

ECONOMIC EFFICIENCY AND FRONTIER TECHNIQUES

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Abstract. Most of the literature related to the measurement of economic efficiency has based its analysis either on parametric or on non-parametric frontier methods. The choice of estimation method has been an issue of debate, with some researchers preferring the parametric and others the non-parametric approach. The aim of this paper is to provide a critical and detailed review of both core frontier methods. In our opinion, no approach is strictly preferable to any other. Moreover, a careful consideration of their main advantages and disadvantages, of the data set utilized, and of the intrinsic characteristics of the framework under analysis will help us in the correct implementation of these techniques. Recent developments in frontier techniques and economic efficiency measurement such as Bayesian techniques, bootstrapping, duality theory and the analysis of sampling asymptotic properties are also considered in this paper.

Keywords. Economic efficiency; Parametric Frontier Techniques; Non-parametric Frontier Techniques; Bootstrapping; Bayesian analysis; Multiple output models

1. Introduction

The measurement of economic efficiency has been intimately linked to the use of frontier functions. The modern literature in both fields begins with the same seminal paper, namely Farrell (1957). Michael J. Farrell, greatly influenced by Koopmans (1951)'s formal definition and Debreu (1951)'s measure of technical efficiency¹ introduced a method to decompose the overall efficiency of a production unit into its technical and allocative components. Farrell characterised the different ways in which a productive unit can be inefficient either by obtaining less than the maximum output available from a determined group of inputs (technically inefficient) or by not purchasing the best package of inputs given their prices and marginal productivities (allocatively inefficient).

The analysis of efficiency carried out by Farrell (1957) can be explained in terms of Figure 1.1,

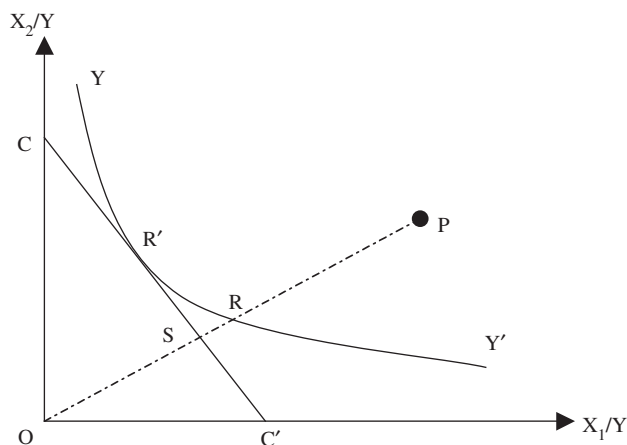


Figure 1.1. Technical and allocative efficiency measures.

Assuming constant returns to scale (*CRS*) as Farrell (1957) initially does in his paper, the technological set is fully described by the unit isoquant YY' that captures the minimum combination of inputs per unit of output needed to produce a unit of output. Thus, under this framework, every package of inputs along the unit isoquant is considered as technically efficient while any point above and to the right of it, such as point P , defines a technically inefficient producer since the input package that is being used is more than enough to produce a unit of output. Hence, the distance RP along the ray OP measures the technical inefficiency of producer located at point P . This distance represents the amount by which all inputs can be divided without decreasing the amount of output. Geometrically, the technical inefficiency level associated to package P can be expressed by the ratio RP/OP , and therefore; the technical efficiency (TE) of the producer under analysis ($1-RP/OP$) would be given by the ratio OR/OP .

If information on market prices is known and a particular behavioural objective such as cost minimization is assumed in such a way that the input price ratio is reflected by the slope of the isocost-line CC' , allocative inefficiency can also be derived from the unit isoquant plotted in Figure 1.1. In this case, the relevant distance is given by the line segment SR , which in relative terms would be the ratio SR/OR . With respect to the least cost combination of inputs given by point R' , the above ratio indicates the cost reduction that a producer would be able to reach if it moved from a technically but not allocatively efficient input package (R) to a both technically and allocatively efficient one (R'). Therefore, the allocative efficiency (AE) that characterises the producer at point P is given by the ratio OS/OR .

Together with the concepts of technical efficiency and allocative efficiency, Farrell (1957) describes a measure of what he termed overall efficiency and later

literature has renamed economic efficiency (EE). This measure comes from the multiplicative interaction of both technical and allocative components,

$$EE = TE \times AE = OR/OP \times OS/OR = OS/OP$$

where the distance involved in its definition (SP) can also be analyzed in terms of cost reduction.

Farrell's efficiency measures described in this section follow an input-oriented scheme. A detailed analysis of output-orientated efficiency measures can be found in Färe, Grosskopf and Lovell (1985&1994). Färe and Lovell (1978) point out that, under CRS, input-oriented and output-oriented measures of technical efficiency are equivalent. Such equivalence as Forsund and Hjalmarsson (1979) and Kopp (1981) state, ceases to apply in the presence of non-constant returns to scale. The analysis of allocative efficiency in an output-oriented problem is also treated in Färe, Grosskopf and Lovell (1985&1994) and Lovell (1993) from a revenue maximization perspective. Kumbhakar (1987), Färe, Grosskopf and Lovell (1994) and Färe, Grosskopf and Weber (1997) approach the analysis of allocative efficiency on the basis of profit maximization, where both cost minimization (input-oriented model) and revenue maximization (output-oriented model) are assumed.

The above literature references constitute some examples of how the initial concept of the unit efficient isoquant developed in Farrell (1957) has evolved into other alternative ways of specifying the technological set of a producer, i.e. production, cost, revenue or profit functions. The use of distance functions² has also spread widely since Farrell's seminal measures for technical and allocative efficiency. In any case, the underlying idea of defining an efficient frontier function against which to measure the current performance of productive units has been maintained during the last fifty years. In that time, different techniques have been utilised to either calculate or estimate those efficient frontiers.

These techniques can be classified in different ways. The criterion followed here distinguishes between parametric and non-parametric methods that is, between techniques where the functional form of the efficient frontier is pre-defined or imposed a priori and those where no functional form is pre-established but one is calculated from the sample observations in an empirical way.

The non-parametric approach has been traditionally assimilated into Data Envelopment Analysis (DEA); a mathematical programming model applied to observed data that provides a way for the construction of production frontiers as well as for the calculus of efficiency scores relatives to those constructed frontiers.

With respect to parametric approaches, these can be subdivided into deterministic and stochastic models. The first are also termed 'full frontier' models. They envelope all the observations, identifying the distance between the observed production and the maximum production, defined by the frontier and the available technology, as technical inefficiency.

The deterministic specification, therefore, assumes that all deviations from the efficient frontier are under the control of the agent. However, there are

some circumstances out of the agent's control that can also determine the sub-optimal performance of units. Regulatory-competitive environments, weather, luck, socio-economic and demographic factors, uncertainty, etc., should not properly be considered as technical efficiency. The deterministic approach does so, however. Moreover, any specification problem is also considered as inefficiency from the point of view of deterministic techniques. On the contrary, stochastic frontier procedures model both specification failures and uncontrollable factors independently of the technical inefficiency component by introducing a double-sided random error into the specification of the frontier model.

A further classification of frontier models can be made according to the tools used to solve them, namely the distinction between mathematical programming and econometric approaches. The deterministic frontier functions can be solved either by using mathematical programming or by means of econometric techniques. The stochastic specifications are estimated by means of econometric techniques only.

Most of the literature related to the measurement of economic efficiency have based their analysis either on any of the above parametric or on non-parametric methods. The choice of estimation method has been an issue of debate, with some researchers preferring the parametric (e.g. Berger, 1993) and others the non-parametric (e.g. Seiford and Thrall, 1990) approach. The main disadvantage of non-parametric approaches is their deterministic nature. Data Envelopment Analysis, for instance, does not distinguish between technical inefficiency and statistical noise effects. On the other hand, parametric frontier functions require the definition of a specific functional form for the technology and for the inefficiency error term. The functional form requirement causes both specification and estimation problems.

The aim of this paper is to provide a critical and detailed review of the core frontier methods, both parametric and non-parametric, for the measurement of economic efficiency. Unlike previous studies such as Kalirajan and Shand (1999) where the authors review various methodologies for measuring technical efficiency, this paper provides the reader with an extensive analysis of not only technical efficiency but also cost efficiency measurement. The introduction of duality theory allows for the joint investigation of both technical and allocative efficiency what guarantees a better and more accurate understanding of the overall efficiency reached by a set of productive units.

Moreover, the examination of the latest advances in Bayesian analysis and bootstrapping theory also contained in this paper, enhances preceding survey literature by presenting final developments in promising research areas such as the introduction of statistical inference or the treatment of stochastic noise within non-parametric frontier models, and the description of more flexible functional forms, the study of multiple outputs technologies or the analysis of undesirable outputs within the context of parametric frontier models.

In doing so, section 2 focuses on the non-parametric approaches discussing a basic model, further extensions and recent advances proposed in the latest literature. Section 3 describes the evolution of parametric techniques and the treatment of

duality tools in both cross-section and panel data frameworks together with a final part devoted to summarising the current research agenda. A brief summary of empirical evidence in terms of comparative analysis is presented in section 4. Section 5 concludes.

2. Non-parametric Frontier Techniques

2.1 *The Basic Model*

The method developed in Farrell (1957) for the measurement of productive efficiency is based on a production possibility set consisting of the convex hull of input-output vectors. This production possibility set was represented by means of a frontier unit-isoquant. According to that specification and the fact that Farrell's efficiency measures are completely data-based, no specific functional form needed to be predefined.

The single-input/output efficiency measure of Farrell is generalised to the multiple-input/output case and reformulated as a mathematical programming problem by Charnes, Cooper and Rhodes (1978). Charnes, Cooper and Rhodes (1981) named the method introduced in Charnes, Cooper and Rhodes (1978) Data Envelopment Analysis. They also described the duality relations and the computational power that Charnes, Cooper and Rhodes (1978) made available. This technique was initially born in operations research for measuring and comparing the relative efficiency of a set of decision-making units (DMUs). Since that seminal paper, numerous theoretical improvements and empirical applications of this technique have appeared in the productive efficiency literature.³

The aim of this non-parametric approach⁴ to the measurement of productive efficiency is to define a frontier envelopment surface for all sample observations. This surface is determined by those units that lie on it, that is the efficient DMUs. On the other hand, units that do not lie on that surface can be considered as inefficient and an individual inefficiency score will be calculated for each one of them. Unlike stochastic frontier techniques, Data Envelopment Analysis has no accommodation for noise, and therefore can be initially considered as a non-statistical technique where the inefficiency scores and the envelopment surface are 'calculated' rather than estimated.

The model developed in Charnes, Cooper and Rhodes (1978), known as the CCR model, imposes three restrictions on the frontier technology: Constant returns to scale, convexity of the set of feasible input-output combinations; and strong disposability of inputs and outputs. The CCR model is next interpreted through a simple example on the basis of Figure 2.1.1.

Here A, B, C, D, E and G are six DMUs that produce output Y with two inputs; X_1 and X_2 . The line DG in Figure 2.1.1 represents the frontier unit isoquant derived by DEA techniques from data on the population of five DMUs,⁵ each one utilising different amounts of two inputs to produce various amounts of a single output. The level of inefficiency of each unit is determined by comparison to a single referent DMU or a convex combination of other referent

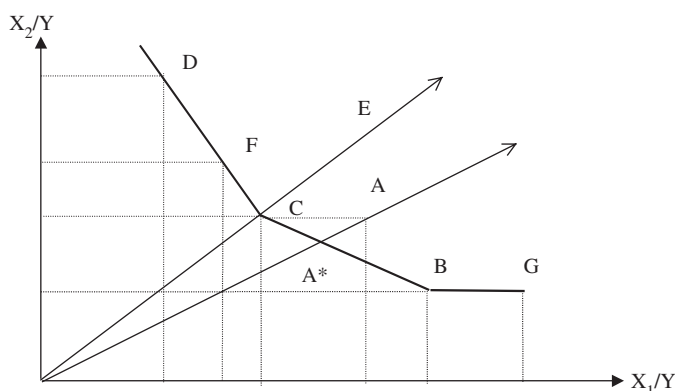


Figure 2.1.1. The CCR model.

units lying on the frontier isoquant line and utilising the same proportions of inputs. Therefore, the technical efficiency of A would be represented by the ratio OA^*/OA where A^* is a linear combination of referents B and C ('peer group') that utilises the inputs in the same proportions as A, since both A and A^* lie on the same ray. The efficiency of E could be directly measured by comparison with C, which is located on the efficient isoquant and on the same ray as C. The ratio OC/OE determines the technical efficiency of E. Finally, although unit G is situated on the efficient frontier, it cannot be considered as technically efficient in a Pareto sense, since it is using the same amount of input X_2 as B, but more input X_1 , to produce the same level of output.⁶

The Data Envelopment Analysis method calculates the efficient frontier by 'finding' the segments DC, CB and BG that envelope all the DMUs' performances. This frontier is not a proper isoquant but a linear approximation in which the observations at vertices (D, C, B, G) represent real DMUs while the units between them (F, A^*) are hypothetical units calculated as weighted averages of inputs. They are thus combinations of the real units. The individual technical efficiency scores will then be calculated (not estimated) by using mathematical programming techniques where the solutions will be required to satisfy inequality constraints in order to be able to increase (decrease) certain outputs (inputs) without worsening the other inputs (outputs).

Hence, in order to determine the efficiency score of each unit, it will be compared with a 'peer group' consisting of a linear combination of efficient DMUs. Given a set of N homogenous DMUs characterized by an input-output vector with m inputs and s outputs, for each unit not located on the efficient frontier we can define a vector $\bar{\mu} = (\mu_1, \dots, \mu_N)$ where each μ_j represents the weight of each DMU within that peer group. The DEA calculations are designed to maximize the relative efficiency score of each DMU, subject to the constraint that the set of weights obtained in this manner for each DMU must

also be feasible for all the other DMUs included in the sample. That efficiency score can be calculated by means of the following mathematical programming formulation,

$$\begin{aligned}
 TE_{CRS} &= \min_{\mu} \psi^0 \\
 \text{s.t.} & \\
 & \sum_{j=1}^n \mu_j X_{ij} \leq \psi X_i^0 \quad i = 1, \dots, m \\
 & \sum_{j=1}^n \mu_j Y_{rj} \geq Y_r^0 \quad r = 1, \dots, s
 \end{aligned} \tag{2.1.1}$$

The solution of this linear program reports the peer group that for each DMU analysed, yields at least the same level of output (second constraint) but consuming just a proportion (ψ) of each of the inputs used by the DMU (first constraint).⁷ The final objective is therefore to determine the linear combination of referents that for each DMU minimizes the value of ψ . The technical efficiency scores will be determined by the optimal ψ^* .

The dual version of the above model is often used in Operation Research techniques. This dual formulation can be obtained as the maximum of a ratio of weighted outputs to weighted inputs subject to the constraint that the similar ratios for every DMU be less than or equal to unity,⁸

$$\begin{aligned}
 Max_{w_r, z_i} H_0 &= \frac{\sum_r w_r Y_{r_0}}{\sum_i z_i X_{i_0}} \\
 \text{s.t.} & \\
 & \frac{\sum_r w_r Y_{rj}}{\sum_i z_i X_{ij}} \leq 1 \quad j = 1, \dots, N \\
 & w_r, z_i > 0 \quad r = 1, \dots, s \quad i = 1, \dots, m
 \end{aligned} \tag{2.1.2}$$

where w_r and z_i are the weights that solve this maximization problem and Y_{rj} and X_{ij} the inputs and outputs attached to each DMU. This ratio formulation ensures that $0 < Max H_0 < 1$. Moreover, a unit will be efficient if and only if this ratio equals unity, otherwise it will be considered as relatively inefficient. As Coelli, Rao and Battese (1998) point out, one problem with this ratio formulation is that it allows for an infinite number of solutions: if w_r and z_i are solutions of the above linear program problem then αw_r and αz_i are also solutions. This can be avoided by imposing the following additional constraint:

$$\sum_i t_i X_{i_0} = 1 \quad (2.1.3)$$

This gives rise to the following multiplier form of the DEA linear program,⁹

$$\begin{aligned} & \text{Max } \omega_r t_i \sum_r \omega_r Y_{r_0} \\ & \text{s.t.} \\ & \sum_i t_i X_{i_0} = 1 \\ & \sum_r \omega_r Y_{r_j} - \sum_i t_i X_{i_j} \leq 0 \\ & \omega_r, t_i > 0 \end{aligned} \quad (2.1.4)$$

2.2 Extensions

The main attributes of Data Envelopment Analysis techniques are their flexibility and adaptability. Indeed, this adaptability has led to the development of a large number of extensions to the initial CCR model and of applications in recent years. We next briefly review some of the most relevant contributions.

The method provided by Farrell (1957) consisted of projecting each observed unit onto an efficient unit-isoquant. The model described above – the CCR model – generalises Farrell's approach to the multiple output case and reformulates the calculus of individual input-saving efficiency measures by solving a linear programming problem for each DMU. This efficient frontier is computed as a convex hull in the 'input space' and is represented by a convex set of facets. Charnes, Cooper and Rhodes (1978) assume Constant Returns to Scale (CRS) in their initial approach. The CRS restriction assumes that all DMU's under analysis are performing at an optimal scale. In the real world, however, this optimal behaviour is often precluded by a variety of circumstances such as different types of market power, constraints on finances, externalities, imperfect competition, etc. In all these cases, the CRS specification given by Charnes, Cooper and Rhodes (1978) yields misleading measures of technical efficiency in the sense that technical efficiency scores reported under that set of constraints are biased by scale efficiencies. This important shortcoming is corrected by Färe, Grosskopf and Lovell (1983), Byrnes, Färe and Grosskopf (1984) and Banker, Charnes and Cooper (1984)¹⁰ who extended DEA to the case of Variable Returns to Scale (VRS). Variable Returns to Scale are modelled by adding the convexity constraint $\sum \mu_j = 1$ to the model formulated in (2.1.1). This final constraint simply guarantees that each DMU is only compared to others of similar size. This mode of operation avoids the damaging effect of scale efficiency on the technical efficiency scores. The resulting linear programming problem can be expressed as:

$$\begin{aligned}
 TE_{VRS} &= \min_{\mu} \psi^0 \\
 \text{s.t.} & \\
 &\sum_{j=1}^n \mu_j X_{ij} \leq \psi X_i^0 \quad i = 1, \dots, m \\
 &\sum_{j=1}^n \mu_j Y_{rj} \geq Y_r^0 \quad r = 1, \dots, s \\
 &\sum_{j=1}^n \mu_j = 1
 \end{aligned} \tag{2.2.1}$$

The main implications of different scale assumptions about the production set will be discussed in reference to Figure 2.2.1,

The ray from the origin through J and B represents the frontier technology associated with the assumption of CRS. According to this specification, only units J and B would be considered technically efficient. Other non-efficient units such as A and D will have the efficiency scores represented by the ratios X_h/X_a and X_j/X_d respectively both less than unity. Under Variable Returns to Scale, the efficiency scores will be calculated from the efficient frontier defined by the line X_a , A, B, C and the horizontal segment to the right of C. Since the constraint set for TE_{CRS} is less restrictive (the convexity constraint is absent) than in the VRS formulation, lower efficiency scores are possible and therefore more units are declared efficient for a VRS envelope surface. In this case, A, B, and C are now fully efficient. On the other hand, unit D is still inefficient but its efficiency score has changed from the ratio X_j/X_d under CRS to X_w/X_d under VRS.

Finally, on the grounds of the above two alternative specifications and their corresponding linear programming formulations, a measure of scale efficiency

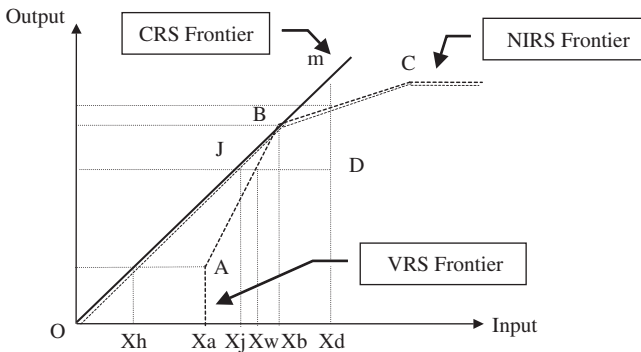


Figure 2.2.1. Constant, variable and non-increasing returns to scale.

can be calculated: unit B is both CRS and VRS technically efficient. In contrast, units A and C are VRS efficient but inefficient with respect to the CRS efficient frontier, which means that the size of these units deviates from optimal scale. As a result, a measure of scale efficiency would be offered by the ratio TE_{CRS}/TE_{VRS} . Finally, unit D is technically inefficient with respect to both efficiency frontiers. In this case, total technical efficiency (TTE) can be decomposed into two components: pure technical efficiency (PTE) and scale efficiency (SE), according to the following relationship:

$$TTE = PTE \times SE$$

where $TTE = X_j/X_d$, $PTE = X_w/X_d$ and $SE = X_j/X_w$.

A major weakness of the procedure described above is the fact that it cannot provide us with an indicator of whether the DMU is operating under increasing or decreasing returns to scale. This can be resolved by solving a linear programming problem where non-increasing returns to scale (NIRS) are assumed by introducing the constraint $\sum \mu_j \leq 1$. In terms of Figure 2.2.1, the NIRS efficiency frontier is represented by the discontinuous line from the origin through units B and C. This envelopment surface allows us to distinguish between different scales in the production structure. Thus, all those units such as C, for which $TE_{NIRS} = TE_{VRS} \neq TE_{CRS}$, are producing at decreasing returns to scale. For those such as B that satisfy $TE_{NIRS} = TE_{VRS} = TE_{CRS}$, production is characterised by constant returns to scale. Finally, A and D are examples of units producing under increasing returns to scale since $TE_{NIRS} \neq TE_{VRS}$. Empirical applications of this approach can be found in Färe, Grosskopf, and Logan (1985) and the Bureau of Industry Economics (1994).

So far, all the preceding analysis has been developed in terms of input-oriented models. However, a DEA model, besides being input oriented, may also be output oriented or even unoriented.¹¹ In oriented models, unlike in unoriented models, one set of variables, either inputs or outputs, precedes the other in its proportional movement toward the efficient frontier. Input oriented models try to maximize the proportional decrease in input variables while remaining within the envelopment space, while output oriented models will maximize the proportional increase in the output vector. The choice of one or the other model might be based on the specific characteristics of the data set analysed. For instance, for regulated sectors such as the electric sector, where output is usually assumed to be exogenous and inputs operate in competitive markets, the use of input oriented rather than output oriented models, seems to be the best choice. In any case, both input and output oriented models will determine the same efficient frontier, which implies that the same DMU's will be recognized as efficient/inefficient for both types of approaches. Furthermore, as Coelli and Perelman (1996b) show, the choice of any particular orientation rarely has more than a minor influence upon the reported efficiency scores.

The derivation of the output-oriented models is straightforward. With the nomenclature used before, they can be formulated in the following way,

$$\begin{aligned}
 TE = \max_{\mu} \psi^0 & & \min_{w_r, z_i} h_0 = \frac{\sum_i z_i X_{i_0}}{\sum_r w_r Y_{r_0}} \\
 \text{s.t.} & & \text{s.t.} \\
 \sum_{j=1}^n \mu_j X_{ij} \leq X_i^0 & & \frac{\sum_i z_i X_{i_0}}{\sum_r w_r Y_{r_0}} \geq 1 \\
 \sum_{j=1}^n \mu_j Y_{rj} \geq \psi Y_r^0 & & w_r, z_i > 0 \\
 j = 1, \dots, n \quad i = 1, \dots, m \quad r = 1, \dots, s & & \\
 \sum_{j=1}^n \mu_j = 1(VRS) & & \sum_{j=1}^n \mu_j \leq 1(NIRS) \quad (2.2.2)
 \end{aligned}$$

The decomposition of total technical efficiency into pure technical efficiency and scale efficiency can also be replicated in terms of output oriented models. In this case, the total technical efficiency with reference to the CRS envelopment surface would be represented by the ratio X_{dD}/X_{dm} , while pure technical efficiency and scale efficiency (both output oriented) would be X_{dD}/X_{dp} and X_{dp}/X_{dm} respectively. Again the relationship $TTE = PTE \times SE$ holds.

The literature so far described deals with model specifications based on a continuous definition for input and output variables. Hence, discrete level variables can not be analysed. The DEA literature refers to this class of variables as categorical variables. Charnes, Cooper and Rhodes (1981) developed a strategy to include the information provided for these variables in the construction of efficient frontiers by simply solving a separate DEA for each category group of observations and assessing any difference in the mean efficiency of the different sub-samples under consideration. Moreover, if a natural nesting or hierarchy of the categories can be assumed, Banker and Morey (1986b) show how, by using mixed integer linear program models¹² each DMU can be compared with those in its same category group or in another operating under worse conditions.

Data Envelopment Analysis has also evolved to treat variables over which DMU's do not have a direct control. In order to get more realistic individual efficiency scores, one might isolate in some way this type of variable, known as non-discretionary variables, and their effects on the final performance of the observed units. For the case of input oriented models for example, it seems quite reasonable not to maximize the proportional decrease in the entire input vector with respect to those input variables not directly controlled by agents but just with respect to the discretionary variables. For the same reason, output oriented models where non-discretionary variable data is available should just deal with the maximization of the proportional increase of those output variables under the control of agents. Banker and Morey (1986a)¹³ adapt the mathematical

programming treatment of DEA models to allow a partial analysis of efficiency on the basis of what they initially termed exogenously and non-exogenously fixed inputs and outputs.

Another example of the flexibility that characterises DEA techniques can be found in the way they deal with the multiplier set. Unlike ratio analysis where the relative importance of inputs and outputs is known, DEA techniques do not impose the weights attached to any input or output. Instead, they will be calculated by solving the mathematical programming problem. This feature allows DEA techniques to generate measures of comparative efficiency in those environments where no a priori weights are known.

However, that complete flexibility can yield extremely high/low multipliers in relation to the underlying economic theory that supports the specification of the model. Several approaches have been developed to remedy this. Among them, the most relevant are the Assurance Region (AR) method developed by Thompson, Singleton, Thrall and Smith (1986) and the Cone-Ratio (CR) method developed by Charnes, Cooper, Wei and Huang (1989) and Charnes, Cooper, Sun and Huang (1990). The AR approach deals with the existence of large differences in input/output weights from DMU to another. The AR approach incorporates into the initial DEA model additional constraints on the relative magnitude of the weights for some particular inputs/outputs. This limits the region of weights to some special area consistent with the underlying economic theory. Further developments of the AR Model can be found in Thompson, Langemeir, Lee and Thrall (1990) and Roll, Cook and Gollany (1991). More general than the AR method, the cone-ratio approach extends the Charnes, Cooper and Rhodes (1978) model by using constrained multipliers, which are constrained to belong to closed cones. Further development of the CR method is found in Brockett, Charnes, Cooper, Huang and Sun (1997) where risk evaluation of bank portfolios is analysed. Pedraja-Chaparro, Salinas-Jimenez and Smith (1997), have studied the role of weight restrictions in DEA techniques and their effects on efficiency estimates. They first survey the theoretical and empirical literature related to this topic, and then propose an approach to the treatment of constrained multipliers on the basis of imposing limits on the part of total costs and total benefits associated to each input and output. They term this contingent virtual weight restriction.

Other extensions to the model initially proposed in Charnes, Cooper and Rhodes (1978) include the introduction of multiplicative measures of relative efficiency through the use of multiplicative envelopment surfaces such as those described in Charnes, Cooper, Seiford, and Stutz (1982&1983) and Banker and Maindiratta (1986); the measurement of allocative efficiency on the basis of price information and the assumption of a behavioural objective such as cost minimization in Ferrier and Lovell (1990), revenue maximisation in Färe, Grosskopf and Lovell (1985) or profit maximization in Färe, Grosskopf and Weber (1997);¹⁴ the treatment of panel data by means of the window analysis developed in Charnes, Clark, Cooper, and Golany (1985) or the Malmquist index approach of Färe, Grosskopf, Lindgren and Roos (1994); and the recent challenging extension

of DEA models to the dynamic context, synthesised in Sengupta (1995a) and further developed in Sengupta (1999a).

2.3 New developments: Statistical inference, Bootstrapping and Stochastic Approaches

As it has been described so far, one of the main drawbacks of non-parametric techniques is their deterministic nature. This is what traditionally has driven specialised literature in this issue to describe them as non-statistical methods. Nevertheless, recent literature has shown it is possible to define a statistical model allowing for the determination of statistical properties of the non-parametric frontier estimators. In this respect, Grosskopf (1996) first provides a good and selective survey of statistical inference in nonparametric, deterministic, linear programming frontier models. Nonparametric regularity tests, sensitivity analysis, and non-parametric statistics tests are also treated in this paper. Finally, on the basis of showing that DEA estimators are maximum likelihood, Grosskopf (1996) analyses the asymptotic properties of these estimators.

In any case, the type of asymptotic results described in Grosskopf (1996) and more recently developed in further references such as Kneip, Park and Simar (1998) or Park, Simar and Weiner (2000) presents some important limitations. These results may be misleading when used in conjunction with small samples. In addition, extra noise is introduced when estimates of the unknown parameters of the limiting distributions are used in constructing estimates of confidence intervals. Finally, the asymptotic sampling distributions presented in Grosskopf (1996) are only available for univariate DEA frameworks, whereas most applications of the DEA estimator usually deal with multivariate frameworks.

It is at this stage when bootstrapping techniques come into their own. The bootstrap¹⁵ provides us with a suitable way to analyze the sensitivity of efficiency scores relative to the sampling variations of the calculated frontier by avoiding the mentioned drawbacks of asymptotic sampling distributions. We next briefly describe some of the most relevant literature regarding bootstrapping and the measurement of economic efficiency by means of non-parametric frontier models.

Thus, Ferrier and Hirschberg (1997) first developed a method for introducing a stochastic element into technical efficiency scores obtained by DEA techniques. They derived confidence intervals for the original efficiency levels by using computational power to obtain empirical distributions for the efficiency measures. Nevertheless, the methodology employed in Ferrier and Hirschberg (1997) is later criticized in Simar and Wilson (1999a, 1999b) by demonstrating that the bootstrap procedure suggested by these authors gives inconsistent estimates.

To avoid this inconsistency, Simar and Wilson (1998) provide an alternative approach by analysing the bootstrap sampling variations of input efficiency measures of a set of electricity plants. In doing so, Simar and Wilson (1998) show how in order to validate the bootstrap it is necessary to define a reasonable data-generating process and to propose a reasonable estimator of it. As Simar

and Wilson (2000a) establish, the procedure described in Simar and Wilson (1998) for constructing confidence intervals depends on using bootstrap estimates of bias to correct for the bias of the DEA estimators.

In addition, the above process requires using these bias estimates to shift obtained bootstrap distribution appropriately. Such use of bias estimates introduce a further source of noise into the process. Simar and Wilson (1999c) overcome this *modus operandi*'s weakness by implementing an improved procedure which automatically corrects for bias without explicit use of a noisy bias estimator. Moreover, the initial methodology proposed in Simar and Wilson (1998) is also extended to a less restrictive framework by allowing heterogeneity in the structure of efficiency in Simar and Wilson (2000b).

Therefore, we might conclude that today statistical inference based on non-parametric frontier approaches to the measurement of economic efficiency is available either by using asymptotic results or by using bootstrap. However, a couple of main issues still remain to be solved, namely the high sensitivity of non-parametric approaches to extreme values and outliers, and also the way for allowing stochastic noise to be considered in a non-parametric frontier framework.

As for the first of these issues, Cazals, Florens and Simar (2002) have recently proposed a non-parametric estimator which is more robust to extreme values, noise or outliers, in the sense that it does not envelope all the data points. This estimator is based on a concept of expected minimum input function. With respect to modelling the stochastic noise within a non-parametric framework, some attempts have also made in recent literature. Among them, Sengupta (1990) can be considered as one of the pioneering ones. Later, Olesen and Petersen (1995) and Kneip and Simar (1996) also proposed alternative versions of a stochastic DEA model. Finally, Sengupta (2000a) and Huang and Li (2001) have developed more refined stochastic DEA models. So, Sengupta (2000a) generalizes the non-parametric frontier approach in the stochastic case, when the input prices and capital adjustment costs vary, and Huang and Li (2001) discusses the relationships of their stochastic DEA models based on utilizing the theory of chance constrained programming, with some conventional DEA models.

3. Parametric Frontier Techniques

3.1 *The cross-sectional framework*

In terms of a cross-sectional production function, a parametric frontier can be represented as:

$$Y_i = f(X_i, \beta) \cdot TE_i \quad (3.1.1)$$

where $i = 1 \dots I$ indexes the producers, Y is the scalar output,¹⁶ X represents a vector of N inputs and $f(\cdot)$ is the production frontier which depends on inputs and on a technological parameter vector, β . Finally, TE_i indicates the output-oriented

technical efficiency of producer i defined as the ratio of the observed output to maximum feasible output,

$$TE_i = \frac{Y_i}{f(X_i, \beta)} \quad (3.1.2)$$

Farrell (1957) assumed what later literature has termed a deterministic frontier function. In terms of this specification, equation 3.1.1 can be rewritten as:

$$y_i = f(X_i, \beta) \cdot \exp(-u_i) \quad u_i \geq 0 \quad (3.1.3)$$

where u_i represents the shortfall of output from the frontier (technical inefficiency) for each producer. The additional restriction imposed on u_i ($u_i \geq 0$) guarantees that $TE_i \leq 1$, which is consistent with equation 3.1.2. Next, assuming that the productive technology adopts a log-linear Cobb-Douglas form,¹⁷ the deterministic frontier production function becomes:

$$\ln Y_i = \beta_0 + \sum_{n=1}^N \beta_n \ln X_{ni} - u_i \quad (3.1.4)$$

Once the production structure has been parameterized, both goal programming and econometric techniques can be applied to either calculate or estimate the parameter vector and also to obtain estimates of u_i and so of TE_i .

Goal programming techniques calculate the technology parameter vector by solving deterministic optimization problems. Aigner and Chu (1968), Timmer (1971), Forsund and Hjalmarsson (1979), Nishimizu and Page (1982) or Bjurek, Hjalmarsson and Forsund (1990) are some of the most relevant references in this research area. The main drawback of these approaches is that the parameters are not estimated in any statistical sense but calculated using mathematical programming techniques. This complicates statistical inference concerning the calculated parameters, and precludes any hypothesis testing.

It is at this stage when econometric analysis of frontier functions comes into its own. In an attempt to accommodate econometric techniques to the underlying economic theory,¹⁸ a wide and challenging literature related to the estimation of frontier functions has proliferated over the last three decades. These attempts can be classified into two main groups according to the specification of the error term, namely deterministic and stochastic econometric approaches.

The deterministic econometric approach employs the technological framework previously introduced by mathematical programming approaches. With an econometric formulation, it is possible to estimate rather than 'calculate' the parameters of the frontier functions. Additionally, statistical inference based on those estimates will be possible. Several techniques such as Modified Ordinary Least Squares (e.g. Richmond, 1974), Corrected Ordinary Least Squares (e.g. Gabrielsen, 1975) and Maximum Likelihood Estimation (e.g. Greene, 1980a) have been developed in the econometric literature in order to estimate these deterministic-full frontier models.

Unlike mathematical programming approaches, the deterministic econometric models accommodate economic efficiency as an explicative factor for the output variations, but still sacrifice the analysis of random shocks. Therefore, neither goal programming models nor deterministic econometric approaches provide accurate measures of the productive structure. So, in the interest of brevity and given that none of the above techniques is being really used in current literature,¹⁹ we next focus on an alternative econometric approach that overcomes the mentioned drawbacks and has become the most popular and widely used parametric approach for the measurement of economic efficiency, namely stochastic frontier models.

3.1.1 Stochastic frontier models

Aigner, Lovell and Schmidt (1977), Meeusen and van den Broeck (1977) and Battese and Corra (1977) simultaneously developed a Stochastic Frontier Model (SFM) that, besides incorporating the efficiency term into the analysis (as do the deterministic approaches) also captures the effects of exogenous shocks beyond the control of the analysed units. Moreover, this type of model also covers errors in the observations and in the measurement of outputs.

For the Cobb-Douglas case, and in logarithmic terms, the single-output²⁰ stochastic frontier can be represented as

$$\ln Y_i = \beta_0 + \sum_{n=1}^N \beta_n \ln X_{ni} + v_i - u_i \quad (3.1.5)$$

The term $v_i - u_i$ is a composed error term where v_i represents randomness (or statistical noise) and u_i represents technical inefficiency.²¹

The error representing statistical noise is assumed to be identical independent and identically distributed. With respect to the one-sided (inefficiency) error, a number of distributions have been assumed in the literature, the most frequently used being the half-normal, exponential, and truncated from below at zero. If the two error terms are assumed independent of each other and of the input variables and one of the above distributions are used, then the likelihood functions can be defined and maximum likelihood estimates can be determined.

In any case, for efficiency measurement analysis, the composed error term needs to be separated. Jondrow, Lovell, Materov and Schmidt (1982) showed that for the half-normal case, the expected value of u_i conditional on the composed error term is

$$E[u_i|e_i] = \frac{\sigma\lambda}{(1 + \lambda^2)} \left[\frac{\phi(e_i\lambda/\sigma)}{\Phi(-e_i\lambda/\sigma)} - \frac{e_i\lambda}{\sigma} \right] \quad (3.1.6)$$

where $\phi(\cdot)$ is the density of the standard normal distribution, $\Phi(\cdot)$ the cumulative density function, $\lambda = \sigma_u/\sigma_v$, $e_i = v_i - u_i$ and $\sigma = (\sigma_u^2 + \sigma_v^2)^{1/2}$.

Once conditional estimates of u_i have been obtained, Jondrow, Lovell, Materov and Schmidt (1982) calculate the technical efficiency of each producer as

$$TE_i = 1 - E[u_i | e_i] \quad (3.1.7)$$

Other authors later pointed to the use of $\exp\{-(u_i | e_i)\}$ as preferable to $1 - E[u_i | e_i]$ for calculating the technical efficiency reached by each productive unit under analysis. The reason lies in the fact that Jondrow, Lovell, Materov and Schmidt (1982)'s conditional estimate is no more than a first-order approximation to the more general infinite series, $\exp\{-(u_i | e_i)\} = 1 - u_i + u_i^2/2 - u_i^3/3! \dots$

Lastly, Battese and Coelli (1988) proposed another alternative point estimator for TE_i . Battese-Coelli's estimate is preferred to others when u_i is not close to zero. It is expressed as

$$E[\exp(-u_i) | e_i] = \frac{1 - \Phi(\delta + (\gamma e_i / \delta))}{1 - \Phi(\gamma e_i / \delta)} \cdot \exp(\gamma e_i + (\delta^2 / 2)) \quad (3.1.8)$$

where $\delta = \frac{\sigma_u \cdot \sigma_v}{\sigma}$ and $\gamma = \frac{\sigma_u^2}{\sigma^2}$.

In any case, whichever point estimator is finally used, all of them share an important defect, namely that although they are unbiased they are not consistent estimates of technical efficiency, since $plim E(u_i | v_i - u_i) - u_i$ is not zero.²² However, as the recent literature has shown, it is possible to get confidence intervals for any of the three alternative technical efficiency point estimates commented on above. Thus, Hjalmarsson, Kumbhakar and Heshmati (1996) propose confidence intervals for the Jondrow, Mateorov, Lovell and Schmidt (1982) technical efficiency estimator, and Bera and Sharma (1996) for the Battese and Coelli (1988) one. Finally, Horrace and Schmidt (1995&1996) derive upper and lower bounds on $\exp\{-(u_i | e_i)\}$ based on lower and upper bounds of $(u_i | e_i)$ respectively. Horrace and Schmidt (1996) describes a method for calculating confidence intervals for efficiency levels, while Horrace and Schmidt (1995) develop the multiple comparisons with the best methodology which are both hot topics in current research with papers as Horrace, Schmidt and Witte (1998), Jensen (2000) or Horrace and Schmidt (2000) leading recent advances in this field.

Jondrow, Lovell, Materov and Schmidt (1982) also computed the expected value of u_i 's conditional on the composed error term for the case in which the asymmetric error term follows an exponential distribution. They provided the following result:

$$E[u_i | e_i] = (e_i - \theta \sigma_v^2) + \frac{\sigma_v \phi[(e_i - \theta \sigma_v^2) / \sigma_v]}{\Phi[(e_i - \theta \sigma_v^2) / \sigma_v]} \quad (3.1.9)$$

where $\theta = 1/\sigma_u$.

Half-normal and exponential distributions both have a mode at zero. This causes conditional technical inefficiency scores, specially in the neighbourhood of zero that can involve artificially high technical efficiency levels. Moreover, these distribution specifications fix a pre-determined shape for the distribution of the disturbances that can also be considered a shortcoming. Stevenson (1980) argued that the zero mean assumed in the Aigner, Lovell and Schmidt (1977) model was an unnecessary restriction and produced some results for a truncated

distribution as opposed to a half-normal distribution. Greene (1993) shows that the conditional technical inefficiencies for the truncated model are obtained by replacing $e_i\lambda/\sigma$ in the expression (3.1.6) for the half-normal case with:

$$u_i^* = \frac{e_i\lambda}{\sigma} + \frac{u_i}{\sigma\lambda} \quad (3.1.10)$$

The two-parameters gamma distribution constitutes another attempt to overcome the half-normal and exponential deficiencies. The gamma frontier model was initially proposed by Greene (1980a) within the framework of a deterministic frontier model:

$$y = f(X, \beta) - \mu \text{ where } \mu \sim G(\Theta, P)$$

$$\text{so } f(\mu) = \frac{\Theta^P}{\Gamma(P)} \mu^{P-1} e^{-\Theta\mu} \quad \mu \geq 0, \Theta, P > 0 \quad (3.1.11)$$

Later, Greene (1990) also applied the gamma density to the stochastic composed error frontier model:

$$y = f(X, \beta) + v - \mu \quad \text{where } v \sim N(0, \sigma^2) \text{ and } \mu \sim G(\Theta, P)$$

Maximum likelihood techniques and a consistent method-of-moments estimator based on OLS are described in Greene (1990) as well as the decomposition of the error term into its efficiency and statistical noise components. In any case, the complexity associated with these estimation procedures seems likely to outweigh their benefits.²³ So, for better or worse, the fixed-shape models, especially the normal-half-normal one,²⁴ have dominated the empirical literature of stochastic frontiers.²⁵

3.2 The Panel data framework

3.2.1 Technical efficiency time-invariant models

Besides the strong distribution assumptions which the cross-sectional stochastic frontier models rely on to provide conditional measures of inefficiency, as has already been shown, their measures although unbiased are non-consistent estimates. However, these two main limitations of cross-sectional stochastic frontier models can be overcome if panel data is available.

Schmidt and Sickles (1984) point out some of the advantages of the Panel Data Stochastic Frontier Models (PDMs) versus the cross-sectional ones. First, while cross-section models assume that the inefficiency term and the input levels (or more generally, whatever exogenous variable) are independent, for panel data estimation that hypothesis is not needed. This is especially useful in order to introduce time-invariant regressors in the specification of the model. Second, by adding temporal observations in the same unit, PDMs yield consistent estimates of the inefficiency term. Third by exploiting the link between the 'one-sided inefficiency term' and the 'firm effect' concepts, Schmidt and Sickles (1984) observed that when panel data are available, there is no need for any distribution

assumption for the inefficiency effect and all the relevant parameters of the frontier technology can be obtained by simply using the traditional estimation procedures for panel data; i.e. fixed-effects model and random-effects model approaches.²⁶ We next briefly analyse these methods in a panel data stochastic frontier framework.

Consider the frontier production function model in equation (3.2.1),²⁷ where statistical noise varies over units and time, but the asymmetric inefficiency error term varies only over units.²⁸ If the inefficiency is considered systematic and therefore the u_i 's are treated as firm-specific constants, a fixed effect model can be implemented.

$$Y_{it} = \beta_0 + \sum_{n=1}^N \beta_n X_{nit} + v_{it} - u_i \quad u_i \geq 0 \quad (3.2.1)$$

Hence, using the 'within-groups' transformation, the above model can be estimated by OLS after all observations have been expressed in terms of deviations from the unit means,

$$(Y_{it} - \bar{Y}_i) = \beta_n (X_{nit} - \bar{X}_{ni}) + v_{it} \quad (3.2.2)$$

An alternative estimation procedure consists of eliminating the intercept term by adding a dummy variable for each of the sample units:

$$Y_{it} = \beta_{0i} + \sum_{n=1}^N \beta_n X_{nit} + v_{it} \quad (3.2.3)$$

Finally, the individual estimated inefficiency terms can be determined by means of the following definitions:

$$\begin{aligned} \hat{\beta}_{0i} &= \max(\hat{\beta}_{0i}) \text{ and} \\ \hat{u}_i &= \hat{\beta}_0 - \hat{\beta}_{0i}, i = 1, 2, \dots, I \end{aligned} \quad (3.2.4)$$

Besides its simplicity, the fixed-effects model provides consistent estimates of the producer-specific technical efficiency. However, the fixed-effects approach to the estimation of a stochastic frontier function presents an important computing drawback in those cases in which the frontier function includes time-invariant regressors:²⁹ the within-groups transformation required for the fixed-effect model implementation precludes those variables once the initial model is expressed in terms of deviations from the unit means. This shortcoming caused the stochastic frontier panel data literature to use a random-effects model where, by assuming the independence of the inefficiency term and the regressors, time-invariant regressors can be included in the analysis. Rewriting equation (3.2.1) as (3.2.5) below, the assumption that the u_i are random rather than fixed allows some of the

regressors to be time-invariant and therefore might be preferable to the fixed-effect approach,

$$Y_{it} = (\beta_0 - E(u_i)) + \sum_{n=1}^N \beta_n X_{nit} + v_{it} - (u_i - E(u_i)) \quad (3.2.5)$$

where both v_{it} and $u_i^* = (u_i - E(u_i))$ are zero mean.

The panel data literature shows how the above model can be estimated by the standard two-step generalized least squares (GLS) method.³⁰ As in the case of the fixed-effect model, consistent estimates of inefficiency under a random-effect framework require the number of both cross-section and temporal observations to tend to infinity. This was first noted by Schmidt and Sickles (1984). Thus, the main advantage of a random-effects model over a fixed-effects models lies in its allowing for time-invariant attributes in the technology specification. As a drawback, in the random-effects model all the u_i s have to be uncorrelated with the regressors, while this condition is not imposed in the fixed effects-approach.³¹

The above panel data techniques avoid the necessity of distribution assumptions in both the specification and the estimation of stochastic frontier functions. However, if the latter are known, similar maximum likelihood techniques to the ones applied to the cross-sectional data can be applied to a stochastic production frontier panel data model in order to get more efficient estimates of the parameter vector and of the technical inefficiency scores for each productive unit. In this respect, Pitt and Lee (1981) derived the normal-half-normal counterpart of Aigner, Lovell and Schmidt's (1977) model for panel data, while Kumbhakar (1987) and Battese and Coelli (1988) extend Pitt and Lee's analysis to the normal-truncated stochastic frontier panel data model. Maximum likelihood techniques are also applied to unbalanced panel data in Battese, Coelli and Colby (1989).

In terms of comparative analysis, the above three alternative approaches (namely fixed-effects model, random effects model and maximum likelihood techniques) present different properties and impose different requirements on the data. This makes it difficult to formulate a clear universal statement about the preferability of one over the others. To a great degree, any such preferability would be subject to the particular circumstances and framework of each analysis. This has been shown in the recent literature in empirical comparisons³² of these alternative approaches to the estimation of panel data stochastic frontier functions. This empirical literature also shows that, despite the differences inherent to each method, the three approaches are likely to generate a similar efficiency ranking, particularly at the top and the bottom of the distribution as Kumbhakar and Lovell (2000) pointed out.

Both the fixed/random-effects approaches and maximum likelihood techniques analysed so far have considered technical inefficiency effects to be time-invariant. However, as the time dimension becomes larger, it seems more reasonable to allow inefficiency to vary over time. As with the time-invariant technical inefficiency model, time-varying technical inefficiency can be estimated by using either fixed or random effects or maximum likelihood techniques.

3.2.2 Technical efficiency time variant models

Cornwell, Schmidt and Sickles (1990) were the first to propose a generalisation of the Schmidt and Sickles (1984) model to account for time-varying inefficiency effects within a stochastic frontier panel data framework. The model used in their paper can be specified as,

$$Y_{it} = \beta_{0t} + \sum_{n=1}^N \beta_n X_{nit} + v_{it} - u_{it} = \beta_{it} + \sum_{n=1}^N \beta_n X_{nit} + v_{it} \quad (3.2.6)$$

where β_{0t} indicates the common production frontier intercept to all cross-sectional productive units in period t and $\beta_{it} = \beta_{0t} - u_{it}$ is the intercept of unit i in period t . Cornwell, Schmidt and Sickles (1990) model the intercept parameters for different cross-section productive units at different time periods as a quadratic function of time in which the time variables are associated to producers' specific parameters. This yields the following specification for the technical inefficiency error term,

$$u_{it} = \Phi_{1i} + \Phi_{2i}t + \Phi_{3i}t^2 \quad (3.2.7)$$

where the Φ s represent cross-section producer specific parameters. Several estimation strategies, including a fixed-effect approach and a random-effects approach are described in Cornwell, Schmidt and Sickles (1990) and again the jump from fixed-effects approaches to random-effects approaches is made on the basis of allowing for the inclusion of time-invariant regressors. Thus, a GLS random-effects estimator and an Efficient Instrumental Variable (EIV) estimator³³ are used for their time-varying technical efficiency model with time-invariant regressors.

Lee and Schmidt (1993) propose an alternative formulation, in which the technical inefficiency effects for each productive unit at a different time period are defined by the product of individual technical inefficiency and time effects,

$$u_{it} = \delta_t u_i \quad (3.2.8)$$

where the δ_t s are the time effects represented by time dummies and the u_i can be either fixed or random producer-specific effects.³⁴

On the other hand, if independence and distributional assumptions are available, Maximum Likelihood techniques can also be applied to the estimation of stochastic frontier panel data models where technical inefficiency depends on time. Kumbhakar (1990) suggests a model in which the technical inefficiency effects assumed to have a half-normal distribution, vary systematically with time according to the following expression,

$$u_{it} = \delta(t)u_i = [1 + \exp(\gamma t + \rho t^2)]^{-1} u_i \quad (3.2.9)$$

where γ and ρ are unknown parameters to be estimated. Finally, Battese and Coelli (1992) proposed an alternative to the Kumbhakar (1990) model assuming

technical inefficiency to be an exponential function of time and where only one additional parameter (η) has to be estimated,

$$u_{it} = \delta(t)u_i = [\exp(-\eta(t - T))]u_i \quad (3.2.10)$$

where the u_i s are assumed to be i.i.d following a truncated-normal distribution.

3.3 Duality theory

Panel data techniques have greatly contributed to a better and more accurate implementation of frontier models. A further step in enabling a greater flexibility in the analysis of economic efficiency by means of parametric techniques is due to Duality Theory. Dual representations of the production technology allow, inter alia, for the treatment of multiple outputs, quasi-fixed inputs, alternative behavioural objectives, and the joint analysis of both technical and allocative efficiency levels.

The duality approach to the estimation of frontier functions, its implications for the measurement of technical and allocative efficiency, and the introduction of a major degree of flexibility by means of multiple equation estimation procedures, have dominated the stochastic frontier literature in recent years. These approaches are intended to yield more asymptotically efficient estimates of technology and efficiency.

The duality problem consists of determining which is preferable: the direct estimation of the structure of the production by means of a production function, or the indirect estimation through a cost function.³⁵ The choice may be based on multiple factors, for example: exogeneity assumptions,³⁶ data availability, specific characteristics of the production set, or the complexity of the estimation procedures.

The econometric literature of 'average' functions has developed several alternative methods to estimate the structure of the production set coherent with the main insights of duality theory. Nerlove (1963) estimates the parameters of a single cost function by OLS. This technique is attractive from the point of view of its simplicity but it ignores the additional information that cost share equations can introduce into the estimation process. Berndt and Wood (1975) estimate those cost shares as a multivariate regression system. This approach also presents some deficiencies.³⁷ Finally, Christensen and Greene (1976) introduced the joint estimation of the cost share equations and the cost function. This procedure allows for the estimation of all relevant parameters that define the production structure.

Dual frontier econometric approaches have also evolved from the estimation of single cost functions³⁸ to multiple equation systems.³⁹ However, as we shall next see, serious specification and estimation problems arise as one moves far from the traditional, well-behaved, and self-dual Cobb-Douglas functional forms.

With respect to the specification problem, the work of Schmidt and Lovell (1979) can be regarded as the first attempt to analyse the duality between stochastic frontier production and cost functions. They exploit the self-duality of the Cobb-Douglas functional form to provide estimates of input-oriented technical inefficiency and input allocative inefficiency.

By using the following Cobb-Douglas technology (expressed in logarithmic terms):

$$\ln Y = A + \sum_{n=1}^N \beta_n \ln X_n + (v - u) \tag{3.3.1}$$

where Y is the output, X_n are the inputs to the production process, v represents statistical noise and u is a non-negative disturbance, reflecting technical inefficiency, and where producer subscripts are omitted to simplify nomenclature, Schmidt and Lovell (1979) get the following dual cost technology representation, where cost minimization and allocative efficiency are assumed:

$$\ln C = K + \frac{1}{r} \ln Y + \sum_{n=1}^N \frac{\beta_n}{r} \ln \omega_n - \frac{1}{r} (v - u) \tag{3.3.2}$$

Here $\omega_n = (\omega_1 \dots \omega_N)$ represents a vector of the input prices the producer is faced with, $\beta_n = (\beta_1 \dots \beta_N)$ is the vector parameter, and $r = \sum_{n=1}^N \beta_n$ indicates the returns to scale.

According to the above dual specification of the structure of the production, the firm can be above its cost frontier only by being below its production frontier (definition of technical inefficiency). The cost of that technical inefficiency is represented by $(1/r) u$, which measures the extra cost of producing below the production frontier.

Under the presence of allocative inefficiency, Schmidt and Lovell (1979) showed that if the producer is assumed to minimize costs, then the first order conditions for the cost minimization problem can be expressed as a system of equations involving (3.3.1) and the following N-1 first-order conditions:

$$\ln \left(\frac{X_1}{X_n} \right) = \ln \left(\frac{\beta_1 \omega_n}{\beta_n \omega_1} \right) + \vartheta_n \quad n = 2, \dots, N \tag{3.3.3}$$

where the term ϑ_n indicating the amount by which the first-order conditions for cost minimization fail to hold,⁴⁰ represents input allocative inefficiency for the input pair X_1 and X_n .

From (3.3.1) and (3.3.3), it is possible to obtain a set of input demand functions, and then the following expression for total cost that includes both the cost of technical efficiency $(1/r) u$ and the cost of input allocative inefficiency $(E - \ln r)$ is recovered as an analogous equation to (3.3.2),

$$\ln C = K + \frac{1}{r} \ln y + \sum_{n=1}^N \frac{\beta_n}{r} \ln \omega_n - \frac{1}{r} (v - u) + (E - \ln r) \tag{3.3.4}$$

where $E = \sum_{m=2}^N \frac{\beta_m}{r} \omega_m + \ln \left[\beta_1 + \sum_{m=2}^N \beta_m \exp(-\vartheta_m) \right]$

The Cobb-Douglas function used in Schmidt and Lovell (1979) imposes stringent separability restrictions on a neoclassical production function. Two functional

forms with no separability restrictions a priori are the Generalised Leontief Production function⁴¹ and the Transcendental Logarithmic Production function⁴² (translog function). Greene (1980b) uses the latter to estimate a frontier production model⁴³ under the following multiple equation system specification:

$$\begin{aligned}\log Y &= \ln \beta_0 + \sum_{n=1}^N \beta_n \ln X_n + \frac{1}{2} \sum_{n=1}^N \sum_{m=1}^N \delta_{nm} \ln X_n \ln X_m - \varepsilon_p \\ S_n &= \beta_i + \sum_{m=1}^N \delta_{nm} \ln X_m + \zeta_p\end{aligned}\quad (3.3.5)$$

where ε_p captures the effects of technical efficiency and ζ_p represents the deviation of the observed cost shares⁴⁴ (S_n) from the theoretical optimum. According to this specification of the model, ζ_p will capture the effects of allocative inefficiency only in the case of homothetic production functions. On the other hand, if the production structure is not homothetic, neither the input ratios of Schmidt and Lovell (1979), nor the factor cost shares are independent of the level of output, and therefore ζ_p can not be understood as allocative inefficiency.

By using duality theory tools, Greene (1980b) specified the following translog cost system:

$$\begin{aligned}\log C &= \beta_0 + \beta_Y \log Y + \frac{1}{2} \gamma_{YY} (\log Y)^2 + \\ &+ \sum_{n=1}^N \beta_n \log \omega_n + \frac{1}{2} \sum_{n=1}^N \sum_{m=1}^N \gamma_{nm} \log \omega_n \log \omega_m \\ &+ \sum_{n=1}^N \gamma_{Yn} \log Y \log \omega_n + \varepsilon_c \\ S_n &= \beta_n + \gamma_{Yn} \log Y + \sum_{m=1}^N \gamma_{nm} \ln \omega_m + \xi_c\end{aligned}\quad (3.3.6)$$

Assuming that firms are allocatively efficient, ε_c will measure the cost of technical inefficiency. If the production function is homogeneous of degree r , then $\varepsilon_c = (1/r)\varepsilon_p$. However, if the production function is not homogeneous, the relationship between them is not so clear, albeit it still depends on the degree of returns to scale. Finally, in a context where allocative inefficiency exists and the production function is homothetic, ε_c will represent the cost of both technical and allocative inefficiencies as in Schmidt and Lovell (1979), and ξ_c will represent purely allocative effects. With this formulation, the relationship between ε_c and ξ_c is not at all clear. These relationships are even more difficult to determine when the production function is not homothetic.⁴⁵

In summary, and assuming the most general case with allocative inefficiency, flexible technology of production, and a stochastic specification, cost systems that allow for cost inefficiency can be written as

$$\ln C = \ln C(Y, \omega) + \ln T + \ln A + v$$

$$S_n = S_i(Y, \omega) + \xi_c$$

where $\ln T \geq 0$ represents the increase in cost due to technical inefficiency, $\ln A \geq 0$ the increase in cost due to allocative inefficiency, v statistical noise, and ξ_c the disturbance on the input shares equations which is a mix of allocative inefficiency and noise.

Besides these specification problems, the other main handicap associated with these models is how to estimate the relationship between the two-sided error term in the input share equations with the nonnegative allocative inefficiency term in the cost equation.⁴⁶ Bauer (1990) distinguished three 'routes' that the literature has followed in order to solve this problem.

A first group of solutions is what Bauer (1990) calls qualitative solutions. They directly ignore the relationship between the allocative inefficiency term in the cost function and the one in the input share equations. This is the approach followed by Greene's (1980b) work.

A second group of solutions -the approximate solutions- model the relationships among the allocative inefficiency disturbances, by means of a function that approximate the real relationship in accordance with the a priori information that one has about its structure. Melfi (1984) and Bauer (1985) are listed in Bauer (1990)'s literature review as examples of this 'route'. A further contribution to this research agenda is the approach described in Kopp and Mullahy (1990) where some of the distributional assumptions that characterize most of the stochastic frontier cost models based on maximum-likelihood techniques are relaxed by applying Generalized method of moments estimation procedures.

Finally, analytic solutions try to look for the exact analytic relationship between the input and producer-specific allocative inefficiency error terms. That was the approach used in Schmidt and Lovell (1979, 1980) for the case in which the production structure is defined in accordance with a Cobb-Douglas technology. Kumbhakar (1988) extends this approach by incorporating multiple outputs and fixed inputs into a stochastic frontier model where allocative inefficiency is modelled as a departure from the first order cost minimization conditions and an additional input-specific technical inefficiency measure is retrieved from the maximum likelihood estimation of the model. Kumbhakar (1989) analyzes these input-specific measures of technical inefficiency in a symmetric-generalised-McFadden cost function framework. Other attempts to solve the Greene problem can be found in Ferrier and Lovell (1990) where both econometric and mathematical programming techniques are comparatively applied for measuring cost efficiency in banking sector.

Duality tools have contributed to a major and more accurate analysis of efficiency measurement. However, important avenues remain still to be explored.⁴⁷ We next briefly summarize some challenging paths for future investigation in this research field.

On the basis of the Atkinson and Cornwell (1993 and 1994a) approach for the determination of parametric representations of output and input technical inefficiency measures in a dual cost frontier framework, Atkinson and Cornwell

(1994b) derive a translog shadow cost system that permits the analyst to obtain and identify joint parametric estimates of input and producer-specific allocative inefficiency, and producer-specific technical inefficiency. Thereby avoiding the restrictive assumptions of previous approaches concerning the technological functional form and the inefficiency error terms' distributions.

Kumbhakar (1997) extends Schmidt and Lovell (1979)'s Cobb-Douglas production frontier specification to a more flexible translog cost frontier framework providing a theoretically and econometrically consistent analysis of allocative inefficiency but still at the cost of assuming that this is invariant across producers.

Finally, a complete analysis of the estimation and decomposition of profit efficiency by means of parametric frontier techniques is reported in Kumbhakar and Lovell (2000) that together with Cornwell and Schmidt (1996) contain a comprehensive and exhaustive treatment of dual panel data frontier techniques. A recent empirical application of these techniques to the banking sector can be found in Kumbhakar, Lozano-Vivas, Lovell, and Hasan (2001).

3.4 *New developments: Bayesian, multiple outputs and undesirable outputs models*

Besides the broad range of both primal and dual parametric approaches for the measurement of economic efficiency so far surveyed in this paper, and of the attempts of recent literature to overcome their main weakness by developing more evolved specification and estimation procedures, the need for these models to assume restrictive parametric assumptions is still a criticism to be surmounted.

In line with above, the use of Bayesian techniques within the efficiency measurement context, provides the researcher with a set of more flexible models. Bayesian models overcome the need to impose a priori sampling distributions on the efficiency term (u) of the composed error term that characterizes conventional stochastic frontier approaches.

Van den Broeck, Koop, Osiewalski and Steel (1994) and Koop, Osiewalski and Steel (1994) first introduced Bayesian analysis in the estimation of cross-section stochastic composed error models. Van den Broeck, Koop, Osiewalski and Steel (1994) treat uncertainty concerning which sampling model to use by mixing over a number of competing inefficiency distributions proposed in the literature with posterior model probabilities as weights. In doing so, they avoid the much criticized two-step procedure of Jondrow, Lovell, Materow and Schmidt (1982) permitting the direct posterior inference on firm-specific efficiencies. Koop, Osiewalski and Steel (1994) describe the use of Gibbs sampling⁴⁸ methods for drawing posterior inferences in a cost frontier model with an asymptotically ideal price aggregator, non-constant returns to scale, and composed error term. These seminal contributions show how the Bayesian analysis of stochastic frontier models is both theoretically and practically feasible.

Later research such as Koop, Osiewalski and Steel (1997) and Fernández, Osiewalski and Steel (1997) extends the use of Bayesian techniques for the measurement of economic efficiency to a panel data framework. Koop, Osiewalski and Steel (1997) show how, by using different prior structures, it is

possible to derive Bayesian analogues to classical fixed and random effects models. So, the Bayesian fixed effects models implemented in their paper are characterized by marginal prior independence between the individual effects, assumed to be constant over time but not linked across firms. And, the set of random effects models are derived assuming prior links between the individual effects, in such a way that their means can be functionally related to certain firm characteristics or alternatively be drawn from a common distribution.

Fernández, Osiewalski and Steel (1997) address the issue of verifying the existence of the posterior distribution and moments before conducting any Bayesian inference. They show that in pure cross-section models, posterior inference is precluded under the usual class of (partly) non-informative prior distributions. However, the disposability of panel data seems to overcome this problem by making use of them in imposing some structure upon the model. In summary, if we have panel data at our disposal as in Koop, Osiewalski and Steel (1997), Fernández, Osiewalski and Steel (1997), and also in Osiewalski and Steel (1998),⁴⁹ further methodological advances may be achieved by both allowing efficiency levels to depend on firm or DMU's characteristics, and providing Bayesian counterparts to the classical fixed and random effects stochastic frontier models.

The above papers have applied different Bayesian models for the measurement of economic efficiency to various data sets, however none of them attempt to implement a systematic comparison of all these techniques for a common data set. Kim and Schmidt (2000) do this. They apply a large number of Bayesian models to three previously-analyzed data sets, and compare the point estimates and confidence intervals for technical efficiency levels. Both classical procedures – including multiple comparisons with the best, based on the fixed effects estimates; a univariate version, marginal comparisons with the best; bootstrapping of the fixed effects estimates; and maximum likelihood given a distributional assumption – and Bayesian procedures – including a Bayesian version of the fixed effects model, and various Bayesian models with informative priors for efficiencies – are implemented in order to try to understand the relationship between the assumptions underlying these models and the empirical results.

Despite its recent incorporation to the body of techniques employed for the measurement of economic efficiency in a frontier framework, Bayesian models are the basis of some of the most recent and successful applied research. Koop, Osiewalski and Steel (1999, 2000), Nottebom, Coeck and van den Broeck (2000), Kleit and Terrell (2001), Tsionas (2001, 2002) are just some examples of this incipient literature. In this respect, Koop, Osiewalski and Steel (1999) use Bayesian stochastic frontier methods to decompose output change into technical, efficiency and input changes. On the same basis, Koop, Osiewalski and Steel (2000) seek to improve understanding of cross-country patterns of economic growth by assuming a production frontier depending on effective inputs rather than measured inputs. Kleit and Terrell (2001) examine the efficiency of electric power generation plants in the United States by using a Bayesian stochastic frontier model that imposes concavity and monotonicity restrictions. Finally, Tsionas (2001) describes the computational aspects of posterior inference and

posterior efficiency measurement using a basic Bayesian model as well as analysing alternative extensions to it. Precisely, one of these extensions, namely a Bayesian stochastic frontier model with random coefficients is the one utilised in Tsionas (2002) to separate technical inefficiency from technological differences across firms, and free the frontier model from the restrictive assumption that all firms must share exactly the same technological possibilities.

Bayesian techniques also allow parametric frontier modelling to deal with multiple outputs and undesirable outputs. The extension of Bayesian models to the case of multiple good outputs is more complicated since multivariate distributions must be used and various ways of defining efficiency exist. Koop (2001), Fernández, Koop and Steel (2000a, 2000b, 2002a) are some of the most recent references in the specification and estimation of multiple output Bayesian stochastic frontier models. Fernández, Koop and Steel (2000b, 2002b) broaden this methodology to the case where some of the outputs produced might be undesirable. This extension as Fernández, Koop and Steel (2002b) point out, not only involves a careful discussion of how to define the production technology for turning inputs into outputs, but also how to measure efficiency relative to this technology, and how to distinguish between technical and environmental efficiency.

Unlike previous literature which either assumes a classical econometric perspective with restrictive functional form assumptions (e.g., Kumbhakar, 1996; Löthgren, 1997; Adams, Berger and Sickles, 1999) or a non-stochastic approach which directly estimates the output distance function (e.g., Färe and Primont, 1990), the above Bayesian approaches calculate exact finite sample properties of all features of interest (including firm-specific efficiency) and surmount some of the statistical problems involved with classical estimation of stochastic frontier models.

Alternatively to Bayesian analysis, other research such as Hailu and Veeman (2000), Coelli and Perelman (2000) or Sickles, Good and Getachew (2002) are also attempting to analyze multiple output technologies by means of parametric frontier models. Hailu and Veeman (2000) employ a parametric input distance that incorporates both desirable and undesirable outputs so more environmentally sensitive productivity and efficiency measures can be obtained. Coelli and Perelman (2000) illustrate the usefulness of econometric distance functions in the analysis of production in multiple output industries, where behavioural assumptions such as cost minimization or profit maximization, are unlikely to be applicable. Finally, Sickles, Good and Getachew (2002) model a multiple output technology using a semi-parametric stochastic distance function where multivariate kernel estimators are introduced to address the endogeneity of multiple outputs.

4. Empirical evidence

A huge applied literature has treated the measurement of economic efficiency by means of parametric and non-parametric frontier techniques. These techniques

have been applied to a wide range of fields in Economics. Hunt-McCool, Koh and Francis (1996) or Stanton (2002) in finance; Adams, Berger and Sickles (1999), Fernández, Koop and Steel (2000a) or Lozano-Vivas and Humprey (2002) in banking; Wadud and White (2000) or Zhang (2002) in agriculture; Reinhard, Lovell and Thijssen (1999) or Amaza and Olayemi (2002) in environmental economics; Perelman and Pestieau (1994) or Worthington and Dollery (2002) in public economics; or Pitt and Lee (1981) and Thirtle, Shankar, Chitkara, Chatterjee and Mohanty (2000) in development economics, are just some recent examples of the relevance economists in diverse applied fields place on efficiency measurement.

However, despite the lack of accuracy that the exclusive use of either parametric or non parametric methods may cause due to their inherent limitations, and besides the important implications that the estimates reported by these methods can have on policy formulations, the few attempts have been made in recent literature to compare the proximity of both types of frontier approaches.

In this respect, one of the pioneering⁵⁰ comparative studies is the one of Ferrier and Lovell (1990). They measure cost efficiency of US banks by using a data set of 575 units with five outputs and three inputs each. For the parametric analysis, they specify a stochastic translog cost frontier dual cost function. The cost frontier is estimated by maximum likelihood procedure. The non-parametric approach is deterministic and follows the DEA model due to Banker, Charnes and Cooper -BCC- (1984). They found a 'lack of close harmony' between the two sets of efficiency scores, but more similar results as regards returns to scale properties. In accordance with their interpretation of the results, the differences between approaches are explained by the fact that a stochastic specification had been compared with a deterministic one.

In view of the research quoted above, Bjurek, Hjalmarsson and Forsund (1990) compare two parametric specifications, a Cobb-Douglas and a flexible quadratic function, both deterministic, with a deterministic non-parametric frontier on the basis of DEA techniques. They use a data set of about 400 social insurance offices, specifying four outputs and one input. In this case, the two parametric models yield quite close results. With respect to the non-parametric approach, DEA envelops the data more closely than the parametric models, resulting in more fully efficient units.

Forsund (1992) also deals with a comparative analysis of both parametric and non parametric approaches to a deterministic frontier. As a result, a homothetic deterministic frontier with a Cobb-Douglas kernel function and a Data Envelopment Analysis are applied to a dataset covering the case of Norwegian ferries in 1988. Forsund (1992)'s findings differ from Ferrier and Lovell (1990)'s ones of similarity as regards scale properties, but not efficiency distributions, where both methods report quite similar distributions, more in accordance with the overall conclusion of Bjurek, Hjalmarsson and Forsund (1990).

Later, Ray and Mukherjee (1995), re-examine Christensen and Greene's (1976) electrical utilities data set. Greene (1990) had previously used the same

data set to estimate individual efficiency scores by means of several alternative stochastic frontier specifications. Ray and Mukherjee (1995) compare Greene (1990)'s results with those derived from the application of DEA techniques. Following Varian (1984) and Banker and Maindiratta (1988), they get upper and lower bounds on cost efficiency for each observation. By using this extension of DEA, Ray and Mukherjee (1995) find that efficiency scores calculated under this procedure are close to those estimated by several parametric techniques.

More recently, Cummins and Zi (1998) have measured cost efficiency for a data set of 445 life insurers over the period 1988–1992, using a variety of parametric and non parametric frontier techniques. They evaluate these alternative techniques according to four criteria: average efficiency levels, rank correlations of efficiency levels, the consistency of methods in identifying best and worst practice units, and the correlation of efficiency scores with conventional performance measures. In doing so, they conclude that the choice of efficiency estimation method can make a significant effect on the conclusions of an efficiency study. In any case, they also recommend the use of more than one method for the measurement of economic efficiency in order to avoid specification errors.

Chakraborty, Biswas, and Lewis (2001) assess technical efficiency in public education using both stochastic parametric and deterministic non parametric methods. They define an educational production function for 40 school districts in Utah with a single output, a set of school inputs associated with the instructional and non instructional activities under the control of the school management, and non school inputs including status of the students and other environmental factors that may influence student productivity. The stochastic specification assumes half and exponential distributions for the inefficiency error term while the deterministic specification uses a two-stage DEA model in which efficiency levels from an output-oriented DEA using controllable school inputs only are regressed on the non school inputs using a Tobit regression model. According to their findings, Chakraborty, Biswas, and Lewis (2001) state that researchers can safely select any of the above methods without great concern for that choice having a large influence on the empirical results.

Finally, Murillo-Zamorano and Vega-Cervera (2001) apply a broad range of econometric and mathematical programming frontier techniques to an industrial organisation setup corresponding to a sample of 70 US (investor-owned) electric utility firms in 1990. Their results suggest that the choice of parametric or non-parametric techniques, deterministic or stochastic approaches, or between different distribution assumptions within stochastic techniques seems not to be relevant if one is interested in ranking productive units in terms of their individual efficiency scores. Murillo-Zamorano and Vega-Cervera (2001) focus on the definition of a framework for the joint use of these techniques in order to avoid the weaknesses inherent in each, and benefit from the strong aspects of the two methods. Their findings also provide encouragement for the continued development of the collaboration between parametric and non-parametric methods.

5. Concluding Remarks

We have analysed a wide range of different techniques dedicated to the measurement of economic efficiency. The main issue throughout was to determine an efficient frontier function or envelopment surface, in order to compare the performance of different units with the one that characterises the efficient geometric site.

These efficiency measurement techniques may be classified in different ways. Our criterion has been to distinguish between parametric and non-parametric methods. A vast literature has treated the measurement of economic efficiency by means of both parametric and non-parametric approaches. In our opinion, and the above empirical studies seem to confirm it, no approach is strictly preferable to any other. As has been shown throughout this survey, each of them has its own advantages and disadvantages. A careful consideration of them, of the data set utilized, and of the intrinsic characteristics of the industry under analysis will help us in the correct implementation of these techniques.

In any case, the present survey calls for more research. The implementation of comparative analysis between parametric and non parametric frontier techniques – such as the ones described in previous section – the integration of the two types of approaches through two-step models like the one used in Sengupta (1995b), further research on misspecification problems (e.g. Smith, 1997) and on the quality (e.g. Pedraja, Salinas and Smith, 1999) of Data Envelopment Analysis, and extra investigation on the measurement of economic efficiency in a dynamic context as the one presented in Sengupta (1999b and 2000b) might constitute the basis for future theoretical and applied research.

In summary, it would be desirable to introduce more flexibility into the parametric frontier approach, as well as to go more deeply into the analysis of stochastic non-parametric methods and their statistical properties. In this respect, some new routes are explored in Kumbhakar and Lovell (2000), Fernández, Koop and Steel (2002a, 2002b), or Sickles, Good and Getachew (2002) regarding the former, and in Sengupta (2000a), Simar and Wilson (2000a, 2000b) and Huang and Li (2001) concerning the latter. These studies constitute a set of alternative, complementary and challenging attempts to achieve better and more reliable efficiency measures. The lunch is served!

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Notes

1. According to the activity analysis developed in Koopmans (1951), a producer is said to be technically inefficient when it can produce the same amount of output with less of at least one input, or can use the same package of inputs to produce more of at

- least one output. This definition establishes the twofold orientation – output augmenting and input reducing – of the technical component of economic efficiency. The measure of technical efficiency introduced in Debreu (1951), and initially termed as the coefficient of resource utilization, is defined as one minus the maximum equiproportionate reduction in all inputs that still allows the production process to continue. A critical discussion of the differences between the Koopmans definition and the Debreu-Farrell measures of technical efficiency can be found in Lovell (1993).
2. See Cornes (1992) for an introductory treatment of this issue and Rodriguez-Alvarez (2001) for an updated survey of recent approaches to the measurement of allocative inefficiency by means of distance functions.
 3. An exhaustive summary with over 400 articles (1978–1995) can be found in Seiford (1996). Detailed reviews are also presented in Seiford and Thrall (1990), Lovell (1993), Ali and Seiford (1993), Lovell (1994), Charnes, Cooper, Lewin and Seiford (1994) and Coelli, Rao and Battese (1998). A later and complete analysis of DEA techniques can be found in Cooper, Seiford and Tone (2000).
 4. Unlike the parametric approach which assumes the existence of a specific transformation technology that determines what maximum amounts of outputs can be produced from different combinations of inputs, the starting point for DEA is the construction, from the observed data, of a piecewise empirical production frontier.
 5. Notice that the DEA method produces only *relative* efficiency measures, since they are generated from actual observations for each DMU.
 6. This is a technical feature of the DEA model; units located at the horizontal and vertical extremes of the efficient isoquant frontier will be assigned efficiency values of unity, but those units are not Pareto efficient. The correct treatment of this issue implies the introduction of slacks into the input or output constraints of the mathematical linear program that solves the DEA model. A detailed analysis of slacks in DEA models can be found in Ali and Seiford (1993) and Ali (1994).
 7. The first constraint also ensures that the projected DMU will utilise inputs in the same proportions as the unit being analysed under constant returns to scale.
 8. This ratio formulation was the one initially presented in Charnes, Cooper and Rhodes (1978).
 9. Note that the change of notation identifies a different linear program. A more detailed analysis of this issue can be found in Lovell (1994).
 10. The variable returns to scale DEA model is usually referred to as – the BCC model – after Banker, Charnes, and Cooper (1984).
 11. The slack-based measure of efficiency (SBM) model developed by Tone (2001) is an example of a non-oriented model.
 12. Further treatment of categorical variables and mixed integer linear programming models can be found in Kamakura (1988) and Rouseeou and Semple (1993).
 13. Although Banker and Morey (1986a) is usually referred to as the main contribution to the analysis of non-discretionary variables, Koop (1981) had treated this issue previously. Färe, Grosskopf and Lovell (1994) also analyze the non-discretionary variables by what they referred to as ‘sub-vector optimisations’.
 14. Cooper, Park and Pastor (1999) have recently extended these approaches in additive models that permit input/output substitutions in allocative efficiency evaluations by developing what they termed as a range-adjusted measure of inefficiency.
 15. The reader is referred to Efron (1979) or Efron and Tibshirani (1993) for an introduction to the bootstrap.

16. In what follows the analysis is made on the basis of single-output production functions. Multiple output functions will be analysed in a later section.
17. The Cobb-Douglas functional form has been widely used in the empirical calculus/estimation of technological frontier functions. However, the Cobb-Douglas form has a number of restrictive properties, such as the constant input elasticities and returns to scale for all the units under analysis or the imposition for elasticities of substitution to be one. These properties have pushed frontier literature to the use of other more flexible alternative functional forms. Among them, the most relevant are the translog and the Zellner-Revankar ones. The latter avoids the returns to scale restriction while the former imposes no restrictions upon returns to scale or substitution possibilities. For a more detailed analysis of these or other functional form properties, the reader is referred to Blackorby, Primont and Russell (1978) or Fuss and McFadden (1978).
18. During at least fifty years and since Cobb and Douglas (1928), most of the empirical studies devoted to the estimation of production functions through econometric techniques were based on an ordinary least squares (OLS) method. OLS techniques for the estimation of production functions allow for the presence of a positive as well as negative residual. As a result, for a long time econometricians were estimating 'average' production functions based on the mean output rather than on the maximum output, despite the efficient nature of such concepts as minimum cost functions, profit-maximizing output supply or cost minimizing input demand functions on which the economic theory relies.
19. The reader is referred to Kalirajan and Shand (1999) and Murillo-Zamorano (2002) for a comprehensive treatment of both mathematical programming and deterministic econometric models.
20. The analysis developed in this section is made on the grounds of a scalar output. Generalization to a multiple output-multiple input is treated in a posterior section. Some recent references in this area of research are Coelli and Perelman (1996a), Fuentes, Grifell-Tatje and Perelman (1997), Reinhard and Thijssen (1997), Atkinson, Färe and Primont (1998) and Atkinson and Primont (1998).
21. Based on OLS results and asymptotic theory, Schmidt and Lin (1984) first and more recently, Coelli (1995) have proposed tests for the presence of technical inefficiency in the data. These tests check the skewness of the composed error term in such a way that a negatively (positively) skewed composed error term suggests the existence (non-existence) of technical inefficiency.
22. 'The inconsistency of the estimator of u_i is unfortunate in view of the fact that the purpose of the exercise to begin with is to estimate inefficiency. It would appear, however, that no improvement on this measure for the single-equation, cross sectional framework considered here is forthcoming' (Greene 1993, p. 81).
23. Normal-gamma models are roundly criticised in Ritter and Simar (1997): 'For maximum likelihood inference, we show here that the problem of estimating the four parameters β , σ^2 , Θ , and P is poorly conditioned for samples of up to several hundreds of observations. As a consequence, estimates of the main quantities of interest suffer from substantial imprecision, are ambiguous, or cannot be calculated at all.' Ritter and Simar (1997, p. 168).
24. Many applied papers such as Battese and Coelli (1988&1992), have tested normal-half-normal models against normal-truncated models and frequently the latter have not rejected the former.

25. The estimation of stochastic frontier functions so far analysed relies on MLE techniques. An alternative approach for the estimation of all parameters involved in the stochastic model invokes the use of the method of moments. Olson, Schmidt and Waldman (1980) describe such a technique in a normal-half normal framework. Harris (1992) utilises the method of moments for the normal-truncated case, and Greene (1993, 1997b) discusses the use of this estimation procedure for the normal-exponential and for the normal-gamma cases.
26. A detailed analysis of these methods can be found in Griffiths, Hill and Judge (1993).
27. Atkinson and Cornwell (1994b) consider single production/cost frontier analysis as *limited information* approaches compared with those *full information* estimates derived from more complex profit/cost systems. Schmidt and Sickles (1984) study belongs to the former cases. We follow their approach in this section. Full information models will be treated later. Kumbhakar (1990, 1991a) and Seale (1990) are some examples of these other models that allow for the estimation of both technical and allocative efficiencies.
28. In a context where both error terms were independent over time as well as across individuals, panel data nature would be irrelevant and a pooled estimation of all observations could be implemented at any time period. Moreover, the technical inefficiency error term can be allowed to vary across producers and over time for each productive unit. These time-variant technical efficiency models are commented on later.
29. Capital stock, location or institutional factors can be some examples of time-invariant attributes.
30. An exhaustive analysis of the two-step generalised least squares treatment for stochastic frontier models can be found in Greene (1997a).
31. Based on Hausman's (1978) test for a fixed-effects estimator versus a GLS estimator, Hausman and Taylor (1981) develop a test of the uncorrelation hypothesis required in stochastic frontier random-effects model. Hausman and Taylor (1981), assuming that the effects are uncorrelated with some but not all of the regressors, also allows for the treatment of time-invariant variables. In that framework, individual effects can be consistently estimated and separated from the intercept term as long as cross-sectional and temporal observations are large enough.
32. Gong and Sickles (1989), Gathon and Perelman (1992), Bauer, Berger and Humphrey (1993), Bauer and Hancock (1993) or Ahmad and Bravo-Ureta (1996) are examples of these comparative analyses.
33. When technical inefficiencies are correlated with the regressors, the EIV approach is preferable to a GLS one in that it provides consistent estimates unlike GLS, which although remaining more efficient than the fixed-effects estimator, is inconsistent if technical inefficiency is correlated with regressors.
34. A generalized method of moments is proposed to estimate the Lee and Schmidt model in Ahn, Lee and Schmidt (1994).
35. Following the assumption most usually made in dual frontier analysis, i.e. cost minimization, the analysis made in this section focuses on cost frontier dual functions. Duality theory also allows for such other alternative indirect representations of the technological set as revenue, profit or distance functions. A complete and updated revision of these alternative representations can be found in Kumbhakar and Lovell (2000).
36. In such regulated industrial sectors as the electric sector, output is usually considered exogenous, as well as are the input prices (competitive markets). In these cases, the use

of cost functions is usually preferable. On the other hand, if inputs levels, not output, are exogenous, the estimation of a production function seems to be the more natural approach.

37. Berndt and Wood (1975) assumed the existence of constant returns to scale. Under that model specification, all relevant parameters can be derived from the cost share equations. However, when constant returns to scale are not imposed, some relevant parameters, such as those which determine the economies of scale, can not be estimated.
38. See for example Greene (1990).
39. Greene (1980b), Ferrier and Lovell (1990), Kumbhakar (1991b).
40. Schmidt and Lovell (1979) initially assumed ϑ_n to be random, with a zero mean. Later, this assumption was relaxed allowing ϑ_n to represent a systematic tendency to over-utilise any input relative to any other. On the other hand, ϑ_n is also assumed to be independent from the u and v error terms. In a subsequent paper, Schmidt and Lovell (1980) analyse the duality issue, allowing them to be correlated.
41. See Diewert (1971).
42. Christensen, Jorgenson and Lau (1971, 1973) and Christensen (1973).
43. The approach of this paper, unlike that of Schmidt and Lovell (1979, 1980) is deterministic. Greene (1980b) considers a gamma density function for the one-sided inefficiency error term. Later, Greene (1982) will use the gamma distribution in a stochastic context.
44. Schmidt and Lovell (1979) used input ratios instead of factor cost shares. Nevertheless, the two formulations are informationally equivalent.
45. As in Schmidt and Lovell (1979), Greene (1980b) treats ε_c and ξ_c as statistically independent.
46. This problem ('the Greene problem') was firstly noted by Greene (1980b) although no way to solve it was offered and, as we have seen, the two error terms were considered independent. Nadiri and Schankerman (1981) also analysed those relationships.
47. Other alternative dual approaches to the measurement of allocative efficiency out of the scope of this survey are the deterministic approach of Kopp and Diewert (1982) based in the analysis of the cost minimizing pure demands implied by Shephard's lemma, the comparative analysis between cost minimizing demands and actual prices developed by Sickles, Good and Johnson (1986) and Eakin and Kniesner (1988), the Distribution Free approach of Berger (1993) or the Thick Frontier analysis developed in Berger and Humphrey (1991&1992) and recently applied in Lang and Welzel (1998) and Gropper, Caudill, Beard, Randolph (1999).
48. The Gibbs sampler is a technique for obtaining a random sample from a joint distribution by taking random draws only from the full conditional distributions. Gelfand and Smith (1990), Casella and George (1992) or Koop (1994) are introductory references for Gibbs sampler. A discussion of the use of Gibbs sampling techniques in stochastic frontier models with cross-section data is given in Koop, Steel and Osiewalski (1995).
49. On the basis of related literature above mentioned, Osiewalski and Steel (1998) describe the use of modern numerical integration methods for implementing posterior inferences in composed error stochastic frontier models for panel data or individual cross-sections.
50. Before, Banker, Conrad and Strauss (1986) compared results based on DEA with an econometrically estimated cost function by means of corrected ordinary least squares. Banker, Charnes, Cooper and Maindiratta (1988), and Banker, Gadh and Gorr (1990) also deal with this type of comparison but based on generating artificial data from prespecified parametric technologies.

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