Designing Stress Scenarios*

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Abstract

We study the optimal design of stress scenarios. A principal manages the unknown risk exposures of agents by asking them to report losses under hypothetical scenarios before taking remedial actions. We apply a Kalman filter to solve the learning problem and we relate the optimal design to the risk environment, the principal's preferences and available interventions. In a banking context, optimal capital requirements cover losses under an adverse scenario while targeted interventions depend on covariances among residual exposures and systematic risks. Our calibration reveals that information is particularly valuable for targeted interventions as opposed to broad capital requirements.

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Introduction

Stress tests are ubiquitous in risk management and financial supervision. Risk officers use stress tests to set and monitor risk limits within their organizations, and financial regulators around the world use stress tests to assess the health of financial institutions. For example, financial firms use stress tests to complement their statistical risk management tools (e.g., Value at Risk); asset managers stress test their portfolios; trading venues stress tests their counter-party exposures; regulators mandate large scale stress tests for banks and insurance companies and use the results to enforce capital requirements and validate dividend policies.[1]

Despite the growing importance of stress testing, and the amount of resources devoted to them, there is little theoretical guidance on exactly how one should design stress scenarios. A theoretical literature has focused on the trade-offs involved in the *disclosure* of supervisory information (see Goldstein and Sapra, 2014 for a review), which range from concerns about the reputation of the regulator (Shapiro and Skeie, 2015) to the importance of having a fiscal backstop (Faria-e-Castro et al., 2017). These papers provide insights into disclosure and regulatory actions but they are silent about the design of forward-looking hypothetical scenarios. In that sense, existing models are models of asset quality reviews (and their disclosure) rather than models of *stress testing.*

The goal of our paper is to start filling this void. Risk management is a two-tier process involving risk discovery (learning) and risk mitigation (intervention). Stress tests belong to the risk discovery phase but one cannot analyze the design of a test without understanding the remedial actions that can be taken once the results are known. We therefore model both the risk discovery stage and the risk mitigation stage.

We consider a principal and a potentially large number of agents. The agents can be traders within a financial firm, or they can be financial firms within a financial system. The principal can be a regulator designing supervisory tests, or a risk officer running an internal stress test. For concreteness we will use the supervisory stress testing analogy in much of the paper. The key issue in all cases is that the principal does not directly observe the exposures of agents to systematic risk factors.

The regulator is risk averse and worries about the financial system experiencing large losses in

[1] Central banks in the United States, Europe, England, Brazil, Chile, Singapore, China, Australia, and New Zealand, as well as the International Monetary Fund in Japan, have recently used stress tests to evaluate the banking sector's solvency and guide banking regulation.

some states of the world. The regulator then designs a set of hypothetical scenarios and asks the banks to report their losses under these scenarios. The regulator uses reported losses across all banks and scenarios to extract information about underlying exposures. Based on this information, the regulator decides how to intervene, i.e., she can ask a set of banks to change their exposures to some factors. Interventions are costly, either directly – by drawing on limited regulatory resources, creating disruptions – or indirectly – by preventing banks from engaging in valuable activities.

Our model allows for two types of interventions: broad capital requirements and targeted risk reductions. We can interpret the targeted interventions as limits on specific asset classes (e.g., LTV ratios on particular mortgages) or as matters requiring attention (MRA) in the language of supervision. With regards to capital requirements, banking stress tests often rely on a particular scenario – usually called the adverse scenario – to deliver "pass/fail" grades. The mechanical link between the scenario and the grade conflates learning and intervening, which are two conceptually separate issues.[2] In our baseline model we assume instead that regulators choose optimal actions conditional on the results of the test, and thus have complete freedom to design the most informative scenarios. In our calibrated model we also analyze the case where one scenario is used to set capital requirements directly.

Our first main insight comes from writing the learning problem as a Kalman filter. The filter gives us a mapping from prior beliefs and test results into posterior beliefs about exposures. The precision of the mapping depends on the scenarios in the stress test. We can then formulate the regulator's problem as an information acquisition problem in which the regulator chooses the precision of her signals about risk exposures. Formally, we map the primitive parameters of the model, such as the priors of the regulator regarding the banks' exposures, to the endogenous feasible set of posteriors beliefs, usually taken as exogenous in the literature on information acquisition. If, for instance, the regulator is worried about a particular risk factor, we can derive the stress test that maximizes learning about exposures to that factor.

The optimal intervention, and hence learning, depend on the model's primitives such as the regulator's prior beliefs and the correlation of risk exposures within and across banks. For instance, the regulator's priors about average exposures – holding constant her uncertainty – have two effects on the optimal stress scenario. A higher expected exposure increases the likelihood of intervention,

[2]For instance, imagine that a bank needs the same level of ex-ante equity to satisfy a 9% capital requirement after scenario 1 or a 7% requirement after scenario 2 (presumably because scenario 2 embodies a higher degree of stress). As far as ex-ante capital adequacy is concerned, these two regulations are equivalent.

which makes accurate information more valuable. This effect pushes the regulator to learn about factors with high expected risk exposures. On the other hand, when the regulator's prior mean is high, her posterior mean is likely to remain high and thus her action is less responsive to new information, which discourages learning along that dimension. This second effect dominates when the expected risk exposure is high. Hence, the weight of a factor in the stress scenario is hump-shaped with respect to the regulator's prior.

When exposures are correlated, learning about one provides information about the others. The regulator therefore stresses more the factors with correlated exposures. This is true for correlated exposures within a bank as well as correlated exposures across banks. Correlated factors are more systemic and our model predicts that they play an outsized role in scenario design. The regulator may focus mostly on these factors if the correlation is high enough, but, due to the convexity of information sets, specialization is usually incomplete and the design tends to put some weight on all factors.

The optimal design approach in our paper allows us to quantify the value of well designed stress tests. In Section (5), we calibrate our model to U.S. stress tests using quarterly bank-level and macroeconomic data. We illustrate how to apply our framework in the context of credit losses, measured as total net charge offs, and a representative bank. Our calibrated model suggests that endogenous new information from stress testing adds relatively little value to the choice of capital requirements compared to a simple rule based on the average adverse scenario. On the other hand, new information is valuable when the regulator uses targeted interventions. Quantitatively, stress tests with optimally designed scenarios with targeted interventions can achieve an increase in welfare of the same order of magnitude as a 10% decrease in the cost of bank capital.

Our calibration results therefore speak to a recurring debate about the use of stress tests. Are they simply a way of implementing Basel-style requirements? Or are they meant to uncover hidden risks in the financial system? Our results suggests that stress tests act mostly as an implementation rule when it comes to capital requirements. However, when it comes to targeted interventions, information is valuable and scenarios should be designed to elicit new information by deviating from the average bad state.

Literature Review

Most of the literature on stress tests focuses on banking. Several recent papers study specifically the trade-offs involved in disclosing stress test results. Goldstein and Leitner (2018) focuses on the Hirshleifer (1971) effect: revealing too much information destroys risk-sharing opportunities between risk neutral investors and (effectively) risk averse bankers. These risk-sharing arrangements also play an important role in Allen and Gale (2000). Shapiro and Skeie (2015) studies the reputation concerns of a regulator when there is a trade-off between moral hazard and runs. Faria-e-Castro et al. (2017) study a model of optimal disclosure where the government trades off Lemon market costs with bank run costs, and show that a fiscal backstop allows government to run more informative stress tests. Schuermann (2014) discusses the principles behind the design and governance for more effective stress test exercises. Schuermann (2016) analyzes how stress testing in crisis times can be adapted to normal times in order to insure adequate lending capacity and other key financial services. Orlov et al. (2023) looks at the optimal disclosure policy when it is jointly determined with capital requirements, while Gick and Pausch (2014), Inostroza and Pavan (2017), and Williams (2017) do so in the context of Bayesian persuasion. Our model's predictions are consistent with the results in Orlov et al. (2023) that the optimal sequential capital requirements involve a precautionary recapitalization of banks followed by a recapitalization contingent on stress test results. Huang (2021) studies the optimal disclosure in banking networks with potential spillovers and contagion among banks. As argued by Goldstein and Leitner (2020), stress test design and disclosure policy are connected. We complement this strand of papers by explicitly modeling the stress scenario design, which allows us to study the kind of information in the optimal stress test–the relative weight of each factor in the optimal scenario–and not only on how much information it contains.

While most of the existing literature on stress testing, theoretical and empirical, analyzes the disclosure of stress test results, some papers have focused on the risk modeling part of stress testing. For example, Leitner and Williams (2023) focuses on the disclosure of the regulator's risk modeling. The paper examines the trade-offs involved in disclosing the model the regulator uses to perform the stress test to banks. Relatedly, Cambou and Filipovic (2017) focuses on how scenarios translate into losses when the regulator and the banks face model uncertainty. On the banks' risk modeling side, Colliard (2019) and Leitner and Yilmaz (2019) focuses on how a regulator should elicit information from banks when banks can lie or hide information. However, none of these papers consider the optimal scenario design, which is the focus of our paper.

Most empirical papers on stress tests focus on the information content at the time of disclosure, using an event study methodology to determine whether stress tests provide valuable information to investors. Petrella and Resti (2013) finds that the 2011 European stress test exercise provided valuable information to market participants. Similarly, Donald et al. (2014) finds significant abnormal stock returns for banks with capital shortfalls following the 2009 U.S. stress tests. Candelon and Sy (2015), Bird et al. (2015), Fernandes et al. (2015), and Flannery et al. (2017) also find significant average cumulative abnormal returns for stress tested BHCs around many of the stress test disclosure dates. Glasserman and Tangirala (2016) shows that the results of U.S. stress tests are somewhat predictable, in the sense that rankings according to projected stress losses in 2013 and 2014 are correlated. Acharya et al. (2014) compares the capital shortfalls from stress tests with the capital shortfalls predicted using the systemic risk model of Acharya et al. (2016) based on equity market data. Camara et al. (2016) studies the quality of the 2014 EBA stress tests using the actual micro data from the tests. Our model clarifies the role of stress tests. Stress tests provide a natural way to objectify capital requirements, by mandating that banks remain well capitalized under a plausible stress scenario. Quantitatively, however, we find that learning about unknown exposures has a small value for pure capital requirements. On the other hand, we show that learning is valuable when the regulator can engage in targeted interventions.

Finally, our paper is related to the large theoretical literature on information acquisition following Verrecchia (1982), Kyle (1989), and especially Van Nieuwerburgh and Veldkamp (2010). In this class of models, the cost of acquiring information pins down the set of feasible precisions and determines whether the signals are complement or substitutes. Vives (2008) and Veldkamp (Veldkamp) provide a comprehensive review of this literature. These papers take the information processing constraint on the signal precisions as given. In contrast, our paper focuses on the design of the signals that the regulator receives.

The rest of the paper is organized as follows. Section 1 describes the environment. Section 2 describes how the regulator learns from stress test. Sections 3 and 4 characterize the optimal intervention policy and the optimal stress scenarios, respectively. Section 5.1 calibrates the model and Section 6 discusses the practical implications of our analysis and concludes.

1 Technology and Preferences

We consider the problem of a principal who wants to manage the risk exposures of a set of agents. The model has several natural interpretations. The principal could be a chief risk officer and the agents could be traders in her firm. The remedial actions could be hedging or downsizing the traders' positions. Alternatively, the principal could be a regulator and the agents could be a set of banks. The remedial actions could be hedging, reducing new deal flows, selling non-performing assets, or raising capital.

To be concrete we use the regulator/banks metaphor when describing the model. The regulator elicits information from the banks in the form of stress tests. In our model, a stress test is a technology used by regulators to ask questions about profits and losses under hypothetical scenarios. The banks cannot evade the questions and have to answer to the best of their abilities. Banks in our model can only lie by omission: they do not have to volunteer information, but they have to provide estimates of their losses under various scenarios.

1.1 Banks and Risks

There is one regulator overseeing N banks indexed by $i \in [1, .., N]$ exposed to systematic risks. The state of the economy is given by the random vector

$$\mathrm{s} = \begin{bmatrix} s_1 \\ \vdots \\ s_J \end{bmatrix},$$

where $\{s_j\}$ are the J macroeconomic risk factors. To simplify the analysis, we normalize the unconditional expectation of s to zero $\mathbb{E}[\mathrm{s}] = 0$. Therefore, the realization of the state s should be interpreted as a deviation from the baseline.[3] The net worth of bank i in state s (bank capital) is

[3]Stress scenarios rely on traditional macroeconomic variables such as GDP, unemployment, and house prices. In a DSGE model these macro variables would themselves be functions of underlying structural shocks such as productivity and risk aversion. Formally, let ϵ^s be the structural shocks and H the solution matrix of the DSGE model, so that $\mathrm{s} = H\epsilon^s$. In a fully specified model, banks' losses would also be functions of the structural shocks: $y_i(\epsilon^{\mathrm{s}}) = \tilde{\mathrm{x}}_{\mathrm{i}}' \epsilon^{\mathrm{s}} + \eta_i$, where $\tilde{\mathrm{x}}_i$ are structural exposures. This equation is equivalent to (2) when H is invertible: $\epsilon^s = H^{-1}\mathrm{s}$ and with $\mathrm{x}_i = H'^{-1}\tilde{\mathrm{x}}_i$ we obtain $y_i(\mathrm{s}) = \mathrm{x}_i'\mathrm{s} + \eta_i$. In practice regulators specify directly the macro variables s to avoid model ambiguity, since H would likely differ among banks, and between banks and regulators.

given by

$$w_i\left(\mathrm{s}\right)=\overline{w}_i-y_i\left(\mathrm{s}\right), \tag{1}$$

where $\overline{w}_i$ is the initial capital and $y_i\left(\mathrm{s}\right)$ represents the bank's cumulative losses if state s is realized. Bank i's losses in state s are given by

$$y_i\left(\mathrm{s}\right)=\sum_{j=1}^{J}x_{i,j}s_j, \tag{2}$$

where $x_{i,j}$ represents the exposure of bank i to factor j.[4] We summarize a bank i's risk exposures in the $J\times 1$ vector x_i and stack the banks' exposures vectors $\{\mathrm{x_i}\}$ in one large $NJ\times 1$ vector $\mathbf{x}$, as follows

$$\mathrm{x}_i=\begin{bmatrix} x_{i,1}\\ \vdots \\ x_{i,J}\end{bmatrix} \quad \text{and} \quad \mathbf{x}\equiv\begin{bmatrix} \mathrm{x}_1\\ \vdots \\ \mathrm{x}_N\end{bmatrix}.$$

The risk exposures in $\mathbf{x}$ are not observed by anyone. We can add bank-specific shocks to Equation (2) but these are not important for our analysis. The aggregate capital of the banking system is

$$W\left(\mathrm{s}\right)\equiv\sum_{i=1}^{N}w_i\left(\mathrm{s}\right)=\overline{W}-\sum_{i=1}^{N}y_i\left(\mathrm{s}\right), \tag{3}$$

where $\overline{W}=\sum_{i=1}^{N}\overline{w}_i$. In what follows, we assume that risk factors and exposures are independent of each other.

1.2 Regulator's Preferences and Interventions

Following Acharya et al. (2016) we assume that the regulator has preferences $U\left(W\right)$ over the aggregate net worth of the banking system.[5] The regulator can affect W in two ways. She can

[4] We use the term "exposure" to denote the relevant elasticity that determines losses under a given realization of the macroeconomic state. Banks and regulators usually agree on the book value of positions. They can disagree about the value of illiquid positions, and in all cases, liquid or not, the impact of a scenario on the loss on that position needs to be estimated. What we call "exposure" combines the position (measured with near certainty in some cases) with its value under stress scenarios (estimated with error).

[5] The general case is $U\left(\left[w_i\right]_{1..N}\right)$, where the idiosyncratic failure of bank i matters regardless of the health of the banking sector as a whole. As in the systemic risk literature, we assume here that only $W=\sum_{i=1}^{N}w_i$ matters. As a result, a financial crisis only happens when the financial system as a whole is under-capitalized. See Philippon and Wang (2021) for a proof of how transfers of assets from under- to well-capitalized banks transform the value function $U\left(\left[w_i\right]_{1..N}\right)$ into $U\left(W\right)$.

impose capital requirements to set $\overline{W}$. She can also intervene to change banks' exposures to specific risks. These targeted interventions include capital and collateral requirements against specific types of loans or specific borrowers (e.g., LTV ratios in commercial real estate), assets sales and divestitures, as well as supervisory communications on matters requiring (immediate) attention (MRA and MRIA).

The most granular description of interventions is at the bank×factor level. In some cases, however, a targeted intervention would affect exposures to several factors. We discuss in detail how we model these constraints in Section 3. We denote the actions as an $NJ \times 1$ vector $\mathbf{a}$ in some feasible set $\mathcal{A} \subset [0,1]^{NJ}$ with the understanding that higher actions reduces exposure more: the vector x_i becomes $(\mathbf{1}_{J\times 1} - \mathbf{a}_i) \circ \mathrm{x}_i$ where $\mathbf{a}_i$ is a $J \times 1$ vector that represents the set of actions taken on bank i. Interventions are costly. We let $\mathcal{C}(\mathbf{a})$ denote the cost of action $\mathbf{a}$. Similarly we let $\mathcal{K}\left(\overline{W}\right)$ denote the cost of bank capital.

Given the regulator's actions $\left\{\overline{W}, \mathbf{a}\right\}$ the aggregate wealth of the banking sector is given by

$$W\left(s, x; \mathbf{a}, \overline{W}\right) = \overline{W} - \mathrm{s} \cdot \left(\sum_{i=1}^{N} (\mathbf{1}_{J\times 1} - \mathbf{a}_i) \circ \mathrm{x}_i, \right), \tag{4}$$

where $\circ$ denotes the Hadamard product.

1.3 Prior beliefs and stress tests

The regulator has prior beliefs over the distribution of exposures within banks and across banks. These prior beliefs come from historical experiences and the regulator's own risk models. We stack the banks' exposures vectors $\{\mathrm{x_i}\}$ in one large $NJ \times 1$ vector as follows

$$\mathbf{x} \equiv \begin{bmatrix} \mathrm{x}_1 \\ \vdots \\ \mathrm{x}_N \end{bmatrix}.$$

We assume that the regulator's prior over the vector of exposures $\mathbf{x}$ is given by

$$\mathbf{x} \sim N\left(\overline{\mathbf{x}}, \Sigma_{\boldsymbol{x}}\right),$$

where the $NJ \times 1$ vector of unconditional means and the $NJ \times NJ$ covariance matrix are, respectively,

$$\overline{\mathbf{x}} = \begin{pmatrix} \overline{\mathrm{x}}_1 \\ \vdots \\ \overline{\mathrm{x}}_N \end{pmatrix} \quad \text{and } \Sigma_{\boldsymbol{x}} = \begin{bmatrix} \Sigma_{\boldsymbol{x}}^1 & \Sigma_{\boldsymbol{x}}^{1,2} & \cdots & \Sigma_{\boldsymbol{x}}^{1,N} \\ \Sigma_{\boldsymbol{x}}^{1,2} & \Sigma_{\boldsymbol{x}}^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \Sigma_{\boldsymbol{x}}^{(N-1),N} \\ \Sigma_{\boldsymbol{x}}^{1,N} & \cdots & \Sigma_{\boldsymbol{x}}^{(N-1),N} & \Sigma_{\boldsymbol{x}}^N \end{bmatrix}$$

where $\Sigma_{\boldsymbol{x}}^i = \mathbb{V}\mathrm{ar}\,(\mathrm{x}_i)$ is the $J \times J$ covariance of exposures of bank i, and $\Sigma_{\boldsymbol{x}}^{i,h} = \mathbb{C}\mathrm{ov}\,(\mathrm{x}_i, \mathrm{x}_h)$ for all $i \neq h$ is the covariance of exposures across banks.[6] If $\Sigma_{\boldsymbol{x}}^i$ is diagonal the regulator expects the exposures of bank i to the different factors to be independent of each other. If $\Sigma_{\boldsymbol{x}}^{i,h} = 0$, the regulator's prior is that the risk exposures of banks i and h are independent. In almost all empirically relevant cases the covariance matrices are not diagonal.

The regulator uses stress tests to learn about the banks' risk exposures and improve the accuracy of her intervention. In a stress test, the regulator asks the banks to estimate and report their losses under a particular realization of the future macroeconomic state. We assume that banks must truthfully report their expected losses (see discussion below). The choice of macroeconomic state is a *scenario* $\hat{s}$.

Definition 1. (Scenario) A scenario $\hat{\mathrm{s}} = (\hat{s}_1, \ldots, \hat{s}_J)'$ is a realization of the vector of states s.

A scenario $\hat{s}$ is a column-vector of size J that represents an aggregate state of the economy. The simplest way to think of J is as exogenously given. We entertain two interpretations of the size of the state space, J. There might be a limited number of macroeconomic variables (GDP, unemployment, house prices) that everyone agrees need to be included in the test. The other way to think about J is as a large number capturing the set of all possible risk factors and in any given tests many have zero loadings. A non-zero weight is then a statement about whether that risk factor is included in the particular stress test. Our model can also then shed light on which risk factors should be used.

Given our normalization of the baseline state to $s = 0$, a scenario close to 0 is a scenario close to the baseline of the economy. A scenario $\hat{s}$ in which element $\hat{s}_j$ is large, represents a large

[6] We assume that the risk exposures of banks are normally distributed in order to apply the Kalman filter. Technically, therefore, it can happen that $x < 0$ but, as usual, we choose parameters to ensure that this is a negligible possibility.

deviation from the baseline along the dimension of factor j. The larger $|\hat{s}_j|$, the more extreme the scenario along dimension j.

Definition 2. (Stress test) A stress test is a collection of M scenarios $\{\hat{\mathrm{s}}^m\}_{m=1}^M$ presented by the regulator and a collection of estimated losses $\{\hat{y}_i^m\}_{i=1..N}^{m=1..M}$ reported by the banks.

When designing a stress test, the regulator specifies a *set* of scenarios for which the banks need to report their losses. For each scenario m, each bank i estimates and reports its net losses $\hat{y}_i^m$ given the input parameters in scenario $\hat{\mathrm{s}}^m$.

1.4 Stress test results

Banks use imperfect models to predict their losses under the stress test scenarios. Bank i's estimated loss under scenario $\hat{\mathrm{s}}^m$ is

$$\hat{y}_i\left(\hat{\mathrm{s}}^m, M\right) = \hat{\mathrm{s}}^m \cdot \mathrm{x}_i + \hat{\epsilon}_{i,m}\left(\|\hat{\mathrm{s}}^m\|, M\right), \tag{5}$$

where the error term $\hat{\epsilon}_i\left(\|\hat{\mathrm{s}}\|, M\right)$ is a random variable that captures measurement error and model uncertainty. When a stress test contains multiple scenarios, the results of the stress test reported by one bank i are summarized in the $M \times 1$ vector

$$\hat{\mathrm{y}}_i\left(\hat{S}\right) = \hat{S}\,\mathrm{x}_i + \hat{\varepsilon}_i, \tag{6}$$

where $\hat{\mathrm{y}}_i\left(\hat{S}\right)$ represents the estimated reported losses for bank i under the $M \times J$ matrix $\hat{S}$ that gathers the M scenarios, and the errors in bank i's reported losses are gathered in the $M \times 1$ vector $\hat{\varepsilon}_i$, i.e.,

$$\hat{\mathrm{y}}_i\left(\hat{S}\right) = \begin{bmatrix} \hat{y}_i\left(\hat{\mathrm{s}}^1, M\right) \\ \vdots \\ \hat{y}_i\left(\hat{\mathrm{s}}^M, M\right) \end{bmatrix}, \quad \hat{S} \equiv \begin{bmatrix} \left(\hat{\mathrm{s}}^1\right)' \\ \vdots \\ \left(\hat{\mathrm{s}}^M\right)' \end{bmatrix}, \quad \text{and} \quad \hat{\varepsilon}_i = \begin{bmatrix} \hat{\epsilon}_{i,1}\left(\left\|\hat{\mathrm{s}}^1\right\|, M\right) \\ \vdots \\ \hat{\epsilon}_{i,M}\left(\left\|\hat{\mathrm{s}}^M\right\|, M\right) \end{bmatrix},$$

where $\hat{\varepsilon}_i$ is a normally distributed random vector with mean zero and variance-covariance $\mathbf{\Sigma}_{\hat{\varepsilon}}^i \equiv \mathbb{V}ar\left[\hat{\varepsilon}_i\right]$, and $\hat{\varepsilon}_i \perp \mathrm{x}_j$ for all i, j. We assume that $\mathbb{V}ar\left(\hat{\epsilon}_i\left(\|\hat{\mathrm{s}}\|, M\right)\right)$ is increasing in the norm of the scenarios $\|\hat{\mathrm{s}}\|$. This assumption is consistent with parameter uncertainty (Barberis, 2000). In a linear world, if banks project their losses using time series regressions $y_t = y_0 + x s_t + e_t$ (assuming that y_0 is known for simplicity), the variance of the projection is $Var\left(\hat{x}\right)\hat{s}^2$, which grows with the squared norm. Since stress tests focus on tail events where non linearities are important, we assume that errors grow more quickly than in the linear case. Formally, $\mathbb{V}ar\left(\hat{\epsilon}_i\left(\|\hat{\mathrm{s}}\|, M\right)\right)$ is continuous and $\lim_{\hat{s}_j \to \infty} \frac{(\hat{s}_j)^2}{\mathbb{V}ar(\hat{\epsilon}_i(\|\hat{\mathrm{s}}\|, M))} = 0$ for all m and for all j. This turns out to be enough to guarantee an interior solution of the scenario choice problem.[7] Moreover, since banks rely on the same historical data, they are likely to make correlated mistakes, captured by the non-diagonal elements of $\boldsymbol{\Sigma}^i_{\hat{\varepsilon}}$.

Remark 1. **(Truthful reporting)** We assume throughout the paper that banks truthfully report their expected losses to regulators for several reasons. A pragmatic reason is that the model without incentives to lie is already rich and complex to understand. Another reason is that actively lying to regulators is unlikely to be a good or even feasible strategy. Running stress tests is a massive undertaking involving many employees and it would be difficult for the top management to convince all of them to deliberately mislead their regulator without getting caught. In addition the regulator can compare results across banks. As long as banks do not collude, the regulator can spot a bank that produces an excessively optimistic loss estimate.

The more relevant issues are lying by omission and statistical bias. The fact that banks can lie by omission is actually supportive of our model. Banks do not need to answer questions that are not asked and this reinforces the importance of designing the right scenarios. Statistical bias is a different matter. Each risk unit within the bank would accept at face value results that look fine, but would have an incentive to investigate and modify a model that delivers a large predicted loss. This issue is similar to p-hacking in academia. While we agree that this is a real concern, it does not necessarily invalidate our approach. The average bias drops out in a rational expectation equilibrium since the regulator can always infer it (Holmström, 1999). What is left is noise, as assumed in the paper. This does not mean that inducing truthful reporting is not important, but it suggests that our approach is useful even in a world of imperfect reporting.

1.5 Timing

To summarize, there are three stages in our model: the scenario design stage, the stress testing stage, and the intervention stage. The regulator first chooses the stress scenarios. The regulator then extract information from the results of the test. The regulator finally chooses her targeted

[7] Formally we assume that the error term variance increases in the number of scenarios but this is not essential for our results.

Scenario Design	Stress Testing	Intervention
Regulator chooses stress scenarios	Banks report stress test results	Regulator chooses capital requirements and direct actions
$\hat{S}$	$\hat{\mathbf{y}}\left(\hat{S}\right)$	$\overline{W}^{\star}, \mathbf{a}^{\star}\left(\hat{\mathbf{y}}\right)$

Figure 1: Timeline

interventions and capital requirements. Figure (1) shows the timeline of the model. The regulator takes into account that her choices affect the informativeness of the test and the efficiency of her interventions.

1.6 Regulator's Problem

The regulator designs the stress test scenarios to maximize her expected utility taking into account her interventions after observing the stress test results will be optimal. Therefore, we can divide the regulator's problem into two: the optimal intervention choice and the optimal scenario choice.

Given her information set $\mathscr{S} = \left\{\{\hat{\mathrm{y}}_i\}_{i=1..N}, \hat{S}\right\}$ after observing the stress test results–her posterior beliefs over the banks' risk exposures–the regulator chooses $\left(\overline{W}, \mathbf{a}\right)$ to maximize her expected utility net of intervention costs

$$\mathbb{E}\left[U\left(W\left(s, x; \mathbf{a}, \overline{W}\right)\right)\middle|\mathscr{S}\right] - \mathcal{C}\left(\mathbf{a}\right) - \mathcal{K}\left(\overline{W}\right), \tag{7}$$

where the aggregate wealth of the banking sector is given by Equation (4). The outcome of this problem consists of optimal policy functions for the interventions as a function of the information set $\mathscr{S}$, i..e, $\left(\overline{W}^{\star}\left(\mathscr{S}\right), \mathbf{a}^{\star}\left(\mathscr{S}\right)\right)$. Taking into account these choices, the regulator chooses the stress scenarios $\hat{S}$ to maximize her ex-ante expected utility given by

$$\mathbb{E}_{\mathscr{S}}\left[\mathbb{E}\left[U\left(W\left(s, x; \mathbf{a}^{\star}\left(\mathscr{S}\right), \overline{W}^{\star}\left(\mathscr{S}\right)\right)\right)\middle|\mathscr{S}\right] - \mathcal{C}\left(\mathbf{a}^{\star}\left(\mathscr{S}\right)\right) - \mathcal{K}\left(\overline{W}^{\star}\left(\mathscr{S}\right)\right)\right], \tag{8}$$

where the first expectation $\mathbb{E}_{\mathscr{S}}$ is taken over the distribution of stress test results, which in turn depend on the stress scenarios. In the next section, we explain in detail the link between the stress test results and the posterior beliefs of the regulator over the banks' risk exposures. With this characterization in hand, we then solve for the regulator's problem in two parts as described above.

2 Learning

The bank's model summarized in Equation (6) implies that reported losses contain information about the true vector of exposures $\mathbf{x}$. The Bayesian regulator updates her prior beliefs before deciding on her interventions. In our linear Gaussian setting, the regulator's learning can be expressed as a filtering problem in which the regulator's optimal updating is given by the Kalman filter.[8] Moreover, in this case, the regulator's posterior distribution over risk exposures is summarized by the posterior mean $\hat{\mathbf{x}}$ and the posterior variance $\hat{\Sigma}_{\mathbf{x}}$.

2.1 A Kalman Filter

To interpret the stress test results as signals about the banks' risk exposures it is useful to define the reported losses and error terms in the $NM \times 1$ vectors

$$\hat{\mathbf{y}} \equiv \begin{bmatrix} \left[\hat{y}_1\left(\hat{S}\right)\right] \\ \vdots \\ \left[\hat{y}_N\left(\hat{S}\right)\right] \end{bmatrix} \quad \text{and} \quad \hat{\boldsymbol{\varepsilon}} = \begin{bmatrix} \left[\hat{\varepsilon}_1\right] \\ \vdots \\ \left[\hat{\varepsilon}_N\right] \end{bmatrix}.$$

Then, using these definitions, the state-space representation of the reported losses is

$$\hat{\mathbf{y}} = \hat{\mathbf{S}}\mathbf{x} + \hat{\boldsymbol{\varepsilon}}, \tag{9}$$

where $\hat{\mathbf{S}} \equiv \left(\mathbf{I}_N \otimes \hat{S}\right)$ simply repeats $\hat{S}$ on its diagonal, and $\hat{\boldsymbol{\varepsilon}} \sim N\left(0, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\varepsilon}}}\right)$. This formulation captures one of our key insights that the stress tests results $\hat{\mathbf{y}}$ can be interpreted as signals about the banks' risk exposures $\mathbf{x}$. Equation (9) shows that the stress test scenarios determine the structure of the signals observed by the regulator. First, the scenarios $\hat{\mathbf{S}}$ control the weight of each exposure in the reported losses. Second, the scenarios also determine the precision of the banks' reported losses in Equation (5). Increasing $|s_j|$ in a scenario makes the results more informative about exposures to factor j, but extreme scenarios reduce the precision of the banks' estimates

[8]See Chapter 2 in Veldkamp (Veldkamp) for a textbook analysis of the Kalman filter.

and the noise might spill over to the measurement of other exposures. Moreover, expressing the stress test results as in equation (9) allows us to apply the Kalman filter and to obtain a full characterization of the posterior beliefs of the Bayesian regulator.

Lemma 1. ***Kalman Filter.*** *After observing the results* $\hat{\mathbf{y}}$ *of the stress test, the posterior beliefs of the regulator regarding the banks' risk exposures are*

$$\mathbf{x}|\,\hat{\mathbf{y}} \sim N\left(\hat{\mathbf{x}}, \hat{\Sigma}_{\mathbf{x}}\right) ,$$

where the posterior mean $\hat{\mathbf{x}}$ *and the posterior covariance matrix* $\hat{\Sigma}_{\mathbf{x}}$ *are given by*

$$\hat{\mathbf{x}} = \left(\mathbf{I}_{NJ} - K\hat{\mathbf{S}}\right)\bar{\mathbf{x}} + K\hat{\mathbf{y}} \quad \textit{and} \tag{10}$$

$$\hat{\Sigma}_{\mathbf{x}} = \Sigma_{\mathbf{x}} - K\hat{\mathbf{S}}\Sigma_{\mathbf{x}}\,, \tag{11}$$

where the $NJ \times MN$ *matrix* $K = \Sigma_{\mathbf{x}}\hat{\mathbf{S}}'\left(\hat{\mathbf{S}}\Sigma_{\mathbf{x}}\hat{\mathbf{S}}' + \mathbf{\Sigma}_{\hat{\varepsilon}}\right)^{-1}$ *is the Kalman gain.*

Lemma 1 is a direct application of the Kalman filter and it shows the link between the stress scenarios and the informational content of stress tests through the Kalman gain K. Equation (10) shows that K captures how much information the stress test results $\hat{\mathbf{y}}$ have about the bank exposures. For example, if there is only one scenario, an entry $K_{1,2} = 0$ implies that the stress test result of bank 2 does not contain any information about the exposure of bank 1 to factor 1. In this case, the posterior $\hat{x}_{1,1}$ is independent of $\hat{y}_2(\hat{\mathrm{s}})$. On the other hand, when $K_{1,2}$ is high, $\hat{x}_{1,1}$ responds a lot to $\hat{y}_2(\hat{\mathrm{s}})$, which implies that the stress test result of bank 2 is quite informative about the exposure of bank 1 to factor 1, $x_{1,1}$.

At the same time, the Kalman gain K represents how much can be learned from the stress test. To see this, note that the extent to which learning takes place is captured *ex ante* by the *distribution of the posterior mean*, given by

$$\hat{\mathbf{x}} \sim N\left(\bar{\mathbf{x}}, \Sigma_{\hat{\mathbf{x}}}\right), \tag{12}$$

where the *expected variance of the posterior mean*, $\Sigma_{\hat{\mathbf{x}}}$, is given by

$$\Sigma_{\hat{\mathbf{x}}} \equiv \Sigma_{\mathbf{x}} - \hat{\Sigma}_{\mathbf{x}} = K\hat{\mathbf{S}}\Sigma_{\mathbf{x}}.$$

The posterior covariance matrix $\hat{\Sigma}_{\mathbf{x}}$ measures the *residual uncertainty* that persists after observing the results of the stress test. Hence, The matrix $\Sigma_{\hat{\mathbf{x}}}$ represents the expected amount of learning from the stress test, which depends on $K\hat{\mathbf{S}}$. If the stress test is pure noise, $K = 0$, the regulator learns nothing, $\Sigma_{\hat{\mathbf{x}}} = 0$, and $\hat{\Sigma}_{\mathbf{x}} = \Sigma_{\mathbf{x}}$. If the test is fully informative, then $\hat{\Sigma}_{\mathbf{x}} = 0$ and the regulator learns exactly all the exposures, i.e., $\Sigma_{\hat{\mathbf{x}}} = \Sigma_{\mathbf{x}}$.

Note from Lemma 1 that the Kalman gain K depends on the scenario choice directly and through the variance covariance matrix of $\hat{\varepsilon}$. When choosing the optimal stress scenario, the regulator takes into account this dependence. Moreover, given the ex-ante distribution of the posterior mean in (12), the extent of the regulator's lending from stress tests depends on K only through the posterior variance $\hat{\Sigma}_{\mathbf{x}}$, which plays a critical role in our analysis as we describe in the next section.

Remark 2. **(Endogenous noise)** If $\sigma_{\hat{\epsilon}}$ is exogenous, then learning is trivially maximized by sending $\hat{s}_1 \to \infty$. In reality extreme scenarios are more difficult to estimate. This is why we assume that $\sigma_{\hat{\epsilon}}$ increases when the scenario deviates more from the baseline. We obtain an interior solution as long as $\sigma_{\hat{\epsilon}}$ is convex enough in $\|\hat{s}\|$.

2.2 Feasible Information Sets

Every set of stress scenarios $\hat{S}$ delivers a unique posterior covariance matrix. More precisely, the Kalman filter provides a mapping of $\frac{JN(JN-1)}{2}$ equations from the $J \times M$ elements in the scenario matrix $\hat{S}$ into the elements of $\Sigma_{\hat{\mathbf{x}}}$. Choosing a set of scenarios $\hat{S}$ is therefore equivalent to choosing a posterior covariance matrix $\hat{\Sigma}_{\mathbf{x}}$ in the set $\mathbf{\Sigma}$ given by Equation (11). [9] The shape of the feasibility set $\mathbf{\Sigma}$ is determined by the regulator's priors $\Sigma_{\boldsymbol{x}}$ and by the errors in banks' models $\mathbf{\Sigma}_{\hat{\boldsymbol{\varepsilon}}}$.

We can therefore think of the scenario design problem as choosing an information point in a feasible set $\mathbf{\Sigma}$ determined by the Kalman filter. Given our assumptions on the error term structure, perfect learning is not feasible, i.e., the budget set does not include a posterior variance of zero. This is because choosing a more extreme scenario has two effects on the amount of information that the regulator can acquire. On the one hand, a higher value of $\hat{s}_i$ increases the weight the bank's stress test results put on the bank's exposure to factor i. On the other hand, more extreme scenarios increase the noise $\mathbf{\Sigma}_{\hat{\boldsymbol{\varepsilon}}}$ in the stress tests result.

[9] When the number of banks is higher than the number of scenarios ($N > M$), the design problem boils down to choosing residual variances about the risk exposures of any M banks since it is equivalent for the regulator to choose the stress scenarios or to choose $J \times M$ elements of the residual covariance matrix.

Prior correlation among risk exposures makes it easier to learn and reduces the posterior variances. When exposures are correlated, the regulator cannot learn about the bank's exposure to a factor without learning about the bank's exposure to other factors. Hence, as it can be seen from Figure B.1 in the Appendix, the boundary of set of feasible posterior precisions, $\mathbf{\Sigma}$, becomes more convex the higher the prior correlation between exposures.

3 Taking Action

The regulator values information from the stress test because it allows her to intervene more accurately. In return, the design of optimal scenarios depends on the actions that the regulator expects to take. Regulators typically have two ways of intervening in the banking sector. They can mandate a broad increase in capital, or they can restrict specific activities, for instance by imposing loan-to-value ratios or collateral requirements.

In our model, broad interventions are captured by the capital required by the regulator $\overline{W}$ and the restriction of specific activities is captured by targeted interventions $\mathbf{a} = \{\mathrm{a}_i\}_{i=1,\ldots,N} = \left\{\{a_{i,j}\}_{j=1..J}\right\}_{i=1,\ldots,N}$, which are bank and risk factor specific. As we discuss in Section (1.2), the regulator chooses actions $\left\{\overline{W}, \mathbf{a}\right\}$ to maximize her expected utility given her information set $\mathscr{S} = \left\{\hat{\mathbf{y}}, \hat{S}\right\}$.

In an interior solution, the first order conditions equate the marginal cost of an intervention to its expected marginal benefit. For capital requirements we obtain

$$\mathcal{K}'\left(\overline{W}\right) = \mathbb{E}\left[U'\left(W\right) \mid \mathscr{S}\right]. \tag{13}$$

Since capital is useful in all states of the world the optimality condition simply states that the marginal cost of banking capital be equal to the expected marginal utility of banking net worth. Similarly, the optimal targeted intervention on the exposure of bank i to factor j is

$$\frac{\partial \mathcal{C}\left(\mathbf{a}\right)}{\partial a_{i,j}} = \mathbb{E}\left[x_{i,j} s_j U'\left(W\right) \mid \mathscr{S}\right]. \tag{14}$$

The expected marginal benefit of reducing the risk exposure to factor j in bank i depends on the covariance between the marginal social utility $U'\left(W\right)$ and the contribution of factor j to bank i's losses, $x_{i,j}s_j$. Risk reduction is more valuable when the regulator expects high losses in states of the world where U' is also large.

3.1 Pseudo Mean-Variance Preferences

The first order conditions (13) and (14) characterize the regulator's solution for a *given* information set. The key point of our model, however, is that the information set is endogenous. The FOCs thus represent only an intermediate step in the regulator's overall optimization problem. Working with a generic utility function renders the analysis quite intractable. To make progress we use the following approximation.

We assume that we can partition the realizations of the shocks s into two separate regions. With probability $1-p$ they land in the "normal" region where utility is linear and marginal utility is (normalized) to 1: $U'(W) = 1$. With probability p they land in the distress region around a bank value of $\tilde{W}$, and we use a second order approximation for the marginal utility:

$$U'(W) = \begin{cases} 1 & \text{with probability} \quad 1-p, \\ 1+\theta-\gamma\left(W-\tilde{W}\right) & \text{with probability} \quad p, \end{cases} \tag{15}$$

where $\theta \equiv U'\left(\tilde{W}\right) - 1 > 0$ and $\gamma \equiv -U"\left(\tilde{W}\right) > 0$. With a slight abuse we say that these preferences are *linear quadratic*. For now it is enough that $\tilde{W}$ be in the distress region. Later, to focus on the incremental value of stress testing, we will define $\tilde{W}$ as the expected value in distress conditional on optimal actions but without information.

For ease of notation we define $\tilde{\mathbb{E}}$ as the expectation conditional on the economy being in distress, i.e., $\tilde{\mathbb{E}}[.] \equiv \mathbb{E}[. \mid \text{Distress}]$. Since we have normalized $\mathbb{E}[s] = 0$ we must have $\tilde{\mathbb{E}}[s] > 0$, and we define $\tilde{\Sigma}_{\text{s}} \equiv \mathbb{V}ar\left[\text{s}\right|\text{Distress}]$. We can then write the expected utility of the regulator as

$$\mathbb{E}[U(W)] = (1-p)\,\mathbb{E}[W \mid \text{NoDistress}] + p\left(U\left(\tilde{W}\right) + (1+\theta)\,\tilde{\mathbb{E}}\left[W-\tilde{W}\right] - \frac{\gamma}{2}\tilde{\mathbb{E}}\left[\left(W-\tilde{W}\right)^2\right]\right). \tag{16}$$

Since $\mathbb{E}[\text{s}] = 0$ we see from (4) that $\mathbb{E}[W] = \overline{W}$. Therefore, with linear quadratic preferences the regulator's intervention problem is equivalent to

$$\max_{\overline{W},\mathbf{a}\in\mathcal{A}} \overline{W} + p\left(\theta\tilde{\mathbb{E}}\left[W-\tilde{W} \mid \mathscr{S}\right] - \frac{\gamma}{2}\tilde{\mathbb{E}}\left[\left(W-\tilde{W}\right)^2 \mid \mathscr{S}\right]\right) - \mathcal{K}\left(\overline{W}\right) - \mathcal{C}(\mathbf{a}), \tag{17}$$

with $W = \overline{W} - \sum_{i=1}^{N}\sum_{j=1}^{J}(1-a_{i,j})\,x_{i,j}s_j$. The first order condition for the optimal capital requirement in Equation (13) becomes

$$\mathcal{K}'\left(\overline{W}^{\star}\right) = 1 + p\theta + p\gamma\left(\tilde{W} + \sum_{i=1}^{N}\sum_{j=1}^{J}(1-a_{i,j})\,\hat{x}_{i,j}\tilde{\mathbb{E}}[s_j] - \overline{W}^{\star}\right) \tag{18}$$

and the first order conditions for optimal actions in Equation (14) are

$$\frac{\partial \mathcal{C}(\mathbf{a})}{\partial a_{i,j}} = p\theta \hat{x}_{i,j} \tilde{\mathbb{E}}[s_j] + p\gamma \tilde{\mathbb{E}}\left[x_{i,j} s_j \left(\tilde{W} - W\right) \mid \mathscr{S}\right]. \tag{19}$$

Capital requirements are increasing in risk aversion γ, in the probability of distress p, and in the estimated risk exposures $\hat{x}$. Capital requirements are mitigated by targeted interventions. For the remainder of the paper we assume the following functional forms.[10]

Assumption L. *The cost of bank capital and the cost of targeted actions are linear, i.e.,*

$$\mathcal{K}\left(\overline{W}\right) = (1+\kappa)\,\overline{W}, \quad \text{and} \quad \mathcal{C}(\mathbf{a}) = \Phi' \mathbf{a},$$

with $\kappa > p\theta$.

3.2 Pure Capital Requirement

To understand the connection between our optimal capital requirement and actual stress tests it is useful to first consider first a model *without* targeted interventions since actual stress tests are conducted "all else equal", i.e., assuming no actions from the regulator. Let us then assume $\mathbf{a} = 0$. We obtain the following lemma.

Lemma 2. ***(Pure capital requirement,*** $\mathbf{a} = 0$***)*** *Optimal capital requirements are a linear function of expected losses under distress:*

$$\overline{W}^{\star} = \sum_{i=1}^{N} \mathbb{E}\left[y_i \mid \textit{Distress}, \mathscr{S}\right] + \tilde{W} - \frac{\kappa - p\theta}{p\gamma}, \tag{20}$$

where $\mathbb{E}\left[y_i \mid \textit{Distress}, \mathscr{S}\right] = \tilde{\mathbb{E}}[\mathrm{s}] \cdot \mathbb{E}[\mathrm{x_i} \mid \mathscr{S}] = \sum_{i=1}^{N} \sum_{j=1}^{J} \hat{x}_{i,j} \tilde{\mathbb{E}}[s_j]$.

The net marginal cost of bank equity $\mathcal{K}'\left(\overline{W}^{\star}\right) - 1$ is compared to the adjusted risk of distress $p\gamma$. Net of this cost, capital requirements are set to cover losses in the adverse scenario. It is important to understand the similarities and the differences between our Lemma 2 and what regulators do in practice. Exactly as in standard stress tests, our model says that the requirements

[10] We assume linear costs for simplicity since the utility function is concave. All results are valid qualitatively with $\mathcal{C}(\mathbf{a}) = \Phi_0' \mathbf{a} + \frac{1}{2} \mathbf{a}' \Phi_1 \mathbf{a}$ or with a convex cost of capital. Also note that Assumption L only needs to hold over the relevant range.

should be set to cover losses under an adverse scenario. The adverse scenario in our model is the expectation of the state conditional on distress, $\tilde{\mathbb{E}}[\mathrm{s}]$.

The main difference is that, in our model, the regulator uses expected exposures from the Kalman filter $\mathbb{E}[\mathrm{x_i} \mid \mathscr{S}]$. In general, therefore $\mathbb{E}[y_i \mid \text{Distress}, \mathscr{S}] \neq \hat{y}_i\left(\tilde{\mathbb{E}}[\mathrm{s}]\right) = \tilde{\mathbb{E}}[\mathrm{s}] \cdot \mathrm{x}_i + \hat{\epsilon}_i(\|\tilde{\mathrm{s}}\|)$: the optimal forecast of losses under the adverse scenario differ from the losses the bank would report under the adverse scenario. Without noise, of course, $\hat{y}_i\left(\tilde{\mathbb{E}}[\mathrm{s}]\right) = \tilde{\mathbb{E}}[\mathrm{s}] \cdot \mathrm{x}_i$ would be exact. Given unavoidable measurement errors and model mis-specifications, however, the regulator optimally uses reported losses from *other* scenarios ($\hat{\mathrm{s}} \neq \tilde{\mathrm{s}}$) and from *other* banks ($j \neq i$) to improve their estimate of $\hat{\mathrm{x}}_\mathrm{i} = \mathbb{E}[\mathrm{x_i} \mid \mathscr{S}]$.

Remark 3. **(Unconditional capital requirements)** Our mean-variance assumption implies a form of certainty equivalence. The *expected* level of capital requirements depends only on the regulator's priors:

$$\mathbb{E}_{\hat{x}}\left[\overline{W}^{\star}\right] = \sum_{i=1}^{N} \tilde{\mathbb{E}}[\mathrm{s}] \cdot \bar{\mathrm{x}}_\mathrm{i} + \tilde{W} - \frac{\kappa - p\theta}{p\gamma}. \tag{21}$$

We can therefore implement the optimal capital requirement in two steps. The regulator sets unconditional requirements based on her priors as in (21) and then makes (mean-zero) adjustments based on the result of the test. The adjustments are potentially valuable because they allow the regulator to tighten requirements when exposures are high and loosen them when they are low. This result is consistent with Orlov et al. (2023) that argues that the optimal sequential stress test consists of a precautionary recapitalization (our expected capital requirement) followed by contingent recapitalizations based on an informative stress test, as needed (our adjustments based on the stress test results). We will show, however, that these improvements are quantitatively small.

3.3 Optimal Interventions and the Distress Uncertainty Matrix

We now explain how targeted interventions depend on average posterior exposures and on a specific covariance matrix. We solve for the optimal interventions $\left(\overline{W}, \mathbf{a}\right)$ by first solving for the optimal capital requirement as a function of the targeted interventions, $\overline{W}^{\star}(\mathbf{a})$ and then solving for $\mathbf{a}$. The first order condition for optimal capital requirement from Equation (18) implies that the expected shortfall $\tilde{\mathbb{E}}\left[W-\tilde{W}\right]=\frac{\kappa-p\theta}{p\gamma}$ is constant. The regulator's problem therefore simplifies to

$$\min_{\mathbf{a}\in\mathcal{A}}\kappa\overline{W}^{\star}\left(\mathbf{a}\right)+\mathcal{C}\left(\mathbf{a}\right)+\frac{p\gamma}{2}\tilde{\mathbb{E}}\left[\left(\frac{\kappa-p\theta}{p\gamma}+\sum_{i=1}^{N}\sum_{j=1}^{J}\left(1-a_{i,j}\right)\left(x_{i,j}s_{j}-\hat{x}_{i,j}\tilde{\mathbb{E}}\left[s_{j}\right]\right)\right)^{2}\mid\mathscr{S}\right].$$

The optimal interventions are therefore given by

$$\mathbf{a}^{\star}=\left(p\gamma\tilde{\mathbb{V}}\right)^{-1}\left(\kappa\left(\mathbf{1}_{N\times N}\otimes\tilde{\mathbb{E}}\left[\mathrm{s}\right]\right)\circ\hat{\mathbf{x}}-\Phi+p\gamma\tilde{\mathbb{V}}\mathbf{1}_{NJ\times1}\right),\tag{22}$$

where $\hat{\mathbf{x}}$ is the posterior mean from the Kalman gain K and $\tilde{\mathbb{V}}$ is the **Distress Uncertainty Matrix** given by

$$\tilde{\mathbb{V}}\equiv\mathbb{COV}\left[\left(\mathbf{1}_{N}\otimes\mathrm{s}\right)\circ\mathbf{x}\mid\mathscr{S},\mathscr{D}=1\right].\tag{23}$$

The matrix $\tilde{\mathbb{V}}$ is the covariance of the NJ vector $\left(\mathbf{1}_{N}\otimes\mathrm{s}\right)\circ\mathbf{x}=\left(x_{i,j}s_{j}\right)_{i=1:N}^{j-1:J}=\left[s_{1}x_{1,1},..,s_{J}x_{1,J},s_{1}x_{2,1}..,s_{J}x_{N,J}\right]$. This covariance is conditional on the stress test results – ex-post mean $\hat{x}$ and residual variance $\hat{\Sigma}_{\mathbf{x}}$ – and on the distribution of risks under distress – $\tilde{\mathbb{E}}\left[\mathrm{s}\right]$ and $\tilde{\Sigma}_{\mathrm{s}}$. The residual uncertainty is known in advance since the evolution of the covariance matrix is deterministic, but the posterior mean depends on the random realization the test itself, since $\hat{\mathbf{x}}=\bar{\mathbf{x}}+K\left(\hat{\mathrm{y}}-\mathrm{s}'\bar{\mathbf{x}}\right)$. Since s and $\hat{\mathbf{x}}$ are independent we can write the covariance matrix as[11]

$$\tilde{\mathbb{V}}=\left(\mathbf{1}_{\boldsymbol{N}\times\boldsymbol{N}}\otimes\tilde{\Sigma}_{\mathrm{s}}\right)\circ\left(\hat{\mathbf{x}}\hat{\mathbf{x}}'\right)+\left(\mathbf{1}_{\boldsymbol{N}\times\boldsymbol{N}}\otimes\left(\tilde{\Sigma}_{\mathrm{s}}+\tilde{\mathbb{E}}\left[\mathrm{s}\right]\tilde{\mathbb{E}}\left[\mathrm{s}\right]'\right)\right)\circ\hat{\Sigma}_{\mathbf{x}}.$$

The first term of $\tilde{\mathbb{V}}$ measures uncertainty about the macro state under distress $\tilde{\Sigma}_{\mathrm{s}}$ interacted with expected exposures. This term is large when estimated exposures $\hat{\mathbf{x}}\hat{\mathbf{x}}'$ are high in states where conditional risk $\tilde{\Sigma}_{\mathrm{s}}$ is also high. The second term captures residual uncertainty about banks' exposures measured by the residual covariance matrix $\hat{\Sigma}_{\mathbf{x}}$ from the Kalman filter.[12]

Equation (22) says that the regulator intervenes more against high and uncertain exposures to bad and uncertain states. Her interventions are limited by the cost Φ and the uncertainty itself.

[11] We have assumed that the error term ε in stress results is independent of future realization of the risk factors s. While this is an obvious assumption to make at this point, we note that it is not without loss of generality if we consider endogenous financial crises. Suppose, for example, that banks are too optimistic about mortgage risk: ε is negative and their perceived exposures are lower than their true exposures. This might lead to excessive lending, real estate price appreciation, and this might increase the probability of a future decrease in real estate prices. This would violate the assumption of independence between ε and $\tilde{\mathrm{s}}$.

[12] The matrix notations are somewhat complicated but in the one dimensional case the formula is simply the variance of a product of independent variables: $\tilde{\mathbb{V}}\left(xs\right)=\tilde{\mathbb{V}}\left(x\right)\tilde{\mathbb{V}}\left(s\right)+\tilde{\mathbb{V}}\left(x\right)\left(\tilde{\mathbb{E}}\left[s\right]\right)^{2}+\tilde{\mathbb{V}}\left(s\right)\left(\tilde{\mathbb{E}}\left[x\right]\right)^{2}$.

High residual uncertainty limits the responsiveness of the targeted interventions to the expected exposures $\hat{\mathbf{x}}$. Better information translates into lower variation in ex-post net exposures since the regulator intervenes more when it is needed, and less when exposures are low.

3.4 Understanding Differential Information Sensitivity

We will show in our calibration that targeted interventions are more information-sensitive than pure capital requirements. A simple example helps illustrate the theory behind this result. Let us consider the case of one bank ($N = 1$), two symmetric uncorrelated factors ($J = 2$, with $s_1 \perp s_2$, $\tilde{\mathbb{E}}[s_1] = \tilde{\mathbb{E}}[s_2] = \tilde{s}$, and $\tilde{\sigma}^2_{s_1} = \tilde{\sigma}^2_{s_2}$). The optimal capital requirement is then

$$\overline{W}^{\star} = \frac{p\theta - \kappa}{p\gamma} + \tilde{W} + (\hat{x}_1 + \hat{x}_2)\,\tilde{s}$$

Suppose that the exposures are perfectly negatively correlated, i.e., $x_1 + x_2 = c$ for some constant c. Then, the optimal capital $\overline{W}^{\star} = \frac{p\theta-\kappa}{p\gamma} + \tilde{W} + c\tilde{s}$ is completely insensitive to new information. A regulator armed only with $\overline{W}^{\star}$ would not pay anything to run a stress test. because the average risk is constant and capital requirements can only target average risk.

However, if the regulator also had access to targeted interventions, she would value the information revealed by stress tests. In this example, the first order condition for a targeted action a_1 is

$$\tilde{\mathbb{E}}\left[x_1 s_1 \left((1-a_1)(x_1 s_1 - \hat{x}_1 \tilde{s}_1) + (1-a_2)(x_2 s_2 - \hat{x}_2 \tilde{s}_2)\right) \mid \mathscr{S}\right] = \frac{\phi_1}{p\gamma} - \hat{x}_1 \tilde{\mathbb{E}}[s_1]\,\frac{\kappa}{p\gamma}.$$

Suppose for simplicity the case where $c = 0$ and $\phi_1 = \phi_2$. Then, given the symmetry of the risk factors, the FOC above simplifies to

$$(1-a_1)\left(\hat{x}^2\tilde{\sigma}^2_s + \hat{\sigma}^2_x \tilde{s}^2 + \tilde{\sigma}^2_s \hat{\sigma}^2_x\right) - (1-a_2)\,\hat{\sigma}^2_x \tilde{s}^2 = \frac{\phi}{p\gamma} - \hat{x}_1 \frac{\tilde{s}\kappa}{p\gamma}$$

and the symmetric equation holds for a_2. Solving for the individual intervention yields

$$a_1 - 1 = \frac{1}{p\gamma}\left(\frac{\tilde{s}\kappa\hat{x}_1}{\tilde{\sigma}^2_s\left(\hat{x}^2 + \hat{\sigma}^2_x\right) + 2\hat{\sigma}^2_x \tilde{s}^2} - \frac{\phi}{\tilde{\sigma}^2_s\left(\hat{x}^2 + \hat{\sigma}^2_x\right)}\right),$$

which shows that targeted interventions depend on the exposures estimated from the stress test. This expression also shows the substitution between capital requirements and targeted interventions: when κ is lower, capital requirements are higher and the need for targeted interventions is lower.[13]

[13] The fact that it appears only in the term $\tilde{s}\kappa\hat{x}_1$ comes from the simplifying assumptions $c = 0$ and $\sigma_{s_{12}} = 0$).

4 Designing Optimal Scenarios

We can finally characterize the design of optimal scenarios. Taking into account the optimal future interventions, the interim utility of the regulator $V(\mathscr{S})$ depends on the scenarios chosen and on the stress test results $\hat{\boldsymbol{y}}$. At the scenario design stage the regulator chooses the stress scenarios $\hat{S}$ to maximize her expected value. The scenario design problem is therefore

$$\hat{S}^{*} = \arg\max_{\hat{S}} \mathbb{E}\left[V(\mathscr{S}) \mid \hat{S}\right]. \tag{24}$$

We could incorporate a cost of creating additional scenarios for the regulator: choosing M scenarios for the stress test could have a cost $\mathscr{C}(M)$. In that case the objective function would simply be $\mathbb{E}_{\hat{\boldsymbol{y}}}\left[V(\mathscr{S}) \mid \hat{S}\right] - \mathscr{C}(M)$ and the regulator would also choose the number of scenarios to include in the stress test. The relevant cost function depends on institutional details (e.g., stress testing insurance portfolios vs. banks' credit books) and we leave this for future applied work. Moreover, using our observations in Section (2) that the scenario choices map into a posterior variance and that the feasible set of such posterior variances is an outcome of the Kalman filter, we can write the regulator's choice of scenario as a choice of posterior variance $\hat{\Sigma}_{\mathbf{x}} \in \boldsymbol{\Sigma}$.

4.1 Scenario Design for Pure Capital Requirements

In the case of pure capital requirements we obtain a particularly simple result.

Lemma 3. ***Scenarios for pure capital requirements*** *(*$\mathbf{a} = 0$*). Under L the regulator designs capital stress scenarios to minimize the Distress Uncertainty Matrix:*

$$\min_{\hat{\Sigma}_{\mathbf{x}} \in \boldsymbol{\Sigma}} \mathbf{1}_{1\times NJ} \mathbb{E}\left[\tilde{\mathbb{V}}\right] \mathbf{1}_{NJ\times 1}, \tag{25}$$

where $\tilde{\mathbb{V}}$ is given by (23) and $\boldsymbol{\Sigma}$ is the set of feasible residual uncertainty implied by the Kalman filter.

The optimal pure capital requirements given by $\overline{W}^{\star} = \tilde{W} + \frac{\theta}{\gamma} - \frac{\kappa}{p\gamma} + \sum_{i=1}^{N} \mathbb{E}\left[y_i \mid \text{Distress}, \mathscr{S}\right]$ are linear in expected losses. Therefore, the *expected* capital cost $\kappa \mathbb{E}\left[\overline{W}^{\star}\right]$ is independent of $\hat{\Sigma}_{\mathbf{x}}$ and the regulator only needs to minimize expected uncertainty. The regulator's problem is then equivalent to $\min_{\hat{\Sigma}_{\mathbf{x}}\in\boldsymbol{\Sigma}} \mathbf{1}_{1\times NJ}\left(\mathbf{1}_{N\times N}\otimes\tilde{\mathrm{s}}\tilde{\mathrm{s}}'\right)\circ\hat{\Sigma}_{\mathbf{x}}\mathbf{1}_{NJ\times 1}$.[14] This implies that the value of learning about factor j depends on the unit cost of exposure $\tilde{\mathrm{s}}\tilde{\mathrm{s}}'$ and on residual exposure uncertainty $\hat{\Sigma}_{\mathbf{x}}$.

Lemma 3 implies a form of envelope theorem for the case of pure capital requirements. For convenience, define total losses as $Y \equiv \sum_{i=1}^{N}\sum_{j=1}^{J} x_{i,j}s_j$. In the case of pure capital requirements ($\mathbf{a}=0$) we have

$$V = p\left(\theta\tilde{\mathbb{E}}\left[W-\tilde{W}\mid\mathscr{S}\right]-\frac{\gamma}{2}\tilde{\mathbb{E}}\left[\left(W-\tilde{W}\right)^2\mid\mathscr{S}\right]\right)-\kappa\overline{W}^{\star}$$

where $W=\overline{W}^{\star}-Y$ and the optimal requirement is $\overline{W}^{\star}=\tilde{W}-\frac{\kappa-p\theta}{p\gamma}+\tilde{\mathbb{E}}\left[Y\mid\mathscr{S}\right]$. Using the expression above and taking expectations with respect to the information set we get

$$\mathbb{E}\left[V\left(\mathscr{S}\right)\right]=\frac{1}{2}\frac{\left(\kappa-p\theta\right)^2}{p\gamma}-\kappa\left(\tilde{W}+\sum_{i=1}^{N}\sum_{j=1}^{J}\bar{x}_{i,j}\tilde{s}_j\right)-p\frac{\gamma}{2}\mathbb{E}\left[\tilde{\mathbb{E}}\left[\left(Y-\tilde{\mathbb{E}}\left[Y\mid\mathscr{S}\right]\right)^2\mid\mathscr{S}\right]\right]$$

where $\bar{x}_{i,j}$ is the prior of the regulator and $\mathbb{E}\left[\tilde{\mathbb{E}}\left[\left(Y-\tilde{\mathbb{E}}\left[Y\mid\mathscr{S}\right]\right)^2\mid\mathscr{S}\right]\right]=\mathbf{1}_{1\times NJ}\mathbb{E}\left[\tilde{\mathbb{V}}\right]\mathbf{1}_{NJ\times 1}$ is the distress uncertainty matrix. The value of information is therefore proportional to the variance reduction. The welfare impact of stress tests is second-order small under pure capital requirements when these tests are expected to reveal a small amount of news (for instance because prior uncertainty is small).

4.2 Scenario Design with Targeted Interventions

Given that $\hat{\mathbf{y}}$ is normally distributed and that risk factors are independent from risk exposures, we can integrate the indirect value function $\mathbb{E}\left[V\left(\mathscr{S}\right)\mid\hat{S}\right]$ and express it as function of the covariance matrices $\hat{\Sigma}_{\mathbf{x}}$ and $\tilde{\Sigma}_s$. To see this, note that the regulator's objective depends on the result of the stress test $\hat{\mathbf{y}}$ only through $\hat{\mathbf{x}}$. Moreover, the optimal targeted interventions depend only on $\hat{\mathbf{x}}$ and on the posterior variance. The regulator then solves[15]

$$\min_{\hat{\Sigma}_{\mathbf{x}}\in\boldsymbol{\Sigma}}\mathbb{E}\left[\kappa\overline{W}^{\star}+\frac{1}{2}\left(\left(1_{NJ\times 1}-\mathbf{a}^{\star}\right)'p\gamma\tilde{\mathbb{V}}\left(1_{NJ\times 1}-\mathbf{a}^{\star}\right)+\Phi'\mathbf{a}^{\star}\right)\right]. \tag{26}$$

[14]Using the definition of $\tilde{\mathbb{V}}$ and $\mathbb{E}\left(\hat{\mathbf{x}}\hat{\mathbf{x}}'\right)=\mathbb{E}\hat{\mathbf{x}}\mathbb{E}\hat{\mathbf{x}}'+\Sigma_{\mathbf{x}}-\hat{\Sigma}_{\mathbf{x}}=\bar{\mathbf{x}}\bar{\mathbf{x}}'+\Sigma_{\mathbf{x}}-\hat{\Sigma}_{\mathbf{x}}$ we get

$$\mathbf{1}_{1\times NJ}\mathbb{E}\left[\tilde{\mathbb{V}}\right]\mathbf{1}_{NJ\times 1}=\mathbf{1}_{1\times NJ}\left\{\left(\mathbf{1}_{\boldsymbol{N}\times\boldsymbol{N}}\otimes\tilde{\Sigma}_{\mathrm{s}}\right)\circ\left\{\bar{\mathbf{x}}\bar{\mathbf{x}}'+\Sigma_{\mathbf{x}}\right\}+\left(\mathbf{1}_{\boldsymbol{N}\times\boldsymbol{N}}\otimes\tilde{\mathrm{s}}\tilde{\mathrm{s}}'\right)\circ\hat{\Sigma}_{\mathbf{x}}\right\}\mathbf{1}_{NJ\times 1},$$

where the term $\left(\mathbf{1}_{\boldsymbol{N}\times\boldsymbol{N}}\otimes\tilde{\Sigma}_{\mathrm{s}}\right)\circ\left\{\bar{\mathbf{x}}\bar{\mathbf{x}}'+\Sigma_{\mathbf{x}}\right\}$ is constant.

[15]Recall that $\overline{W}^{\star}-\tilde{W}-\left(\mathbf{1}_N\otimes\tilde{\mathrm{s}}\right)\circ\mathbf{x}\left(1_{NJ\times 1}-\mathbf{a}^{\star}\right)=\frac{\theta}{\gamma}-\frac{\kappa}{p\gamma}$.

It is useful to compare (26) with (25). If we force $\mathbf{a}^\star = 0$ in (26) we obtain (25) since, when $\mathbf{a}^\star = 0$, $\mathbb{E}\left[\overline{W}^\star\right]$ is independent of $\hat{\Sigma}_\mathbf{x}$. Three changes occur when $\mathbf{a}^\star$ is optimally chosen. First $\mathbb{E}\left[\overline{W}^\star\right]$ now depends on $\hat{\Sigma}_\mathbf{x}$ via $\mathbf{a}^\star$. Second, $\mathbf{a}^\star$ mitigates the cost of uncertainty, as seen in the middle term by limiting the ex-post exposures to the macro factors. Finally, the cost of targeted action appears as $\Phi'\mathbf{a}^\star$. As usual in linear-quadratic problems, we can substitute the optimal controls (actions and capital requirements) to re-write the optimal scenario design problem.

Lemma 4. *The regulator's scenario design with capital and targeted interventions solves*

$$\min_{\hat{\Sigma}_\mathbf{x} \in \boldsymbol{\Sigma}} \mathbb{E}_{\hat{\mathbf{x}}}\left[\kappa \overline{W}^\star + \Phi \mathbf{a}^\star\right], \tag{27}$$

where $\mathbf{a}^\star$ is given by (22), $\overline{W}^\star$ by (18), and $\boldsymbol{\Sigma}$ is the set of feasible residual uncertainty implied by the Kalman filter.

The simplicity of Equation (27) comes from the linear cost of functions and the quadratic benefits of targeted actions. Consider for simplicity the one dimensional case, $NJ = 1$. Then we have $\overline{W}^\star = \tilde{W} + \theta - \frac{\kappa}{p\gamma} + \hat{x}\tilde{s}\left(1 - a^\star\right)$ and the program is $\min \mathbb{E}\left[\kappa \overline{W}^\star + \phi a^\star + \frac{1}{2} p\gamma \tilde{\mathbb{V}}\left(1 - a^\star\right)^2\right]$. The optimal action $a^\star = \frac{p\gamma\tilde{\mathbb{V}} - \phi + \kappa\hat{x}\tilde{s}}{p\gamma\tilde{\mathbb{V}}}$ implies $p\gamma\tilde{\mathbb{V}}\left(1 - a^\star\right)^2 = \left(\phi\left(1 - a^\star\right) - \left(1 - a^\star\right)\kappa\hat{x}\tilde{s}\right)$ and therefore the program is equivalent to $\min \mathbb{E}\left[\kappa\hat{x}\tilde{s}\left(1 - a^\star\right) + \phi a^\star\right]$.

The regulator anticipates that she will intervene optimally after observing the results of the test and she chooses a posterior covariance matrix $\hat{\Sigma}_\mathbf{x}$ to maximize the accuracy of her interventions weighted by the relevant costs. The set $\boldsymbol{\Sigma}$ restricts the feasible information set. The benefit of learning more about exposure j, i.e. of decreasing $\hat{\Sigma}^j_\mathbf{x}$, is given by the derivative of the objective in (27) with respect to the posterior variance, which reduces to

$$\mathbb{E}\left[\left(\mathbf{1}_{NJ\times 1} - \mathbf{a}^{\star\prime}\right)\left(\kappa\left(\mathbf{1}_{N\times 1} \otimes \tilde{\mathrm{s}}\right) \circ \frac{\partial \hat{\mathbf{x}}}{\partial \hat{\Sigma}^j_\mathbf{x}}\right) + \left(\Phi - \kappa\left(\mathbf{1}_{N\times 1} \otimes \tilde{\mathrm{s}}\right) \circ \hat{\mathbf{x}}\right)\frac{d\mathbf{a}^{\star\prime}}{d\hat{\Sigma}^j_\mathbf{x}}\right]. \tag{28}$$

The value of learning depends on the responsiveness of interventions to new information. The first term in Equation (28) represents the impact of information on the posterior expected risk exposure. The more precise this information, the more sensitive $\hat{\mathbf{x}}$ is to the new information in the stress test. Since $\frac{\partial \overline{W}^\star}{\partial \hat{\mathbf{x}}} = \left(\mathbf{1}_{NJ\times 1} - \mathbf{a}^{\star\prime}\right)\left(\mathbf{1}_{N\times 1} \otimes \tilde{\mathrm{s}}\right)$, the first term in Equation (28) represents the reduction in the cost of the capital requirements when the test improves information along dimension j. The second term in Equation (28) captures the benefit of changing the targeted interventions when $\hat{\Sigma}^j_\mathbf{x}$ is lower. The effect of $\hat{\Sigma}^j_\mathbf{x}$ on $\mathbf{a}^\star$ deserves further attention. Since $\mathbf{a}^\star\left(\hat{\mathbf{x}}, \tilde{\mathbb{V}}\right)$

we have

$$\frac{d\mathbf{a}^\star}{d\hat{\Sigma}^j_{\mathbf{x}}} = \frac{\partial \mathbf{a}^\star}{\partial \tilde{\mathbb{V}}} \frac{d\tilde{\mathbb{V}}}{d\hat{\Sigma}^j_{\mathbf{x}}} + \frac{\partial \mathbf{a}^\star}{\partial \hat{\mathbf{x}}} \frac{d\hat{\mathbf{x}}}{d\hat{\Sigma}^j_{\mathbf{x}}} \tag{29}$$

The first term in Equation (29) captures changes in targeted interventions in response to changes in residual uncertainty. The second term measures the value of intervening more accurately. It depends on the sensitivity of targeted interventions to the ex-post expected exposures $\frac{\partial \mathbf{a}^\star}{\partial \hat{\mathbf{x}}}$ and on how new information changes this posterior mean, $\frac{d\hat{\mathbf{x}}}{d\hat{\Sigma}^j_{\mathbf{x}}}$.

It is worth noting that if the regulator's priors are very precise, only extreme realizations of $\hat{\mathbf{y}}$ can move her beliefs and the sensitivity of the intervention policy to new information, $\frac{d\hat{\mathbf{x}}}{d\hat{\Sigma}^j_{\mathbf{x}}}$, is low. In this case, increasing the precision of the test along dimension j does not improve the accuracy of interventions and the value of learning is low. Similarly, the value of information is low if interventions are too costly to be responsive to new information.

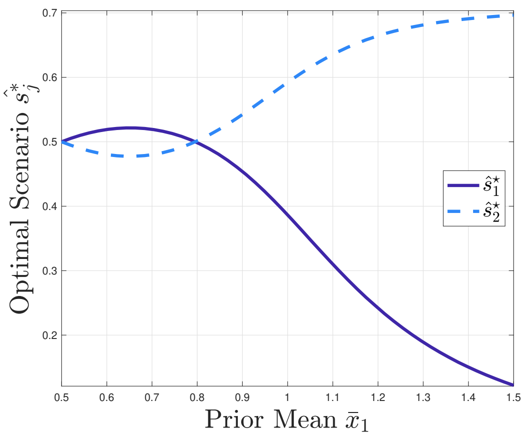
4.3 Comparative Statics

The weights of the different risk factors in the optimal scenarios depend on how targeted interventions respond to new information, which in turn depend on prior beliefs and intervention costs. In this section, we provide comparative statics to illustrate the forces that link the model's primitive to the optimal stress scenarios. The Appendix contains additional comparative statics. Most examples below use two factors and one bank and, unless explicitly stated otherwise, we assume that the regulator's priors about exposures are uncorrelated across factors, and we use the following functional form for the variance of the error term in the stress test: $\sigma_{\hat{\epsilon}} = \alpha + \beta \left\|\hat{s}\right\|^2 + \beta_\tau e^{\tau\|\hat{s}\|}$.[16] The Appendix contains additional comparative statics.

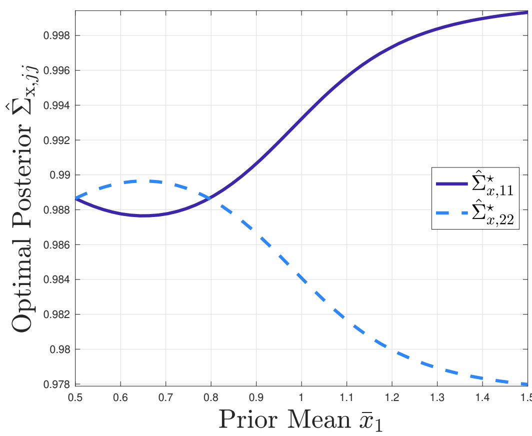
Prior mean exposure

There are two opposing forces that connect the prior mean exposures and the optimal scenario design. On the one hand, targeted interventions are increasing in the prior mean exposure.s. This makes uncertainty about high exposures costly and increases the value of learning about factors with high expected exposures to prevent wasteful interventions. On the other hand, a high

[16] If the regulator's prior expectation is that exposures are correlated, it may still be beneficial for her to stress factor j even if doing so does not improve the accuracy of her intervention along dimension j. In this case, stressing factor j would only be valuable to learn about the exposures to other factors and improve the accuracy of the targeted interventions along these other dimensions.



(a) Optimal scenario choice

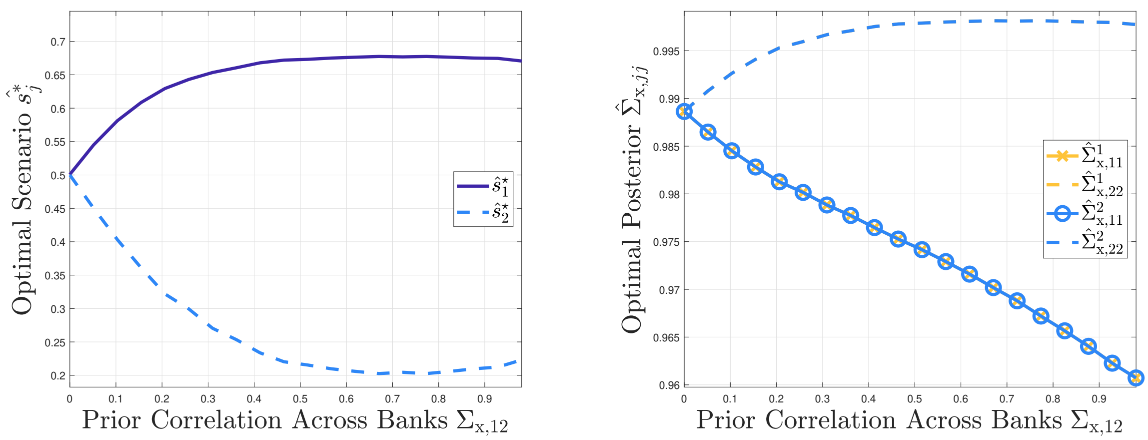


(b) Optimal posterior variances

Figure 2: Optimal scenario and information choice as a function of the prior mean $\bar{x}_1$.

Note: The parameters used are $M = 1$, $N = 1$, $J = 2$, $\theta = 0$, $p\gamma = 0.1$, $\Phi = [0.75, 0.75]'$, $\bar{x} = [1, 1]'$, $\Sigma_{\boldsymbol{x}} = \boldsymbol{I}_J$, $\alpha = 0$, $\beta = 5$, $\beta_\tau = 7$, $\tau = 2$, $\mathbb{E}[s] = [0, 0]'$, $\tilde{\mathbb{E}}[s] = [1, 1]'$, $\Sigma_s = \boldsymbol{I}_J$, $\tilde{\mathbb{E}}[s] = 3 \times \boldsymbol{I}_J$, $\kappa = 0.1$, $\tilde{W} = 100$, and $\mathbb{E}[\epsilon\epsilon'] = \boldsymbol{I}_N$.

prior mean exposure to factor i implies that the regulator's posterior mean about x_i is not very sensitive to the information produced by the stress test. More specifically, the higher the prior mean exposure, the more likely it is that posterior mean exposure is high regardless of the result of the tests–the posterior mean is anchored around the prior mean. This effect makes learning about x_i less valuable as the prior mean increases. Panels (a) and (b) in Figure 2 illustrate how these two forces may lead to non-monotonic comparative statics with respect to the prior mean. The figure shows that the second effect dominates when the expected mean exposure to factor 1 is high enough. Intervention costs also affect the sensitivity of interventions to new information and imply similar comparative statics to those of prior means, as it can be seen from Panels (a) and (b) in Figure B.2 in the Appendix.



(a) Optimal scenario choice

(b) Optimal posterior variance

Figure 3: Two factors, Two banks. Optimal scenario and information choice as a function of the prior correlation between the bank's risk exposures to factor 1, $\Sigma^{11}_{\mathbf{x},12}$.

Note: Figure 3 illustrates the regulator's optimal choice of scenario and the implied posterior variance as a function of prior correlation in risk exposures. The parameters used are $N = 2$, $J = 2$, $\theta = 0$, $p\gamma = 0.1$, $\Phi = [0.05, 0.05]'$, $\bar{x} = [1, 1]'$, $\Sigma_{\boldsymbol{x}} = \boldsymbol{I}_J$, $\alpha = 0$, $\beta = 5$, $\beta_\tau = 7$, $\tau = 2$, $\mathbb{E}[s] = [0, 0]'$, $\tilde{\mathbb{E}}[s] = [1, 1]'$, $\Sigma_s = \boldsymbol{I}_J$, $\tilde{\Sigma_s} = 3 \times \boldsymbol{I}_J$, $\kappa = 0.1$, $\tilde{W} = 100$, , and $\mathbb{E}[\epsilon\epsilon'] = \boldsymbol{I}_N$.

Correlated exposures

Consider the case in which the regulator's prior is that exposures to a factor are correlated across banks. In this case, a bank's stress tests will contain information about its own exposures and about the exposures of other banks that are correlated to them. Therefore, stressing factors to which banks' have correlated exposures is more efficient for the regulator as it provides multiple signals on them.

Figure 3 shows the optimal stresses as a function of the prior correlation between exposures to factor 1 among two banks when there are two factors and two banks. When the exposures to factor 1 are correlated across banks, reported losses from one bank contain information about that bank's exposures but also about the other bank's exposure to the correlated factor. Learning about factor 1 becomes more valuable. When the correlation is strong the regulator barely learns about the other factor. The posterior variance of risk exposures to factor 2 tends towards the prior variance. A similar result is shown in the Appendix in Figure 3a for the case in which prior exposures are correlated within a bank.

5 Calibration

Our model suggests that well-designed tests can increase the efficiency of financial supervision but stress testing is ultimately a quantitative exercise. This section therefore proposes a calibration of the model in the U.S. banking context. We derive quantitative insights regarding the role of multiple scenarios and the value of information for targeted interventions.

5.1 Calibration

We calibrate our model of U.S. stress tests using quarterly bank-level and macroeconomic data. Given data availability we model only credit losses, measured as total net charge offs. To choose our risk factors we follow the Capital and Loss Assessment under Stress Scenarios (CLASS) model and data developed in Hirtle et al. (2014). We regress banks' total net charge off (NCO) rate on standard macroeconomic variables: GDP growth, short-term and long-term interest rates, unemployment rate, housing prices, equity prices, and credit spreads.[17] We implement this regression at the aggregate banking system level and at the bank level for the panel of banks participating in Dodd-Frank Act Stress Test (DFAST) exercises. We find that GDP growth rate and real estate prices explain more than 80% of the variation in NCO rates in our sample.[18] We therefore calibrate our model using two macro risk factors: GDP growth rate and a real estate price index that puts equal weights on residential and commercial properties. We get the distribution of macroeconomic states and the probability of distress from historical data. To make our results easy to interpret we standardize the risk factors to have mean zero and unit variance. Stress scenario magnitudes therefore represents units of standard deviations. We set the mean of the risk factors under distress to $\tilde{\mathbb{E}}[s] = 1$ and with a conditional volatility of 1.5 times the historical one to take into account that uncertainty is higher in distress.

[17] We use macro historical data from 1991-2013 for this exercise. In line with the CLASS model, we include a one-quarter lag of the dependent variable and cluster standard errors over time.

[18] We include the cumulative GDP growth rate of the previous four quarters, the 1-quarter lag of the level of the real estate price index and the four-quarter lag of the dependent variable in this regression.

Table 1: Summary of 2015 DFAST

(%)	Ex-Ante	Adverse	Severe. Adv.
Tier 1 Capital Ratio	13.5	10.4	8.4
Tier 1 Leverage	8.8	7.2	5.9
Loan Loss Rate	n.a.	4.1	6.1

We use the results from the 2015 DFAST to calibrate exposures and the parameters of capital requirements, as summarized in Table 1.

Equation 21 allows us to interpret these numbers: $\mathbb{E}_{\hat{x}}\left[\overline{W}^{\star}\right] = \sum_{i=1}^{N} \tilde{\mathbb{E}}\left[\mathrm{s}\right] \cdot \overline{\mathrm{x}}_{\mathrm{i}} + \tilde{W} - \frac{\kappa - p\theta}{p\gamma}$. We identify the distress region with the adverse scenario. We use the Tier capital ratios for our calibration, rounding down for simplicity. Ex-ante average capital is $\overline{W}^{\star} = 13\%$ and it goes down to 10% in the stress region with a loss rate of 4%. This is consistent with a 1% loss rate in normal times and 3% additional losses in the stress region (recall that our model is written in deviation from the benchmark scenario). With two risk factors and our normalization $\tilde{\mathbb{E}}\left[\mathrm{s}\right] = 1$ this implies $\overline{\mathrm{x}}_1 + \overline{\mathrm{x}}_2 = 3\%$. This further implies that $\tilde{W} - \frac{\kappa - p\theta}{p\gamma} = 10\%$. We set the marginal cost of capital $\kappa = 0.3$ to match the shadow cost of capital estimated in Kisin and Manela (2016). We set the probability of a crisis at $p = 0.1$. We know that $\kappa > p\theta$ and that the difference should be small. We thus set the marginal utility at $\theta = 2.9$, which means that one extra dollar of capital in distress is worth 2.9 dollars. There are several way to calibrate γ. Capital in the severely adverse scenario is 0.02 below capital in the adverse scenario where the marginal utility is 2.9. If we want to capture a meaningful difference between these two states we can require that marginal utility increases by 1, which implies $\gamma \times .02 = 1$ or $\gamma = 50$. Alternatively we can see that the elasticity of ex-ante requirements $\overline{W}^{\star}$ to capital costs κ is $p\gamma$. With $p\gamma = 5$ and $\kappa = 0.3$, a 10% decrease in the marginal cost of bank equity increases the optimal capital requirement by $.03/5 = 0.6pp$. These numbers imply $13\% = 3\% + \tilde{W} - \frac{0.3 - 0.1 * 2.9}{5}$ or $\tilde{W} = 10\% + \frac{0.01}{5} = 10.2\%$.

The priors of the regulator are an important determinant of optimal scenarios. We use the CLASS model to learn how regulators think about the link between macroeconomic scenarios and bank outcomes. Based on the regressions in the CLASS model for NCO rates, we use the asymptotic variances of the coefficients in a regression of NCO rates on our two factors to back out the regulator's prior uncertainty.

We measure targeted interventions in deviations from the optimum based on the priors of the regulator (i.e., what she would do without stress testing information). Formally we let $\bar{a}$ be the optimal choice based on the priors: $\frac{\partial \mathcal{C}(\bar{\mathbf{a}})}{\partial a_j} = \mathbb{E}\left[x_j s_j U'(W)\right]$, where $\mathcal{C}$ is convex. We then interpret our cost function as a linear approximation: $\mathcal{C}(\mathbf{a}) = \mathcal{C}(\bar{\mathbf{a}}) + \Phi(\mathbf{a} - \bar{\mathbf{a}})$ and we calibrate $\Phi_j = \mathbb{E}\left[x_j s_j U'(W)\right]$. Deviations $\mathbf{a} - \bar{\mathbf{a}}$ can then be positive or negative.[19] The implied value is $\phi \in [0.0077, 0.0078]$, which implies that the maxium cost associated with targeted interventions is in the order of 40bps per factor.

Finally, we require a parameterization of the error in the stress test responses. To do so we compute the forecasting errors of the CLASS model in the cross section of banks and regress their standard deviation on the squared norm of the macro risk factors. Table 2 summarizes the calibrated parameters.

To summarize we assume a baseline loss rate of 1%, which increases by 3*pp* in the adverse scenario and by 5*pp* in the severely adverse scenario. The standard deviation of the estimation errors is 1*pp* in the adverse scenario and 2.3*pp* in the severely adverse scenario.

5.2 Results

We consider four "problems" for the regulator. The first three problems focus on capital requirements without targeted interventions. The fourth problem studies targeted interventions.

- Problem 1: pure capital requirements and two scenarios chosen optimally. This is the scenario design problem studied earlier where the regulator chooses both scenarios to optimize her learning.

- Problem 2: pure capital requirements with one scenario fixed at $\tilde{\mathbb{E}}[\mathrm{s}]$ and one scenario chosen optimally. This problem captures an important feature of actual stress tests where one scenario is often used to set capital requirements under a *plausible* adverse scenario. We therefore constrain one scenario to be the expected adverse state $\hat{\mathrm{s}}^1 = \tilde{\mathbb{E}}[\mathrm{s}]$, and we let the regulator optimize over the second scenario. This allows us to quantify the efficiency loss of using one scenario in this mechanical way.

[19] An informative stress test can reduce precautionary interventions (for instance, a regulator might impose a LTV constraint most of the time if she has no information, but only some of the time if she has access to stress test information). In our calibration we assume $\mathcal{A} = [-0.5, 0.5]$..

Parameter	Description	Value
N	number of banks	1
J	number of risk factors	2
$[s_1, s_2]$	cumulative GDP growth over 4 quarters and real estate price index	normalized, mean zero and variance ~ 1
Σ_s	covariance matrix of normalized risk factors	$\begin{bmatrix} 0.990 & 0.664 \\ 0.664 & 1.046 \end{bmatrix}$
$\left[\tilde{\mathbb{E}}\left[s_1\right], \tilde{\mathbb{E}}\left[s_2\right]\right]$	mean under distress	$[1, 1]$
$\tilde{\Sigma}_s$	covariance matrix of normalized risk factors under distress	$3 \times \begin{bmatrix} 0.990 & 0.664 \\ 0.664 & 1.046 \end{bmatrix}$
Regulator priors	computed from CLASS model	
$\overline{\mathbf{x}}$	prior mean exposures to risk factors	$\begin{bmatrix} 0.015 \\ 0.015 \end{bmatrix}$
$\Sigma_{\mathbf{x}}^{1/2}$	prior covariance matrix of exposures to risk factors	$\begin{bmatrix} .006 & 0 \\ 0 & .006 \end{bmatrix}^2$
Regulator preferences		
p	probability of distress	0.1
θ	marginal value of capital in distress	2.9
γ	curvature in distress	50
$\tilde{W}$	average capital in distress region	10.2%
κ	marginal cost of bank capital	0.3
Φ	marginal cost of targeted action	$[0.0077, 0.0078]$
Stress test results		
$\sigma\left(\hat{\epsilon}_i\left(\left\Vert\hat{\mathrm{s}}\right\Vert, M\right)\right)$	standard deviation of model error	$\alpha + \beta \left\Vert\hat{\mathrm{s}}\right\Vert^2$
$[\alpha, \beta]$	model error parameter	$[0.55\%, 0.11\%]$

Table 2: Calibration

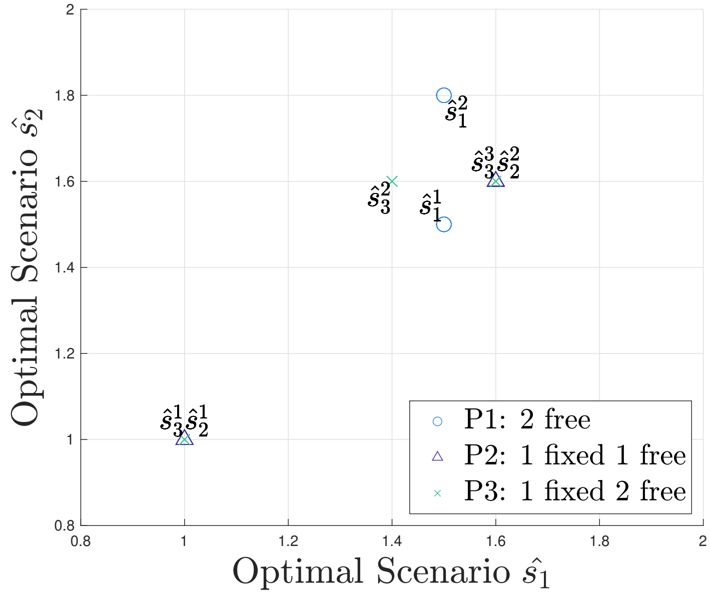


Figure 4: Optimal scenarios

- Problem 3: pure capital requirements with one scenario fixed at $\tilde{\mathbb{E}}[\mathsf{s}]$ and two scenarios chosen optimally. The first scenario is the expected adverse state. Scenarios 2 and 3 are chosen optimally. This allows us to quantity the value of adding one exploratory scenario to the standard stress test of problem 2.

- Problem 4: capital requirements with targeted interventions and two chosen optimally scenarios. This allows us to quantify the value of targeted interventions.

As discussed earlier, targeted interventions can take the form of notices of Matters Requiring (Immediate) Attention (MR(I)A) used by bank examiners to communicate concerns about a bank's management.[20] They represent an important part of the supervision process but are sometimes weakly enforced. We can use problem 4 to gauge the potential value of MRAs, especially if they are enforceable and not just mere suggestions brought up for consideration by the bank's management or board of directors.

Figure 4 shows that we obtain sensible magnitudes for our optimal scenarios in Problems 1 (circles), 2 (triangles) and 3 (crosses). In Problems 2 and 3 the first scenario is fixed $\tilde{\mathbb{E}}[\mathsf{s}] = 1$.

[20]https://www.federalreserve.gov/supervisionreg/srletters/sr1313a1.pdf

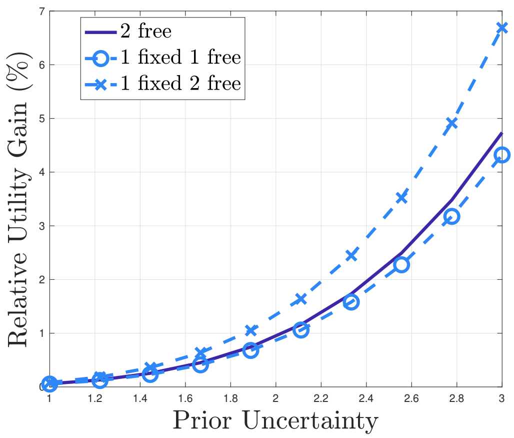
In Problem 1 both scenarios are chosen optimally. The optimal range of scenario is between 1 and 2 additional historical sigmas of negative shocks. Our calibration yields almost symmetric risk factors and symmetric priors for the regulator. This symmetry, paired with the convexity of the choice set for posterior variance given by the Kalman filter, implies that both risk factor are stressed in a similar magnitude in the optimal stress test.

Figures 5a and 5b show the welfare gains from various types of stress tests, relative to the no learning case, as a function of uncertainty in the distress states. The horizontal axis is the multiplicative factor λ on the prior standard deviation of exposures, i.e., the prior variance is $\lambda^2 \Sigma_x$. Welfare gains are small when uncertainty is small and the historical calibration can give a false sense of confidence in estimated exposures, especially during distress. The vertical axis show the welfare gains from the stress tests normalized by the welfare gain from a hypothetical decrease in the cost of capital.

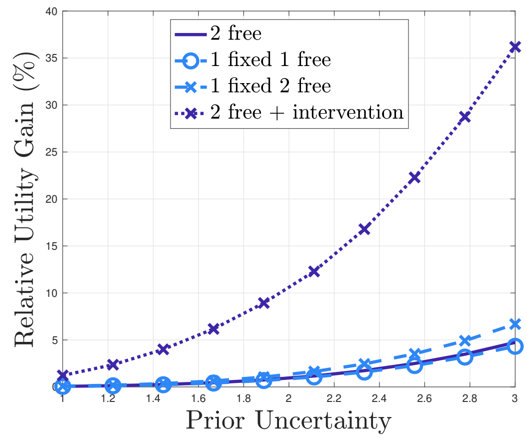
To provide an intuitive interpretation of our results we use as a benchmark a well defined experiment: the welfare gains from lowering capital costs κ by some fraction. The baseline capital requirement is calibrated at 13%. In figure 5 the benchmark is a 10% decrease in κ which corresponds to an increase in the optimal capital ratio from 13% to 13.6%. Figures 5a and 5b allow us to compare the welfare gains from the stress test to this simple benchmark.

Figure 5a shows that the ex-ante welfare gains from learning are relatively small when the regulator can *only* adjust overall capital requirements. The option to increase the requirements when risks build up is worth less than 7% of the gain from a 10% lower capital cost. The reason is that requirements are correct on average and that large shocks are unlikely. As explained in Section 4, an envelope theorem applies to the value of information revealed by the stress tests. Of course this does not invalidate the usefulness of stress tests as a way to set the benchmark requirement ex-ante as in Lemma 2. The regulator can always *define* the capital requirement as the level of equity that guarantees that a bank remains well-capitalized in an adverse scenario. Our results here only refer to the value of updating the regulator's prior based on *new* information revealed by the stress tests.

Figure 5b, on the other hand, shows that information is valuable when the regulator can make targeted interventions. In this figure we recalibrate the cost of targeted interventions so that for each λ, optimal targeted interventions are zero without learning. The welfare gains from stress testing are an order of magnitude larger than under pure capital requirements. The reason is that the regulator can now tailor its interventions to the specific information revealed by the tests and



(a) Pure capital requirements



(b) Targeted interventions

Figure 5: Utility Gain Relative to Gains Achieved by $\kappa' = 0.9 \times \kappa$

change exposures to individual risks as necessary.

Let us discuss some caveats to conclude this section. One caveat is that we focus on aggregate credit risk, which turns out to be well described with only two macro factors. Taking into account trading and derivative losses as well as interest rate risk might increase the welfare gains from stress testing as well as highlight the role of other factors. The second main caveat is that, following the literature on systemic risk, our welfare function assumes a simple aggregation of capital. This limits the role of heterogeneity across banks, which may be important in the case of contagious runs.

On the other hand, we have found the main quantitative insight – that information is much more valuable when the regulator can engage in targeted interventions – to be robust across various calibrations. We can scale up or down the cost of systemic financial distress and the ex-ante cost of equity ($p\theta$ and γ). This affects the magnitudes of the absolute welfare gains, but not their relative size: in all cases targeted interventions increase the welfare value of information by an order of magnitude compared to pure capital requirements.

6 Discussion and Conclusion

Despite the growing importance of stress testing for financial regulation and risk management, economists still lack a theory of the design of stress scenarios. We study the joint problem of optimal design of stress scenarios and optimal interventions following the test. We model stress testing as a learning mechanism and show how to map the scenario choice problem into an information acquisition problem. In this framework, we derive optimal scenarios and characterize how their design depends on the cost of interventions, the prior beliefs of the regulator, the precision of regulatory information, the uncertainty about the risk factors, and the volatility of systemic risk factors.

Our comparative static exercises above shed light on the optimal stress scenario design in the presence of systemic factors, in times of distress, over time, and its relation to capital requirements. We conclude our paper by highlighting some important findings.

Correlation and Uncertainty Can Lead to Specialization Our analysis on correlated exposures among banks suggests that the optimal stress scenario would put relatively more weight on risk factors that lead to correlated losses among banks. Moreover, if the correlation of the banks' exposures to some factor is high enough, the optimal stress scenario may put weight *only* on this factor.

Similarly we find that it is optimal to focus the stress test on factors that are more uncertain. Moreover, an across the board increase in uncertainty or in the regulator's risk aversion can lead the optimal design to put more weight on fewer factors. This implies a positive correlation between the uncertainty in the economy and the focus and severity of the stress scenarios.

Testing to Learn or to Set Capital Requirements? Our model sheds light on the debate about the fundamental value of stress tests. We find that learning is of limited value when the regulator can only adjust capital requirements. In this case, one can think of stress tests as a sensible way to set ex-ante capital requirements using plausible adverse scenarios, in line with current practice.

On the other hand, our quantitative analysis highlights the complementarities between learning and specific interventions. Designing scenarios explicitly to learn about exposures is valuable when the regulator can make targeted interventions. For instance, if the response to excessive duration exposure is to require specific hedging or modifications to loan maturities, then it is valuable to design a specific scenario to learn about these exposures. This scenario is likely to be quite different from the plausible adverse scenario used to set overall capital requirements.

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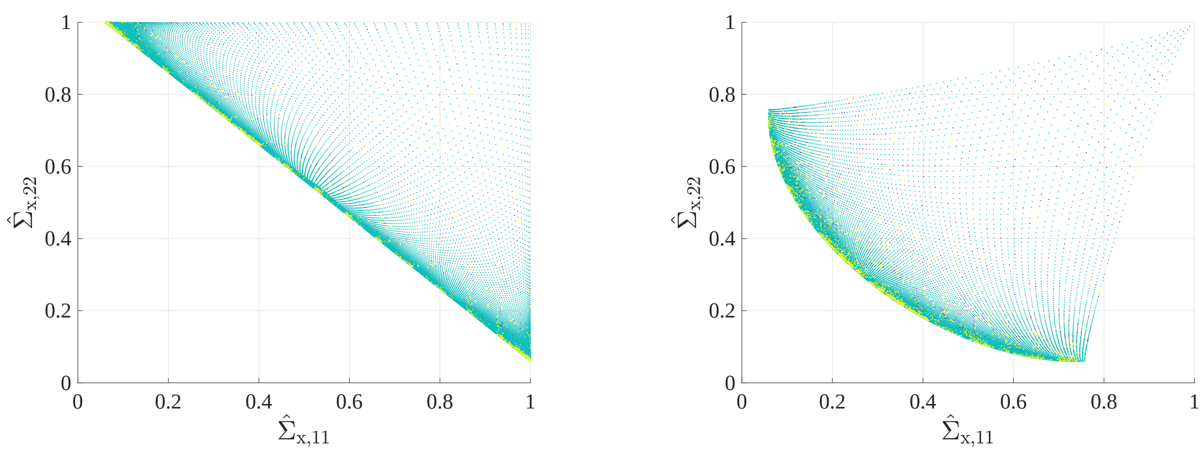
Appendix

The Appendix contains some auxiliary calculation for formulas in the text as well as omitted proofs. It also includes the additional comparative static exercises referred to in Section 4.3.

A Feasible Information set

Figure (B.1) shows the feasible set of posterior variances $\left\{\hat{\Sigma}_{\mathbf{x},11}, \hat{\Sigma}_{\mathbf{x},22}\right\}$ in a model with one bank and two risk factors, for different values of prior correlations among risk exposures. Note that the boundary of set of feasible posterior precisions, $\boldsymbol{\Sigma}$, becomes more convex the higher the prior correlation between exposures. When the prior correlation is high, the boundary of the feasible set slopes up in the tails. In the figure, when s_1 is already large, it can become more efficient to learn about x_1 by increasing s_2 instead of s_1.



(a) Prior correlation in exposures $\Sigma_{x,12} = 0$ (b) Prior correlation in exposures $\Sigma_{x,12} = 0.2$

Figure B.1: Feasible set of residual variances, $\left(\hat{\Sigma}_{\mathbf{x},11}, \hat{\Sigma}_{\mathbf{x},22}\right)$ when there are two factors and one representative bank for different values of the regulator's prior correlation among the bank's risk exposures to factors 1 and 2.

Note: Figures 1 illustrates the set of feasible posterior variances, $\boldsymbol{\Sigma}$ for different values of prior correlations among risk exposures when $\sigma^2_{\hat{\epsilon}} = \beta \left\|\hat{s}\right\|^2 + \beta_\tau e^{\tau \left\|\hat{s}\right\|^2}$. The parameters used are $M = 1$, $N = 1$, $J = 2$, $\theta = 0$, $\bar{x} = [1, 1]'$, $\Sigma_{\boldsymbol{x},\mathbf{11}} = \Sigma_{\boldsymbol{x},\mathbf{22}} = 1$, $\beta = 5$, $\beta_\tau = 5$, $\tau = 2$, and $\mathbb{E}\left[\epsilon^2\right] = 1$.

B Proofs

This section contains proofs omitted in the main text and preliminary calculations for them.

Preliminaries

Under linear quadratic preferences, the first order conditions that characterize the optimal capital requirement is

$$\mathcal{K}'\left(\overline{W}^{\star}\right) = 1 + p\theta - p\gamma\tilde{\mathbb{E}}\left[\left(W - \tilde{W}\right) \mid \mathscr{S}\right] \tag{A.1}$$

and the first order condition that characterizes the regulator's optimal targeted intervention policy is

$$\frac{\partial \mathcal{C}\left(\mathbf{a}^{\star}\right)}{\partial a_{i,j}} = p\theta \hat{x}_{i,j}\tilde{\mathbb{E}}\left[s_j\right] - p\gamma\tilde{\mathbb{E}}\left[x_{i,j}s_j\left(W - \tilde{W}\right) \mid \mathscr{S}\right], \tag{A.2}$$

where $W = \overline{W}^{\star} - \left(\left(\mathbf{1}_{N\times 1} \otimes \mathrm{s}\right) \circ \mathbf{x}\right)' \left(\mathbf{1}_{NJ\times 1} - \mathbf{a}^{\star}\right)$ and we used that $\mathbb{E}\left[s_j\right] = 0$. Using Equation (A.1) in Equation (14) and Assumption L, we get

$$W - \tilde{W} = \frac{p\theta - \kappa}{p\gamma} - \sum_{i=1}^{N}\sum_{j=1}^{J}\left(1 - a_{i,j}\right)\left(x_{i,j}s_j - \hat{x}_{i,j}\tilde{\mathbb{E}}\left[s_j\right]\right),$$

which implies the first order conditions for optimal actions in Equation (14) are

$$\frac{\phi_{h,l}}{p\gamma} = \frac{\kappa}{p\gamma}\hat{x}_{i,j}\tilde{\mathbb{E}}\left[s_j\right] + \sum_{i=1}^{N}\sum_{j=1}^{J}\left(1 - a_{i,j}\right)\tilde{\mathbb{C}}ov\left[x_{h,l}s_l, x_{i,j}s_j \mid \mathscr{S}\right] \quad \forall h, l.$$

Proof of Lemma 3

Under linear costs, the objective function of the regulator is given by

$$\begin{aligned}
\mathbb{E}_{\hat{\mathbf{x}}}\left[V\right] &= \mathbb{E}_{\hat{\mathbf{x}}}\left[\mathbb{E}_{\mathrm{s},\eta,\mathrm{x}}\left[U\left(W\left(\mathbf{a}^{\star}, \overline{W}^{\star}\right)\right) \mid \mathscr{S}\right] - \mathcal{C}\left(\mathbf{a}^{\star}\right) - \mathcal{K}\left(\overline{W}^{\star}\right)\right] \\
&= \mathbb{E}_{\hat{\mathbf{x}}}\left[\mathbb{E}_{\mathrm{s},\eta,\mathrm{x}}\left[W\left(\mathbf{a}^{\star}, \overline{W}^{\star}\right) \mid \mathscr{S}\right] - \frac{p\gamma}{2}\tilde{\mathbb{E}}_{\mathrm{s},\eta,\mathrm{x}}\left[\left(W\left(\mathbf{a}^{\star}, \overline{W}^{\star}\right) - \tilde{W}\right)^2 \mid \mathscr{S}\right] - \mathcal{C}\left(\mathbf{a}^{\star}\right) - \left(1 + \kappa\right)\overline{W}^{\star}\right].
\end{aligned}$$

Then, the FOC in Equation A.1 that characterizes the optimal capital requirement in the absence of targeted actions is

$$\overline{W}^{\star} = \tilde{W} + \frac{\theta}{\gamma} - \frac{\kappa}{p\gamma} + \left(\mathbf{1}_{N\times 1} \otimes \tilde{\mathbb{E}}\left[\mathrm{s}\right]\right)' \hat{\mathbf{x}}.$$

The total wealth of the regulator is then

$$W = \tilde{W} + \frac{\theta}{\gamma} - \frac{\kappa}{p\gamma} - \left(\left(\mathbf{1}_{N\times 1} \otimes \mathrm{s}\right)' \mathbf{x} - \left(\mathbf{1}_{N\times 1} \otimes \tilde{\mathbb{E}}\left[\mathrm{s}\right]\right)' \hat{\mathbf{x}}\right),$$

which implies

$$\mathbb{E}\left[W\right] = \overline{W}^{\star}$$

$$\tilde{\mathbb{E}}\left[W \mid \mathscr{S}\right] = \tilde{W} + \frac{\theta}{\gamma} - \frac{\kappa}{p\gamma}$$

and

$$\tilde{\mathbb{E}}\left[\left(W - \tilde{W}\right)^2 \mid \mathscr{S}\right] = \left(\frac{\theta}{\gamma} - \frac{\kappa}{p\gamma}\right)^2 + \mathbf{1}_{1\times NJ}\tilde{\mathbb{V}}\mathbf{1}_{NJ\times 1},$$

where

$$\tilde{\mathbb{V}} \equiv \tilde{\mathbb{C}ov}\left[(\mathbf{1}_{N\times 1}\otimes \mathrm{s})\circ \mathbf{x} \mid \mathscr{S}\right] = \left(\left(\mathbf{1}_{\boldsymbol{N}\times\boldsymbol{N}}\otimes \tilde{\Sigma}_{\mathrm{s}}\right)\circ \hat{\Sigma}_{\mathbf{x}}\right) + \left(\mathbf{1}_{\boldsymbol{N}\times\boldsymbol{N}}\otimes \tilde{\Sigma}_{\mathrm{s}}\right)\circ (\hat{\mathbf{x}}\hat{\mathbf{x}}') + (\mathbf{1}_{N\times N}\otimes \tilde{\mathrm{s}}\tilde{\mathrm{s}}')\circ \hat{\Sigma}_{\mathbf{x}}.$$

Then, the objective function of the regulator becomes

$$\begin{aligned}\mathbb{E}\left[V\right] &= \mathbb{E}\left[\mathbb{E}\left[W\,|\mathscr{S}\right] + p\theta\tilde{\mathbb{E}}\left[W-\tilde{W}\,|\mathscr{S}\right] - \frac{p\gamma}{2}\tilde{\mathbb{E}}\left[\left(W-\tilde{W}\right)^2\,|\mathscr{S}\right] - (1+\kappa)\,\overline{W}^{\star}\right]\\ &= -\kappa\left(\tilde{W} - \frac{\kappa}{p\gamma} + \left(\mathbf{1}_{N\times 1}\otimes \tilde{\mathbb{E}}\left[\mathrm{s}\right]\right)'\bar{\mathbf{x}}\right) + p\theta\left(\frac{\theta}{\gamma} - \frac{\kappa}{p\gamma}\right) - \frac{p\gamma}{2}\left(\frac{\theta}{\gamma}-\frac{\kappa}{p\gamma}\right)^2 - \frac{p\gamma}{2}\mathbf{1}_{1\times NJ}\mathbb{E}\left[\tilde{\mathbb{V}}\right]\mathbf{1}_{NJ\times 1}.\end{aligned}$$

Note that the only term that the regulator can affect by choosing a stress scenario, or alternatively a posterior covariance matrix, is $\tilde{\mathbb{V}}$. Therefore, the regulator's objective in the design problem is to minimize her residual uncertainty $\mathbb{E}\left[\tilde{\mathbb{V}}\right]$, which is given by

$$\begin{aligned}\mathbb{E}\left[\tilde{\mathbb{V}}\right] &\equiv \mathbb{E}\left[\tilde{\mathbb{C}ov}\left[(\mathbf{1}_{N\times 1}\otimes \mathrm{s})'\,\mathbf{x}\,|\mathscr{S}\right]\right] = \left(\mathbf{1}_{N\times N}\otimes \tilde{\Sigma}_s\right)\circ \mathbb{E}\left[\mathbb{E}\left[\mathbf{x}\mathbf{x}'\mid \mathscr{S}\right]\right] + \left(\mathbf{1}_{N\times N}\otimes \left(\tilde{\Sigma}_s + \tilde{\mathrm{s}}\tilde{\mathrm{s}}'\right)\right)\circ \hat{\Sigma}_{\mathbf{x}},\\ &= \left(\mathbf{1}_{N\times N}\otimes \tilde{\Sigma}_s\right)\circ\left(\Sigma_{\mathbf{x}} + \bar{\mathbf{x}}\bar{\mathbf{x}}'\right) + \left(\mathbf{1}_{N\times N}\otimes \tilde{\mathrm{s}}\tilde{\mathrm{s}}'\right)\circ \hat{\Sigma}_{\mathbf{x}}\end{aligned}$$

which is separable in $\bar{\mathbf{x}}\bar{\mathbf{x}}'$ and $\hat{\Sigma}_{\mathbf{x}}$ because

$$\mathbb{E}\left[\mathbb{E}\left[\mathbf{x}\mathbf{x}'\mid \mathscr{S}\right]\right] = \mathbb{E}\left[\mathbf{x}\mathbf{x}'\right] = \Sigma_{\mathbf{x}} + \bar{\mathbf{x}}\bar{\mathbf{x}}'.$$

Proof of Lemma 4

When utility is linear quadratic, and targeted intervention costs and capital requirement costs are linear, the FOC conditions in Equations (A.1) and (14) imply that the optimal capital requirement is given by

$$\overline{W}^{\star} = \tilde{W} + \frac{\theta}{\gamma} - \frac{\kappa}{p\gamma} + \left(\mathbf{1}_{N\times 1}\otimes \tilde{\mathbb{E}}\left[s\right]\right)'\left(\hat{\mathbf{x}}\circ\left(1_{NJ\times 1} - \mathbf{a}^{\star}\right)\right),$$

and the optimal targeted interventions are given by

$$\mathbf{a}^{\star} = \left(p\gamma\tilde{\mathbb{V}}\right)^{-1}\left(\kappa\left(\mathbf{1}_{N\times 1}\otimes \tilde{\mathrm{s}}\right)\circ \hat{\mathbf{x}} - \Phi + p\gamma\tilde{\mathbb{V}}\mathbf{1}_{NJ\times 1}\right).$$

Moreover, the regulator's expected utility is given by

$$\mathbb{E}_{\hat{\mathbf{x}}}\left[-\kappa\overline{W}^{\star} + p\theta\tilde{\mathbb{E}}\left(W-\tilde{W}\,|\mathscr{S}\right) - \frac{p\gamma}{2}\tilde{\mathbb{E}}\left(\left(W-\tilde{W}\right)^2\,|\mathscr{S}\right) - \Phi'\mathbf{a}^{\star}\right].$$

Note that given the expression for $\overline{W}^{\star}$above, we have

$$\tilde{\mathbb{E}}\left(W-\tilde{W}\,|\mathscr{S}\right) = \frac{\theta}{\gamma} - \frac{\kappa}{p\gamma}$$

and

$$\tilde{\mathbb{E}}\left(\left(W-\tilde{W}\right)^2\,|\mathscr{S}\right) = \left(\frac{\theta}{\gamma}-\frac{\kappa}{p\gamma}\right)^2 + \left(1_{NJ\times 1} - \mathbf{a}^{\star}\right)'\tilde{\mathbb{V}}\left(1_{NJ\times 1} - \mathbf{a}^{\star}\right),$$

where we used that

$$\tilde{\mathbb{C}ov}\left[\left(\mathbf{1}_{N\times 1}\otimes \mathrm{s}\right)'\left(\mathbf{x}\circ\left(1_{NJ\times 1}-\mathbf{a}^{\star}\right)\right)|\mathscr{S}\right]=\left(1_{NJ\times 1}-\mathbf{a}^{\star}\right)'\tilde{\mathbb{V}}\left(1_{NJ\times 1}-\mathbf{a}^{\star}\right).$$

Moreover, note that the expression for $\mathbf{a}^{\star}$ above implies

$$\left(1_{NJ\times 1}-\mathbf{a}^{\star}\right)'p\gamma\tilde{\mathbb{V}}\left(1_{NJ\times 1}-\mathbf{a}^{\star}\right)=\left(-\kappa\left(\mathbf{1}_{N\times 1}\otimes\tilde{\mathrm{s}}\right)\circ\hat{\mathbf{x}}+\Phi\right)\left(1_{NJ\times 1}-\mathbf{a}^{\star}\right).$$

Therefore, the regulators expected utility can be written as

$$\mathbb{E}_{\hat{\mathbf{x}}}\left[-\frac{1}{2}\kappa\overline{W}^{\star}-\frac{1}{2}\left(\tilde{W}+\frac{\theta}{\gamma}-\frac{\kappa}{p\gamma}\right)+p\theta\left(\frac{\theta}{\gamma}-\frac{\kappa}{p\gamma}\right)-\frac{p\gamma}{2}\left(\frac{\theta}{\gamma}-\frac{\kappa}{p\gamma}\right)^{2}-\frac{1}{2}\Phi' 1_{NJ\times 1}-\frac{1}{2}\Phi'\mathbf{a}^{\star}\right]$$

and therefore, the regulator's problem is equivalent to minimizing

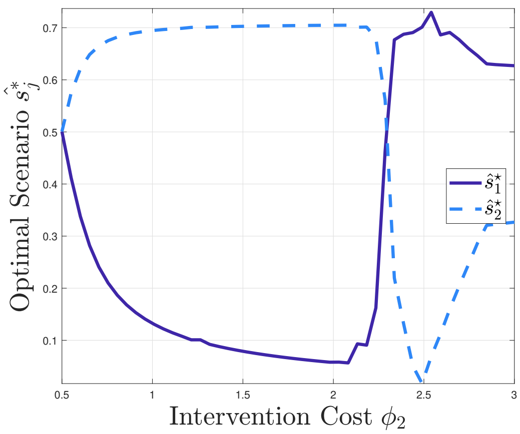
$$\mathbb{E}_{\hat{\mathbf{x}}}\left[\kappa\overline{W}^{\star}+\Phi'\mathbf{a}^{\star}\right].$$

C Additional comparative statics

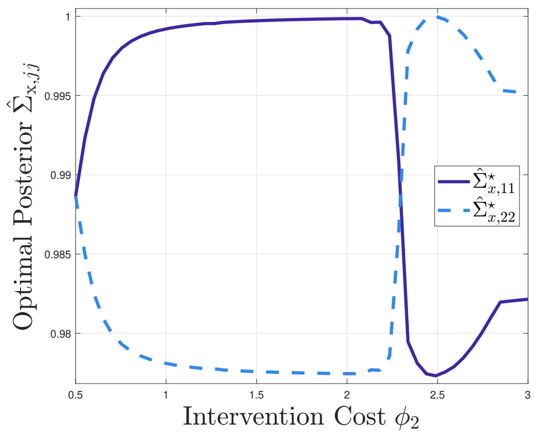
In this section, we include additional comparative statics to showcase how the optimal scenario choice depends on the regulator's priors and the costs of intervention.

C.1 Intervention costs

The first important point is that intervention costs have a non monotone impact on scenario design. When intervention costs are low, the regulator can intervene preemptively to reduce exposures. Inaccurate interventions are not too costly and the regulator cares little about learning about that factor. When the intervention costs are intermediate, interventions are sensitive to the information produced by the stress tests and the regulator values learning to avoid wasteful interventions. Finally, when the intervention costs are high, the ex-post interventions hit the boundary and learning is less valuable for the regulator.



(a) Optimal scenario choice

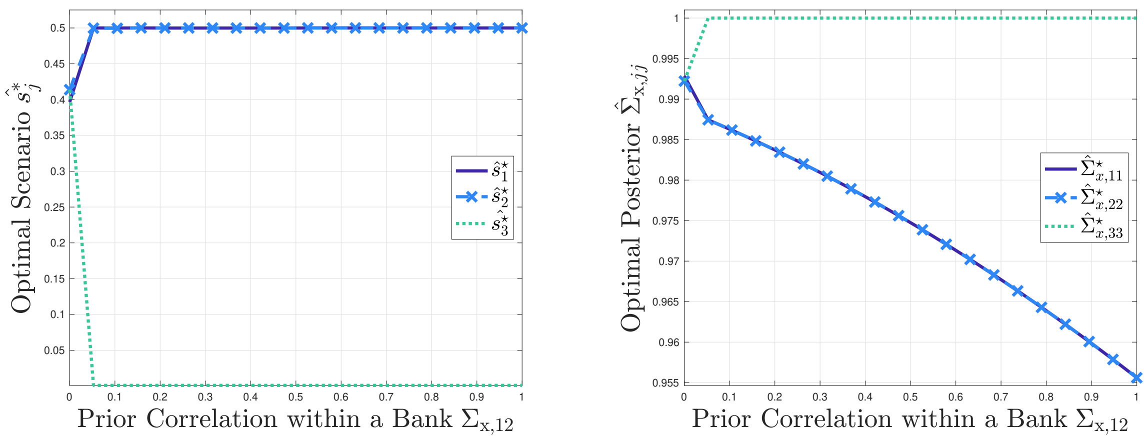


(b) Optimal posterior variances

Figure B.2: Optimal scenario and information choice as a function of the intervention cost Φ_2

Note: Figure B.2 illustrates the regulator's optimal choice of scenario and the implied posterior variance as a function of the intervention cost Φ_2. The parameters used are $N = 2$, $J = 2$, $\theta = 0$, $p\gamma = 0.3$, $\bar{x} = [1, 1]'$, $\Sigma_{\boldsymbol{x}} = \boldsymbol{I}_J$, $\alpha = 0$, $\beta = 5$, $\beta_\tau = 7$, $\tau = 2$, $\mathbb{E}[s] = [0, 0]'$, $\tilde{\mathbb{E}}[s] = [1, 1]'$, $\Sigma_s = \boldsymbol{I}_J$, $\tilde{\Sigma_s} = 3 \times \boldsymbol{I}_J$, $\kappa = 0.1$, $\tilde{W} = 100$, , and $\mathbb{E}[\epsilon\epsilon'] = \boldsymbol{I}_N$, $\mathcal{A} = [-0.7, 1.25]$.

C.2 Correlated exposures within banks



(a) Optimal scenario choice

(b) Optimal posterior variance

Figure B.3: One bank, Three Risk Factors. Optimal scenario and information as a function of $\Sigma_{\mathbf{x},12}$, the prior correlation among exposures 1 and 2.

Note: The parameters used are $M = 1$, $N = 1$, $J = 3$, $\theta = 0$, $p\gamma = 0.1$, $\Phi = [0.1, 0.1, 0.1]'$, $\bar{x} = [1, 1]'$, $\Sigma_{\boldsymbol{x}} = \boldsymbol{I}_J$, $\alpha = 0$, $\beta = 5$, $\beta_\tau = 7$, $\tau = 2$, $\mathbb{E}[s] = [0, 0]'$, $\tilde{\mathbb{E}}[s] = [1, 1]'$, $\Sigma_s = \boldsymbol{I}_J$, $\tilde{\Sigma_s} = 3 \times \boldsymbol{I}_J$, $\kappa = 0.1$, $\tilde{W} = 100$, , and $\mathbb{E}[\epsilon\epsilon'] = \boldsymbol{I}_N$.

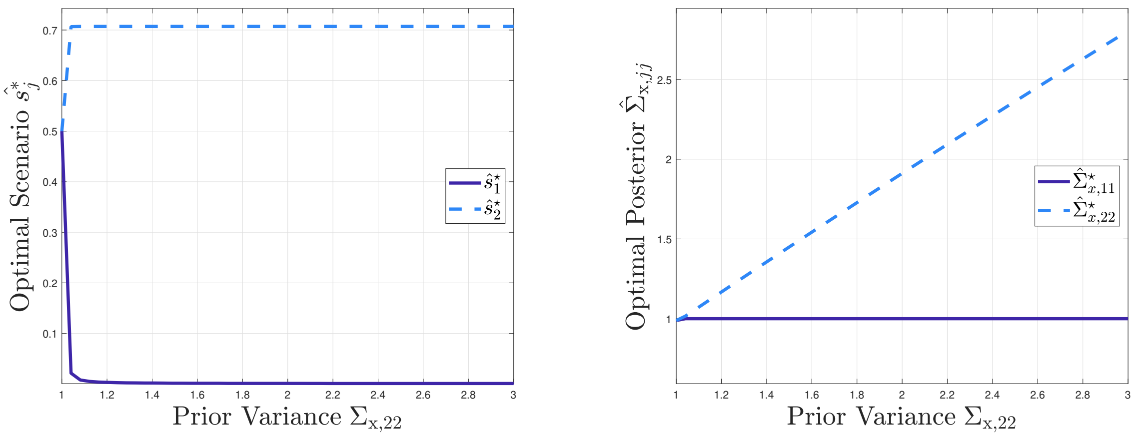
Let us now consider the role of correlation among risk exposures within a bank. Figure (B.1) shows that prior correlations affect the shape of $\boldsymbol{\Sigma}$, the feasible set of posterior precisions. When correlations are low, stressing one risk factor conveys little information about other risk exposures. When correlation are high, signals about one exposure contain information about the others.

Panel (a) in Figure B.3 plots the optimal stresses among 3 factors as a function of prior correlation among the first two exposures. Panel (b) shows that the amount of information that the regulator can learn increases with the prior correlation. Note that if the correlation among two factors is high enough, the regulator may choose not to learn about the other independent dimension.

C.3 Uncertainty

Two dimensions of uncertainty shape the regulator's choice of stress scenarios: uncertainty about risk exposures and uncertainty about risk factors. The regulator intervenes more along dimensions about which she is more uncertain. When the regulator is more uncertain about exposures to risk factor j, her targeted intervention along dimension j is more responsive to the information contained in the stress test results and information is more valuable.

Figure B.4 shows the effect of prior uncertainty regarding exposures to factor 2, $\Sigma_{\mathbf{x},22}$, on the optimal stress on factors 1 and 2. When $\Sigma_{\mathbf{x},22}$ is high the regulator stresses factor 2 to improve the efficiency



(a) Optimal scenario choice

(b) Optimal posterior variances

Figure B.4: Optimal scenario and information choice as a function of the regulator's prior uncertainty of the exposure to factor 2, $\Sigma_{\mathbf{x},22}$.

Note: Figure B.4 illustrates the regulator's optimal choice of scenario and the implied posterior variance as a function of the regulator's prior variance of the exposure to factor 2, $\Sigma_{\mathbf{x},22}$. The parameters used are $N = 2$, $J = 2$, $\theta = 0$, $p\gamma = 0.3$, $\bar{x} = [1, 1]'$, $\Sigma_{\boldsymbol{x}} = \boldsymbol{I}_J$, $\alpha = 0$, $\beta = 5$, $\beta_\tau = 7$, $\tau = 2$, $\mathbb{E}[s] = [0, 0]'$, $\Phi = [0.1, 0.1]'$, $\Sigma_s = \boldsymbol{I}_J$, $\tilde{\Sigma}_s = 3 \times \boldsymbol{I}_J$, $\kappa = 0.1$, $\tilde{W} = 100$, , and $\mathbb{E}[\epsilon\epsilon'] = \boldsymbol{I}_N$, $\mathcal{A} = [-0.7, 1.25]$.

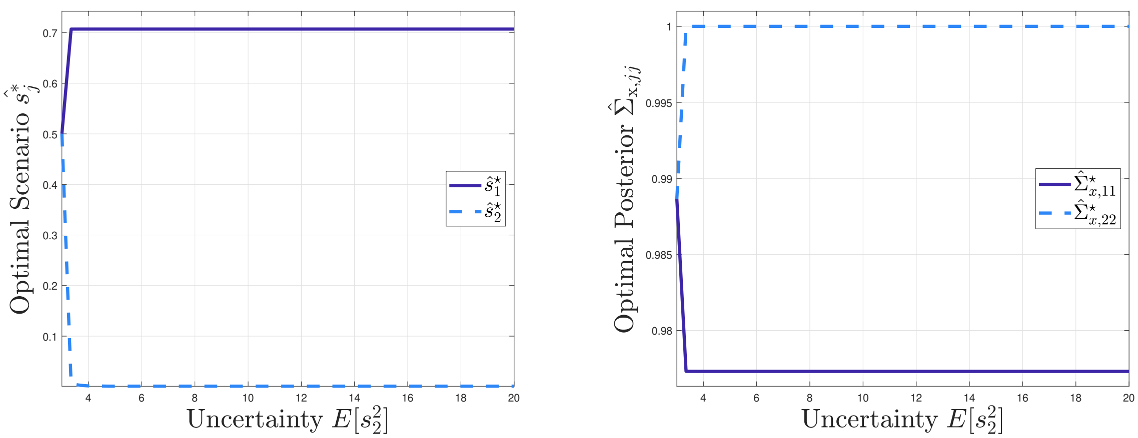
of her expected interventions. The consequences of uncertainty about the risk factors themselves are similar, as shown in Figure B.5 in the Appendix.

C.4 Uncertainty about risk factors

If one risk factor has a very low variance and will stay close to the baseline, then it is less valuable to learn about the exposures to it and to intervene to reduce them. In this case, the factor's weight on the expected losses will be small and uncertainty about the exposure to it is less costly. However, if the variance of a risk factor is large it has the potential to be an important driver of bank losses depending on the risk exposures to it. In this case, the regulator has more incentives to learn and intervene along the dimension of this factor to curve its potential impact on losses. Therefore, the regulator will stress a risk factor more in the optimal scenario the highest the uncertainty about it. Figures B.5 show the optimal scenario choice as a function of the uncertainty about risk factor 2, $\mathbb{E}[s_2^2]$.

D Calibration

We replicate CLASS model regressions based on historical quarterly bank-level data and code available from Hirtle et al. (2014). In order to identify the most important macroeconomic variables, we regress



(a) Optimal scenario choice

(b) Optimal posterior

Figure B.5: Optimal scenario and information choice as a function of the uncertainty about risk factor 2, $\mathbb{E}\left[s_2^2\right]$.

Note: Figure B.5 illustrates the regulator's optimal choice of scenario and the implied posterior variance as a function of the uncertainty about risk factor 2, $\mathbb{E}\left[s_2^2\right]$. The parameters used are $N = 1$, $J = 2$, $\theta = 0$, $p\gamma = 0.1$, $\Phi = [0.75, 0.75]'$, $\bar{x} = [1, 1]'$, $\Sigma_{\boldsymbol{x}} = \boldsymbol{I}_J$, $\alpha = 0$, $\beta = 5$, $\beta_\tau = 7$, $\tau = 2$, $\mathbb{E}[s] = [0, 0]'$, $\tilde{\mathbb{E}}[s] = [1, 1]'$, $\Sigma_s = \boldsymbol{I}_J$, $\tilde{\tilde{\mathbb{E}}}[s] = 3 \times \boldsymbol{I}_J$, $\kappa = 0.1$, $\tilde{W} = 100$, and $\mathbb{E}[\epsilon\epsilon'] = \boldsymbol{I}_N$.

banks' net charge-offs to total loans rate (annualized NCO rate (in %)) on the one quarter lag of all domestic macroeconomic variables for the time-series of 1991-2013. In line with the CLASS model, we include a one-quarter lag of the dependent variable and cluster standard errors over time. We implement the regression for the aggregate banking system, the panel of banks participating in DFAST exercises, as well as bank-individual time-series regressions. For the panel regression, we additionally include a bank-fixed-effect. Consistent across all three specifications, the macroeconomic variables with highest explanatory power are the GDP growth rate, and growth rate of commercial and residential real estate indexes, which we combine into an equal weighted real estate index.

We standardize both macro factors to be able to interpret the magnitudes in terms of standard deviations. To calibrate the regulator's beliefs, we regress the mean NCO rate on the cumulative GDP growth rate over the previous four quarters and a one-quarter lag of the combined Real-Estate Index. This specification captures 69 percent of the variation in the NCO rate. The coefficient for both factors is similar, -0.3571737 and -0.3027348, respectively. The asymptotic variance of these estimates is 0.00370976 and 0.00350896, respectively. We round the estimates of the prior mean exposures to 0.36 and their variance to 0.0036.

We assume the following functional form for standard deviation of the bank model error term.

$$\sigma_{\hat{\varepsilon}} = \alpha + \beta \|s\|^2.$$

We base our calibration of the parameters α and β on the coefficients from a regression of the standard deviation of the NCO rate on the squared norm of both factors, shown in the table below. We set $\alpha = 0.55$ and $\beta = 0.11$, to take into account that the value of β recovered from the regression is based on normal times and hence overestimates the correlation between the macro factors and the model errors.

	$\sigma_{\hat{\varepsilon}}$
α	.5491956***
	(.0570604)
β	.2262904***
	(.0269786)
N	80
R-squared	0.4742
Adj. R-squared	0.4675

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
t statistics in parentheses

Table 3: Regression to recover the standard deviation of the error term of the banks' model