

Econometric Analysis of Panel Data

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Assignment 6

Nonlinear Models for Panel Data

Part I. A Concentrated Log Likelihood

Consider an exponential regression model with fixed effects, The density is

$$f(y_{it}|\mathbf{x}_{it}) = [1/\theta_{it}] \exp(-y_{it} / \theta_{it}), y_{it} \geq 0, \text{ where } \theta_{it} = \exp(\alpha_i + \mathbf{x}_{it}'\boldsymbol{\beta}), i = 1, \dots, n; t = 1, \dots, T.$$

It will prove convenient to let $\gamma_i = \exp(\alpha_i)$ so $\theta_{it} = \gamma_i \exp(\mathbf{x}_{it}'\boldsymbol{\beta}) = \gamma_i \lambda_{it}$.

The log likelihood for this exponential regression model with fixed effects is

$$(*) \quad \log L = \sum_{i=1}^n \sum_{t=1}^T (-\log \theta_{it} - y_{it} / \theta_{it})$$

(a) Obtain the first order condition for maximizing $\log L$ with respect to γ_i . Note, there is one of these for each i , so you need only differentiate

$$\log L_i = \sum_{t=1}^T (-\log \theta_{it} - y_{it} / \theta_{it})$$

with respect to γ_i and equate it to zero. You will gain some convenience by defining $a_{it} = y_{it}/\lambda_{it}$.

With the suggestion, the contribution of individual i to the log likelihood becomes

$$\begin{aligned}\log L_i &= \sum_{t=1}^T (-\log \theta_{it} - y_{it} / \theta_{it}) \\ &= \sum_{t=1}^T (-\log \gamma_i - \log \lambda_{it} - (1 / \gamma_i)(y_{it} / \lambda_{it}))\end{aligned}$$

The first derivative with respect to γ_i is

$$(**) \quad \frac{\partial \log L_i}{\partial \gamma_i} = \sum_{t=1}^T -\frac{1}{\gamma_i} + \frac{1}{\gamma_i^2} a_{it}$$

Equating this to zero, then multiplying both sides of the equation, we get

$$\gamma_i = \bar{a}_i$$

(This is the solution to b.)

(b) Now, treating $\boldsymbol{\beta}$ as if it were known, show that the implicit solution of this likelihood equation for γ_i in terms of $\boldsymbol{\beta}$ is

$$\gamma_i = \left(\frac{\sum_{t=1}^T [y_{it} / \exp(\mathbf{x}'_{it} \boldsymbol{\beta})]}{T} \right) = \frac{\sum_{t=1}^T a_{it}}{T} = \bar{a}_i = \gamma_i(\boldsymbol{\beta})$$

See above

(c) It follows that at the solution for the MLE, it will be true that $\gamma_i(\boldsymbol{\beta}) = \bar{a}_i$ where \bar{a}_i is the sample mean of a_{it} . Denote $\theta_{it}^c = \bar{a}_i \lambda_{it}$. Insert this solution back into the log likelihood function, to obtain the *concentrated log likelihood function*

$$(***) \quad \log L^c = \sum_{i=1}^n \sum_{t=1}^T (-\log \theta_{it}^c - y_{it} / \theta_{it}^c).$$

Note that this is a function of $\boldsymbol{\beta}$ but not of γ_i . To obtain the maximum likelihood estimator of $\boldsymbol{\beta}$, we can now maximize this function with respect to $\boldsymbol{\beta}$. This is equivalent to maximizing the whole log likelihood function, while considering only the solutions for γ_i that satisfy $\gamma_i = \gamma_i(\boldsymbol{\beta})$ as shown above. When we find $\boldsymbol{\beta}$, we can then compute γ_i . (No assignment for this part.)

(d) With this in hand, it is now possible to maximize the function with respect to β . Show that the likelihood equation will be

$$(\text{****}) \quad \frac{\partial \log L^c}{\partial \beta} = \sum_{i=1}^n \sum_{t=1}^T \left(-\frac{1}{\theta_{it}^c} + \frac{y_{it}}{(\theta_{it}^c)^2} \right) \frac{\partial \theta_{it}^c}{\partial \beta} = \sum_{i=1}^n \sum_{t=1}^T \frac{1}{\theta_{it}^c} \left(\frac{y_{it}}{\theta_{it}^c} - 1 \right) \frac{\partial \theta_{it}^c}{\partial \beta}.$$

You now need the derivative, $\partial \theta_{it}^c / \partial \beta$. Continuing, show that

$\partial \theta_{it}^c / \partial \beta = \bar{a}_i \lambda_{it} \mathbf{x}_{it} - \frac{1}{T} \sum_{t=1}^T a_{it} \lambda_{it} \mathbf{x}_{it}$ Hint: $\partial \lambda_{it} / \partial \beta = \lambda_{it} \mathbf{x}_{it}$. Insert your result in the log likelihood equation to obtain the implicit solution for β ,

$$\frac{\partial \log L^c}{\partial \beta} = \sum_{i=1}^n \sum_{t=1}^T \frac{1}{\theta_{it}^c} \left(\frac{y_{it}}{\theta_{it}^c} - 1 \right) \left[\lambda_{it} \bar{a}_i \mathbf{x}_{it} - \frac{1}{T} \sum_{t=1}^T \lambda_{it} a_{it} \mathbf{x}_{it} \right] = \mathbf{0}.$$

This involves a lot of tedious calculus. (***) results from simply inserting the solution for γ_i that was obtained in (a) into the original log likelihood function, (*). Because γ_i has been eliminated, this is the “concentrated” log likelihood. Likewise, the derivative in (****) just uses the chain rule in (**). The remaining complication is finding $\partial \theta_{it}^c / \partial \beta$, which is shown above.

Part II. Solving for FE in Panel Probit

For the binary fixed effects panel probit model,

$$\text{Prob}(y_{it} = 1 \mid \mathbf{x}_{it}) = \Phi(\alpha_i + \mathbf{x}_{it}'\beta),$$

$$\text{Prob}(y_{it} = 0 \mid \mathbf{x}_{it}) = 1 - \text{Prob}(y_{it} = 1 \mid \mathbf{x}_{it}) = \Phi(-\alpha_i - \mathbf{x}_{it}'\beta).$$

a. Write out the full log likelihood function.

The full log likelihood function is

$$\log L = \sum_{i=1}^N \left\{ \left[\sum_{t, y_{it}=0} \log \Phi(-\alpha_i - \mathbf{x}_{it}'\beta) \right] + \left[\sum_{t, y_{it}=1} \log \Phi(\alpha_i + \mathbf{x}_{it}'\beta) \right] \right\}$$

A very useful simplification is to write $q_{it} = 2y_{it} - 1$, so that $q_{it} = 1$ when $y_{it} = 1$ and -1 when $y_{it} = 0$. Then,

$$\log L = \sum_{i=1}^N \sum_{t=1}^T \log \Phi[q_{it}(\alpha_i + \mathbf{x}_{it}'\beta)]$$

b. Write out the first order condition for maximizing the function with respect to α_i , taking β as known. Take this derivation as far as possible – you will ultimately find that unlike the exponential model we examined in class, in this model, there is no explicit solution for α_i in terms of β and the data.

In the expression above, only T terms in the inner sum involve α_i . That part of the log likelihood function is

$$\log L_i = \sum_{t=1}^T \log \Phi[q_{it}(\alpha_i + \mathbf{x}'_{it}\beta)]$$

$$\frac{\partial \log L_i}{\partial \alpha_i} = \sum_{t=1}^T \frac{1}{\Phi[q_{it}(\alpha_i + \mathbf{x}'_{it}\beta)]} \phi[q_{it}(\alpha_i + \mathbf{x}'_{it}\beta)] q_{it}$$

No further simplification is possible.

c. Show that regardless of the finding in b, there is no solution for α_i when y_{it} is always 1 or always 0 within a given group (i).

In the derivative, both ϕ and Φ are always positive. The terms are negative when $y_{it} = 0$ and positive when $y_{it} = 1$. So, if y_{it} is always equal to 1, then the sum has to be positive and you cannot equate it to 0. If y_{it} is always 0, then the sum has to be negative and, again, you cannot equate it to zero.

Part III. The Incidental Parameters Problem.

This is a purely empirical exercise. It will involve some computations using the German health care data.

As we discussed in class, for the binary logit model, there are two estimators for the fixed effects model

$$\text{Prob}(y_{it} = 1 | \mathbf{x}_{it}) = \Lambda(\alpha_i + \beta' \mathbf{x}_{it}), i = 1, \dots, n, t = 1, \dots, T.$$

The ‘brute force’ approach maximizes the whole log likelihood for $\alpha_i, i = 1, \dots, n$ and β . This estimator is known to suffer from the ‘incidental parameters problem;’ when T is small, the estimator is biased away from zero. The best known result is that when T = 2, there is a 100% bias. The other approach is the Rasch/Andersen/Chamberlain method, which computes a conditional MLE using the probabilities conditioned on the sum of the y_{it} s for each group. This estimator is known to be consistent. For this exercise, we will see if the effect is visible in a sample, using precisely the estimators described.

a. We first see if we can observe Hsiao/Abrevaya's finding when $T = 2$. The following commands compute the estimates of the logit model both ways. Estimate the equations, and report your results. Do the empirical results seem to conform to the theory?

```

Sample ; All $
Reject ; _groupti # 2 $
Namelist ; x = hhninc,age,married,working$
Create ; y = doctor $
Logit ; Lhs = y ; Rhs = x ; pds = 2 ; Fixed$
Logit ; Lhs = y ; Rhs = x ; pds = 2 $
POOLED
-----
Logit      Regression Start Values for Y
Dependent variable      Y
Log likelihood function  -1378.51484
Estimation based on N =  2158, K =   5
Information Criteria: Normalization=1/N
-----
      Y | Coefficient      Standard      Prob.      Mean
      Y | Coefficient      Error      z      z>|Z|      of X
-----+-----
      HHNINC | -.29336      .26910     -1.09     .2756     .35678
      AGE    | .02488***     .00403      6.18     .0000     38.9222
      MARRIED | -.03152      .09869      -.32     .7494     .61538
      WORKING | -.10627      .10416     -1.02     .3076     .62326
Constant | -.17427      .19791      -.88     .3786
-----
BRUTE FORCE
-----
FIXED EFFECTS Logit Model
Dependent variable      Y
Log likelihood function  -474.89737
Estimation based on N =  2158, K = 349
Sample is 2 pds and 1079 individuals
Skipped 734 groups with inestimable ai <=====
LOGIT (Logistic) probability model
-----
      Y | Coefficient      Standard      Prob.      Mean
      Y | Coefficient      Error      z      z>|Z|      of X
-----+-----
      Index function for probability
      HHNINC | -1.79046      1.14804     -1.56     .1189     .35928
      AGE    | .00177      .05837      .03     .9758     36.2652
      MARRIED | .52879      .45144      1.17     .2415     .56667
      WORKING | .71611*      .37067      1.93     .0534     .69275
-----
CONDITIONAL
-----
Logit Model for Panel Data
Dependent variable      Y
Log likelihood function  -237.44868
Estimation based on N =  2158, K =   4
Fixed Effect Logit Model for Panel Data
-----
      Y | Coefficient      Standard      Prob.
      Y | Coefficient      Error      z      z>|Z|
-----+-----
      HHNINC | -.89523      .81179     -1.10     .2701
      AGE    | .00089      .04127      .02     .9829
      MARRIED | .26440      .31921      .83     .4075
      WORKING | .35806      .26210      1.37     .1719
-----

```

Three sets of results are given. The pooled results look quite far off compared to the third set of results which we know are from a consistent estimator. Comparing the second set of results to the third, it looks like the theory is working very well. The unconditional estimates are almost exactly twice the conditional estimates.

b. A second result that seems intuitively reasonable is that the IP bias diminishes as T increases. Is this the case? Change the three 2s in the command set above to 3s and redo the experiment. What do you find? Now, change the 2s to 7s and repeat the experiment. In each case, report your findings and your conclusions.

Here is the full experiment with T=2,3,4,5,6,7

WITH T=2

Y	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
Index function for probability					
HHNINC	-1.79046	1.14804	-1.56	.1189	.35928
AGE	.00177	.05837	.03	.9758	36.2652
MARRIED	.52879	.45144	1.17	.2415	.56667
WORKING	.71611*	.37067	1.93	.0534	.69275
HHNINC	-.89523	.81179	-1.10	.2701	
AGE	.00089	.04127	.02	.9829	
MARRIED	.26440	.31921	.83	.4075	
WORKING	.35806	.26210	1.37	.1719	

WITH T=3

Y	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
Index function for probability					
HHNINC	.59013	.70428	.84	.4021	.36352
AGE	.12338***	.03485	3.54	.0004	40.4401
MARRIED	.65030*	.35210	1.85	.0648	.74479
WORKING	.09112	.27020	.34	.7359	.66146
HHNINC	.40128	.57716	.70	.4869	
AGE	.08214***	.02841	2.89	.0038	
MARRIED	.42203	.28313	1.49	.1361	
WORKING	.05916	.22023	.27	.7882	

WITH T=4

Y	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
Index function for probability					
HHNINC	-1.27207**	.53695	-2.37	.0178	.36115
AGE	.17885***	.02282	7.84	.0000	41.6709
MARRIED	-.32722	.25114	-1.30	.1926	.73537
WORKING	.17151	.19681	.87	.3835	.68236
HHNINC	-.94006**	.45951	-2.05	.0408	
AGE	.13319***	.01955	6.81	.0000	
MARRIED	-.24378	.21610	-1.13	.2593	
WORKING	.12651	.16941	.75	.4552	

WITH T=5

Y	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
Index function for probability					
HHNINC	-.38665	.38487	-1.00	.3151	.35799
AGE	.11721***	.01661	7.06	.0000	42.9664
MARRIED	-.17447	.22345	-.78	.4349	.78049
WORKING	.18235	.16158	1.13	.2591	.72052
HHNINC	-.30921	.34404	-.90	.3688	
AGE	.09371***	.01483	6.32	.0000	
MARRIED	-.14012	.19971	-.70	.4829	
WORKING	.14476	.14422	1.00	.3155	

WITH T=6

Y	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
Index function for probability					
HHNINC	.76162**	.35975	2.12	.0343	.35924
AGE	.08686***	.01337	6.49	.0000	44.1876
MARRIED	-.44181*	.22544	-1.96	.0500	.80129
WORKING	-.52941***	.14991	-3.53	.0004	.74873
HHNINC	.63224*	.32811	1.93	.0540	
AGE	.07233***	.01220	5.93	.0000	
MARRIED	-.36869*	.20575	-1.79	.0731	
WORKING	-.44155***	.13683	-3.23	.0013	
WITH T=7					
Y	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
Index function for probability					
HHNINC	-.03468	.33461	-.10	.9175	.34717
AGE	.08870***	.01193	7.43	.0000	43.8163
MARRIED	-.04349	.21200	-.21	.8374	.84234
WORKING	-.09784	.14573	-.67	.5020	.76362
HHNINC	-.03061	.30848	-.10	.9210	
AGE	.07604***	.01103	6.89	.0000	
MARRIED	-.03777	.19604	-.19	.8472	
WORKING	-.08394	.13482	-.62	.5336	

The estimators are clearly getting closer together as T increases. With T=7, they seem to be pretty close. The general experience suggests that if T is 10 or larger, the difference is small enough to be comfortable with the unconditional estimator. One might think that this is not necessarily a useful result, since we can always use the conditional estimator. But, the conditional estimator only exists for the logit model. If you want to use a probit model, you can only use the unconditional estimator, so this is a useful guide for that case.

c. What do you conclude about the fixed effects model?

See above.

Part IV. A Common Effects Probit Model

In this exercise, you will fit a probit model with common effects, and develop the appropriate model based on your findings. The probit model we will use is

$$\text{Prob}(y_{it} = 1 | x_{it}) = \Phi(c_i + \beta'x_{it})$$

y_{it} = Public_{it} = whether or not the individual chose public health insurance in that year.

x_{it} = **one,age,educ,hhninc,handper,working,hsat**

1. Suppose, for the moment, we ignore the heterogeneity, c_i and just pool the data and fit a simple probit model. Is the estimator consistent? What assumptions are necessary for the pooled estimator to be a consistent estimator of β ?

The only way for the pooled estimator to be a consistent estimator of β is for c_i to equal zero for every observation. Otherwise, it is inconsistent in all cases.

2. All of the suggested covariates in the model are time varying. Fit a random effects model and a fixed effects model (this can only be done by brute force). Report your results.

The pooled, random and fixed effects results are shown below.

```
--> probit;lhs=public;rhs=xit;random;pds=_groupti$
```

Binomial Probit Model					
Dependent variable		PUBLIC			
Log likelihood function		-8294.31338			
Restricted log likelihood		-9711.25153			
Chi squared [6 d.f.]		2833.87629			
Significance level		.00000			
McFadden Pseudo R-squared		.1459069			
Estimation based on N =		27326, K = 7			

PUBLIC	Coefficient	Standard Error	z	Prob. z> Z	Mean of X

Index function for probability					
Constant	3.86731***	.08308	46.55	.0000	
AGE	-.00034	.00106	-.32	.7498	43.5257
EDUC	-.16849***	.00407	-41.37	.0000	11.3206
HHNINC	-.96505***	.05584	-17.28	.0000	.35208
HANDPER	.00114	.00070	1.64	.1014	7.01229
WORKING	-.01036	.02533	-.41	.6825	.67705
HSAT	-.03834***	.00532	-7.21	.0000	6.78543

Normal exit: 24 iterations. Status=0. F=				4868.491	

Random Effects Binary Probit Model

Dependent variable PUBLIC

Log likelihood function -4868.49090

Restricted log likelihood -8294.31338

Chi squared [1 d.f.] 6851.64496

Significance level .00000

Unbalanced panel has 7293 individuals

PUBLIC	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
Constant	11.5697***	.33006	35.05	.0000	
AGE	-.00291	.00362	-.80	.4214	43.5257
EDUC	-.56234***	.01755	-32.04	.0000	11.3206
HHNINC	-1.46850***	.13213	-11.11	.0000	.35208
HANDPER	.00254	.00205	1.24	.2157	7.01229
WORKING	.10990*	.06136	1.79	.0733	.67705


```

      HSAT |      -.03535**      .01383      -2.56      .0106      6.78543
      Rho  |      .90503***      .00416      217.77      .0000
-----+-----
--> probit;lhs=public;rhs=xit;fem;pds=_groupti$
-----+-----
FIXED EFFECTS Probit Model
Dependent variable      PUBLIC
Log likelihood function      -1346.22838
Estimation based on N = 27326, K =1236
Information Criteria: Normalization=1/N
                        Normalized      Unnormalized
AIC                      .18899      5164.45676
Fin.Smpl.AIC             .19328      5281.66570
Bayes IC                  .56060      15318.93084
Hannan Quinn             .30876      8437.17555
Model estimated: Feb 10, 2010, 07:04:22
Unbalanced panel has 7293 individuals
Skipped 6063 groups with inestimable ai
PROBIT (normal) probability model
-----+-----
      PUBLIC |      Coefficient      Standard      z      Prob.      Mean
              |      Error      z>|z|      of X
-----+-----
      Index function for probability
      AGE      |      -.04549***      .01178      -3.86      .0001      42.1510
      EDUC      |      -.30999***      .08493      -3.65      .0003      12.5433
      HHNINC     |      -.94060***      .25739      -3.65      .0003      .39798
      HANDPER     |      .00017      .00437      .04      .9692      5.53937
      WORKING     |      .09822      .11254      .87      .3828      .78911
      HSAT      |      -.02955      .02123      -1.39      .1639      7.17090
-----+-----
Note: ***, **, * ==> Significance at 1%, 5%, 10% level.
-----+-----

```

3. We are interested in deciding which is preferred, fixed or random effects. I propose to use a variable addition test. Add the group means to the model, then carry out a likelihood ratio test of the hypothesis that the coefficients on the group means are all zero. What do you find? What do you conclude is the preferred model?

```

sample;all$
namelist;xit = one,age,educ,hhninc,handper,working,hsat$
probit;lhs=public;rhs=xit;fem;pds=_groupti$
calc;logl0=logl$
create ; agebar=GroupMean(age,pds=_groupti) $
create ; edbar=GroupMean(educ,pds=_groupti) $
create ; incbar=GroupMean(hhninc,pds=_groupti) $
create ; handbar=GroupMean(handper,pds=_groupti) $
create ; workbar=GroupMean(working,pds=_groupti) $
create ; hsatbar=GroupMean(hsat,pds=_groupti) $
namelist;xb=agebar,edbar,incbar,handbar,workbar,hsatbar$
probit ; lhs=public;rhs=xit,xb;random;pds=_groupti $
calc;logl1=logl$
calc;list;chisq=2*(logl1-logl0);ctb(.95,6)$
matrix;bm=b(8:13);vm=varb(8:13,8:13)
;list;waldstat=bm'<vm>bm$

```

Here are the results for the model with the group means added. The log likelihood without the group means is -4868.49090. With the group means, it is -4830.45663. Twice the difference is 76.06854. This is a chi squared statistic with 6 degrees of freedom. The 95% critical value is 12.5915, so the hypothesis that the coefficients on the group means are all zero would be rejected. This makes the fixed effects model the preferred specification. The Wald statistic is 172.82994 which leads to the same inference.

```

-----
Random Effects Binary Probit Model
Dependent variable      PUBLIC
Log likelihood function  -4830.45663
Restricted log likelihood -8231.41932
Chi squared [ 1 d.f.]   6801.92539
Significance level       .00000
McFadden Pseudo R-squared .4131684
Estimation based on N = 27326, K = 14
Information Criteria: Normalization=1/N
                        Normalized  Unnormalized
AIC                    .35457      9688.91326
Fin.Smpl.AIC           .35457      9688.92863
Bayes IC               .35878      9803.93157
Hannan Quinn           .35592      9725.98289
Model estimated: Feb 10, 2010, 07:14:13
Unbalanced panel has 7293 individuals

```

PUBLIC	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
Constant	12.7702***	.42367	30.14	.0000	
AGE	-.03087***	.00836	-3.69	.0002	43.5257
EDUC	-.24376***	.05281	-4.62	.0000	11.3206
HHNINC	-.67317***	.19869	-3.39	.0007	.35208
HANDPER	.00037	.00356	.10	.9167	7.01229
WORKING	.05389	.08190	.66	.5105	.67705
HSAT	-.01904	.01723	-1.10	.2693	6.78543
AGEBAR	.03372***	.00888	3.80	.0001	43.5257
EDBAR	-.26090***	.05530	-4.72	.0000	11.3206
INCBAR	-3.26485***	.33218	-9.83	.0000	.35208
HANDBAR	.00519	.00453	1.15	.2517	7.01229
WORKBAR	-.00795	.13099	-.06	.9516	.67705
HSATBAR	-.17811***	.03074	-5.79	.0000	6.78543
Rho	.90640***	.00445	203.59	.0000	

Matrix WALDSTAT has 1 rows and 1 columns.

```

      1
+-----+
1| 172.82994
+-----+

```

4. Suppose it were hypothesized that the previous year's choice of whether or not to choose public insurance were on the right hand side of the equation. That is,

$$\text{Prob}(y_{it} = 1 | x_{it}) = \Phi(c_i + \beta'x_{it} + \gamma y_{i,t-1})$$

What would this imply for how one (you) should go about estimating the parameters of the model. What issues should you be concerned with for a dynamic model?

The new specification creates a problem for estimation. Even if it is argued that c_i is uncorrelated with x_{it} , it can't be uncorrelated with $y_{i,t-1}$. So, some alternative approach would be called for. Wooldridge's suggestion is a combination of Mundlak's approach and a separate model for the initial conditions, y_{i0} . In one way or another, the model will have to account for the endogeneity of the lagged dependent variable in the equation.