Dependency Rates and Savings Rates: Further Comment

By ARTHUR S. GOLDBERGER*

The empirical results on dependency rates and savings rates reported by Nathaniel Leff (1969) cannot be correct.

For several cross-country samples, Leff estimates pairs of equations of the form

$$(1) \quad y_1 = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + e_1$$

$$(2) \quad y_2 = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + e_2$$

where

$y_1 = ln\ S/Y = ln$ domestic savings ratio
$y_2 = ln\ S/N = ln$ per capita savings
$x_1 = ln\ Y/N = ln$ per capita income
$x_2 = g =$ growth rate of per capita income
$x_3 = ln\ D_1 = ln$ percentage of population aged 14 or less
$x_4 = ln\ D_2 = ln$ percentage of population aged 65 or more,

and a and b are least-squares regression coefficients, and e_1 and e_2 are least-squares residuals.

As noted by Leff, $S/N \equiv (S/Y)(Y/N)$. Consequently, $y_2 = y_1 + x_1$. Least-squares regression being what it is, a proper computation of (2) should produce

$$(3) \quad y_2 = a_0 + (1 + a_1)x_1 + a_2x_2 + a_3x_3 + a_4x_4 + e_1$$

That is, regressing y_2 on the x should give the same coefficients and the same residuals as occur when y_1 is regressed on the x, except for the coefficient of x_1, which should increase by exactly 1.

Furthermore, if regression coefficients are guaranteed to be equal, their standard errors, and hence their t-ratios, must be equal. If regression coefficients are guaranteed to differ by unity, their standard errors must be equal, and hence their t-ratios must be related by

$$b_1/s_{b_1} = (a_1/s_{a_1})((1 + a_1)/a_1)$$

But the results Leff reports do not satisfy these arithmetic requirements. For example, consider the upper panel of his Table 1, p. 891, which refers to a sample of 47 underdeveloped countries. In the present notation we find:

Explanatory variable $j=$	1	2	3	4
Coefficients:				
a_j	.1292	.0227	−1.2297	−.4455
b_j	1.1167	.0239	−1.3122	−.4469
t-ratios:				
a_j/s_{a_j}	1.8487	2.8079	2.7636	2.1554
b_j/s_{b_j}	16.8355	3.1204	2.9400	2.2783

Something must be wrong.

For the same reasons, the empirical results reported by Kanhaya Gupta cannot be correct. Gupta fits (1) and (2) to 3 subgroups of Leff's sample, using Leff's data. His results also fail to satisfy the arithmetic requirements noted above. For example, consider the first panel of his Table 2, p. 470, which refers to 9 underdeveloped countries. In the present notation we find:

Explanatory variable $j=$	1	2	3	4
Coefficients:				
a_j	.4548	.1263	−.7685	−.6475
b_j	1.6112	.1309	−1.0487	−.5585
t-ratios:				
a_j/s_{a_j}	.3768	.3737	.1481	.9446
b_j/s_{b_j}	1.6008	.4644	.2424	.9770

The discrepancies are somewhat more striking in this subsample.

Perhaps none of the discrepancies noted here is large enough to vitiate the substantive analyses of Leff and Gupta. But it is disturbing that figures, some of which are

* Professor of economics, University of Wisconsin, Madison.

transparently wrong in the first decimal place, are reported to four decimal places.

One's first reaction is to attribute such discrepancies to computational inaccuracies. (Indeed, G. M. Mullet and T. W. Murray recently proposed fitting equation pairs like (1)–(2) to check the accuracy of computer programs.) But in the present case, the explanation lies elsewhere. The basic data constructed by Leff, and used by him and Gupta, were internally inconsistent. Scanning the series, which were kindly provided to me by Leff, one finds that, country by country, there are substantial discrepancies between per capita savings and the product of the savings ratio times per capita income. For example, for the first two countries in the list, we find

S/N	S/Y	Y/N	$(S/Y)(Y/N)$
43.7	.101	585	59.1
20.6	.094	271	25.5

Since $y_2 \neq y_1 + x_1$, the argument which led us to (3) loses its force. But it's difficult to see how an economic analysis of the relationship between savings rates and dependency rates can be based on data which are so internally inconsistent.

REFERENCES

K. L. Gupta, "Dependency Rates and Savings Rates: Comment," *Amer. Econ. Rev.*, June 1971, *61*, 469–71.

N. H. Leff, "Dependency Rates and Savings Rates," *Amer. Econ. Rev.*, Dec. 1969, *59*, 886–96.

———, "Dependency Rates and Savings Rates: Reply," *Amer. Econ. Rev.*, June 1971, *61*, 476–80.

G. M. Mullett and T. W. Murray, "A New Method for Examining Rounding Error in Least-Squares Computer Programs," *J. Amer. Statist. Ass.*, Sept. 1971, *66*, 496–98.