Econometrics I

Professor William Greene
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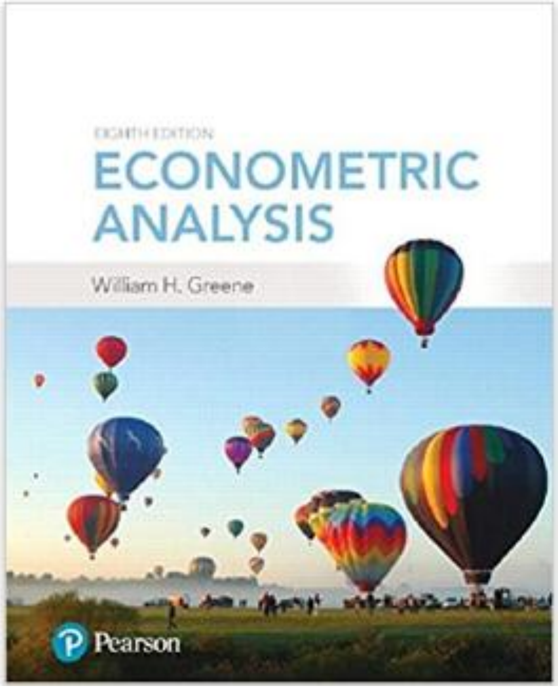
Department of Economics



Econometrics I

Part 15 – Panel Data-1

Panel Data Sets

- Longitudinal data
  - British household panel survey (BHPS)
  - Panel Study of Income Dynamics (PSID)
  - … many others
- Cross section time series
  - Penn world tables
- Financial data by firm, by year
  - $r_{it} - r_{ft} = \beta_i(r_{mt} - r_{ft}) + \varepsilon_{it}$, i = 1,…,many; t=1,…many
  - Exchange rate data, essentially infinite T, large N

Benefits of Panel Data

- Time and individual variation in behavior unobservable in cross sections or aggregate time series
- Observable and unobservable individual heterogeneity
- Rich hierarchical structures
- More complicated models
- Features that cannot be modeled with only cross section or aggregate time series data alone
- Dynamics in economic behavior

OFFICE OF FAIR TRADING

Evaluation of an OFT intervention

Independent fee-paying schools

May 2012

Figure 12: Average annual boarding fees of SS Schools: actual and expected in absence of OFT intervention

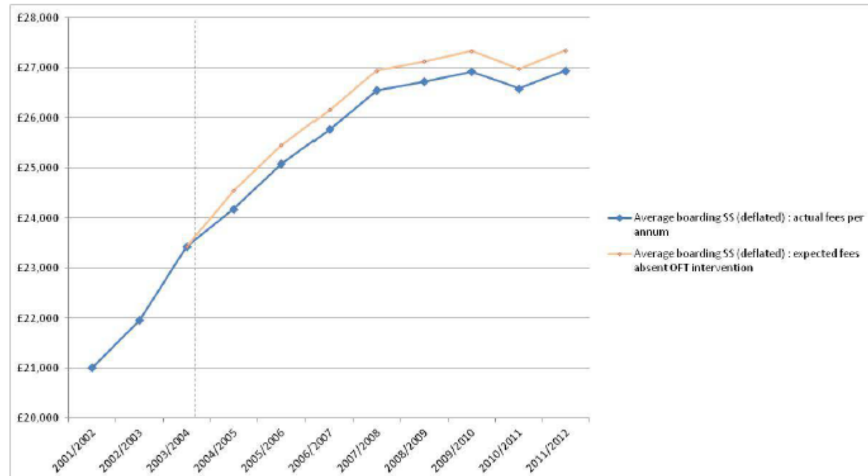
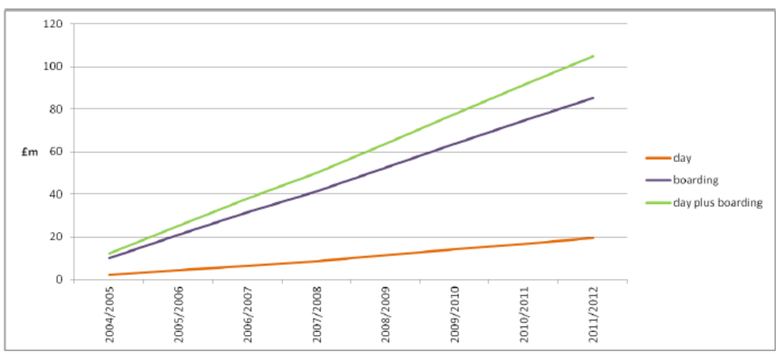


Figure 13: Cumulative savings in fees to the consumer from OFT intervention, 2010 prices, £m, discounted to present



In this context, the OFT's evaluation team has evaluated the impact of the intervention addressing the anti-competitive practice of 50 independent fee-paying schools in the setting of fees during academic years 2001/02 to 2003/04. This research has been carried out by OFT economists and independently reviewed by Professor Stephen Davies.[1]

The main aim is to understand whether the OFT intervention had an impact, and to estimate this impact in terms of reduced school fees. To do so we have collected data on the evolution of school fees and other variables before and after the OFT's intervention.

www.oft.gov.uk/shared_oft/reports/Evaluating-OFTs-work/oft1416.pdf

PSID

A national study of socioeconomics and health over lifetimes and across generations

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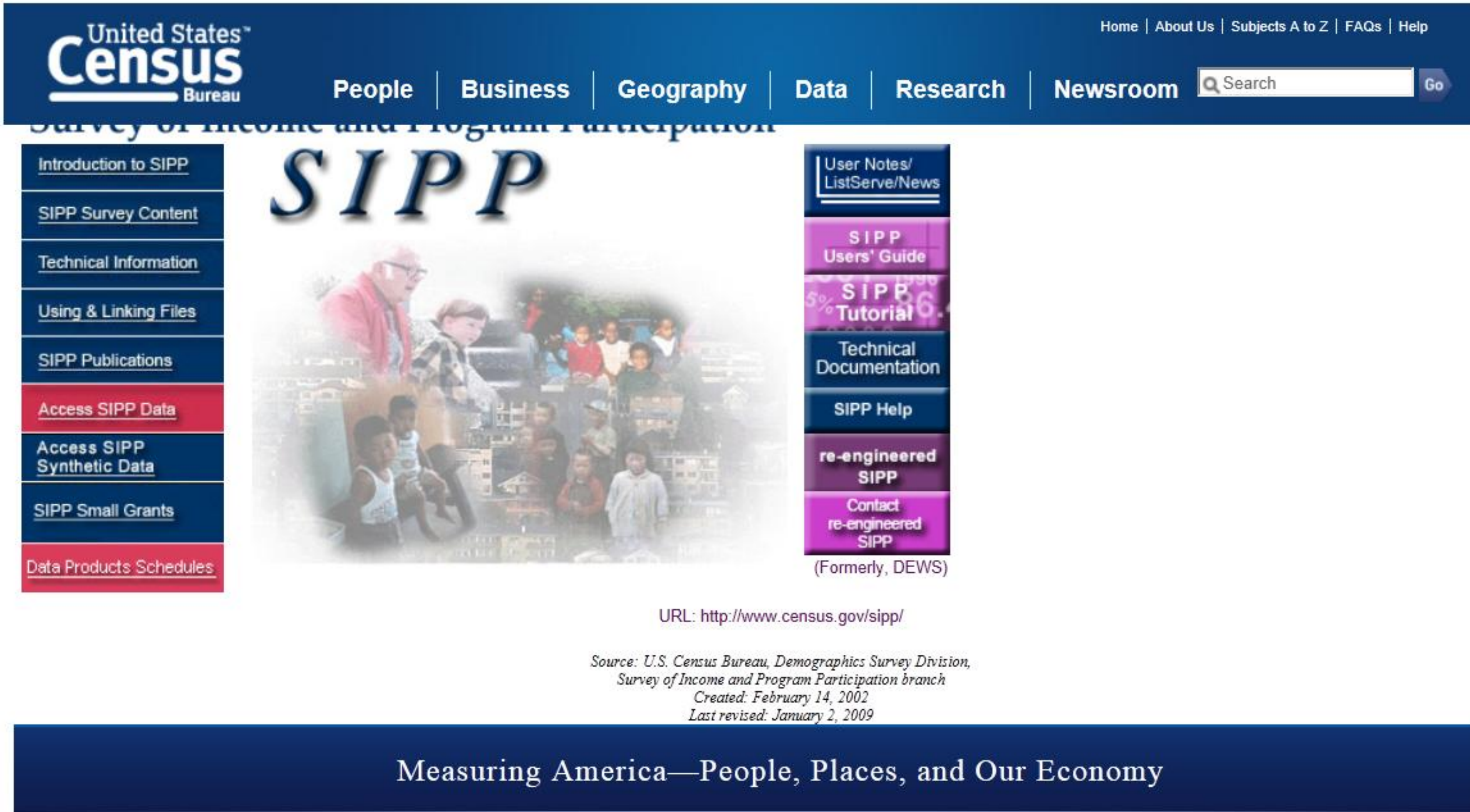
The Panel Study of Income Dynamics - PSID - is the longest running longitudinal household survey in the world.

The study began in 1968 with a nationally representative sample of over 18,000 individuals living in 5,000 families in the United States. Information on these individuals and their descendants has been collected continuously, including data covering employment, income, wealth, expenditures, health, marriage, childbearing, child development, philanthropy, education, and numerous other topics. The PSID is directed by faculty at the University of Michigan, and the data are available on this website without cost to researchers and analysts.

The data are used by researchers, policy analysts, and teachers around the globe. Over 3,000 peer-reviewed publications have been based on the PSID. Recognizing the importance of the data, numerous countries have created their own PSID-like studies that now facilitate cross-national comparative research. The National Science Foundation recognized the PSID as one of the 60 most significant advances funded by NSF in its 60 year history.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Contact re-engineered SIPP
(Formerly, DEWS)
URL: http://www.census.gov/sipp/
Source: U.S. Census Bureau, Demographics Survey Division,
Survey of Income and Program Participation branch
Created: February 14, 2002
Last revised: January 2, 2009
Measuring America—People, Places, and Our Economy

British Household Panel Survey

BHPS

BHPS British Household Panel Survey

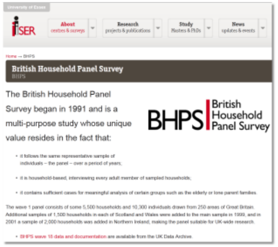
The British Household Panel Survey began in 1991 and is a multi-purpose study whose unique value resides in the fact that:

- it follows the same representative sample of individuals – the panel – over a period of years;
- it is household-based, interviewing every adult member of sampled households;
- it contains sufficient cases for meaningful analysis of certain groups such as the elderly or lone parent families.

The wave 1 panel consists of some 5,500 households and 10,300 individuals drawn from 250 areas of Great Britain. Additional samples of 1,500 households in each of Scotland and Wales were added to the main sample in 1999, and in 2001 a sample of 2,000 households was added in Northern Ireland, making the panel suitable for UK-wide research.

- BHPS wave 18 data and documentation are available from the UK Data Archive.

BHPS Has Evolved



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The Household, Income and Labour Dynamics in Australia (HILDA) Survey

HILDA Survey

The Household, Income and Labour Dynamics in Australia (HILDA) Survey is a household-based panel study which began in 2001. It has the following key features:

- It collects information about economic and subjective well-being, labour market dynamics and family dynamics.
- Special questionnaire modules are included each wave.
- The wave 1 panel consisted of 7,682 households and 19,914 individuals. In wave 11 this was topped up with an additional 2,153 households and 5,477 individuals.
- Interviews are conducted annually with all adult members of each household.
- The panel members are followed over time.
- The funding has been guaranteed for sixteen waves, though the survey is designed to continue for longer than this.
- Academic and other researchers can apply to use the General Release datasets for their research.

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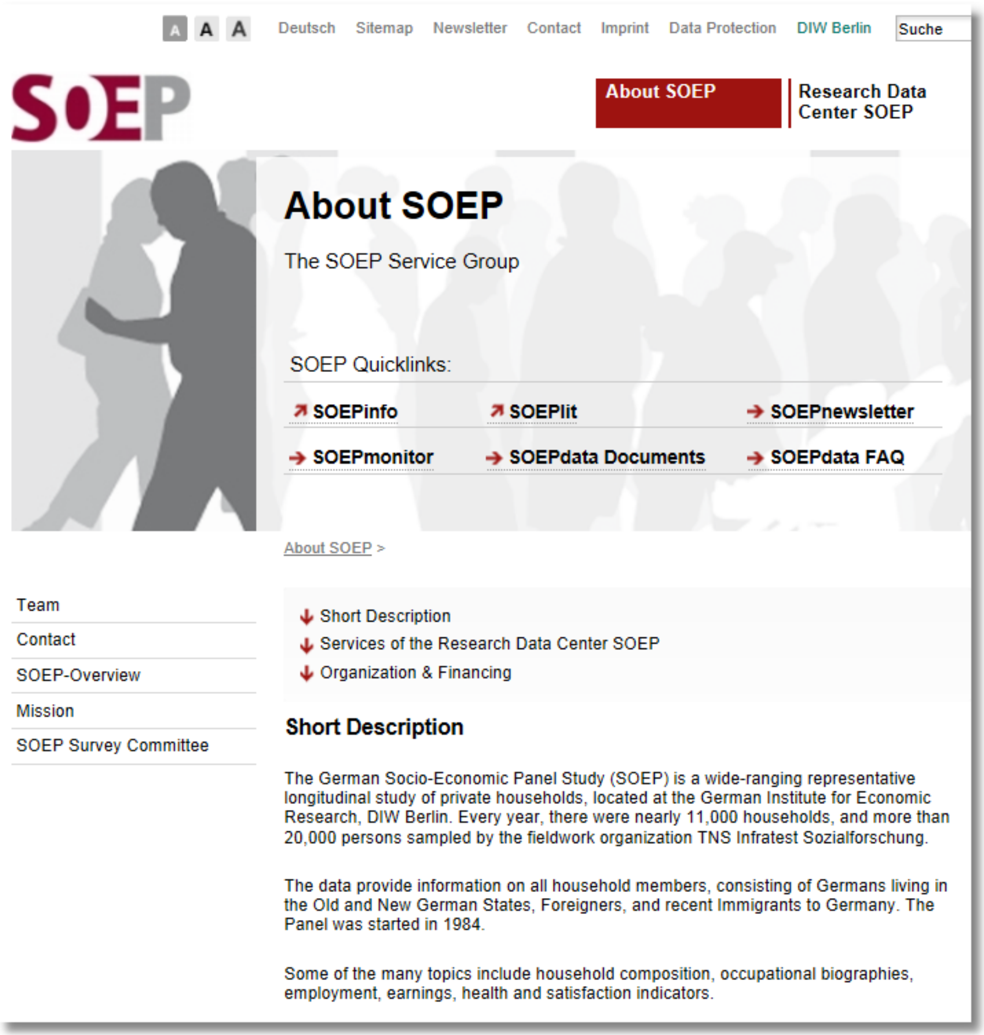
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Short Description

The German Socio-Economic Panel Study (SOEP) is a wide-ranging representative longitudinal study of private households, located at the German Institute for Economic Research, DIW Berlin. Every year, there were nearly 11,000 households, and more than 20,000 persons sampled by the fieldwork organization TNS Infratest Sozialforschung.

The data provide information on all household members, consisting of Germans living in the Old and New German States, Foreigners, and recent Immigrants to Germany. The Panel was started in 1984.

Some of the many topics include household composition, occupational biographies, employment, earnings, health and satisfaction indicators.

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European Community Household Panel (ECHP)

ECHP microdata for scientific purposes: how to obtain them?

Description of dataset

The European Community Household Panel (ECHP) is a panel survey in which a sample of households and persons have been interviewed year after year.

These interviews cover a wide range of topics concerning living conditions. They include detailed income information, financial situation in a wider sense, working life, housing situation, social relations, health and biographical information of the interviewed.

The total duration of the ECHP was 8 years, running from 1994-2001 (8 waves).

ECHP based data in the database

99% of the "income and living conditions" domain under theme "Population and social conditions" is derived from ECHP. This includes many indicators of relative monetary poverty and of income inequality, analysed in different ways (eg. different cut-off thresholds, by age, gender, activity status, tenure status...).

It also includes a selection of indicators of social exclusion and non-monetary deprivation derived from ECHP, notably on housing.

Of these, 4 have been chosen as structural indicators, namely the at-risk-of-poverty rate before cash social transfers, the persistent at-risk-of-poverty rate and the s80/s20 income quintile share ratio. The at-risk-of-poverty rate after social transfers is a headline indicator.

A selection of indicators in the "health status" and "health care" collections of the "public health" domain also under the above-mentioned same theme are derived from ECHP as well.

See Also

- Additional information on ECHP
- Income, Social Inclusion and Living Conditions

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National Longitudinal Surveys

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The **National Longitudinal Surveys (NLS)** are a set of surveys designed to gather information at multiple points in time on the labor market activities and other significant life events of several groups of men and women. For more than 4 decades, NLS data have served as an important tool for economists, sociologists, and other researchers.

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NLS General Overviews

- National Longitudinal Survey of Youth 1997 (NLSY97)-- Survey of young men and women born in the years 1980-84; respondents were ages 12-17 when first interviewed in 1997.
- National Longitudinal Survey of Youth 1979 (NLSY79)-- Survey of men and women born in the years 1957-64; respondents were ages 14-22 when first interviewed in 1979.
- NLSY79 Children and Young Adults-- Survey of the biological children of women in the NLSY79.
- National Longitudinal Surveys of Young Women and Mature Women (NLSW)-- The Young Women's survey includes women who were ages 14-24 when first interviewed in 1968. The Mature Women's survey includes women who were ages 30-44 when first interviewed in 1967. These surveys were discontinued in 2003.
- National Longitudinal Surveys of Young Men and Older Men-- The Young Men's survey, which was discontinued in 1981, includes men who were ages 14-24 when first interviewed in 1966. The Older Men's survey, which was discontinued in 1990, includes men who were ages 45-59 when first interviewed in 1966.

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Current Population Survey (CPS)

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- Definitions
- Frequently Asked Questions
- Technical Documentation

A Joint Effort Between the Bureau of Labor Statistics and the Census Bureau

The Current Population Survey (CPS), sponsored jointly by the U.S. Census Bureau and the U.S. Bureau of Labor Statistics (BLS), is the primary source of labor force statistics for the population of the United States. The CPS is the source of numerous high-profile economic statistics, including the national unemployment rate, and provides data on a wide range of issues relating to employment and earnings. The CPS also collects extensive demographic data that complement and enhance our understanding of labor market conditions in the nation overall, among many different population groups, in the states and in substate areas.

Related Sites

- Bureau of Labor Statistics
- DataFerrett
- DataWeb FTP page
- Information for CPS Participants

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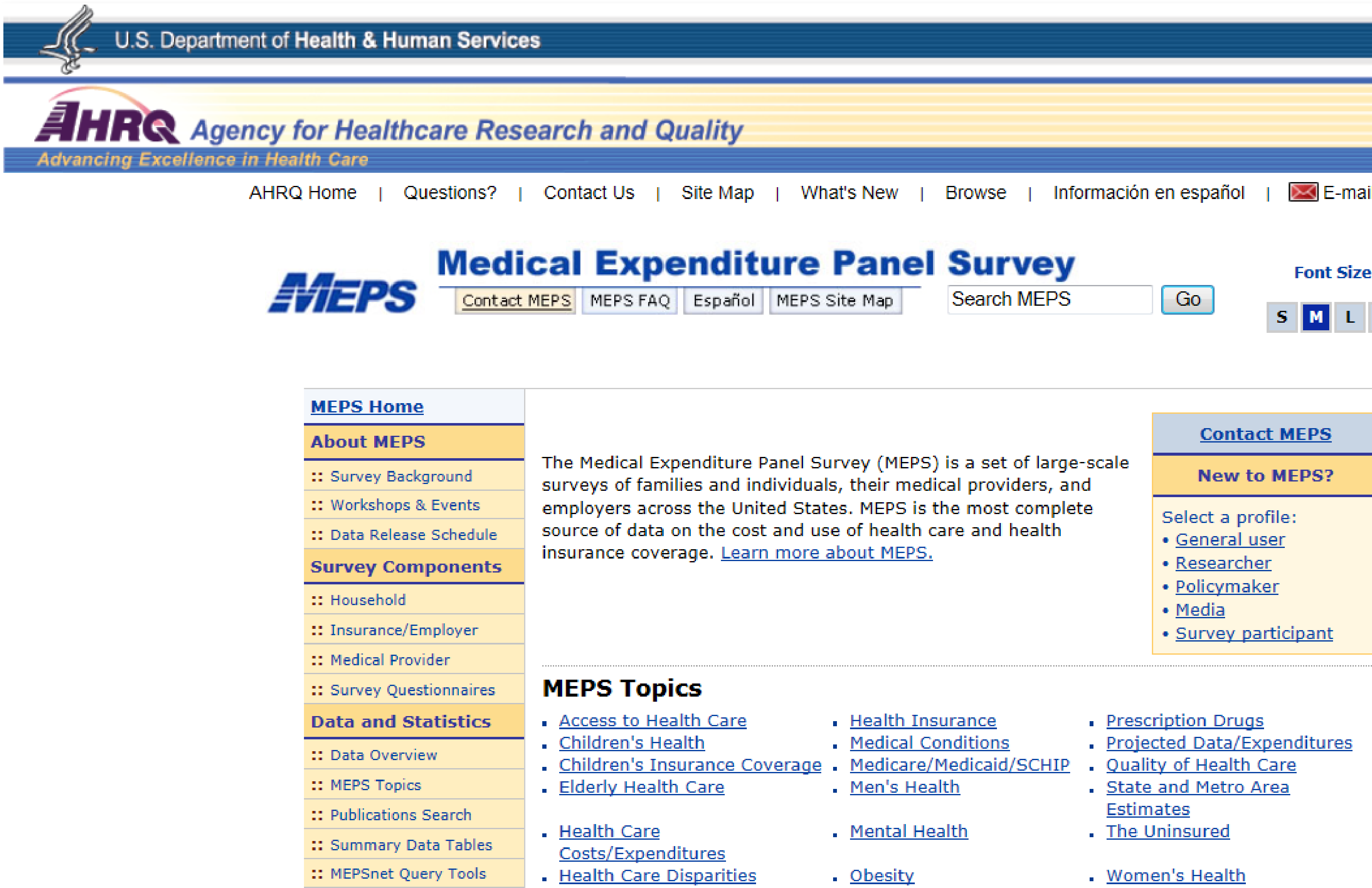
Overview

The annual Agricultural Resource Management Survey (ARMS) is USDA's primary source of information on the financial condition, production practices, and resource use of America's farm businesses and the economic well-being of America's farm households. ARMS data are essential to USDA, congressional, administration, and industry decision makers when weighing alternative policies and programs that touch the farm sector or affect farm families.

Sponsored jointly by ERS and the National Agricultural Statistics Service (NASS), ARMS is the only national survey that provides observations of field-level farm practices, the economics of the farm businesses operating the field (or dairy herd, green house, nursery, poultry house, etc.), and the characteristics of farm operators and their households (age, education, occupation, farm and off-farm work, types of employment, family living expenses, etc.)--all collected in a representative sample. Information about crop production, farm production, business, and households includes data for selected surveyed States where available. See more background on ARMS....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The Medical Expenditure Panel Survey (MEPS) is a set of large-scale surveys of families and individuals, their medical providers, and employers across the United States. MEPS is the most complete source of data on the cost and use of health care and health insurance coverage. Learn more about MEPS.
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Medicare/Medicaid/SCHIP
Men's Health
Mental Health
Obesity
Prescription Drugs
Projected Data/Expenditures
Quality of Health Care
State and Metro Area Estimates
The Uninsured
Women's Health

```
Dairy Farm Data, N = 247, T = 6
Variables
FARM = Farm ID
YEAR = year
Inputs
  COWS,  X1 = log of, deviations from means (logs)
  LAND,  X2 = same
  LABOR, X3 = same
  FEED,  X4 = same
  Translog terms, X11, X22, X33, X44,
                  X12, X13, X14,
                  X23, X24,
                  X34
            = squares and cross products
  YEAR93,...,YEAR98 = year dummy variables
Output
  MILK = farm output
  YIT = log of MILK production
```

Panel Data on 247 Spanish Dairy Farms Over 6 Years

Data Editor

29/900 Vars; 11111 Rows: 1482 Obs Cell: 1

	FARM	AGEL	YEAR	COWS	LAND	MILK	LABOR	FEED
1 »	1	10	93	15.3	8	73647	2	33435.7
2 »	1	10	94	18.1	8	91260	2	36869
3 »	1	10	97	17.1	7	110419	2	51013.6
4 »	1	10	96	17.3	8	111454	2	50711.6
5 »	1	10	95	17.8	8	118498	2	54153.6
6 »	1	10	98	19.5	7.2	131197	2	59038.7
7 »	2	10	93	20.3	9	118149	2	53875.9
8 »	2	10	94	20.3	10.4	127742	2	51991
9 »	2	10	95	22	10.7	146490	2	61379.3
10 »	2	10	96	23.3	10.7	163434	2	71093.8
11 »	2	10	97	23.3	10.6	163603	2	69204.1
12 »	2	10	98	25	9.4	169540	3	73580.4
13 »	3	10	93	19.6	11	102445	2.5	42412.2
14 »	3	10	94	22.2	11	129938	2.5	63149.9
15 »	3	10	96	24.7	11	132594	2.5	54893.9
16 »	3	10	95	25.4	12	134282	2.5	58681.2
17 »	3	10	97	25.3	13.5	140581	2.5	55810.7
18 »	3	10	98	26.1	14.5	182037	2.5	93567
19 »	4	10	93	55.4	22	405042	2.5	196445
20 »	4	10	94	63.5	22	489134	2.5	212773
21 »	4	10	95	[illegible]	22	[illegible]	2.5	[illegible]

Cornwell and Rupert Data

Cornwell and Rupert Returns to Schooling Data, 595 Individuals, 7 Years (Extracted from NLSY.) **Variables in the file are**

EXP	= work experience
WKS	= weeks worked
OCC	= occupation, 1 if blue collar,
IND	= 1 if manufacturing industry
SOUTH	= 1 if resides in south
SMSA	= 1 if resides in a city (SMSA)
MS	= 1 if married
FEM	= 1 if female
UNION	= 1 if wage set by union contract
ED	= years of education
LWAGE	= log of wage = dependent variable in regressions

These data were analyzed in Cornwell, C. and Rupert, P., "Efficient Estimation with Panel Data: An Empirical Comparison of Instrumental Variable Estimators," Journal of Applied Econometrics, 3, 1988, pp. 149-155. See Baltagi, page 122 for further analysis. The data were downloaded from the website for Baltagi's text.

Data Editor

28/900 Vars; 11111 Rows: 4165 Obs   Cell: 0

	LOGWAGE	EDUC
1 »	5.56068	9
2 »	5.72031	9
3 »	5.99645	9
4 »	5.99645	9
5 »	6.06146	9
6 »	6.17379	9
7 »	6.24417	9
8 »	6.16331	11
9 »	6.21461	11
10 »	6.2634	11
11 »	6.54391	11
12 »	6.69703	11
13 »	6.79122	11
14 »	6.81564	11
15 »	5.65249	12
16 »	6.43615	12
17 »	6.54822	12
18 »	6.60259	12
19 »	6.6958	12
20 »	6.77878	12
21 »	6.86066	12

Balanced and Unbalanced Panels

- Distinction: Balanced vs. Unbalanced Panels
- A notation to help with mechanics

$$z_{i,t},\ i = 1,\ldots,N;\ \boxed{t = 1,\ldots,T_i}$$

- The role of the assumption
  - Mathematical and notational convenience:
    - Balanced, n=NT
    - Unbalanced: $\boxed{n = \sum_{i=1}^{N} T_i}$
  - Is the fixed T_i assumption ever necessary? Almost never.
- Is unbalancedness due to **nonrandom** attrition from an otherwise balanced panel? This would require special considerations.

Application: Health Care Usage

German Health Care Usage Data, 7,293 Individuals, Varying Numbers of Periods

This is an unbalanced panel with 7,293 individuals. There are altogether 27,326 observations. The number of observations ranges from 1 to 7.

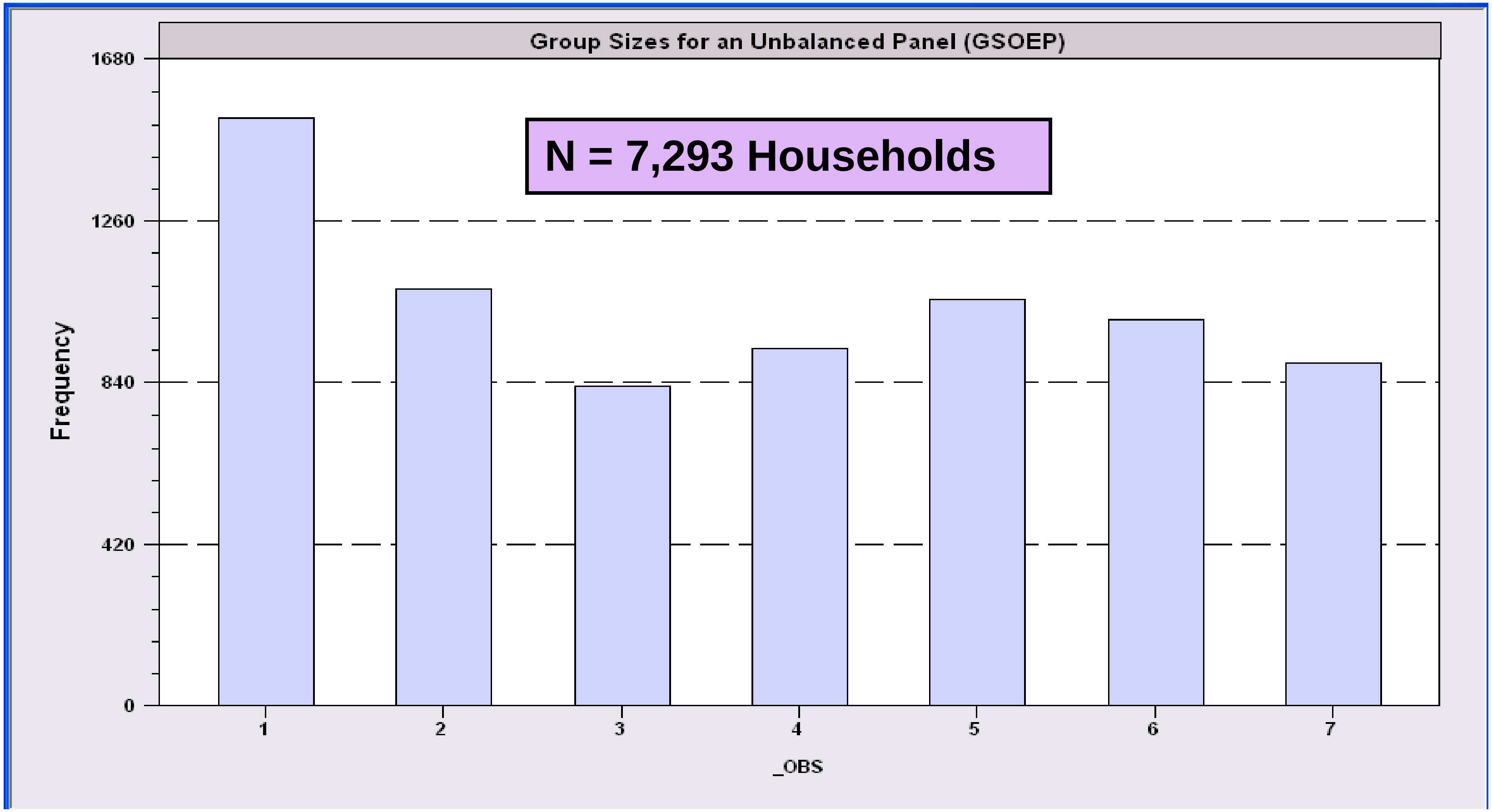
(Frequencies are: 1=1525, 2=2158, 3=825, 4=926, 5=1051, 6=1000, 7=987).

(Downloaded from the JAE Archive)

Variables in the file are

DOCTOR	=	**1(Number of doctor visits > 0)**
HOSPITAL	=	**1(Number of hospital visits > 0)**
HSAT	=	**health satisfaction, coded 0 (low) - 10 (high)**
DOCVIS	=	**number of doctor visits in last three months**
HOSPVIS	=	**number of hospital visits in last calendar year**
PUBLIC	=	**insured in public health insurance = 1; otherwise = 0**
ADDON	=	**insured by add-on insurance = 1; otherswise = 0**
HHNINC	=	**household nominal monthly net income in German marks / 10000**. (4 observations with income=0 were dropped)
HHKIDS	=	**children under age 16 in the household = 1; otherwise = 0**
EDUC	=	**years of schooling**
AGE	=	**age in years**
MARRIED	=	**marital status**

An Unbalanced Panel: RWM's GSOEP Data on Health Care



A Basic Model for Panel Data

- Unobserved individual effects in regression: $E[y_{it} \mid \mathbf{x}_{it}, c_i]$

  Notation: $y_{it} = \mathbf{x}_{it}'\beta + c_i + \varepsilon_{it}$

$$\mathbf{X}_i = \begin{bmatrix} \mathbf{x}_{i1}' \\ \mathbf{x}_{i2}' \\ \vdots \\ \mathbf{x}_{iT_i}' \end{bmatrix} \quad T_i \text{ rows, K columns}$$

- Linear specification:

  Fixed Effects: $E[c_i \mid \mathbf{X}_i] = g(\mathbf{X}_i)$. $Cov[\mathbf{x}_{it}, c_i] \neq \mathbf{0}$
  effects are correlated with included variables.

  Random Effects: $E[c_i \mid \mathbf{X}_i] = 0$. $Cov[\mathbf{x}_{it}, c_i] = \mathbf{0}$

Convenient Notation

- Fixed Effects – the ‘dummy variable model’

$$y_{it} = \alpha_i + \mathbf{x}_{it}'\boldsymbol{\beta} + \varepsilon_{it}$$

Individual specific constant terms.

- Random Effects – the ‘error components model’

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + \varepsilon_{it} + u_i$$

Compound (“composed”) disturbance

Explaining Fixed Effects: Random Effects modelling of Time-Series Cross-Sectional and Panel Data

Andrew Bell and Kelvyn Jones

School of Geographical Sciences

Centre for Multilevel Modelling

University of Bristol

Last updated: 11th Sept 2013

http://people.stern.nyu.edu/wgreene/Econometrics/Bell-Jones-Fixed-vs-Random-Sept-2013.pdf

Estimating β

- **β** is the partial effect of interest

- Can it be estimated (consistently) in the presence of (unmeasured) c_i?
  - Does pooled least squares "work?"
  - Strategies for "controlling for c_i" using the sample data

Assumptions for Asymptotics

- Convergence of moments involving cross section $\mathbf{X}_i$.
- N increasing, T or T_i assumed fixed.
  - "Fixed T asymptotics" (see text, p. 348)
  - Time series characteristics are not relevant (may be nonstationary – relevant in Penn World Tables)
  - If T is also growing, need to treat as multivariate time series.
- Ranks of matrices. $\mathbf{X}$ must have full column rank. ($\mathbf{X}_i$ may not, if $T_i < K$.)
- Strict exogeneity and dynamics. If $\mathbf{x}_{it}$ contains $y_{i,t-1}$ then $\mathbf{x}_{it}$ cannot be strictly exogenous. $\mathbf{X}_{it}$ will be correlated with the unobservables in period t-1. (To be revisited later.)
- Empirical characteristics of microeconomic data

The Pooled Regression

- Presence of omitted effects

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + c_i + \varepsilon_{it}, \text{ observation for person i at time t}$$

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + c_i\mathbf{i} + \boldsymbol{\varepsilon}_i, \ T_i \text{ observations in group i}$$

$$= \mathbf{X}_i\boldsymbol{\beta} + \mathbf{c}_i + \boldsymbol{\varepsilon}_i, \text{ note } \mathbf{c}_i = (c_i, c_i, \ldots, c_i)'$$

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{c} + \boldsymbol{\varepsilon}, \ \Sigma_{i=1}^{N} T_i \text{ observations in the sample}$$

- Potential bias/inconsistency of OLS – depends on ‘fixed’ or ‘random’

OLS in the Presence of Individual Effects

$$\mathbf{b}=(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

$$=\boldsymbol{\beta}+\left[(1/N)\Sigma_{i=1}^{N}\mathbf{X}_i'\mathbf{X}_i\right]^{-1}\left[(1/N)\Sigma_{i=1}^{N}\mathbf{X}_i'\mathbf{c}_i\right] \text{ (part due to the omitted } c_i\text{)}$$

$$+\left[(1/N)\Sigma_{i=1}^{N}\mathbf{X}_i'\mathbf{X}_i\right]^{-1}\left[(1/N)\Sigma_{i=1}^{N}\mathbf{X}_i'\boldsymbol{\varepsilon}_i\right] \text{ (covariance of } \mathbf{X} \text{ and } \boldsymbol{\varepsilon} \text{ will = 0)}$$

The third term vanishes asymptotically by assumption

$$\text{plim } \mathbf{b} = \boldsymbol{\beta} + \text{plim}\left[\frac{1}{N}\Sigma_{i=1}^{N}\mathbf{X}_i'\mathbf{X}_i\right]^{-1}\left[\Sigma_{i=1}^{N}\frac{T_i}{N}\bar{\mathbf{x}}_i c_i\right] \text{ (left out variable formula)}$$

So, what becomes of $\left[\Sigma_{i=1}^{N} w_i \bar{\mathbf{x}}_i c_i\right]$?

plim **b** = $\boldsymbol{\beta}$ if the covariance of $\bar{\mathbf{x}}_i$ and c_i converges to zero.

Estimating the Sampling Variance of **b**

- $s^2(\mathbf{X}'\mathbf{X})^{-1}$? Inappropriate because
  - Correlation across observations (certainly)
  - Heteroscedasticity (possibly)

- A ‘robust’ covariance matrix
  - Robust estimation (in general)
  - The White estimator
  - A Robust estimator for OLS.

Cluster Estimator

Robust variance estimator for Var[**b**]

Est.Var[**b**]

$$= (\mathbf{X'X})^{-1}\left[\Sigma_{i=1}^{N}(\Sigma_{t=1}^{T_i}\mathbf{x}_{it}\hat{v}_{it})(\Sigma_{t=1}^{T_i}\mathbf{x}_{it}'\hat{v}_{it})\right](\mathbf{X'X})^{-1}$$

$$= (\mathbf{X'X})^{-1}\left[\Sigma_{i=1}^{N}\left(\Sigma_{t=1}^{T_i}\Sigma_{s=1}^{T_i}\hat{v}_{it}\hat{v}_{is}\mathbf{x}_{it}\mathbf{x}_{is}'\right)\right](\mathbf{X'X})^{-1}$$

$$\hat{v}_{it} = \text{ a least squares residual } = \varepsilon_{it} + c_i$$

(If T_i = 1, this is the White estimator.)

```
----------------------------------------------------------------------------
Ordinary    least squares regression ............
LHS=YIT     Mean                 =       11.57749
            Standard deviation   =         .64344
----------  No. of observations  =           1482  DegFreedom   Mean square
Regression  Sum of Squares       =        584.056           4     146.01403
Residual    Sum of Squares       =        29.0957        1477        .01970
Total       Sum of Squares       =        613.152        1481        .41401
----------  Standard error of e  =         .14035  Root MSE           .14012
Fit         R-squared            =         .95255  R-bar squared      .95242
Model test  F[  4,  1477]        =     7412.18529  Prob F > F*        .00000
--------+-------------------------------------------------------------------
        |                Clustered                Prob.      95% Confidence
     YIT|  Coefficient   Std.Error       z       |z|>Z*         Interval
--------+-------------------------------------------------------------------
Constant|    11.5775***     .01599    723.88    .0000     11.5461   11.6088
      X1|     .59518***     .05821     10.22    .0000      .48109    .70926
      X2|     .02305        .02692       .86    .3918     -.02971    .07581
      X3|     .02319        .02816       .82    .4102     -.03201    .07839
      X4|     .45176***     .03768     11.99    .0000      .37791    .52561
--------+-------------------------------------------------------------------
***, **, * ==>  Significance at 1%, 5%, 10% level.
Standard errors clustered on AGEL     (     7 clusters)
----------------------------------------------------------------------------

--------+-------------------------------------------------------------------
        |                Standard                 Prob.      95% Confidence
     YIT|  Coefficient     Error         z       |z|>Z*         Interval
--------+-------------------------------------------------------------------
Constant|    11.5775***     .00365   3175.52    .0000     11.5703   11.5846
      X1|     .59518***     .01958     30.39    .0000      .55679    .63356
      X2|     .02305**      .01122      2.05    .0400      .00105    .04505
      X3|     .02319*       .01303      1.78    .0751     -.00235    .04873
      X4|     .45176***     .01078     41.89    .0000      .43062    .47290
--------+-------------------------------------------------------------------
***, **, * ==>  Significance at 1%, 5%, 10% level.
----------------------------------------------------------------------------
```

Alternative OLS Variance Estimators

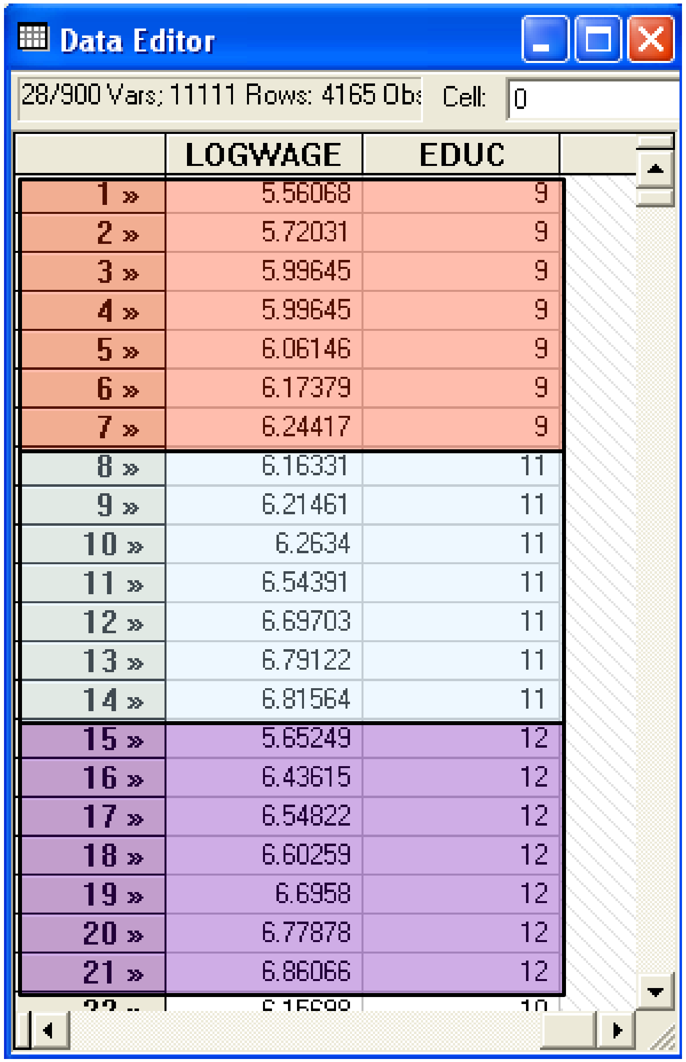
Cluster correction increases SEs

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]
Constant	5.40159723	.04838934	111.628	.0000
EXP	.04084968	.00218534	18.693	.0000
EXPSQ	-.00068788	.480428D-04	-14.318	.0000
OCC	-.13830480	.01480107	-9.344	.0000
SMSA	.14856267	.01206772	12.311	.0000
MS	.06798358	.02074599	3.277	.0010
FEM	-.40020215	.02526118	-15.843	.0000
UNION	.09409925	.01253203	7.509	.0000
ED	.05812166	.00260039	22.351	.0000
Robust				
Constant	5.40159723	.10156038	53.186	.0000
EXP	.04084968	.00432272	9.450	.0000
EXPSQ	-.00068788	.983981D-04	-6.991	.0000
OCC	-.13830480	.02772631	-4.988	.0000
SMSA	.14856267	.02423668	6.130	.0000
MS	.06798358	.04382220	1.551	.1208
FEM	-.40020215	.04961926	-8.065	.0000
UNION	.09409925	.02422669	3.884	.0001
ED	.05812166	.00555697	10.459	.0000

```
Namelist ; x=one,exp,expsq,occ,smsa,ms,fem,union,ed$
Regress  ; Lhs = lwage ; rhs=x ; cluster=7$
```

Results of Bootstrap Estimation

```
--------------------------------------------------------------------------------
Results of bootstrap estimation of model.
Model has been reestimated      20 times.
Coefficients shown below are the original
model estimates based on the full sample.
Bootstrap samples have 4165 observations.
Estimated parameter vector is B       .
Estimated variance matrix saved as VARB.
--------+-----------------------------------------------------------------------
        |                  Standard            Prob.      95% Confidence
BootStrp|  Coefficient       Error        z    |z|>Z*        Interval
--------+-----------------------------------------------------------------------
    B001|    5.66098***      .05161   109.68  .0000     5.55982   5.76214
    B002|    -.11220***      .01202    -9.34  .0000     -.13576   -.08864
    B003|     .15504***      .01497    10.35  .0000      .12570    .18439
    B004|     .09569***      .01641     5.83  .0000      .06352    .12786
    B005|    -.39478***      .02054   -19.22  .0000     -.43505   -.35452
    B006|     .05688***      .00327    17.38  .0000      .05046    .06330
    B007|     .01044***      .00064    16.38  .0000      .00919    .01169
--------+-----------------------------------------------------------------------
Constant|    5.66098***      .04686   120.81  .0000     5.56914   5.75282
     OCC|    -.11220***      .01464    -7.66  .0000     -.14090   -.08350
    SMSA|     .15504***      .01234    12.57  .0000      .13086    .17922
      MS|     .09569***      .02133     4.49  .0000      .05387    .13751
     FEM|    -.39478***      .02603   -15.16  .0000     -.44581   -.34376
      ED|     .05688***      .00268    21.24  .0000      .05163    .06213
     EXP|     .01044***      .00054    19.26  .0000      .00938    .01150
--------+-----------------------------------------------------------------------
```



	LOGWAGE	EDUC
1 »	5.56068	9
2 »	5.72031	9
3 »	5.99645	9
4 »	5.99645	9
5 »	6.06146	9
6 »	6.17379	9
7 »	6.24417	9
8 »	6.16331	11
9 »	6.21461	11
10 »	6.2634	11
11 »	6.54391	11
12 »	6.69703	11
13 »	6.79122	11
14 »	6.81564	11
15 »	5.65249	12
16 »	6.43615	12
17 »	6.54822	12
18 »	6.60259	12
19 »	6.6958	12
20 »	6.77878	12
21 »	6.86066	12

Bootstrap variance for a panel data estimator

- **Panel Bootstrap = Block Bootstrap**
- **Data set is N groups of size T_i**
- **Bootstrap sample is N groups of size T_i drawn with replacement.**

LWAGE	Coefficient	Standard Error	z	Prob. |z|>Z*	95% Confidence Interval		
							OLS
Constant	5.66098***	.04686	120.81	.0000	5.56914	5.75282	
OCC	-.11220***	.01464	-7.66	.0000	-.14090	-.08350	
SMSA	.15504***	.01234	12.57	.0000	.13086	.17922	
MS	.09569***	.02133	4.49	.0000	.05387	.13751	
FEM	-.39478***	.02603	-15.16	.0000	-.44581	-.34376	
ED	.05688***	.00268	21.24	.0000	.05163	.06213	
EXP	.01044***	.00054	19.26	.0000	.00938	.01150	
							Bootstrap
B001	5.66098***	.04683	120.89	.0000	5.56920	5.75276	
B002	-.11220***	.01326	-8.46	.0000	-.13820	-.08620	Assumes no correlation within groups
B003	.15504***	.01205	12.87	.0000	.13143	.17866	
B004	.09569***	.01953	4.90	.0000	.05742	.13396	
B005	-.39478***	.01863	-21.19	.0000	-.43129	-.35827	
B006	.05688***	.00325	17.52	.0000	.05052	.06324	
B007	.01044***	.00053	19.67	.0000	.00940	.01148	
							Cluster
Constant	5.66098***	.10026	56.46	.0000	5.46447	5.85750	
OCC	-.11220***	.02653	-4.23	.0000	-.16421	-.06020	Accounts for within group correlation
SMSA	.15504***	.02540	6.10	.0000	.10526	.20483	
MS	.09569**	.04657	2.05	.0399	.00442	.18696	
FEM	-.39478***	.05319	-7.42	.0000	-.49904	-.29052	
ED	.05688***	.00568	10.01	.0000	.04574	.06802	
EXP	.01044***	.00132	7.93	.0000	.00786	.01302	
							Block Bootstrap
B001	5.66098***	.09497	59.61	.0000	5.47484	5.84712	
B002	-.11220***	.02617	-4.29	.0000	-.16349	-.06092	Mimics results of panel correction
B003	.15504***	.02351	6.60	.0000	.10897	.20112	
B004	.09569***	.03542	2.70	.0069	.02627	.16511	
B005	-.39478***	.04287	-9.21	.0000	-.47880	-.31077	
B006	.05688***	.00536	10.61	.0000	.04637	.06739	
B007	.01044***	.00138	7.57	.0000	.00774	.01314	

Using First Differences

$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + c_i + \varepsilon_{it}$, observation for person i at time t

Eliminating the heterogeneity

$$\Delta y_{it} = y_{it} - y_{i,t-1} = (\Delta \mathbf{x}_{it}')\boldsymbol{\beta} + \Delta c_i + \Delta \varepsilon_{it}$$
$$= (\Delta \mathbf{x}_{it}')\boldsymbol{\beta} + w_{it}$$

Note: Time invariant variables become zero

Time trend becomes the constant term

OLS with First Differences

With strict exogeneity of ($\mathbf{X}_i$,c_i), OLS regression of Δy_{it} on $\Delta \mathbf{x}_{it}$ is unbiased and consistent but inefficient.

$$\text{Var}\begin{pmatrix} \varepsilon_{i,2}-\varepsilon_{i,1} \\ \varepsilon_{i,3}-\varepsilon_{i,2} \\ \vdots \\ \varepsilon_{i,T_i}-\varepsilon_{i,T_i-1} \end{pmatrix} = \begin{bmatrix} 2\sigma_\varepsilon^2 & -\sigma_\varepsilon^2 & 0 & 0 \\ -\sigma_\varepsilon^2 & 2\sigma_\varepsilon^2 & -\sigma_\varepsilon^2 & \vdots \\ 0 & -\sigma_\varepsilon^2 & \ddots & -\sigma_\varepsilon^2 \\ 0 & \cdots & -\sigma_\varepsilon^2 & 2\sigma_\varepsilon^2 \end{bmatrix}$$

GLS is unpleasantly complicated. Use OLS in first differences and use Newey-West with one lag.

The Fixed Effects Model

$\mathbf{y}_i = \mathbf{X}_i\beta + \mathbf{d}_i\alpha_i + \boldsymbol{\varepsilon}_i$, for each individual

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{pmatrix} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{d}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{X}_2 & \mathbf{0} & \mathbf{d}_2 & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_N & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{d}_N \end{bmatrix} \begin{pmatrix} \boldsymbol{\beta} \\ \boldsymbol{\alpha} \end{pmatrix} + \boldsymbol{\varepsilon}$$

$$= [\mathbf{X}, \mathbf{D}] \begin{pmatrix} \boldsymbol{\beta} \\ \boldsymbol{\alpha} \end{pmatrix} + \boldsymbol{\varepsilon}$$

$$= \mathbf{Z}\boldsymbol{\delta} + \boldsymbol{\varepsilon}$$

$E[c_i \mid \mathbf{X}_i] = g(\mathbf{X}_i)$; Effects are correlated with included variables.

$Cov[\mathbf{x}_{it}, c_i] \neq \mathbf{0}$

The Within Groups Transformation Removes the Effects

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + c_i + \varepsilon_{it}$$

$$\bar{y}_i = \bar{\mathbf{x}}_i'\boldsymbol{\beta} + c_i + \bar{\varepsilon}_i$$

$$y_{it} - \bar{y}_i = (\mathbf{x}_{it} - \bar{\mathbf{x}}_i)'\boldsymbol{\beta} + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

Use least squares to estimate $\boldsymbol{\beta}$.

Useful Analysis of Variance Notation

Decomposition of Total variation:

$$\Sigma_{i=1}^{N}\Sigma_{t=1}^{T_i}(z_{it}-\bar{\bar{z}})^2 = \Sigma_{i=1}^{N}\left[\Sigma_{t=1}^{T_i}(z_{it}-\bar{z}_i.)^2\right] + \Sigma_{i=1}^{N}T_i\left[\bar{z}_i.-\bar{\bar{z}}\right]^2$$

Total variation = Within groups variation

+ Between groups variation

WHO Data

The model used by the researchers at WHO was

$$\ln DALE_{it} = \alpha_i + \beta_1 \ln Health\ Expenditure_{it} + \beta_2 \ln Education_{it} + \beta_3 \ln Education_{it}^2 + \varepsilon_{it}$$

The analysis of variance for a variable x_{it} is based on the decomposition

$$\sum_{i=1}^{n}\sum_{t=1}^{T_i}(x_{it} - \bar{\bar{x}})^2 = \sum_{i=1}^{n}\sum_{t=1}^{T_i}(x_{it} - \bar{x}_{i.})^2 + \sum_{t=1}^{n} T_i(\bar{x}_{i.} - \bar{\bar{x}})^2 .$$

Analysis of Variance for WHO Data on Health Care Attainment

Variable	Within Groups Variation	Between Groups Variation
DALE	5.645%	94.355%
COMP	0.150%	99.850%
Expenditure	0.635%	99.365%
Education	0.178%	99.822%

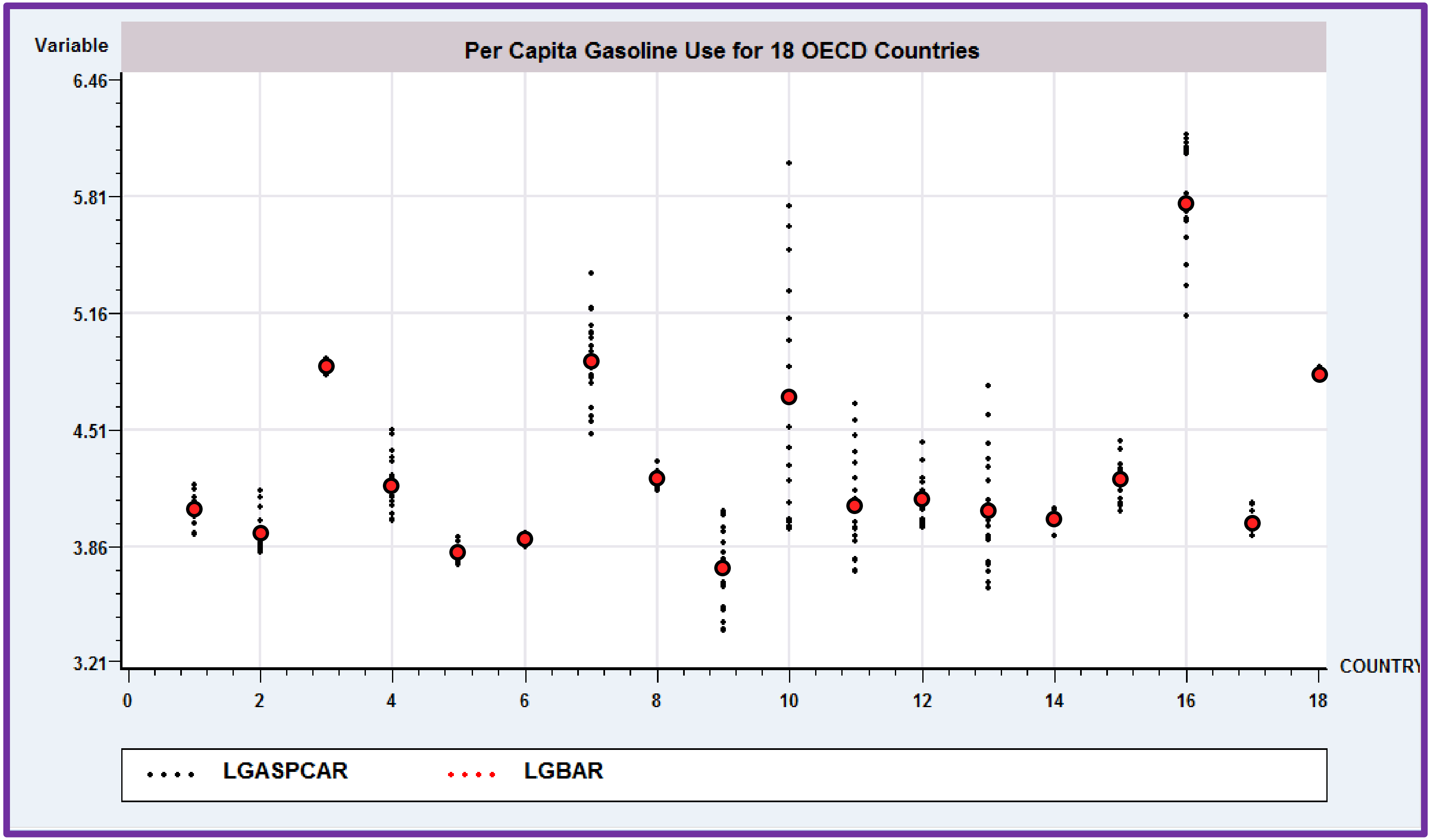
Baltagi and Griffin’s Gasoline Data

World Gasoline Demand Data, 18 OECD Countries, 19 years
Variables in the file are

COUNTRY = name of country
YEAR = year, 1960-1978
LGASPCAR = log of consumption per car
LINCOMEP = log of per capita income
LRPMG = log of real price of gasoline
LCARPCAP = log of per capita number of cars

See Baltagi (2001, p. 24) for analysis of these data. The article on which the analysis is based is Baltagi, B. and Griffin, J., "Gasolne Demand in the OECD: An Application of Pooling and Testing Procedures," European Economic Review, 22, 1983, pp. 117-137. The data were downloaded from the website for Baltagi's text.

Analysis of Variance



Analysis of Variance

```
+----------------------------------------------------------------------------+
| Analysis of Variance for           LGASPCAR                                |
| Stratification Variable            _STRATUM                                |
| Observations weighted by           ONE                                     |
| Total Sample Size                                    342                   |
| Number of Groups                                      18                   |
| Number of groups with no data                          0                   |
| Overall Sample Mean                            4.2962420                   |
| Sample Standard Deviation                       .5489071                   |
| Total Sample Variance                           .3012990                   |
|                                                                            |
| Source of Variation             Variation     Deg.Fr.       Mean Square   |
| Between Groups                85.68228007          17           5.04013   |
| Within  Groups                17.06068428         324            .05266   |
| Total                        102.74296435         341            .30130   |
| Residual S.D.                  .22946990                                   |
| R-squared                      .83394791     MSB/MSW          21.96425   |
| F ratio                      95.71734806     P value            .00000   |
+----------------------------------------------------------------------------+
```

Estimating the Fixed Effects Model

- The FEM is a plain vanilla regression model but with many independent variables
- Least squares is unbiased, consistent, efficient, but inconvenient if N is large.

$$\begin{pmatrix} \mathbf{b} \\ \mathbf{a} \end{pmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{D} \\ \mathbf{D}'\mathbf{X} & \mathbf{D}'\mathbf{D} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{D}'\mathbf{y} \end{bmatrix}$$

Using the Frisch-Waugh theorem

$$\mathbf{b} = [\mathbf{X}'\mathbf{M}_\mathbf{D}\mathbf{X}]^{-1}\left[\mathbf{X}'\mathbf{M}_\mathbf{D}\mathbf{y}\right]$$

Fixed Effects Estimator (cont.)

$$\mathbf{M_D} = \begin{bmatrix} \mathbf{M_D^1} & \mathbf{0} & & \mathbf{0} \\ \mathbf{0} & \mathbf{M_D^2} & & \mathbf{0} \\ & & & \\ \mathbf{0} & \mathbf{0} & & \mathbf{M_D^N} \end{bmatrix}$$ (The dummy variables are orthogonal)

$$\mathbf{M_D^i} = \mathbf{I_{T_i}} - \mathbf{d_i}(\mathbf{d_i'd_i})^{-1}\mathbf{d_i'} = \mathbf{I_{T_i}} - (1/T_i)\mathbf{d_i d_i'}$$

$$\mathbf{X'M_DX} = \Sigma_{\mathbf{i=1}}^{\mathbf{N}}\mathbf{X}_i'\mathbf{M_D^i}\mathbf{X}_i, \quad \left\{\mathbf{X}_i'\mathbf{M_D^i}\mathbf{X}_i\right\}_{k,l} = \Sigma_{t=1}^{T_i}(x_{it,k}\text{-}\bar{x}_{i.,k})(x_{it,l}\text{-}\bar{x}_{i.,l})$$

$$\mathbf{X'M_Dy} = \Sigma_{\mathbf{i=1}}^{\mathbf{N}}\mathbf{X}_i'\mathbf{M_D^i}\mathbf{y}_i, \quad \left\{\mathbf{X}_i'\mathbf{M_D^i}\mathbf{y}_i\right\}_{k} = \Sigma_{t=1}^{T_i}(x_{it,k}\text{-}\bar{x}_{i.,k})(y_{it}\text{-}\bar{y}_{i.})$$

If all groups have the same T_i, $\mathbf{M_D} = \mathbf{M}^0 \otimes \mathbf{I}$ where $\mathbf{M}^0 = \mathbf{I_T} - (1/T)\mathbf{dd'}$

$$\mathbf{X'M_DX} = \mathbf{X'}[\mathbf{M}^0 \otimes \mathbf{I}]\mathbf{X} \text{ and } \mathbf{b} = \left(\mathbf{X'}[\mathbf{M}^0 \otimes \mathbf{I}]\mathbf{X}\right)^{-1}\mathbf{X'}[\mathbf{M}^0 \otimes \mathbf{I}]\mathbf{y}.$$

The Within Transformation Removes the Effects

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + c_i + \varepsilon_{it}$$

$$\bar{y}_i = \bar{\mathbf{x}}_i'\boldsymbol{\beta} + c_i + \bar{\varepsilon}_i$$

$$y_{it} - \bar{y}_i = (\mathbf{x}_{it} - \bar{\mathbf{x}}_i)'\boldsymbol{\beta} + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

$$\ddot{y}_{it} = \ddot{\mathbf{x}}_{it}'\boldsymbol{\beta} + \ddot{\varepsilon}_{it}$$

Wooldridge notation for data in deviations from group means

Least Squares Dummy Variable Estimator

- **b** is obtained by **'within' groups least squares** (group mean deviations)

- **a** is estimated using the normal equations:

  $\mathbf{D'Xb+D'Da=D'y}$

$$\mathbf{a = (D'D)^{-1}D'(y - Xb)}$$

$$a_i=(1/T_i)\Sigma_{t=1}^{T_i}(y_{it}-\mathbf{x}_{it}'\mathbf{b})=\overline{e}_i$$

Inference About LSDV

- Assume strict exogeneity: Cov[ε_{it},($\mathbf{x}_{js}$,c_j)]=0. Every disturbance in every period for each person is uncorrelated with variables and effects for every person and across periods.
- Now, it's just least squares in a classical linear regression model.
- Asy.Var[**b**] = $(\sigma_\varepsilon^2 / \Sigma_{i=1}^N T_i)\text{plim}[(1/\Sigma_{i=1}^N T_i)\Sigma_{i=1}^N \mathbf{X}_i'\mathbf{M}_\mathbf{D}^i\mathbf{X}_i]^{-1}$

  which is the usual estimator for OLS

  $$\hat{\sigma}_\varepsilon^2 = \frac{\Sigma_{i=1}^N \Sigma_{t=1}^{T_i}(y_{it} - a_i - x_{it}'\mathbf{b})^2}{\left(\Sigma_{i=1}^N T_i \; - \; N \; - \; K\right)}$$

  (Note the degrees of freedom correction)

Application Cornwell and Rupert

```
+----------------------------------------------------+
| Panel Data Analysis of LWAGE       [ONE way]       |
|           Unconditional ANOVA (No regressors)      |
| Source        Variation    Deg. Free.    Mean Square |
| Between         646.254        594.       1.08797    |
| Residual        240.651       3570.      .674093E-01 |
| Total           886.905       4164.       .212994    |
+----------------------------------------------------+

+----------------------------------------------------+
| OLS Without Group Dummy Variables                  |
| LHS=LWAGE     Mean                 =   6.676346    |
|               Standard deviation   =   .4615122    |
| Model size    Parameters           =          5    |
|               Degrees of freedom   =       4160    |
| Residuals     Sum of squares       =   651.7870    |
|               Standard error of e  =   .3958277    |
| Fit           R-squared            =   .2650993    |
|               Adjusted R-squared   =   .2643927    |
| Model test    F[  4,  4160] (prob) = 375.16 (.0000) |
+----------------------------------------------------+
+---------+--------------+----------------+--------+--------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+--------------+----------------+--------+--------+----------+
 OCC          -.29227536       .01259221    -23.211   .0000     .51116447
 SMSA          .17712491       .01327104     13.347   .0000     .65378151
 MS            .35695474       .01610229     22.168   .0000     .81440576
 EXP           .00746892       .00057035     13.095   .0000   19.8537815
 Constant     6.27095389       .02041864    307.119   .0000
```

LSDV Results

```
+----------------------------------------------------+
| Least Squares with Group Dummy Variables           |
| LHS=LWAGE    Mean                  =   6.676346    |
|              Standard deviation    =    .4615122   |
| Model size   Parameters            =        599    |
|              Degrees of freedom    =       3566    |
| Residuals    Sum of squares        =   83.88505    |
|              Standard error of e   =    .1533740   |
| Fit          R-squared             =    .9054182   |
|              Adjusted R-squared    =    .8895573   |
| Model test   F[598,  3566] (prob) =  57.08 (.0000) |
+----------------------------------------------------+
+----------------------------------------------------+
| Panel:Groups    Empty       0,    Valid data   595 |
|                 Smallest    7,    Largest        7 |
|                 Average group size            7.00 |
+----------------------------------------------------+
```

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]	Mean of X
OCC	-.02021384	.01374007	-1.471	.1412	.51116447
SMSA	-.04250645	.01950085	-2.180	.0293	.65378151
MS	-.02946444	.01913652	-1.540	.1236	.81440576
EXP	.09665711	.00119162	81.114	.0000	19.8537815

Pooled OLS

Coefficient	Standard Error
-.29227536	.01259221
.17712491	.01327104
.35695474	.01610229
.00746892	.00057035

Note huge changes in the coefficients. SMSA and MS change signs. Significance changes completely!

The Effect of the Effects

Test Statistics for the Classical Model

	Model	Log-Likelihood	Sum of Squares	R-squared
(1)	Constant term only	-2688.80597	.8869049390D+03	.0000000
(2)	Group effects only	27.58464	.2406511943D+03	.7286618
(3)	X - variables only	-2047.35445	.6517870323D+03	.2650993
(4)	X and group effects	2222.33376	.8388505089D+02	.9054182

Hypothesis Tests

	Likelihood Ratio Test			F Tests			
	Chi-squared	d.f.	Prob.	F	num.	denom.	Prob value
(2) vs (1)	5432.781	594	.00000	16.140	594	3570	.00000
(3) vs (1)	1282.903	4	.00000	375.157	4	4160	.00000
(4) vs (1)	9822.279	598	.00000	57.085	598	3566	.00000
(4) vs (2)	4389.498	4	.00000	1666.054	4	3566	.00000
(4) vs (3)	8539.376	594	.00000	40.643	594	3566	.00000

Robust Counterpart to White Estimator?

Assumes Var[$\boldsymbol{\varepsilon}_i$] = $\boldsymbol{\Omega}_i \neq \sigma^2 \mathbf{I}_{Ti}$

$\mathbf{e}_i = \mathbf{y}_i - a_i \mathbf{i}_{Ti} - \mathbf{X}_i \mathbf{b} = \mathbf{M_D y}_i - \mathbf{M_D X}_i \mathbf{b}$

(T_i x 1 vector of group residuals)

$$\text{Est.Asy.Var}[\mathbf{b}] = \left[\Sigma_{i=1}^{N} \mathbf{X}_i' \mathbf{M}_D^i \mathbf{X}_i\right]^{-1} \left[\Sigma_{i=1}^{N} (\mathbf{X}_i' \mathbf{M}_D^i \mathbf{e}_i)(\mathbf{e}_i' \mathbf{M}_D^i \mathbf{X}_i)\right] \left[\Sigma_{i=1}^{N} \mathbf{X}_i' \mathbf{M}_D^i \mathbf{X}_i\right]^{-1}$$

$$= \mathbf{H}^{-1} \left[\Sigma_{i=1}^{N} \left\{\Sigma_{t=1}^{T_i} (\mathbf{x}_{it} - \bar{\mathbf{x}}_i) e_{it}\right\} \left\{\Sigma_{t=1}^{T_i} (\mathbf{x}_{it} - \bar{\mathbf{x}}_i) e_{it}\right\}'\right] \mathbf{H}^{-1}$$

$$\mathbf{H} = \left[\Sigma_{i=1}^{N} \Sigma_{t=1}^{T_i} (\mathbf{x}_{it} - \bar{\mathbf{x}}_i)(\mathbf{x}_{it} - \bar{\mathbf{x}}_i)'\right]$$

Resembles (and is based on) White, but treats a full vector of disturbances at a time. Robust to heteroscedasticity and autocorrelation (within the groups).

```
----------------------------------------------------------------------------
LSDV         least squares with fixed effects ....
LHS=LWAGE    Mean                 =      6.67635
             Standard deviation   =       .46151
----------   No. of observations  =         4165  DegFreedom   Mean square
Regression   Sum of Squares       =      804.638        603        1.33439
Residual     Sum of Squares       =      82.2673       3561         .02310
Total        Sum of Squares       =      886.905       4164         .21299
----------   Standard error of e  =       .15199  Root MSE          .14054
Fit          R-squared            =       .90724  R-bar squared     .89154
Estd. Autocorrelation of e(i,t)   =      .146506
--------------------------------------------------
Panel:Groups Empty       0,      Valid data      595
             Smallest    7,      Largest           7
             Average group size in panel        7.00
               Effects a(i)        Residuals e(i,t)
Variances          1.068764                 .023102
Std.Devs.          1.033810                 .151994
Rho =        Residual variation due to ai 97.884%
Within groups variation in LWAGE          .24065D+03
R squared based on within group variation  .658147
Between group variation in LWAGE          .64625D+03
Constant term (group 1) [se]     5.2942 [     .0574]
--------+------------------------------------------------------------------------
        |                   Standard             Prob.      95% Confidence     Clustered
   LWAGE|  Coefficient        Error       z     |z|>Z*         Interval         Std.Error
--------+------------------------------------------------------------------------
     EXP|     .11321***      .00247     45.81   .0000      .10837     .11805      .00437
   EXPSQ|    -.00042***    .5459D-04    -7.66   .0000     -.00053    -.00031    .8905D-04
     WKS|     .00084         .00060      1.39   .1633     -.00034     .00201      .00094
     OCC|    -.02148         .01378     -1.56   .1192     -.04849     .00554      .02052
     IND|     .01921         .01545      1.24   .2136     -.01106     .04948      .02450
   SOUTH|    -.00186         .03430      -.05   .9567     -.06909     .06536      .09646
    SMSA|    -.04247**       .01943     -2.19   .0288     -.08055    -.00439      .03185
      MS|    -.02973         .01898     -1.57   .1174     -.06693     .00748      .02902
   UNION|     .03278**       .01492      2.20   .0280      .00354     .06203      .02708
--------+------------------------------------------------------------------------
```

The Within (LSDV) Estimator is an IV Estimator

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + (\mathbf{D}\boldsymbol{\alpha} + \boldsymbol{\varepsilon})$$
$$= \mathbf{X}\boldsymbol{\beta} + \mathbf{w}$$

Regression of y on X is inconsistent because X is correlated with w. The data in group mean deviations is

$$\mathbf{Z} = \mathbf{M_D X} = \mathbf{X} - \mathbf{D}(\mathbf{D}'\mathbf{D})^{-1}\mathbf{D}'\mathbf{X}$$

The inconsistent OLS estimator is $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ (omits $\mathbf{D}$)

The IV estimator $\mathbf{b}_{\mathbf{LSDV}} = (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{y} = [(\mathbf{X}'\mathbf{M_D})\mathbf{X})]^{-1}[\mathbf{X}'\mathbf{M_D}]\mathbf{y}$.

$$= [(\mathbf{X}'\mathbf{M_D})(\mathbf{M_D X})]^{-1}(\mathbf{X}'\mathbf{M_D})(\mathbf{M_D y})$$

This is OLS using data in mean deviations, i.e., LSDV.

LSDV – As Usual

```
----------------------------------------------------------------------------
LSDV         least squares with fixed effects ....
LHS=LWAGE    Mean                     =      6.67635
             Standard deviation       =       .46151
----------   No. of observations      =         4165  DegFreedom   Mean square
Regression   Sum of Squares           =      804.313          598      1.34501
Residual     Sum of Squares           =      82.5915         3566       .02316
Total        Sum of Squares           =      886.905         4164       .21299
----------   Standard error of e      =       .15219  Root MSE          .14082
Fit          R-squared                =       .90688  R-bar squared     .89126
Model test   F[598,  3566]            =     58.07241  Prob F > F*       .00000
Estd. Autocorrelation of e(i,t)       =      .146762
--------------------------------------------------
Panel:Groups Empty       0,     Valid data       595
             Smallest    7,     Largest            7
             Average group size in panel        7.00
Variances    Effects a(i)          Residuals e(i,t)
                 1.068377                   .023161
Rho squared: Residual variation due to ai   .978781
Within groups variation in LWAGE           240.6512
R squared based on within group variation   .656800
Within groups variation in LWAGE           646.2537
--------+-------------------------------------------------------------------
        |                  Standard            Prob.      95% Confidence
   LWAGE|  Coefficient       Error       z    |z|>Z*          Interval
--------+-------------------------------------------------------------------
     WKS|      .00087       .00060     1.45  .1476     -.00031    .00205
     EXP|      .11363***    .00247    45.98  .0000      .10879    .11847
   EXPSQ|     -.00042***  .5465D-04   -7.72  .0000     -.00053   -.00032
     OCC|     -.01813       .01365    -1.33  .1841     -.04489    .00862
--------+-------------------------------------------------------------------
```

2SLS Using $\mathbf{Z}=\mathbf{M}_D\mathbf{X}$ as Instruments

```
|-> create ; devwks=groupdevs(wks,pds=7)$
|-> create ; devexp=groupdevs(exp,pds=7)$
|-> create ; devexpsq=groupdevs(expsq,pds=7)$
|-> create ; devocc=groupdevs(occ,pds=7)$
|-> name;x=wks,exp,expsq,occ$
|-> name;z=devwks,devexp,devexpsq,devocc$
|-> 2sls;lhs=lwage;rhs=x;inst=z$
-----------------------------------------------------------------------------
Two stage    least squares regression ............
LHS=LWAGE    Mean                  =        6.67635
             Standard deviation    =         .46151
             Number of observs.    =           4165
Model size   Parameters            =              4
             Degrees of freedom    =           4161
Residuals    Sum of squares        =        92801.3
             Standard error of e   =        4.72257
Fit          R-squared             =     -103.73560
             Adjusted R-squared    =     -103.81111
Not using OLS or no constant. Rsqrd & F may be < 0
Instrumental Variables:
DEVWKS     DEVEXP     DEVEXPSQ  DEVOCC
--------+--------------------------------------------------------------------
        |                 Standard            Prob.       95% Confidence
   LWAGE|  Coefficient      Error       z    |z|>Z*          Interval
--------+--------------------------------------------------------------------
     WKS|    .00087       .01862      .05   .9628     -.03563      .03737
     EXP|    .11363       .07669     1.48   .1384     -.03669      .26395
   EXPSQ|   -.00042       .00170     -.25   .8034     -.00375      .00290
     OCC|   -.01813       .42359     -.04   .9659     -.84835      .81209
--------+--------------------------------------------------------------------
```

A Caution About Stata and R^2

$$\text{R squared} = 1 - \frac{\text{Residual Sum of Squares}}{\text{Total Sum of Squares}}$$

Or is it? What is the total sum of squares?

$$\text{Conventional: Total Sum of Squares} = \sum_{i=1}^{N} \sum_{t=1}^{T_i} (y_{it} - \bar{y})^2$$

$$\text{"Within Sum of Squares"} = \sum_{i=1}^{N} \sum_{t=1}^{T_i} (y_{it} - \bar{y}_i)^2$$

For the FE model above,

$R^2 = 0.90542$

$R^2 = 0.65142$

Which should appear in the denominator of R^2

The coefficient estimates and standard errors are the same. The calculation of the R^2 is different. In the **areg** procedure, you are estimating coefficients for each of your covariates plus each dummy variable for your groups. In the **xtreg, fe** procedure the R^2 reported is obtained by only fitting a mean deviated model where the effects of the groups (all of the dummy variables) are assumed to be fixed quantities. So, all of the effects for the groups are simply subtracted out of the model and no attempt is made to quantify their overall effect on the fit of the model.

Since the SSE is the same, the $R^2=1-SSE/SST$ is very different. The difference is real in that we are making different assumptions with the two approaches. In the **xtreg, fe** approach, the effects of the groups are fixed and **unestimated quantities are subtracted out of the model** before the fit is performed. In the **areg** approach, the group effects are estimated and affect the total sum of squares of the model under consideration.

Robust Covariance Matrix for LSDV
Cluster Estimator for Within Estimator

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]	Mean of X
OCC	-.02021	.01374007	-1.471	.1412	.5111645
SMSA	-.04251**	.01950085	-2.180	.0293	.6537815
MS	-.02946	.01913652	-1.540	.1236	.8144058
EXP	.09666***	.00119162	81.114	.0000	19.853782

Covariance matrix for the model is adjusted for data clustering.
Sample of 4165 observations contained 595 clusters defined by
7 observations (fixed number) in each cluster.

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]	Mean of X
DOCC	-.02021	.01982162	-1.020	.3078	.00000
DSMSA	-.04251	.03091685	-1.375	.1692	.00000
DMS	-.02946	.02635035	-1.118	.2635	.00000
DEXP	.09666***	.00176599	54.732	.0000	.00000

A Caution About Stata and Fixed Effects

```
. xtreg yit x1 x2 x3 x4,fe

Fixed-effects (within) regression               Number of obs      =     1,482
Group variable: farm                            Number of groups   =       247

R-sq:                                           Obs per group:
     within  = 0.8359                                         min =          6
     between = 0.9615                                         avg =        6.0
     overall = 0.9513                                         max =          6

                                                F(4,1231)          =   1568.11
corr(u_i, Xb)  = 0.1089                         Prob > F           =    0.0000

------------------------------------------------------------------------------
         yit |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .6620012   .0246784    26.83   0.000     .6135847    .7104177
          x2 |   .0373524   .0161331     2.32   0.021      .005701    .0690038
          x3 |   .0303996   .0232078     1.31   0.190    -.0151316    .0759307
          x4 |   .3825104   .0120169    31.83   0.000     .3589345    .4060862
       _cons |   11.57749   .0021151  5473.85   0.000     11.57334    11.58164
-------------+----------------------------------------------------------------
     sigma_u |  .12198441
     sigma_e |  .08142265
         rho |  .69178541   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0: F(246, 1231) = 12.84                  Prob > F = 0.0000
```

```
. xtreg yit x1 x2 x3 x4,fe r

Fixed-effects (within) regression               Number of obs      =     1,482
Group variable: farm                            Number of groups   =       247

R-sq:                                           Obs per group:
     within  = 0.8359                                         min =          6
     between = 0.9615                                         avg =        6.0
     overall = 0.9513                                         max =          6

                                                F(4,246)           =    607.68
corr(u_i, Xb)  = 0.1089                         Prob > F           =    0.0000

                                (Std. Err. adjusted for 247 clusters in farm)
------------------------------------------------------------------------------
             |               Robust
         yit |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .6620012   .0341774    19.37   0.000     .5946836    .7293188
          x2 |   .0373524   .0171459     2.18   0.030     .0035808    .0711239
          x3 |   .0303996   .0243227     1.25   0.213    -.0175077    .0783069
          x4 |   .3825104   .0172933    22.12   0.000     .3484485    .4165722
       _cons |   11.57749   8.40e-11  1.4e+11   0.000     11.57749    11.57749
-------------+----------------------------------------------------------------
     sigma_u |  .12198441
     sigma_e |  .08142265
         rho |  .69178541   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

```
----------------------------------------------------------------------
LSDV         least squares with fixed effects ....
LHS=YIT      Mean                 =          11.57749
--------+-------------------------------------------------------------
        |                  Standard            Prob.      95% Confidence
     YIT|   Coefficient     Error         z    |z|>Z*        Interval
--------+-------------------------------------------------------------
      X1|     .66200***     .02468     26.83   .0000      .61363    .71037
      X2|     .03735**      .01613      2.32   .0206      .00573    .06897
      X3|     .03040        .02321      1.31   .1902     -.01509    .07589
      X4|     .38251***     .01202     31.83   .0000      .35896    .40606
--------+-------------------------------------------------------------
```

Time Invariant Regressors

- Time invariant $\mathbf{x}_{it}$ is defined as invariant for all i. E.g., sex dummy variable, FEM and ED (education in the Cornwell/Rupert data).

- If $\mathbf{x}_{it,k}$ is invariant for all t, then the group mean deviations are all 0.

FE With Time Invariant Variables

```
+----------------------------------------------------+
| There are  2 vars. with no within group variation. |
| FEM      ED                                        |
+----------------------------------------------------+
+--------+--------------+----------------+--------+--------+----------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|
+--------+--------------+----------------+--------+--------+----------+
 EXP     |      .09671227       .00119137    81.177   .0000   19.8537815
 WKS     |      .00118483       .00060357     1.963   .0496   46.8115246
 OCC     |     -.02145609       .01375327    -1.560   .1187    .51116447
 SMSA    |     -.04454343       .01946544    -2.288   .0221    .65378151
 FEM     |        .000000     ......(Fixed Parameter).......
 ED      |        .000000     ......(Fixed Parameter).......
+--------------------------------------------------------------------+
|              Test Statistics for the Classical Model               |
+--------------------------------------------------------------------+
|       Model              Log-Likelihood    Sum of Squares  R-squared |
|(1)   Constant term only     -2688.80597         886.90494     .00000 |
|(2)   Group effects only        27.58464         240.65119     .72866 |
|(3)   X - variables only     -1688.12010         548.51596     .38154 |
|(4)   X and group effects     2223.20087          83.85013     .90546 |
+--------------------------------------------------------------------+
```

Drop The Time Invariant Variables Same Results

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]	Mean of X
EXP	.09671227	.00119087	81.211	.0000	19.8537815
WKS	.00118483	.00060332	1.964	.0495	46.8115246
OCC	-.02145609	.01374749	-1.561	.1186	.51116447
SMSA	-.04454343	.01945725	-2.289	.0221	.65378151

Test Statistics for the Classical Model

Model	Log-Likelihood	Sum of Squares	R-squared
(1) Constant term only	-2688.80597	886.90494	.00000
(2) Group effects only	27.58464	240.65119	.72866
(3) X - variables only	-1688.12010	548.51596	.38154
(4) X and group effects	2223.20087	83.85013	.90546

No change in the sum of squared residuals

Appendix

Fixed Effects Vector Decomposition

Efficient Estimation of Time Invariant and Rarely Changing Variables in Finite Sample Panel Analyses with Unit Fixed Effects

Thomas Plümper and Vera Troeger
Political Analysis, 2007

Introduction

[T]he FE model … does not allow the estimation of time invariant variables. A second drawback of the FE model … results from its **inefficiency** in estimating the effect of variables that have very little within variance.

This article discusses a remedy to the related problems of estimating time invariant and rarely changing variables in FE models with unit effects

The Model

$$y_{it} = \alpha_i + \sum_{k=1}^{K} \beta_k x_{kit} + \sum_{m=1}^{M} \gamma_m z_{mi} + \varepsilon_{it}$$

where α_i denote the N unit effects.

Fixed Effects Vector Decomposition

Step 1: Compute the fixed effects regression to get the "estimated unit effects." "We run this FE model with the sole intention to obtain estimates of the unit effects, α_i."

$$\hat{\alpha}_i = \bar{y}_i - \sum_{k=1}^{K} b_k^{FE} \bar{x}_{ki}$$

Step 2

Regress a_i on $\mathbf{z}_i$ and compute residuals

$$a_i = \sum_{m=1}^{M} \gamma_m z_{im} + h_i$$

h_i is orthogonal to $\mathbf{z}_i$ (since it is a residual)

Vector $\mathbf{h}_i$ is expanded so each element h_i is replicated T_i times - $\mathbf{h}$ is the length of the full sample.

Step 3

Regress y_{it} on a constant, **X**, **Z** and **h** using ordinary least squares to estimate α, **β**, **γ**, δ.

$$y_{it} = \alpha + \sum_{k=1}^{K} \beta_k x_{kit} + \sum_{m=1}^{M} \gamma_m z_{mi} + \delta h_i + \varepsilon_{it}$$

Notice that α_i in the original model has become $\alpha + \delta h_i$ in the revised model.

Step 1 (Based on full sample)

```
These  2 variables have no within group variation.
FEM      ED
F.E. estimates are based on a generalized inverse.
--------+--------------------------------------------------
        |               Standard            Prob.      Mean
   LWAGE| Coefficient     Error         z   z>|Z|      of X
--------+--------------------------------------------------
     EXP|     .09663***    .00119    81.13  .0000   19.8538
     WKS|     .00114*      .00060     1.88  .0600   46.8115
     OCC|    -.02496*      .01390    -1.80  .0724    .51116
     IND|     .02042       .01558     1.31  .1899    .39544
   SOUTH|    -.00091       .03457     -.03  .9791    .29028
    SMSA|    -.04581**     .01955    -2.34  .0191    .65378
   UNION|     .03411**     .01505     2.27  .0234    .36399
     FEM|      .000    .....(Fixed Parameter).....   .11261
      ED|      .000    .....(Fixed Parameter).....  12.8454
--------+--------------------------------------------------
```

Step 2 (Based on 595 observations)

UHI	Coefficient	Standard Error	z	Prob. z>|Z|	Mean of X
Constant	2.88090***	.07172	40.17	.0000	
FEM	-.09963**	.04842	-2.06	.0396	.11261
ED	.14616***	.00541	27.02	.0000	12.8454

Step 3!

LWAGE	Coefficient	Standard Error	z	Prob. z>|Z|	Mean of X
Constant	2.88090***	.03282	87.78	.0000	
EXP	.09663***	.00061	157.53	.0000	19.8538
WKS	.00114***	.00044	2.58	.0098	46.8115
OCC	-.02496***	.00601	-4.16	.0000	.51116
IND	.02042***	.00479	4.26	.0000	.39544
SOUTH	-.00091	.00510	-.18	.8590	.29028
SMSA	-.04581***	.00506	-9.06	.0000	.65378
UNION	.03411***	.00521	6.55	.0000	.36399
FEM	-.09963***	.00767	-13.00	.0000	.11261
ED	.14616***	.00122	120.19	.0000	12.8454
HI	1.00000***	.00670	149.26	.0000	-.103D-13

The Magic

Step 1 / **Step 3**

LWAGE	Step 1 Coefficient	Step 1 Standard Error	Step 3 Coefficient	Step 3 Standard Error
			2.88090***	.03282
EXP	.09663***	.00119	.09663***	.00061
WKS	.00114*	.00060	.00114***	.00044
OCC	-.02496*	.01390	-.02496***	.00601
IND	.02042	.01558	.02042***	.00479
SOUTH	-.00091	.03457	-.00091	.00510
SMSA	-.04581**	.01955	-.04581***	.00506
UNION	.03411**	.01505	.03411***	.00521
			-.09963***	.00767
			.14616***	.00122
			1.00000***	.00670

Step 2

UHI	Coefficient	Standard Error
Constant	2.88090***	.07172
FEM	-.09963**	.04842
ED	.14616***	.00541

What happened here?

$$y_{it} = \alpha_i + \sum_{k=1}^{K} \beta_k x_{kit} + \sum_{m=1}^{M} \gamma_m z_{mi} + \varepsilon_{it}$$

where α_i denote the N unit effects.

An assumption is added along the way

$\text{Cov}(\alpha_i, Z_i) = \mathbf{0}$. This is exactly the number of orthogonality assumptions needed to identify γ. It is not part of the original model.

Econometrics Beat: Dave Giles' Blog

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Tuesday, June 12, 2012

Fixed-Effects Vector Decomposition

Warning! Avoid the so-called "Fixed-Effects Vector Decomposition" (FEVD) estimator, introduced by Plümper and Troeger in a 2007 issue of *Political Analysis*.

A recent "Symposium on Fixed-Effects Vector Decomposition" in the 2011 volume of that journal, which included critiques by William Greene and by Trevor Breusch *et al.*, reveals just what this estimator is.... and isn't!

Greene (2011, p.135) hits the nail on the head:

> "The FEVD estimator is so simple it seems like magic. Like magic, the estimator is illusory."

Ouch!

http://davegiles.blogspot.com/2012/06/fixed-effects-vector-decomposition.html

Symposium on Fixed-Effects Vector Decomposition

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