Department of Economics

Econometric Analysis of Panel Data

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www.stern.nyu.edu/~wgreene/Econometrics/PanelDataEconometrics.htm

Assignment 4
Parameter Heterogeneity in Linear Models: RPM and HLM

The estimation parts of this assignment will be based on the Baltagi and Griffin gasoline market and the Cornwell and Rupert labor market data sets that are posted on the course website.

We will begin with the gasoline market.The basic linear regression model in use will be

$$y_{it} = \beta_1 + \beta_2 x_{it,1} + \beta_3 x_{it,2} + \beta_4 x_{it,3} + w_{it}$$

where
and i = 1,…,18 OECD countries
t = 1,…,19 years (1960 to 1978).
y_{it} = *lgaspcar* = log of per capita gasoline use
$x_{it,1}$ = *lincomep* = log of per capita income
$x_{it,2}$ = *lrpmg* = log of gasoline price index
$x_{it,3}$ = *lcarpcap* = log of cars per capita
w_{it} = a disturbance that may have have both permanent (time invariant) components and time varying components, and may, under some circumstances, be correlated with $\mathbf{x}_{it}$.

Denote $\mathbf{x}_{it}$ = $(1,x_{it,1},x_{it,2},x_{it,3})$ and
$\mathbf{X}_i$ = the 19×4 matrix containing all the data on $\mathbf{x}_{it}$ for country i.

Part I. Parameter Variation in the Gasoline Market

A. Homogeneous parameters: To begin, we *assume* that all parameters, including the constant term, are homogeneous across countries and through time and that $w_{it} = \varepsilon_{it}$, a classical zero mean, homoscedastic disturbances.

1. Under these assumptions, what are the properties of the pooled OLS estimator?

Under the assumptions, the model is a classical linear regression, so OLS is unbiased, consistent, and efficient by the Gauss Markov Theorem. The asymptotic distribution is mormal with mean β and asymptotic covariance matrix $\sigma_\varepsilon^2/(nT) \times \text{plim}[X'X/nT]^{-1}$.

2. Estimate the parameters of the model using OLS and report your results.

```
--> NAMELIST ; X = one,lincomep,lrpmg,lcarpcap $
--> REGRESS ; Lhs = lgaspcar ; Rhs =X $

-----------------------------------------------------------------
Ordinary     least squares regression ............
LHS=LGASPCAR Mean                 =       4.29624
             Standard deviation   =        .54891
             Number of observs.   =           342
Model size   Parameters           =             4
             Degrees of freedom   =           338
Residuals    Sum of squares       =      14.90436
             Standard error of e  =        .20999
Fit          R-squared            =        .85494
             Adjusted R-squared   =        .85365
Model test   F[  3,   338] (prob) =   664.0(.0000)
Diagnostic   Log likelihood       =      50.49288
             Restricted(b=0)      =    -279.63574
             Chi-sq [  3]  (prob) =   660.3(.0000)
Info criter. LogAmemiya Prd. Crt. =      -3.10976
             Akaike Info. Criter. =      -3.10977
             Bayes Info. Criter.  =      -3.06491
--------+--------------------------------------------------------
        |              Standard            Prob.        Mean
LGASPCAR| Coefficient       Error       t    t>|T|        of X
--------+--------------------------------------------------------
Constant|    2.39133***      .11693    20.45  .0000
LINCOMEP|     .88996***      .03581    24.86  .0000   -6.13943
   LRPMG|    -.89180***      .03031   -29.42  .0000    -.52310
LCARPCAP|    -.76337***      .01861   -41.02  .0000   -9.04180
--------+--------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------
```

3. As a first cut at assessing whether the assumptions are correct, compute the robust, cluster (country) corrected standard errors for the least squares estimator. Do they appear to be the same, or close to the same, as the uncorrected OLS standard errors? What do you conclude about the disturbances in the equation?

```
--> REGRESS ; Lhs = lgaspcar ; Rhs =X ; Cluster=Country$
+---------------------------------------------------------------+
| Covariance matrix for the model is adjusted for data clustering.    |
| Sample of    342 observations contained     18 clusters defined by  |
| variable COUNTRY  which identifies by a value a cluster ID.        |
+---------------------------------------------------------------+

-----------------------------------------------------------------
Ordinary     least squares regression ............
LHS=LGASPCAR Mean                 =       4.29624
             Standard deviation   =        .54891
             Number of observs.   =           342
Model size   Parameters           =             4
             Degrees of freedom   =           338
Residuals    Sum of squares       =      14.90436
             Standard error of e  =        .20999
Fit          R-squared            =        .85494
             Adjusted R-squared   =        .85365
Model test   F[  3,   338] (prob) =   664.0(.0000)
Diagnostic   Log likelihood       =      50.49288
             Restricted(b=0)      =    -279.63574
             Chi-sq [  3]  (prob) =   660.3(.0000)
Info criter. LogAmemiya Prd. Crt. =      -3.10976
             Akaike Info. Criter. =      -3.10977
             Bayes Info. Criter.  =      -3.06491
Model was estimated on Feb 09, 2010 at 09:52:38 PM
--------+--------------------------------------------------------
        |                  Standard            Prob.        Mean
LGASPCAR| Coefficient        Error       t    t>|T|        of X
--------+--------------------------------------------------------
Constant|    2.39133***      .44167     5.41  .0000
LINCOMEP|     .88996***      .17248     5.16  .0000   -6.13943
   LRPMG|    -.89180***      .14578    -6.12  .0000    -.52310
LCARPCAP|    -.76337***      .06985   -10.93  .0000   -9.04180
--------+--------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------
```

The standard errors have increased substantially. I suspect that this indicates that there is substantial correlation across observations within a country.

B. Heterogeneous Constant Terms: Now, consider fixed and random effects formulations of the model. We write the model as

$$y_{it} = \beta_{1i} + \beta_2 x_{it,1} + \beta_3 x_{it,2} + \beta_4 x_{it,3} + \varepsilon_{it}$$

where

$$\beta_{1i} = \beta_1 + u_i \text{ and } E[u_i] = 0.$$

Thus, this is a model with a random constant term. By substituting the second equation into the first, you can see that it is the “effects” model we have discussed in class.

1. (**Fixed Effects**) Using the OECD gasoline data, estimate the parameters of the model under the assumption that $E[u_i|\mathbf{X}_i] = g(\mathbf{X}_i)$ for some nonzero function $g(.)$. Explain the estimator and the motivation for using it. Display your results with the OLS estimates so that you (and your reader) can see the difference between the two. Note that $E[u_i|\mathbf{X}_i] = g(\mathbf{X}_i)$ is consistent with $E[u_i] = 0$. When averaged over $\mathbf{X}_i$, the overall mean is zero, but the mean is not zero for a specific $\mathbf{X}_i$. This implies that u_i and $\mathbf{X}_i$ are correlated.

This would be the fixed effects model. The parameters are estimated by using the within estimator – including the country dummy variables in the equation.

```
----------------------------------------------------------------------
OLS Without Group Dummy Variables.................
Ordinary     least squares regression ............
LHS=LGASPCAR Mean                 =       4.29624
             Standard deviation   =        .54891
             Number of observs.   =           342
Model size   Parameters           =             4
             Degrees of freedom   =           338
Residuals    Sum of squares       =      14.90436
             Standard error of e  =        .20999
Fit          R-squared            =        .85494
             Adjusted R-squared   =        .85365
Model test   F[  3,   338] (prob) =   664.0(.0000)
Diagnostic   Log likelihood       =      50.49288
             Restricted(b=0)      =    -279.63574
             Chi-sq [  3]  (prob) =   660.3(.0000)
Info criter. LogAmemiya Prd. Crt. =      -3.10976
             Akaike Info. Criter. =      -3.10977
             Bayes Info. Criter.  =      -3.06491
Panel Data Analysis of LGASPCAR          [ONE way]
               Unconditional ANOVA (No regressors)
Source         Variation  Deg. Free.   Mean Square
Between         85.68228         17.       5.04013
Residual        17.06068        324.        .05266
Total          102.74296        341.        .30130
--------+-------------------------------------------------------------
        |                  Standard            Prob.       Mean
LGASPCAR| Coefficient       Error       t    t>|T|       of X
--------+-------------------------------------------------------------
LINCOMEP|     .88996***      .03581    24.86  .0000   -6.13943
   LRPMG|    -.89180***      .03031   -29.42  .0000    -.52310
LCARPCAP|    -.76337***      .01861   -41.02  .0000   -9.04180
Constant|    2.39133***      .11693    20.45  .0000
--------+-------------------------------------------------------------
```

```
----------------------------------------------------------------------
Least Squares with Group Dummy Variables..........
Ordinary     least squares regression ............
LHS=LGASPCAR Mean                 =       4.29624
             Standard deviation   =        .54891
             Number of observs.   =           342
Model size   Parameters           =            21
             Degrees of freedom   =           321
Residuals    Sum of squares       =       2.73649
             Standard error of e  =        .09233
Fit          R-squared            =        .97337
             Adjusted R-squared   =        .97171
Model test   F[ 20,   321] (prob) =   586.6(.0000)
Diagnostic   Log likelihood       =     340.33399
             Restricted(b=0)      =    -279.63574
             Chi-sq [ 20]  (prob) =  1239.9(.0000)
Info criter. LogAmemiya Prd. Crt. =      -4.70517
             Akaike Info. Criter. =      -4.70533
             Bayes Info. Criter.  =      -4.46986
Estd. Autocorrelation of e(i,t)   =       .775557
Panel:Groups Empty      0,     Valid data       18
             Smallest  19,     Largest          19
             Average group size in panel     19.00
--------+-------------------------------------------------------------
        |                  Standard            Prob.       Mean
LGASPCAR| Coefficient       Error       t    t>|T|       of X
--------+-------------------------------------------------------------
LINCOMEP|     .66225***      .07339     9.02  .0000   -6.13943
   LRPMG|    -.32170***      .04410    -7.29  .0000    -.52310
LCARPCAP|    -.64048***      .02968   -21.58  .0000   -9.04180
--------+-------------------------------------------------------------
```

Fixed effects results are somewhat differerent. But, notice that the estimate price elasticity has fallen by 50%, a large change.

2. (**Random Effects**) Estimate the parameters of the model under the more restrictive assumption that $E[u_i|\mathbf{X}_i] = 0$.

```
-----------------------------------------------------------------------
Random Effects Model: v(i,t)    = e(i,t) + u(i)
Estimates:  Var[e]              =       .008525
            Var[u]              =       .035571
            Corr[v(i,t),v(i,s)] =       .806673
Lagrange Multiplier Test vs. Model (3) =1465.55
( 1 degrees of freedom, prob. value =  .000000)
(High values of LM favor FEM/REM over CR model)
Baltagi-Li form of LM Statistic =       1465.55
            Sum of Squares            27.266173
            R-squared                   .734618
--------+--------------------------------------------------------------
        |                  Standard           Prob.       Mean
LGASPCAR| Coefficient        Error       z    z>|Z|       of X
--------+--------------------------------------------------------------
LINCOMEP|     .55270***      .05651     9.78  .0000   -6.13943
   LRPMG|    -.42500***      .03842   -11.06  .0000    -.52310
LCARPCAP|    -.60631***      .02446   -24.78  .0000   -9.04180
Constant|    1.98508***      .17572    11.30  .0000
--------+--------------------------------------------------------------
```

The random effects results resemble the fixed effects results.

3. Use the Wu/Mundlak variable addition test to test for the assumption of the (null) random effects model against the (alternative) fixed effects model. Report your results and your conclusions.

```
Create  ; x1bar = GroupMean(lincomep,str=country)$
Create  ; x2bar = GroupMean(lrpmg,str=country)$
Create  ; x3bar = GroupMean(lcarpcap,str=country)$
REGRESS ; Lhs = lgaspcar
        ; Rhs = x1bar,x2bar,x3bar,X
        ; Str=Country ; Random Effects$
-----------------------------------------------------------------------
Random Effects Model: v(i,t)    = e(i,t) + u(i)
Estimates:  Var[e]              =       .008605
            Var[u]              =       .030281
            Corr[v(i,t),v(i,s)] =       .778708
Lagrange Multiplier Test vs. Model (3) =1864.35
( 1 degrees of freedom, prob. value =  .000000)
(High values of LM favor FEM/REM over CR model)
Baltagi-Li form of LM Statistic =       1864.35
            Sum of Squares            13.027071
            R-squared                   .873207
--------+--------------------------------------------------------------
        |                  Standard           Prob.       Mean
LGASPCAR| Coefficient        Error       z    z>|Z|       of X
--------+--------------------------------------------------------------
   X1BAR|     .30533*        .15712     1.94  .0520   -6.13943
   X2BAR|    -.64185***      .12649    -5.07  .0000    -.52310
   X3BAR|    -.15482*        .07933    -1.95  .0510   -9.04180
LINCOMEP|     .66225***      .07373     8.98  .0000   -6.13943
   LRPMG|    -.32170***      .04431    -7.26  .0000    -.52310
LCARPCAP|    -.64048***      .02982   -21.48  .0000   -9.04180
Constant|    2.54163***      .46953     5.41  .0000
--------+--------------------------------------------------------------
--> Matrix  ; bm = b(1:3) ; vm=varb(1:3,1:3)
    ; List ; WaldStat = bm'<vm>bm $

Matrix WALDSTAT has  1 rows and  1 columns.
              1
        +-------------+
       1|   31.31621
        +-------------+
```

The wald statistic is much larger than the critical value with 3 degrees of freedom. This suggests that the fixed effects specification is the preferred model.

C. General parameter heterogeneity: Let $\mathbf{x}_{it}$ denote $(1, lincomep, lrpmg, lcarpcap)_{it}$. We now consider the possibility that there are differences across countries. Write the model

(1) $y_{it} = \boldsymbol{\beta}_i'\mathbf{x}_{it} + \varepsilon_{it}$.

Absent any further assumptions about the variation in the parameters across countries, how would you proceed to examine the relationship between per capita gasoline consumption, y_{it} and the other variables, $\mathbf{x}_{it}$?

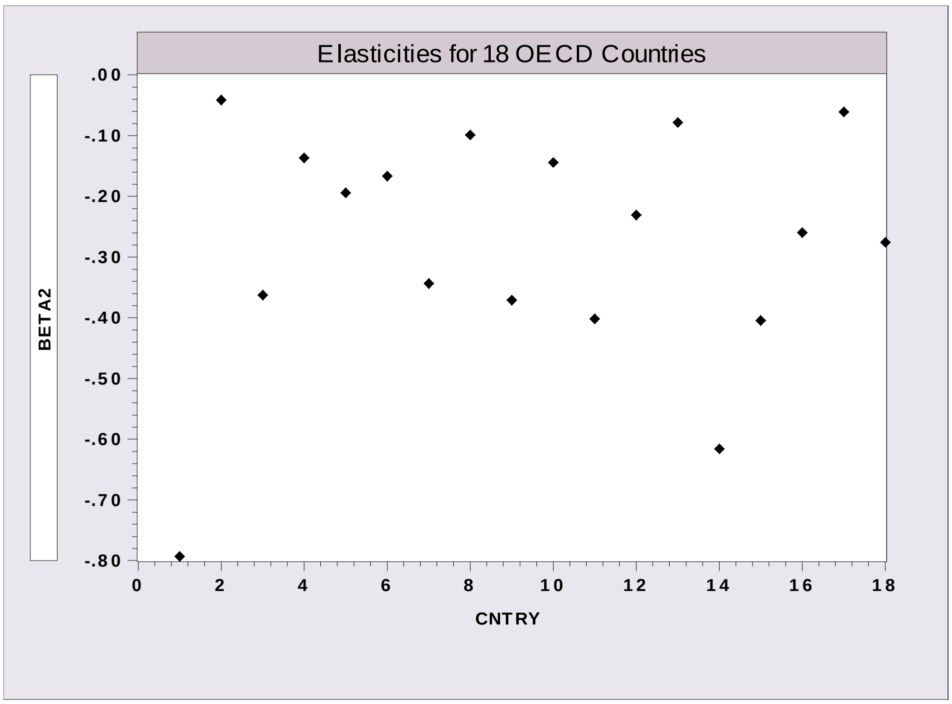
1. Suppose we now assume that all the parameters, not just the constant, are random;

(2) $\boldsymbol{\beta}_i = \boldsymbol{\beta} + \mathbf{u}_i$

where $\mathbf{u}_i$ has an overall mean of zero, however $E[\mathbf{u}_i|\mathbf{X}_i] = g(\mathbf{X}_i)$, where $\mathbf{X}_i$ is the 19 years of data on $\mathbf{x}_{it}$ for country i? Note that the assumption of the overall mean of zero states only that $\boldsymbol{\beta}_i$ varies around a mean.

We are particularly interested in the price elasticity of the demand for gasoline, the coefficient on *lrpmg*. To explore the cross country variation, compute the linear regression model for each of the 19 countries (separately). Display in a graph or a well labeled table the results of your estimation, to describe the variation in the estimated coefficients on *lrpmg*. Note that the assumption about $\mathbf{u}_i$ is equivalent to the "fixed effects" case, but here we are considering the entire parameter vector, not just the constant term.

```
Matrix ; beta2 = Init(18,1,0)$
Calc ; i1=1 ; i = 0 $
Procedure
Calc ; i2 = i1 + 18 ; i = i + 1 $
Sample  ; i1-i2 $
Regress ; quietly ; Lhs = lgaspcar ; Rhs = x $
Matrix  ; beta2(i)=b(3) $
Calc    ; i1=i1+19 $
EndProc
Execute ; N = 18 $
Matrix  ; cntry=[1/2/3/4/5/6/7/8/9/10/11/12/13/14/15/16/17/18]$
Mplot   ; Lhs = cntry ; Rhs = beta2
        ;Title=Elasticities for 18 OECD Countries $
```



2. If we add to A. the assumption $E[\mathbf{u}_i|\mathbf{X}_i] = 0$, the model turns into a "random effects" model, though note, once again, we are considering the entire parameter vector. Under this new assumption, what are the properties of the pooled ordinary least squares estimator? What does **b** estimate in this case? For a useful step in the analysis, insert (2) into (1) and analyze the implied model.

Under the random effects assumption, the model turns into a heteroscedastic linear regression. I.e.,

$$y_{it} = \beta_i' x_{it} + \varepsilon_{it}, \quad \beta_i = \beta + u_i,$$

so

$$y_{it} = \beta' x_{it} + \varepsilon_{it} + u_i' x_{it}.$$

This is a linear regression with mean $\beta' x_{it}$ and variance $\sigma_\varepsilon^2 + x_{it}'\Sigma x_{it}$ where Σ is the 4×4 covariance matrix of the random parameters. Since it is a linear regression and the distirbances, though heteroscedastic, are not correlated with x_{it}, it can be fit by OLS, though inefficiently. Note, $u_i' x_{it}$ is not correlated with x_{it}. To see, just note that $E[u_i' x_{it}|x_{it}] = E[u_i'] \times x_{it} = 0$. (Looks like a trivial proof. But, it's right.) So, pooled OLS has the properties of OLS in the presence of heteroscedasticity. Unbiased (possibly), consistent, asymptotic normal, not efficient.

Part II. Theory and an Example for Simulation Based Estimation:

This theoretical exercise will begin to suggest how simulation based estimation works. Consider a simple regression model

$$y_{it} = \beta_i x_{it} + \varepsilon_{it}$$

There is only one variable and no constant in the model. Assume that $\varepsilon_{it} \sim N[0,\sigma^2]$. We suppose as well that β_i is random; $\beta_i = \beta + w u_i$ where $w_i \sim N[0,\theta^2]$. A simpler way to write this is

$$\beta_i = \beta + \theta u_i \text{ where } u_i \sim N[0,1].$$

Putting θ specifically in the equation simplifies the derivation a bit. The contribution of individual *i* to the likelihood function is the product of the normal densities,

$$L_i = \prod_{t=1}^{T} \frac{1}{\sigma}\phi\left(\frac{y_{it} - \beta_i x_{it}}{\sigma}\right)$$

This is not useable for maximum likelihood estimation because $\beta_i = \beta + \theta u_i$ which means that the log likelihood to be maximized involves the unobserved u_i;

$$L_i = \prod_{t=1}^{T} \frac{1}{\sigma}\phi\left(\frac{y_{it} - \beta x_{it} - \theta u_i x_{it}}{\sigma}\right)$$

In principle, we would now maximize log$L$ = Σ_i logL_i with respect to (β,θ,σ). The problem is that the unobserved u_i is in the equation and must be integrated out to proceed.. The contribution of individual i to the *unconditional* log likelihood function is

$$\log L_i = \log\int_{-\infty}^{\infty}\left[\prod_{t=1}^{T}\frac{1}{\sigma}\phi\left(\frac{y_{it}-\beta x_{it}-\theta u_i x_{it}}{\sigma}\right)\right]\phi(u_i)du_i$$

where $\phi(u_i)$ is the standard normal density. The integral of the product above does not exist in closed form, so we will approximate it by simulation. (It could be approximated with quadrature.) Adding up the individual contributions, the *simulated* log likelihood is

$$\log L_S = \sum_{i=1}^{n}\log\left\{\frac{1}{R}\sum_{r=1}^{R}\left[\prod_{t=1}^{T}\frac{1}{\sigma}\phi\left(\frac{y_{it}-\beta x_{it}-\theta u_{ir}x_{it}}{\sigma}\right)\right]\right\}$$

where u_{ir} is a set of R random draws on the standard normal population for each individual i. (The random draws are reused every time the function or its derivatives are computed. There are a total of nR random draws used in the simulation.) An additional simplification is obtained by using $\gamma = 1/\sigma$. (We make use of the invariance principle for maximum likelihood estimation.) Then,

$$\log L_S = \sum_{i=1}^{n}\log\left\{\frac{1}{R}\sum_{r=1}^{R}\left[\prod_{t=1}^{T}\gamma\phi\left(\gamma\left(y_{it}-\beta x_{it}-\theta u_{ir}x_{it}\right)\right)\right]\right\}.$$

The maximum simulated likelihood estimator is the (β,θ,γ) that maximizes this function.

1. Derive the necessary (first order) conditions for maximizing this function. Hint: your derivation is simplified greatly by using the result $d\phi(t)/dt = -t\phi(t)$. You can then just use the chain rule.

$$\frac{\partial\log L_S}{\partial\gamma} = \sum_{i=1}^{n}\frac{\frac{1}{R}\sum_{r=1}^{R}\left[\prod_{t=1}^{T}\gamma\phi\left(\gamma\left(y_{it}-\beta x_{it}-\theta u_{ir}x_{it}\right)\right)\right]\sum_{t=1}^{T}\frac{\partial\log\gamma\phi\left(\gamma\left(y_{it}-\beta x_{it}-\theta u_{ir}x_{it}\right)\right)}{\partial\gamma}}{\left\{\frac{1}{R}\sum_{r=1}^{R}\left[\prod_{t=1}^{T}\gamma\phi\left(\gamma\left(y_{it}-\beta x_{it}-\theta u_{ir}x_{it}\right)\right)\right]\right\}}$$

(Note I used a trick above, $\partial f/\partial x = f\times\partial\log f/\partial x$. The derivatives with respect to β and θ have the same form save for the derivative in the rightmost sum. These three derivatives are

$$\frac{\partial\log\gamma\phi\left(\gamma\left(y_{it}-\beta x_{it}-\theta u_{ir}x_{it}\right)\right)}{\partial\begin{pmatrix}\gamma\\ \beta\\ \theta\end{pmatrix}} = \begin{pmatrix}1/\gamma\\ 0\\ 0\end{pmatrix} + \frac{1}{\phi(.)}(-.)\phi(.)\begin{pmatrix}(y_{it}-\beta x_{it}-\theta u_{ir}x_{it})\\ -\gamma x_{it}\\ -\gamma u_{ir}x_{it}\end{pmatrix}$$

$$\frac{\partial \log \gamma\phi\left(\gamma\left(y_{it}-\beta x_{it}-\theta u_{ir}x_{it}\right)\right)}{\partial\begin{pmatrix}\gamma\\ \beta\\ \theta\end{pmatrix}}=\begin{pmatrix}1/\gamma\\ 0\\ 0\end{pmatrix}-\gamma\left(y_{it}-\beta x_{it}-\theta u_{ir}x_{it}\right)\begin{pmatrix}(y_{it}-\beta x_{it}-\theta u_{ir}x_{it})\\ -\gamma x_{it}\\ -\gamma u_{ir}x_{it}\end{pmatrix}$$

2. How would you obtain asymptotic standard errors for your estimator?

Notice that the derivatives have to be simulated just like the log likelihood. In principle, you would differentiate this function again and use the negative inverse of the second derivatives. The derivatives are extremely complicated. This looks like a very good candidate for the BHHH estimator. The first derivative vector above, looking at it from the outside, takes the form

$$\frac{\partial \log L_S}{\partial\begin{pmatrix}\gamma\\ \beta\\ \theta\end{pmatrix}}=\sum_{i=1}^{n}\mathbf{g}_i$$

where g_i is the vector in the messy expression after the first summation. An estimator that will work asymptotically is just

$$V=\left[\sum_{i=1}^{n}\mathbf{g}_i\mathbf{g}_i'\right]^{-1}$$

Complicated, but not so bad as the full Hessian.

3. The following small exercise will show this computation at work. This application estimates the parameters of a model that precisely satisfies the assumptions of the model above. Execute these commands and report all of your results

```
?  1,000 observations in total will be n=100, T=10.  The x(i,t) is normally distributed
?  with mean zero and standard deviation 1.  Variable i is the 1,1,1,1...,2,2,2,2... etc.
Sample ; 1 - 1000 $
Create ; xit = Rnn(0,1) ; i = Trn(10,0) $
? We generate b(i) = 0.5 + u(i) where u(i) is normal with mean 0, standard deviation
? .5.  Then, y(i,t) = b(i)*x(i,t) + e(i,t) where e(i,t) is normally distributed with zero
? mean and standard deviation 1.
Matrix ; bi = init(100,1,.5) + .5*rndm(100)$
Create ; yit = bi(i)*xit + rnn(0,1) $
? This command estimates the random parameters model exactly as shown in
? part 2. above.
Regress ; lhs = yit ; rhs = xit ;rpm ; fcn=xit(n)
        ; pds=10 ; pts=100 ; halton $
```

```
--> CALC;DELETE I$
--> Sample ; 1 - 1000 $
--> Create ; xit = Rnn(0,1) ; i = Trn(10,0) $
--> Matrix ; bi = init(100,1,.5) + .5*rndm(100)$
--> Create ; yit = bi(i)*xit + rnn(0,1) $
--> Regress ; lhs = yit ; rhs = xit ;rpm ; fcn=xit(n)
    ; pds=10 ; pts=100 ; halton $

-----------------------------------------------------------------
OLS Starting values for random parameters model...
Ordinary     least squares regression ............
LHS=YIT      Mean                 =       -.03481
             Standard deviation   =       1.21392
             Number of observs.   =          1000
Model size   Parameters           =             1
             Degrees of freedom   =           999
Residuals    Sum of squares       =    1222.62727
             Standard error of e  =       1.10628
Fit          R-squared            =        .16948
             Adjusted R-squared   =        .16948
Model test   F[  1,   999] (prob) =   203.9(.0000)
Diagnostic   Log likelihood       =    -1519.43957
             Restricted(b=0)      =    -1612.29192
             Chi-sq [  1]  (prob) =   185.7(.0000)
Info criter. LogAmemiya Prd. Crt. =        .20300
             Akaike Info. Criter. =        .20300
             Bayes Info. Criter.  =        .20791
--------+--------------------------------------------------------
        |                  Standard            Prob.       Mean
     YIT| Coefficient        Error       z    z>|Z|       of X
--------+--------------------------------------------------------
     XIT|     .48868***      .03414    14.31  .0000    -.00495
--------+--------------------------------------------------------
Normal exit:   9 iterations. Status=0. F=    1451.252

-----------------------------------------------------------------
Random Coefficients  LinearRg Model
Dependent variable                  YIT
Log likelihood function     -1451.25191
Estimation based on N =   1000, K =   3
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC              2.90850     2908.50381
Fin.Smpl.AIC     2.90853     2908.52791
Bayes IC         2.92323     2923.22708
Hannan Quinn     2.91410     2914.09968
Sample is 10 pds and    100 individuals
LINEAR regression model
Simulation based on    100 Halton draws
--------+--------------------------------------------------------
        |                  Standard            Prob.       Mean
     YIT| Coefficient        Error       z    z>|Z|       of X
--------+--------------------------------------------------------
        |Means for random parameters
     XIT|     .48041***      .03006    15.98  .0000    -.00495
        |Scale parameters for dists. of random parameters
     XIT|     .63417***      .02593    24.45  .0000
        |Variance parameter given is sigma
Std.Dev.|     .96500***      .01908    50.57  .0000
--------+--------------------------------------------------------
```

Estimation begins with OLS (as usual). The OLS (consistent) estimate is 0.48868. The RP estimate of the mean of the distribution is .48041. The standard deviation of the distribution of β_i is 0.63417. The true value is 0.5. The estimate of σ_ε is 0.965. The true value is 1.0.

Part III. Random Parameters Models

This exercise will demonstrate the computation of a fairly elaborate, hierarchical linear model. The computations are based on the Cornwell and Rupert data. Note that the simulations below are based on Halton sequences, not pseudorandom random numbers. As such, the results you obtain below are replicable – in principle, you and I (and your colleagues) should all get the same results. Also, if you fit these equations more than once, you will get the same answers.

1. A simple RPM with one random coefficient. The first model is the regression model discussed in class, now with a random coefficient on education. After fitting the random parameters model, this program computes the posterior estimates of $E[\beta_{i,Ed}|y_i,X_i]$ and plots the distribution with a kernel density estimator and a histogram. Estimate the model and report all results. (Note, you can copy/paste the figure into a Word document.)

```
Sample ; All $
Regress ; Lhs = Lwage ;Rhs = One,Exp,Occ,Ind,South,SMSA,MS,FEM,Union,Ed,Blk
;Pds=7 ;RPM ; Halton ; Pts=100 ;Fcn = Ed(N) ;Parameters ; Maxit = 20 $
Sample ; 1 - 595 $
Create ; Ed_Coeff = 0 $
Create ; Ed_Coeff = beta_i $
Kernel ; Rhs = Ed_Coeff$
Histogram ; Rhs = Ed_Coeff $
```

```
--> Sample ; All $
--> Regress ; Lhs = Lwage
    ; Rhs = One,Exp,Occ,Ind,South,SMSA,MS,FEM,Union,Ed,Blk
    ;Pds=7 ;RPM ; Halton ; Pts=100 ;Fcn = Ed(N)
    ;Parameters $
```

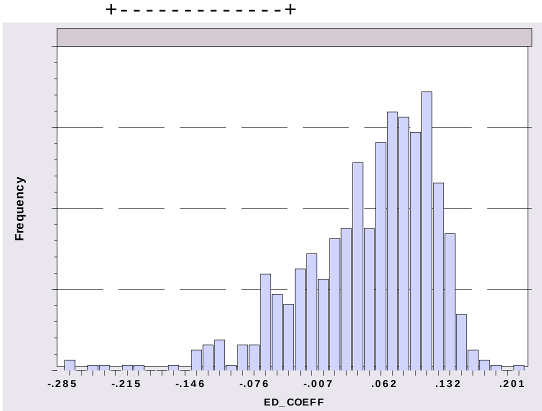
```
-----------------------------------------------------------------
OLS Starting values for random parameters model...
Ordinary     least squares regression ............
LHS=LWAGE    Mean                 =       6.67635
             Standard deviation   =        .46151
             Number of observs.   =          4165
Model size   Parameters           =            11
             Degrees of freedom   =          4154
Residuals    Sum of squares       =     533.91105
             Standard error of e  =        .35851
Fit          R-squared            =        .39801
             Adjusted R-squared   =        .39656
Model test   F[ 10,  4154] (prob) =   274.6(.0000)
Diagnostic   Log likelihood       =   -1631.91946
             Restricted(b=0)      =   -2688.80603
             Chi-sq [ 10]  (prob) =  2113.8(.0000)
Info criter. LogAmemiya Prd. Crt. =      -2.04896
             Akaike Info. Criter. =      -2.04896
             Bayes Info. Criter.  =      -2.03223
Model was estimated on Feb 09, 2010 at 10:51:13 PM
--------+--------------------------------------------------------
        |                  Standard            Prob.        Mean
   LWAGE| Coefficient        Error       z    z>|Z|        of X
--------+--------------------------------------------------------
Constant|    5.68035***      .04783   118.77  .0000
     EXP|     .01024***      .00054    19.11  .0000     19.8538
     OCC|    -.14664***      .01502    -9.76  .0000      .51116
     IND|     .05525***      .01208     4.57  .0000      .39544
   SOUTH|    -.05298***      .01285    -4.12  .0000      .29028
    SMSA|     .14851***      .01236    12.02  .0000      .65378
      MS|     .06783***      .02107     3.22  .0013      .81441
     FEM|    -.36013***      .02569   -14.02  .0000      .11261
   UNION|     .09158***      .01290     7.10  .0000      .36399
     BLK|    -.16824***      .02262    -7.44  .0000      .07227
      ED|     .05669***      .00268    21.16  .0000     12.8454
--------+--------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------
```

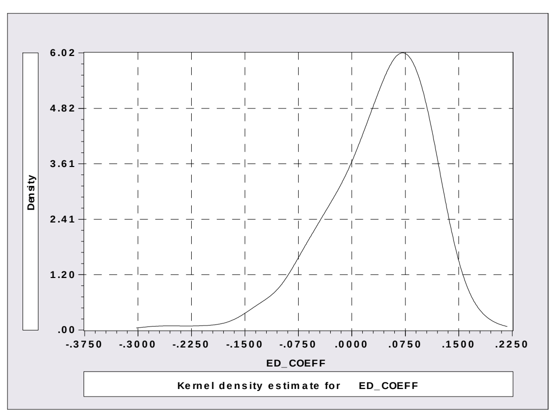
```
Maximum of   100 iterations. Exit iterations with status=1.

-----------------------------------------------------------------
Random Coefficients  LinearRg Model
Dependent variable               LWAGE
Log likelihood function     -264.53571
Estimation based on N =   4165, K =  13
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC               .13327      555.07141
Fin.Smpl.AIC      .13329      555.15910
Bayes IC          .15304      637.41954
Hannan Quinn      .14026      584.20182
Model estimated: Feb 09, 2010, 22:54:10
Sample is  7 pds and    595 individuals
LINEAR regression model
Simulation based on    100 Halton draws
--------+--------------------------------------------------------
        |                  Standard            Prob.       Mean
   LWAGE| Coefficient        Error       z    z>|Z|        of X
--------+--------------------------------------------------------
        |Nonrandom parameters
Constant|    4.57949***      .02054   222.92  .0000
     EXP|     .07444***      .00045   166.45  .0000     19.8538
     OCC|    -.05447***      .00590    -9.23  .0000      .51116
     IND|    -.04841***      .00483   -10.01  .0000      .39544
   SOUTH|     .10661***      .00513    20.79  .0000      .29028
    SMSA|     .07319***      .00478    15.32  .0000      .65378
      MS|     .00116         .00788      .15  .8832      .81441
     FEM|    -.09641***      .01056    -9.13  .0000      .11261
   UNION|     .17620***      .00489    36.02  .0000      .36399
     BLK|    -.16604***      .01027   -16.17  .0000      .07227
        |Means for random parameters
      ED|     .11889***      .00114   103.87  .0000     12.8454
        |Scale parameters for dists. of random parameters
      ED|    1.34686***      .00162   829.14  .0000
        |Variance parameter given is sigma
Std.Dev.|     .15915***      .00075   213.52  .0000
--------+--------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------


Implied standard deviations of random parameters

Matrix S.D_Beta has  1 rows and  1 columns.
               1
        +-------------+
       1|    1.34686
        +-------------+
```





2. The second model is a typical hierarchical model. The model is

$$y_{it} = \beta_{1,i} + \beta_{2,i}Exp_{it} + \beta_3 OCC_{it} + ... + \beta_8 Union_{it} + \varepsilon_{it}$$
$$\beta_{1,i} = \beta_1 + \delta_{1,1}Fem_i + \delta_{1,2}Ed_i + \delta_{1,3}Blk_i + u_{1i},$$
$$\beta_{2,i} = \beta_2 + \delta_{2,1}Fem_i + \delta_{2,2}Ed_i + \delta_{2,3}Blk_i + u_{2i},$$

This is a common sort of model in which the regression of interst is based on the time varying attributes and the variation in the parameters is explained by the randomness, $u_{k,i}$ and by the demographics that do not vary across time, here Gender, Education and Race. Fit the model and report all results, identifying what parameter is what in your report.

```
Sample ; All $
Regress ; Lhs = Lwage
;Rhs = One,Exp,Occ,Ind,South,SMSA,MS,Union
;Pds=7
;RPM=Fem,Ed,Blk
;Halton ; Pts=100
;Fcn = one(n),exp(n)
;Parameters $
-----------------------------------------------------------------
OLS Starting values for random parameters model...
Ordinary     least squares regression ............
LHS=LWAGE    Mean                 =         6.67635
             Standard deviation   =          .46151
             Number of observs.   =            4165
Model size   Parameters           =               8
             Degrees of freedom   =            4157
Residuals    Sum of squares       =       633.19640
             Standard error of e  =          .39028
Fit          R-squared            =          .28606
             Adjusted R-squared   =          .28486
Model test   F[  7,   4157] (prob) =    237.9(.0000)
Diagnostic   Log likelihood       =     -1987.09274
             Restricted(b=0)      =     -2688.80603
             Chi-sq [   7]  (prob) =  1403.4(.0000)
Info criter. LogAmemiya Prd. Crt. =        -1.87985
             Akaike Info. Criter. =        -1.87985
             Bayes Info. Criter.  =        -1.86768
Model was estimated on Feb 09, 2010 at 10:57:31 PM
--------+--------------------------------------------------------
        |                  Standard            Prob.        Mean
   LWAGE| Coefficient        Error        z    z>|Z|        of X
--------+--------------------------------------------------------
     OCC|    -.32393***      .01373   -23.59  .0000      .51116
     IND|     .04033***      .01301     3.10  .0019      .39544
   SOUTH|    -.11104***      .01373    -8.09  .0000      .29028
    SMSA|     .15575***      .01327    11.73  .0000      .65378
      MS|     .33545***      .01611    20.83  .0000      .81441
   UNION|     .06787***      .01398     4.86  .0000      .36399
Constant|    6.31844***      .02106   300.02  .0000
     EXP|     .00705***      .00057    12.44  .0000     19.8538
--------+--------------------------------------------------------
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------

Warning   141: Iterations:current or start estimate of sigma is nonpositiv
Maximum of    20 iterations. Exit iterations with status=1.

-----------------------------------------------------------------
Random Coefficients  LinearRg Model
Dependent variable                 LWAGE
Log likelihood function        468.34115
Restricted log likelihood         .00000
Chi squared [   8 d.f.]        936.68230
Significance level                .00000
Estimation based on N =    4165, K =  17
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC              -.21673     -902.68230
```

```
Fin.Smpl.AIC     -.21670     -902.53472
Bayes IC         -.19088     -794.99628
Hannan Quinn     -.20758     -864.58870
Model estimated: Feb 09, 2010, 22:59:14
Sample is  7 pds and    595 individuals
LINEAR regression model
Simulation based on    100 Halton draws
--------+-------------------------------------------------------------
        |                  Standard            Prob.       Mean
   LWAGE| Coefficient        Error       z    z>|Z|       of X
--------+-------------------------------------------------------------
        |Nonrandom parameters
     OCC|    -.02885***      .00575    -5.02  .0000      .51116
     IND|     .05175***      .00455    11.38  .0000      .39544
   SOUTH|    -.03755***      .00494    -7.60  .0000      .29028
    SMSA|     .00458         .00477      .96  .3364      .65378
      MS|    -.06154***      .00814    -7.56  .0000      .81441
   UNION|     .05017***      .00477    10.52  .0000      .36399
        |Means for random parameters
Constant|    5.10687***      .02754   185.43  .0000
     EXP|     .02578***      .00085    30.33  .0000    19.8538
        |Scale parameters for dists. of random parameters
Constant|     .19776***      .00509    38.88  .0000
     EXP|     .00070***   .8794D-04     7.98  .0000
        |Heterogeneity in the means of random parameters
cONE_FEM|    -.13431***      .01417    -9.48  .0000
 cONE_ED|    -.01659***      .00180    -9.19  .0000
cONE_BLK|     .36096***      .01871    19.29  .0000
cEXP_FEM|    -.00572***      .00056   -10.19  .0000
 cEXP_ED|     .00460***   .6620D-04    69.50  .0000
cEXP_BLK|    -.01732***      .00073   -23.58  .0000
        |Variance parameter given is sigma
Std.Dev.|     .15238***      .00075   204.11  .0000
--------+-------------------------------------------------------------
Note: nnnnn.D-xx or D+xx => multiply by 10 to -xx or +xx.
Note: ***, **, * ==>  Significance at 1%, 5%, 10% level.
----------------------------------------------------------------------
```

3. Construct a different random parameters specification, modify the command above accordingly and fit your model. Report your results and interpret the estimates you obtain.