**Chapter 20**

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**Serial Correlation**

**Exercises**

1. For the first order autoregressive model, the autocorrelation is ρ. Consider the first difference, *vt* =

ε*t* ‑ ε*t*-1 which has Var[*vt*] = 2Var[ε*t*] ‑ 2Cov[(ε*t*,ε*t*-1)] = 2σ*u*2[1/(1 - ρ2) - ρ/(1 - ρ2)] = 2σ*u*2/(1 + ρ) and Cov[*vt*,*vt-*1] = 2Cov[ε*t*,ε*t*-1] ‑ Var[ε*t*] ‑ Cov[ε*t*,ε*t*-1] = σ*u*2[1/(1 - ρ2)][2ρ - 1 - ρ2] = σ*u*2[(ρ - 1)/(1 + ρ)]. Therefore, the autocorrelation of the differenced process is Cov[*vt*,*vt*-1] / Var[*vt*] = (ρ ‑ 1) / 2. As the figure below on the left shows, first differencing reduces the absolute value of the autocorrelation coefficient when ρ is greater than 1/3. For economic data, this is likely to be fairly common.



1



For the moving average process, the first order autocorrelation is Cov[(ε*t*,ε*t*-1)]/Var[ε*t*] = ‑λ/(1 + λ2). To obtain the autocorrelation of the first difference, write ε*t* ‑ ε*t*-1 = *ut* ‑ (1 + λ)*ut*-1 + λ*ut*-2 and ε*t*-1 ‑ ε*t*-2 =

*ut-*1 ‑ (1 + λ)*ut*-2 + λ*ut*-3. The variance of the difference is Var[ε*t* ‑ ε*t*-1] = σ*u*2[(1 + λ)2 + (1 + λ2)]. The covariance can be found by taking the expected product of terms with equal subscripts. Thus, Cov[ε*t* ‑ ε*t*-1,ε*t*-1 ‑ εt-2] = ‑σ*u*2(1 + λ)2. The autocorrelation is Cov[ε*t* ‑ ε*t*-1,ε*t*-1 ‑ ε*t*-2]/Var[ε*t* ‑ ε*t*-1] = ‑ (1 + λ)2/[(1 + λ)2 + (1 + λ2)]. A plot of the relationship between the differenced and undifferenced series is shown in the right panel above. The horizontal axis plots the autocorrelation of the original series. The values plotted are the absolute values of the difference between the autocorrelation of the differenced series and the original series. The results are similar to those for the AR(1) model. For most of the range of the autocorrelation of the original series, differencing increases autocorrelation. But, for most of the range of values that are economically meaningful, differencing reduces autocorrelation.

2. Derive the disturbance covariance matrix for the model *yt*  = **β′x***t* + ε*t*, ε*t* = ρε*t*-1 + u**t** ‑ λu*t*-1. What parameter is estimated by the regression of the ordinary least squares residuals on their lagged values?

Solve the disturbance process in its moving average form. Write the process as ε*t* ‑ ρε*t*-1 = *ut* ‑ λ*ut*-1 or, using the lag operator, ε*t*(1 ‑ ρ*L*) = *ut* ‑ λ*ut*-1 or εt = *u*t/(1 ‑ ρ*L*) ‑ λ*u*t-1/(1 ‑ ρ*L*). After multiplying these out, we obtain ε*t*  = *ut* + ρ*ut*-1 + ρ2*ut*-2 + ρ3*ut*-3 + ... ‑ λ*ut*-1 ‑ ρλ*ut*-2 ‑ ρ2λ*ut-*3 ‑ ...

= *ut* + (ρ‑λ)*ut-*1 + ρ(ρ‑λ)*ut*-2 + ρ2(ρ‑λ)*ut-*3 + ...

Therefore, Var[ε*t*] = σ*u*2(1 + (ρ‑λ)2)(1 + ρ2 + ρ4 + ...) = σ*u*2(1 + (ρ‑λ)2/(1 ‑ ρ2))

= σ*u*2(1 + λ2 ‑ 2ρλ)/(1 ‑ ρ2)

Cov[ε*t*,ε*t*-1] = ρVar[ε*t*-1] + Cov[ε*t*-1,*ut*] ‑ λCov[ε*t*-1,*ut*-1].

To evaluate this expression, write

ε*t-*1 = *ut-*1 + (ρ‑λ)*ut-*2 + ρ(ρ‑λ)*ut*-3 + ρ2(ρ‑λ)*ut*-4+ ...

Therefore, the middle term is zero and the third is simply λσu2. Thus,

Cov[ε*t*,ε*t*-1] = σ*u*2{[ρ(1 + λ2 ‑ 2ρλ)]/(1 ‑ ρ2) ‑ λ]} = σ*u*2[(ρ - λ)(1 - λρ)/(1 ‑ ρ2)]

For lags greater than 1, Cov[ε*t*,ε*t-j*] = ρCov[ε*t*-1,ε*t-j*] + Cov[ε*t-j*,*ut*] ‑ λCov[ε*t-j*,*ut*-1].

Since ε*t-j* involves only *u*s up to its current period, ε*t-j* is uncorrelated with *ut* and *ut*-1 if *j* is greater than 1. Therefore, after the first lag, the autocovariances behave in the familiar fashion, Cov[ε*t*,ε*t-j*] = ρCov[ε*t*,ε*t*-*j*+1]

The autocorrelation coefficient of the residuals estimates Cov[ε*t*,ε*t*-1]/Var[ε*t*] = (ρ - λ)(1 - ρλ)/(1 + λ2 - 2ρλ).

3. Since the regression contains a lagged dependent variable, we cannot use the Durbin‑Watson statistic directly. The *h* statistic in (15‑34) would be *h* = (1 ‑ 1.21/2)[21 / (1 ‑ 21(.182)]1/2 = 3.201. The 95% critical value from the standard normal distribution for this one‑tailed test would be 1.645. Therefore, we would reject the hypothesis of no autocorrelation.

4. It is commonly asserted that the Durbin‑Watson statistic is only appropriate for testing for first order autoregressive disturbances. What combination of the coefficients of the model is estimated by the Durbin‑Watson statistic in each of the following cases: AR(1), AR(2), MA(1)? In each case, assume that the regression model does not contain a lagged dependent variable. Comment on the impact on your results of relaxing this assumption.

In each case, plim *d* = 2 ‑ 2ρ1 where ρ1 = Corr[ε*t*,ε*t*-1]. The first order autocorrelations are as follows: AR(1): ρ (see (15‑9)) and AR(2): θ1/(1 ‑ θ2). For the AR(2), a proof is as follows: First, ε*t* = θ1ε*t*-1 + θ2ε*t*-2 + *ut*. Denote Var[ε*t*] as *c*0 and Cov[ε*t*,ε*t*-1] as *c*1. Then, it follows immediately that *c*1 = θ1*c*0 + θ2*c*1 since *ut* is independent of ε*t*-1. Therefore ρ1 = *c*1/*c*0 = θ1/(1 - θ2). For the MA(1): ‑λ / (1 + λ2) (See (15‑43)). To prove this, write ε*t* = *ut* - λ*ut*-1. Then, since the *u*s are independent, the result follows just by multiplying out ρ1 = Cov[ε*t*,ε*t*-1]/Var[ε*t*] = -λVar[*ut*-1]/{Var[*ut*] + λ2Var[*ut*-1]} = -λ/(1 + λ2).

**Applications**

1. Phillips Curve

--> date;1950.1$

--> peri;1950.1-2000.4$

--> crea;dp=infl-infl[-1]$

--> crea;dy=loggdp-loggdp[-1]$

--> peri;1950.3-2000.4$

--> regr;lhs=dp;rhs=one,unemp$;ar1;res=u$

+-----------------------------------------------------------------------+

| Ordinary least squares regression Weighting variable = none |

| Dep. var. = DP Mean= -.1926996283E-01, S.D.= 2.818214558 |

| Model size: Observations = 202, Parameters = 2, Deg.Fr.= 200 |

| Residuals: Sum of squares= 1592.321197 , Std.Dev.= 2.82163 |

| Fit: R-squared= .002561, Adjusted R-squared = -.00243 |

| Model test: F[ 1, 200] = .51, Prob value = .47449 |

| Diagnostic: Log-L = -495.1583, Restricted(b=0) Log-L = -495.4173 |

| LogAmemiyaPrCrt.= 2.084, Akaike Info. Crt.= 4.922 |

| Autocorrel: Durbin-Watson Statistic = 2.82755, Rho = -.41378 |

+-----------------------------------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant .4918922148 .74047944 .664 .5073

UNEMP -.9013159906E-01 .12578616 -.717 .4745 5.6712871

**--> peri;1951.2-2000.4$**

**--> regr;lhs=u;rhs=one,u[-1],u[-2]$**

+-----------------------------------------------------------------------+

| Ordinary least squares regression Weighting variable = none |

| Dep. var. = U Mean= -.3890391012E-01, S.D.= 2.799476915 |

| Model size: Observations = 199, Parameters = 3, Deg.Fr.= 196 |

| Residuals: Sum of squares= 1079.052269 , Std.Dev.= 2.34635 |

| Fit: R-squared= .304618, Adjusted R-squared = .29752 |

| Model test: F[ 2, 196] = 42.93, Prob value = .00000 |

| Diagnostic: Log-L = -450.5769, Restricted(b=0) Log-L = -486.7246 |

| LogAmemiyaPrCrt.= 1.721, Akaike Info. Crt.= 4.559 |

| Autocorrel: Durbin-Watson Statistic = 1.99273, Rho = .00363 |

+-----------------------------------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant -.5048615289E-01 .16633422 -.304 .7618

U[-1] -.5946344724 .65920584E-01 -9.020 .0000 -.10234931E-01

U[-2] -.3824653303 .65904378E-01 -5.803 .0000 -.14370453E-01

(Note: E+nn or E-nn means multiply by 10 to + or -nn power.)

**--> calc;list;lm=n\*rsqrd$**

LM = .60618960968412850D+02

+---------------------------------------------+

| AR(1) Model: e(t) = rho \* e(t-1) + u(t) |

| Initial value of rho = -.41378 |

| Maximum iterations = 100 |

| Method = Prais - Winsten |

| Iter= 1, SS= 1299.275, Log-L=-474.710175 |

| Final value of Rho = -.413779 |

| Iter= 1, SS= 1299.275, Log-L=-474.710175 |

| Durbin-Watson: e(t) = 2.827557 |

| Std. Deviation: e(t) = 2.799716 |

| Std. Deviation: u(t) = 2.548799 |

| Durbin-Watson: u(t) = 2.340706 |

| Autocorrelation: u(t) = -.170353 |

| N[0,1] used for significance levels |

+---------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant .4704274598 .47671946 .987 .3237

UNEMP -.8709854633E-01 .80962277E-01 -1.076 .2820 5.6712871

RHO -.4137785986 .64213081E-01 -6.444 .0000

Regression results are almost unchanged. Autocorrelation of transformed residuals is -.17, less than -.41 in original model.

2. (Improved Phillips curve model)

**--> crea;newecon=dmy(1974.1,2000.4)$**

**--> regr;lhs=dp;rhs=one,unemp,newecon;plot$**

+-----------------------------------------------------------------------+

| Ordinary least squares regression Weighting variable = none |

| Dep. var. = DP Mean= -.1926996283E-01, S.D.= 2.818214558 |

| Model size: Observations = 202, Parameters = 3, Deg.Fr.= 199 |

| Residuals: Sum of squares= 1586.260338 , Std.Dev.= 2.82332 |

| Fit: R-squared= .006357, Adjusted R-squared = -.00363 |

| Model test: F[ 2, 199] = .64, Prob value = .53017 |

| Diagnostic: Log-L = -494.7731, Restricted(b=0) Log-L = -495.4173 |

| LogAmemiyaPrCrt.= 2.091, Akaike Info. Crt.= 4.928 |

| Autocorrel: Durbin-Watson Statistic = 2.83473, Rho = -.41737 |

+-----------------------------------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant .5507626279 .74399306 .740 .4600

UNEMP -.9835166981E-01 .12621412 -.779 .4368 5.6712871

NEWECON -2.474910396 2.8382661 -.872 .3843 .49504950E-02

3. (GARCH Models)

.a. We used LIMDEP with the macroeconomics data in table F5.1. The rate of inflation was computed with all observations, then observations 6 to 204 were used to remove the missing data due to lags. Least squares results were obtained first. The residuals were then computed and squared. Using observations 15-204, we then computed a regression of the squared residual on a constant and 8 lagged values. The chi-squared statistic with 8 degrees of freedom is 28.24. The critical value from the table for 95% significance and 8 degrees of freedom is 15.51, so at this level of significance, the hypothesis of no GARCH effects is rejected.

crea;pt=100\*log(cpi\_u/cpi\_u[-1])$

crea;pt1=pt[-1];pt2=pt[-2];pt3=pt[-3];pt4=pt[-4]$

samp;6-204$

regr;lhs=pt;rhs=one,pt1,pt2,pt3,pt4;res=et$$

crea;vt=et\*et$

crea;vt1=vt[-1];vt2=vt[-2];vt3=vt[-3];vt4=vt[-4];vt5=vt[-5];vt6=vt[-6];vt7=vt[-7];vt8=vt[-8]$

samp;15-204$

regr;lhs=vt;rhs=one,vt1,vt2,vt3,vt4,vt5,vt6,vt7,vt8$

calc;list;lm=n\*rsqrd$

+-----------------------------------------------------------------------+

| Ordinary least squares regression Weighting variable = none |

| Dep. var. = PT Mean= .9589185961 , S.D.= .8318268241 |

| Model size: Observations = 199, Parameters = 5, Deg.Fr.= 194 |

| Residuals: Sum of squares= 61.97028507 , Std.Dev.= .56519 |

| Fit: R-squared= .547673, Adjusted R-squared = .53835 |

| Model test: F[ 4, 194] = 58.72, Prob value = .00000 |

| Diagnostic: Log-L = -166.2871, Restricted(b=0) Log-L = -245.2254 |

| LogAmemiyaPrCrt.= -1.116, Akaike Info. Crt.= 1.721 |

| Autocorrel: Durbin-Watson Statistic = 1.80740, Rho = .09630 |

+-----------------------------------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant .1296044455 .67521735E-01 1.919 .0564

PT1 .2856136998 .69863942E-01 4.088 .0001 .97399582

PT2 .1237760914 .70647061E-01 1.752 .0813 .98184918

PT3 .2516837602 .70327318E-01 3.579 .0004 .99074774

PT4 .1824670634 .69251374E-01 2.635 .0091 .98781131

LM = .28240022492847690D+02

For the second step, we need an estimate of α0, which is the unconditional variance if there are no ARCH effects. We computed this based on the ARCH specification by a regression of et2 – (8/36)et-12 - … - (1/36)et-82 on just a constant term. This produces a negative estimate of α0, but this is not the variance, so we retain the result. We note, the problem that this reflects is probably the specific, doubtless unduly restrictive, ARCH structure assumed.

samp;6-204$

crea;vt=et\*et$

crea;ht=vt-8/36\*vt[-1]-7/36\*vt[-2]-6/36\*vt[-3]-5/36\*vt[-4]-4/36\*vt[-5]-3/36\*vt[-6]-2/36\*vt[-7]-1/36\*vt[-8]$

samp;15-204$

calc;list;a0=xbr(ht)$

samp;6-204$

crea;qt=a0+8/36\*vt[-1]+7/36\*vt[-2]+6/36\*vt[-3]+5/36\*vt[-4]+4/36\*vt[-5]+3/36\*vt[-6]+2/36\*vt[-7]+1/36\*vt[-8]$

samp;15-204$

plot;rhs=qt$

crea;wt=1/qt$

regr;lhs=pt;rhs=one,pt1,pt2,pt3,pt4;wts=wt$

regr;lhs=pt;rhs=one,pt1,pt2,pt3,pt4;model=garch(1,1)$

Once we have an estimate of α0 in hand, we then computed the set of variances according to the ARCH(8) model, using the lagged squared residuals. Finally, we used these variance estimators to compute a weighted least squares regression accounting for the heteroscedasticity. This regression is based on observations 15-204, again because of the lagged values. Finally, using the same sample, a GARCH(1,1) model is fit by maximum likelihood.

+-----------------------------------------------------------------------+

| Ordinary least squares regression Weighting variable = WT |

| Dep. var. = PT Mean= .8006997687 , S.D.= .6327877239 |

| Model size: Observations = 190, Parameters = 5, Deg.Fr.= 185 |

| Residuals: Sum of squares= 38.67492770 , Std.Dev.= .45722 |

| Fit: R-squared= .488964, Adjusted R-squared = .47791 |

| Model test: F[ 4, 185] = 44.25, Prob value = .00000 |

| Diagnostic: Log-L = -147.7324, Restricted(b=0) Log-L = -211.5074 |

| LogAmemiyaPrCrt.= -1.539, Akaike Info. Crt.= 1.608 |

| Autocorrel: Durbin-Watson Statistic = 1.90310, Rho = .04845 |

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant .1468553158 .60127085E-01 2.442 .0155

PT1 .9760051110E-01 .88469908E-01 1.103 .2714 .77755556

PT2 .3328520370 .86772549E-01 3.836 .0002 .76745308

PT3 .1428889148 .85420554E-01 1.673 .0961 .76271761

PT4 .2878686524 .84090832E-01 3.423 .0008 .74173558

The 8 period ARCH model produces quite a substantial change in the estimates. Once again, this probably results from the restrictive assumption about the lag weights in the ARCH model. The GARCH model follows.+---------------------------------------------+

| GARCH MODEL |

| Maximum Likelihood Estimates |

| Model estimated: Jul 31, 2002 at 01:19:14PM.|

| Dependent variable PT |

| Weighting variable None |

| Number of observations 190 |

| Iterations completed 22 |

| Log likelihood function -135.5043 |

| Restricted log likelihood -147.6465 |

| Chi squared 24.28447 |

| Degrees of freedom 2 |

| Prob[ChiSqd > value] = .5328953E-05 |

| GARCH Model, P = 1, Q = 1 |

| Wald statistic for GARCH = 521.483 |

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+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Regression parameters

Constant .1308478127 .61887183E-01 2.114 .0345

PT1 .1749239917 .70912277E-01 2.467 .0136 .98810078

PT2 .2532191617 .73228319E-01 3.458 .0005 .98160455

PT3 .1552879436 .68274176E-01 2.274 .0229 .97782066

PT4 .2751467919 .63910272E-01 4.305 .0000 .97277700

Unconditional Variance

Alpha(0) .1005125676E-01 .11653271E-01 .863 .3884

Lagged Variance Terms

Delta(1) .8556879884 .89322732E-01 9.580 .0000

Lagged Squared Disturbance Terms

Alpha(1) .1077364862 .60761132E-01 1.773 .0762

Equilibrium variance, a0/[1-D(1)-A(1)]

EquilVar .2748082674 2.0559946 .134 .8937

**Chapter 21**

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**Exercise**

1. The autocorrelations are simple to obtain just by multiplying out vt2, vtvt-1 and so on. The autocovariances are 1+θ12 + θ22, -θ2(1 - θ2), -θ2, 0, 0, 0... which provides the autocorrelations by division by the first of these. The partial autocorrelations are messy, and can be obtained by the Yule Walker equations. Alternatively (and much more simply), we can make use of the observation in Section 21.2.3 that the partial autocorrelations for the MA(2) process mirror tha autocorrelations for an AR(2). Thus, the results in Section 21.2.3 for the AR(2) can be used directly.

**Applications**

1. ADF Test

+-----------------------------------------------------------------------+

| Ordinary least squares regression Weighting variable = none |

| Dep. var. = R Mean= 8.212678571 , S.D.= .7762719558 |

| Model size: Observations = 56, Parameters = 6, Deg.Fr.= 50 |

| Residuals: Sum of squares= .9651001703 , Std.Dev.= .13893 |

| Fit: R-squared= .970881, Adjusted R-squared = .96797 |

| Model test: F[ 5, 50] = 333.41, Prob value = .00000 |

| Diagnostic: Log-L = 34.2439, Restricted(b=0) Log-L = -64.7739 |

| LogAmemiyaPrCrt.= -3.846, Akaike Info. Crt.= -1.009 |

| Autocorrel: Durbin-Watson Statistic = 1.91589, Rho = .04205 |

+-----------------------------------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant .2565690959 .47172815 .544 .5889

T .4401352136E-03 .25092142E-02 .175 .8615 32.500000

R1 .9653227410 .48183346E-01 20.034 .0000 8.2305357

DR1 .5600009441 .14342088 3.905 .0003 -.12321429E-01

DR2 -.1739775168 .14781417 -1.177 .2448 -.20535714E-01

DR3 -.7792177815E-03 .11072916 -.007 .9944 -.11607143E-01

(Note: E+nn or E-nn means multiply by 10 to + or -nn power.)

**--> wald;fn1=b\_r1-1$**

+-----------------------------------------------+

| WALD procedure. Estimates and standard errors |

| for nonlinear functions and joint test of |

| nonlinear restrictions. |

| Wald Statistic = .51796 |

| Prob. from Chi-squared[ 1] = .47171 |

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+---------+--------------+----------------+--------+---------+

|Variable | Coefficient | Standard Error |b/St.Er.|P[|Z|>z] |

+---------+--------------+----------------+--------+---------+

Fncn(1) -.3467725900E-01 .48183346E-01 -.720 .4717

The unit root hypothesis is definitely not rejected.

2. Macroeconomic Model

**--> samp;1-204$**

**--> crea;c=log(realcons);y=log(realdpi)$**

**--> crea;c1=c[-1];c2=c[-2]$**

**--> samp;3-204$**

**--> regr;lhs=c;rhs=one,y,c1,c2$**

+-----------------------------------------------------------------------+

| Ordinary least squares regression Weighting variable = none |

| Dep. var. = C Mean= 7.889033683 , S.D.= .5102401315 |

| Model size: Observations = 202, Parameters = 4, Deg.Fr.= 198 |

| Residuals: Sum of squares= .1519097328E-01, Std.Dev.= .00876 |

| Fit: R-squared= .999710, Adjusted R-squared = .99971 |

| Model test: F[ 3, 198] =\*\*\*\*\*\*\*\*, Prob value = .00000 |

| Diagnostic: Log-L = 672.4019, Restricted(b=0) Log-L = -150.2038 |

| LogAmemiyaPrCrt.= -9.456, Akaike Info. Crt.= -6.618 |

| Autocorrel: Durbin-Watson Statistic = 1.89384, Rho = .05308 |

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+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant .8165780259E-03 .10779352E-01 .076 .9397

Y .7869591065E-01 .29020268E-01 2.712 .0073 7.9998985

C1 .9680839747 .72732869E-01 13.310 .0000 7.8802520

C2 -.4701660339E-01 .70076193E-01 -.671 .5030 7.8714299

**--> crea;e1=e[-1];e2=e[-3];e3=e[-3]$**

**--> crea;e1=e[-1];e2=e[-2];e3=e[-3]$**

**--> regr;lhs=e;rhs=one,e1,e2,e3$**

+-----------------------------------------------------------------------+

| Ordinary least squares regression Weighting variable = none |

| Dep. var. = E Mean= -.6947138134E-15, S.D.= .8693502258E-02 |

| Model size: Observations = 202, Parameters = 4, Deg.Fr.= 198 |

| Residuals: Sum of squares= .1339943625E-01, Std.Dev.= .00823 |

| Fit: R-squared= .117934, Adjusted R-squared = .10457 |

| Model test: F[ 3, 198] = 8.82, Prob value = .00002 |

| Diagnostic: Log-L = 685.0763, Restricted(b=0) Log-L = 672.4019 |

| LogAmemiyaPrCrt.= -9.581, Akaike Info. Crt.= -6.743 |

| Autocorrel: Durbin-Watson Statistic = 1.85371, Rho = .07314 |

+-----------------------------------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant .2437121418E-04 .57884755E-03 .042 .9665

E1 -.2553462753E-01 .70917392E-01 -.360 .7192 -.21497022E-04

E2 .3385045374 .66904365E-01 5.060 .0000 -.56959898E-04

E3 .6894158132E-01 .71101163E-01 .970 .3334 -.81793147E-04

**--> calc;list;chisq=n\*rsqrd$**

CHISQ = .23822731697405480D+02

Matrix Result has 2 rows and 2 columns.

1 2

+----------------------------

1| 1.0688 .0000000D+00

2| 19.8378 .0000000D+00

Short run multiplier is β = .07869. Long run is β/(1-γ1 - γ2) = 12.669. (Not very plausible.)

3. ADF Test. To carry out the test, the rate of inflation is regressed on a constant, a time trend, the previous year’s value of the rate of inflation, and three lags of the first difference. The test statistic for the ADF is (0.7290534455-1)/.011230759 = -2.373. The critical value in the lower part of Table 20.4 with about 100 observations is -3.45. Since our value is large than this, it follows that the hypothesis of a unit root cannot be rejected.

4. Reestimated model in example 13.1.

**--> samp;1-204$**

**--> crea;ddp1=infl[-1]-infl[-2]$**

**--> crea;ddp2=ddp1[-1]$**

**--> crea;ddp3=ddp1[-2]$**

**--> crea;dp=infl[-1]$**

**--> samp;97-204$**

**--> crea;t=trn(1,1)$**

**--> regr;lhs=infl;rhs=one,t,dp,ddp1,ddp2,ddp3$**

+-----------------------------------------------------------------------+

| Ordinary least squares regression Weighting variable = none |

| Dep. var. = INFL Mean= 4.907672727 , S.D.= 3.617392978 |

| Model size: Observations = 108, Parameters = 6, Deg.Fr.= 102 |

| Residuals: Sum of squares= 608.5020156 , Std.Dev.= 2.44248 |

| Fit: R-squared= .565403, Adjusted R-squared = .54410 |

| Model test: F[ 5, 102] = 26.54, Prob value = .00000 |

+-----------------------------------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant 2.226039717 1.1342702 1.963 .0524

T -.1836785769E-01 .11230759E-01 -1.635 .1050 54.500000

DP .7290534455 .11419140 6.384 .0000 4.9830886

DDP1 -.4744389916 .12707255 -3.734 .0003 -.58569323E-01

DDP2 -.4273030624 .11563482 -3.695 .0004 -.46827528E-01

DDP3 -.2248432703 .98954483E-01 -2.272 .0252 -.86558444E-02

**--> wald;fn1=b\_dp-1$**

+---------+--------------+----------------+--------+---------+

|Variable | Coefficient | Standard Error |b/St.Er.|P[|Z|>z] |

+---------+--------------+----------------+--------+---------+

Fncn(1) -.2709465545 .11419140 -2.373 .0177

**--> samp;1-204$**

**--> crea;ct=realcons;yt=realgdp;gt=realgovt;rt=tbilrate$**

**--> crea;ct1=ct[-1];yt1=yt[-1]$**

**--> samp;2-204$**

**--> samp;1-204$**

**--> crea;ct=realcons;yt=realgdp;gt=realgovt;rt=tbilrate;it=realinvs$**

**--> crea;ct1=ct[-1];yt1=yt[-1]$**

**--> crea;dy=yt-yt1$**

**--> samp;2-204$**

**--> name;x=one,rt,ct1,yt1,gt$**

**--> 2sls;lhs=ct;rhs=one,yt,ct1;inst=x;res=ec$**

**--> 2sls;lhs=it;rhs=one,rt,dy;inst=x;res=ei$**

**--> iden;rhs=ec;pds=10$**

**--> iden;rhs=ei;pds=10$**

+-----------------------------------------------------------------------+

| Two stage least squares regression Weighting variable = none |

| Dep. var. = CT Mean= 3008.995074 , S.D.= 1456.900152 |

| Model size: Observations = 203, Parameters = 3, Deg.Fr.= 200 |

| Residuals: Sum of squares= 96595.67529 , Std.Dev.= 21.97677 |

| Fit: R-squared= .999771, Adjusted R-squared = .99977 |

| (Note: Not using OLS. R-squared is not bounded in [0,1] |

| Model test: F[ 2, 200] =\*\*\*\*\*\*\*\*, Prob value = .00000 |

| Diagnostic: Log-L = -913.8005, Restricted(b=0) Log-L = -1766.2087 |

| LogAmemiyaPrCrt.= 6.195, Akaike Info. Crt.= 9.033 |

| Autocorrel: Durbin-Watson Statistic = 1.61078, Rho = .19461 |

+-----------------------------------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant 6.666079115 8.6211817 .773 .4394

YT -.2932041745E-01 .35260653E-01 -.832 .4057 4577.1882

CT1 1.051478712 .51482187E-01 20.424 .0000 2982.9744

+-----------------------------------------------------------------------+

| Two stage least squares regression Weighting variable = none |

| Dep. var. = IT Mean= 654.5295567 , S.D.= 391.3705005 |

| Model size: Observations = 203, Parameters = 3, Deg.Fr.= 200 |

| Residuals: Sum of squares= 54658669.31 , Std.Dev.= 522.77466 |

| Fit: R-squared= -.793071, Adjusted R-squared = -.81100 |

| (Note: Not using OLS. R-squared is not bounded in [0,1] |

| Diagnostic: Log-L = -1557.1409, Restricted(b=0) Log-L = -1499.3832 |

| LogAmemiyaPrCrt.= 12.533, Akaike Info. Crt.= 15.371 |

| Autocorrel: Durbin-Watson Statistic = 1.49055, Rho = .25473 |

+-----------------------------------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Constant -141.8297176 103.57113 -1.369 .1709

RT 52.04340559 12.971223 4.012 .0001 5.2499007

DY 13.80361384 1.7499250 7.888 .0000 37.898522

Time series identification for EC

Box-Pierce Statistic = 40.8498 Box-Ljung Statistic = 41.7842

Degrees of freedom = 10 Degrees of freedom = 10

Significance level = .0000 Significance level = .0000

\* => |coefficient| > 2/sqrt(N) or > 95% significant.

PACF is computed using Yule-Walker equations.

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

Lag | Autocorrelation Function |Box/Prc| Partial Autocorrelations X

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

1 | .194\*| |\*\* | 7.65\*| .194\*| |\*\* X

2 | .264\*| |\*\*\* | 21.82\*| .236\*| |\*\*\* X

3 | .273\*| |\*\*\* | 36.93\*| .207\*| |\*\* X

4 | .067 | |\* | 37.85\*|-.063 | \* | X

5 | .054 | |\* | 38.44\*|-.068 | \* | X

6 | .073 | |\* | 39.52\*| .018 | |\* X

7 | .009 | |\* | 39.53\*| .003 | |\* X

8 |-.078 | \*| | 40.78\*|-.109 | \* | X

9 | .019 | |\* | 40.85\*| .023 | |\* X

10 | .002 | |\* | 40.85\*| .050 | |\* X

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

Time series identification for EI

Box-Pierce Statistic = 27.4753 Box-Ljung Statistic = 28.3566

Degrees of freedom = 10 Degrees of freedom = 10

Significance level = .0022 Significance level = .0016

\* => |coefficient| > 2/sqrt(N) or > 95% significant.

PACF is computed using Yule-Walker equations.

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

Lag | Autocorrelation Function |Box/Prc| Partial Autocorrelations X

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

1 | .244\*| |\*\*\* | 12.13\*| .244\*| |\*\*\* X

2 | .143\*| |\*\* | 16.27\*| .096 | |\* X

3 | .037 | |\* | 16.55\*|-.019 | \* | X

4 |-.001 | \*| | 16.55\*|-.017 | \* | X

5 |-.066 | \*| | 17.42\*|-.078 | \* | X

6 | .003 | |\* | 17.43\*| .043 | |\* X

7 |-.042 | \*| | 17.79\*|-.033 | \* | X

8 |-.107 | \*| | 20.10\*|-.107 | \* | X

9 | .108 | |\* | 22.46\*| .194\*| |\*\* X

10 | .157\*| |\*\* | 27.48\*| .142\*| |\*\* X

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**Exercises**

1. For the first, the mean lag is .55(.02)(0) + .55(.15)(1) + ... + .55(.17)(4) = 1.31 periods. The impact multiplier is .55(.02) = .011 while the long run multiplier is the sum of the coefficients, .55.

For the second, the coefficient on *x*t is .6, so this is the impact multiplier. The mean lag is found by applying (18‑9) to *B*(*L*) = [.6 + 2*L*]/[1 - .6*L* + .5*L*2] = *A*(*L*)/*D*(*L*). Then, *B′*(1)/*B*(1) =

{[*D*(1)*A*′(1) - *A*(1)*D*′(1)]/[*D*(1)]2} / [*A*(1)/*D*(1)] = *A*′(1)/*A*(1) - *D*′(1)/*D*(1) = (2/2.6) / (.4/.9) = 1.731 periods.

The long run multiplier is *B*(1) = 2.6/.9 = 2.888 periods.

For the third, since we are interested only in the coefficients on *xt*, write the model as

*yt* = α + β*xt*[1 + γ*L* + γ2*L*2 + ...] + δz*t*\* + *ut*. The lag coefficients on *x*t are simply β times powers of γ.

2. We would regress *y*t on a constant, *x*t, *x*t-1, ..., *x*t-6. Constrained least squares using

1 ‑5 10 ‑10 5 ‑1 0 0 0

**R** = 0 1 ‑5 10 ‑10 5 ‑1 0 , **q** = 0

0 0 1 ‑5 10 ‑10 5 ‑1 0

would produce the PDL estimates.

3. The ratio of polynomials will equal *B*(*L*) = [.6 + 2*L*]/[1 - .6*L* + .5*L*2]. This will expand to

*B*(*L*) = β0 + β1*L* + β2*L*2 + .... Multiply both sides of the equation by (1 ‑ .6*L* + .5*L*2) to obtain

(β0 + β1*L* + β2*L*2 + ....)(1 ‑ .6*L* + .5*L*2) = .6 + 2*L*. Since the two sides must be equal, it follows that

β0 = .6 (the only term not involving *L*) ‑.6β0 + β1 = 2 (the only term involving only *L*. Therefore, β1 = 2.36. All remaining terms, involving *L*2, *L*3, ... must equal zero. Therefore, β*j* ‑ .6β*j*-1 + .5β*j*-2 = 0 for all *j* > 1, or β*j* = .6β*j-*1 ‑ .5β*j-*2. This provides a recursion for all remaining coefficients. For the specified coefficients, β2 = .6(2.36) ‑ .5(.3) = 1.266. β3 = .6(1.266) ‑ .5(2.36) = ‑.4204, β4 = .6(‑.4204) ‑ .5(1.266) = ‑.88524 and so on.

4. By multiplying through by the denominator of the lag function, we obtain an autoregressive form

*yt* = α(1+δ1+δ2) + β*xt* + γ*xt*-1 - δ1*yt*-1 - δ2*yt-*2 + ε*t* + δ1ε*t*-1 + δ2ε*t*-2

= α(1+δ1+δ2) + β*xt* + γ*xt-*1 - δ1*yt*-1 - δ2*yt*-2 + *vt*

The model cannot be estimated consistently by ordinary least squares because there is autocorrelation in the presence of a lagged dependent variable. There are two approaches possible. Nonlinear least squares could be applied to the moving average (distributed lag) form. This would be fairly complicated, though a method of doing so is described by Maddala and Rao (1973). A much simpler approach would be to estimate the model in the autoregressive form using an instrumental variables estimator. The lagged variables *xt*-2 and *xt*-3 can be used for the lagged dependent variables. ~

5.The model can be estimated as an autoregressive or distributed lag equation. Consider, first, the autoregressive form. Multiply through by (1 - γ*L*)(1 - φ*L*) to obtain

y*t* = α(1-γ)(1-φ) + β*xt* - (βφ)*xt*-1 + δ*zt* - (δγ)*zt*-1 + (γ + φ)*yt*-1 - (γφ)*yt*-2 + εt -(γ+φ)ε*t*-1 + (γφ)ε*t*-2.

Clearly, the model cannot be estimated by ordinary least squares, since there is an autocorrelated disturbance and a lagged dependent variable. The parameters can be estimated consistently, but inefficiently by linear instrumental variables. The inefficiency arises from the fact that the parameters are overidentified. The linear estimator estimates seven functions of the five underlying parameters. One possibility is a GMM estimator. Let *vt* = ε*t* -(γ+φ)ε*t-*1 + (γφ)ε*t-*2. Then, a GMM estimator can be defined in terms of, say, a set of moment equations of the form E[*vtwt*] = 0, where *wt* is current and lagged values of *x* and *z*. A minimum distance estimator could then be used for estimation.

The distributed lag approach might be taken, instead. Each of the two regressors produces a recursions *xt*\* = *xt*  + γ*xt-*1\* and *zt*\* = *zt* + γ*zt*-1\*. Thus, values of the moving average regressors can be built up recursively. Note that the model is linear in 1,  *xt*\*, and *zt*\*. Therefore, an approach is to search a grid of values of (γ,φ) to minimize the sum of squares. ~

**Applications**

1. The long run multiplier is β0 + β1 + ... + β6. The model is a classical regression, so it can be estimated by ordinary least squares. The estimator of the long run multiplier would be the sum of the least squares coefficients. If the sixth lag is omitted, then the standard omitted variable result applies, and all the coefficients are biased. The orthogonality result needed to remove the bias explicitly fails here, since xt is an AR(1) process. All the lags are correlated. Since the form of the relationship is, in fact, known, we can derive the omitted variable formula. In particular, by construction, xt will have mean zero. By implication, yt will also, so we lose nothing by assuming that the constant term is zero. To save some cumbersome algebra, we’ll also assume with no loss of generality that the unconditional variance of xt is 1. Let X1 = [xt,xt-1,...,xt-5] and X2 = xt-6. Then, for the regression of y on X1, we have by the omitted variable formula,



We can derive a formal solution to the bias in this estimator. Note that the column that is to the right of the inverse matrix is r times the last column matrix. Therefore, the matrix product is r times the last column of an identity matrix. This gives us the complete result,



Therefore, the first 5 coefficients are unbiased, and the last one is an estimator of β5 + rβ6. Adding these up, we see that when the last lag is omitted from the model, the estimator of the long run multiplier is biased downware by (1-r)β6. For part d, we will use a similar construction. But, now there are five variables in X1 and xt-5 and xt-6 in X2. The same kind of computation will show that the first four coefficients are unbiased while the fifth now estimates β4 + rβ5 + r2β6. The long run multiplier is estimated with downward bias equal to (1-r)β5 + (1-r2)β6.

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

XT .9726595701 1.9258818 .505 .6141 8.3384522

XT1 .7709686332 3.1555811 .244 .8072 8.3301663

XT2 .5450409860 3.1761465 .172 .8639 8.3218191

XT3 -.6061007409 3.1903388 -.190 .8495 8.3134324

XT4 -.2272352746 3.1729930 -.072 .9430 8.3050260

XT5 -1.916555094 3.1414210 -.610 .5425 8.2964570

XT6 1.218771893 1.8814874 .648 .5179 8.2878393

Matrix LRM has 1 rows and 1 columns.

1

+--------------

1| .7575

XT 1.101551478 1.9126777 .576 .5653 8.3384522

XT1 .6941982792 3.1485851 .220 .8257 8.3301663

XT2 .5287939572 3.1712435 .167 .8677 8.3218191

XT3 -.7300170198 3.1797815 -.230 .8187 8.3134324

XT4 -.5552651191 3.1275848 -.178 .8593 8.3050260

XT5 -.2826674399 1.8697065 -.151 .8800 8.2964570

Matrix LRM has 1 rows and 1 columns.

1

+--------------

1| .7566

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

XT 1.077633667 1.9012923 .567 .5715 8.3384522

XT1 .7070443138 3.1394606 .225 .8221 8.3301663

XT2 .5633400685 3.1549830 .179 .8585 8.3218191

XT3 -.6608149939 3.1386871 -.211 .8335 8.3134324

XT4 -.9304013056 1.8990464 -.490 .6247 8.3050260

Matrix LRM has 1 rows and 1 columns.

1

+--------------

1| .7568

**--> calc;list;cor(xt,xt1)$**

Result = .99978740920470700D+00

The results of the three suggested regressions are shown above. We used observations 7 - 204 of the logged real investment and real GDP data in deviations from the means for all regressions. Note that although there are some large changes in the estimated individual parameters, the long run multiplier is almost identical in all cases. Looking at the analytical results we can see why this would be the case. The correlation between current and lagged log gdp is r = 0.9998. Therefore, the biases that we found, (1-r)β6 and (1-r)β5 + (1-r2)β6 are trivial.

2. Because the model has both lagged dependent variables and autocorrelated disturbances, ordinary least squares will be inconsistent. Consistent estimates could be obtained by the method of instrumental variables. We can use *x*t-1 and *x*t-2 as the instruments for *yt*-1 and *yt*-2. Efficient estimates can be obtained by a two step procedure. We write the model as *yt* ‑ ρ*yt*-1 = α(1‑ρ) + β(*xt* ‑ ρ*xt*-1) + γ(*yt*-1 ‑ ρ*yt*-2) + δ(*yt*-2 ‑ ρ*yt*-3) + *ut*. With a consistent estimator of ρ, we could use FGLS. The residuals from the *IV* estimator can be used to estimate ρ. Then OLS using the transformed data is asymptotically equivalent to GLS. The method of Hatanaka discussed in the text is another possibility.

Using the real consumption and real disposable income data in Table F5.1, we obtain the following results: Estimated standard errors are shown in parentheses. (The estimated autocorrelation based on the IV estimates is .9172.) All three sets of estimates are based on the last 201 observations, 1950.4 to 2000.4

OLS IV 2 Step FGLS

 -1.4946 -64.5073 -4.6614

(3.8291) (46.1075) (3.2041)

 .007569 .7003 .3477

(.001662) (.4910) (.0432)

 1.1977 .5726 .2332

(.006921) (.9043) (.05933)

 ‑0.1988 ‑.3324 .4072

(.07109) (.4962) (.05500)

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There are no exercises or applications in Chapter 22.