbf{s}')}\right) \tag{12}$$

where as before, $\tilde{\pi}(\mathbf{s})$ is the value in terms of current consumption of dividends distributed by the firm at the end of the period. Using forward substitution and the law of iterated expectations, we then have:

$$\frac{\kappa}{q(\mathbf{s}_t)} = \beta E_t \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} \tilde{\pi}(\mathbf{s}_{t+j}) \tag{13}$$

This equation says that an increase in the expected sum of future dividends (properly discounted) must induce a reduction in the current value of q, the probability of a vacancy being filled, because κ is fixed. The fall in q requires an increase in the number of vacancies which in turn increases the next period employment.

Equation (13) provides intuition on how changes in the interest rate affect the employment rate. As long as an expected fall in the future interest rates generates an increase in the future expected dividends, they induce an increase in employment. Also notice that the future inflation rates play an important role as $\tilde{\pi}_{t+j} = \beta P_{t+j} \pi_{t+j} / P_{t+j+1}$, for $j \geq 1$. On the other hand, the current dividend π_t and the next period inflation rate P_{t+1}/P_t do not enter equation (13). These observations are important for understanding the impact of monetary policy interventions on employment and the properties of the optimal monetary policy as characterized in later sections.

2 Defining the optimal monetary policy

Now that the economic environment has been described, we can define the optimal monetary policy under the two policy regimes: absence of commitment (time-consistence) and full commitment (Ramsey allocation). Section 2.1 defines the time-consistent policy and section 2.2 defines the optimal policy under commitment.

2.1 Optimal and time-consistent monetary policy

In this section we define the optimal policy when the monetary authority chooses the growth rate of money on a period-by-period basis and cannot credibly commit to the choice of future rates. Thus, we define policies that are time-consistent. We restrict the analysis to policies that are Markov stationary, that is, policy rules that are functions of the current aggregate states of the economy. Given $\mathbf{s}$ the current states, a policy rule will be denoted by $g = \Psi(\mathbf{s})$.

The procedure we follow to derive the time-consistent policy consists of two steps. In the first step we define a recursive equilibrium where the policy maker follows an arbitrary policy rule $\Psi(\mathbf{s})$. In the second step we ask what the optimal growth rate of money would be today, if the policy maker anticipates that from tomorrow on it will follow the arbitrary rule $\Psi(\mathbf{s})$. This allows us to derive the optimal current g as a function of the current states and the policy rule that will be followed from tomorrow on. We will denote the function that gives the optimal current policy by $g = \psi(\Psi; \mathbf{s})$. If the current policy rule ψ is the same as the policy rule that will be followed starting from tomorrow, that is, $\psi(\Psi; \mathbf{s}) = \Psi(\mathbf{s})$ for all $\mathbf{s}$, then Ψ is an optimal and time consistent policy rule. We describe these two steps in detail in the next two subsections.

2.1.1 The household's problem given the policy function Ψ

Assume that the policy maker commits to the policy rule $g = \Psi(\mathbf{s})$. Then, using a recursive formulation, we will describe the household's problem and define a competitive equilibrium conditional on this policy rule. In order to use a recursive formulation, we normalize all nominal variables by the pre-transfer stock of money M. The aggregate states of the economy are the technology shock, z, the normalized pre-transfer stock of nominal deposits,

D, and the number of employed workers, N. Therefore, $\mathbf{s} = (z, D, N)$. The individual states are the occupational status χ, the normalized pre-transfer stock of nominally denominated assets m, the normalized pre-transfer stock of nominal deposits d, and the number of firms' shares n owned by the household. We denote the set of individual states by $\hat{\mathbf{s}} = (\chi, m, d, n)$. The household's problem is:

$$\Omega(\Psi; \mathbf{s}, \hat{\mathbf{s}}) = \max_{n', d'} \left\{ c - \chi a + \beta\, E\, \Omega(\Psi; \mathbf{s}', \hat{\mathbf{s}}') \right\} \tag{14}$$

subject to

$$c \leq \frac{m - d}{P} - \frac{(n' - (1 - \lambda)n)\kappa}{q} \tag{15}$$

$$m' = \frac{(d + g)(1 + R) + P(\chi w + n\pi)}{(1 + g)} \tag{16}$$

$$\mathbf{s}' = H(\Psi; \mathbf{s}) \tag{17}$$

$$g = \Psi(\mathbf{s}) \tag{18}$$

Notice that in order to have n' shares of active firms in the next period, the household needs to buy $(n' - (1 - \lambda)n)$ new shares. Given the matching probability for a new vacancy, q, the creation of a new firm requires the posting of $1/q$ new vacancies, each of which costs κ. Therefore, the total investment in new firm shares is $i = (n' - (1 - \lambda)n)\kappa/q$. In solving this problem, the household takes as given the policy rule Ψ and the law of motion for the aggregate states H defined in equation (17). To make clear that this problem is conditional on the particular policy rule Ψ, this function has been included as an extra argument in the household's value function and in the aggregate law of motion.

A solution for this problem is given by the state contingent functions $n'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$ for next period firms' shares and $d'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$ for bank deposits. As for the value function, we make explicit the dependence of these decisions on the policy rule Ψ.

In equilibrium, households are indifferent about the allocation of cash between the purchase of consumption goods and the purchase of firms' shares, independently of their employment status. This derives from the assumption that the utility function is linear in consumption. Consequently, we have multiple equilibria corresponding to different distributions of firms' shares among households. Because the aggregate behavior of the economy is independent of this distribution, we concentrate on a particular equilibrium. This is the equilibrium in which all agents make the same portfolio choices of deposits and shares of firms. This implies that differences in earned wages between employed and unemployed workers give rise to different consumption levels rather than differences in asset holdings. We refer to this particular equilibrium as the *symmetric* equilibrium. We then have the following definition.

Definition 2.1 (Symmetric equilibrium given Ψ) *A recursive symmetric competitive equilibrium, given the policy rule Ψ, is defined as a set of functions for (i) household decisions $n'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$, $d'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$, and value function $\Omega(\Psi; \mathbf{s}, \hat{\mathbf{s}})$; (ii) intermediate input $x(\Psi; \mathbf{s})$; (iii) wage $w(\Psi; \mathbf{s})$; (iv) loans $L(\Psi; \mathbf{s})$; (v) interest rate $R(\Psi; \mathbf{s})$ and nominal price $P(\Psi; \mathbf{s})$; (vi) law of motion $H(\Psi; \mathbf{s})$. Such that: (i) the household's decisions are optimal solutions to the household's problem (14); (ii) the intermediate input x maximizes the surplus of the match; (iii) the wage is such that the worker obtains a fraction η of the surplus; (iv) the market for loans clears, that is $D + g = L(\Psi; \mathbf{s})$, and $R(\Psi; \mathbf{s})$ is the equilibrium interest* *rate; (v) the law of motion $H(\Psi; \mathbf{s})$ for the aggregate states is consistent with the individual decisions of households and firms; (vi) all agents choose the same holdings of deposits and firms shares (symmetry).*

Differentiating with respect n', we get:

$$\frac{\kappa}{q} = \beta\, E\left(\frac{\beta P'\pi'}{P''(1+g')}\right) + \beta\, E\left(\frac{(1-\lambda)\kappa}{q'}\right) \tag{19}$$

which is equivalent to equation (12) found before. The first order condition with respect to d' is:

$$E\left(\frac{1}{P'}\right) = \beta E\left(\frac{1+R'}{P''(1+g')}\right) \tag{20}$$

which is the Euler equation in standard dynamic models with money when agents are risk neutral.

2.1.2 One-shot optimal policy and the fixed point of the policy problem

In the previous section we derived the household's decision rules $n'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$, $d'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$ and the value function $\Omega(\Psi; \mathbf{s}, \hat{\mathbf{s}})$, for a given policy rule Ψ. We now ask what the optimal policy is today, if the policy maker anticipates that from tomorrow on, it will follow the arbitrary policy Ψ.

Defining the optimality of a particular policy requires the definition of a welfare objective. Given that in this economy there is heterogeneity due to unemployment, the definition of a social welfare function is not trivial. One possibility is to assume that the policy maker attributes equal weight to households in the economy, independently of whether they are employed or unemployed, and maximizes the sum of all households' utility. An alternative would be to assume that workers enter insurance contracts so that they enjoy the same level of utility independent of their employment status. In this framework, the monetary authority will maximize the welfare of the representative household. The social welfare function is exactly the same in the two cases, and therefore, it does not affect the type of monetary policy undertaken by the policy maker.[5] Given the equivalence between the two assumptions, to simplify the notation we assume that workers insure themselves against unemployment risks and the monetary authority maximizes the welfare of the representative household.

To determine the optimal growth rate of money today, the policy makers need to determine how this growth rate affects the households' welfare. We need to derive a function that links the households' welfare to g. To derive this function, consider the following household's problem:

$$\mathbf{V}(\Psi;\mathbf{s},\hat{\mathbf{s}},g) = \max_{n',d'}\left\{c-\chi a+\beta\, E\,\Omega(\Psi;\mathbf{s}',\hat{\mathbf{s}}')\right\} \tag{21}$$

subject to

$$c \leq \frac{m-d}{P}-\frac{(n'-(1-\lambda)n)\kappa}{q} \tag{22}$$

$$m' = \frac{(d+g)(1+R)+P(w_{\chi}+n\pi)}{(1+g)} \tag{23}$$

$$\mathbf{s}' = \tilde{H}(\Psi;\mathbf{s},g) \tag{24}$$

[5]Of course, this derives from the assumption that agents are risk-neutral. With risk-averse agents, the welfare function would differ.

where the function $\Omega(\Psi; \mathbf{s}', \hat{\mathbf{s}}')$ is the next period value function conditional on the policy rule Ψ as defined in the previous section. The new function $\mathbf{V}(\Psi; \mathbf{s}, \hat{\mathbf{s}}, g)$ is the value function for the representative household when the current growth rate of money is g and future growth rates are determined according to the policy rule Ψ.[6]

After solving for this problem and imposing the aggregate consistency condition $\hat{\mathbf{s}} = \mathbf{s}$,[7] we are able to derive the function $\mathbf{V}(\Psi; \mathbf{s}, \mathbf{s}, g)$. This is the welfare level reached by the representative household, when the current growth rate of money is g and the future rates are determined by the policy rule Ψ. Because the objective of the policy maker is to choose g to maximize the welfare of the representative household, the optimal current value of g is determined by the solution to the problem:

$$g^{OPT} \;=\; \arg\max_{g}\; \mathbf{V}(\Psi; \mathbf{s}, \mathbf{s}, g) \;=\; \psi(\Psi; \mathbf{s}) \tag{25}$$

We then have the following definition of an optimal and time-consistent monetary policy rule.

Definition 2.2 *The optimal monetary policy rule $\Psi^{OPT}(\mathbf{s})$ is the fixed point of the mapping $\psi(\Psi; \mathbf{s})$, that is:*

$$\Psi^{OPT}(\mathbf{s}) = \psi(\Psi^{OPT}; \mathbf{s})$$

[6]In the budget constraint we have used the subscript χ to differentiate the wages received by employed and non-employed workers. Even though they sign insurance contracts, the income received by employed workers differ from the income received by non-employed workers because the former face a disutility from working.

[7]In writing $\hat{\mathbf{s}} = \mathbf{s}$ we make an abuse of notation as the vector $\hat{\mathbf{s}}$ includes different variables than the vector $\mathbf{s}$. What we mean in writing $\hat{\mathbf{s}} = \mathbf{s}$ is that $m = 1$, $d = D$, and $n = N$.

The basic idea behind this definition is that, when the agents in the economy (households, firms and the monetary authority) expect that future values of g are determined according to the policy rule Ψ^{OPT}, the optimal value of g today is the one predicted by the same policy rule Ψ^{OPT} that will determine the future values. This property assures that, in the future, the policy maker will continue to use the same policy rule, so it is rational to assume that future values of g will be determined according to this rule.

2.2 Optimal policy with commitment

With commitment, the policy maker chooses at time zero a sequence of money growth, as a function of future history realizations of the shock and the initial states. The equilibrium allocation associated with this policy is usually referred to as the Ramsey equilibrium.

Let $\mathbf{h}^t$ be the history of shock realizations from time zero up to time t and let $\mathbf{H}^t$ be the collection of all possible histories. A monetary policy with commitment can then be expressed as $g_t = g(N_0, \mathbf{h}^t)$, for all $\mathbf{h}^t \in \mathbf{H}^t$ and $t \geq 0$. Similarly, the realization of the interest rate induced by this policy can be expressed as a function of N_0 and $\mathbf{h}^t$, that is, $R_t = R(N_0, \mathbf{h}^t)$. The policy maker will choose $g(N_0, \mathbf{h}^t)$ to maximize the expected discounted utility of the representative household obtained under the competitive allocation induced by the policy $g(N_0, \mathbf{h}^t)$. If we define $C(N_0, \mathbf{h}^t | g(N_0, \mathbf{h}^t))$ to be the aggregate consumption induced by the policy $g(N_0, \mathbf{h}^t)$ in the competitive equilibrium and $N(N_0, \mathbf{h}^t | g(N_0, \mathbf{h}^t))$ to be the employment rate also induced by the policy $g(N_0, \mathbf{h}^t)$ in the competitive equilibrium, the optimal policy with commitment is defined as:

$$\arg \max_{\{\{g(N_0,\mathbf{h}^t)\}_{\mathbf{h}^t \in \mathbf{H}^t}\}_{t\geq 0}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \Big[C(N_0, \mathbf{h}^t | g(N_0, \mathbf{h}^t)) - aN(N_0, \mathbf{h}^t | g(N_0, \mathbf{h}^t)) \Big] \tag{26}$$

In characterizing the optimal policy with commitment we follow the primal approach which consists of choosing the optimal allocation among the set of all competitive allocations that can be induced by a feasible policy $g(N_0, \mathbf{h}^t)$. See Chari, Christiano, & Kehoe (1996) for details about the primal approach.

3 Properties of the optimal policy

After defining the optimal policies with and without commitment, we are now in a position to characterize them. Before analyzing the model with portfolio rigidity, it is instructive to study the optimal monetary policy when agents are able to freely adjust their portfolio of deposits at any moment. The next section studies this simplified version of the economy. The characterization of the optimal policy with portfolio rigidity follows that.

3.1 Optimal monetary policy without portfolio rigidity

Consider the case in which there is no portfolio rigidity and households are able to adjust their stock of deposits at any moment. The following proposition characterizes the optimal and time-consistent policy for this simplified version of the economy.

Proposition 3.1 (Time-consistent policy) *In absence of portfolio rigidity, any policy rule $g = \Psi(\mathbf{s})$ is an optimal and time-consistent policy in the class of Markov-stationary policies.*

Proof 3.1 *The proof follows the two steps used in the definition of the time consistent policy. With portfolio flexibility, the interest rate is determined by $1 = (1+R)\beta E(P/P'(1+g))$. If the term $P/(1+g)$ does not change when g changes, then the interest rate is not affected by changes in g. To show that this is the case we have to show that the current price level is proportional to $1+g$. Consider the aggregate budget constraint. This is equal* *to* $1 + g = PN[X(1+R) + W + \Pi] = PNAX^{\nu}$. *Given* R, X *is determined and any increase in* g *will give rise to a proportional increase in price. Therefore,* $P/(1+g)$ *does not depend on* g. *Because* g *also does not affect* P', *changes in* g *cannot affect* R *and the real variables in the economy. This implies that, whatever the policy* Ψ *assumed for the future, the policy maker does not have incentive to deviate from it. In making the argument we did not consider the equilibrium in the loans market, that is,* $D + g = PNX$. *However, because households are able to change their deposits at any moment, they will readjust* D *so that this condition will always be satisfied.*

This result derives from the fact that the current growth rate of money, g, does not have any impact on the price change between today and tomorrow, and therefore, cannot affect the nominal interest rate. It affects the current *level* of prices, but it does not affect their future changes. Accordingly, the current growth rate of money does not have any real impact on the economy. Future growth rate of money would have real effects, but the policy maker today cannot choose the future rates. Consequently, given any arbitrary policy rule $\Psi(\mathbf{s})$ adopted in the future, there is not incentive today to deviate from this rule. Of course, the policy maker may desire to change the future rule, but that would require commitment.

Consider now the case with commitment. In this situation the policy maker chooses at time zero a sequence of growth rates of money as function of the history of the realization of the shocks, given the initial employment rate N_0. We have the following proposition.

Proposition 3.2 (Optimal policy with commitment) *Assume no portfolio rigidity. If* $\eta \geq 1 - \alpha$, *the optimal policy with commitment implies* $R(N_0, \mathbf{h}^t) = 0$ *for all* $\mathbf{h}^t \in \mathbf{H}^t$ *and* $t \geq 0$. *If* $\eta < 1 - \alpha$, *the optimal policy with commitment implies* $R(N_0, \mathbf{h}^t) = 0$ *at* $t = 0$, *and* $R(N_0, \mathbf{h}^t) > 0$ *for all* $\mathbf{h}^t \in \mathbf{H}^t$ *and* $t \geq 1$.

Proof 3.2 *See appendix.*

According to this proposition, if the sharing fraction of the workers is too small, then the optimal policy under commitment induces a sequence of non-zero interest rates. The only exception is at time zero.

In this economy there are two sources of inefficiency. The first inefficiency derives from the cost of financing the intermediate input x. This cost is determined by the nominal interest rate R. On this dimension, a zero nominal interest rate would be efficient. To obtain a zero interest rate, the expected inflation rate must be negative. The second source of inefficiency derives from frictions inherent the search and matching framework. If the worker's share of the surplus is smaller than $1-\alpha$, the high profitability of a match for the firm induces an excessive creation of vacancies. The policy maker can reduce the profitability of a match by increasing the inflation and interest rates. However, it is important to point out that the decision to create new vacancies is not affected by either the current inflation rate or by the next period inflation rate (change in prices between today and tomorrow). What affects the return on a new vacancy is the change in prices two periods from now. This can be seen by looking at equation (13), which for simplicity we rewrite below:[8]

$$\frac{\kappa}{q(\mathbf{s}_0)} = \beta E_0 \sum_{t=1}^{\infty} [\beta(1-\lambda)]^{t-1} \pi(\mathbf{s}_t) \frac{\beta P(\mathbf{s}_t)}{P(\mathbf{s}_{t+1})} \tag{27}$$

Notice that the infinite sum on the right hand side of this equation starts at $t = 1$ so the creation of new vacancies does not depend on the current and next period inflation rates. On the other hand, the next period inflation rate affects the interest rate, as shown

[8]Notice that the surplus of the match depends on the next period interest rate, but the next period interest rate depends on the inflation rate two periods from now.

in equation (20). Because the next period inflation rate does not affect the creation of vacancies but does affect the current interest rate, the optimal inflation rate in the next period will be set to obtain $R_0 = 0$. Future inflation rates, instead, will be set taking into consideration the possibility of correcting for the second source of inefficiency. Under the condition $\eta < 1 - \alpha$, this requires a higher inflation rate which induces a higher nominal interest rate. If $\eta > 1 - \alpha$, then it would be optimal from the planner's point of view to have an even lower inflation rate and a negative interest rate. A negative interest rate, however, is not compatible with a competitive equilibrium and the optimal interest rate is the lowest possible rate, that is, $R = 0$.

Corollary 3.1 *If $\eta \geq 1-\alpha$, the optimal growth rate of money with commitment, $g(N_0, \mathbf{h}^t)$, is not unique. If $\eta < 1 - \alpha$, the policy $g(N_0, \mathbf{h}^t)$ is not determined at $t = 0$, but is unique for $t \geq 1$.*

The policy indeterminacy under the condition $\eta \geq 1 - \alpha$ derives from the fact that with a zero nominal interest rate the cash-in-advance constraints of households and firms are not binding and several sequences of money growth can induce a zero interest rate. However, when $R > 0$ the cash-in-advance constraints is binding, and this allows for the uniqueness of the optimal sequences of policies $g(N_0, \mathbf{h}^t)$. The initial indeterminacy derives from the fact that the initial growth rate of money does not affect the current interest rate as observed above.

The optimality of a zero nominal interest rate is a common feature of several monetary models. However, in this model the optimality of a zero interest rate is a general result *only* when the policy maker cannot commit to future policies. With commitment, the optimality of a zero interest rate depends on the sharing parameter η. In this respect the searching and matching frictions play a crucial role.

These results stand in contrast to previous studies of the optimal monetary policy under the regime with and without commitment. As discussed in Sargent (1998), the inability of the central bank to commit induces an equilibrium outcome with higher inflation and lower employment. In this paper, however, if $\eta < 1 - \alpha$ we find exactly the opposite result: the ability to commit can induce an equilibrium with higher inflation and lower employment. This, however, does not imply that the equilibrium allocation with commitment is Pareto inferior. In both models, of course, the equilibrium allocation under commitment is Pareto superior.

3.2 Optimal monetary policy with portfolio rigidity

We have argued in the previous section that, absent commitment, the optimal and time-consistent monetary policy, and hence the sequence of nominal interest rates, are indeterminant. The introduction of portfolio rigidity eliminates this indeterminacy.

Proposition 3.3 *Assume one-period portfolio rigidity. Then, without commitment, the optimal and time-consistent policy maintains the nominal interest rate equal to zero in any state of the economy.*

Proof 3.3 *See appendix.*

The portfolio rigidity eliminates the indeterminacy in the nominal interest rate because the current growth rate of money is now able to affect the current interest rate. Because the current stock of deposits cannot be immediately changed, the current nominal interest rate is no longer dependent on the next period inflation rate. Instead, it depends on the current growth rate of money which changes the liquidity available to make loans to firms. Because the policy maker is not indifferent about the current level of the interest rate, it will use the policy instrument (growth rate of money) to set the nominal interest rate at the optimal level $R = 0$.

With policy commitment, the results obtained in the previous section extend to the case of portfolio rigidity. Formally:

Proposition 3.4 (Optimal policy with commitment) *Assume one-period portfolio rigidity. If $\eta \geq 1-\alpha$, the optimal policy with commitment implies $R(N_0, \mathbf{h}^t) = 0$ for all $\mathbf{h}^t \in \mathbf{H}^t$ and $t \geq 0$. If $\eta < 1 - \alpha$, the optimal policy with commitment implies $R(N_0, \mathbf{h}^t) = 0$ at $t = 0$, and $R(N_0, \mathbf{h}^t) > 0$ for all $\mathbf{h}^t \in \mathbf{H}^t$ and $t \geq 1$.*

Proof 3.4 *The proof follows the argument of proposition 3.2.*

The intuition for this result is similar to the intuition provided in the previous section when we considered the case without portfolio rigidity. Also, the corollary stated in the previous section extends to the case of one-period rigidity. The only change is for the optimal growth rate of money at time zero, when $\eta < 1 - \alpha$. In this case the first period optimal growth rate of money is uniquely determined by the zero interest rate target.[9]

The result that a constant interest rate rule is the optimal policy, has some similarities with the result of Carlstrom & Fuerst (1995). They compare an interest rate rule with a growth rate rule in a cash-in-advance model, and find that the first rule welfare dominates the second rule. In their analysis, however, they do not study the first best policy but simply compare two simple rules: interest rate rule versus growth rate rule.

Although the optimal interest rate becomes determined once we introduce the portfolio rigidity, in the environment without commitment this interest rate can be obtained with a

[9]More precisely, there is a lower bound to the optimal growth rate of money at time zero. Higher growth rates are still compatible with a zero interest rate if we allow the intermediary to retain liquidity.

multiplicity of time consistent policies $g = \Psi(\mathbf{s})$. In what follows we restrict the analysis to a particular policy rule, that is, the policy rule under which the whole quantity of money is used for transactions. This is equivalent to imposing that the cash-in-advance constraints for households and firms are binding. This policy rule would be unique if we assume that there is some cost associated with unused money.

Proposition 3.5 (Procyclical policy) *Assume one-period portfolio rigidity. The optimal and time-consistent policy compatible with full use of money is given by* $g = \beta E_{-1}(1+g_Y) - 1$, *where* $E_{-1}(1+g_Y)$ *is the expected gross growth rate of output before the observation of the shock.*

Proof 3.5 *See the appendix.*

According to the proposition above, the current growth rate of money depends only on the predictable part (before the shock) of the current growth rate of output. Therefore, current technology innovations do not affect the optimal growth rate of money. This is because in the current period the nominal interest rate is determined by the equilibrium condition $1 + R = \nu(1 + g)/(D + g)$ (see the proof of proposition 3.3). Because in the current period D cannot be changed, the constancy of R requires the constancy of g.

This property extends to the case of full policy commitment and $\eta \geq 1 - \alpha$, but it does not apply to the case in which $\eta < 1 - \alpha$. In this case the zero interest rate is optimal only for the initial period as stated in proposition 3.4.

The fact that the optimal growth rate of money follows the predictable growth rate of output, implies that the persistence of g depends on the persistence of the growth rate of output. In this respect the matching frictions play an important role in the model. In a limited participation model with a neoclassical production technology, the response of output to shocks is not hump-shape. The matching framework is able to generate a hump-shaped response of output as shown by Andolfatto (1996), Den-Haan, Ramey, & Watson (1997) and Merz (1995). Consequently, in this paper, the response of the growth rate of money to an $AR(1)$ technology shock displays some persistence that is absent in the limited participation model with a neoclassical technology. With matching frictions, the optimal growth rate of money will be above the steady state level for more than one period. This is shown in Figure 1 that reports the impulse responses for the growth rate of money, interest rate, employment, and output to a technology shock. Two policy regimes are considered. In the first regime the growth rate of money is maintained constant. We will refer to this case as the "passive policy regime". In the second case, instead, the monetary authority reacts optimally to shocks and implements the optimal and time-consistent policy. We will refer to this case as the "optimal policy regime". The technology shock follows a first order autoregressive process with an autocorrelation coefficient of 0.95. The values assigned to the other parameters of the model will be discussed in the next section, when we study the quantitative properties of the model.

As can be seen for the figure, the optimal policy prevents the interest rate from rising and allows for a more persistent response of employment and output. The optimal growth rate of money will be above the steady state for more than one period. Specifically, it will stay above the steady state during the periods in which output grows. As observed above, because of the matching frictions, employment and output continue to grow beyond the first period and this requires the growth rate of money to stay above the steady-state level for more than one period. Consequently, the model generates a pattern for money growth that is serially correlated as in the data.

4 Quantitative properties of the time-consistent policy

In this section we analyze the quantitative cyclical properties of the optimal monetary policy when the monetary authority cannot commit to future policies (time-consistency). The analysis of the previous section showed that the optimal and time-consistent policy maintains a zero interest rate and a negative inflation rate. These features are clearly counterfactual. Given that the objective of this section is to calibrate the model to investigate its quantitative cyclical properties, it would be desirable for the model to be consistent with the long-run inflation rate of the U.S. economy. Therefore, in what follows we introduce an extra feature that allows for the optimality of a long-term positive nominal interest rate, but does not change the cyclical (short-term) properties of the model. An easy way to do this is by assuming that there is a negative externality associated with production. We assume that each firm generates a negative externality of the form ξx, where x is the intermediate input used in production and ξ is constant.[10] With the introduction of this externality, propositions 3.3 and 3.5 become:

Proposition 4.1 *Assume one-period portfolio rigidity. Then without commitment, the optimal and time-consistent policy maintains the nominal interest rate equal to ξ in any state of the economy and $g = \beta(1 + \xi)E_{-1}(1 + g_Y) - 1$.*

Proof 4.1 *The proof follows the same steps of propositions 3.5 taking into account the*

[10] Of course, we are not claiming that this is an interesting way to explain positive values for the optimal inflation and interest rates. It simply represents a modeling strategy to facilitate the calibration of the model. Given that in this section we are interested in the cyclical properties of the optimal policy, rather than its long-run behavior, and this cyclical properties are not affected by the presence of ξ, this is an acceptable way to proceed.

externality ξxN in the objective of the policy maker.

The introduction of the externality implies that the optimal interest rate is positive but it does not change the constancy of it. At the same time, the positiveness of the interest rate implies that the cash-in-advance constraint is always binding and the optimal policy rule Ψ has the same cyclical properties as the optimal rule analyzed in proposition 3.5. Differently from the case in which there is no externality, this policy rule is now unique.[11]

4.1 Calibration

The period in the model is one quarter and we fix the discount factor at $\beta = 0.99$. The externality parameter ξ is set at 0.018. This implies that in the steady state the optimal growth rate of money is 0.008 per quarter and the quarterly nominal interest rate is 0.018.

The production function is characterized by the parameter ν and the stochastic properties of the shock. The fraction of nominally denominated assets used by households for transaction purposes (money), as a fraction of their total nominally denominated assets is $(M - D)/M(1 + g)$. This is also equal to $(1 - \nu) + RD/M(1 + g)$. Because $RD/M(1 + g)$ is a small number, we take $1 - \nu$ to be the approximate fraction of transaction funds used by households. A proxy for $1 - \nu$ is then given by the stock of $M1$ used by households as a fraction of aggregate $M3$ that they own. The value chosen is $\nu = 0.9$.

The technology shock z follows the first-order autoregressive process $z' = \rho_z z + \epsilon'_z$, with $\epsilon_z \sim N(0, \sigma_z^2)$. The parameter ρ_z is assigned the value 0.95, which is in the order of magnitude commonly used in the business cycle studies. The parameter σ_z, instead, is

[11] Without the externality, the optimal rule is unique in terms of the nominal interest rate, but it is not unique in terms of the growth rate of money due to the fact that with $R = 0$ the cash-in-advance constraint is not binding.

chosen so that the volatility of output is similar to what is observed in the data. We set $\sigma_z = 0.00045$. Of course, the evaluation of the performance of the model will not be based on the ability to match the volatility of output.

The workers share of the surplus, η, is set to 0.2. This value is about half the value that would guarantee an efficient creation of new vacancies. After fixing η, the disutility from working, a, is chosen so that the steady state capital income share is 18 percent. This value is lower than the values usually used in business cycle models. This is motivated by the fact that in this economy the depreciation of capital is smaller than in these other models.[12] However, the capital income share, net of depreciation, is similar to the values usually used in other business cycle models. To evaluate the importance of η for the performance of the model, we will report the simulation results also for other values of this parameter.

The search and matching framework is characterized by four parameters: the parameters of the matching technology, μ and α, the probability of exogenous separation, λ, and the cost of creating a new vacancy, κ. We set $\alpha = 0.6$ which is consistent with the estimate of Blanchard & Diamond (1989). Then to calibrate the parameters μ, λ and κ, we follow Andolfatto (1996) and we impose the following steady state values for the labor sector of the model: (a) the fraction of the population that is employed equals 0.57;[13] (b) the

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[12] In this economy the steady state depreciation is equal to the steady state cost of posting new vacancies.

[13] This implies that the steady state fraction of searching workers is 0.43, which is obviously larger than in the data. However, this larger fraction is imposed in order to reduce the impact that changes in the number of employed workers have on the probability that an advertised vacancy is filled. When the fraction of steady state searchers is small, a small percentage increase in the number of employed workers corresponds to a large percentage fall in the number of searching workers, which in turn implies a large fall in the probability with which new vacancies are filled. This can be interpreted as the simpler way

probability that a vacancy is successfully filled is $q = 0.9$; and (c) the transition probability from employment to non-employment is 0.15.

4.2 Properties of the calibrated economy

Figures 1a-1f plot the impulse responses of several variables to a positive productivity shocks under two assumptions about the determination of policy. With the passive policy the growth rate of money is kept constant. With the optimal policy the monetary authority chooses the growth rate of money optimally on a period-by-period basis.[14] The most important feature to note is that the responses of employment and output are amplified under the optimal policy regime. This is because the optimal policy prevents the interest rate from rising and allows the economy to take full advantage of the higher productivity.

Table 1 reports some business cycle statistics of the calibrated economy. As expected from the impulse responses plotted in Figures 1c and 1d, the volatility of employment and output is larger under the optimal policy regime. The model generates volatility of money stock and money growth that are not very different from the data. It also generates positive correlations of the stock of money with employment and output. Employment and output are also positively correlated with the growth rate of money. Notice that the model generates the positive correlation of money growth with output only under the optimal policy regime. With a passive policy this correlation is negative. This is because, in the passive regime, when output expands prices fall and inflation is negative. This fall in price

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to capture the reaction of the labor force participation to the economic conditions in the labor market; specially if there is some lag in the response of the labor force participation to the conditions in the labor market.

[14]The results also hold for the case of commitment if $\eta > 1 - \alpha$.

is not allowed in the optimal policy regime, as the optimal policy is pro-cyclical. As it is shown in figure 1e, the fall in prices arises only in the first period. But in the first period, the increase in output is small (see figure 1d). Therefore, the fall in inflation arises when output is small and this generates the positive correlation of inflation and output.

Another important feature of the model is the autocorrelation of the optimal growth rate of money as reported in the lower section of the table. This autocorrelation is positive and close to the value found in the data. This positive serial correlation is possible because the response of output to shocks is hump-shaped. The hump-shaped response occurs because of the searching and matching frictions. Without these frictions, the response of output would not be hump-shaped and the autocorrelation of the optimal growth rate of money would be negative.

Table 1: Business cycle properties of the calibrated economy under passive and optimal policies.

	$\eta = 0.2$		$\eta = 0.1$		$\eta = 0.3$		**U.S.**
	Passive	Optimal	Passive	Optimal	Passive	Optimal	**economy**
Standard deviation							
Employment	0.46	1.26	0.48	1.35	0.46	1.19	0.99
Output	0.91	1.64	0.92	1.72	0.90	1.57	1.67
Consumption	0.86	1.54	0.86	1.61	0.86	1.49	1.39
Price index	0.73	0.57	0.74	0.57	0.73	0.57	1.39
Inflation	0.45	0.43	0.45	0.43	0.45	0.43	0.57
Interest rate	1.68	0.00	1.69	0.00	1.67	0.00	1.29
Money stock	0.00	1.21	0.00	1.30	0.00	1.15	1.52
Money growth	0.00	0.52	0.00	0.56	0.00	0.49	0.73
Correlation of employment with							
Prices	-0.77	-0.61	-0.77	-0.61	-0.76	-0.61	-0.30
Inflation	0.35	0.36	0.35	0.36	0.35	0.36	0.51
Money stock	0.00	0.99	0.00	0.99	0.00	0.99	0.49
Money growth	0.00	0.18	0.00	0.18	0.00	0.18	0.33
Correlation of final output with							
Prices	-0.98	-0.81	-0.98	-0.81	-0.98	-0.82	-0.30
Inflation	-0.15	0.12	-0.15	0.13	-0.15	0.11	0.51
Money stock	0.00	0.96	0.00	0.96	0.00	0.96	0.49
Money growth	0.00	0.34	0.00	0.33	0.00	0.34	0.33
Autocorrelation of money growth							
	0.00	0.49	0.00	0.49	0.00	0.49	0.59

NOTES: Statistics for the model economy are computed on HP detrended data generated by simulating the model for 240 periods and repeating the simulation 1000 times. The statistics are averages over these 1000 simulations. Statistics for the U.S. economy are computed using HP detrended data from 1959.1 through 1996.4. Consumption includes consumer expenditures in non-durable and services. The price index is the CPI index.

A limitation of the model is that it does not generate any volatility in the nominal interest rate. One way to allow fluctuations in the nominal interest rate is by assuming that the monetary authority controls the growth rate of money only imperfectly. Given that we interpret M as a broad monetary aggregate, it is reasonable to assume that the monetary authority controls this aggregate only indirectly and the channel through which the monetary instruments affect this aggregate is subject to shocks. Formally, we can assume that $g = \hat{g} + \varepsilon$, where $\hat{g}$ is controlled by the monetary authority and ε is some shock that is realized after the choice of $\hat{g}$. In this modified model, the interest rate would not be constant. Table 2 reports the business cycle statistics when the economy is subject to the monetary shock, in addition to the technology shock. The monetary shock is assumed to be independently and identically distributed according to a normal variate with standard deviation σ_ε. It is important to observe that the results are robust to other assumptions about the distribution of the monetary shock. Even if the shock is serially correlated, it is only the unpredictable component (the i.i.d. part) that has an impact in the economy. The predictable component will be compensated by a change in $\hat{g}$ controlled by the monetary authority. The statistics are for different values of σ_ε.

As can be seen from the table, the quantitative properties of the model change only slightly. The important difference is in the behavior of the nominal interest rate which now displays some variability. The monetary shock also reduces the autocorrelation of the growth rate of money. This is because after a positive monetary shock, the optimal policy decreases the next period growth rate of money. Remember that the optimal policy is such that $\hat{g}$ increases when the expected growth rate of output increases and decreases when the growth rate of output declines. After an unexpected monetary shock, output increases above the steady state and the next period it returns to the steady state level. Because output is expected to decline in the next period, the expected next period growth rate of

Table 2: Business cycle properties of the calibrated economy when there are monetary shocks.

	$\sigma_\varepsilon = 0.0$		$\sigma_\varepsilon = 0.001$		$\sigma_\varepsilon = 0.002$		**U.S.**
	Passive	Optimal	Passive	Optimal	Passive	Optimal	**economy**
Standard deviation							
Employment	0.46	1.26	0.46	1.26	0.46	1.26	0.99
Output	0.91	1.64	0.91	1.64	0.91	1.64	1.67
Consumption	0.86	1.54	0.86	1.54	0.86	1.54	1.39
Price index	0.73	0.57	0.74	0.57	0.78	0.57	1.39
Inflation	0.45	0.43	0.46	0.43	0.51	0.43	0.57
Interest rate	1.68	0.00	1.82	0.70	2.20	1.41	1.29
Money stock	0.00	1.21	0.13	1.22	0.00	1.23	1.52
Money growth	0.00	0.52	0.09	0.54	0.00	0.60	0.73
Correlation of employment with							
Prices	-0.77	-0.61	-0.75	-0.61	-0.72	-0.61	-0.30
Inflation	0.35	0.36	0.34	0.36	0.31	0.36	0.51
Money stock	0.00	0.99	0.00	0.99	0.00	0.98	0.49
Money growth	0.00	0.18	0.00	0.17	0.00	0.15	0.33
Correlation of final output with							
Prices	-0.98	-0.81	-0.97	-0.82	-0.93	-0.81	-0.30
Inflation	-0.15	0.12	-0.15	0.12	-0.14	0.11	0.51
Money stock	0.00	0.96	0.01	0.96	0.01	0.95	0.49
Money growth	0.00	0.34	0.02	0.32	0.04	0.30	0.33
Autocorrelation of money growth							
	0.00	0.49	-0.07	0.41	-0.07	0.23	0.49

NOTES: Statistics for the model economy are computed on HP detrended data generated by simulating the model for 240 periods and repeating the simulation 1000 times. The statistics are averages over these 1000 simulations. Statistics for the U.S. economy are computed using HP detrended data from 1959.1 through 1996.4. Consumption includes consumer expenditures in non-durable and services. The price index is the CPI index.

money falls. This generates the negative serial correlation in the growth rate of money. If the monetary shock is very large, this effect dominates the positive autocorrelation induced by the technology shock, and the serial correlation of g becomes negative.

5 Conclusion

In this paper we have analyzed the properties of an optimal monetary policy in a world where inflationary monetary interventions have expansionary effects in the economy through the liquidity channel. We find that in this economy, if business cycle fluctuations are driven by shocks to technology, then the optimal monetary policy is pro-cyclical: It increases the stock of money when employment and output are high and reduces the stock of money when employment and output are low.

The economy in which the monetary authority acts optimally is able to capture many of the features of the real economy. From this we conclude that the monetary policy implemented by the Federal Reserve Bank is broadly consistent with the notion of optimality as defined in this paper.

We have also analyzed the long-run properties of an optimal and time-consistent policy and compared it to the optimal policy under commitment. The finding is that, under some conditions, the ability to commit can lead to higher inflation and lower employment. This is in contrast to many studies concluding that the lack of commitment induces more inflation and lower employment.

Finally, we should note that the optimality of a pro-cyclical policy derives from the assumption that technology shocks are the main source of business cycle fluctuations. Different conclusions may be reached if other sources of business fluctuations are considered. For example, if we assume that firms need to finance only a fraction of their input and this fraction changes stochastically (shocks to the velocity of money), then the optimal policy would be counter-cyclical. This is because an increase in velocity reduces the interest rate and expands output. To prevent the fall in the nominal interest rate the monetary authority has to implement a contractionary policy that reduces the stock of money. Other forms of shocks can also be considered, like demand shocks resulting from changes in government spending. The analysis of all such possibilities, however, is beyond the scope of this paper.

A Proof of proposition 3.2

Consider the following planner's problem in the choice of vacancies V and current deposits D when the stock of deposits can be changed at any moment:[15]

$$\Omega(A,N) = \max_{V,D}\left\{C - aN + \beta\, E\,\Omega(A',N')\right\} \tag{28}$$

subject to

$$\begin{aligned}
C &= \frac{1-D}{P} - V\kappa && (29)\\
1+g &= (D+g)(1+R) + PN(W+\Pi) && (30)\\
(D+g) &= PXN && (31)\\
W+\Pi &= (1-\nu)AX^{\nu} && (32)\\
X &= \left(\frac{\nu A}{1+R}\right)^{\frac{1}{1-\nu}} && (33)\\
N' &= (1-\lambda)N + m(V, 1-N) && (34)
\end{aligned}$$

Equation (29) is the aggregate cash-in-advance for households; (30) is the aggregate budget constraint, (31) is the equilibrium condition in the loan market, (32) states that the sum of dividends and wages is equal to the period surplus of the firm-worker match, and (33) defines the intermediate input X as derived in proposition 1.1. Finally, (34) is the law of motion for the next period employment. The first order conditions for the planner problem are:

$$-\kappa + \beta m_1 E\left[R'X' + \Pi' + W' - a + \frac{\kappa}{m_1'}(1-\lambda-m_2')\right] = 0 \tag{35}$$

$$-1 + \left[1 + \frac{R}{1-\nu}\right] = 0 \tag{36}$$

[15] All variables are denoted in capital letters as they are aggregate variables.

The first order conditions for the household in the competitive equilibrium are:

$$\frac{\kappa}{q} = \beta E\left(\frac{\beta P' \pi'}{P''(1+g')}\right) + \beta E\left(\frac{(1-\lambda)\kappa}{q'}\right) \tag{37}$$

$$1 = (1+R) E\left(\frac{\beta P}{P'(1+g)}\right) \tag{38}$$

These can be derived by setting the problem as in (14) taking into consideration that now the household chooses the current stock of deposits rather than the next period stock.

After some substitutions condition (37) can be written as:

$$-\kappa + \beta(1-\eta) q E\left[(\Pi' + W') E\left(\frac{\beta P'}{P''(1+g')}\right) - a + \frac{\kappa}{(1-\eta)q'}(1-\lambda-\eta h')\right] = 0 \tag{39}$$

Equation (36) implies that $R = 0$, which means that for the planner the optimal interest rate is zero. In order to have an equilibrium interest rate equal to zero, the term $E(\beta P / P'(1+g))$ must be equal to 1 (see equation (38)). If $R = 0$ and $\eta = 1 - \alpha$, it can be verified that (19) is exactly equal to (35) and by following a policy that maintains the interest rate equal to zero the first best allocation is obtained. If $\eta > 1 - \alpha$, the policy that keeps the interest rate equal to zero ($R = 0$) does not reach the first best allocation. The first best allocation requires $R < 0$, which however is not compatible with a competitive equilibrium. In this case the planner will keep the interest rate at the lower feasible level, which is $R = 0$. If $\eta < 1 - \alpha$, a zero interest rate policy generates an excessive creation of vacancies (the value of q in equation (39) will be smaller than in equation (35)). In order to change the number of vacancies, the planner must decrease the values of $E\left(\frac{\beta P_{t+j}}{P_{t+j+1}(1+g_{t+j})}\right) = 1/(1+R_{t+j})$ and the dividend π_{t+j}, for $j \geq 1$. The increase in R_{t+j} for $j \geq 1$ is equivalent to reducing the future values of $P/P'(1+g)$ and the dividends π. At time zero, however, because the interest rate does not affect the rate of new vacancies, $R = 0$ is still optimal.

B Proof of proposition 3.3

Combining the aggregate budget constraint with the equilibrium in the loans market, $D + g = PXN$, we can show that the interest rate is determined by the condition $1 + R = \nu(1+g)/(D+g)$. Studying the sign of the first derivative of this condition, it can be verified that R depends negatively on g and the policy maker can use g to affect the current nominal interest rate. Notice that this is possible because D cannot be changed in the current period.

Now consider again the planner problem (28) with first order conditions (39) and (38). The first order conditions for the household in the competitive equilibrium, instead, are given by:

$$-\kappa + \beta(1-\eta)qE\left[(\Pi' + W')E\left(\frac{\beta P'}{P''(1+g')}\right) - a + \frac{\kappa}{(1-\eta)q'}(1-\lambda-\eta h')\right] = 0 \tag{40}$$

$$-E\left(\frac{1}{P'}\right) + \beta E\left(\frac{(1+R')}{P''(1+g')}\right) = 0 \tag{41}$$

From equation (38) we have that the optimal interest rate is $R = 0$. With portfolio inflexibility, the policy maker can use g to set $R = 0$. With $R = 0$ equation (39) is not necessarily equal to (40), that is, the optimal number of new vacancies is not necessarily optimal. However, equation (40) does not depend on the current value of g, but only on the future values of the growth rate of money. The planner will be able to affect the rate of vacancy creation only if it can credibly commit to g'' today. Without commitment the value of g'' chosen today will not be optimal tomorrow. Consequently, the zero interest rate policy is an optimal and time-consistent policy. However, the optimal policy in terms of the growth rate of money is not necessarily unique because at $R = 0$ the cash-in-advance constraint is not binding. To show that the zero interest rate policy is unique (but it can be implemented with a non unique policy Ψ), it is enough to show that the policy maker will always deviate from a policy that do not implement $R = 0$. Fix g to be the current growth rate of money chosen by the policy Ψ which is assumed to determine the future growth rates, that is, $g = \Psi(\mathbf{s})$. Given (g, Ψ) the equilibrium condition implies either $R = 0$ or $R > 0$ (the interest rate cannot be negative). In the first case the planner does not need to change g to obtain $R = 0$. If instead $R > 0$, then it will change g to obtain $R = 0$. Therefore, the planner will deviate from any policy that allows for a positive nominal interest rate.

C Proof of proposition 3.5

Under the condition that the cash-in-advance constraint is satisfied with equality, we have $PY = 1 + g$, where $Y = NAX^{\nu}$ is aggregate gross output. This must be satisfied in each period. Therefore, it must be that $P'Y' = 1 + g'$ and $P''Y'' = 1 + g''$ are also satisfied. Using these two relations the following condition can be derived:

$$E\left(\frac{Y''}{Y'(1+g'')}\frac{1}{P'}\right) = E\left(\frac{1}{P''(1+g')}\right) \tag{42}$$

Assume that the optimal policy is $g_t = \beta E_{t-1} Y_t / Y_{t-1} - 1$. To verify that this policy implements a zero nominal interest rate, substitute this policy in equation (42). This gives:

$$E\left(\frac{1}{P'}\right) = \beta E\left(\frac{1}{P''(1+g')}\right) \tag{43}$$

From the household's first order conditions, equation (41), we observe that this condition implies a zero nominal interest rate. Therefore, the policy $g_t = \beta E_{t-1} Y_t / Y_{t-1} - 1$ is optimal and time-consistent. We want to show now that this policy is unique. Notice that any policy that implements a zero nominal interest rate must satisfies condition (43). Then given a policy that satisfies this condition, the next period stock of deposits is determined and it cannot be changed in the next period. If the optimal policy rule is different from $g_t = \beta E_{t-1} Y_t / Y_{t-1} - 1$, it must have some stochastic component that is unpredictable so that condition (43) is still satisfied. But then ex-post, this stochastic component may induce an interest rate different from zero and the policy maker will deviate from this policy. Therefore, the policy this stochastic component cannot be part of the optimal policy. Moreover, because the growth rate of final output $(1-\nu)Ax^{\nu} + Rx$ is equal to the growth rate of gross output $Y = Ax^{\nu}$ when $R = 0$, then $g = \beta E_{-1}(1 + g_Y) - 1$.

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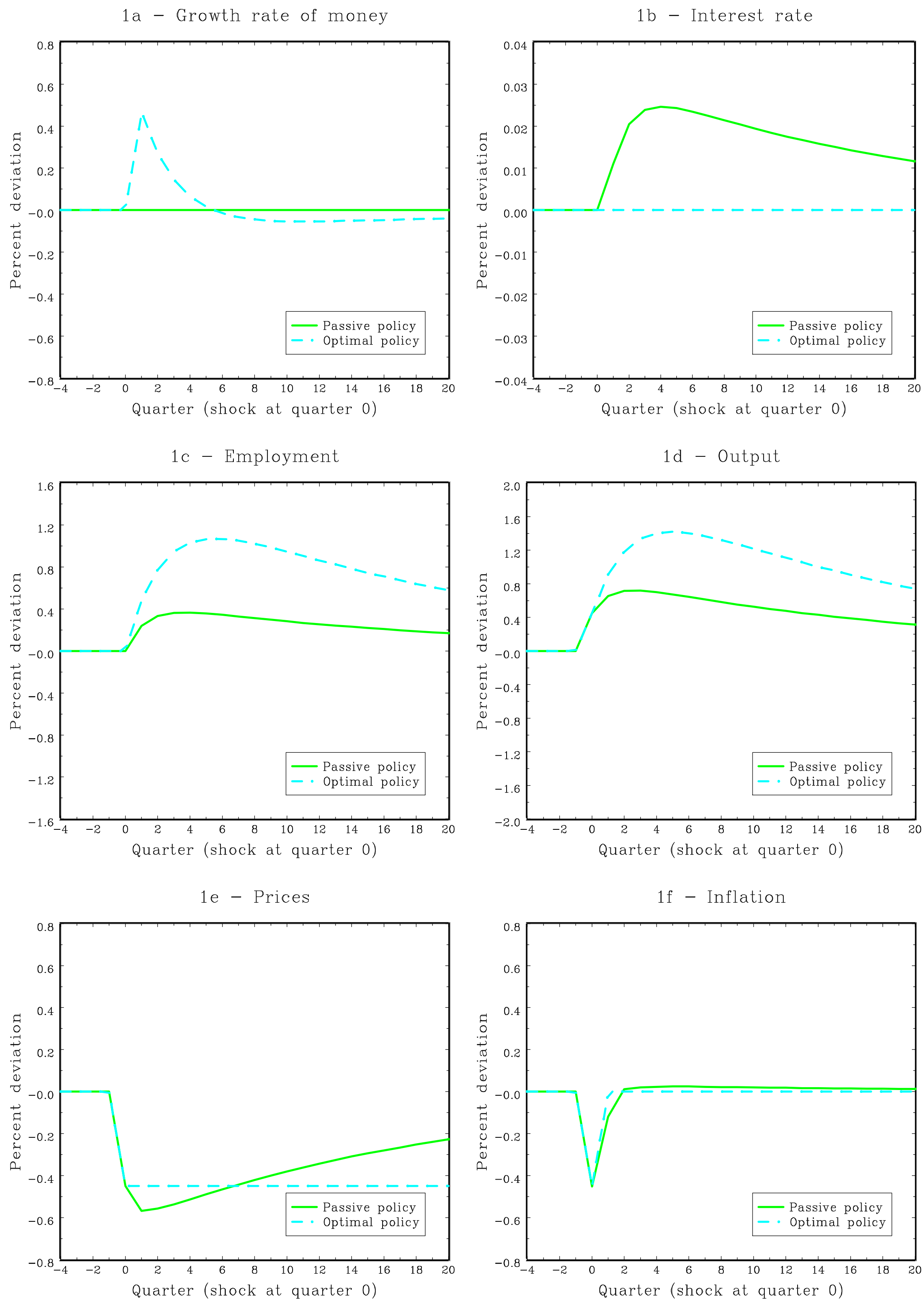
1a – Growth rate of money
Percent deviation
Quarter (shock at quarter 0)
Passive policy
Optimal policy
1b – Interest rate
Percent deviation
Quarter (shock at quarter 0)
Passive policy
Optimal policy
1c – Employment
Percent deviation
Quarter (shock at quarter 0)
Passive policy
Optimal policy
1d – Output
Percent deviation
Quarter (shock at quarter 0)
Passive policy
Optimal policy
1e – Prices
Percent deviation
Quarter (shock at quarter 0)
Passive policy
Optimal policy
1f – Inflation
Percent deviation
Quarter (shock at quarter 0)
Passive policy
Optimal policy