

## **ECONOMETRICS I Take Home Final Examination**

## Fall 2015

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Today is Tuesday, December 8, 2015. This exam is due by 10AM, Tuesday, December 22, 2015.

Please do not include a copy of the exam questions with your submission; submit only your answers to the questions.

NOTE: In the empirical results below, a number of the form .nnnnnE+aa means multiply the number .nnnnn by 10 to the aa power. E-aa implies multiply 10 to the minus aa power. Thus, .123456E-04 is 0.0000123456. Note, as well, D+nn or D-nn or e+nn or e-nn all mean the same as E+nn or E-nn.

This test comprises 275 points. The allocation of points to the 10 questions is as follows:

1. 20 2. 20 3. 40 4. 20 5. 20 6. 25 7. 40 8. 25 9. 40 10. 25

## 1. Properties of the least squares estimator

- a. Show (algebraically) how the ordinary least squares coefficient estimator, **b**, and the estimated asymptotic covariance matrix are computed.
- b. What are the finite sample properties of this estimator? Make your assumptions explicit.
- c. What are the asymptotic properties of the least squares estimator? Again, be explicit about all assumptions, and explain your answer carefully.
- d. How would you compare the properties of the least absolute deviations (LAD) estimator to those of the ordinary least squares (OLS) estimator? Which is a preferable estimator?

2. The OLS regression results given below are based on the Baltagi-Griffin OECD gasoline market data. The LHS variable is log(per capita gasoline consumption). The RHS variables are logs of per capita income, the price index of gasoline and the per capita number of cars in the country. In the first set of results, the standard errors are computed using White's heteroscedasticity consistent, robust estimator of the covariance matrix. The second set use the conventional estimator,  $s^2(\mathbf{X}'\mathbf{X})^{-1}$ . The third set are "clustered" at the country level. (There are 18 countries and 19 years of data (1960-1978).)

- a. Explain what is meant by the term "robust covariance estimator." Why would one report "robust standard errors?"
- b. How is the White estimator computed?
- c. Looking at these results, would you conclude that there is evidence of heteroscedasticity in these data?
- d. How is the cluster corrected covariance matrix computed? Can you draw a conclusion about the data based on these results?

							-
Ordinary	least squares	s regressio	n				
LHS=LGASE	PCAR Mean	=	4.	29624			
	Standard devi	ation =		54891			
	Number of obs	servs. =		342			
Model siz	e Parameters	=		4			
	Degrees of fr	reedom =	338				
Residuals	s Sum of square	es =	14	.9044			
	Standard erro	or of e =		20999			
Fit	R-squared	=		85494			
	Adjusted R-so	uared =		85365			
Model tes	st F[ 3, 338]	(prob) =	664.0(.	0000)			
White het	eroscedasticity r	obust cova	riance ma	trix.			
Br./Pagar	n LM Chi-sq [ 3]	(prob) =	.83 (.	8427)			
							-
		Standard		Prob.	95% Cor	nfidence	
LGASPCAR	Coefficient	Error	t	t >T*	Inte	erval	
Congtont	2 20122***	11705	20 27		2 16015	2 62250	-
TNOMED	2.39133***	.11/95	20.27	.0000	2.10015	2.02250	
LINCOMEP	.00990***	.04429	20.09	.0000	.00315	.9/0//	
LRPMG	89180^^^	.03891	-22.92	.0000	90800	81334	
LCARPCAP	/033/^^^	.02153	-35.40	.0000	80557	/2118	_
***, **,	* ==> Significar	nce at 1%,	5%, 10% 1	evel.			-
Conventio	onal Standard Erro	ors					_
Constant	2.39133***	.11693	20.45	.0000	2.16214	2.62051	
LINCOMEP	.88996***	.03581	24.86	.0000	.81978	.96014	
LRPMG	89180***	.03031	-29.42	.0000	95121	83238	
LCARPCAP	76337***	.01861	-41.02	.0000	79984	72690	
							_

+   Covaria   Sample   variabl	ance matrix for the of 342 observation 342 observation of the second sec	he model is ations conta identifies	adjusted nined by a val	for dat 18 clus ue a clu	ta clustering sters defined ister ID.	J. J by
Constant	2.39133***	.44167	5.41	.0000	1.52567	3.25698
LINCOMEP	.88996***	.17248	5.16	.0000	.55190	1.22803
LRPMG	89180***	.14578	-6.12	.0000	-1.17753	60607
LCARPCAP	76337***	.06985	-10.93	.0000	90028	62647

3. The regressions on the next page are computed using the gasoline data used in question 2 above. The estimated asymptotic covariance matrix is shown with each set of regression results. (Note the use of scientific notation in the covariance matrix). The first regression is the same as reported in question 2. I suspect that the effect of (log) income is nonlinear in the model, so in the second regression, I have added a quadratic term in log income to the model.

- a. Show how each of the values in the box above the coefficient estimates in the first regression is computed, and interpret the value given.
- b. Using the results given, form a confidence interval for the true value of the coefficient log price variable.
- c. The second set of results given includes the quadratic term in log income. In the first regression, as we might have expected, log income is highly significant. Looking at the second regression, I might conclude that the quadratic model has revealed that income is not significant. Would this be the correct conclusion? Explain.
- d. Test the null hypothesis of the log-linear model against the alternative of the log-quadratic model. Do the test in three ways: 1. Use a Wald test; 2. Use an F test. 3. Use a likelihood ratio test assuming that the disturbances are normally distributed.
- e. I am interested in the partial of log Income. As such, the quantity

 $\delta = \partial E[IGasPcar \mid \mathbf{x}] / \partial \text{ lincomep}$ 

is of interest. Obtain the expression for this function based on the second regression. Estimate this at the average value of lincomep = -6.139425. Form a confidence interval for the estimate of  $\delta$ .

f. Describe in detail how to use the method of Krinsky and Robb to obtain the standard error needed to compute the confidence interval in part f.

Ordinary	least square	s regress	ion				
LHS=LGASI	PCAR Mean		_	4.29	9624		
	Standard dev	viation	-	.54	4891		
	No. of obser	vations	_		342	DegFreedom	Mean square
Regressio	on Sum of Squar	es	_	87.8	3386	3	29.27954
Residual	Sum of Squar	es.		14.0	2044	338	.04410
Total	Sum of Squar	200		102	743	341	30130
	- Standard er	or of e		202	1000	Poot MCE	20876
Rit.	Standard eri	9 10 10		.20	5404	ROOL MSE	.20070
Model to	K-Squared			.0.	022	R-Dai Squareo	.05505
Diagnost	st fl 3, 330			E0 40	222	Prop F > F"	- 2 10077
Diagnost.	Dogtrigtod (	(h=0)		270 6	203	ARAIKE I.C.	= -3.10977
	Restricted (	D=0)		.2/9.03		Dayes I.C.	= -3.00491
	Chi squared	[ 3]	-	660.2	5/26	Prod $C_2 > C_2^*$	= .00000
	+ I	<b>C L u u d u u</b>	 a			050 0	6 / A
		Standar	a		Prob	. 95% Con	ifidence
LGASPCAR	Coefficient	Error		t	t >T	* Inte	erval
Constant	2.39133***	.1169	3 20	).45	.0000	2.16214	2.62051
LINCOMEP	.88996***	.0358	1 24	.86	.0000	.81978	.96014
LRPMG	89180***	.0303	1 -29	.42	.0000	95121	83238
LCARPCAP	76337***	.0186	1 -41	.02	.0000	79984	72690
	+						
***, **,	* ==> Significa	nce at 1%	, 5%, 1	.0% lev	vel.		
	+						
Cov.[b^]	ONE	LINCO	MEP	]	RPMG	LCARPCAF	)
ONE	.0136736	.00236	569	772904	4E-03	635905E-04	
LINCOMEP	.00236569	.00128	206	86747	LE-03	558696E-03	}
LRPMG	772904E-03	867471E	-03	918984	4E-03	.450369E-03	
LCARPCAP	635905E-04	558696E	-03	45036	9E-03	.346269E-03	
Ordinary	least square	es regress	ion	•••••	••••		
LHS=LGASI	PCAR Mean		-	4.29	9624		
	Standard dev	viation	-	.54	1891		
	No. of obser	vations	-		342	DegFreedom	Mean square
Regressio	on Sum of Squar	es	-	87.9	9772	4	21 99431
Residual							21.00101
	Sum of Squar	res	=	14.	/65/	337	.04382
Total	Sum of Squar Sum of Squar	res	=	14.	.743	337 341	.04382
Total	Sum of Squar Sum of Squar Standard err	es ces cor of e	=	14. 102 .20	,743 932	337 341 Root MSE	.04382 .30130 .20779
Total  Fit	Sum of Squar Sum of Squar Standard err R-squared	es cor of e	= = =	14. 102 .20 .85	743 932 5628	337 341 Root MSE R-bar squared	.04382 .30130 .20779 I
Total  Fit Model tes	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337	res cor of e 7]	= = = =	14. 102. .20 .89 501.97	743 932 5628 7922	337 341 Root MSE R-bar squared Prob F > F*	.04382 .30130 .20779 L .85458 .00000
Total Fit Model tes Diagnost:	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho	res for of e 7] pod		14. 102 .20 .85 501.97 52.09	743 932 5628 7922 9097	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C.	.04382 .30130 .20779 l .85458 .00000 = -3.11326
Total Fit Model tes Diagnost:	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted (	res res for of e /] bod b=0)	= = = = = = = = =	14. 102 .20 .89 501.97 52.09	7657 743 0932 5628 7922 9097 3574	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C.	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720
Total Fit Model tes Diagnost:	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared	res for of e bood (b=0) [ 4]	= = = = = = = = = =	14. 102 .20 .85 501.95 52.09 .279.65 663.45	743 932 5628 7922 9097 3574 5343	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2*	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720 7 = .00000
Total Fit Model tes Diagnost:	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared	res for of e 20 20 20 20 20 20 20 20 20 20 20 20 20	= = = = = = = = =	14. 102 .20 .89 501.97 52.09 -279.63 663.49	.743 0932 5628 7922 9097 3574 5343	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2*	$ \begin{array}{r} .04382\\.30130\\.20779\\.85458\\.00000\\=-3.11326\\=-3.05720\\\end{array} $
Total Fit Model tes Diagnost:	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared	res for of e bod b=0) [ 4] Standar	= = = = = = 	14. 102 .20 .89 501.97 52.09 .279.63 663.49	743 932 5628 7922 9097 3574 5343 Prob	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720 = .00000
Total Fit Model tes Diagnost: LGASPCAR	Sum of Squar Sum of Squar Standard err R-squared st F[4, 337 ic Log likeliho Restricted ( Chi squared Coefficient	res for of e bod b=0) [ 4] Standar Error	= = = = = = d	14. 102 .2( .8! 501.9 52.09 -279.63 663.4! 	.743 )932 5628 7922 9097 3574 5343  Prob  t >T	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720 c = .00000 
Total Fit Model tes Diagnost: LGASPCAR	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared 	res for of e d b=0) [ 4] Standar Error	= = = = = d	14. 102 .20 .85 501.95 52.09 -279.65 663.45 t	7657 743 9932 5628 7922 9097 3574 5343 Prob  t >T	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720 f = .00000
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Total Fit Model tes Diagnost: LGASPCAR Constant LINCOMEP	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared 	res for of e b=0) [ 4] Standar Error 1.1899	= = = = = - - d 1 9	14. 102 .2( .8! 501.9' 52.09 -279.63 663.4! t .24 .67	7657 743 9932 5628 7922 9097 3574 5343 Prob  t >T .8109 .5024	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte -2.04732 46968	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720 = .00000 
Total Fit Model tes Diagnost: LGASPCAR Constant LINCOMEP	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared 	res for of e b=0) [ 4] Standar Error 1.1899 .3644	= = = = =  d -  0 MRD ^2	14. 102 .2( .8! 501.9 <sup>5</sup> 52.09 -279.62 663.4! t .24 .67 0	7657 743 9932 5628 7922 9097 3574 5343 Prob  t >T .8109 .5024	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte -2.04732 46968	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720 = .00000 fidence mval 2.61705 .95910
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Total Fit Model tes Diagnost: LGASPCAR  Constant LINCOMEP Intrct01 LEDMC	Sum of Squar Sum of Squar Sum of Squar R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared Coefficient .28487 .24471 Constructed vari 05083* _ 92413***	res res ror of e () b=0) [ 4] Standar Error 1.1899 .3644 .able LINC .0285 0352	= = = = =  d  0 MEP^2. 7 -1 6 -20	14. 102 .21 .8! 501.9' 52.09 663.4! t .24 .67 0 .78 .21	.743 .743 .932 .5628 .7922 .097 .574 .574 .574 .574 .8109 .5024 .0762	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte -2.04732 46968 10683 _99324	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720 = .00000 .0000 ffidence erval 2.61705 .95910 .00518 85501
Total Fit Model tes Diagnost: LGASPCAR 	Sum of Squar Sum of Squar Sum of Squar R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared 	res for of e () () () () () () () () () () () () ()	= = = = =  d  1 9 OMEP^2. 7 -1 6 -26 7 -26	14. 102 .88 501.9° 52.09 279.63 663.49 t .24 .67 0 .78 .21 .67 .0 .78 .21 .67 .0 .0 .0 .0 .0 .0 .0 .0 .0 .0	.0762 .0762 .0762	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte -2.04732 46968 10683 99324	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720 ? = .00000 .0000 .95910 .00518 85501 -73229
Total Fit Model te; Diagnost: LGASPCAR  Constant LINCOMEP Intrct01 LRPMG LCARPCAP	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared 	res for of e () (4) (4) (4) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1	= = = = = 	14. 102 .21 .8! 501.9' 52.09 .279.6: 663.4! t .24 .67 0 .78 ;.21 .61	.743 .743 .932 .5628 .7922 .0097 .3574 .5343 	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte -2.04732 46968 10683 99324 80767	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720 f = .00000 .0000 .0000 .95910 .00518 85501 73329
Total Fit Model tes Diagnost: LGASPCAR  Constant LINCOMEP Intrct01 LRPMG LCARPCAP	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared Coefficient .28487 .24471 Constructed vari 05083* 92413*** 77048***	res for of e () (b=0) [ 4] (c) (c) (c) (c) (c) (c) (c) (c) (c) (c)	= = = = = - - - - - - - - - - - - - - -	14. 102 .21 .8! 501.9' 52.09 .279.63 663.4! t .24 .67 .0 .78 .21 .161	743         .743         .743         .932         .6628         .7922         .9097         .3574         .343         .7062         .0000         .0000         .0000	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte -2.04732 46968 10683 99324 80767	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720 fidence prval 2.61705 .95910 .00518 85501 73329
Total Fit Model tes Diagnost: LGASPCAR  Constant LINCOMEP Intrct01 LRPMG LCARPCAP  Cov.[b <sup>*</sup> ]	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared .28487 .24471 Constructed vari 05083* 92413*** 77048***	res for of e () () () () () () () () () () () () ()	= = = = = - - - - - - - - - - - - - - -	14. 102 .20 .501.9 <sup>5</sup> 52.09 .279.63 .663.49 .24 .67 .0 78 21 	743       743       932       9628       7922       9097       3574       5343       Prob        t >T       .8109       .5024       .0762       .0000       .0000       .0000       .cct01	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* 95% Con * Inte -2.04732 46968 10683 99324 80767 LRPMC	.04382 .30130 .20779 1 .85458 .00000 = -3.11326 = -3.05720 5 = .00000 .0000 .0000 .00518 85501 73329
Total Fit Model tes Diagnost: LGASPCAR Constant LINCOMEP Intrct01 LRPMG LCARPCAP Cov.[b^]	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared .28487 .24471 Constructed vari 05083* 92413*** 77048***	res for of e () () () () () () () () () () () () ()	= = = = = d 1 9 0MEP^22. 7 -1 6 -26 7 -40 MEP	14. 102 .2( .8! 501.9' 52.0! 279.6: 663.4! .24 .67 .0 .78 .21 .61 .111	<pre>// 5743 .743 .9932 .5628 .7922 .9097 .3574 .5343 </pre>	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte -2.04732 46968 10683 99324 80767 LRPMC	.04382 .30130 .20779 .85458 .00000 = -3.11326 = -3.05720 = .00000 .0000 .0000 .0000 .00518 85501 73329 .004566655
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Total Fit Model tes Diagnost: LGASPCAR Constant LINCOMEP Intrct01 LRPMG LCARPCAP Cov.[b^] ONE LINCOMEP	Sum of Squar Sum of Squar Sum of Squar R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared .28487 .24471 Constructed vari 05083* 92413*** 77048*** ONE 1.41589 .431902 .22252	res for of e () () () () () () () () () () () () ()	= = = = = = = = = = = = = = = = = = =	14. 102 .2( .8 501.9' 52.0! -279.63 663.4! t .24 .67 .0 .78 5.21 .178 5.21 .61 .033 .010	<pre>/537 743 9932 5628 7922 9097 3574 5343  Prob 1t &gt;T .8109 .5024 .0762 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000</pre>	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte -2.04732 46968 .99324 80767 LRPMC .0207538 .00573056	.04382 .30130 .20779 .85458 .00000 = -3.11326 = -3.05720 = .00000 .00518 85501 7329 .00466965 .894615E-03
Total Fit Model tex Diagnost: LGASPCAR  Constant LINCOMEP Intrct01 LRPMG LCARPCAP  Cov.[b^]  ONE LINCOMEP Intrct01	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared .28487 .24471 Constructed vari 05083* 92413*** ONE 1.41589 .431902 .0338359	res for of e () (4) (4) (4) (5) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1	= = = = = = = = = = = = = = = = = = =	14. 102 .8 501.9 52.0 .279.6 663.4 t .24 .67 .0 .78 5.21 .61 .03 .010 816422	<pre>/537 .743 .9932 .5628 7922 .9097 .3574 .5343 </pre>	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte -2.04732 46968 .99324 80767 LRPMO .0207538 .00573056 .519295E-03	.04382 .30130 .20779 .85458 .00000 = -3.11326 = -3.05720 ? = .00000 .00518 85501 73329 .00466965 .894615E-03 .114198E-03
Total Fit Model te; Diagnost: LGASPCAR Constant LINCOMEP Intrct01 LRPMG LCAPCAP Cov.[b^] ONE LINCOMEP Intrct01 LRPMG ONE	Sum of Squar Sum of Squar Standard err R-squared st F[ 4, 337 ic Log likeliho Restricted ( Chi squared .28487 .24471 Constructed vari 05083* 92413*** 77048*** ONE 1.41589 .431902 .0338359 .0207538	res for of e () (4) (4) (4) (5tandar Error 1.1899 .3644 .able LINC .0285 .0352 .0189  LINCO .431 .132 .0103 .00573	= = = = = = = = = = = = = = = = = = =	14. 102 .24 .8! 501.9' 52.0! -279.6: 663.4! t .24 .67 .0 .78 .21 .61 .03: .010 816422 519299	743         .743         .932         .628         .9922         .997         .3574         .343         .91097         .8109         .5024         .0000	337 341 Root MSE R-bar squared Prob F > F* Akaike I.C. Bayes I.C. Prob C2 > C2* . 95% Con * Inte -2.04732 46968 .10683 .99324 80767 LRPMG .0207538 .00573056 .519295E-03 .00124344	.04382 .30130 .20779 .85458 .00000 = -3.11326 = -3.05720 = .00000 .00518 85501 73329 .00466965 .894615E-03 .114198E-03 .520140E-03

4. In our discussion of the generalized regression model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \, \mathrm{E}[\boldsymbol{\epsilon}|\mathbf{X}] = 0, \, \mathrm{E}[\boldsymbol{\epsilon}\boldsymbol{\epsilon}' \ |\mathbf{X}] = \sigma^2 \boldsymbol{\Omega}$$

we arrived at the result of the Aitken theorem, that the optimal estimator of  $\beta$  was GLS,

 $\mathbf{b}_{GLS} = (\mathbf{X'} \mathbf{\Omega}^{-1} \mathbf{X})^{-1} \mathbf{X'} \mathbf{\Omega}^{-1} \mathbf{y}$ 

The estimator is optimal in the sense that the Gauss Markov theorem applies – its variance is smaller than any other linear unbiased estimator, including OLS,

$$\mathbf{b}_{\text{OLS}} = (\mathbf{X'X})^{-1} \mathbf{X'y}.$$

In the case of the random effects model for panel data,

$$y_{it} = \mathbf{x}_{it}' \boldsymbol{\beta} + \varepsilon_{it} + u_i, i = 1,...,n, t = 1,...,T$$
 (balanced panel)

The covariance matrix for the disturbances is block diagonal with each block in the matrix equal to the T×T matrix

$$\Omega_{i} = \sigma^{2} \begin{bmatrix} 1 & \rho & \dots & \rho \\ \rho & 1 & \rho & \dots \\ \dots & \rho & \dots & \rho \\ \rho & \rho & \dots & 1 \end{bmatrix}, \ \sigma^{2} = \sigma_{\varepsilon}^{2} + \sigma_{u}^{2}, \ \rho = \sigma_{u}^{2} / \sigma^{2}.$$

The disturbances are homoscedastic but every pair of disturbances within the ith group has the same correlation.

a. Prove that the generalized least squares estimator is obtained by regressing  $y_{it} - \theta \overline{y}_i$  on  $x_{it} - \theta \overline{x}_i$ . (Hint, it suffices to prove that  $\Omega_i^{-1}$  = the result given in your text, then multiply the ith group,  $y_i$  and  $X_i$  by the resulting matrix.

5. The three sets of results below show the least squares estimates of the model

Health =  $\beta_1 + \beta_2 \operatorname{Age} + \beta_3 \operatorname{Educ} + \beta_4 \operatorname{Income} + \varepsilon$ 

(We are ignoring the problems with this equation that are discussed in question 8.) Results are given for male headed households (female=0), female headed households (female=1) and all households (female=\*).

- a. Theory 1 states that the coefficient vectors are the same for the two genders. Is there an optimal way that I could combine these two estimators to form a single efficient estimator of the model parameters? How should I do that? Describe the computations in detail.
- b. Use a Chow test to test the hypothesis that the two coefficient vectors are the same. Explain the computations in full detail so that I know exactly how you obtained your result.
- c. Show in detail how to use a Wald test to test the hypothesis that the coefficients are the same.
- d. Is there any particular reason to use the Wald test or the Chow test i.e., one and not the other? What assumptions would justify each. Do the regression results suggest that one or the other test might be appropriate? Explain.

> regr;f	or[female=*,0,1];	lhs=hsat;r	hs=x;cov\$		
Setting up The model of this var	an iteration over command will be er riable. In the cu	r the value kecuted for urrent samp	s of FEMALE 2 values le of 27322		
Subservation Subsample FEMALE = FEMALE =	Observations 0 14240 **** 27322	Subsample FEMALE =	Observations 1 13082		
Subsample	analyzed for this	command is	FEMALE =	-  0 	
Ordinary LHS=HSAT	least squares n Mean	regression =	6.92428		
	Standard deviat No. of observat	tion = tions =	$2.25170 \\ 14240$	DegFreedom	Mean square
Regression Residual Total	Sum of Squares Sum of Squares Sum of Squares	= = =	5386.20 66807.6 72193.8	3 14236 14239	1795.40049 4.69286 5.07014
Fit Model test	Standard error R-squared F[ 3, 14236]	of e = = =	2.16630 .07461 382.58133	Root MSE R-bar squared Prob F > F*	2.16600 .07441 .00000
+-   HSAT	Coefficient	Standard Error	Prob z  z >Z	. 95% Con * Inte	fidence rval
Constant   AGE	7.90985*** 04923***	.11779 .00162	67.15 .0000 -30.33 .0000	7.67898 05241	8.14071 04605
EDUC HHNINC	.07188*** .75521***	.00785 .11001	9.15 .0000 6.87 .0000	.05649 .53960	.08727 .97082
Note: ***,	**, * ==> Signii	ficance at	1%, 5%, 10% 10	evel.	
Cov.[b^]	ONE	AGE	EDUC	HHNINC	
ONE AGE	.0138745 123167E-03	123167E-03 263389E-05	688251E-03 .139160E-05	610555E-03 153205E-04	
EDUC HHNINC	688251E-03 .: 610555E-03:	139160E-05 153205E-04	.616681E-04 262788E-03	262788E-03 .0121018	
Subsample	analyzed for this	command is	FEMALE =	1	
Ordinary	least guiares i	regression			
LHS=HSAT	Mean Standard deviat	= tion =	6.63407 2.32957		
Regression	No. of observat Sum of Squares	tions = =	13082 $4241.14$	DegFreedom 1 3	Mean square 1413.71331
Residual Total	Sum of Squares Sum of Squares Standard error	= = of e =	66748.2 70989.3 2 25917	13078 13081 Root MSE	5.10385 5.42690 2.25883
Fit Model test	R-squared F[ 3, 13078]	=	.05974 276.98943	R-bar squared Prob F > F*	.05953
HSAT	Coefficient	Standard Error	Prob z  z >Z	. 95% Con * Inte	fidence rval
Constant	7.31752***	.14966	48.89 .0000	7.02419	7.61085
AGE   EDUC	04138*** .08589***	.00180 .00989	-22.96 .0000 8.68 .0000	04491 .06650	03785 .10527
HHNINC	.64726***	.11231	5.76 .0000	.42714	.86739
Note: ***,	**, * ==> Signit	ficance at	1%, 5%, 10% le	evel.	
Cov.[b^]	ONE	AGE	EDUC	HHNINC	

ONE	.0223980	192149E-03	00117683	00192307
AGE	192149E-03	.324844E-05	.432051E-05	.197074E-05
EDUC	00117683	.432051E-05	.978490E-04	230813E-03
HHNINC	00192307	.197074E-05	230813E-03	.0126137

Full pooled sample is used for this iteration.

\_\_\_\_\_

Ordinary LHS=HSAT	least squares Mean Standard devi	regression = ation =	 6. 2	 78532 29386		
	No of observ	ations =	2.	27322	DegFreedom	Mean square
Regression	Sum of Square	s =	10	033 8	2	3344 60308
Residual	Sum of Square	s =	13	3724	27318	4 89507
Total	Sum of Square	s =	14	3757	27321	5 26179
	Standard erro	rofe =	2	21248	Root MSE	2 21232
Fit	R-squared	- 1010	2.	06980	R-bar squared	1 06969
Model test	F[ 3, 27318]	=	683.	25957	Prob F > F*	.00000
		Standard		Prob	. 95% Cor	nfidence
HSAT	Coefficient	Error	Z	z   >Z	* Inte	erval
Constant	7.61481***	.09226	82.54	.0000	7.43399	7.79563
AGE	04585***	.00120	-38.21	.0000	04821	04350
EDUC	.08170***	.00606	13.48	.0000	.06982	.09358
HHNINC	.68599***	.07850	8.74	.0000	.53213	.83986
Note: ***,	**, * ==> Sign	ificance at	1%, 5%,	10% 10	evel.	
Cov.[b^]	ONE	AGE		EDUC	HHNINC	-

ONE	.00851141	757535E-04	427036E-03	570189E-03
AGE	757535E-04	.143995E-05	.127524E-05	386028E-05
EDUC	427036E-03	.127524E-05	.367549E-04	126496E-03
HHNINC	570189E-03	386028E-05	126496E-03	.00616291

6. The results below show OLS, fixed effects and random effects estimates of a loglinear cost function of the form

$$\log(C_{it} / w_5) = \alpha + \sum_{k=1}^{4} \beta_k \log(w_k / w_5) + \sum_{m=1}^{5} \gamma_m \log q_m + \varepsilon_{it} + u_i$$

Where C is total costs, there are 5 inputs with input prices  $w_k$  and 5 outputs denoted  $q_m$ , m = 1,...,5. The cost variable and the four inputs in the equation are divided by  $w_5$  to enforce the linear homogeneity constraint  $\beta_1+\beta_2+\beta_3+\beta_4+\beta_5 = 1$ . The sample is a set of 500 banks observed for 5 years.

- a. Test the hypothesis of 'no effects' vs. 'some effects' using the results given below.
- b. Explain in precise detail the difference between the fixed and random effects models.
- c. What is the result of the Hausman test for fixed vs. random effects? Report your conclusion. Carefully explain what you are doing in this test. Based on your result, which is the preferred model, fixed effects or random effects?
- d. In the context of the fixed effects model, test the hypothesis that there are no effects i.e., that all banks have the same constant term. (The statistics you need to carry out the test are given in the results.)
- e. In the final regression below, I have added the bank (5 period) means of the log price ratios and log outputs to the model and reestimated the random effects model. I then used a Wald statistic to test the joint hypothesis that the coefficients on the 9 group means are jointly equal to zero. Are the results consistent or inconsistent with the results in part c? Explain.

OLS Witho	out Group Dummy Variable	s			
Ordinary	least squares regre	ssio	n		
LHS=C	Mean	=	11.4	46039	
	Standard deviation	=	1.2	17411	
	Number of observs.	=		2500	
Model siz	ze Parameters	=		10	
	Degrees of freedom	=		2490	
Residuals	s Sum of squares	=	156.3	19259	
	Standard error of e	=		25046	
Fit	R-squared	=		95466	
Adjusted R-squared		=		95450	
Model tes	st F[ 9, 2490] (prob	) =	5825.4(.)	(000	
Diagnosti	lc Log likelihood	=	-81.	15108	
-	Restricted(b=0)	=	-3948.	12242	
	Chi-sq [ 9] (prob	) =	7733.9(.)	(000	
Panel Dat	a Analysis of C		[ONE	way]	
	Unconditional ANC	VA (	No regres	sors)	
Source	Variation Deg. F	ree.	Mean So	quare	
Between	668.14287	499.	1.	33896	
Residual	2776.81463 2	000.	1.	38841	
Total	3444.95749 2	499.	1.	37853	
+	+				
	Stand	ard		Prob.	Mean
C	Coefficient Err	or	Z	z> Z	of X
+ w1	40201+++ 01	 771			
M T I	.42321	000	23.90	.0000	1 00057
∠w w2	.03050**** .00	100	4.52	.0000	1.00257
W.3   W.4	10602*** 01	17/	11.00	.0000	- 69155
01	.10002 .01	1/4 7/5	12 07	.0000	00155 0 E0762
	.10335*** .00	745	13.07	.0000	0.00/03
Q2	.37493**** .00	709	10 01	.0000	10.0932
Q3	.09658*** .00	400	10.01	.0000	9.71949
Q4	.05624*** .00	400	14.05	.0000	7.78290
Congtant	56264*** 10	211	49.51 4 57	.0000	1.13/10
constant	.50304^^^ .12	544	4.5/	.0000	
Note: ***	*, **, * ==> Significan	 ce a	 t 1%, 5%,	10% le	vel.
	-				

Least Sou	uares	with Group	Dumm	v Variab	les			
Ordinary		least squar	es re	aression				
LUS-C		Mean	00 10	-	11	16039		
5-0110		Ctandard do		- -	1 .	17/11		
		Stalluaru ue	VIALI	.011 =	1.1	2500		
		Number of o	bserv	rs. =		2500		
Model siz	ze	Parameters		=		509		
		Degrees of	freed	lom =		1991		
Residuals	5	Sum of squa	res	=	120.8	39104		
		Standard er	ror o	ofe =		24641		
Fit		R-squared		=		96491		
110		Adjusted P-	causr	- be		25505		
Model to	~ <del>+</del>		11 /m	rah) -	107 0/ /	2000		
Model Les	51	F[500, 199	1) (þ	= (d010	107.0(.0	0000)		
Diagnost	lC	Log likelin	ooa	=	239.0	19913		
		Restricted(	b=0)	=	-3948.1	L2242		
		Chi-sq [508	] (p	rob) =	8374.4(.(	)000)		
Estd. Aut	tocor	relation of	e(i,	t) =	23	33637		
Panel:Gro	oups	Empty	Ο,	Valid	data	500		
	-	Smallest	5.	Larges	t.	5		
		Average gro	un ei	ze in na	nel	5 00		
		Average gro	up si	ze in pa	lici	5.00		
	 I		 0+	andard		Dreb	Moon	
a		EE1	SL			PLOD.	Mean	
C	Coe	IIIClent		Error	Z	z> 2	OI X	
	+							
Wl		.40903***		.01951	20.97	.0000	6.73864	
W2		.04421***		.00891	4.96	.0000	1.88257	
W3		.18063***		.01652	10.93	.0000	23288	
W4	i	.11294***		.01289	8.77	.0000	68155	
01	i	10693***		00820	13.04	0000	8.58763	
02	i	27667***		00785	47 99	0000	10 0932	
Q2	1	10027+++		.00705		.0000	10.0932	
Q3		.1003/***		.01070	9.38	.0000	9.71949	
Q4		.05536***		.00440	12.57	.0000	7.78290	
Q5		.27849***		.01086	25.64	.0000	7.13716	
	+							
Note: ***	*, **	, * ==> Si	gnifi	cance at	1%, 5%,	10% lev	vel.	
+								+
		Test Stati	stics	for the	Classica	al Model	1	
+								+
1	Model		Log-	Likeliho	od Sur	n of Squ	lares R-square	d
(1) Cor	nstan	t term only	_	3948.122	38	3444.9	95749 .0000	0 İ
(2) Gro	oup e	ffects only	_	3678.613	49	2776.8	31463	5 İ
(3) X -	- var	iables only		-81 151	04	156	19259 9546	6
(J) X	nd a	rour offort	~	220 000	16	120.0	20104 0640	1
(4) A C	anu g	roup errect	5	239.099	10	120.0	.9049	±
+								+
	T 21-	-141 D	нуро	chesis i	ests			
	LIK	erinooa Rat	то ле	st .	F1.6	SUS		ļ
	Chi	-squared	a.t.	Prob	F	num	denom P valu	e
(2) vs	(1)	539.02	499	.1042	.96	499	2000 .7082	1
(3) vs	(1)	7733.94	9	.0000	5825.45	9	2490 .0000	0
(4) vs	(1)	8374.44	508	.0000	107.77	508	1991 .0000	οİ
(4) vs	(2)	7835.43	9	.0000	4860.16	9	1991 .0000	οĺ
(4) ve	(3)	640 50	499	0000	1 17	499	1991 0062	5
1 ( ± ) VB	()/				· · · /			

Estimates	Var[e]	=	.0607 0020	19	
	Corr[v(i,t).v	(i.s)] =	.0020	30	
Lagrange	Multiplier Test	vs. Model (3	) = 4.	93	
( 1 degre	es of freedom, p	rob. value =	.02643	6)	
(High val	ues of LM favor I	FEM/REM over	CR mode	1)	
Fixed vs.	Random Effects	(Hausman)	= 10.	17	
( 9 degre	es of freedom, p	rob. value =	.33705	1)	
(High (lc	ow) values of H fa	avor F.E.(R.	E.) mode	1)	
	Sum of Squares	5	156.1949	72	
+	R-squared		.9546		
		Standard		Prob.	Mean
C	Coefficient	Error	Z	z> Z	of X
+ w1	// 21 57 * * *	01769	22 05	0000	6 72961
W1  W2	.42157	00807	4 64	0000	1 88257
W2	.17802***	.01495	11.90	.0000	- 23288
W4	.10687***	.01172	9.12	.0000	68155
01	.10376***	.00744	13.95	.0000	8.58763
Q2	.37513***	.00708	52.98	.0000	10.0932
Q3	.09700***	.00964	10.07	.0000	9.71949
Q4	.05614***	.00400	14.05	.0000	7.78290
Q5	.28516***	.00969	29.42	.0000	7.13716
Constant	.57111***	.12316	4.64	.0000	
+ Note: ***	, **, * ==> Sign	nificance at	1%, 5%,	10% lev	rel.
Random EI	IECTS MODEL: V(1	,t) = e(1	,t) + u(	1)	
				<b>•</b> •	
Estimates	: Var[e]	=	.0609	94	
Estimates	Var[e] Var[u]	= =	.0609	94 79	
Estimates	: Var[e] Var[u] Corr[v(i,t),v	= = (i,s)] =	.0609 .0016 .0267	94 79 94	
Estimates Lagrange	: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v	= = (i,s)] = vs. Model (3 rob. value =	.0609 .0016 .0267 ) = 3.	94 79 94 97	
Estimates Lagrange ( 1 degre (High val	: Var[e] Var[u] Corr[v(i,t),v Multiplier Test ees of freedom, pr ues of LM favor 1	= = (i,s)] = vs. Model (3 rob. value = FEM/REM over	.0609 .0016 .0267 ) = 3. .04640 CR mode	94 79 94 97 3)	
Estimates Lagrange ( 1 degre (High val	: Var[e] Var[u] Corr[v(i,t),v Multiplier Test es of freedom, pr ues of LM favor 1	= (i,s)] = vs. Model (3 rob. value = FEM/REM over	.0609 .0016 .0267 ) = 3. .04640 CR mode	94 79 94 97 3) 1)	
Estimates Lagrange ( 1 degre (High val + C	: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr uses of LM favor 1  Coefficient	= = (i,s)] = vs. Model (3 rob. value = FEM/REM over  Standard Error	.0609 .0016 .0267 ) = 3. .04640 CR mode	94 79 94 97 3) 1)  Prob. z> Z	Mean of X
Estimates Lagrange ( 1 degre (High val + C   +	: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 Coefficient	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error	.0609 .0016 .0267 ) = 3. .04640 CR mode 	94 79 94 97 3) 1) Prob. z> Z	Mean of X
Estimates Lagrange ( 1 degre (High val + C   C   + W1B	: Var[e] Var[u] Corr[v(i,t),v Multiplier Test ees of freedom, pr ues of LM favor 1 Coefficient .06958	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02112	.0609 .0267 ) = 3. .04640 CR mode  z 	94 79 94 97 3) 1) Prob. z> z  .1337	Mean of X 6.73864
Estimates Lagrange ( 1 degre (High val + C   C   + W1B W2B W2D	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113	.0609 .0267 ) = 3. .04640 CR mode  z  1.50 -1.68	94 79 94 97 3) 1) Prob. z> Z  .1337 .0927	Mean of X 6.73864 1.88257 22000
Estimates Lagrange ( 1 degre (High val + C   C   + W1B W2B W3B W3P	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr uses of LM favor 1 	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 02102	.0609 .0267 ) = 3. .04640 CR mode  1.50 -1.68 29	94 79 94 97 3) 1) Prob. z> Z  .1337 .0927 .7721 1752	Mean of X 6.73864 1.88257 23288 69155
Estimates Lagrange ( 1 degre (High val + C   C   W1B W2B W3B W4B W4B	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr uses of LM favor 1 	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 01932	.0609 .0267 ) = 3. .04640 CR mode  z  1.50 -1.68 29 -1.36 36	94 79 94 97 3) 1) Prob. z> Z  .1337 .0927 .7721 .1753 3912	Mean of X 6.73864 1.88257 23288 68155 8.58763
Estimates Lagrange ( 1 degre (High val + C   C   W1B W2B W3B W4B Q1B 02P	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr uses of LM favor 1 	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01932	.0609 .0267 ) = 3. .04640 CR mode  1.50 -1.68 29 -1.36 86 86 46	94 79 94 97 3) 1) Prob. z> z  .1337 .0927 .7721 .1753 .3912 .6435	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932
Estimates Lagrange ( 1 degre (High val + C   C   + W1B  W2B  W3B  W4B  Q1B  Q2B  O3B	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01821 .02463	.0609 .0267 ) = 3. .04640 CR mode CR mode  1.50 -1.68 29 -1.36 86 46 62	94 79 94 97 3) 1) Prob. z> Z  .1337 .0927 .7721 .1753 .3912 .6435 .5362	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949
Estimates Lagrange ( 1 degre (High val + C   C   W1B  W2B  W3B  W4B  Q1B  Q2B  Q3B  Q4B  04B	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01932 .02463 .01042	.0609 .0267 ) = 3. .04640 CR mode CR mode 	94 79 94 97 3) 1) Prob. z> Z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290
Estimates Lagrange ( 1 degre (High val + C   C   W1B  W2B  W3B  W4B  Q1B  Q2B  Q3B  Q4B  Q5B	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01821 .02463 .01042 .02409	.0609 .0267 ) = 3. .04640 CR mode  2 	94 79 94 97 3) 1) Prob. z> Z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716
Estimates Lagrange ( 1 degre (High val + C   C   W1B  W2B  W3B  W4B  Q1B  Q2B  Q3B  Q4B  Q4B  Q5B  W1	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01821 .02463 .01042 .02409 .01955	.0609 .0267 ) = 3. .04640 CR mode  z  1.50 -1.68 29 -1.36 86 46 62 .42 1.55 20.92	94 79 94 97 3) 1) Prob. z> z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .0000	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864
Estimates Lagrange ( 1 degre (High val + C   C   W1B  W2B  W3B  W4B  Q1B  Q2B  Q3B  Q4B  Q4B  Q5B  W1  W2	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01821 .02463 .01042 .02409 .01955 .00893	.0609 .0267 ) = 3. .04640 CR mode  z  1.50 -1.68 29 -1.36 46 62 .42 1.55 20.92 4.95	94 79 94 97 3) 1) Prob. z> Z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .0000 .0000	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864 1.88257
Estimates Lagrange ( 1 degre (High val + C   C   W1B  W2B  W3B  W4B  Q1B  Q2B  Q3B  Q4B  Q4B  Q5B  W1  W2  W3	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01821 .02463 .01042 .02409 .01955 .00893 .01656	.0609 .0267 ) = 3. .04640 CR mode  1.50 -1.68 29 -1.36 86 46 62 .42 1.55 20.92 4.95 10.91	94 79 97 3) 1) Prob. z >  Z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .0000 .0000 .0000	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864 1.88257 23288
Estimates ( 1 degre (High val + C   C   W1B  W2B  W4B  Q1B  Q2B  Q3B  Q4B  Q5B  W1  W2  W3  W4	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01821 .02463 .01042 .02409 .01955 .00893 .01656 .01291	.0609 .0267 ) = 3. .04640 CR mode CR mode CR mode  1.50 -1.68 68 46 62 .42 1.55 20.92 4.95 10.91 8.75	94 79 97 3) 1) Prob. z >  Z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .0000 .0000 .0000 .0000	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864 1.88257 23288 68155
Estimates ( 1 degre ( High val + C   C   W1B  W2B  W3B  W4B  Q1B  Q2B  Q3B  Q4B  Q2B  Q3B  Q4B  Q5B  W1  W2  W3  W4  Q2	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test y ees of freedom, pr ues of LM favor 1 .ues of	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01932 .01821 .02463 .01042 .02409 .01955 .00893 .01656 .01291 .00822	.0609 .0267 ) = 3. .04640 CR mode CR mode  1.50 -1.68 29 -1.36 46 62 .42 1.55 20.92 4.95 10.91 8.75 13.01	94 79 94 97 3) 1) Prob. z> z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .0000 .0000 .0000 .0000 .0000	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864 1.88257 23288 68155 8.58763
Estimates Lagrange ( 1 degre (High val + W1B W2B W3B W4B Q1B Q2B Q3B Q4B Q2B Q3B Q4B Q5B W1 W2 W3 W4 Q1 W2 W1 W2 Q2 W1 W3 W4 W1 W2 W3 W4 W2 W1 W4 W3 W4 W2 W3 W4 W3 W4 W3 W4 W4 W3 W4 W3 W4 W3 W4 W3 W4 W3 W4 W3 W4 W4 W3 W4 W4 W4 W4 W4 W4 W4 W4 W4 W4	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test y ees of freedom, pr ues of LM favor 1 .ues of LM favor 1 .ues of LM favor 1 .06958 03553* 01122 04204 01657 00843 01524 .00441 .03734 .40903*** .11294*** .11294*** .10693***	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01932 .01821 .02463 .01042 .02409 .01955 .00893 .01656 .01291 .00822 .00787	.0609 .0267 ) = 3. .04640 CR mode  1.50 -1.68 29 -1.36 46 62 .42 1.55 20.92 4.95 10.91 8.75 13.01 47.88	94 79 97 3) 1) Prob. z> z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .0000 .0000 .0000 .0000 .0000 .0000	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864 1.88257 23288 68155 8.58763 10.0932
Estimates ( 1 degre ( High val + C   C   W1B  W2B  W2B  W2B  W2B  Q2B  Q3B  Q4B  Q2B  Q3B  Q4B  Q5B  W1  W2  W1  W2  Q2  Q3  W4  W2  W2  W3  W4  W2  W3  W4  W2  W3  W4  W2  W3  W4  W4  W3  W4  W4  W4  W3  W4  W4  W4  W4  W4  W4  W4  W4	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 .ues of LM favor 1 .ues of LM favor 1 .06958 03553* 01122 04204 01657 00843 01524 .00441 .03734 .40903*** .18063*** .11294*** .10693*** .37667*** .0037***	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01932 .01821 .02463 .01042 .02409 .01955 .00893 .01656 .01291 .00822 .00787 .01073	.0609 .0267 ) = 3. .04640 CR mode CR mode  1.50 -1.68 29 -1.36 46 62 .42 1.55 20.92 4.95 10.91 8.75 13.01 47.88 9.36	94 79 97 3) 1) Prob. z> z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.7949
Estimates Lagrange ( 1 degre (High val + C   C   W1B  W2B  W4B  Q1B  Q2B  Q3B  Q4B  Q5B  W1  W22  Q3B  W4  W22  W3  W4  W22  W3  W4  W22  W3  W4  W22  W3  W4  W22  W3  W4  W22  W3  W4  W3  W4  W3  W4  W3  W4  W3  W4  W3  W4  W3  W4  W3  W4  W3  W4  W3  W3  W4  W3  W3  W3  W4  W3  W3  W4  W3  W3  W3  W4  W3  W3  W4  W3  W4  W3  W3  W3  W4  W3  W3  W4  W3  W3  W3  W4  W3  W3  W3  W3  W3  W3  W3  W3	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 .ues	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01821 .02463 .01042 .02463 .01042 .02409 .01955 .00893 .01656 .01291 .00822 .00787 .01073 .00441	.0609 .0267 ) = 3. .04640 CR mode  1.50 -1.68 29 -1.36 46 62 .42 1.55 20.92 4.95 10.91 8.75 13.01 47.88 9.36 12.55	94 79 97 3) 1) Prob. z> z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.178290
Estimates ( 1 degre ( High val + C   C   C   W1B  W2B  W2B  W2B  W2B  Q2B  Q3B  Q4B  Q2B  Q3B  Q4B  Q5  Constant	:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 .ues	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01932 .01821 .02463 .01042 .02409 .01955 .00893 .01656 .01291 .00822 .00787 .01073 .00441 .01089 .29913	.0609 .0267 ) = 3. .04640 CR mode  1.50 -1.68 29 -1.36 46 62 .42 1.55 20.92 4.95 10.91 8.75 13.01 47.88 9.36 12.55 25.58	94 79 97 3) 1) Prob. z> z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .3718	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716
Estimates ( 1 degre ( 1 degre ( High val + C   C   W1B  W2B  W4B  Q1B  Q2B  Q3B  Q4B  Q5B  W1  W2  W3  W4  Q1  Q2  Q3  W4  Q2  Q3  Q4  Q2  Q3  Q4  Q2  Q3  Q4  Q4  Q2  Q3  Q4  Q4  Q5  Constant  +	<pre>:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 </pre>	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01932 .01821 .02463 .01042 .02409 .01955 .00893 .01656 .01291 .00822 .00787 .01073 .0073 .01073 .00441 .01089 .29913	.0609 .0267 ) = 3. .04640 CR mode  1.50 -1.68 46 62 .42 1.55 20.92 4.95 10.91 8.75 13.01 47.88 9.36 12.55 25.58 .89	94 79 97 3) 1) Prob. z >  Z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .00000 .00000 .00000 .00000 .000000	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716
Estimates Lagrange ( 1 degre (High val 	<pre>:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr uses of LM favor 1 .ues of LM favor</pre>	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01932 .01932 .01942 .02463 .01042 .02409 .01955 .00893 .01656 .01291 .00822 .00787 .01073 .00441 .01089 .29913 .016120	.0609 .016 .0267 ) = 3. .04640 CR mode CR mode 	94 79 94 97 3) 1) Prob. z> Z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .00000 .00000 .00000 .000000	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 rel.
Estimates Lagrange ( 1 degre (High val 	<pre>:: Var[e] Var[u] Corr[v(i,t),v Multiplier Test v ees of freedom, pr ues of LM favor 1 .ues of LM favor</pre>	= (i,s)] = vs. Model (3 rob. value = FEM/REM over Standard Error .04640 .02113 .03875 .03102 .01932 .01932 .01932 .01925 .00893 .01656 .01291 .00822 .00787 .01073 .00421 .01089 .29913 .01656.at .01291 .0089 .29913 .01656.at .01089 .01089 .0108 .01089 .0108 .01089 .0108	.0609 .016 .0267 ) = 3. .04640 CR mode CR mode 	94 79 94 97 3) 1) Prob. z> Z  .1337 .0927 .7721 .1753 .3912 .6435 .5362 .6724 .1212 .00000 .00000 .00000 .00000 .00000 .00000 .000000	Mean of X 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 6.73864 1.88257 23288 68155 8.58763 10.0932 9.71949 7.78290 7.13716 

1| 10.32455

7. The data for this exercise are parked on the web on my home page in 4 formats:

http://people.stern.nyu.edu/wgreene/labor.lpj (limdep or nlogit project file) http://people.stern.nyu.edu/wgreene/labor.dta (stata data file) http://people.stern.nyu.edu/wgreene/labor.csv (Excel comma separated values) http://people.stern.nyu.edu/wgreene/labor.txt (Text format. (Same as csv))

All three files contain 753 observations on 5 variables relating to a study of labor market behavior of married women.

LFP = the dependent variable, labor force participation coded 0 and 1 AGE = age WE=wife's education in years FAMINC=family income KIDS = dummy variable for whether there are kids in the home.

The raw data look like these, which are the first few observations.

LFP,AGE,WE,FAMINC,KIDS

1.00000	32.0000	12.0000	16310.0	1.00000
1.00000	30.0000	12.0000	21800.0	1.00000
1.00000	35.0000	12.0000	21040.0	1.00000
1.00000	34.0000	12.0000	7300.00	1.00000
1.00000	31.0000	14.0000	27300.0	1.00000
1.00000	54.0000	12.0000	19495.0	.000000
1.00000	37.0000	16.0000	21152.0	1.00000
1.00000	54.0000	12.0000	18900.0	.000000

You will need a statistical package to do this part of the exam. The .csv file can be read directly into Excel without conversion. The .lpj file is an nlogit or limdep project file. The .txt and .dta files are suitable for export to Stata. The .csv or .txt file can be exported to R.

- a. Your assignment is to estimate a binary choice model using these data. Your model should explain LFP using age, education and family income. (You may fit a probit model or a logit model your choice. Indicate in your report which form you used.) As part of your analysis, compute the partial effect on the probability of participation in the formal labor market of an additional year of education and of an additional thousand dollars in family income.. Report your result, and interpret it. Use least squares to fit a "linear probability model," and compare your results to your maximum likelihood based results for the probit or logit model.
- b. One might think that the presence of children in the household would completely change the labor force participation decision. Split the sample based on the KIDS dummy variable, and compute the two probit (or logit) models for the subsamples. Carry out a likelihood ratio test of the hypothesis that pooling is valid versus the alternative that separate models apply to the two subsamples.

8. I propose to estimate the model

E[Income|Age,Education,Health] =  $\exp(\beta_1 + \beta_2 Age + \beta_3 Educ + \beta_4 Health)$ 

However, I have been convinced by my colleagues that in a model of income determination, Health would be endogenous. So, added to the complication that my model is nonlinear is the complication that one of the right hand side variables is endogenous.

a. Explain what is meant by 'endogenous' in this discussion. Why would endogeneity be a problem?

b. I have data on income (hhninc = household income), Age, Education, and HSAT = health satisfaction. I will take HSAT, the individuals assessment of their health, as a reliable proxy for their actual health. At least for purposes of this exercise, I will take Age and Educ to be exogenous. I also have a set of instrumental variables, Z = (married, hhkids, working, female). (All are dummy variables, in fact.) Two sets of results are given below. This first are nonlinear least squares estimates that ignore the endogeneity question. The second are nonlinear instrumental variables estimates.

(i) Explain how the nonlinear least squares estimates are computed.

(ii) Explain how the nonlinear instrumental variables are computed.

(iii) I might have improved my estimator by using GMM. Explain how I would use GMM to estimate the parameters of this model.

User Defi	ned Optimization.						
Nonlinear	least squares	regression					
LHS=HHNIN	C Mean	=		35214			
	Standard devi	ation =		17687			
	Number of obs	ervs. =		27322			
Model siz	e Parameters	=		4			
	Degrees of fr	reedom =		27318			
Residuals	Sum of square	s =	79	2.505			
	Standard erro	or of e =		17031			
Fit	R-squared	=	•	07275			
+		Standard		Prob.	95% Co	nfidence	
UserFunc	Coefficient	Error	z	z >Z*	Int	erval	
Constant	-1.82573***	.02135	-85.50	.0000	-1.86759	-1.78388	
Age	.00260***	.00027	9.72	.0000	.00208	.00312	
Educ	.05108***	.00106	48.26	.0000	.04900	.05315	
HSAT	.01219***	.00135	9.05	.0000	.00955	.01483	
Instrumen	tal Variables (NL	IV)					
Nonlinear	least squares	regression					
Residuals	Sum of square	s =	21	44.34			
	Standard erro	or of e =		28015			
Fit	R-squared	=	-1.	50894			
+		Standard		Prob.	95% Co	nfidence	
UserFunc	Coefficient	Error	z	z >Z*	Int	erval	
Constant	-4.61690***	.70523	-6.55	.0000	-5.99913	-3.23467	
Age	.01437***	.00225	6.38	.0000	.00995	.01878	
Educ	.03105***	.00336	9.25	.0000	.02447	.03763	
HSAT	.34524***	.07902	4.37	.0000	.19037	.50011	
+							

## 9. Maximum Likelihood Estimation of a Loglinear Model



In analyzing skewed income data such as those shown in the histogram below,

it is customary to analyze logs of income with conventional regression methods. Suppose, in an attempt to impress my colleagues with my facility with 'loglinear models,' I propose, instead to analyze *Income*, not log*Income*, in the context of a *gamma regression model*. That is a model in which the conditional density for *Income* is

$$f(Income_i | \mathbf{x}_i) = \frac{\lambda_i^P Income_i^{P-1} \exp(-\lambda_i Income_i)}{\Gamma(P)}, \ \lambda_i = \exp(\boldsymbol{\beta}' \mathbf{x}_i), \ Income_i \ge 0, \ P > 0.$$

The model is 'loglinear' in that  $E[Income_i | \mathbf{x}_i] = P/\lambda_i$ , so that the log of the mean is

$$\log E[Income_i | \mathbf{x}_i] = \log P - \log \lambda_i = \alpha - \beta' \mathbf{x}_i.$$

It will be assumed that  $\mathbf{x}_i$  contains a constant term as well as covariates such as *age*, *education* and *gender*, so that the log of the mean is  $\delta' \mathbf{x}_i$  where the element in  $\delta$  that corresponds to a constant is  $\gamma = \log P - \beta_0$  and the other elements are  $-\beta_k$ . (Note the sign change.) The parameters to be estimated are P and the elements of  $\boldsymbol{\beta}$ . (P is known as the 'shape parameter.' If P is less than or equal to 1, then the distribution looks like the exponential while if it is greater than one, it looks like chi squared.)

a. Derive the log likelihood function for maximum likelihood estimation of P and  $\beta$ . (Note, the log likelihood involves the function log $\Gamma(P)$ . You can just leave it in this form.

b. Obtain the likelihood equations for estimation of P and  $\beta$ . (Hint: Use the chain rule. Obtain the derivative with respect to  $\lambda_i$ . Then, the derivative of  $\lambda_i$  with respect to  $\beta$  is  $\lambda_i \mathbf{x}_i$ .)

c. Use the likelihood equations to show that  $E[Income_i | \mathbf{x}_i] = P/\lambda_i$  and  $E[logIncome_i | \mathbf{x}_i] = \Psi(P) - log\lambda_i$ where  $\Psi(P)$  (which is called the 'psi function' or the 'digamma function') is  $dlog\Gamma(P)/dP$ .

d. Contining to manipulate the first order conditions, show that the solution for P is

 $P = (1/n)\Sigma_i \lambda_i$  Income<sub>i</sub>.

Insert this solution for P into the log likelihood function to obtain the *concentrated log likelihood* which is only a function of the data and the unknown  $\beta$ . (Note that if  $\lambda_i$  were a constant, the solution would be  $P/\lambda = \overline{Income}$ , which makes sense.

e. Derive an estimator for the asymptotic covariance matrix of the MLE of  $(P,\beta)$ . Hint: this will involve  $d^2\log\Gamma(P)/dP^2 = \Psi'(P)$ . This is called the 'trigamma function.' Just leave the function in this form.

Several sets of results are given below, where the estimated models are based on the German health care data that we have discussed in class. The dependent variable is hhninc = household income. The first two are maximum likelihood estimates of the gamma loglinear model. Use the results provided to answer the following questions:

f. The first set of results provides unrestricted estimates of the gamma model using the full sample (less the four observations for which hhninc = 0). Using these results, test the hypothesis that AGE is not a significant determinant of income.

g. Your colleague who is skeptical of nonlinear models to begin with points out that in your first set of results, the reported value for the 'Pseudo- $R^2$ ' is -10.9599753. There is obviously something drastically wrong here – proportions of variance explained are between 0 and 1. On this basis, they dismiss your nonlinear model as obviously wrong (and implore you to use ordinary least squares). How would you answer this criticism?

h. Using the first set of results, test the hypothesis that all five slope coefficients in the model are jointly equal to zero.

i. Using the first set of results, test the hypothesis of the exponential model as a restriction on the gamma model. The restriction is P = 1. The third set of results is the maximum likelihood estimates of the exponential model – that is, the restriction P = 1 is imposed. Using both the first and third set of results (and a different type of test from that used in part g), test the hypothesis of the exponential model.

j. Show that the set of partial effects in this gamma regression model are

$$\partial E[Income|x]/\partial x = -E[Income|x] \times \beta$$

That is, the slopes of the mean are equal to the negative of beta times the mean.

k. Means of the variables in the model are given at the beginning of the results. Using the estimated parameters, compute the partial effects for AGE, EDUC and MARRIED at the means of the data. (Hint: MARRIED is a dummy variable.)

1. Show how you would compute standard errors for the partial effects in part j. (You don't actually have to do the computations. Just show precisely how it would be done.)

m. The second set of results below adds a quadratic term in AGE to the gamma model. I am interested in the age profile of incomes. At what age does Income reach its maximum? (Hint: the log function is a monotonic function of Income, so you can answer this by finding the AGE at which the log of expected Income reaches its maximum. The expression given earlier for log  $E[Income_i | \mathbf{x}_i]$  will be extremely useful. Now that you have found AGE\*, the AGE at which income is maximized, use the delta method to compute an asymptotic standard error for your estimator of AGE\*.

n. The final set of results given below shows the linear regression of Income on the constant and the same variables used in the first model. The coefficient estimates in regression 5 are completely different from those in regression 1 - in fact, the signs are all opposite and the magnitudes are different. Your critical colleagues is by now really upset – something is obviously drastically wrong. OLS is always robust, and your coefficients all have the wrong signs! Can you suggest what might explain this semingly contradictory finding?

Descriptive	Statistics	for	5	variables
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Variable	Mean	Std.Dev.	Minimum	Maximum	Cases	Missing
AGE	43.52719	11.33032	25.0	64.0	27322	0
Age*Age	2022.988	1004.087	625.0	4096.0	27322	0
EDUC	11.32018	2.324347	7.0	18.0	27322	0
MARRIED	.758693	.427884	0.0	1.0	27322	0
FEMALE	.478808	.499560	0.0	1.0	27322	0
HHKIDS	.402716	.490453	0.0	1.0	27322	0

1. Gamma (Loglinear) Regression Model Dependent variable HHNINC Log likelihood function 14293.00214 Restricted log likelihood( $\beta$ =0)1195.06953 Chi squared [ 6 d.f.] 26195.86522 Significance level .00000 Significance level.00000McFadden Pseudo R-squared-10.9599753 Estimation based on N = 27322, K = 7 Inf.Cr.AIC = -28572.0 AIC/N = -1.046

HHNINC	Coefficient	Standard Error	Z	Prob. $ z  > Z^*$	95% Confidence Interval		
	Parameters in cond	itional me	an funct	ion			. –
Constant	3.40841***	.02154	158.21	.0000	3.36618	3.45063	
AGE	.00205***	.00028	7.41	.0000	.00151	.00260	
EDUC	05572***	.00120	-46.50	.0000	05807	05337	
MARRIED	26341***	.00692	-38.04	.0000	27698	24984	
FEMALE	00542	.00545	99	.3198	01611	.00526	
HHKIDS	.06512***	.00618	10.54	.0000	.05302	.07723	
	Scale parameter fo	r gamma mo	del				
P_shape	5.12486***	.04250	120.59	.0000	5.04157	5.20815	
Note: ***, **, * ==> Significance at 1%, 5%, 10% level.							

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🖽 Matri	III Matrix - VARB								
[7, 7]	Cell:								
	1	2	3	4	5	6	7		
1	0.000464108	-3.46813e-006	-1.8497e-005	-4.27673e-006	-2.39817e-005	-3.26011e-005	0.000352398		
2	-3.46813e-006	7.68273e-008	3.42455e-008	-6.66724e-007	-7.38979e-008	6.89342e-007	-4.19768e-013		
3	-1.8497e-005	3.42455e-008	1.43625e-006	2.8321e-007	1.15748e-006	-5.28885e-008	-3.62485e-012		
4	-4.27673e-006	-6.66724e-007	2.8321e-007	4.79481e-005	9.71058e-008	-1.57257e-005	1.76474e-011		
5	-2.39817e-005	-7.38979e-008	1.15748e-006	9.71058e-008	2.97137e-005	-5.10296e-007	1.09561e-011		
6	-3.26011e-005	6.89342e-007	-5.28885e-008	-1.57257e-005	-5.10296e-007	3.81658e-005	1.90277e-011		
7	0.000352398	-4.19768e-013	-3.62485e-012	1.76474e-011	1.09561e-011	1.90277e-011	0.00180599		

2. Gamma	a (Loglinear) Reg	ression Mode	el			
Dependen	t variable	HHN	INC			
Log like	lihood function	14709.584	448			
Restrict	ed log likelihood	l 1195.069	953			
Chi squa:	red [ 7 d.f.]	27029.029	989			
Significa	ance level	.000	000			
McFadden	Pseudo R-squared	l -11.3085	595			
Estimati	on based on $N =$	27322, K =	8			
Inf.Cr.A	IC = -29403.2 AI	C/N = -1.0	076			
	+ 	Standard		Prob.	95% Cot	fidence
HHNINC	Coefficient	Error	z	$ z  > Z^*$	Inte	erval
	, +					
	Parameters in co	nditional me	ean funct	ion		
Constant	4.59487***	.04470	102.79	.0000	4.50726	4.68249
AGE	05827***	.00208	-28.04	.0000	06234	05419
	1		~~ ~~			
AGE ^ AGE	.00069***	.2360D-04	29.29	.0000	.00065	.00074
AGE AGE EDUC	.00069*** 05323***	.2360D-04 .00119	29.29 -44.92	.0000 .0000	.00065 05556	.00074 05091
AGE*AGE EDUC MARRIED	.00069*** 05323*** 22994***	.2360D-04 .00119 .00694	29.29 -44.92 -33.13	.0000 .0000 .0000	.00065 05556 24354	.00074 05091 21634
AGE*AGE EDUC MARRIED FEMALE	.00069***  05323***  22994***  00068	.2360D-04 .00119 .00694 .00538	29.29 -44.92 -33.13 13	.0000 .0000 .0000 .8993	.00065 05556 24354 01122	.00074 05091 21634 .00986
AGE * AGE EDUC MARRIED FEMALE HHKIDS	.00069*** 05323*** 22994*** 00068 .10563***	.2360D-04 .00119 .00694 .00538 .00627	29.29 -44.92 -33.13 13 16.85	.0000 .0000 .0000 .8993 .0000	.00065 05556 24354 01122 .09334	.00074 05091 21634 .00986 .11792
AGE AGE EDUC MARRIED FEMALE HHKIDS	.00069*** 05323*** 22994*** 00068 .10563*** Scale parameter	.2360D-04 .00119 .00694 .00538 .00627 for gamma mo	29.29 -44.92 -33.13 13 16.85 odel	.0000 .0000 .0000 .8993 .0000	.00065 05556 24354 01122 .09334	.00074 05091 21634 .00986 .11792

Note: \*\*\*, \*\*, \* ==> Significance at 1%, 5%, 10% level. \_\_\_\_\_ \_\_\_\_\_ \_\_\_\_\_

Ш М	atrix -	VARB							x
[7, 7]		Cell:							
		1	2	3	4	5	6	7	
1		0.00199829	-8.44201e-005	9.30409e-007	-1.44653e-005	4.20678e-005	-1.70356e-005	2.45288e-005	$\square$
2	-{	8.44201e-005	4.31772e-006	-4.86415e-008	-1.49224e-007	-3.064e-006	-3.97192e-007	-2.26723e-006	
3		9.30409e-007	-4.86415e-008	5.57059e-010	2.08149e-009	2.77308e-008	3.70895e-009	3.35675e-008	$\sim$
4	-1	1.44653e-005	-1.49224e-007	2.08149e-009	1.40424e-006	3.57818e-007	1.13815e-006	9.20511e-008	$\square$
5	4	4.20678e-005	-3.064e-006	2.77308e-008	3.57818e-007	4.81601e-005	3.25504e-007	-1.3768e-005	
6	-1	1.70356e-005	-3.97192e-007	3.70895e-009	1.13815e-006	3.25504e-007	2.89066e-005	-3.74216e-007	
7	2	2.45288e-005	-2.26723e-006	3.35675e-008	9.20511e-008	-1.3768e-005	-3.74216e-007	3.93173e-005	

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3. Expon	ential (Loglinear)	Regressic	n Model			
Dependent	variable	HHNI	NC			
Log likel	ihood function	1550.075	36			
Restricte	d log likelihood	1195.069	53			
Chi squar	ed [ 5 d.f.]	710.011	.66			
Significa	nce level	.000	000			
+		 Standard		Prob.	95% Cor	nfidence
HHNINC	Coefficient	Error	Z	z >Z*	Interval	
+	Parameters in cond	itional me	an funct	ion		
Constant	1.77430***	.04501	39.42	.0000	1.68608	1.86253
AGE	.00205***	.00063	3.27	.0011	.00082	.00328
EDUC	05572***	.00271	-20.54	.0000	06104	05040
MARRIED	26341***	.01568	-16.80	.0000	29413	23269
FEMALE	00542	.01234	44	.6603	02961	.01876
ннктрс	06512***	01399	4 66	0000	03771	09254

HHKIDS	.06512***	.01399	4.66	.0000	.03771	.09254	

4. Gamma	a (Loglinear) Regres	sion Mode	el			
Gamma (Lo	oglinear) Regression	Model				
Dependent	z variable	HHN	INC			
LM Stat.	at start values	9618.48	555			
LM statis	stic kept as scalar	LMSTA	C			
Log likel	Lihood function	1550.07	538			
Restricte	ed log likelihood	1195.069	953			
Chi squar	red [ 6 d.f.]	710.01	L69			
Significa	ance level	.000	000			
	 S	tandard		Prob.	95% Co	nfidence
HHNINC	Coefficient	Error	Z	z >Z*	Int	erval
	Parameters in condi	tional me	ean funct	 ion		
Constant	1.77430***	.04564	38.88	.0000	1,68485	1.86375
AGE	.00205***	.00063	3.27	.0011	.00082	.00328
EDUC	05572***	.00271	-20.54	.0000	06104	05040
MARRIED	26341***	.01568	-16.80	.0000	29413	23269
FEMALE	00542	.01234	44	.6603	02961	.01876
HHKIDS	.06512***	.01399	4.66	.0000	.03771	.09254
	Scale parameter for	gamma mo	odel			
P_shape	1.0***	.00753	132.74	.0000	.98523D+00	.10148D+01
	least squar	es regres	sion			
LHS=HHNIN	JC Mean	=		35214		
	Standard deviat	ion =		17687		
	No. of observat	ions =		27322	DegFreedom	Mean square
Regressio	on Sum of Squares	= =	92	.1956	5	18,43911
Residual	Sum of Squares	=	76	2.486	27316	.02791
Total	Sum of Squares	=	85	4.682	27321	.03128
	Standard error	ofe =	00	16707	Root MSE	16706

=

z

8.16

-2.90

47.75

33.43

2.41

-8.37

Standard

Error

\_\_\_\_

.00760

.00010

.00045

.00260

.00206

.00238

Fit

Model test

HHNINC

AGE

EDUC

\_\_\_\_+

Constant

MARRIED

FEMALE

HHKIDS

R-squared

Coefficient

F[ 5, 27316]

.06198\*\*\*

-.00030\*\*\*

.02144\*\*\*

.08705\*\*\*

.00497\*\*

-.01993\*\*\*

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10. This question involves a small amount of "library" research. (You can do it on the web, of course.) Locate an empirical (applied) paper (study) in any field (political science, economics, finance, management, accounting, pharmacology, environment, energy, urban economics, etc.) in which a model that involved an endogenous variable on the right hand side is estimated. (This should be easy to find – most of the contemporary applied literature deals with such situations.) Report (a) what empirical issue the study was about; (b) what the model was; (c) what estimation technique the author used; (d) (briefly) what results they obtained. In part (d), describe the actual statistics that the author reported, and what conclusion they drew. This entire essay should not exceed one double spaced page.

.10787 R-bar squared

660.57945 Prob F > F\*

Prob.

|z|>Z\*

.0000

.0037

.0000

.0000

.0160

.0000

.10771

.00000

.07687

-.00010

.02232

.09215

.00901

-.01527

95% Confidence

Interval

.04709

-.00050

.02056

.08194

.00093

-.02460