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The replacement problem

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Abstract

A prototypical vintage capital model of economic growth is developed, where the decision to replace old technologies with new ones is modeled explicitly. Technological change is investment specific. Depreciation in this environment is an economic, not a physical, concept. The vintage capital economy's balanced-growth paths and transitional dynamics are analyzed. The transitional dynamics are markedly different from the standard neoclassical growth model.

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1. Introduction

1.1. Observations

Consider the following observations about capital accumulation in the US economy:

1. Investment at the plant level occurs infrequently and in bursts. A recent study by Doms and Dunne (1994) of 33,000 plants over a 17-year period confirms

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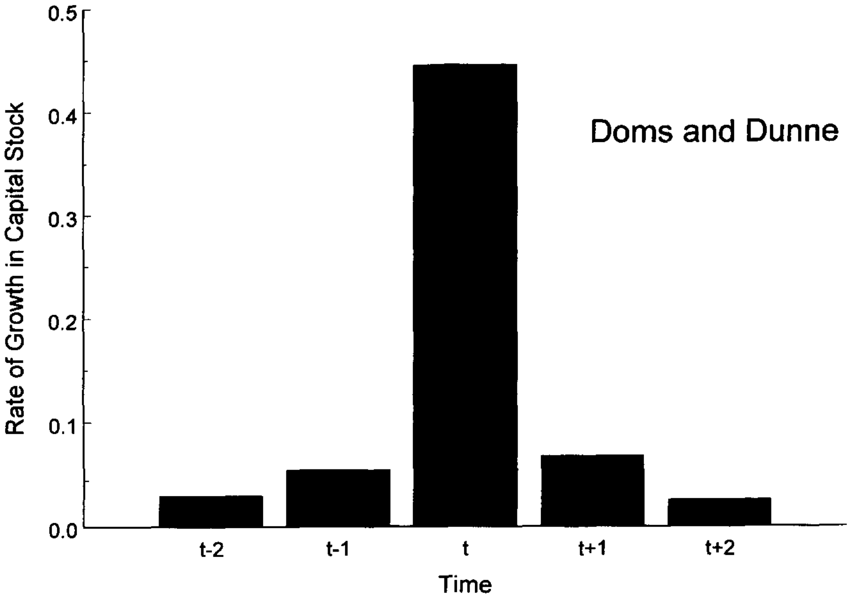

Fig. 1. Investment spikes.

this picture. Fig. 1 presents some of their findings. Denote the date of a plant's highest rate of investment by t. The figure plots the average rate of investment at this date. It also shows the average rate of investment for the two previous and subsequent years. Investment has a distinct spiked pattern. When attention was restricted to the 13,000 plants that were around for the entire 17 year sample period, they found that 25% of a plant's investment was concentrated in a single year, and about 50% was concentrated in 3 years.[1]

2. Technological progress is investment specific; that is, technological progress is embodied in the form of *new* capital goods. This is evidenced by the fact that the relative price for (an efficiency unit of) equipment has declined fairly steadily and rapidly in the postwar US economy. Thus, over time a unit of forgone consumption can buy ever-increasing quantities of equipment. Further, the ratio of equipment to output has increased steadily. These facts have been used by Greenwood et al. (1997) to argue that as much as 60% of postwar US growth can be accounted for by investment-specific technological

[1] Recent empirical work has further clarified the pattern of investment at the level of the plant or firm: (i) Abel and Eberly (1996) find significant nonconvexities in firm-level investment, (ii) Caballero et al. (1995) report evidence of irreversibilities in plant-level investment, and (iii) Cooper et al. (1995) find that lumpy investment at the plant level is more likely to occur the older is the existing capital stock.

progress. Additionally, there is microeconomic evidence that investment-specific technological progress may be important for growth. Bahk and Gort (1993), using a cross section of more than 2000 firms from 41 industries, find that a one year change in the average age of capital is associated with a 2.5–3.5% change in output. These facts suggest that a successful model of capital accumulation should treat the investment and consumption goods sectors separately, and should link the process of growth with investment in new technologies.

3. Employment and investment are related. Dunne 1994, (Table 3 and 4) finds that firms using the newest technology have more employees. Employment at the plant level is a $\cap$-shaped function of age; employment increases during the first five years of a plant's life and decreases thereafter, a fact documented by Davis et al., 1996 (Table 3.5). Employment is less, therefore, when technologies or plants are old.

These observations suggest that a successful model of investment will have to be of the vintage capital variety. Moreover, the observations on plant-level employment behavior suggest that standard putty-clay models will not be adequate to capture employment dynamics. In this paper a vintage capital model is developed that is consistent with these observations. The vintage capital framework naturally suggests certain questions: What determines the efficiency of new capital goods? When do new vintages of capital get adopted and old ones get replaced? How is economic growth tied to the decision to replace old capital goods with new ones? How effective are policies designed to stimulate the adoption of new capital goods? Are the dynamics of a vintage capital economy much different from the standard neoclassical growth? These questions are addressed here.

An economy is developed where technological change is embodied in new capital goods. The firm in the model economy must decide when to replace its existing capital with a new vintage. Investment is a lumpy decision and depreciation is an economic concept, not a physical one. The firm produces consumption and investment goods using capital and two kinds of labor, designated as skilled and unskilled. A distinguishing feature of this environment is that growth results from the ability to produce evermore efficient capital goods. This occurs because skilled agents in the economy make continuing investments in human capital. In this setting, the age distribution of the capital stock, economic growth, and the distribution of income between skilled and unskilled workers are endogenously determined. Also, the relative price of new capital goods declines, and the capital-to-income ratio increases, over time. In addition, the economy has a government which taxes factor incomes, offers tax credits for new investment, and rebates its net revenues to households.

Clearly, the incentives to develop (through R&D) and to adopt (through replacement) more efficient capital goods will be integrally connected. Therefore, it seems worth exploring how an economy's long-run growth may be affected by the adoption-replacement decision. In fact, the notion that new technology is embodied in investment and that the adoption of new technologies is an important factor in economic growth has been enshrined in US fiscal policy since the early part of this century. With the exception of a few short-term reversals, the tax treatment of capital income has become more generous, particularly with respect to policies regarding depreciation. This leads to a final observation about capital accumulation in US economy:

> The average age of the capital stock has declined for most of the postwar period.[2]

While a large part of the dramatic decline in the average age of the aggregate capital stock over the postwar period is undoubtedly due to modernizing the capital stock in the aftermath of World War II, some of it is attributable to this trend toward leniency in the tax treatment of capital income. In the later part of the paper the response of the model economy to changes in the tax treatment of capital is studied. This serves to illustrate the model's mechanics. The dynamics for the vintage capital model differ dramatically from the standard neoclassical growth model.

1.2. Relationship to the literature

The classic vintage capital models where technological change is embodied in new capital goods were developed by Robert Solow. In Solow (1960) new capital goods incorporate the latest technology. Capital can be combined with a variable amount of labor and depreciates at a geometric rate. At any point in time plants with new and old capital coexist, but Solow (1960) illustrated how this world with heterogeneity could be represented in terms of the standard growth model with a single aggregate stock of capital. In Solow (1962) capital has a fixed lifetime and the amount of labor allocated to given unit of capital is fixed at the time it is introduced (the technology is 'putty-clay'). The current analysis is different from previous vintage capital models in several important respects. First, the decision to replace old capital with new more efficient capital is modeled explicitly. In contrast, the typical vintage capital model treats depreciation as exogenous. Old capital never becomes obsolete; it either vanishes gradually due to the assumed fixed rate of capital consumption or it dies suddenly because of a fixed lifetime. In the environment described here capital only disappears because of replacement; depreciation is an economic, not

[2] See Table A7 in *Fixed Reproducible Tangible Wealth in the United States*, 1925–1989, a publication of the US Department of Commerce.

a physical, concept. Second, consistent with observations at the microeconomic level, labor is allocated efficiently across vintages so that older technologies have less labor assigned to them. Finally, growth is modeled as endogenous rather than exogenous, as in the Solow models.

The model developed bears some resemblance to the vintage human capital models of Zeckhauser (1968) and Parente (1994). Zeckhauser (1968) considers the case of an immortal craftsman who must decide when to switch from an old to a new technique. There is bounded learning by doing within a technique. New techniques are more productive than older ones, but upon upgrading from an old technique to a new one the craftsman realizes a drop in productivity until he learns from experience. This decision is related to the adoption-replacement problem studied here. Parente (1994) studies a variant of this problem in general equilibrium. His model results in an equilibrium distribution of knowledge or skills across agents that is similar to equilibrium age distribution of capital over plants produced by the current model. The current work is also related to Campbell's (1997) model of the relationship between the adoption of new technologies over the business cycle, and the exit and entry decisions of plants. He finds that investment-specific technological change is an important source of business cycle fluctuations. The exit decision of a plant has aspects that are similar to adoption-replacement choice modeled in the current analysis.

2. The economic environment

Imagine an economy inhabited by two types of households, a firm and a government. The firm produces consumption and investment goods using three factor inputs, namely, capital and two types of labor. Households earn income by supplying labor to the firm, by lending funds to finance the firm's acquisition of capital, and from their ownership claim to the stream of profits on the firm's activity. There is a government in the economy that taxes both labor and capital income (here interest and profits). This revenue is used to give households transfer payments and to provide the firm with an investment subsidy and a capital consumption allowance.

2.1. The firm

The firm undertakes production at a fixed number of plants, which are distributed uniformly over the unit interval. In any given period, a plant can produce one of two types of goods: consumption goods and capital goods. Production of these two goods requires the input of capital and labor. Each plant has associated with it a capital stock of a certain age or vintage. At each point in time, the operator of the firm must decide whether to replace the existing capital stock in each plant with the latest vintage. Since capital has a maximum life of N years, replacement is inevitable. The question to be addressed here is when? Let p_i represent the fraction of plants that are currently using capital of age i; clearly, then $\sum_{i=1}^{N} p_i = 1$. The assumption that production is done at a fixed number of locations is equivalent to supposing that it takes a fixed amount of land, or number of managers, etc., to operate a plant, and that such factors are in inelastic supply.

Consider a representative plant of vintage i, i.e., a plant using capital of age i. Let this plant have k_i *efficiency* units of vintage-i capital at its disposal. This plant can be used to produce either consumption or capital goods. Consumption goods can be produced according to the technology

$$c_i = k_i^{\alpha} l_i^{\beta}, \quad 0 \leq \alpha, \beta, \alpha + \beta < 1, \tag{1}$$

where c_i is the output of consumption goods and k_i, l_i represent the inputs of capital and unskilled labor.

Growth in the economy results from the ability to produce evermore efficient capital goods over time. The development of new capital goods requires the use of skilled, in addition to unskilled, labor. New capital goods are produced according to the technology

$$x_i = k_i^{\alpha} b_i^{\xi} (\eta h_i)^{\zeta}, \quad 0 \leq \alpha, \xi, \zeta, \alpha + \xi + \zeta < 1. \tag{2}$$

Here x_i represents the amount of new capital goods produced by plant i using k_i units of capital, while b_i and ηh_i denote the quantities employed of unskilled and skilled labor. Assume that labor's share of income is the same in both sectors so that $\beta = \xi + \zeta$.

At any point in time the firm maximizes the present value of profits. Now, suppose that plant i produces consumption goods in the current period. It should hire unskilled labor to maximize plant profits, π_i. Specifically, it should solve the problem

$$P(k_i; w) \equiv \max_{l_i} \; [\pi_i = k_i^{\alpha} l_i^{\beta} - w l_i], \tag{P1}$$

where w is the wage rate for unskilled labor. The first-order condition associated with the problem is

$$\beta k_i^{\alpha} l_i^{\beta - 1} = w. \tag{3}$$

By making use of (3) in (P1) it is straightforward to deduce that the profits accruing from this location can be expressed as $\pi_i = (1 - \beta) k_i^{\alpha} l_i^{\beta}$. Alternatively, the plant could be assigned to the production of capital goods. Let q represent the price of new capital goods in terms of consumption goods. Now, the maximization problem for the plant would be

$$\max_{b_i, \eta h_i} \; [\pi_i = q k_i^{\alpha} b_i^{\xi} (\eta h_i)^{\zeta} - w b_i - v \eta h_i], \tag{P2}$$

where v is the wage rate for skilled labor. The first-order conditions tied to this problem are

$$q\xi k_i^{\alpha} b_i^{\xi-1}(\eta h_i)^{\zeta} = w \tag{4}$$

and

$$q\zeta k_i^{\alpha} b_i^{\xi}(\eta h_i)^{\zeta-1} = v. \tag{5}$$

The profits derived from this activity are $\pi_i = (1-\xi-\zeta)qk_i^{\alpha}b_i^{\xi}(\eta h_i)^{\zeta}$.

Observe that no plant has a *comparative* advantage in producing one type of good over the other, given the equality of labor's share of income across sectors.[3] Since each plant is free to choose the production activity in which to engage in, it must be true that there is indifference between these choices. Hence,

$$(1-\beta)k_i^{\alpha}l_i^{\beta} = (1-\xi-\zeta)qk_i^{\alpha}b_i^{\xi}(\eta h_i)^{\zeta}. \tag{6}$$

Without loss of generality, assume that the fraction f of each type of plant will produce consumption goods in the current period.

The manager of the firm must decide how many plants of each vintage to operate and how much new capital to place into the plants that are being modernized. New capital formation is subsidized by the government at rate τ_x. There is also a capital consumption allowance in place. In particular, the owners of the firm can write off from their taxes, in equal installments over a Δ-period time horizon, any investment spending (net of the investment subsidy) that is undertaken. The manager undertakes these decisions in line with the dynamic programming problem shown below:[4]

$$\begin{aligned} V(p_1,\ldots,k_1,\ldots;s) = \max_{\{p'_i\}_{i=1}^{N},k'_1}\Big\{ & \sum_{i=1}^{N} p_i(1-\tau_k)P(k_i;w) \\ & -(1-\tau_x)(1-d)qp'_1k'_1 \\ & + V(p'_1,\ldots,k'_1,\ldots;s')/[1+(1-\tau_k)r']\Big\} \end{aligned} \tag{P3}$$

subject to

$$\sum_{i=1}^{N} p'_i \le 1, \tag{7}$$

$$p'_{i+1} \le p_i, \tag{8}$$

$$k'_{i+1} = k_i. \tag{9}$$

[3] If a plant of type i decided to produce consumption goods its profits would be $(1-\beta)[\beta^{\beta}k_i^{\alpha} w^{-\beta}]^{1/(1-\beta)}$. Alternatively, if it produced investment goods it would earn $(1-\xi-\zeta)[\xi^{\xi}\ \zeta^{\zeta}\ qk_i^{\alpha} w^{-\xi}\ v^{-\zeta}]^{1/(1-\xi-\zeta)}$. Given that labor's share of income is the same across the two activities, or $\beta = \xi + \zeta$, the ratio of profits is the same for all i.

[4] The manager of the firm maximizes its present value from the owner's perspective. This implies that after-tax profits should be discounted using the after-tax interest rate.

In the above, r' represents the interest rate between today and tomorrow and s denotes the aggregate state-of-the-world in the current period (where a precise definition for s is given in the appendix.) The variable d is a proxy for the present value of the capital consumption allowance on a unit of investment spending; its value in period t reads $d_t \equiv (\tau_k/\Delta)\{1 + \sum_{j=1}^{\Delta-1} 1/(\prod_{m=1}^{j} [1 + (1 - \tau_k)r_{t+m}])\}$.[5] The first constraint given by (7) limits the number of plants that can be operated next period. Next, the number of plants using capital of age-$i + 1$ next period must be no bigger than the number using age-i capital this period. This is what (8) states. Similarly, capital that is i periods old today will be $i + 1$ periods old tomorrow, cf. (9). Note that once installed, the quantity of capital remains fixed in place until the next replacement date.

The upshot of this dynamic programming problem is the following set of efficiency conditions:

$$(1 - \tau_x)(1 - d)qk'_1 - \frac{[V_1(\cdot') - V_i(\cdot')]}{[1 + (1 - \tau_k)r']} \begin{cases} \leq 0 & \text{if } p'_i = 0, \\ = 0 & \text{if } 0 < p'_i < p_{i-1}, \\ \geq 0 & \text{if } p'_i = p_{i-1}, \end{cases} \tag{10}$$

for $i = 2, \ldots, N$, with

$$V_i(\cdot') = (1 - \tau_k)P(k'_i; w') + \max\left\{ -(1 - \tau_x)(1 - d')q'k''_1 + \frac{V_1(\cdot'')}{[1 + (1 - \tau_k)r'']}, \frac{V_{i+1}(\cdot'')}{[1 + (1 - \tau_k)r'']} \right\} \tag{11}$$

and

$$(1 - \tau_x)(1 - d)p'_1 q = V_{N+1}(\cdot')/[1 + (1 - \tau_k)r'], \tag{12}$$

with

$$V_{N+i}(\cdot') = (1 - \tau_k)p'_i P_1(k'_i; w') + V_{N+i+1}(\cdot'')/[1 + (1 - \tau_k)r'']. \tag{13}$$

Equation (10) determines how many plants of vintage i should be operated next period.[6]

Suppose that the firm decides to replace the age-i capital in a plant with new capital for next period. There are two costs associated with doing this. First is

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[5] Time subscripts are added in standard fashion, as needed. Thus, for instance, the amount of capital in an age-j plant in period t would be denoted by $k_{j,t}$. In the formulae for d_t, r_{t+m} denotes the interest rate bridging periods $t + m - 1$ and $t + m$.

[6] The notation $V_i(\cdot')$ is used to signify that the function V_i is being evaluated at next period's values for its arguments.

the direct cost, $(1-\tau_x)(1-d)qk'_1$, of buying the new capital. Second is the opportunity cost associated with junking the old capital, $V_i(\cdot')/[1+(1-\tau_k)r']$. From equation (11) this can be seen to equal the aftertax present value of the profits over the life of the plant that would obtain if this replacement decision is delayed a period. The benefit of replacing the age-i capital is $V_1(\cdot')/[1+(1-\tau_k)r']$, or the aftertax present value of profits that would be derived from new capital. Equation (10) states that (a) if all vintage-i plants are to be upgraded then these benefits exceed the costs, (b) if only some are renovated there must be indifference between these options at the margin, and (c) if none of these plants are to be refitted then the cost must exceed the benefits. Equation (12) determines the amount of new capital that will be placed in each plant that is modernized. The aftertax cost of supplying an extra unit of capital for all p'_1 newly renovated plants is $(1-\tau_x)(1-d)p'_1q$ while the benefit is $V_{N+1}(\cdot')/[1+(1-\tau_k)r']$, which from (13) is the present value of the marginal product of capital over its economic life.[7]

It is interesting to note that the firm's replacement decision is driven by the lure of earning increased rents at plants. In the absence of rents from modernization, the firm will never update the stock of capital in a plant before it is N years old. This is easy to see from equation (10). Consider a plant with capital of age $i < N$. Now, suppose there are no rents from modernization in the sense that the aftertax profits derived from updating a plant, $V_1(\cdot')/[1+(1-\tau_k)r']$, exactly equal the direct renovation costs, $(1-\tau_x)(1-d)qk'_1$. The plant will not be updated, since the firm loses the forgone rents derived from the age-i capital, $V_i(\cdot')/[1+(1-\tau_k)r']$, which exceed the (zero) net profits that will be realized from the new capital. This is always the case if the production technologies exhibit constant returns to scale.

Lemma 1. If $\alpha+\beta=\alpha+\xi+\zeta=1$ then $V_1(\cdot')/[1+(1-\tau_k)r']=(1-\tau_x)(1-d)qk'_1$.

Proof. Consider the T-period horizon version of problem (P3). Let $V^{T+1-t}(\cdot_t)$ represent the firm's value function for period $t \geq 1$.[8] Clearly, $V^0(\cdot_{T+1})=$

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[7] Solving (13) forward yields $V_{N+1}(\cdot_{t+1})=(1-\tau_k)\{p_{1,t+1}P_1(k_{1,t+1};w_{t+1})+\sum_{j=1}^{N-1}p_{j+1,t+j+1}P_1(k_{j+1,t+j+1};w_{t+j+1})/[\Pi_{m=1}^{j}(1+(1-\tau_k)r_{t+1+m})]\}$. The notation $V_{N+1}(\cdot_{t+1})$ is used to signify that the function V_{N+1} is being evaluated at its arguments for date $t+1$.

[8] The period-t dynamic programming problem is

$$V^{T+1-t}(\cdot_t)=\max_{\{p_{i,t+1}\}^N_{i=1},k_{1,t+1}}\left\{\sum_{i=1}^{N}p_{i,t}(1-\tau_k)P(\cdot_i;\cdot_t)-(1-\tau_x)(1-d_t)q_tp_{1,t+1}k_{1,t+1}+V^{T-t}(\cdot_{t+1})/[1+(1-\tau_k)r_{t+1}]\right\}$$

subject to (7), (8) and (9).

$V_1^0(\cdot_{T+1}) = 0$.The proof now proceeds by induction. Pick any $t \in \{1,\dots,T-1\}$. Now, suppose that $V_1^{T-t-j}(\cdot_{t+j+1})/[1+(1-\tau_k)r_{t+j+1}] - (1-\tau_x)(1-d_{t+j})q_{t+j}k_{1,t+j+1} \leq 0$ for all $j \in \{1,\dots,T-t\}$. From the T-horizon analog to equation (10) this implies that $p_{j+1,t+j+1} = p_{1,t+1}$ (for $j < T-t$ and N). Also, the analog to equation (11) would give

$$V_1^{T-t}(\cdot_{t+1}) = (1-\tau_k)\left[P(\cdot_1;\cdot_{t+1}) + \sum_{j=1}^{N-1} \frac{P(\cdot_{j+1};\cdot_{t+j+1})}{\Pi_{m=1}^{j}[1+(1-\tau_k)r_{t+1+m}]}\right]. \tag{14}$$

Next, substitute (13) into (12) and multiply both sides of the resulting expression by $k_{1,t+1}$ to get

$$\begin{aligned}(1-\tau_x)(1-d_t)p_{1,t+1}q_tk_{1,t+1} &= (1-\tau_k)\Bigg\{p_{1,t+1}P_1(\cdot_1;\cdot_{t+1})k_{1,t+1} \\ &\quad + \sum_{j=1}^{N-1} p_{j+1,t+j+1}\frac{P_1(\cdot_{j+1};\cdot_{t+j+1})k_{1,t+1}}{\Pi_{m=1}^{j}[1+(1-\tau_k)r_{t+1+m}]}\Bigg\}/[1+(1-\tau_k)r_{t+1}] \\ &= (1-\tau_k)\Bigg\{p_{1,t+1}P(\cdot_1;\cdot_{t+1}) \\ &\quad + \sum_{j=1}^{N-1} p_{1,t+1}\frac{P(\cdot_1;\cdot_{t+j+1})}{\Pi_{m=1}^{j}[1+(1-\tau_k)r_{t+1+m}]}\Bigg\}/[1+(1-\tau_k)r_{t+1}],\end{aligned} \tag{15}$$

where use has been made of the fact that $P_1(\cdot_{j+1};\cdot_{t+j+1})k_{1,t+1} = P_1(\cdot_{j+1};\cdot_{t+j+1})k_{j+1,t+j+1} = P(\cdot_{j+1};\cdot_{t+j+1})$ (for $j \geq 0$). Substituting (14) into (15) then yields $V_1^{T-t}(\cdot_{t+1})/[1+(1-\tau_k)r_{t+1}] = (1-\tau_x)(1-d_t)q_tk_{1,t+1}$. The desired result is obtained by letting $T \to \infty$. □

2.2. Households

There are two types of household in the economy, described as skilled and unskilled. There are M times more unskilled workers than skilled ones. Each period unskilled workers decide how much to consume, c, work, l, and save in the form of one period bonds, a'. These agents derive income from working, wl, saving (interest income, ra) and from government lump-sum transfer payments, τ. Labor and interest income are taxed at the rates, τ_l and τ_k. The dynamic programming problem for unskilled agents is

$$J(a;s) = \max_{c,l,a'}\{U(c,l;\lambda) + \rho J(a';s')\} \tag{P4}$$

subject to

$$c + a' = (1-\tau_l)wl + [1+(1-\tau_k)r]a + \tau. \tag{16}$$

The momentary utility function $U(\cdot)$ is given by

$$U(c,l;\lambda) = \ln\left(c - \lambda\Theta\frac{l^{1+\theta}}{1+\theta}\right), \quad \theta, \Theta > 0, \tag{17}$$

where the term λ represents the state of technological advance in the household sector.[9] Its adoption simplifies the analysis since the economy's general equilibrium will not be affected by the distribution of income across skilled and unskilled workers. The first-order conditions associated with the unskilled household's problem are

$$U_1(c,l;\lambda) = \rho[1 + (1-\tau_k)r']U_1(c',l';\lambda') \tag{18}$$

and

$$U_1(c,l;\lambda)(1-\tau_l)w = U_2(c,l;\lambda). \tag{19}$$

Skilled agents in this economy own the firm. This means that decisions concerning R&D (human capital investment) and the replacement of old technologies are made by the owner/operators of the firm. Assume that the firm's current indebtedness is b. The firm will then owe rb in interest. The firm's current profits after paying off this interest will be $\sum_{i=1}^{N} p_i P(k_i;w) - rb$. Additionally, recall that the firm is intending to spend $(1-\tau_x)qp'_1k'_1$ on new capital. This can be financed by issuing new debt, b'. Like unskilled agents, skilled agents must decide in each period how much to consume, z, and how much to work, h. They must further allocate their effort, however, across two activities: working in plants, $h - e$, and human capital formation, e. The skilled agent's dynamic programming problem is[10]

$$X(b,\eta;s) = \max_{z,h,e,b'}\{W(z,h;\lambda) + \rho X(b',\eta';s')\} \tag{P5}$$

subject to the flow budget constraint,

$$\begin{aligned} z = {} & (1-\tau_l)v\eta(h-e) + (1-\tau_k)\sum_{i=1}^{N}[p_iP(k_i;w) - rb] \\ & - (1-\tau_x)qp'_1k'_1 + (\tau_k/\Delta)\left\{(1-\tau_x)\left[qp'_1k'_1 + \sum_{i=0}^{\Delta-2} q_{-i-1}p_{1,-i}k_{1,-i}\right]\right\} \\ & + b' - b + \tau, \end{aligned} \tag{20}$$

[9] This form for the utility function has been successfully used in applied work; an example is Hercowitz and Sampson (1991).

[10] Let x_{-i} denote the value that x had i periods ago.

and the law of motion for human capital,

$$\eta' = H(e)\eta. \tag{21}$$

The efficiency conditions associated with this problem are

$$W_1(z,h;\lambda) = \rho[1 + (1-\tau_k)r']W_1(z',h';\lambda'), \tag{22}$$

$$W_1(z,h;\lambda)(1-\tau_l)\eta v = -W_2(z,h;\lambda) \tag{23}$$

and

$$\rho\left[W_1(z',h';\lambda')(1-\tau_l)v'(h'-e') - W_2(z',h';\lambda')\frac{H(e')}{H_1(e')\eta'}\right]H_1(e)\eta$$
$$= -W_2(z,h;\lambda). \tag{24}$$

In the subsequent analysis, the functions W and H will be restricted to the forms

$$W(z,h;\lambda) = \ln\left(z - \lambda\Omega\frac{h^{1+\omega}}{1+\omega}\right), \quad \omega,\Omega > 0, \tag{25}$$

and

$$H(e) = 1 + \chi e^{\phi}, \quad 0 < \phi < 1.$$

2.3. Government

The last actor in the economy is the government. As mentioned, it taxes labor income at rate τ_l, and interest and profits (net of the capital consumption) at τ_k. It uses the revenues raised from these taxes to provide lump-sum transfer payments, τ, to subsidize gross investment at the rate, τ_x and to give a capital consumption allowance. The government's budget constraint reads

$$(M+1)\tau + \tau_x q p'_1 k'_1 = \tau_l[Mwl + v\eta(h-e)] + \tau_k \sum_{i=1}^{N} p_i P(k_i;w)$$
$$- (\tau_k/\Delta)(1-\tau_x)[q p'_1 k'_1 + \sum_{i=0}^{\Delta-2} q_{-i-1} p_{1,-i} k_{1,-i}]. \tag{26}$$

2.4. Market-clearing conditions

Last, in competitive equilibrium the markets for both consumption and capital goods must clear so that[11]

$$Mc + z = f\sum_{i=1}^{N} p_i k_i^{\alpha} l_i^{\beta}, \tag{27}$$

$$p'_1 k'_1 = (1-f)\sum_{i=1}^{N} p_i k_i^{\alpha} b_i^{\xi} (\eta_i h_i)^{\zeta}. \tag{28}$$

Likewise, the market-clearing conditions for the unskilled and skilled labor markets imply that

$$\sum_{i=1}^{N} p_i[f l_i + (1-f) b_i] = Ml, \tag{29}$$

$$\sum_{i=1}^{N} p_i(1-f) h_i = h - e. \tag{30}$$

3. Balanced growth

The balanced-growth path for the economy can now be characterized. Clearly, along a balanced-growth path some variables, such as consumption, will be growing at some fixed rate, while others, such as aggregate employment, will remain constant. Some basic properties of the economy's balanced growth path will now be derived in a heuristic fashion.

To begin with, it seems reasonable to conjecture that along a balanced-growth path the labor variables l_i, b_i, h_i, l, h, and e will all be constant. Given this conjecture, equation (21) implies that the stock of human capital grows at some constant rate, say γ_η. Second, it seems likely that in balanced growth the age distribution of plants $\{p_i\}_{i=1}^{N}$ will be constant. Using (28) it is then straightforward to compute the rate, γ_k, at which the economy's distribution of capital shifts to the right over time. One finds

$$\gamma_k = (\gamma_\eta)^{\zeta/(1-\alpha)}.$$

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[11] At this point, it may be worth noting that (1) and (2) could have been written as $c_i = (\kappa_i \tilde{k}_i)^{\alpha} l_i^{\beta}$ and $\tilde{x}_i = (\kappa_i \tilde{k}_i)^{\alpha} b_i^{\xi} h_i^{\zeta}$, where $\kappa_i = \eta_{-i}^{\zeta}$, $\tilde{k}_i = k_i/\kappa_i$, and $\tilde{x}_i = x/\kappa'_1$. While more messy notationally, this representation of the model highlights the embodied nature of technological change. Technological progress evolves according to $\kappa'_1 = H(e_{-1})^{\zeta}\kappa_1$ and $\kappa'_{i+1} = \kappa_i$ [cf. (21)]. For more on the equivalence between these two representations of technological change, see Greenwood et al., 1997 (Appendix B).

Note that $k_i = \gamma_k^{1-i} k_1$. Next, from the above condition and (6) it follows that the relative price of capital must grow at the rate, γ_q, given by

$$\gamma_q = \gamma_\eta^{-\zeta} < 1.$$

Thus, the price of capital declines in balanced growth. Finally, let γ_y represent the constant rate at which aggregate consumption grows. Condition (27) restricts this rate of growth to be

$$\gamma_y = \gamma_\eta^{\alpha\zeta/(1-\alpha)} < \gamma_k.$$

Note that aggregate investment spending when measured in terms of consumption goods, or $qp_1' k_1'$, grows at this rate too, a fact that follows from the formulae for γ_k and γ_q. Therefore, γ_y is the rate at which aggregate output – or $c + z + qp_1' k_1'$ when taking consumption as the numeraire – grows. To take stock of the discussion so far, observe that in balanced growth the relative price of capital goods falls at the same time as capital-to-income ratio rises.

Moving on, it is easy to deduce how wages, the interest rate, and profits, etc., behave along a balanced-growth path. From (3) and (5), it is transparent that the wage rates per unit of time worked by skilled and unskilled labor, or w and $v\eta$, grow at rate γ_y. The wage rate, v, for an *efficiency* unit of skilled labor, however, increases at the lower rate $\gamma_y/\gamma_\eta < 1$. For leisure to remain constant in balanced growth, productivity in the household sector must grow at the same rate as in the consumption sector. This is readily apparent from the forms of (17), (19), (25), and (23). To insure that this is the case let $\lambda = \eta^{\alpha\zeta/(1-\alpha)}$. Note that it is the relentless rise in real wages that motivates capital replacement in the economy. As wages increase, the profits for a plant using old capital are continuously shrinking. To increase these dwindling profits the plant must invest in new capital.

As is readily observable from either (18) or (22), the aftertax interest rate, $1 + (1 - \tau_k)r$, remains constant at γ_y/ρ. The profits, $P(k_i,w)$, made by a plant in the ith period of its life rise at rate γ_y along a balanced-growth path.[12] Equation (11) implies that the present value of a vintage-i plant's profits, or V_i, are growing at this rate too. Last, the contribution that an extra unit of capital makes to a plant's profits, $P_1(k_i,w)$, increases at rate $\gamma_q < 1$, or declines over time. Thus, using (13) the marginal product in terms of the present value of profits derived from a unit of new capital, or V_{N+1}, also declines at this rate. Should not this decline in the productivity of new capital eventually choke off capital accumulation and hence growth? The answer is no: observe that while the marginal unit of new capital is becoming less productive over time, the cost of purchasing it is falling at the same rate.

[12] See footnote 3.

The steady-state age distribution of capital across plants has a simple characterization. Two cases can obtain. In the first, *only* the plants with the oldest vintage of capital are modernized. All of these plants are updated. In the second case, *some* next-to-oldest vintage plants are renovated as well. The following lemma makes this characterization precise, where the oldest capital has an age of M.

Lemma 2. $p_1 = p_2 = \cdots = p_{M-1} \geq p_M > 0$.

Proof. There are two cases to consider: either $p_{M-1} = p_M$ or $p_{M-1} > p_M$. First, suppose that $p_{M-1} > p_M$. Here equation (10) holds with equality for p_M. Now, assume, for the moment, that $V'_{i+1} < V'_i$, for all $i \geq 1$. Then if the right-hand side of (10) is equal to zero for p_M it must exceed zero for $p_{M-1}, p_{M-2}, \ldots, p_2$. Consequently, $p_1 = p_2 = \cdots = p_{M-1} > p_M$. It remains to be established that $V'_{i+1} < V'_i$ (for $i \leq N$). This can be shown by induction. To begin with, recall that in balanced growth $V_i = V'_i/\gamma_y$. Next, suppose $V'_{i+2} < V'_{i+1}$. Then equation (11) implies that $V'_{i+1} < V'_i$. To start the induction hypothesis off, note from (11) that $V'_{N+1} < V'_N$ since capital has a maximum physical life of N years.[13] The first case where $p_{M-1} = p_M$ can be analyzed the same way. □

4. Calibration

The next step in the analysis is to choose values for the model's parameters. As is now conventional, as many parameter values as possible are chosen on the basis of either (i) a priori information or (ii), so that along the model's balanced-growth path various endogenous variables assume the long-run values that are observed in the US data. The parameters in question are

Utility: θ, Θ, ω, Ω, ρ,

Technology: α, β, ξ, ζ, χ, ϕ,

Government: τ_l, τ_k, τ_x, Δ.

A time period in the model corresponds to one year.

Six parameters are determined on the basis of a priori information. Over the postwar period labor's share of income had an average of 0.65 in the US economy. This dictates setting $\beta = 0.65$ and imposing the restriction

$$\xi + \zeta = 0.65. \tag{31}$$

[13] There is an abuse of notation here. In the lemma V'_{N+1} denotes next period's value of a plant with age-$N+1$ capital. Elsewhere, V'_{N+1} represents the derivative of V' with respect to its $(N+1)$th argument.

The tax rates on labor and capital income, τ_l and τ_k, were set at 0.30 and 0.30, respectively.[14] The investment subsidy has changed considerably over the postwar period. A value of 0.10 was picked for the investment subsidy, τ_x.[15] Finally, one of the most volatile elements of the tax treatment of capital over the postwar period has been the capital consumption allowance. An accounting life of 20 years was chosen for capital in the model; i.e., $\Delta = 20.0$.[16]

The values of $1/\theta$ and $1/\omega$ correspond to the labor supply elasticities for unskilled and skilled labor. A value of 0.6 was chosen for θ and ω, which implies a value of 1.7 for the labor supply elasticities.[17]

The rest of the parameters were chosen so that the model's growth path shares certain characteristics with the long-run US data. To begin with, the average growth rate of output per hour was 1.24% between 1954 and 1990. Thus, the model should satisfy the restriction

$$\gamma_y = 1.0124. \tag{32}$$

The average ratio of hours worked to non-sleeping hours of the working-age population is 0.25. This implies that

$$l = 0.25 \tag{33}$$

and

$$h = 0.25. \tag{34}$$

Evidence on the amount of time devoted to human capital formation in R&D activities for the US economy is scant. As a result, the following arbitrary restriction is imposed on the model:

$$e = 0.1. \tag{35}$$

This condition implies that approximately 0.4% of working time is spent on R&D activities. This is roughly in accord with Birdsall and Rhee's (1993) calculation that approximately 0.2% of the population are involved in R&D activity. It also implies that about 3.2% of GNP is spent on R&D, a number very close to the 3% estimate reported by Jovanovic (1997).

[14] This is the baseline tax rate structure used in the classic Auerbach et al. (1983) study.

[15] This is the rate reported by Fullerton and Gordon (1983) for after 1975.

[16] This is the approximate average accounting life over the postwar period, based upon calculations using data presented in Gravelle (1994), Table B.2, and *Fixed Reproducible Tangible Wealth in the US*, 1925–1989, Table A4.

[17] This is in the range of elasticity values found by Heckman and MaCurdy (1980) and MaCurdy (1981).

US income distribution statistics indicate that the top 1% of the population earn approximately 8 times that of the bottom 99%.[18] Let skilled labor be identified as representing the top 1% of the income distribution. Then, $M = 99$. Next, assume that the top 1% of the population earn 8 times more labor income than the bottom 99%. This yields the condition

$$\frac{q\zeta k_1^{\alpha} b_1^{\xi} h_1^{\zeta-1} \eta^{\zeta}(h-e)}{\beta k_1^{\alpha} l_1^{\beta-1} l} = 8.0. \tag{36}$$

Additionally, let skilled workers have 8 times the wealth of unskilled workers implying

$$z/c = 8.0. \tag{37}$$

In 1989 the average age of capital in the US was 11.9 years.[19] This implies that the model's balanced-growth path should obey the restriction

$$\frac{\sum_{j=1}^{N} j p_j \gamma_k^{1-j} k_1}{\sum_{j=1}^{N} p_j \gamma_k^{1-j} k_1} = 11.9. \tag{38}$$

Finally, the aftertax real interest rate is taken to be 7%. Therefore,

$$\rho\gamma_y^{-1} = 1/1.07. \tag{39}$$

Now, the conjectured solution for the model's balanced-growth path suggests deflating the nonstationary variables by functions of η to render them stationary. To this end, let $\hat{c} = c/\eta^{\alpha\zeta/(1-\alpha)}$, $\hat{z} = z/\eta^{\alpha\zeta/(1-\alpha)}$, $\hat{k}_i = k_i/\eta^{\zeta/(1-\alpha)}$, for $i = 1, \ldots, N$, $\hat{q} = q/\eta^{-\zeta}$, etc., where the circumflex over a variable denotes its transformed value. Note that (3)–(7), (9)–(13), (19), (23), (24), and (27)–(30) can be rewritten as a system of $6N + 8$ equations in $6N + 8$ unknowns: the firm's variables l_j, b_j, h_j, p_j, $\hat{V}_j$, $\hat{k}_j$, for $j = 1, \ldots N$, and $\hat{V}_{N+1}$; the households' variables $M\hat{c} + \hat{z}$, l, h, and e; the market variables r, $\hat{q}$, and f. (See the appendix for more detail on the transformed system.) Observe that the balanced-growth path is invariant to the distribution of income between skilled and unskilled agents in the sense that the solution to the above system of equations can be determined independently of the breakdown of aggregate consumption, $M\hat{c} + \hat{z}$, between $\hat{c}$ and $\hat{z}$. This

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[18] This estimate is taken from Gomme and Greenwood, 1995 (Appendix B), who fit a Pareto distribution to the tail of the US income distribution.

[19] This number is from Table A7 in *Fixed Reproducible Tangible Wealth in the United States, 1925–1989*. Average age is computed using current-cost estimates for the gross stock of capital. (38) is based on this valuation method.

breakdown depends upon the long-run distribution of wealth represented by $\hat{a}$. By appending the nine long-run restrictions (31) to (39) to the above system the eight parameters α, Θ, Ω, ζ, ξ, ϕ, ρ, and χ can also be solved for simultaneously in addition to $\hat{a}$. Doing this yielded the following parameter values: $\alpha = 0.2$, $\Theta = 0.39$, $\Omega = 5.26$, $\zeta = 0.504$, $\xi = 0.146$, $\phi = 0.407$, $\rho = 0.9462$, and $\chi = 0.2623$.

5. Quantitative properties of balanced-growth paths

In this section the balanced-growth path for the vintage capital economy under study is analyzed. The analysis begins by categorizing the types of steady states that can occur. Next, some implications of introducing plant-level learning by doing into the model are discussed. Then the results of several experiments that highlight the role of the adoption-replacement decision in the vintage capital economy are reported. In particular, the balanced-growth paths for economies characterized by various tax policies for capital income are compared. The section concludes with a discussion explaining how the various features of the model under study contribute to the end results.

5.1. Steady-state taxonomy

In line with Lemma 2, depending on the particular configuration of tax rates, the steady state can be characterized by one of two cases. In the first case *only* the plants with the oldest vintage of capital are modernized. All of these plants are updated. The plant renovation equation (10) is slack in this situation. In the second case *some* next-to-oldest vintage plants are renovated as well. Equation (10) now holds with equality for the next-to-oldest vintage of plants. Fig. 2 illustrates the two cases (for the transformed economy). In the zones marked 'intensive' the first case transpires, while in the one labeled 'extensive' the second case occurs. This figure traces out the effect that the capital income tax has on the amount of investment in a plant, k'_1, the number of plants renovated, p_1, and vintages of capital, M.

Consider taxes in the [0.25, 0.28] interval. Here the economy is in the first case. At any point in time, there are 34 vintages of capital in existence and $p_1 = p_2 = \cdots = p_{34}$. At higher rates of capital income taxation the amount of investment in a new plant declines, as one would expect from equation (12) governing physical capital accumulation. All adjustment is along the intensive margin here, since p_1 remains fixed. Now, in the zone under consideration, the left-hand side of equation (10) is strictly negative for p_{35}. As the rate of capital income taxation increases new plants become less profitable relative to old ones, or $(1 - \tau_k)(1 - d)qk'_1 - [V_1(\cdot') - V_{35}(\cdot')]/[1 + (1 - \tau_k)r']$ rises, resulting in the left-hand side eventually becoming positive. Then, it pays to delay modernization by one period. This happens as tax rates move into the [0.28, 0.35] range.

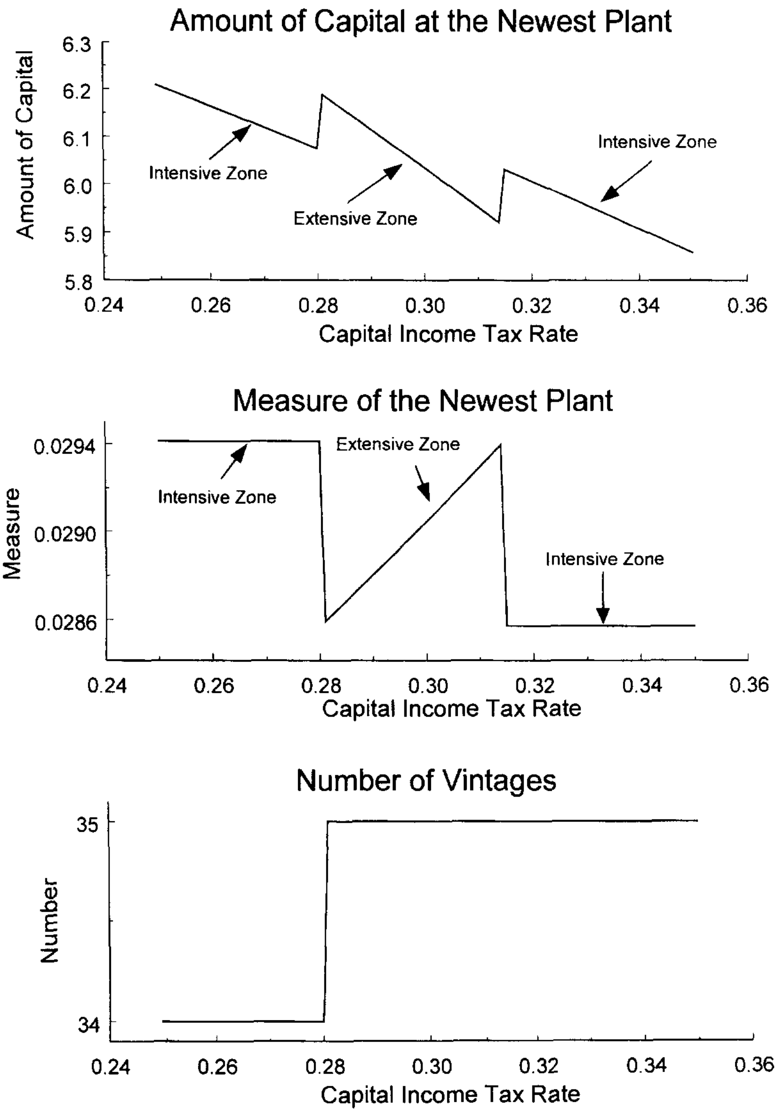

Fig. 2. Steady-state taxonomy, extensive and intensive zones.

Observe that at the point where modernization is postponed by a period investment in new plants takes an upward jump. This makes intuitive sense. Since renovation is more infrequent, investment should be larger since it must make do for an extra period – a fact transparent from (13).

Now, consider taxes in the interval [0.28, 0.314]. The economy is in the second case here. There are 35 vintages of capital in existence with $p_1 = p_2 = \cdots = p_{34} > p_{35}$. Equation (10) holds tightly here for p_{35}. In this zone k'_1 and p_1 move in opposite directions. Again, as the capital income tax rate rises investment in a renovated plant, k'_1, drops. This lowers the cost of renovating an old plant. Hence, the value of a new plant, net of renovation cost, increases vis-à-vis an old one. Consequently, p_1 increases implying adjustment along the extensive margin. The amount of equilibrium investment, $p_1 k'_1$, and thus the relative price of capital, q, still decrease over this range. There is a limit to how far this process can go since p_1 is bounded above by 1/34. This limit is reached as τ_k approaches 0.314. At this point the number of newly renovated plants takes a plunge. This is associated with an upward surge in the amount of new capital placed in a renovated plant. The economy is now in the first case, analyzed previously.

5.2. Learning by doing

Empirical evidence suggests that it may take some time to get a new plant operating to peak efficiency. Bahk and Gort (1993) find evidence of significant learning-by-doing effects at the plant level. They find that, on average, a plant's output increases by about 1% per year over the first 14 years of its life [the length of the data set by Bahk and Gort (1993)] due to learning-by-doing effects. To capture these learning-by-doing effects, rewrite the production functions (1) and (2) as

$$c_i = \mu_i k_i^{\alpha} l_i^{\beta}, \quad 0 \leq \alpha, \beta, \alpha + \beta \leq 1$$

and

$$x_i = \mu_i k_i^{\alpha} b_i^{\xi} (\eta h_i)^{\zeta}, \quad 0 \leq \alpha, \xi, \zeta, \alpha + \xi + \zeta \leq 1,$$

where μ_i represents the productivity level of an age-i plant.[20] Suppose that the μ_i's are distributed in line with Fig. 3. Here a plant's productivity increases by approximately 15% [or $(1.01^{14} - 1) \times 100\%$] over the first 14 years of life.

The presence of learning by doing causes the time profile of employment at the plant level to be $\cap$ shaped. Employment initially increases along with the rise in a plant's productivity. Eventually, though, as a plant ages the increases in total factor productivity can not compensate for the fact that it is using older, less efficient, capital. Employment then decreases. Observation 3, in the

[20] Klenow (1993) analyzes the effects of plant-level learning by doing in a similar way. The focus of his study is on the cyclical behavior of manufacturing.

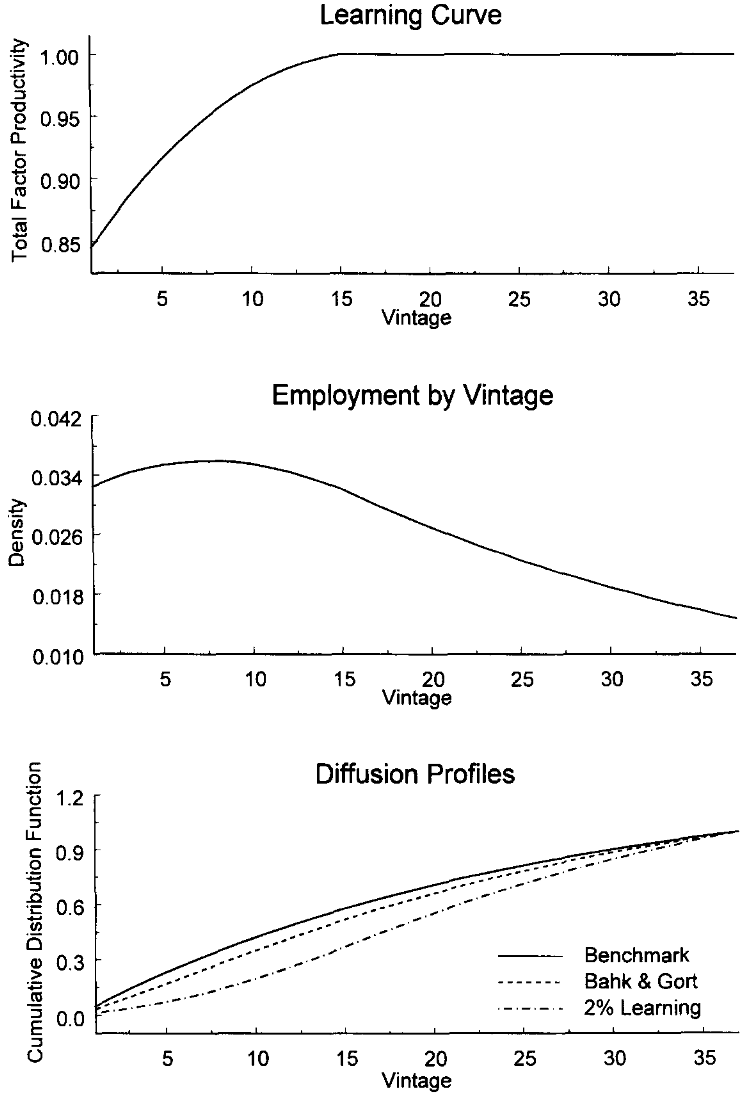

Fig. 3. Learning by doing.

introduction, suggests that this life-cycle pattern of plant employment may characterize the data. Since learning by doing increases the total factor productivity of old plants relative to young plants, it shifts the cumulative distribution of employment by vintage to the right, as in shown in Fig. 3. Notice that with the introduction of learning by doing the cumulative distribution function for employment begins to display an S-shaped diffusion pattern. This pattern becomes more pronounced as the learning effects are strengthened (say to an average of 2% per year over the first 14 years).

5.3. Economic depreciation and the tax treatment of capital

To illustrate the mechanics of the vintage capital model developed here, several tax experiments will now be undertaken. Since the agents in these economies predicate their behavior on the certain belief that tax rates are constant over time, the findings should be viewed as a comparison across economies characterized by different fiscal policies. The reference point for the first set of experiments is the benchmark tax policy presented in the previous section. This tax policy assumes that capital income is taxed at the rate of 30%, labor income of both skilled and unskilled workers is taxed at the rate of 30%, and there is a 10% investment tax credit. In addition, capital is assumed to have an accounting life for depreciation purposes of 20 years. This implies that capital tax revenues as a fraction of capital's share of income is about 21%, a number that is close to that found in US data for the 1980s.

One of the most volatile elements of US fiscal policy toward capital has been the variation in the tax treatment of depreciation. Gravelle (1994) argues that the history of the tax treatment of depreciation is one of a steadily more lenient policy toward depreciation up until 1980. Table 1 shows the effect on this economy of varying the capital income tax rate and the accounting life of capital.

The age distribution of the capital stock is characterized here by the average age of capital and the number of vintages in existence. Intuitively, higher tax rates should lessen the demand for new capital: both aggregate investment and investment per new plant fall with the tax rate. The upper left-hand panel of Table 1, and Fig. 4 and Fig. 5 show the effect that varying depreciation policy and the capital income tax rate have on the age distribution of the capital stock. The response of the average age of capital to changes in the accounting life of capital is much greater at higher tax rates.

Now, for illustrative purposes consider the following experiment: Suppose that in 1960 the capital income tax rate was 47%, the accounting life for capital was 27 years, and that there was no investment tax credit. Likewise, assume that in 1985 there was a 41% capital income tax rate, an 11-year accounting life, and a 10% investment tax credit. These numbers are based upon data presented in Gravelle (1994). The model predicts that the average age of capital should be 13.6 years under the first system, and 11.8 in the second, a difference of 1.8 years. Between 1960 and 1985 the average age of capital fell, by 2.5 years, from 13.9 to 11.4 years. Thus, given the trend toward a more lenient treatment of capital income in the postwar period, the model predicts a decline in the average age of capital in line with Observation 4 presented in the introduction.

Table 1
Response to tax and depreciation policies

	Average age			Growth rate			Welfare costs		
	τ_k			τ_k			τ_k		
Accounting life (yr)	0.10	0.30	0.40	0.10	0.30	0.40	0.10	0.30	0.40
5	10.98	10.83	10.65	1.35	1.34	1.33	− 2.22	− 1.88	− 1.56
10	11.40	11.38	11.70	1.34	1.29	1.26	− 1.90	− 1.10	− 0.48
20	11.47	11.90	12.58	1.32	1.24	1.18	− 1.60	0	1.11
	Price decline			Excess burden			Income distribution		
	τ_k			τ_k			τ_k		
	0.10	0.30	0.40	0.10	0.30	0.40	0.10	0.30	0.40
5	5.24	5.18	5.13	1.18	1.21	1.23	10.11	8.90	8.29
10	5.17	5.01	4.88	1.30	1.30	1.32	9.91	8.49	7.64
20	5.12	4.81	4.60	1.40	1.43	1.46	9.78	8.00	7.03

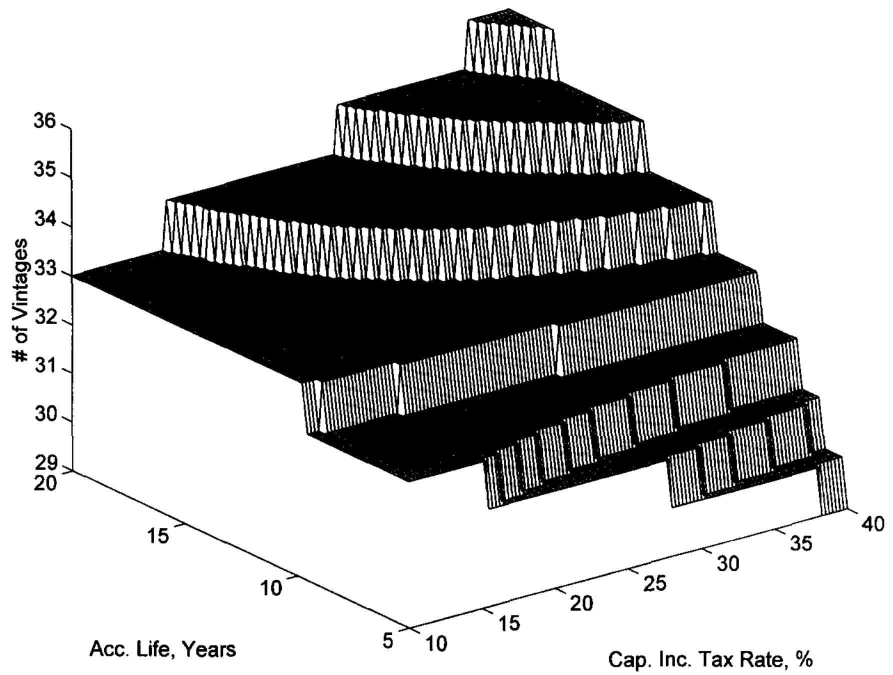

Fig. 4. Capital income taxation and the number of vintages.

Fig. 6 and Fig. 7, and the corresponding panels of Table 1, show the effects of varying depreciation policy on the growth rate of output and the decline in the relative price of new capital. One would expect that as the demand for new capital falls the benefits of investing in the advancement of knowledge will diminish. This should cause the economy's growth rate to drop. Likewise, the decline in the relative price of capital should abate. The magnitude of the growth rate effects in this economy, however, are small. This is a consequence of the law of motion for human capital accumulation used in the analysis in conjunction with the calibrated value for the elasticity of labor supply. This result accords with recent empirical work, though, which has not found the presence of a strong link between taxes and growth – see Mendoza et al. (1997), for instance. The effect of the capital tax rate on growth rates is negligible when the accounting life is short and is much stronger when accounting lives are longer. Fig. 6 shows that accounting life and tax rates have very similar effects on the growth rate.

While the growth rate effects of these different policies are small, the welfare consequences are quite sizable. The welfare consequences of taxes are measured

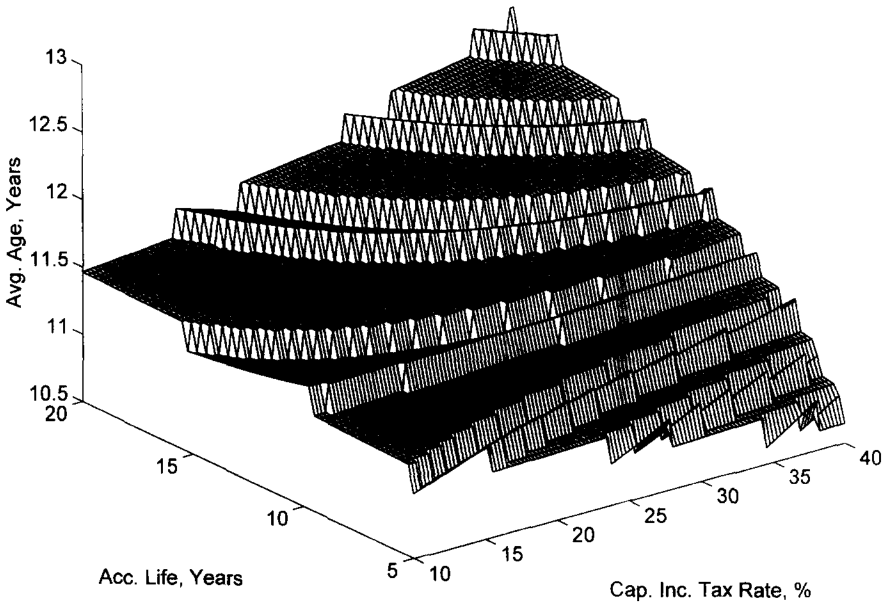

Fig. 5. Capital income taxation and the average age of capital.

by calculating the percentage increment to consumption that would be necessary to leave agents as well off with the new policy as they would have been under the benchmark policy regime. Table 1 and Fig. 8 show the weighted aggregate welfare loss and gains (negative numbers) relative to the calibrated growth path measured as a percentage of output. Raising the tax rate from 30 to 40% with a 20 year accounting life lowers welfare by 1.11% of output. Decreasing the accounting life of capital produces welfare gains of similar magnitudes. The welfare measures just presented do not take account of the fact that government revenue changes with different mixes of taxes. Accordingly, it seems important to consider the fiscal effectiveness of the different taxes. This is measured here by the ratio of dollars of welfare lost to dollars of revenue gained, or 'excess burden'. Table 1 and Fig. 9 show that the excess burden changes much more with changes in accounting life than it does with tax rates. Shorter accounting lives have much smaller excess burden and therefore are more efficient in this sense.

The distribution of income has long been thought to be integrally connected with economic growth, although the direction of the relationship has been the subject of debate. In the environment considered in this paper, the distribution of income is endogenous and responds to the mix of taxes on factor incomes. Table 1 and Fig. 10 show that the response of the distribution of labor

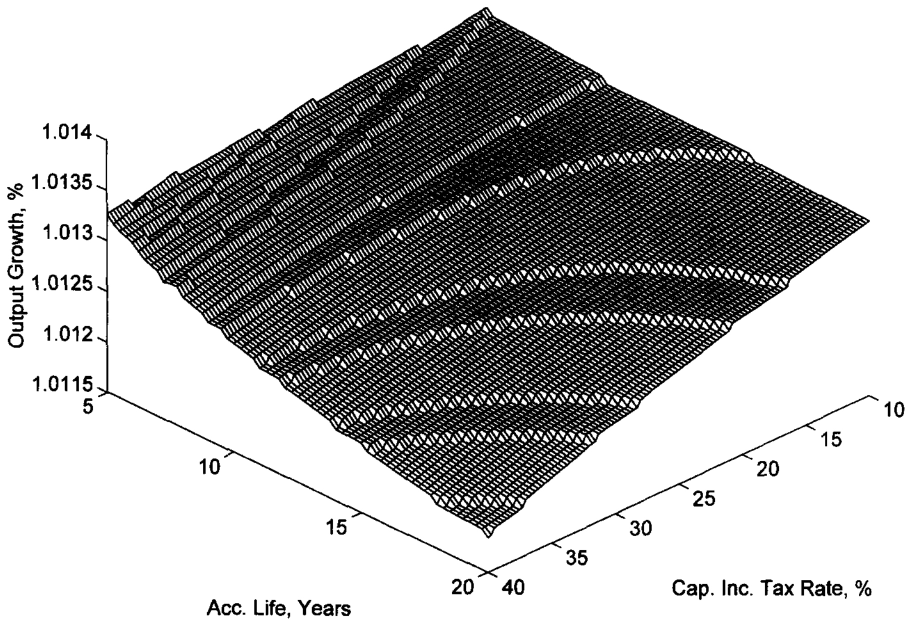

Fig. 6. Capital income taxation and growth.

income – as measured by $v\eta(h-e)/wl$ – to changes in the tax rates is quite pronounced. Raising the capital income tax rate to 40% produces the most equal distribution of income while increasing the accounting life also yields a more equal distribution of income. Higher capital income taxation hurts skilled vis-à-vis unskilled labor because it lessens the demand for capital and therefore reduces the demand for skills.

5.3.1. Discussion on the growth effects of taxes

So, why are the growth effects of tax changes small? A first guess might be that the stock of human capital is not responsive enough to changes in learning. If so, increasing the elasticity of the human capital production might overcome this. To test this, the model was recalibrated assuming that $e = 0.15$. Now, approximately 0.6% of working time and 7.3% of GDP is spent on R&D activities. With this recalibration $\phi = 0.92$, so that the human capital formation function, $H(e)$, is almost linear. Moving from the benchmark tax regime where $\tau_k = 0.30$ and $\Delta = 20$ to the much more lenient tax system where $\tau_k = 0.10$ and $\Delta = 5$ now leads to an increase in growth from 1.24% to 1.77%. While this 0.53 percentage point increase is large compared with the 0.11 percentage point one obtained before, it is still small.

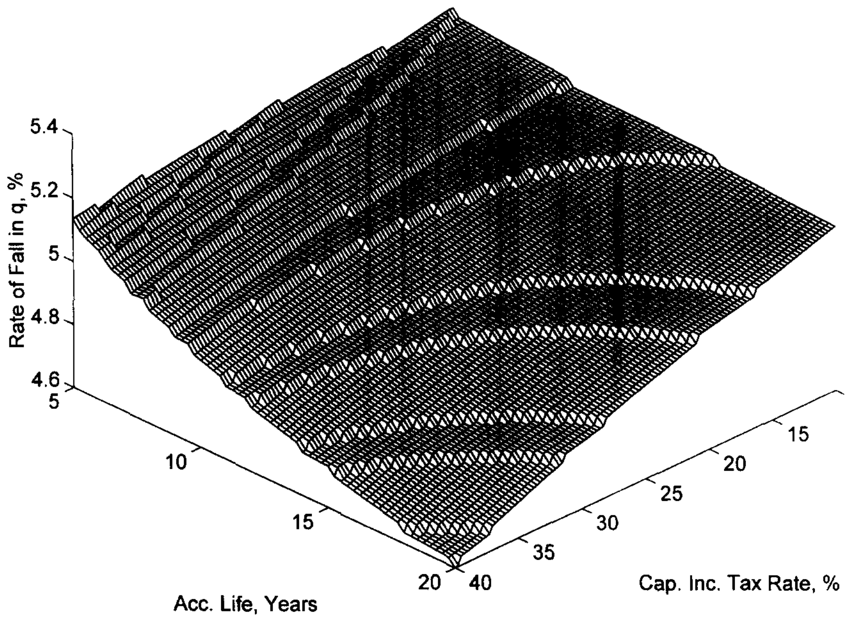

Fig. 7. Capital income taxation and the rate of decline in the price of capital goods.

The reason for these small growth effects lies with the fact that human capital formation is not taxed directly. Substituting (23) into (24) yields the following efficiency condition for human capital formation:

$$\rho W_1(z', h'; \lambda')\left[v'(h' - e') + \frac{v' H(e')}{H_1(e')}\right] H_1(e) = W_1(z, h; \lambda) v.$$

Observe that taxes do not enter this condition.[21] Along a balanced path this condition reads

$$\gamma_y = [(h - e)\phi \chi e^{\phi - 1} \rho/(1 - \rho)]^{\alpha\xi/(1-\alpha)}.$$

Note that output growth is actually decreasing in e. This is not as surprising as it may seem because γ_y/ρ represents the equilibrium aftertax interest rate, or

[21] Suppose that the agent decides to increase his stock of human capital for the *next period only* by spending an extra unit of time in learning today. The agent will realize a loss in leisure today worth $W_1(z, h; \lambda)(1 - \tau_l)v\eta$, in terms of consumption units. Next period he will realize a gain in labor income of $\rho W_1(z', h'; \lambda')(1 - \tau_l)v'(h' - e')H_1(e)\eta$, measured again in consumption units. He will also realize a benefit of $\rho W_1(z', h'; \lambda')(1 - \tau_l)v'[H(e')/H_1(e')]H_1(e)\eta$ in terms of leisure, since he can invest less in human capital next period. Note that taxes will wash out of any calculation to invest in human capital since they affect the costs and benefits symmetrically.

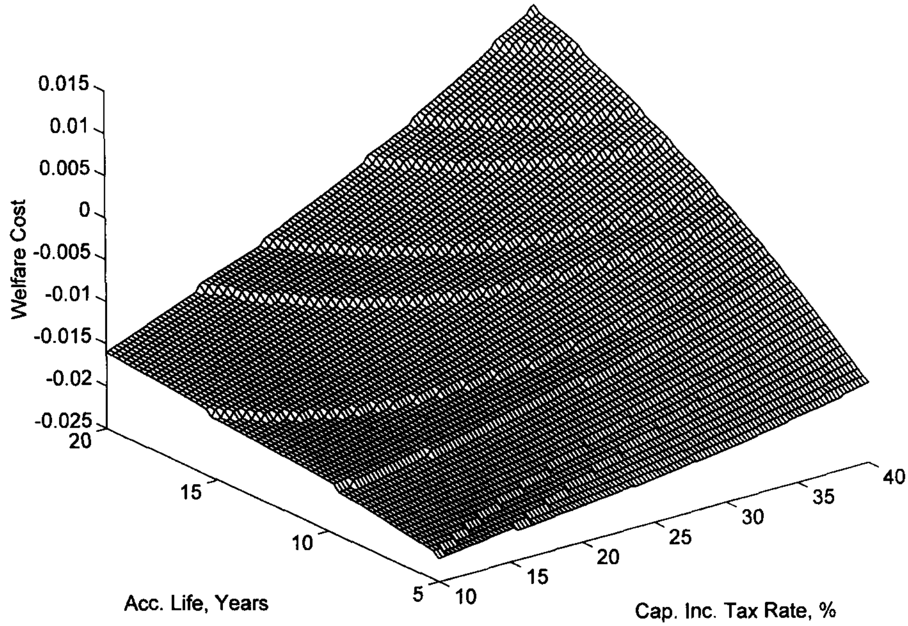

Fig. 8. Welfare cost of capital income taxation.

$1 + (1 - \tau_k)r$. Human capital formation is the engine of growth in the model and its equilibrium rate of return should be a decreasing function of investment in knowledge. Thus, lower taxes will promote growth only to the extent that they elicit more work effort, h. When moving from the benchmark tax regime, $\tau_k = 0.30$ and $\Delta = 20$, to the more lenient one indexed by, $\tau_k = 0.10$ and $\Delta = 5$, h increases from 0.25 to 0.33, an 8 percentage point increase. Large growth effects will therefore require very large elasticities of labor supply.[22]

[22] Consider the Rk model à la Rebelo (1991). Let tastes be given by $\sum_{t=1}^{\infty}\rho^{t-1}\ln c_t$, production be specified as $y = Rk$, and the economy's resource constraint read $c + k' - (1 - \delta)k = y$. Suppose that the government taxes income at the rate τ and rebates the proceeds back via lump-sum transfers. Growth is given by $\gamma_y = \rho[(1 - \tau)R + (1 - \delta)]$. Here taxes have a large effect on output growth because they tax the production of the factor that generates growth, capital. If capital entered the law of motion for human capital formation (21), as in Gomme (1993), then the effects of taxation on growth could be larger. For reasonably calibrated specifications the growth effects will be small, since capital's share of output in any sector is small. This is why Gomme (1993) finds small effects of inflation on growth. This is not necessarily a bad thing. Mendoza et al. (1997) in study of 18 OECD countries covering the period 1965–1991 find that while taxes affect a country's investment-to-output ratio they have little affect on its growth. Their reasoning for this is analogous to that given here.

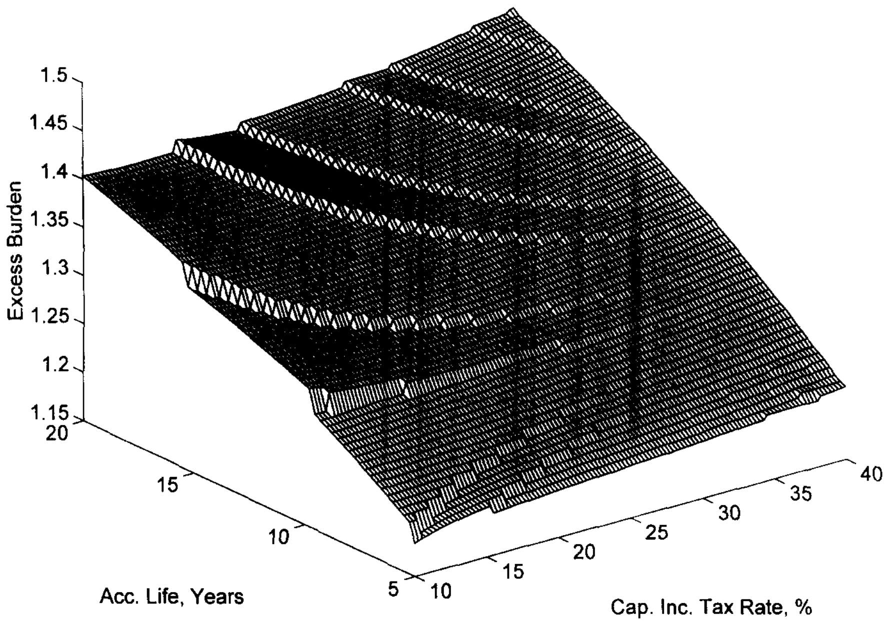

Fig. 9. Excess burden of capital income taxation.

5.4. Discussion on the model's ingredients

It may be worthwhile to take stock of how the model's various ingredients lend flavor to the analysis. Observation 2 in the introduction suggested that a considerable amount of technological change is embodied in the form of new capital goods. This was modelled here by using a two-sector model. The first sector produces consumption goods, the second capital goods. Capital goods production uses skilled labor. Due to the continual advancement in skilled labor's knowledge, the second sector could produce evermore efficient capital goods. In the model the relative price of capital falls over time, while the equipment-to-GDP ratio rises, features found in the postwar data.

Investment at the plant level is lumpy, a fact highlighted by Observation 1. To capture this feature in the model, production is assumed to be undertaken at a fixed number of locations or plants. At each point in time, the owner of a plant must decide whether or not to replace his old capital with more efficient new capital. Wages rise over time in the model. This squeezes the locational rents earned from operating a plant with a given capital stock. Eventually, it pays to replace the old capital stock with new, more efficient capital in order to increase the site's dwindling profits. There are, of course, other ways of capturing Observation 1. As in Solow (1962), production could be subject to a putty-clay

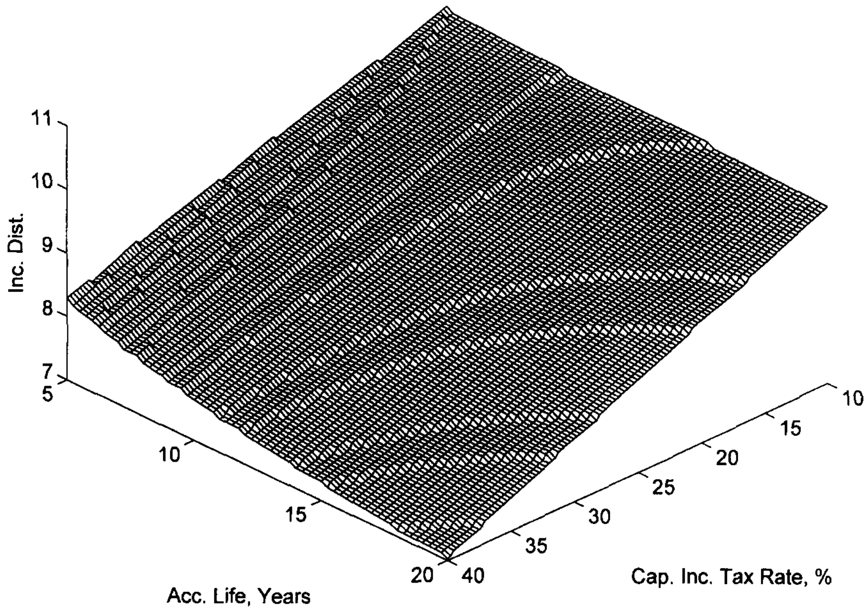

Fig. 10. Capital income taxation and the distribution of income.

technology, where the operation of a plant requires fixed inputs of capital and labor. Here too as wages rise, eventually it will become unprofitable to hire labor to work with the old fixed stock of capital. This setup is patently inconsistent, however, with the life-cycle pattern of plant employment, noted in Observation 3.[23] Alternatively, one could assume that there is a fixed cost associated with operating a plant. In a sense, though, the analysis here is modelling the micro-foundations for such a fixed cost. Essentially, the analysis is assuming that it takes a fixed amount of land, number of managers, or other things, to operate a plant and that these factors are in fixed supply. In the current analysis the price of these factors, or the fixed cost, is allowed to vary with state of the economy, as one would expect in general equilibrium.

[23] It is worth pointing out that a two-sector model with fixed proportions (putty-clay technology) is more complicated too. Suppose that a plant can switch between producing capital and consumption goods – so that there is only one age distribution for capital. It is easy to show that the newer plants will produce capital goods while the older ones will manufacture consumption goods. Firms must now decide both on when to replace old machines, and at what time to switch plants over from producing investment goods to consumption goods.

6. Transitional dynamics

The local dynamics of the vintage capital model are now analyzed. As will be seen, the economy behaves very differently depending upon whether it is operating within an extensive or intensive zone. For the first experiment consider a situation where the economy moves from an initial steady state with a capital income tax rate of 29% toward the benchmark steady state with a 30% capital income tax rate. Here the economy is operating in an extensive zone. To compute the transitional dynamics the transformed model is linearized around the benchmark steady state – the full details are in the appendix. The difference equation system characterizing the model's dynamics has $2N - 1$ eigenvalues with modulus less than one in line with the model's $2N - 1$ state variables $p_1, p_2, \ldots, p_{N-1}, \hat{k}_1, \hat{k}_2, \ldots, \hat{k}_N$.[24] Hence, the transition path is both stable and unique. The transitional dynamics displayed by the vintage capital model are markedly different from those shown by the standard one, say as typified by King et al., 1988 (Fig. 1).

In response to the increase in the capital income tax rate, the economy runs down its capital stock in the transition to the new steady state. This is shown in Fig. 11. The aggregate capital stock in the vintage model is defined by $\hat{k} = \sum_{j=1}^{N} p_j \hat{k}_j$. The vertical distance portrays the deviation away from the terminal steady state as a percentage of the discrepancy that needs to be covered. That is, in a figure that plots the time path for some variable x the vertical distance measures $100 \times (x - x^{**})/|x^{**} - x^*|\%$, where x^* and x^{**} denote the starting and terminal values for x. Observe that the aggregate capital stock behaves non-monotonically. It overshoots its long-run value. This overshooting is due to the dramatic initial decline in aggregate investment that occurs. When the capital income tax rate is raised, aggregate investment in the economy drops below the new steady-state value. Now, recall that aggregate investment, $\gamma_k \hat{q} p'_1 \hat{k}'_1$, is the product of capital per new plant, $\hat{k}'_1$, and the number of newly renovated plants, p'_1. If $\hat{k}'_1$ drops by a factor of $\lambda < 1$ while p'_1 falls by a factor $\epsilon < 1$, then $\gamma_k \hat{q} p'_1 \hat{k}'_1$ would decline by the amplified factor of $\lambda\epsilon < \min(\lambda, \epsilon)$; i.e., the proportional decline in $\gamma_k \hat{q} p'_1 \hat{k}'_1$ is larger than the proportional declines in p'_1 and $\hat{k}'_1$. The impact effect of the increase in the capital income tax rate is to cause both capital per new plant and the number of newly renovated plants to decline and this has an amplified effect on aggregate investment. It is interesting to note that this overshooting behavior in the aggregate capital stock is absent when the economy is operating in the intensive, as opposed to the extensive, region. Finally, associated with the overshooting behavior in the capital stock there is overshooting in consumption and output – see Fig. 12. The initial decline in investment spending allows consumption to rise in the short run.

[24] Note that p_N can be eliminated from the model's state since $p_N = 1 - \sum_{j=1}^{N-1} p_j$.

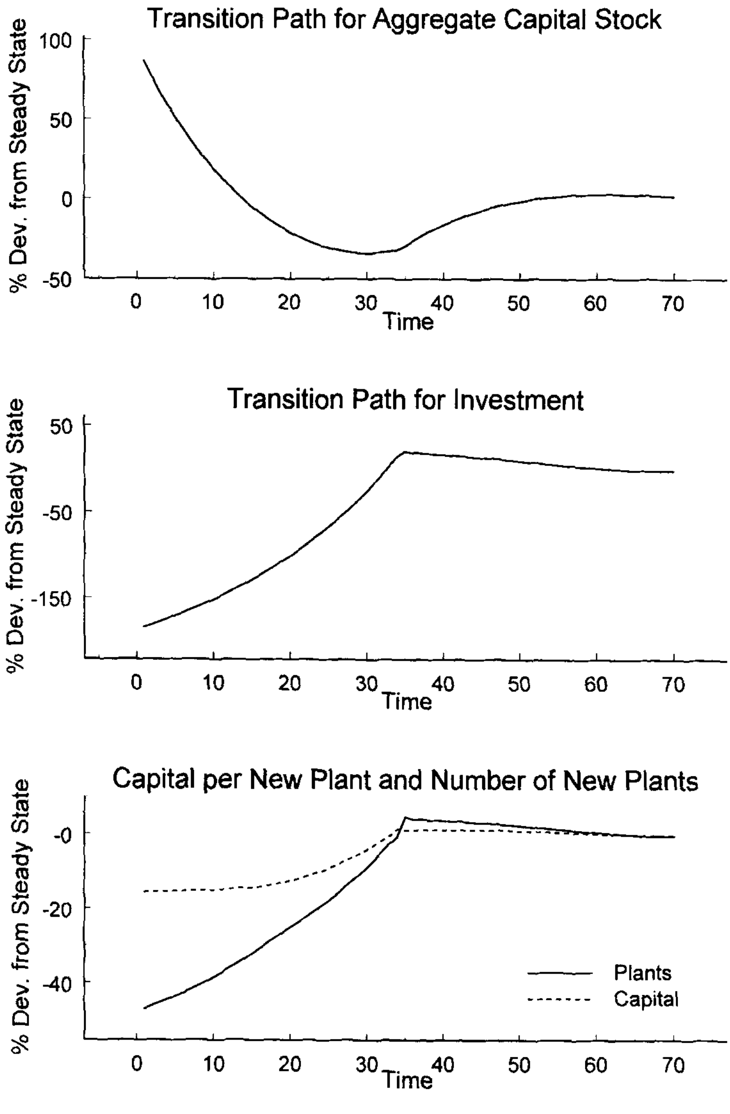

Fig. 11. Transitional dynamics, extensive zone.

It takes the vintage capital model much longer to adjust to the new capital income tax rate than does the standard model. As a measure of the speed of adjustment, define the cumulative λ-life to be the time T at which fraction λ of the total adjustment along the transition path for some *deviation* x of interest

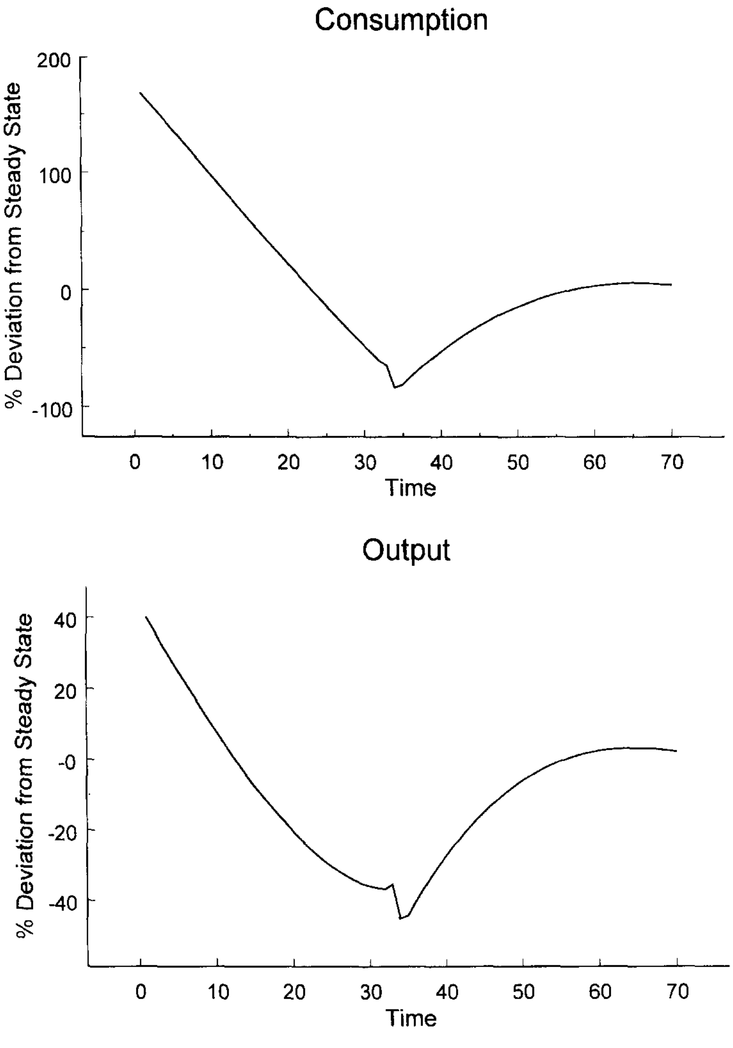

Fig. 12. Transitional dynamics, extensive zone.

has been undertaken. Thus, T solves $\min_T|\sum_{t=0}^{T}|x_t| - \lambda\sum_{t=0}^{\infty}|x_t||$, where T is some nonnegative integer. The speed of adjustment for the vintage model, reported in Table 2, is much slower than the standard neoclassical growth model. This is of interest since the standard neoclassical growth model has often been criticized for its high speed of adjustment or lack of propagation.

Modelling transitional dynamics tends to lower the welfare effects of tax changes. This transpires because consumption initially rises along the

Table 2
Speed of adjustment – extensive zone

λ-life	Capital stock	Investment	Output
25%	5 yr	6 yr	19 yr
50	22	12	30
75	32	20	37

Table 3
Speed of adjustment – intensive zone

λ-life	Capital stock	Investment	Output
25%	4 yr	3 yr	4 yr
50	8	7	9
75	15	12	16

adjustment path as the high initial capital stock is worked off. Consumption will fall below its initial level at some future date, but this is discounted. In the vintage model this process is extended even further. Taking transitional dynamics into account reduces the welfare cost of the tax hike by 60%.

Next, consider intensive-zone transitional dynamics. To do this, suppose that the economy is initially in the position associated with a 25% capital income tax rate and is moving toward one associated with a 26% one. The dynamics in this zone are similar to the standard neoclassical growth model, as Fig. 13 exemplifies. The speed of adjustment for the vintage model is somewhat slower, though, as the cumulative λ-lives reported in Table 3 illustrate.

6.1. *Learning by doing*

How does the introduction of learning by doing at the plant level affect transitional dynamics? Intuitively, one would expect now that any changes in investment would take longer to work themselves through the system, since the effects on a plant's flow of output are delayed by the learning curve. This intuition is confirmed in Fig. 14 which plots the first experiment for various learning curves. As can be seen, modest learning-by-doing effects can slow down the transitional dynamics considerably. For instance, if plant-level output increases 1.5% a year due to learning by doing [only slightly higher than the data set by Bahk and Gort (1993) estimate of 1.2%] the half and three-quarters λ-lives for adjustment are lengthened by 4 and 8 years.

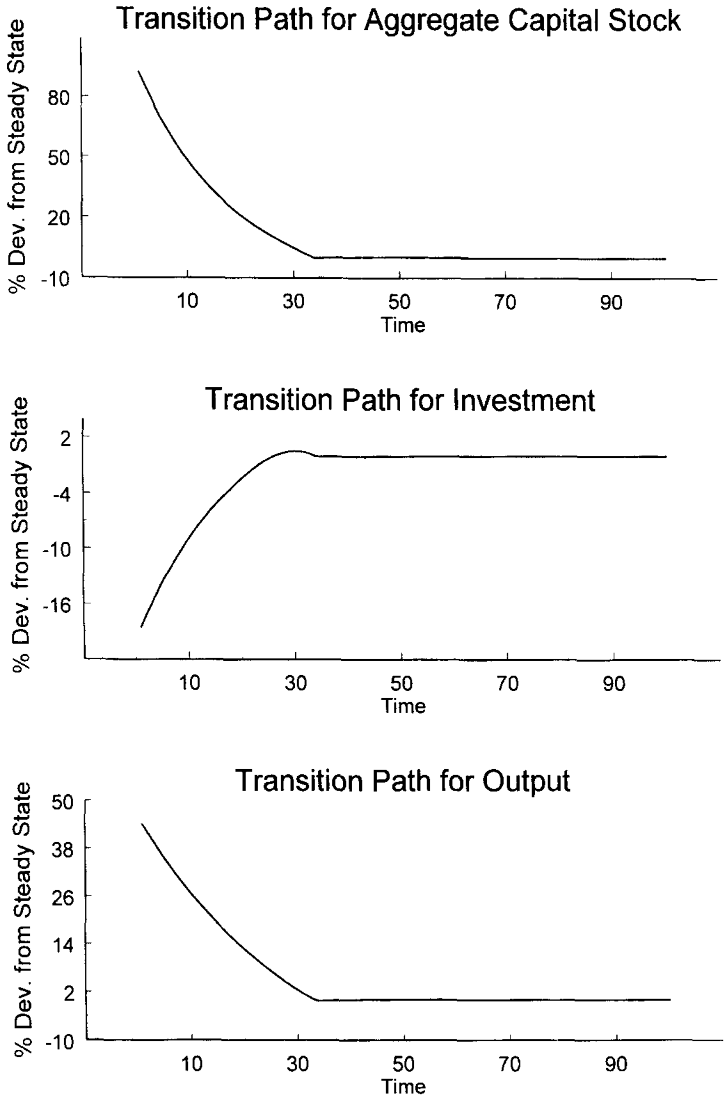

Fig. 13. Transitional dynamics, intensive zone.

7. Conclusions

7.1. Summary

A dynamic general equilibrium model of vintage capital is developed in this paper. Production in the economy is undertaken at a variety of locations or

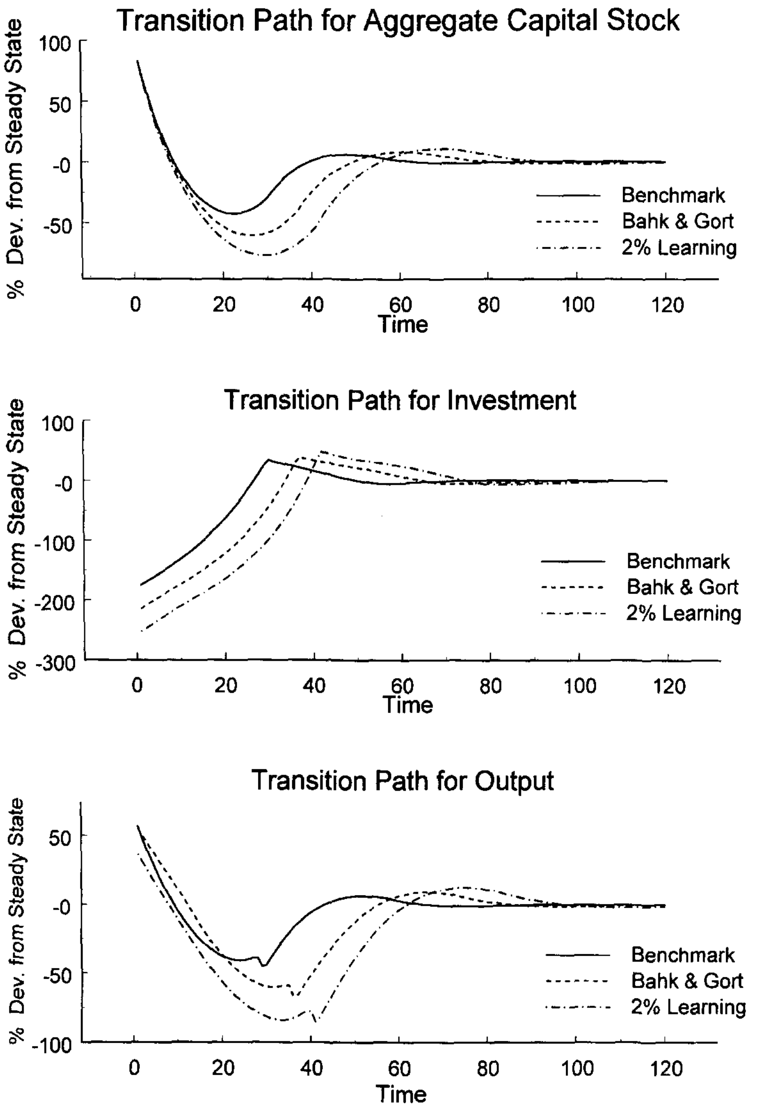

Fig. 14. Transitional dynamics with learning by doing.

plants. At each point in time, the owners of plants must decide whether to replace their existing old capital with more efficient new capital. In equilibrium, some plants will replace and others will not. This feature of the model is consistent with microevidence suggesting that investment at the plant level is lumpy. Additionally, there is macro and micro evidence that technological change is embodied in the introduction of new capital goods. To capture this feature, plants in the analysis could produce either consumption or capital goods using capital and two types of labor, skilled and unskilled. Over time more efficient capital goods can be produced because of investment in human capital by skilled agents. As a result, the relative price of capital declines and the equipment-to-GDP ratio rises in the model, as they do in data. Last, employment at the plant level declines as the capital stock ages and becomes obsolete relative to the latest vintage. This, too, accords with observation on a plant's life-cycle pattern of employment.

The quantitative analysis suggests that tax policy can have a significant effect on welfare, the distribution of income, and the age distribution of the capital stock. Over the postwar period the average age of the capital stock has declined. At the same time the tax treatment of capital income has become more lenient. The model predicts that a more lenient tax treatment of capital income should be associated with a decline in the average age of capital, ceteris paribus. The effects on economic growth are more moderate in this economy, but empirical work on the relationship between taxes and growth has not established the presence of a strong link. The transitional dynamics for a vintage capital economy are very different from, and can be more sluggish than, the standard neoclassical growth model. Learning by doing slows down transitional dynamics. It may be worthwhile to explore the model's dynamics further. To begin with, the global dynamics for the model could be analyzed. Imagine starting the system off from some initial condition that lies far away from the balanced-growth path. In the transition towards balanced growth, the system could travel through both intensive and extensive zones. This would lead to much richer dynamics. In the current version of the model plants can switch freely between producing consumption and capital goods. Presumably, the model would exhibit even more sluggish dynamics if, at the time of refitting, a plant had to commit to producing either one of these goods.

7.2. Some uses of vintage capital models

The prototypical vintage capital model developed here has many potential uses. For instance, Yorukoglu (1997) uses a simplified version of the framework to study the information technology productivity paradox. He allows for two types of capital: conventional and information technology. Plants are allowed to upgrade, or add on to, their existing capital stocks with new, more efficient capital. The compatibility between new and old capital is taken be a decreasing function of the rate of technological change and the age difference between

[25] Gort and Boddy (1967) present an early model where old and new capital can be imperfectly mixed together.

the two capital stocks.[25] The model predicts that information technology investment should be much more lumpy than investment in conventional capital; it is in the real world, too. The lumpy nature of this investment, together with learning by doing, is shown to bias downward conventional econometric estimates of its productivity. Greenwood and Yorukoglu (1997) use a variant of the framework to address the question of industrial revolutions, which they claim are times of rapid investment-specific technological change, rising income inequality, and productivity slowdowns. They model skill as being essential for the learning process associated with the adoption of new technologies. They find that the diffusion of a radically new technology, such as the steam engine or computers, is associated with a rise in the skill premium and a productivity slowdown (due to unmeasured investment in learning).

Last, the model developed here stresses the continual displacement of old technologies by more efficient new ones. This will have implications for (un)employment. Perhaps labor trains to work a particular technology. Thus, there would be vintage human, as well as physical, capital. As the technology becomes obsolete so do workers. When the technology is displaced so are they. This would have the flavor of recent work by Aghion and Howitt (1994) and Caballero and Hammour (1996). Similarly, Jovanovic (1998) develops a model where machines get matched to workers. When capital and workers are Edgeworth–Pareto complements in production, the best machines should be matched to the best workers. Each time a new generation of capital goods arrives the older generations of capital get rematched to lower skilled workers. Such continual churning could be costly. Do firms reallocate capital over their existing labor force, or do they reallocate workers over their existing capital stocks? The setup developed above could be adapted to model the (un)employment process as workers get reallocated from old to new plants.

Acknowledgements

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Appendix A.

The description of the economy outlined in Section 2 will be completed with a definition of the competitive equilibrium. First, the aggregate state-of-the-world is given by the vector $s = (p_1, \ldots, p_N, k_1, \ldots, k_N, \eta)$. Second, the equilibrium wage and interest rates, the price of capital goods, and individual transfer payments are expressed as functions of the aggregate state vector as follows: $w = \mathscr{W}(s)$, $v = \mathscr{V}(s)$, $r' = \mathscr{R}(s)$, $q = \mathscr{Q}(s)$ and $\tau = \mathscr{T}(s)$. Next, suppose that the aggregate state variables evolve according to $p_i' = \mathscr{P}_i(s)$, $k_i' = \mathscr{K}_i(s)$, and $\eta' = \mathscr{N}(s)$. Hence, the law of motion for s is $s' = \mathscr{S}(s) \equiv (\mathscr{P}_1(s), \ldots, \mathscr{K}_1(s), \ldots, \mathscr{N}(s))$. Finally, it is easy to see that the above expression imply that d (the value of the capital consumption allowance) can be represented as $d = \mathscr{D}(s)$.

Definition. A competitive equilibrium is a set of allocation rules $l_i = \mathscr{L}_i(s)$, $b_i = \mathscr{B}_i(s)$, $h_i = \mathscr{H}_i(s)$, $p_i' = \mathscr{P}_i(s)$, $k_i' = \mathscr{K}_i(s)$, for $i = 1, \ldots, N$, $c = \mathscr{C}(s)$, $l = \mathscr{L}(s)$, $z = \mathscr{Z}(s)$, $h = \mathscr{H}(s)$, $e = \mathscr{E}(s)$, and set of pricing and transfer payments $w = \mathscr{W}(s)$, $v = \mathscr{V}(s)$, $r' = \mathscr{R}(s)$, $q = \mathscr{Q}(s)$, $d = \mathscr{D}(s)$ and $\tau = \mathscr{T}(s)$, and an aggregate law of motion $s' = \mathscr{S}(s)$, such that

1. Consumption goods plants, capital goods plants, and the firm solve problems P(1), P(2), and P(3), respectively, taking as given the aggregate state-of-the-world s and the form of the functions $\mathscr{W}(\cdot)$, $\mathscr{V}(\cdot)$, $\mathscr{R}(\cdot)$, $\mathscr{Q}(\cdot)$, $\mathscr{D}(\cdot)$, and $\mathscr{S}(\cdot)$, with the equilibrium solutions to these problems satisfying $l_i = \mathscr{L}_i(s)$, $b_i = \mathscr{B}_i(s)$, $h_i = \mathscr{H}_i(s)$, $p_i' = \mathscr{P}_i(s)$, $k_i' = \mathscr{K}_i(s)$, for $i = 1, \ldots, N$.
2. Unskilled workers solve problem P(4), taking as given the aggregate state-of-the-world s and the form of the functions $\mathscr{W}(\cdot)$, $\mathscr{R}(\cdot)$, $\mathscr{T}(\cdot)$, and $\mathscr{S}(\cdot)$, with the equilibrium solution to this problem satisfying $c = \mathscr{C}(s)$, and $l = \mathscr{L}(s)$.
3. Skilled workers solve problem P(5), taking as given the aggregate state-of-the-world s and the form of the functions $\mathscr{W}(\cdot)$, $\mathscr{V}(\cdot)$, $\mathscr{R}(\cdot)$, $\mathscr{Q}(\cdot)$, $\mathscr{D}(\cdot)$, $\mathscr{T}(\cdot)$ and $\mathscr{S}(\cdot)$, with the equilibrium solution to this problem satisfying $z = \mathscr{Z}(s)$, $h = \mathscr{H}(s)$, $\eta' = \mathscr{N}(s)$, where $\mathscr{N}(s) = H(\mathscr{E}(s))\eta$.
4. All goods and factor markets must clear, so that equations (27) to (30) are met. Additionally, the production arbitrage condition (6) must hold.

Next, as was mentioned in Section 3, a key step in solving the model is to deflate all nonstationary period-t variables by functions of η_t to render them stationary. Specifically, let $\hat{c}_t = c_t/\eta_t^{\alpha\zeta/(1-\alpha)}$, $\hat{z}_t = z_t/\eta_t^{\alpha\zeta/(1-\alpha)}$, $\hat{k}_{i,t} = k_{i,t}/\eta_t^{\zeta/(1-\alpha)}$, for $i = 1, \ldots, N$, and $\hat{q}_t = q_t/\eta_t^{-\zeta}$, where the circumflex over a variable denotes its transformed value. Let $\gamma_{\eta,t+1} = \eta_{t+1}/\eta_t = H(s_t)$, $\gamma_{y,t+1} = \gamma_{\eta,t+1}^{\alpha\zeta/(1-\alpha)}$, and $\gamma_{k,t+1} = \gamma_{\eta,t+1}^{\zeta/(1-\alpha)}$; observe that in general $\gamma_{y,t+1} \neq y_{t+1}/y_t$ and $\gamma_{k,t+1} \neq k_{t+1}/k_t$. Finally, note that $w_t/\eta_t^{\alpha\zeta/(1-\alpha)} = \beta\hat{k}_{1,t}^{\alpha} l_{1,t}^{1-\beta}$, $v_t/\eta_t^{[\alpha\zeta/(1-\alpha)]-1} = \hat{q}_t\zeta\hat{k}_{1,t}^{\alpha} b_{1,t}^{\xi} h_{1,t}^{\zeta-1}$, $P(\cdot_i;\cdot_t)/\eta_t^{\alpha\zeta/(1-\alpha)} = \hat{P}(\cdot_i;\cdot_t)$, $V_i(\cdot_t)/\eta_t^{\alpha\zeta/(1-\alpha)} = \hat{V}_i(\cdot_t)$, $P_1(\cdot_i;\cdot_t)/\eta_t^{-\zeta} = \hat{P}_1(\cdot_i;\cdot_t)$, and $V_{N+1}(\cdot_t)/\eta_t^{-\zeta} = \hat{V}_{N+1}(\cdot_t)$.[26]

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[26] The notation $\hat{F}(\cdot_t)$ indicates that the arguments of $F(\cdot_t)$ are being evaluated at their transformed values in period t.

Using the above facts, the equations governing the model's dynamics can be represented in the form shown below.

Labor allocations [cf. (3)–(5)]:

$$l_{j,t} = [\hat{k}_{j,t}/\hat{k}_{1,t}]^{\alpha/(1-\beta)} l_{1,t} \quad \text{for } j = 2, \ldots, N, \tag{A.1}$$

$$b_{j,t} = [\hat{k}_{j,t}/\hat{k}_{1,t}]^{\alpha/(1-\xi-\zeta)} b_{1,t} \quad \text{for } j = 2, \ldots, N, \tag{A.2}$$

$$h_{j,t} = [\hat{k}_{j,t}/\hat{k}_{1,t}]^{\alpha/(1-\xi-\zeta)} h_{1,t} \quad \text{for } j = 2, \ldots, N, \tag{A.3}$$

$$\hat{q}\xi b_{1,t}^{\xi-1}(h_{1,t})^{\zeta} = \beta l_{1,t}^{\beta-1}. \tag{A.4}$$

Euler equation for aggregate consumption [cf. (18) and (22)]:

$$\rho[1 + (1 - \tau_k) r_{t+1}]\gamma_{y,t+1}^{-1}\left[M\hat{c}_t + \hat{z}_t - M\frac{\Theta}{1+\theta} l_t^{1+\theta} - \frac{\Omega}{1+\omega} h_t^{1+\omega}\right]$$
$$= \left[M\hat{c}_{t+1} + \hat{z}_{t+1} - M\frac{\Theta}{1+\theta} l_{t+1}^{1+\theta} - \frac{\Omega}{1+\omega} h_{t+1}^{1+\omega}\right]. \tag{A.5}$$

Price of capital [cf. (6)]:

$$\hat{q}_t = \frac{(1-\beta) l_{1,t}^{\beta}}{(1-\xi-\zeta) b_{1,t}^{\xi} h_{1,t}^{\zeta}}. \tag{A.6}$$

Plant renovation [cf. (7), (9), (10) and (11)]:

$$(1-\tau_x)(1-d_t)\hat{q}_t(\gamma_{\eta,t+1})^{\zeta/(1-\alpha)}\hat{k}_{1,t+1}$$
$$- \frac{1}{[1+(1-\tau_k)r_{t+1}]}\gamma_{y,t+1}[\hat{V}_{1,t+1} - \hat{V}_{i,t+1}]$$
$$\begin{cases} \leq 0 & \text{if } p_{i+1,t+1} = 0, \\ = 0 & \text{if } 0 < p_{i+1,t+1} < p_{i,t}, \\ \geq 0 & \text{if } p_{i+1,t+1} = p_{i,t}, \end{cases}$$
$$\text{for } i = 2, \ldots, N, \tag{A.7}$$

with

$$\hat{V}_i(\cdot_{t+1}) = (1-\tau_k)\hat{P}(\cdot_i; \cdot_{t+1})$$
$$+ \max[-(1-\tau_x)(1-d_{t+1})\hat{q}_{t+1}(\gamma_{\eta,t+2})^{\zeta/(1-\alpha)}\hat{k}_{1,t+2}$$
$$+ \frac{1}{[1+(1-\tau_k)r_{t+2}]}\gamma_{y,t+2}\hat{V}_1(\cdot_{t+2}), \frac{1}{[1+(1-\tau_k)r_{t+2}]}$$
$$\times \gamma_{y,t+2}\hat{V}_{i+1}(\cdot_{t+2})] \tag{A.8}$$

and

$$\sum_{j=1}^{N} p_{j,t} = 1, \tag{A.9}$$

$$\gamma_{\eta,t+1}^{\zeta/(1-\alpha)}\hat{k}_{j+1,t+1} = \hat{k}_{j,t} \quad \text{for } j = 1, \ldots, N-1. \tag{A.10}$$

Physical capital accumulation [cf. (12) and (13)]:

$$(1-\tau_x)(1-d_t)p_{1,t+1}\hat{q}_t = \frac{1}{[1+(1-\tau_k)r_{t+1}]}(H(e_t))^{-\zeta}(1-\tau_k)\hat{V}_{N+1}(\cdot_{t+1}), \tag{A.11}$$

with

$$\hat{V}_{N+1}(\cdot_{t+1}) = p_{1,t+1}\alpha\hat{k}_{1,t+1}^{\alpha-1}l_{1,t+1}^{\beta} + \sum_{j=1}^{N-1} p_{j+1,t+j+1}\frac{\alpha\hat{k}_{1,t+1}^{\alpha-1}l_{j+1,t+j+1}^{\beta}}{\Pi_{m=1}^{j}[1+(1-\tau_k)r_{t+m+1}]}. \tag{A.12}$$

Labor-leisure choices [cf. (19) and (23)]:

$$(1-\tau_l)\overbrace{\beta\hat{k}_{1,t}^{\alpha}l_{1,t}^{\beta-1}}^{\hat{w}_t} = \Theta l_t^{\theta}, \tag{A.13}$$

$$(1-\tau_l)\overbrace{\beta\hat{k}_{1,t}^{\alpha}l_{1,t}^{\beta-1}}^{\hat{w}_t} = \Theta l_t^{\theta}, \tag{A.14}$$

Human capital accumulation [cf. (24)]:

$$\gamma_y\frac{[(1-\tau_l)\hat{q}_t\zeta\hat{k}_{1,t}^{\alpha}b_{1,t}^{\xi}h_{1,t}^{\zeta-1} + \Omega h_{t+1}^{\omega}H(e_{t+1})/H_1(e_{t+1})]H_1(e_t)/H(e_t)}{[1+(1-\tau_k)r_{t+1}]} = \Omega h_t^{\omega}. \tag{A.15}$$

Resource constraints [cf. (27), (28), (29) and (30)]:

$$M\hat{c}_t + \hat{z}_t = f_t\sum_{j=1}^{N} p_j\hat{k}_{j,t}^{\alpha}l_{j,t}^{\beta}, \tag{A.16}$$

$$p_{1,t+1}\gamma_{\eta,t+1}^{\zeta/(1-\alpha)}\hat{k}_{1,t+1} = (1-f_t)\sum_{j=1}^{N} p_j\hat{k}_{j,t}^{\alpha}b_{j,t}^{\xi}h_{j,t}^{\zeta}, \tag{A.17}$$

$$\sum_{j=1}^{N} p_{j,t}[f_tl_{j,t} + (1-f_t)b_{j,t}] = Ml_t, \tag{A.18}$$

$$\sum_{j=1}^{N} p_{j,t}(1-f_t)h_{j,t} = h_t - e_t. \tag{A.19}$$

At time t the state of the transformed system is given by the $2N$ vector $\hat{s}_t = (p_{1,t}, \ldots, p_{N,t}, \hat{k}_{1,t}, \ldots, \hat{k}_{N,t})$. Determined at this point in time, as functions of the state of the world, s_t, are: the firm's variables $l_{j,t}$, $b_{j,t}$, $h_{j,t}$, $p_{j,t+1}$, $\hat{V}_{j,t+1}$, $\hat{k}_{j,t+1}$, for $j = 1, \ldots, N$, and $\hat{V}_{N+1,t+1}$; the households' variables $M\hat{c}_t + \hat{z}_t$, l_t, h_t, e_t; the market variables r_{t+1}, q_t, and f_t. The model's balanced-growth path can be solved for using these equations. In balanced growth $\hat{x}_t = \hat{x}_{t+1}$, for all time t and variables $\hat{x}$. Equations (A.1) to (A.2) represent a system of $6N + 8$ equations in $6N + 8$ unknowns. A difficulty associated with computing the balanced-growth path is that equation (A.7) does not have to hold with equality; however, Lemma 2 places considerable structure on the range possibilities that can occur. To solve for the model's local dynamics, this system of equations is linearized around the balanced-growth path. The resulting set of linearized equations is then represented as system of first-order linear difference equations. The dynamic path will be (locally) stable and unique provided that the system has associated with it exactly $2N - 1$ eigenvalues with modulus less than one – the number of state variables once p_N is solved out using (A.9). This was the case for all examples studied. While the number of vintages remains fixed along a transition path, given the local nature of the analysis, the number of old plants renovated may vary depending on which of the two zones the economyis operating in. Finally, note that when computing the equilibrium path for the model there is no need to solve for $\hat{c}_t$ and $\hat{z}_t$ separately. All that matters is aggregate consumption, $M\hat{c}_t + \hat{z}_t$, which appears in (A.5) and (A.16). The aggregate Euler equation obtains from summing across the individual Euler equations, (18) and (22), and is a consequence of the assumed form for the momentary utility functions, (17) and (25). The equilibrium path for the model is independent of the distribution of wealth between skilled and unskilled agents.

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