Econometrics I

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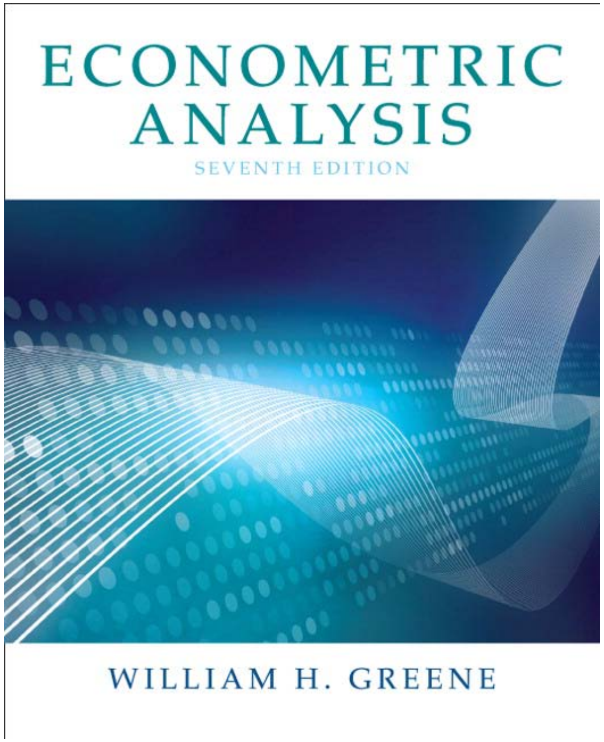


Econometrics I

Part 25 – Time Series

Modeling an Economic Time Series

- Observed y_0, y_1, …, y_t,…
- What is the “sample”
- Random sampling?
- The “observation window”

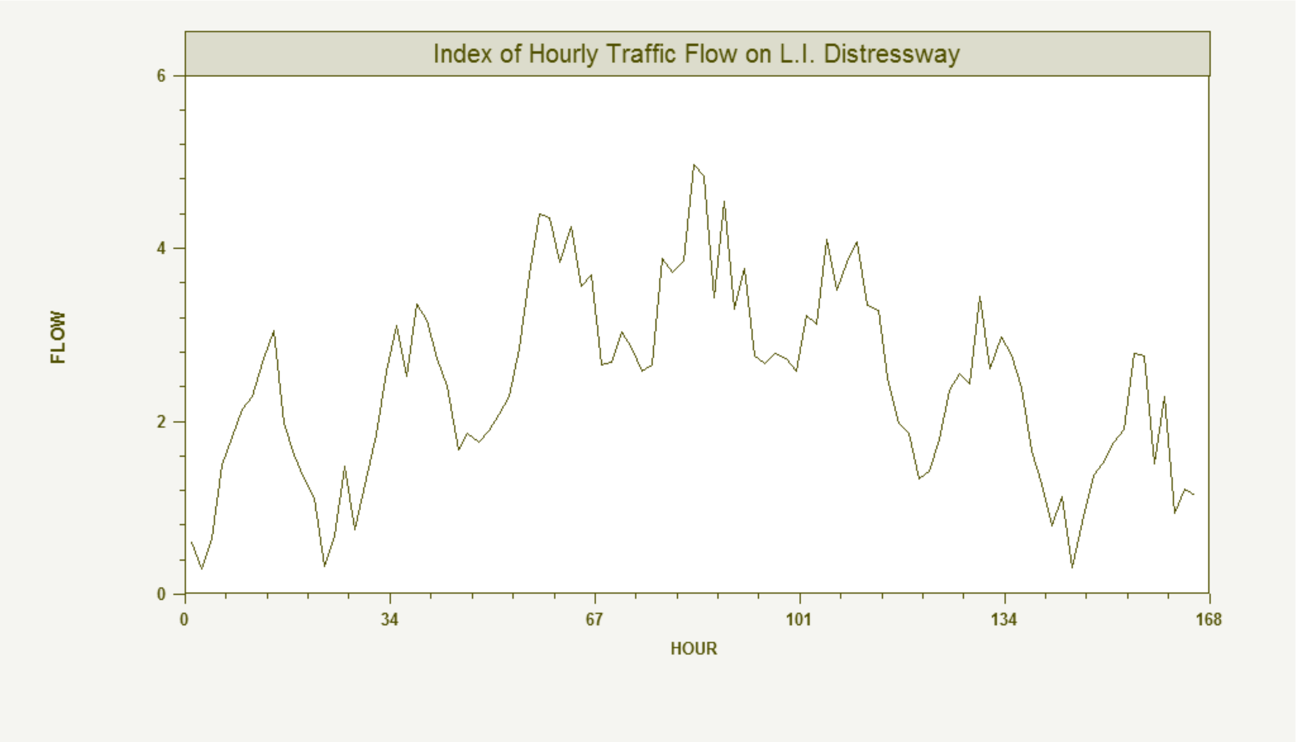
Estimators

- Functions of sums of observations
- Law of large numbers?
  - Nonindependent observations
  - What does “increasing sample size” mean?
- Asymptotic properties? (There are no finite sample properties.)

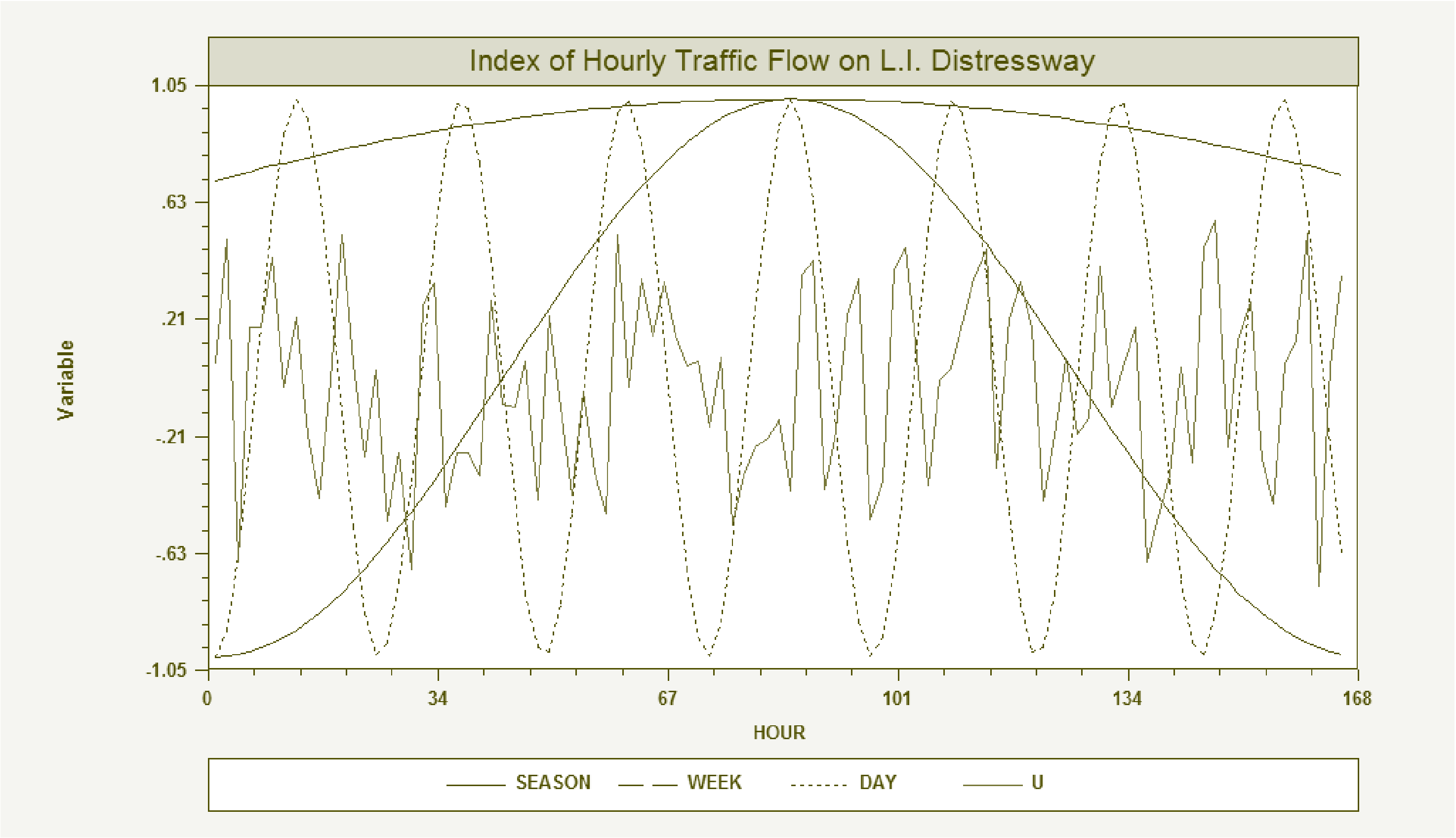
Interpreting a Time Series

- Time domain: A "process"
  - y(t) = ax(t) + by(t-1) + …
  - Regression like approach/interpretation
- Frequency domain: A sum of terms
  - y(t) = $\sum_j \beta_j Cos(\alpha_j t) + \varepsilon(t)$
  - Contribution of different frequencies to the observed series.
- ("High frequency data and financial econometrics – "frequency" is used slightly differently here.)

For example,…


Index of Hourly Traffic Flow on L.I. Distressway
FLOW
0
2
4
6
0
34
67
101
134
168
HOUR

In parts…



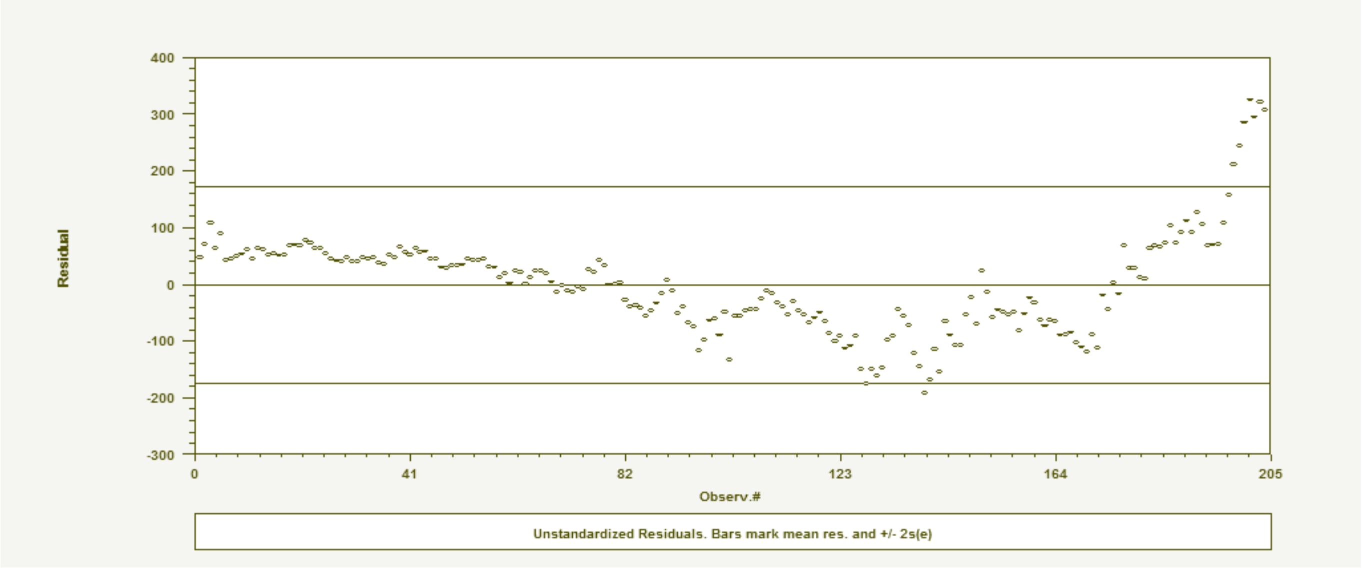
Index of Hourly Traffic Flow on L.I. Distressway
1.05
.63
.21
-.21
-.63
-1.05
Variable
0
34
67
101
134
168
HOUR
SEASON
WEEK
DAY
U

Studying the Frequency Domain

- Cannot identify the number of terms
- Cannot identify frequencies from the time series
- Deconstructing the variance, autocovariances and autocorrelations
  - Contributions at different frequencies
  - Apparent large weights at different frequencies
  - Using Fourier transforms of the data
  - Does this provide “new” information about the series?

Autocorrelation in Regression

- $Y_t = b'x_t + \varepsilon_t$
- $Cov(\varepsilon_t, \varepsilon_{t-1}) \neq 0$
- Ex. $RealCons_t = a + bRealIncome + \varepsilon_t$ U.S. Data, quarterly, 1950-2000



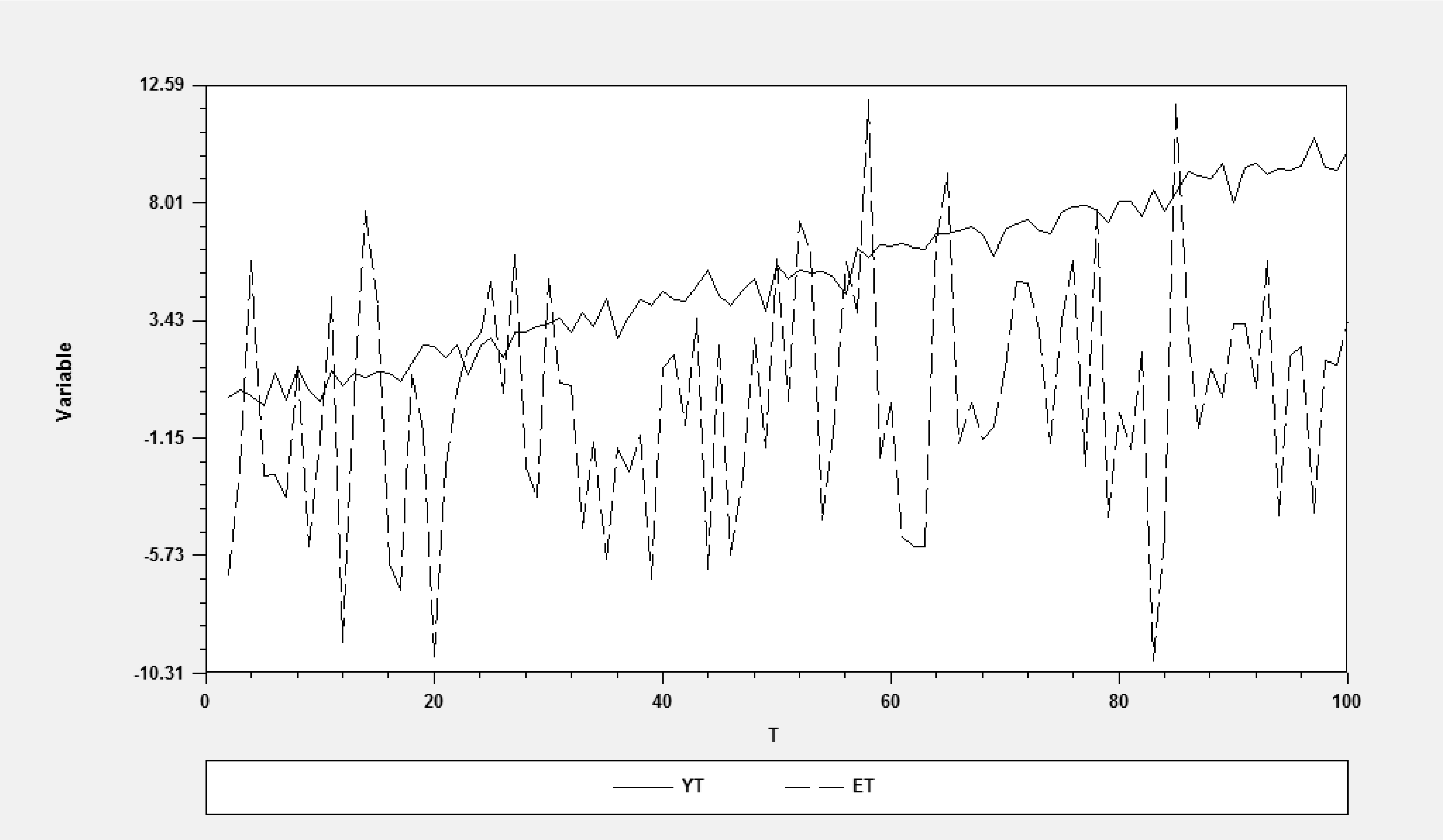
Autocorrelation

- How does it arise?
- What does it mean?
- Modeling approaches
  - Classical – direct: **corrective**
    - Estimation that accounts for autocorrelation
    - Inference in the presence of autocorrelation
  - Contemporary – **structural**
    - Model the source
    - Incorporate the time series aspect in the model

Stationary Time Series

- $z_t = b_1 y_{t-1} + b_2 y_{t-2} + \ldots + b_P y_{t-P} + e_t$
- Autocovariance: $\gamma_k = \text{Cov}[y_t, y_{t-k}]$
- Autocorrelation: $\rho_k = \gamma_k / \gamma_0$
- Stationary series: γ_k depends only on k, not on t
  - Weak stationarity: $E[y_t]$ is not a function of t, $E[y_t * y_{t-s}]$ is not a function of t or s, only of |t-s|
  - Strong stationarity: The joint distribution of $[y_t, y_{t-1}, \ldots, y_{t-s}]$ for any window of length s periods, is not a function of t or s.
- A condition for weak stationarity: The smallest root of the characteristic polynomial: $1 - b_1 z^1 - b_2 z^2 - \ldots - b_P z^P = 0$, is greater than one.
  - The unit circle
  - Complex roots
  - Example: $y_t = \rho y_{t-1} + e_e$, $1 - \rho z = 0$ has root $z = 1/\rho$, $|z| > 1 \Rightarrow |\rho| < 1$.

Stationary vs. Nonstationary Series



The Lag Operator

- $Lx_t = x_{t-1}$
- $L^2 x_t = x_{t-2}$
- $L^P x_t + L^Q x_t = x_{t-P} + x_{t-Q}$
- Polynomials in L: $y_t = B(L)y_t + e_t$
- $A(L)\,y_t = e_t$
- Invertibility: $y_t = [A(L)]^{-1}\,e_t$

Inverting a Stationary Series

- $y_t = \rho y_{t-1} + e_t \Rightarrow (1-\rho L)y_t = e_t$
- $y_t = [1-\rho L]^{-1} e_t = e_t + \rho e_{t-1} + \rho^2 e_{t-2} + \dots$

$$\frac{1}{1-\rho L} = 1 + (\rho L) + (\rho L)^2 + (\rho L)^3 + .$$

  - Stationary series can be inverted
  - Autoregressive vs. moving average form of series

Regression with Autocorrelation

- $y_t = x_t'b + e_t$, $e_t = \rho e_{t-1} + u_t$
- $(1-\rho L)e_t = u_t \Rightarrow e_t = (1-\rho L)^{-1}u_t$
  - $E[e_t] = E[(1-\rho L)^{-1}u_t] = (1-\rho L)^{-1}E[u_t] = 0$
  - $Var[e_t] = (1-\rho L)^{-2}Var[u_t] = 1+\rho^2\sigma_u^2 + \ldots = \sigma_u^2/(1-\rho^2)$
  - $Cov[e_t,e_{t-1}] = Cov[\rho e_{t-1} + u_t, e_{t-1}] =$

    $=\rho Cov[e_{t-1},e_{t-1}]+Cov[u_t,e_{t-1}] = \rho\, \sigma_u^2/(1-\rho^2)$

OLS vs. GLS

- OLS
  - Unbiased?
  - Consistent: (Except in the presence of a lagged dependent variable)
  - Inefficient
- GLS
  - Consistent and efficient

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+---------------------------------------------------+
| Ordinary    least squares regression              |
| LHS=REALCONS Mean                 =   2999.436    |
| Autocorrel   Durbin-Watson Stat.  =   .0920480    |
|              Rho = cor[e,e(-1)]   =   .9539760    |
+---------------------------------------------------+
+---------+-------------+---------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+-------------+---------------+--------+---------+----------+
 Constant     -80.3547488     14.3058515    -5.617   .0000
 REALDPI        .92168567      .00387175   238.054   .0000    3341.47598
| Robust VC    Newey-West, Periods  =         10      |
 Constant     -80.3547488     41.7239214    -1.926   .0555
 REALDPI        .92168567      .01503516    61.302   .0000    3341.47598
+---------------------------------------------+
| AR(1) Model:     e(t) = rho * e(t-1) + u(t) |
| Final value of Rho    =              .998782 |
| Iter=  6, SS= 118367.007, Log-L=-941.371914 |
| Durbin-Watson:   e(t) =              .002436 |
| Std. Deviation:  e(t) =           490.567910 |
| Std. Deviation:  u(t) =            24.206926 |
| Durbin-Watson:   u(t) =             1.994957 |
| Autocorrelation: u(t) =              .002521 |
| N[0,1] used for significance levels         |
+---------------------------------------------+
+---------+-------------+---------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+-------------+---------------+--------+---------+----------+
 Constant      1019.32680     411.177156     2.479   .0132
 REALDPI        .67342731      .03972593    16.952   .0000    3341.47598
 RHO            .99878181      .00346332   288.389   .0000
```

Detecting Autocorrelation

- Use residuals
  - Durbin-Watson d= $\dfrac{\sum_{t=2}^{T}(e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2} \approx 2(1-r)$
  - Assumes normally distributed disturbances strictly exogenous regressors
- Variable addition (Godfrey)
  - $y_t = \beta' x_t + \rho\varepsilon_{t-1} + u_t$
  - Use regression residuals e_t and test $\rho = 0$
  - Assumes consistency of b.

A Unit Root?

- How to test for $\rho = 1$?
- By construction: $\varepsilon_t - \varepsilon_{t-1} = (\rho - 1)\varepsilon_{t-1} + u_t$
  - Test for $\gamma = (\rho - 1) = 0$ using regression?
  - Variance goes to 0 faster than 1/T. Need a new table; can't use standard t tables.
  - Dickey – Fuller tests
- Unit roots in economic data. (Are there?)
  - Nonstationary series
  - Implications for conventional analysis

Reinterpreting Autocorrelation

Regression form

$$y_t = \beta' x_t + \varepsilon_t,\ \varepsilon_t = \rho\varepsilon_{t-1} + u_t$$

Error Correction Form

$$y_t - y_{t-1} = \beta'(x_t - x_{t-1}) + \alpha(y_{t-1} - \beta' x_{t-1}) + u_t, (\alpha = \rho - 1)$$

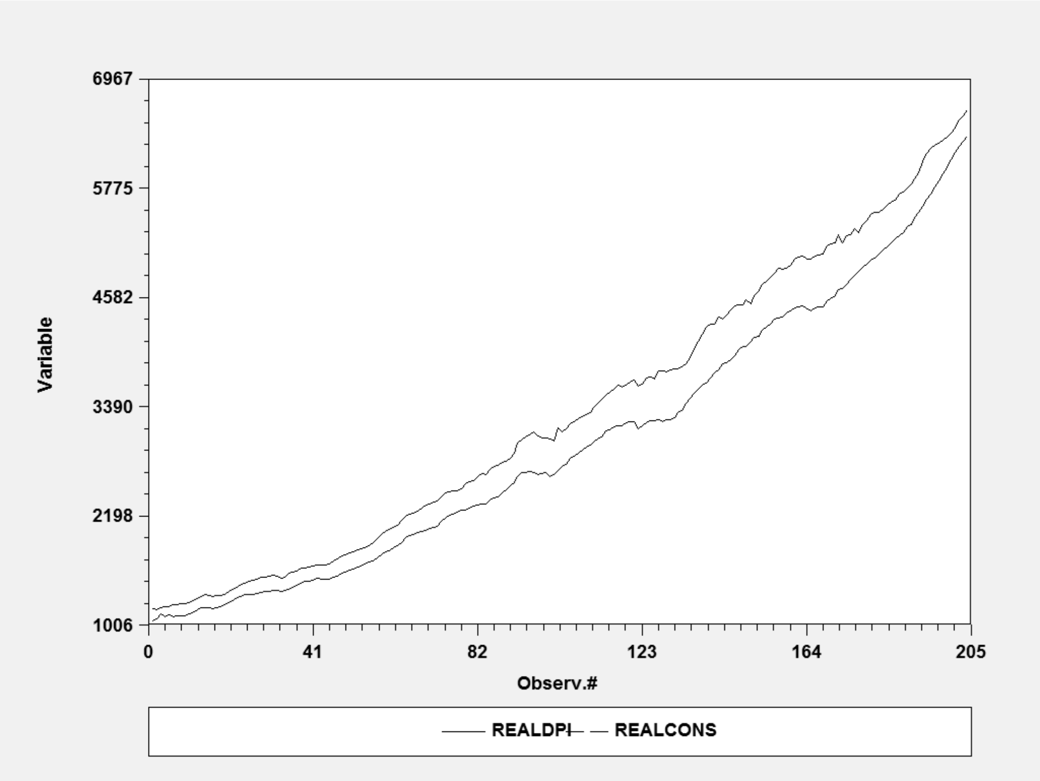
$\beta' x_t =$ the equilibrium

The model describes adjustment of y_t to equilibrium when x_t changes.

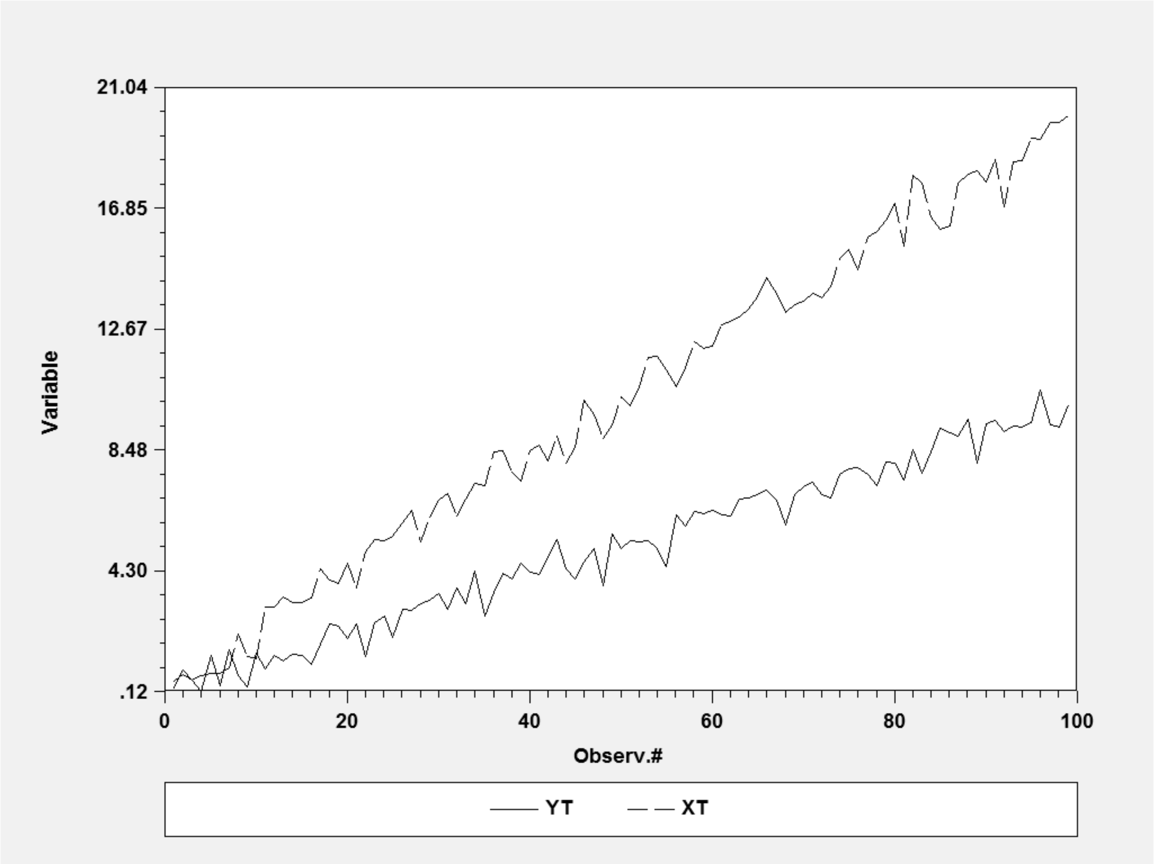
Integrated Processes

- Integration of order (P) when the P'th differenced series is stationary
- Stationary series are I(0)
- Trending series are often I(1). Then $y_t - y_{t-1} = \Delta y_t$ is I(0). [Most macroeconomic data series.]
- Accelerating series might be I(2). Then $(y_t - y_{t-1}) - (y_t - y_{t-1}) = \Delta^2 y_t$ is I(0) [Money stock in hyperinflationary economies. Difficult to find many applications in economics]

Cointegration: Real DPI and Real Consumption



Cointegration – Divergent Series?



Cointegration

- X(t) and y(t) are obviously I(1)
- Looks like any linear combination of x(t) and y(t) will also be I(1)
- Does a model y(t) = bx(t) + u(u) where u(t) is I(0) make any sense? How can u(t) be I(0)?
- In fact, there is a linear combination, [1,-β] that is I(0).
- y(t) = .1*t + noise, x(t) = .2*t + noise
- y(t) and x(t) have a **common trend**
- y(t) and x(t) are cointegrated.

Cointegration and I(0) Residuals