NYU STERN
NEW YORK UNIVERSITY LEONARD N. STERN SCHOOL OF BUSINESS

Department of Economics

ECONOMETRICS I
Take Home Final Examination

Fall 2005

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Today is Thursday, December 8, 2005. This exam is due by 4PM, Monday, December 19, 2005. Feel free to turn it in early, of course. You may submit your answers to me electronically as an attachment to an e-mail if you wish. Your submission for this examination is to be a single authored project – you are assumed to be working alone. Please submit this page, with your signature, with your answers to the questions.

NOTE: In the empirical results below, a number of the form .nnnnnnE+aa means multiply the number .nnnnnn by 10 to the aa power. E-aa implies multiply 10 to the minus aa power. Thus, .123456E-02 is 0.00123456.

This test comprises 100 points. The 8 parts below are each worth 10% of the exam. The remaining 20% is given for the correct entry in the signature line below.

This course is governed by the Stern honor code:

I will not lie, cheat or steal to gain an academic advantage, or tolerate those who do.

Signature ____

Part 1.

In the first assigned problem set for this course, we used a data set on production by a sample of Spanish dairy farms. The data are a panel, T=6 years (1993 – 1998) on N=247 farms, a total of 1,482 observations. The variables in the data set are:

MILK = liters of output;	YIT = log(milk)
COWS	X1 = log(COWS)
LAND	X2 = log(LAND)
LABOR	X3 = log(LABOR)
FEED	X4 = log(FEED)

The data have been transformed so that the means of X1, X2, X3 and X4 are all zero. (Note, means of the logs). We are also using additional variables in this exercise,

X11, X12, X13, X14	= X1*X1, X1*X2, X1*X3, X1*X4
X22, X23, X24	
X33, X34	
X44	and so on. Squares and cross products
T = Time = -2, -1, 0, 1, 2, 3	
T2 = Time^2	

The Cobb-Douglas model of production specifies $\log Y = \alpha + \Sigma_k \beta_k \log X_k$. The production function displays constant returns to scale if $\Sigma_k \partial \log Y/\partial \log X_k = 1 = \Sigma_k \beta_k$

The translog model of production specifies $\log Y = \alpha + \Sigma_k \beta_k \log X_k + \Sigma_k \Sigma_{m \geq k} \gamma_{km} \log X_k \log X_m$

Technical change in a logarithmic production function is represented by $\log Y = {*}{*}{*} + \delta_0 t + \delta_1 t^2$.
The rate of technical change is then $\partial \log Y/\partial t = \delta_0 + 2\delta_1 t$. A constant rate of technical change is imposed by constraining $\delta_1 = 0$.

Results for various specifications of the production function, estimated by ordinary least squares, are presented below. Your responses to this part of the test (and a few others) will be based on these results.

a. Test the null hypothesis that the technology is Cobb-Douglas against the alternative that it is translog. Show exactly how you are doing the test and on what statistic you are basing your conclusion.

b. Test the hypothesis of constant returns to scale in the Cobb-Douglas model using (1) a simple t-test, (2) a Wald test, (3) an F test and (4) a likelihood ratio test. Explain in excruciatingly clear detail exactly how you are carrying out the tests. In your presentation, state clearly what assumptions about the model underlie the test statistics.

c. Form a 95% confidence interval for the “cows elasticity,” β_1. As usual, explain clearly what you are doing.

d. I want to test the hypothesis that the rate of technical change is zero in the fourth year, but not necessarily in the other years. Show how to carry out the test. Do so. Now, I want to test the hypothesis that technical change is zero in every year. Again, explain the test procedure, and carry out the test.

Regressions for Part 1.

```
+-----------------------------------------------------+
| Ordinary    least squares regression                 |
| LHS=YIT      Mean                 =   11.57749       |
|              Standard deviation   =   .6434377       |
| WTS=none     Number of observs.   =       1482       |
| Model size   Parameters           =          5       |
|              Degrees of freedom   =       1477       |
| Residuals    Sum of squares       =   29.09570       |
|              Standard error of e  =   .1403538       |
| Fit          R-squared            =   .9525473       |
|              Adjusted R-squared   =   .9524188       |
| Model test   F[  4,  1477] (prob) =7412.19 (.0000) |
| Diagnostic   Log likelihood       =   809.6761       |
|              Restricted(b=0)      =  -1448.908       |
|              Chi-sq [  4]  (prob) =4517.17 (.0000) |
+-----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant      11.5774868      .00364586  3175.515   .0000
 X1             .59517558      .01958331    30.392   .0000    0
 X2             .02305014      .01122274     2.054   .0400    0
 X3             .02319244      .01303099     1.780   .0751    0
 X4             .45175783      .01078465    41.889   .0000    0
Covariance Matrix
          1             2             3             4             5
 +----------------------------------------------------------------------
1|  .1329230D-04        0             0             0             0
2|       0           .00038      -.00011    -.5457328D-04    -.00017
3|       0          -.00011       .00013     .1001475D-04  .2018432D-04
4|       0       -.5457328D-04  .1001475D-04      .00017  -.1404125D-04
5|       0          -.00017     .2018432D-04 -.1404125D-04    .00012

+-----------------------------------------------------+
| Linearly restricted regression                       |
| LHS=YIT      Mean                 =   11.57749       |
|              Standard deviation   =   .6434377       |
| WTS=none     Number of observs.   =       1482       |
| Model size   Parameters           =          4       |
|              Degrees of freedom   =       1478       |
| Residuals    Sum of squares       =   30.19235       |
|              Standard error of e  =   .1429260       |
| Fit          R-squared            =   .9507588       |
|              Adjusted R-squared   =   .9506588       |
| Model test   F[  3,  1478] (prob) =9512.50 (.0000) |
| Diagnostic   Log likelihood       =   782.2604       |
|              Restricted(b=0)      =  -1448.908       |
|              Chi-sq [  3]  (prob) =4462.34 (.0000) |
| Restrictns.  F[  1,  1477] (prob) =  55.67 (.0000) |
| Not using OLS or no constant. Rsqd & F may be < 0. |
| Note, with restrictions imposed,  Rsqd may be < 0. |
+-----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant      11.5774868      .00371268  3118.366   .0000
 X1             .56677311      .01956182    28.973   .0000     0
 X2            -.00282763      .01086898     -.260   .7947     0
 X3            -.04325035      .00968783    -4.464   .0000     0
 X4             .47930487      .01031870    46.450   .0000     0
Matrix Cov.Mat. has  5 rows and  5 columns.
         1             2             3             4             5
 +----------------------------------------------------------------------
1|  .1378397D-04       0             0             0             0
2|       0           .00038      -.00013     -.9174458D-04    -.00016
3|       0          -.00013       .00012     -.2164277D-04  .3420965D-04
4|       0       -.9174458D-04 -.2164277D-04  .9385403D-04  .1953333D-04
5|       0          -.00016     .3420965D-04  .1953333D-04     .00011
```

```
+-----------------------------------------------------+
| Ordinary    least squares regression                |
| Model was estimated Dec 07, 2005 at 10:35:58AM      |
| LHS=YIT      Mean                 =   11.57749      |
|              Standard deviation   =   .6434377      |
| WTS=none     Number of observs.   =       1482      |
| Model size   Parameters           =          7      |
|              Degrees of freedom   =       1475      |
| Residuals    Sum of squares       =   28.94244      |
|              Standard error of e  =   .1400785      |
| Fit          R-squared            =   .9527973      |
|              Adjusted R-squared   =   .9526053      |
| Model test   F[  6,  1475] (prob) =4962.20 (.0000) |
| Diagnostic   Log likelihood       =   813.5895      |
|              Restricted(b=0)      =  -1448.908      |
|              Chi-sq [  6]  (prob) =4525.00 (.0000) |
| Info criter. LogAmemiya Prd. Crt. =  -3.926392      |
|              Akaike Info. Criter. =  -3.926392      |
| Autocorrel   Durbin-Watson Stat.  =   .8081626      |
|              Rho = cor[e,e(-1)]   =   .5959187      |
+-----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant      11.5799428      .00543433  2130.888   .0000
 X1             .59987916      .01962113    30.573   .0000     0
 X2             .02390494      .01120536     2.133   .0329     0
 X3             .02737137      .01309626     2.090   .0366     0
 X4             .44479994      .01106948    40.183   .0000     0
 T              .00745280      .00267869     2.782   .0054     .50000000
 T2            -.00195232      .00146170    -1.336   .1817    3.16666667
Estimated Asymptotic covariance matrix.
```

	1	2	3	4	5	6	7
1	2.95E-05	-6.16E-08	6.64E-07	-1.32E-07	5.48E-07	3.34E-06	-5.67E-06
2	-6.16E-08	0.000385	-0.00011	-5.17E-05	-0.00017	4.39E-06	-6.74E-07
3	6.64E-07	-0.00011	0.000126	1.04E-05	1.94E-05	8.41E-07	-3.43E-07
4	-1.32E-07	-5.17E-05	1.04E-05	0.000172	-1.80E-05	3.89E-06	-5.73E-07
5	5.48E-07	-0.00017	1.94E-05	-1.80E-05	0.000123	-6.46E-06	8.48E-07
6	3.34E-06	4.39E-06	8.41E-07	3.89E-06	-6.46E-06	7.18E-06	-2.19E-06
7	-5.67E-06	-6.74E-07	-3.43E-07	-5.73E-07	8.48E-07	-2.19E-06	2.14E-06

```
+----------------------------------------------------+
| Ordinary    least squares regression               |
| Model was estimated Dec 07, 2005 at 10:40:26AM     |
| LHS=YIT      Mean                 =   11.57749     |
|              Standard deviation   =   .6434377     |
| WTS=none     Number of observs.   =       1482     |
| Model size   Parameters           =         15     |
|              Degrees of freedom   =       1467     |
| Residuals    Sum of squares       =   28.28359     |
|              Standard error of e  =   .1388520     |
| Fit          R-squared            =   .9538718     |
|              Adjusted R-squared   =   .9534316     |
| Model test   F[ 14,  1467] (prob) =2166.83 (.0000) |
| Diagnostic   Log likelihood       =   830.6529     |
|              Restricted(b=0)      =  -1448.908     |
|              Chi-sq [ 14]  (prob) =4559.12 (.0000) |
| Info criter. LogAmemiya Prd. Crt. =  -3.938623     |
|              Akaike Info. Criter. =  -3.938623     |
| Autocorrel   Durbin-Watson Stat.  =   .8282196     |
|              Rho = cor[e,e(-1)]   =   .5858902     |
+----------------------------------------------------+
```

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]	Mean of X
Constant	11.5688643	.00726864	1591.613	.0000	
X1	.60693062	.02186356	27.760	.0000	0
X2	.01351936	.01169178	1.156	.2476	0
X3	.02384747	.01355872	1.759	.0786	0
X4	.45379253	.01199340	37.837	.0000	0
X11	.47329234	.14309910	3.307	.0009	.11926376
X22	-.08046399	.04929507	-1.632	.1026	.10386698
X33	-.04840054	.09251157	-.523	.6008	.05873631
X44	.17968695	.04555562	3.944	.0001	.28859636
X12	-.08379987	.06166837	-1.359	.1742	.15316763
X13	.18430294	.07247569	2.543	.0110	.09495984
X14	-.28574011	.07559639	-3.780	.0002	.33056414
X23	-.00815564	.04326168	-.189	.8505	.05287598
X24	.05222215	.03095889	1.687	.0916	.19230806
X34	-.05821369	.04040862	-1.441	.1497	.14262419

Part 2.

a. Ordinary least squares was used to compute the regressions analyzed in Part 1. Show (algebraically) how the least squares coefficient estimator, **b**, and the estimated asymptotic covariance matrix are computed. (Theoretically, not for each regression.)

b. Show how each of the values in the box above the coefficient estimates in the first regression is computed, and interpret the value given. (Again, theoretically.)

c. What are the finite sample properties of the least squares estimator? Make your assumptions explicit.

d. What are the asymptotic properties of the least squares estimator? Again, be explicit about all assumptions, and explain your answer carefully.

Part 3.

As noted, the Spanish dairy data are a panel, with six years of data. The results below show the six separate regressions when the Cobb-Douglas model is fit separately in each year.

a. Theory 1 states that the coefficient vectors are the same in the all periods. Is there an optimal way that I could combine these six estimators to form a single efficient estimator of the model parameters? How should I do that? Show the computations explicitly. (Show the theoretical result, not the numbers.)

b. Use a Chow test to test the hypothesis that the six coefficient vectors are the same. Explain the computations in full detail so that I know exactly how you obtained your result.

c. The test in the preceding question could be done with a Wald test. Is there any particular reason to use the Wald test or the Chow test – i.e., one and not the other in this setting? What assumptions would justify each? Do the regression results suggest that one or the other test might be appropriate? Explain. (You need not carry out the Wald test. This question asks about the test, in principle.)

d. The residual vectors from the six regressions described here are collected after we compute the least squares coefficients. This produces 6 sets of 247 observations. The correlations of these six residuals are

	E93	E94	E95	E96	E97	E98
E93	1.00000	.71657	.65440	.63681	.60213	.56930
E94	.71657	1.00000	.79441	.69008	.63613	.57305
E95	.65440	.79441	1.00000	.77181	.68184	.62782
E96	.63681	.69008	.77181	1.00000	.67485	.66965
E97	.60213	.63613	.68184	.67485	1.00000	.74868
E98	.56930	.57305	.62782	.66965	.74868	1.00000

These are obviously not zero. Treating each year as a separate equation, suggest how the information here could be used to construct a more efficient estimator than equation by equation least squares. Show the estimator you propose to use in complete detail.

Regressions for Part 3

```
--> include;new;year93=1$
+----------------------------------------------------+
| LHS=YIT      Mean                 =   11.37137     |
|              Standard deviation   =   .5741918     |
| WTS=none     Number of observs.   =        247     |
| Model size   Parameters           =          5     |
|              Degrees of freedom   =        242     |
| Residuals    Sum of squares       =   4.828938     |
|              Standard error of e  =   .1412597     |
| Fit          R-squared            =   .9404609     |
|              Adjusted R-squared   =   .9394767     |
+----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant      11.5451542      .01020751  1131.045   .0000
 X1             .67127435      .04640328    14.466   .0000    -.10034736
 X2            -.02051104      .02652538     -.773   .4401    -.03237374
 X3             .02515508      .03348824      .751   .4533     .00367797
 X4             .38883467      .02677635    14.522   .0000    -.27565075
--> include;new;year94=1$
+----------------------------------------------------+
| LHS=YIT      Mean                 =   11.47123     |
|              Standard deviation   =   .5935680     |
| WTS=none     Number of observs.   =        247     |
| Model size   Parameters           =          5     |
|              Degrees of freedom   =        242     |
| Residuals    Sum of squares       =   4.736023     |
|              Standard error of e  =   .1398940     |
| Fit          R-squared            =   .9453566     |
|              Adjusted R-squared   =   .9444534     |
+----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant      11.5692455      .00921237  1255.839   .0000
 X1             .62146662      .04577958    13.575   .0000    -.05794187
 X2             .00884371      .02699075      .328   .7435    -.02396359
 X3             .03644040      .03192103     1.142   .2548    -.00850757
 X4             .42051389      .02749437    15.295   .0000    -.14620599
--> include;new;year95=1$
+----------------------------------------------------+
| LHS=YIT      Mean                 =   11.55324     |
|              Standard deviation   =   .6240203     |
| WTS=none     Number of observs.   =        247     |
| Model size   Parameters           =          5     |
|              Degrees of freedom   =        242     |
| Residuals    Sum of squares       =   5.064954     |
|              Standard error of e  =   .1446705     |
| Fit          R-squared            =   .9471259     |
|              Adjusted R-squared   =   .9462520     |
+----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant      11.5855294      .00923661  1254.306   .0000
 X1             .60484401      .04837102    12.504   .0000    -.03079058
 X2             .04572600      .02771820     1.650   .1003    -.00774750
 X3            -.00435116      .03291678     -.132   .8949    -.00850784
 X4             .43360824      .02709758    16.002   .0000    -.03078562
```

```
--> include;new;year96=1$
+----------------------------------------------------+
| LHS=YIT      Mean                 =   11.63496     |
|              Standard deviation   =   .6479655     |
| WTS=none     Number of observs.   =        247     |
| Model size   Parameters           =          5     |
|              Degrees of freedom   =        242     |
| Residuals    Sum of squares       =   4.729280     |
|              Standard error of e  =   .1397944     |
| Fit          R-squared            =   .9542115     |
|              Adjusted R-squared   =   .9534547     |
+----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant      11.5821246      .00900367  1286.378   .0000
 X1             .58637409      .04892288    11.986   .0000     .02860376
 X2             .03589870      .02837273     1.265   .2070    -.01317060
 X3             .00405593      .03224129      .126   .9000     .00181355
 X4             .47513989      .02686625    17.685   .0000     .07687632
--> include;new;year97=1$
+----------------------------------------------------+
| LHS=YIT      Mean                 =   11.68610     |
|              Standard deviation   =   .6578417     |
| WTS=none     Number of observs.   =        247     |
| Model size   Parameters           =          5     |
|              Degrees of freedom   =        242     |
| Residuals    Sum of squares       =   4.391745     |
|              Standard error of e  =   .1347134     |
| Fit          R-squared            =   .9587467     |
|              Adjusted R-squared   =   .9580648     |
+----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant      11.5788492      .00884093  1309.687   .0000
 X1             .56735320      .04922175    11.526   .0000     .06587095
 X2             .03422551      .02785101     1.229   .2203     .02440856
 X3             .06784837      .03052456     2.223   .0272    -.00173056
 X4             .46078641      .02686617    17.151   .0000     .15009731
--> include;new;year98=1$
+----------------------------------------------------+
| LHS=YIT      Mean                 =   11.74802     |
|              Standard deviation   =   .6852052     |
| WTS=none     Number of observs.   =        247     |
| Model size   Parameters           =          5     |
|              Degrees of freedom   =        242     |
| Residuals    Sum of squares       =   4.697669     |
|              Standard error of e  =   .1393264     |
| Fit          R-squared            =   .9593270     |
|              Adjusted R-squared   =   .9586547     |
+----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |t-ratio |P[|T|>t] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant      11.5844443      .00944006  1227.157   .0000
 X1             .53130880      .05112715    10.392   .0000     .09460511
 X2             .04776864      .02783742     1.716   .0874     .05284686
 X3             .03305216      .03153783     1.048   .2957     .01325446
 X4             .48898533      .02854318    17.131   .0000     .22566872
```

Part 4.

We now return to the panel data set examined in question 1. The results below show OLS, fixed effects and random effects estimates for the Cobb-Douglas model.

a. Test the hypothesis of ‘no effects’ vs. ‘some effects’ using the results given below.

b. Explain in precise detail the difference between the fixed and random effects model.

c. Carry out the Hausman test for fixed effects vs. random effects and report your conclusion. Carefully explain what you are doing in this test. Hint: Transcribing and entering the matrices may be a pain. Here are some matrix commands for LIMDEP – these should also be easily transportable to Stata or Matlab as well – that should make life a little easier. Also, do note, if you carry out your test using either Stata or LIMDEP’s automatic procedures for the panel data models, you will get a somewhat different answer for the statistic. The reason is that you are not using all the internal digits of the computed matrices when you do it using this “hint’ while you probably are using the internal procedures.

```
matrix;vfe=[
     .00061   ,-.7143086D-04 , -.2953098D-04,      -.00020/
 -.7143086D-04,     .00026   , -.5828137D-05,-.1381531D-04/
 -.2953098D-04, -.5828137D-05,     .00054   , .6528495D-05/
    -.00020   ,-.1381531D-04 ,  .6528495D-05,       .00014]$
matrix;vre=[
     .00043   ,-.9201395D-04 ,-.6666412D-04,       -.00017/
 -.9201395D-04,     .00018   ,-.4320561D-05,  .8595354D-06/
 -.6666412D-04, -.4320561D-05,     .00030  , -.2862694D-05/
    -.00017   , .8595354D-06 ,-.2862694D-05,     .00012   ]$
matrix;bfe=[
.66200103/
.03735244/
.03039947/
.38251038]$
matrix;bre=[
.65025754/
.03004298/
.03506960/
.39954471]$
```

d. In the context of the fixed effects model, test the hypothesis that there are no effects – i.e., that all individuals have the same constant term. (The statistics you need to carry out the test are given in the results.)

e. In the second set of results, we have added a set of year dummy variables, YEAR93, etc., to the model to allow for time variation as well as for variation across farms. Test the hypothesis that there is no separate time variation using these and the first set of results.

Regression Results for Part 4.

```
--> regress ; lhs=yit;rhs=cobbdgls;panel;pds=6;pri$
+-----------------------------------------------------+
| OLS Without Group Dummy Variables                   |
| Ordinary    least squares regression                |
| Model was estimated Dec 07, 2005 at 11:22:00AM      |
| LHS=YIT      Mean                 =   11.57749      |
|              Standard deviation   =   .6434377      |
| WTS=none     Number of observs.   =       1482      |
| Model size   Parameters           =          5      |
|              Degrees of freedom   =       1477      |
| Residuals    Sum of squares       =   29.09570      |
|              Standard error of e  =   .1403538      |
| Fit          R-squared            =   .9525473      |
|              Adjusted R-squared   =   .9524188      |
| Model test   F[  4,  1477] (prob) =7412.19 (.0000)  |
| Diagnostic   Log likelihood       =   809.6761      |
|              Restricted(b=0)      =  -1448.908      |
|              Chi-sq [  4]  (prob) =4517.17 (.0000)  |
|              Akaike Info. Criter. =  -3.923810      |
+-----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 X1              .59517558      .01958331    30.392   .0000  -.188987D-14
 X2              .02305014      .01122274     2.054   .0400  -.272590D-14
 X3              .02319244      .01303099     1.780   .0751   .124737D-14
 X4              .45175783      .01078465    41.889   .0000   .779238D-14
 Constant      11.5774868      .00364586  3175.515   .0000
+-----------------------------------------------------+
| Least Squares with Group Dummy Variables            |
| Model was estimated Dec 07, 2005 at 11:22:00AM      |
| LHS=YIT      Mean                 =   11.57749      |
|              Standard deviation   =   .6434377      |
| WTS=none     Number of observs.   =       1482      |
| Model size   Parameters           =        251      |
|              Degrees of freedom   =       1231      |
| Residuals    Sum of squares       =   8.161094      |
|              Standard error of e  =   .8142264E-01  |
| Fit          R-squared            =   .9866899      |
|              Adjusted R-squared   =   .9839868      |
| Model test   F[250,  1231] (prob) = 365.02 (.0000)  |
| Diagnostic   Log likelihood       =   1751.644      |
|              Restricted(b=0)      =  -1448.908      |
|              Chi-sq [250]  (prob) =6401.11 (.0000)  |
+-----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 X1              .66200103      .02467845    26.825   .0000  -.188987D-14
 X2              .03735244      .01613309     2.315   .0206  -.272590D-14
 X3              .03039947      .02320776     1.310   .1902   .124737D-14
 X4              .38251038      .01201690    31.831   .0000   .779238D-14
```

```
Matrix Cov.Mat. has  4 rows and  4 columns.
+---------------------------------------------------------
1|     .00061    -.7143086D-04 -.2953098D-04    -.00020
2| -.7143086D-04      .00026   -.5828137D-05 -.1381531D-04
3| -.2953098D-04 -.5828137D-05      .00054    .6528495D-05
4|    -.00020    -.1381531D-04  .6528495D-05      .00014
+-----------------------------------------------------------------------+
|                Test Statistics for the Classical Model                |
|        Model            Log-Likelihood    Sum of Squares    R-squared |
| (1)  Constant term only    -1448.90832   .6131518321D+03     .0000000 |
| (2)  Group effects only      412.25944   .4974526192D+02     .9188696 |
| (3)  X - variables only      809.67611   .2909570093D+02     .9525473 |
| (4)  X and group effects    1751.64437   .8161093811D+01     .9866899 |
|                                Hypothesis Tests                       |
|               Likelihood Ratio Test               F Tests             |
|          Chi-squared   d.f.  Prob.         F    num. denom. Prob value |
| (2) vs (1)  3722.336    246     .00000    56.859  246  1235     .00000 |
| (3) vs (1)  4517.169      4     .00000  7412.185    4  1477     .00000 |
| (4) vs (1)  6401.105    250     .00000   365.021  250  1231     .00000 |
| (4) vs (2)  2678.770      4     .00000  1568.114    4  1231     .00000 |
| (4) vs (3)  1883.937    246     .00000    12.836  246  1231     .00000 |
+-----------------------------------------------------------------------+
+----------------------------------------------------+
| Random Effects Model: v(i,t) = e(i,t) + u(i)       |
| Estimates:  Var[e]              =   .662965D-02    |
|             Var[u]              =   .130695D-01    |
|             Corr[v(i,t),v(i,s)] =   .663456        |
| Lagrange Multiplier Test vs. Model (3) = 1582.16   |
| ( 1 df, prob value =  .000000)                     |
| (High values of LM favor FEM/REM over CR model.)   |
| Fixed vs. Random Effects (Hausman)     =   ????    |
| ( 4 df, prob value =  .014557)                     |
| (High (low) values of H favor FEM (REM).)          |
|             Sum of Squares          .296237D+02    |
+----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 X1             .65025754      .02082134    31.230   .0000  -.188987D-14
 X2             .03004298      .01334238     2.252   .0243  -.272590D-14
 X3             .03506960      .01732942     2.024   .0430   .124737D-14
 X4             .39954471      .01084665    36.836   .0000   .779238D-14
 Constant     11.5774868      .00757539  1528.302   .0000
Matrix Cov.Mat. has  5 rows and  5 columns.
+-----------------------------------------------------------------------
1|     .00043    -.9201395D-04 -.6666412D-04    -.00017          0
2| -.9201395D-04      .00018   -.4320561D-05  .8595354D-06       0
3| -.6666412D-04 -.4320561D-05      .00030   -.2862694D-05       0
4|    -.00017    .8595354D-06 -.2862694D-05      .00012          0
5|      0             0             0              0    .5738657D-04
```

```
+----------------------------------------------------+
| OLS Without Group Dummy Variables                  |
| Model was estimated Dec 07, 2005 at 11:24:55AM     |
| LHS=YIT      Mean                 =   11.57749     |
|              Standard deviation   =   .6434377     |
| WTS=none     Number of observs.   =       1482     |
| Model size   Parameters           =         10     |
|              Degrees of freedom   =       1472     |
| Residuals    Sum of squares       =   28.91153     |
|              Standard error of e  =   .1401463     |
| Fit          R-squared            =   .9528477     |
|              Adjusted R-squared   =   .9525594     |
| Model test   F[  9,  1472] (prob) =3305.11 (.0000) |
| Diagnostic   Log likelihood       =   814.3815     |
|              Restricted(b=0)      =  -1448.908     |
|              Chi-sq [  9]  (prob) =4526.58 (.0000) |
+----------------------------------------------------+
```

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]	Mean of X
X1	.60073303	.01964659	30.577	.0000	-.188987D-14
X2	.02358238	.01121736	2.102	.0355	-.272590D-14
X3	.02727236	.01310340	2.081	.0374	.124737D-14
X4	.44443130	.01107955	40.113	.0000	.779238D-14
YEAR93	-.03446509	.01301167	-2.649	.0081	.16666667
YEAR94	-.01746941	.01281015	-1.364	.1727	.16666667
YEAR95	-.00345211	.01269074	-.272	.7856	.16666667
YEAR96	-.00541532	.01264493	-.428	.6685	.16666667
YEAR97	-.00999072	.01262022	-.792	.4286	.16666667
Constant	11.5892856	.00901935	1284.935	.0000	

```
+----------------------------------------------------+
| Least Squares with Group Dummy Variables           |
| Model was estimated Dec 07, 2005 at 11:24:55AM     |
| LHS=YIT      Mean                 =   11.57749     |
|              Standard deviation   =   .6434377     |
| WTS=none     Number of observs.   =       1482     |
| Model size   Parameters           =        256     |
|              Degrees of freedom   =       1226     |
| Residuals    Sum of squares       =   7.379538     |
|              Standard error of e  =   .7758349E-01 |
| Fit          R-squared            =   .9879646     |
|              Adjusted R-squared   =   .9854613     |
| Model test   F[255,  1226] (prob) = 394.67 (.0000) |
| Diagnostic   Log likelihood       =   1826.239     |
|              Restricted(b=0)      =  -1448.908     |
|              Chi-sq [255]  (prob) =6550.29 (.0000) |
+----------------------------------------------------+
+----------------------------------------------------+
| Panel:Groups   Empty       0,   Valid data     247 |
|                Smallest    6,   Largest          6 |
|                Average group size             6.00 |
+----------------------------------------------------+
```

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]	Mean of X
X1	.63796531	.02379854	26.807	.0000	-.188987D-14
X2	.04127557	.01544463	2.672	.0075	-.272590D-14
X3	.02819226	.02217322	1.271	.2036	.124737D-14
X4	.30816028	.01322571	23.300	.0000	.779238D-14
YEAR93	-.09400525	.00892438	-10.534	.0000	.16666667
YEAR94	-.06108644	.00813834	-7.506	.0000	.16666667
YEAR95	-.03263851	.00761097	-4.288	.0000	.16666667
YEAR96	-.02205545	.00721130	-3.058	.0022	.16666667
YEAR97	-.01870213	.00704283	-2.655	.0079	.16666667

```
+----------------------------------------------------------------------------+
|                Test Statistics for the Classical Model                     |
|                                                                            |
|        Model            Log-Likelihood    Sum of Squares    R-squared      |
| (1)  Constant term only    -1448.90832   .6131518321D+03     .0000000      |
| (2)  Group effects only      412.25944   .4974526192D+02     .9188696      |
| (3)  X - variables only      814.38155   .2891152515D+02     .9528477      |
| (4)  X and group effects    1826.23878   .7379537558D+01     .9879646      |
|                                                                            |
|                                Hypothesis Tests                            |
|               Likelihood Ratio Test                F Tests                 |
|           Chi-squared   d.f.  Prob.         F    num. denom. Prob value    |
| (2) vs (1)  3722.336    246     .00000    56.859  246  1235     .00000     |
| (3) vs (1)  4526.580      9     .00000  3305.109    9  1472     .00000     |
| (4) vs (1)  6550.294    255     .00000   394.667  255  1226     .00000     |
| (4) vs (2)  2827.959      9     .00000   782.048    9  1226     .00000     |
| (4) vs (3)  2023.714    246     .00000    14.542  246  1226     .00000     |
+----------------------------------------------------------------------------+
+----------------------------------------------------+
| Random Effects Model: v(i,t) = e(i,t) + u(i)       |
| Estimates:  Var[e]              =   .601920D-02    |
|             Var[u]              =   .136218D-01    |
|             Corr[v(i,t),v(i,s)] =   .693539        |
| Lagrange Multiplier Test vs. Model (3) = 1621.50   |
| ( 1 df, prob value =  .000000)                     |
| (High values of LM favor FEM/REM over CR model.)   |
| Baltagi-Li form of LM Statistic =        1621.50   |
| Fixed vs. Random Effects (Hausman)     =     .00   | <-- Not computable
| ( 9 df, prob value = 1.000000)                     |
| (High (low) values of H favor FEM (REM).)          |
|             Sum of Squares          .310345D+02    |
|             R-squared               .949385D+00    |
+----------------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 X1             .66230811      .02020885    32.773   .0000  -.188987D-14
 X2             .03781071      .01301123     2.906   .0037  -.272590D-14
 X3             .05518575      .01709150     3.229   .0012   .124737D-14
 X4             .35287326      .01159807    30.425   .0000   .779238D-14
 YEAR93       -.06688085      .00805543    -8.303   .0000     .16666667
 YEAR94       -.04042410      .00756342    -5.345   .0000     .16666667
 YEAR95       -.01774170      .00725994    -2.444   .0145     .16666667
 YEAR96       -.01371576      .00708616    -1.936   .0529     .16666667
 YEAR97       -.01431767      .00700864    -2.043   .0411     .16666667
 Constant      11.6030002      .00908962  1276.512   .0000
```

Part 5

This question is based on the original Mroz data used in his 1975 study of female labor supply. In the following model, we analyze the number of children in the family household using a Poisson regression model. The model is

$$\text{Prob}[\text{Nkids} = K_i] = \exp(-\lambda_i)\, \lambda_i^{K_i} / K_i!$$

where

$$\lambda_i = \exp(\beta_1 + \beta_2 AGE_i + \beta_3 AGE^2_i + \beta_4 WE_i + \beta_5 INCOME_i)$$

K_i = the number of kids
Age = age in years
Age2 = age^2
WE = wife's education in years
INCOME = family income in $10,000

Maximum likelihood Poisson regression results appear below.

a. Test the hypothesis that the number of children is unrelated to AGE using a Wald test.
b. Compute the marginal effect of an additional year in age on the expected number of kids.
c. Prove that the sample mean of the estimated λ_is (that is, the estimates of λ_i when you plug in the data and the maximum likelihood estimates of the parameters) equals the sample mean of K_i. (Note, this is a common result in 'loglinear' models such as this.)
d. Carry out a likelihood ratio test of the hypothesis that the four coefficients on AGE, AGE2, WE and INCOME are all zero.
e. Show exactly how to compute a Lagrange multiplier test statistic for testing the hypothesis that the coefficient on HA, the husband's age, is zero. Note that HA is not in the model, and I want to know if it has been inappropriately omitted. When I do this test, the actual test value that is computed is 5.873. Should the hypothesis that the coefficient on HA in this model is zero be rejected? Explain your answer precisely.

```
+---------------------------------------------+
| Poisson Regression                          |
| Model estimated: Nov 30, 2004 at 04:42:02PM.|
| Dependent variable               NKIDS      |
| Number of observations             753      |
| Iterations completed                 7      |
| Log likelihood function      -1083.397      |
| Number of parameters                 5      |
| Restricted log likelihood    -1279.522      |
+---------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
 Constant     -7.64180956     1.14268278    -6.688   .0000
 AGE            .49624655      .05663388     8.762   .0000    42.5378486
 AGE2         -.00686403       .00069963    -9.811   .0000    1874.54847
 WE           -.03430021       .01448182    -2.369   .0179    12.2868526
 INCOME         .01193400      .02569902      .464   .6424    2.30805950


Matrix Cov.Mat. has  5 rows and  5 columns.
        1             2             3             4             5
 +--------------------------------------------------------------------
1|    1.30572      -.06373       .00078      -.00319       .00284
2|    -.06373       .00321   -.3948059D-04  .3794861D-04    -.00012
3|     .00078   -.3948059D-04  .4894781D-06 -.3460068D-06  .1216040D-05
4|    -.00319    .3794861D-04 -.3460068D-06      .00021      -.00014
5|     .00284      -.00012    .1216040D-05    -.00014       .00066
```

Part 6

In homework 8, you examined a model that might be used for lifetimes of electric or electronic parts, the exponential regression model,

$$f(y_i|x_i,\alpha,\gamma) = \theta_i \exp(-\theta_i y_i),\ \theta_i = \exp(\alpha + \gamma x_i).$$

The regression aspect of this model emerges when we note that $E[y_i|x_i] = 1/\theta_i$.

a. Though we estimated the parameters of the model by using maximum likelihood in homework 8, we could also have used nonlinear least squares. The nonlinear least squares estimates, with the MLEs are shown below. They are, of course, similar. Noting that neither is actually more difficult to compute than the other, is there a statistical reason to prefer one estimator or the other? Explain.

```
+-----------------------------------------------------+
| User Defined Optimization                           |
| Nonlinear   least squares regression                |
| Model was estimated Dec 07, 2005 at 02:29:21PM      |
| LHS=YI        Mean                 =   .5717307     |
|               Standard deviation   =   .6128446     |
| WTS=none      Number of observs.   =         50     |
| Model size    Parameters           =          2     |
|               Degrees of freedom   =         48     |
| Residuals     Sum of squares       =   17.78242     |
|               Standard error of e  =   .5963626     |
| Fit           R-squared            =   .3373985E-01 |
|               Adjusted R-squared   =   .5306506E-01 |
| Model test    F[  1,    48] (prob) =   1.68 (.2016) |
| Diagnostic    Log likelihood       =  -45.10161     |
|               Restricted(b=0)      =  -45.95966     |
|               Chi-sq [  1]  (prob) =   1.72 (.1902) |
+-----------------------------------------------------+
+---------+--------------+----------------+--------+---------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] |
+---------+--------------+----------------+--------+---------+
 AL             .17403168      .26918380      .647   .5179
 CL             .77328951      .54264646     1.425   .1541
Maximum Likelihood Estimates
+----------------------------------------------+
| Exponential (Loglinear) Regression Model     |
| Maximum Likelihood Estimates                 |
| Model estimated: Dec 08, 2005 at 03:58:46PM.|
| Dependent variable                    YI     |
| Weighting variable                  None     |
| Number of observations                50     |
| Iterations completed                   5     |
| Log likelihood function        -21.31650     |
| Number of parameters                   2     |
| Info. Criterion: AIC =            .93266     |
|   Finite Sample: AIC =            .93777     |
| Info. Criterion: BIC =           1.00914     |
| Info. Criterion:HQIC =            .96178     |
| Restricted log likelihood      -22.04564     |
| Chi squared                     1.458283     |
| Degrees of freedom                     1     |
| Prob[ChiSqd > value] =          .2272035     |
+----------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
          Parameters in conditional mean function
 Constant       .25008087      .30543726      .819   .4129
 XI             .61138675      .51150525     1.195   .2320     .52927082
```

b. The model is heteroscedastic. $\mathrm{Var}[y_i|x_i] = 1/\theta_i^2$. Could you improve on the nonlinear least squares estimator with this knowledge? Show how to do generalized least squares in this model.

c. As a model for lifetimes, the exponential model has a number of shortcomings. The most oft noted is its property of 'lack of memory.' Regardless of how long the part has lasted (y_i), the probability it will fail in the next interval Δy_i is the same. A common alternative to the model is the gamma model we examined in class. A slightly simpler one is the Weibull model, which adds a scale parameter, P, to the model;

$$f(y_i|x_i,P,\alpha,\gamma) = P\theta_i\, y_i^{P-1} \exp(-\theta_i y_i^P)\;,\; \theta_i = \exp(\alpha + \gamma x_i),\; P > 0.$$

The exponential model is the special case, with P = 1. Estimates of the parameters of this model using the data from Assignment 8 are shown below:

```
+---------------------------------------------+
| Weibull (Loglinear) Regression Model        |
| Maximum Likelihood Estimates                |
| Model estimated: Dec 07, 2005 at 02:39:27PM.|
| Dependent variable                 YI       |
| Weighting variable               None       |
| Number of observations             50       |
| Iterations completed                9       |
| Log likelihood function     -21.31393       |
| Number of parameters                3       |
| Info. Criterion: AIC =         .97256       |
|   Finite Sample: AIC =         .98299       |
| Info. Criterion: BIC =        1.08728       |
| Info. Criterion:HQIC =        1.01624       |
| Restricted log likelihood   -22.04564       |
| Chi squared                  1.463416       |
| Degrees of freedom                  2       |
| Prob[ChiSqd > value] =       .4810867       |
+---------------------------------------------+
+---------+--------------+----------------+--------+---------+----------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|
+---------+--------------+----------------+--------+---------+----------+
          Parameters in conditional mean function
 Constant       .24858901      .31142980      .798   .4247
 XI             .61583268      .50392591     1.222   .2217     .52927082
          Scale parameter for Weibull model
 P_scale       1.00772871      .16215101     6.215   .0000
```

(1) Derive the log likelihood and the likelihood equations for estimation of α, γ and P for the Weibull model.
(2) Sketch a proof of the consistency of the maximum likelihood estimator. Note, this can be in general terms, as your results will include this model, since it does satisfy the regularity conditions.
(3) Sketch a proof of the asymptotic normality of this maximum likelihood estimator.
(4) Derive the asymptotic covariance matrix for the MLE of (α,γ,P). Derive the BHHH estimator of the asymptotic covariance matrix.
(4) Test the hypothesis of the exponential model (null) against the Weibull (alternative) using the results given here. Use a Wald test and a likelihood ratio test. Be explicit about how you are doing your computations.

Part 7

In the Bertschek and Lechner paper discussed in class, the model analyzed by the authors was

$$y_{it}^* = \boldsymbol{\beta}'\mathbf{x}_{it} + \varepsilon_{it},\ t = 1,...,5 \text{ and } i = i,...,N$$

$$y_{it} = 1 \text{ if } y_{it}^* > 0$$

where $\varepsilon_{i1},\varepsilon_{i2},...,\varepsilon_{i5}$ have a 5-variate normal distribution with means zero, variances 1, and full correlation matrix, R. They would like to do maximum likelihood estimation of $\boldsymbol{\beta}$ and the 4(5)/2 = 10 free correlation coefficients in R. However, this will involve doing 5 dimensional integration of the normal distribution, which is (was) not technologically possible for them. They proposed, instead, to develop a GMM estimator that will make it unnecessary to estimate R altogether. The idea is this: Note, first of all, this is a probit model. In every period, for every observation,

$$\text{Prob}[y_{it} = 1|\mathbf{x}_{it}] = \Phi(\boldsymbol{\beta}'\mathbf{x}_{it}),\ \text{Prob}[y_{it} = 0\ |\mathbf{x}_{it}] = 1 - \Phi(\boldsymbol{\beta}'\mathbf{x}_{it})$$

where $\Phi(\boldsymbol{\beta}'\mathbf{x}_{it})$ is the standard normal CDF. They could just pool the data and use maximum likelihood to estimate β. This would be consistent, but would waste a large amount of information. They do have the following moment equations:

$$E[(y_{it} - \Phi(\boldsymbol{\beta}'\mathbf{x}_{it}))\ \mathbf{x}_{is}] = 0,\ s,t = 1,...,5.$$

Note that this is actually 25 sets of moment equations, because, for example, in period 1, $(y_{i1} - \Phi(\beta' x_{i1}))$ is uncorrelated with (orthogonal to) $\mathbf{x}_{i1}$, $\mathbf{x}_{i2}$, ..., $\mathbf{x}_{i5}$. The same is true for periods 2 through 5. So, suppose there are K regressors. In each period, there are 5K moment conditions, and there are 5 periods. So, this provides 5×5×K moment conditions for estimating K parameters. The parameter vector is vastly overidentified.

Explain how to use this model to obtain GMM estimators of the model parameters. Be precise and detailed on the computations that you will do. Include in your description exactly what computations you will do to obtain the estimator and also how you will estimate the asymptotic covariance matrix for your estimator

Part 8

This question involves some "library" research. (You can do it on the web, of course.) Locate an empirical (applied) paper (study) in any field (political science, economics, finance, management, accounting, pharmacology, environment, etc.) that is an application of a discrete choice model – Poisson, probit or other binary choice, multinomial logit, ordered probit, or something else if you prefer. Report (a) what empirical issue the study was about; (b) what the model was; (c) what estimation technique the author used; (d) (briefly) what results they obtained. In part (d), describe the actual statistics that the author reported, and what conclusion they drew. This entire essay should not exceed one double spaced page.