Chapter 1

Econometrics

There are no exercises or applications in Chapter 1.

Chapter 2

The Linear Regression Model

There are no exercises or applications in Chapter 2.

Chapter 3

Least Squares

**Exercises**

1. Let .

(a) The normal equations are given by (3-12), (we drop the minus sign), hence for each of the columns of **X**, **x***k*, we know that **x***k*′**e** = 0. This implies that and.

(b) Use  to conclude from the first normal equation that .

(c) We know that  and . It follows then that because

. Substitute *ei* to obtain 

or 

Then, 

(d) The first derivative vector of **e′e** is -2**X′e**. (The normal equations.) The second derivative matrix is

∂2(**e′e**)/∂**b**∂**b′** = 2**X′X**. We need to show that this matrix is positive definite. The diagonal elements are 2*n* and which are clearly both positive. The determinant is [(2*n*)( )] - ()2

= -4()2 = . Note that a much simpler proof appears after (3-6).

2. Write **c** as **b** + (**c** ‑ **b**). Then, the sum of squared residuals based on **c** is

(**y** ‑ **Xc**)**′**(**y** ‑ **Xc**) = [**y** ‑ **X**(**b** + (**c** ‑ **b**))] **′**[**y** ‑ **X**(**b** + (**c** ‑ **b**))] = [(**y** ‑ **Xb**) + **X**(**c** ‑ **b**)] **′**[(**y** ‑ **Xb**) + **X**(**c** ‑ **b**)]

= (**y** ‑ **Xb**) **′**(**y** ‑ **Xb**) + (**c** ‑ **b**) **′X′X**(**c** ‑ **b**) + 2(**c** ‑ **b**) **′X′**(**y** ‑ **Xb**).

But, the third term is zero, as 2(**c** ‑ **b**) **′X′**(**y** ‑ **Xb**) = 2(**c** ‑ **b**)**X′e** = **0**. Therefore,

(**y** ‑ **Xc**) **′**(**y** ‑ **Xc**) = **e′e** + (**c** ‑ **b**) **′X′X**(**c** ‑ **b**)

or (**y** ‑ **Xc**) **′**(**y** ‑ **Xc**) ‑ **e′e** = (**c** ‑ **b**) **′X′X**(**c** ‑ **b**).

The right hand side can be written as **d′d** where **d** = **X**(**c** ‑ **b**), so it is necessarily positive. This confirms what we knew at the outset, least squares is least squares.

3. In the regression of **y** on **i** and **X**, the coefficients on **X** are **b** = (**X′M0X**)-1**X′M**0**y**. **M**0 = **I** ‑ **i**(**i′i**)-1**i′** is the matrix which transforms observations into deviations from their column means. Since **M**0 is idempotent and symmetric we may also write the preceding as [(**X′M**0**′**)(**M**0**X**)]-1(**X′M**0**′**)(**M**0**y**) which implies that the regression of **M**0**y** on **M**0**X** produces the least squares slopes. If only **X** is transformed to deviations, we would compute [(**X′M**0**′**)(**M**0**X**)]-1(**X′M**0**′**)**y** but, of course, this is identical. However, if only **y** is transformed, the result is (**X′X**)-1**X′M**0**y** which is likely to be quite different.

4. What is the result of the matrix product **M**1**M** where **M**1 is defined in (3‑19) and **M** is defined in (3‑14)?

**M**1**M** = (**I** ‑ **X**1(**X**1**′X**1)-1**X**1**′**)(**I** ‑ **X**(**X′X**)-1**X′**) = **M** ‑ **X**1(**X**1**′X**1)-1**X**1**′M**

There is no need to multiply out the second term. Each column of **MX**1 is the vector of residuals in the regression of the corresponding column of **X**1 on all of the columns in **X**. Since that **x** is one of the columns in **X**, this regression provides a perfect fit, so the residuals are zero. Thus, **MX**1 is a matrix of zeroes which implies that **M**1**M** = **M**.

5. The original **X** matrix has *n* rows. We add an additional row, **x***s*′. The new **y** vector likewise has an additional element. Thus,  The new coefficient vector is

**b***n,s* = (**X***n,s*′ **X***n,s*)-1(**X***n,s*′**y***n,s*). The matrix is **X***n,s*′**X***n,s* = **X***n***′X***n* + **x***s***x***s*′. To invert this, use (A -66);

. The vector is

(**X***n,s*′**y***n,s*) = (**X***n*′**y***n*) + **x***sys*. Multiply out the four terms to get

(**X***n,s*′ **X***n,s*)-1(**X***n,s*′**y***n,s*) =

**b***n* – +  **x***sys*  **x***sys*

=

**b***n* +  **x***sys* – –

**b***n* + –

**b***n* + –

**b***n* + 

6. Define the data matrix as follows:  (The subscripts on the parts of **y** refer to the “observed” and “missing” rows of **X**. We will use Frish-Waugh to obtain the first two columns of the least squares coefficient vector.  **b**1=(**X**1′**M**2**X**1)-1(**X**1′**M**2**y**). Multiplying it out, we find that **M**2 = an identity matrix save for the last diagonal element that is equal to 0.

**X**1′**M**2**X**1 = . This just drops the last observation. **X**1′**M**2**y** is computed likewise. Thus, the coeffients on the first two columns are the same as if *y*0 had been linearly regressed on **X**1. The denomonator of *R*2 is different for the two cases (drop the observation or keep it with zero fill and the dummy variable). For the first strategy, the mean of the *n*-1 observations should be different from the mean of the full *n* unless the last observation happens to equal the mean of the first *n*-1.

For the second strategy, replacing the missing value with the mean of the other *n*-1 observations, we can deduce the new slope vector logically. Using Frisch-Waugh, we can replace the column of *x*’s with deviations from the means, which then turns the last observation to zero. Thus, once again, the coefficient on the *x* equals what it is using the earlier strategy. The constant term will be the same as well.

7. For convenience, reorder the variables so that **X** = [**i**, **P***d*, **P***n*, **P***s*, **Y**]. The three dependent variables are **E***d*, **E***n*, and **E***s*, and **Y** = **E***d* + **E***n* + **E***s*. The coefficient vectors are

**b***d* = (**X′X**)-1**X′E***d*,

**b***n* = (**X′X**)-1**X′E***n*, and

**b***s* = (**X′X**)-1**X′E***s*.

The sum of the three vectors is

**b** = (**X′X**)-1**X**′[**E***d* + **E***n* + **E***s*] = (**X′X**)-1**X**′**Y**.

Now, **Y** is the last column of **X**, so the preceding sum is the vector of least squares coefficients in the regression of the last column of **X** on all of the columns of **X**, including the last. Of course, we get a perfect fit. In addition, **X′**[**E***d* + **E***n* + **E***s*] is the last column of **X′X**, so the matrix product is equal to the last column of an identity matrix. Thus, the sum of the coefficients on all variables except income is 0, while that on income is 1.

8. Let  denote the adjusted *R*2 in the full regression on *K* variables including **x***k*, and letdenote the adjusted *R*2 in the short regression on *K*‑1 variables when **x***k* is omitted. Let and denote their unadjusted counterparts. Then,

= 1 ‑ **e′e**/**y′M**0**y**

= 1 ‑ **e**1**′e**1/**y′M**0**y**

where **e′e** is the sum of squared residuals in the full regression, **e**1**′e**1 is the (larger) sum of squared residuals in the regression which omits **x***k*, and **y′M**0**y** = Σ*i* (*yi* -)2.

Then,= 1 ‑ [(*n*‑1)/(*n*‑*K*)](1 ‑ )

and= 1 ‑ [(*n*‑1)/(*n*‑(*K*‑1))](1 ‑).

The difference is the change in the adjusted *R*2 when **x***k* is added to the regression,

- = [(*n*-1)/(*n*-*K*+1)][**e**1**′e**1/**y′M**0**y**] - [(*n*-1)/(*n*-*K*)][**e′e**/**y′M**0**y**].

The difference is positive if and only if the ratio is greater than 1. After cancelling terms, we require for the adjusted *R*2 to increase that **e**1**′e**1/(*n-K*+1)]/[(*n-K*)/**e′e**] > 1. From the previous problem, we have that **e**1**′e**1 = **e′e** + *bK2*(**x***k***′M**1**x***k*), where **M**1 is defined above and *bk* is the least squares coefficient in the full regression of **y** on **X**1 and **x***k*. Making the substitution, we require [(**e′e** + *bK2*(**x***k***′M**1**x***k*))(*n*-*K*)]/[(*n*-*K*)**e′e** + **e′e**] > 1. Since **e′e** = (*n*‑*K*)*s*2, this simplifies to [**e′e** + *bK2*(**x***k***′M**1**x***k*)]/[**e′e** + *s*2] > 1. Since all terms are positive, the fraction is greater than one if and only *bK2*(**x***k***′M**1**x***k*) > *s*2 or *bK2*(**x***k***′M**1**x***k*/*s*2) > 1. The denominator is the estimated variance of *bk*, so the result is proved.

9. This *R*2 must be lower. The sum of squares associated with the coefficient vector which omits the constant term must be higher than the one which includes it. We can write the coefficient vector in the regression without a constant as **c** = (0,**b**\*) where **b**\* = (**W′W**)-1**W′y**, with **W** being the other *K*‑1 columns of **X**. Then, the result of the previous exercise applies directly.

10. We use the notation ‘Var[.]’ and ‘Cov[.]’ to indicate the sample variances and covariances. Our information is Var[*N*] = 1, Var[*D*] = 1, Var[*Y*] = 1.

Since *C* = *N* + *D*, Var[*C*] = Var[*N*] + Var[*D*] + 2Cov[*N*,*D*] = 2(1 + Cov[*N*,*D*]).

From the regressions, we have

Cov[*C*,*Y*]/Var[*Y*] = Cov[*C*,*Y*] = .8.

But, Cov[*C*,*Y*] = Cov[*N*,*Y*] + Cov[*D*,*Y*].

Also, Cov[*C*,*N*]/Var[*N*] = Cov[*C*,*N*] = .5,

but, Cov[*C*,*N*] = Var[*N*] + Cov[*N*,*D*] = 1 + Cov[*N*,*D*], so Cov[*N*,*D*] = ‑.5,

so that Var[*C*] = 2(1 + ‑.5) = 1.

And, Cov[*D*,*Y*]/Var[*Y*] = Cov[*D*,*Y*] = .4.

Since Cov[*C*,*Y*] = .8 = Cov[*N*,*Y*] + Cov[*D*,*Y*], Cov[*N*,*Y*] = .4.

Finally, Cov[*C*,*D*] = Cov[*N*,*D*] + Var[*D*] = ‑.5 + 1 = .5.

Now, in the regression of *C* on *D*, the sum of squared residuals is (*n*‑1){Var[*C*] ‑ (Cov[*C*,*D*]/Var[*D*])2Var[*D*]}

based on the general regression result Σ*e*2 = Σ(*yi* -**)2 ‑ *b*2Σ(*xi* -)2. All of the necessary figures were obtained above. Inserting these and *n*‑1 = 20 produces a sum of squared residuals of 15.

11. The relevant submatrices to be used in the calculations are

Investment Constant GNP Interest

Investment \* 3.0500 3.9926 23.521

Constant 15 19.310 111.79

GNP 25.218 148.98

Interest 943.86

The inverse of the lower right 3×3 block is (**X′X**)-1,

7.5874

(**X′X**)-1 = ‑7.41859 7.84078

.27313 ‑.598953 .06254637

The coefficient vector is **b** = (**X′X**)-1**X′y** = (‑.0727985, .235622, ‑.00364866)′. The total sum of squares is **y′y** = .63652, so we can obtain **e′e** = **y′y** ‑ **b′X′y**. **X′y** is given in the top row of the matrix. Making the substitution, we obtain **e′e** = .63652 ‑ .63291 = .00361. To compute *R*2, we require Σ*i* (*yi -*)2 =

.63652 ‑ 15(3.05/15)2 = .01635333, so *R*2 = 1 ‑ .00361/.0163533 = .77925.

12.The results cannot be correct. Since log *S/N* = log *S/Y* + log *Y/N* by simple, exact algebra, the same result must apply to the least squares regression results. That means that the second equation estimated must equal the first one plus log *Y/N*. Looking at the equations, that means that all of the coefficients would have to be identical save for the second, which would have to equal its counterpart in the first equation, plus 1. Therefore, the results cannot be correct. In an exchange between Leff and Arthur Goldberger that appeared later in the same journal, Leff argued that the difference was simple rounding error. You can see that the results in the second equation resemble those in the first, but not enough so that the explanation is credible. Further discussion about the data themselves appeared in subsequent idscussion. [See Goldberger (1973) and Leff (1973).]

**Application**

?=======================================================================

? Chapter 3 Application 1

?=======================================================================

Read $

(Data appear in the text.)

Namelist ; X1 = one,educ,exp,ability$

Namelist ; X2 = mothered,fathered,sibs$

?=======================================================================

? a.

?=======================================================================

Regress ; Lhs = wage ; Rhs = x1$

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=WAGE Mean = 2.059333 |

| Standard deviation = .2583869 |

| WTS=none Number of observs. = 15 |

| Model size Parameters = 4 |

| Degrees of freedom = 11 |

| Residuals Sum of squares = .7633163 |

| Standard error of e = .2634244 |

| Fit R-squared = .1833511 |

| Adjusted R-squared = -.3937136E-01 |

| Model test F[ 3, 11] (prob) = .82 (.5080) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| 1.66364000 .61855318 2.690 .0210

EDUC | .01453897 .04902149 .297 .7723 12.8666667

EXP | .07103002 .04803415 1.479 .1673 2.80000000

ABILITY | .02661537 .09911731 .269 .7933 .36600000

?=======================================================================

? b.

?=======================================================================

Regress ; Lhs = wage ; Rhs = x1,x2$

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=WAGE Mean = 2.059333 |

| Standard deviation = .2583869 |

| WTS=none Number of observs. = 15 |

| Model size Parameters = 7 |

| Degrees of freedom = 8 |

| Residuals Sum of squares = .4522662 |

| Standard error of e = .2377673 |

| Fit R-squared = .5161341 |

| Adjusted R-squared = .1532347 |

| Model test F[ 6, 8] (prob) = 1.42 (.3140) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| .04899633 .94880761 .052 .9601

EDUC | .02582213 .04468592 .578 .5793 12.8666667

EXP | .10339125 .04734541 2.184 .0605 2.80000000

ABILITY | .03074355 .12120133 .254 .8062 .36600000

MOTHERED| .10163069 .07017502 1.448 .1856 12.0666667

FATHERED| .00164437 .04464910 .037 .9715 12.6666667

SIBS | .05916922 .06901801 .857 .4162 2.20000000

?=======================================================================

? c.

?=======================================================================

Regress ; Lhs = mothered ; Rhs = x1 ; Res = meds $

Regress ; Lhs = fathered ; Rhs = x1 ; Res = feds $

Regress ; Lhs = sibs ; Rhs = x1 ; Res = sibss $

Namelist ; X2S = meds,feds,sibss $

Matrix ; list ; Mean(X2S) $

Matrix Result has 3 rows and 1 columns.

1

+--------------

1| -.1184238D-14

2| .1657933D-14

3| -.5921189D-16

The means are (essentially) zero. The sums must be zero, as these new variables are orthogonal to the columns of X1. The first column in X1 is a column of ones, so this means that these residuals must sum to zero.

?=======================================================================

? d.

?=======================================================================

Namelist ; X = X1,X2 $

Matrix ; i = init(n,1,1) $

Matrix ; M0 = iden(n) - 1/n\*i\*i' $

Matrix ; b12 = <X'X>\*X'wage$

Calc ; list ; ym0y =(N-1)\*var(wage) $

Matrix ; list ; cod = 1/ym0y \* b12'\*X'\*M0\*X\*b12 $

Matrix COD has 1 rows and 1 columns.

1

+--------------

1| .51613

Matrix ; e = wage - X\*b12 $

Calc ; list ; cod = 1 - 1/ym0y \* e'e $

+------------------------------------+

COD = .516134

The R squared is the same using either method of computation.

Calc ; list ; RsqAd = 1 - (n-1)/(n-col(x))\*(1-cod)$

+------------------------------------+

RSQAD = .153235

? Now drop the constant

Namelist ; X0 = educ,exp,ability,X2 $

Matrix ; i = init(n,1,1) $

Matrix ; M0 = iden(n) - 1/n\*i\*i' $

Matrix ; b120 = <X0'X0>\*X0'wage$

Matrix ; list ; cod = 1/ym0y \* b120'\*X0'\*M0\*X0\*b120 $

Matrix COD has 1 rows and 1 columns.

1

+--------------

1| .52953

Matrix ; e0 = wage - X0\*b120 $

Calc ; list ; cod = 1 - 1/ym0y \* e0'e0 $

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

COD = .515973

The R squared now changes depending on how it is computed. It also goes up, completely artificially.

?=======================================================================

? e.

?=======================================================================

The R squared for the full regression appears immediately below.

? f.

Regress ; Lhs = wage ; Rhs = X1,X2 $

+----------------------------------------------------+

| Ordinary least squares regression |

| WTS=none Number of observs. = 15 |

| Model size Parameters = 7 |

| Degrees of freedom = 8 |

| Fit R-squared = .5161341 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| .04899633 .94880761 .052 .9601

EDUC | .02582213 .04468592 .578 .5793 12.8666667

EXP | .10339125 .04734541 2.184 .0605 2.80000000

ABILITY | .03074355 .12120133 .254 .8062 .36600000

MOTHERED| .10163069 .07017502 1.448 .1856 12.0666667

FATHERED| .00164437 .04464910 .037 .9715 12.6666667

SIBS | .05916922 .06901801 .857 .4162 2.20000000

Regress ; Lhs = wage ; Rhs = X1,X2S $

+----------------------------------------------------+

| Ordinary least squares regression |

| WTS=none Number of observs. = 15 |

| Model size Parameters = 7 |

| Degrees of freedom = 8 |

| Fit R-squared = .5161341 |

| Adjusted R-squared = .1532347 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| 1.66364000 .55830716 2.980 .0176

EDUC | .01453897 .04424689 .329 .7509 12.8666667

EXP | .07103002 .04335571 1.638 .1400 2.80000000

ABILITY | .02661537 .08946345 .297 .7737 .36600000

MEDS | .10163069 .07017502 1.448 .1856 -.118424D-14

FEDS | .00164437 .04464910 .037 .9715 .165793D-14

SIBSS | .05916922 .06901801 .857 .4162 -.592119D-16

In the first set of results, the first coefficient vector is

**b**1 = (**X**1′**M**2**X**1)-1**X**1′**M**2**y** and

b2 = (**X**2′**M**1**X**2)-1**X**2′M1**y**

In the second regression, the second set of regressors is M1X2, so

**b**1 = (**X**1′**M**12 **X**1)-1**X**1′**M**12**y** where **M**12 = **I** – (**M**1**X**2)[(M1**X**2)′(**M**1**X**2)]-1(**M**1**X**2)′

Thus, because the “M” matrix is different, the coefficient vector is different. The second set of coefficients in the second regression is

**b**2 = [(**M**1**X**2)′**M**1(**M**1**X**2)]-1 (**M**1**X**2)**M**1**y** = (**X**2′**M**1**X**2)-1**X**2′**M**1**y** because **M**1 is idempotent.

Chapter 4

The Least Squares Estimator

**Exercises**

1. Consider the optimization problem of minimizing the variance of the weighted estimator. If the estimate is to be unbiased, it must be of the form *c*1+ *c*2where *c*1 and *c*2 sum to 1. Thus, *c*2 = 1 ‑ *c*1. The function to minimize is Min***c****L*\* = *c*12*v*1 + (1 ‑ *c*1)2*v*2. The necessary condition is ∂*L*\*/∂*c*1 = 2*c*1*v*1 ‑ 2(1 ‑ *c*1)*v*2 = 0 which implies *c*1 = *v*2 / (*v*1 + *v*2). A more intuitively appealing form is obtained by dividing numerator and denominator by *v*1*v*2 to obtain *c*1 = (1/*v*1) / [1/*v*1 + 1/*v*2]. Thus, the weight is proportional to the inverse of the variance. The estimator with the smaller variance gets the larger weight.

2. First, = **c′y** = **c′x** + **c′ε**. So *E*[] = β**c′x** and Var[] = σ2**c′c**. Therefore,

MSE[] = β2[**c′x** ‑ 1]2 + σ2**c′c**. To minimize this, we set ∂MSE[]/∂**c** = 2β2[**c′x** ‑ 1]**x** + 2σ2**c** = **0**.

Collecting terms, β2(**c′x** ‑ 1)**x** = ‑σ2**c**

Premultiply by **x**′ to obtain β2(**c′x** ‑ 1)**x**′**x** = ‑σ2**x′c**

or  **c′x** = **β**2**x**′**x** / (σ2 + β2**x**′**x**).

Then,  **c** = [(‑β2/σ2)(**c′x** ‑ 1)]**x**,

so  **c** = [1/(σ2/β2 + **x**′**x**)]**x**.

Then, = **c′y** = **x′y** / (σ2/β2 + **x**′**x**).

The expected value of this estimator is

*E*[] = β**x**′**x** / (σ2/β2 + **x**′**x**)

so  *E*[] ‑ β = β(‑σ2/β2) / (σ2/β2 + **x**′**x**)

= ‑(σ2/β) / (σ2/β2 + **x**′**x**)

while its variance is Var[**x**′(**x**β + **ε**) / (σ2/β2 + **x**′**x**)] = σ2**x**′**x** / (σ2/β2 + **x**′**x**)2

The mean squared error is the variance plus the squared bias,

MSE[] = [σ4/β2 + σ2**x′x**]/[σ2/β2 + **x**′**x**]2.

The ordinary least squares estimator is, as always, unbiased, and has variance and mean squared error

MSE(*b*) = σ2/**x**′**x**.

The ratio is taken by dividing each term in the numerator

 = 

= [σ2**x′x**/β2 + (**x′x**)2]/(σ2/β2 + **x**′**x**)2

= **x′x**[σ2/β2 + **x′x**]/(σ2/β2 + **x**′**x**)2

= **x′x**/(σ2/β2 + **x′x**)

Now, multiply numerator and denominator by β2/σ2 to obtain

MSE[]/MSE[*b*] = β2**x′x**/σ2/[1 + β2**x′x**/σ2] = τ2/[1 + τ2]

As τ→∞, the ratio goes to one. This would follow from the result that the biased estimator and the unbiased estimator are converging to the same thing, either as σ2 goes to zero, in which case the MMSE estimator is the same as OLS, or as **x**′**x** grows, in which case both estimators are consistent.

3. The OLS estimator fit without a constant term is *b* = **x′y** / **x′x**. Assuming that the constant term is, in fact, zero, the variance of this estimator is Var[*b*] = σ2/**x′x**. If a constant term is included in the regression, then,  *b*′ = /

The appropriate variance is σ2/as always. The ratio of these two is

Var[*b*]/Var[*b*′] = [σ2/**x′x**] / [σ2/]

But, = **x′x** + *n*2

so the ratio is Var[*b*]/Var[*b*′] = [**x′x** + *n*2]/**x′x** = 1 - *n*2/**x′x** = 1 - { *n*2/[*Sxx* + *n*2]} < 1

It follows that fitting the constant term when it is unnecessary inflates the variance of the least squares estimator if the mean of the regressor is not zero.

4. We could write the regression as *yi* = (α + λ) + β*xi* + (ε*i* - λ) = α\* + β*xi* + ε*i*\*. Then, we know that *E*[ε*i*\*] = 0, and that it is independent of *xi*. Therefore, the second form of the model satisfies all of our assumptions for the classical regression. Ordinary least squares will give unbiased estimators of α\* and β. As long as λ is not zero, the constant term will differ from α.

5. Let the constant term be written as *a* = Σ*idiyi* = Σ*idi*(α + β*xi* + ε*i*) = αΣ*idi* + βΣ*idixi* + Σ*idi*ε*i*. In order for *a* to be unbiased for all samples of *xi*, we must have Σ*idi* = 1 and Σ*idixi* = 0. Consider, then, minimizing the variance of *a* subject to these two constraints. The Lagrangean is

*L*\* = Var[*a*] + λ1(Σ*idi* - 1) + λ2Σ*idixi* where Var[*a*] = Σ*i*σ2*di*2.

Now, we minimize this with respect to *di*, λ1, and λ2. The (*n*+2) necessary conditions are

∂*L*\*/∂*di* = 2σ2*di* + λ1 + λ2*xi*, ∂*L*\*/∂λ1 = Σ*i**di* ‑ 1, ∂*L*\*/∂λ2 = Σ*i**dixi*

The first equation implies that *di* = [‑1/(2σ2)](λ1 + λ2*xi*).

Therefore, Σ*i**d*i = 1 = [‑1/(2σ2)][*n*λ1 + (Σ*i**x*i)λ2]

and Σ*i**dixi* = 0 = [‑1/(2σ2)][(Σ*i**xi*)λ1 + (Σ*i**xi*2)λ2].

We can solve these two equations for λ1 and λ2 by first multiplying both equations by ‑2σ2 then writing the resulting equations asThe solution is

Note, first, that Σ*i**xi* = *n* . Thus, the determinant of the matrix is *n*Σ*i**xi*2 ‑ (*n*)2 = *n*(Σ*i**xi*2 ‑ *n*2) = *nSxx* where *Sxx*. The solution is, therefore, 

or λ1 = (-2σ2)(Σ*i**xi*2/*n*)/*Sxx*

λ2 = (2σ2)/*Sxx*

Then, *di* = [Σ*i**xi*2/*n* - *xi*]/*Sxx*

This simplifies if we writeΣ*xi*2 = *Sxx* + *n*2, so Σ*i**xi*2/*n* = *Sxx*/*n* +2. Then,

*di* = 1/*n* + ( - *xi*)/*Sxx*, or, in a more familiar form, *di* = 1/*n* - (*xi* -)/*Sxx*.

This makes the intercept term Σ*idiyi* = (1/*n*)Σ*iyi* - /*Sxx* =  - *b* which was to be shown.

6. Let *q* = *E*[*Q*]. Then,  *q* = α + β*P*, or *P* = (‑α/β) + (1/β)*q*.

Using a well known result, for a linear demand curve, marginal revenue is *MR* = (‑α/β) + (2/β)*q*. The profit maximizing output is that at which marginal revenue equals marginal cost, or 10. Equating *MR* to 10 and solving for *q* produces *q* = α/2 + 5β, so we require a confidence interval for this combination of the parameters.

The least squares regression results are  = 20.7691 ‑ .840583. The estimated covariance matrix of the coefficients is . The estimate of *q* is 6.1816. The estimate of the variance of  is (1/4)7.96124 + 25(.056436) + 5(‑.0624559) or 0.278415, so the estimated standard error is 0.5276. The 95% cutoff value for a *t* distribution with 13 degrees of freedom is 2.161, so the confidence interval is 6.1816 ‑ 2.161(.5276) to 6.1816 + 2.161(.5276) or 5.041 to 7.322.

7. a. The sample means are (1/100) times the elements in the first column of **X'X**. The sample covariance matrix for the three regressors is obtained as (1/99)[(**X′X**)*ij* ‑100].

Sample Var[**x**] =  The simple correlation matrix is



b. The vector of slopes is (**X′X**)-1**X′y** = [‑.4022, 6.123, 5.910, ‑7.525]**′**.

c. For the three short regressions, the coefficient vectors are

(1) one, *x*1, and *x*2: [‑.223, 2.28, 2.11]**′**

(2) one, *x*1, and *x*3 [‑.0696, .229, 4.025]**′**

(3) one, *x*2, and *x*3: [‑.0627, ‑.0918, 4.358]**′**

d. The magnification factors are

for *x*1: [(1/(99(1.01727)) / 1.129]2 = .094

for *x*2: [(1/99(.75596)) / 1.11]2 = .109

for *x*3: [(1/99(.496969))/ 4.292]2 = .068.

e. The problem variable appears to be *x*3 since it has the lowest magnification factor. In fact, all three are highly intercorrelated. Although the simple correlations are not excessively high, the three multiple correlations are .9912 for *x*1 on *x*2 and *x*3, .9881 for *x*2 on *x*1 and *x*3, and .9912 for *x*3 on *x*1 and *x*2.

8. We consider two regressions. In the first, **y** is regressed on *K* variables, **X**. The variance of the least squares estimator, **b** = (**X′X**)-1**X′y**, Var[**b**] = σ2(**X′X**)-1. In the second, **y** is regressed on **X** and an additional variable, **z**. Using results for the partitioned regression, the coefficients on **X** when **y** is regressed on **X** and **z** are **b**.z = (**X′M**z**X**)-1**X′M**z**y** where **M**z = **I** ‑ **z**(**z′z**)-1**z**′. The true variance of **b**.z is the upper left *K*×*K* matrix in Var[**b**,*c*] = *s*2. But, we have already found this above. The submatrix is Var[**b**.z] = *s*2(**X′M**z**X**)-1. We can show that the second matrix is larger than the first by showing that its inverse is smaller. (See (A-120).) Thus, as regards the true variance matrices (Var[**b**])-1 ‑ (Var[**b**.z])-1 = (1/σ2)**z**(**z′z**)-1**z′**

which is a nonnegative definite matrix. Therefore Var[**b**]-1 is larger than Var[**b**.z]-1, which implies that Var[**b**] is smaller.

Although the true variance of **b** is smaller than the true variance of **b**.z, it does not follow that the estimated variance will be. The estimated variances are based on *s*2, not the true σ2. The residual variance estimator based on the short regression is *s*2 = **e′e**/(*n* ‑ *K*) while that based on the regression which includes **z** is *s*z2 = **e**.z**′e**.z/(*n* ‑ *K* - 1). The numerator of the second is definitely smaller than the numerator of the first, but so is the denominator. It is uncertain which way the comparison will go. The result is derived in the previous problem. We can conclude, therefore, that if *t* ratio on *c* in the regression which includes **z** is larger than one in absolute value, then *s*z2 will be smaller than *s*2. Thus, in the comparison, Est.Var[**b**] = *s*2(**X′X**)-1 is based on a smaller matrix, but a larger scale factor than Est.Var[**b**.z] = *s*z2(**X′M**z**X**)-1. Consequently, it is uncertain whether the estimated standard errors in the short regression will be smaller than those in the long one. Note that it is not sufficient merely for the result of the previous problem to hold, since the relative sizes of the matrices also play a role. But, to take a polar case, suppose **z** and **X** were uncorrelated. Then, **XNM**z**X** equals **XNX**. Then, the estimated variance of **b**.z would be less than that of **b** without **z** even though the true variance is the same (assuming the premise of the previous problem holds). Now, relax this assumption while holding the *t* ratio on c constant. The matrix in Var[**b**.z] is now larger, but the leading scalar is now smaller. Which way the product will go is uncertain.

9. The *F* ratio is computed as [**b′X′Xb**/*K*]/[**e′e**/(*n* ‑ *K*)]. We substitute **e** = **Mε,** and

**b** = **β** + (**X′X**)-1**X**′**ε** = (**X′X**)-1**X**′**ε**. Then, *F* = [**ε′X**(**X′X**)-1**X′X**(**X′X**)-1**X′ε**/*K*]/[**ε ′Mε**/(*n* ‑ *K*)] =

[**ε′(I - M)ε**/*K*]/[**ε′Mε**/(*n* ‑ *K*)].

The exact expectation of *F* can be found as follows: *F* = [(*n*-*K*)/*K*][**ε′(I - M)ε**]/[**ε′Mε**]. So, its exact expected value is (*n*‑*K*)/*K* times the expected value of the ratio. To find that, we note, first, that **Mε** and

(**I - M**)ε are independent because **M**(**I - M**) = **0**. Thus, *E*{[**ε′(I - M)ε**]/[**ε′Mε**]} = *E*[**ε′(I- M)ε**]×*E*{1/[**ε′Mε**]}.

The first of these was obtained above, *E*[**ε′**(**I ‑ M**)**ε**] = *K*σ2. The second is the expected value of the reciprocal of a chi‑squared variable. The exact result for the reciprocal of a chi‑squared variable is

*E*[1/χ2(*n*‑*K*)] = 1/(*n* ‑ *K* ‑ 2). Combining terms, the exact expectation is *E*[*F*] = (*n* ‑ *K*) / (*n* ‑ *K* ‑ 2). Notice that the mean does not involve the numerator degrees of freedom.

10. We write **b** = β + (**X′X**)-1**X**′**ε**, so **b′b** = **β**′**β** +  **ε′X**(**X′X**)-1(**X′X**)-1**X′ε** + 2**β′**(**X′X**)-1X′ε. The expected value of the last term is zero, and the first is nonstochastic. To find the expectation of the second term, use the trace, and permute **ε′X** inside the trace operator. Thus,

E[**β′β**] = **β′β** + *E*[**ε′X**(**X′X**)-1(**X′X**)-1**X′ε**]

= **β′β** + *E*[*tr*{**ε′X**(**X′X**)-1(**X′X**)-1**X′ε**}]

= **β′β** + *E*[*tr*{(**X′X**)-1**X′εε′X**(**X′X**)-1}]

= **β′β** + *tr*[*E*{(**X′X**)-1**X′εε′X**(**X′X**)-1}]

= **β′β** + *tr*[(**X′X**)-1**X′***E*[**εε′**]**X**(**X′X**)-1]

= **β′β** + *tr*[(**X′X**)-1**X′**(σ2**I**)**X**(**X′X**)-1]

= **β′β** + σ2*tr*[(**X′X**)-1**X′X**(**X′X**)-1]

= **β′β** + σ2*tr*[(**X′X**)-1]

= **β′β** + σ2Σ*k* (1/λ*k* )

The trace of the inverse equals the sum of the characteristic roots of the inverse, which are the reciprocals of the characteristic roots of **X′X**.

11. The *F* ratio is computed as [**b′X′Xb**/*K*]/[**e′e**/(*n* ‑ *K*)]. We substitute **e** = **M,** and

**b** = **β** + (**X′X**)-1**X**′**ε** = (**X′X**)-1**X**′**ε**. Then, *F* = [**ε′X**(**X′X**)-1**X′X**(**X′X**)-1**X′ε**/*K*]/[**ε ′Mε**/(*n* ‑ *K*)] =

[**ε′(I - M)ε**/*K*]/[**ε′Mε**/(*n* ‑ *K*)]. The denominator converges to σ2 as we have seen before. The numerator is an idempotent quadratic form in a normal vector. The trace of (**I** ‑ **M**) is *K* regardless of the sample size, so the numerator is always distributed as σ2 times a chi‑squared variable with *K* degrees of freedom. Therefore, the numerator of *F* does not converge to a constant, it converges to σ2/*K* times a chi‑squared variable with *K* degrees of freedom. Since the denominator of *F* converges to a constant, σ2, the statistic converges to a random variable, (1/*K*) times a chi‑squared variable with *K* degrees of freedom.

12. We can write *ei* as *ei* = *yi* ‑ **b′x***i*  = (**β′x***i* + εi) ‑ **b′x***i* = ε*i*  + (**b** ‑ **β**)**′x***i*

We know that plim **b** = **β**, and **x***i* is unchanged as *n* increases, so as *n*→∞, *ei* is arbitrarily close to ε*i*.

13.The estimator is = (1/*n*)Σ*i**yi* = (1/*n*)Σ*i*(μ + ε*i*) = μ + (1/*n*)Σ*i*ε*i.* Then, *E*[] = μ+ (1/*n*)Σ*i**E*[εi] = μ

and Var[]= (1/*n*2)Σ*i*Σ*j*Cov[ε*i*,ε*j*] = σ2/*n*. Since the mean equals μ and the variance vanishes as *n*→∞, is mean square consistent. In addition, sinceis a linear combination of normally distributed variables, has a normal distribution with the mean and variance given above in every sample. Suppose that ε*i* were not normally distributed. Then, (-μ) = (1/)(Σ*i*ε*i*) satisfies the requirements for the central limit theorem. Thus, the asymptotic normal distribution applies whether or not the disturbances have a normal distribution.

For the alternative estimator, = Σ*i**w*i*y*i, so *E*[] = Σ*i**wiE*[*y*i] = Σ*i**wi*μ = μΣ*i**wi* = μ and Var[]= Σ*i**wi*2σ2 = σ2Σ*i**wi*2. The sum of squares of the weights is Σ*iwi*2 = Σ*i**i*2/[Σ*i**i*]2 = [*n*(*n*+1)(2*n*+1)/6]/[*n*(*n*+1)/2]2 = [2(*n*2 + 3*n*/2 + 1/2)]/[1.5*n*(*n*2 + 2*n* + 1)]. As *n*→∞, the fraction will be dominated by the term (1/*n*) and will tend to zero. This establishes the consistency of this estimator. The last expression also provides the asymptotic variance. The large sample variance can be found as Asy.Var[] = (1/*n*)lim *n*→∞Var[(- μ)]. For the estimator above, we can use Asy.Var[] = (1/*n*)lim *n*→∞*n*Var[- μ] = (1/*n*)lim *n*→∞σ2[2(*n*2 + 3*n*/2 + 1/2)]/[1.5(*n*2 + 2*n* + 1)] = 1.3333σ2. Notice that this is unambiguously larger than the variance of the sample mean, which is the ordinary least squares estimator.

14. a. To solve this, we will use an extension of Exercise 5 in Chapter 3 (adding one row of data), and the necessary matrix result, (A-66b) in which **B**will be **X***m* and **C** will be **I**. Bypassing the matrix algebra, which will be essentially identical to the earlier exercise, we have

**b***c,m* = **b***c* + [**I** + **X**m(**X**c′**X**c)-1**X**m]-1(**X**c′**X**c)-1**X**m′(**y**m – **X**m**b**c)

But, in this case, **y**m is precisely **X**m**b**c, so the ending vector is zero. Thus, the coefficient vector is the same. b. The model applies to the first *n*c observations, so **b**c is the least squares estimator for those observations. Yes, it is unbiased.

c. The residuals at the second step are **e**c and (**X***m***b***c* – **X***m***b***c*) = (**e***c*′, **0**′)′. Thus, the sum of squares is the same at both steps.

d. The numerator of *s*2 is the same in both cases, however, for the second one, the degrees of freedom is larger. The first is unbiased, so the second one must be biased downward.

15. The proof is actually trivial. The result above (4-17) holds when *K*2 elements of **** are zero. That is all that is required.

16. The coefficient vector is **d** = (**Z′Z**)-1**Z′y**. As assumed, **Z** = **XC***L*, so **d** = (**C***L***′X′XC***L*)-1 **C***L***′X′y**. **y** = **X** + **ε**. Therefore, **d** = (**C***L***′X′XC***L*)-1 **C***L***′X′X** + (**C***L***′X′XC***L*)-1 **C***L***′X′ε**. The second term has expectation zero. Write the characteristic roots of **X′X** that partner with **C***L* as **Λ***L*. Then, by construction, (**C***L***′X′XC***L*) = **Λ***L* and **C***L***′X′X** = **Λ***L***C***L***′**. Therefore, *E*[**d**] = *E*[**Λ***L*-1**Λ***L***C***L***′**]**** + **0** = **C***L***′**.

17. Using the results in Table 4.6 and the data given for the exercise,

matrix ; c = [-8.42653 / 1.33373 / -0.16537] $

matrix ; v = [0.37434 / -0.05429, 0.00823 / -0.00974, -0.00075, 0.01626] $

calc ; s2 = 1.10266^2 $

calc ; h=35 ; w=39.4 $

calc ; la = log(h\*w) ; ar = h/w $

matrix ; x0=[1/la/ar] $

?For log area

calc ; list ; logyf = x0'c $

matrix ; list ; varyf = s2 + x0'\*v\*x0 $

calc ; list ; lower = logyf - 1.96\*sqr(varyf)

; upper = logyf + 1.96\*sqr(varyf) $

[CALC] LOWER = -1.0977513

[CALC] UPPER = 3.2342588

calc ; list ; mu0 = logyf $

calc ; list ; sp0 = sqr(varyf) $

? Naive confidence interval

calc ; list ; LO = exp(lower) $

calc ; list ; L1 = exp(lower) ; U1 = exp(upper) $

[CALC] MU0 = 1.0682538

[CALC] SP0 = 1.1051046

[CALC] LO = .3336204

[CALC] L1 = .3336204 <=== This is the naïve interval.

[CALC] U1 = 25.3875488

? Iterations, by trial and error.

calc ; delta = .005 $

proc $

calc ; LO = LO - delta $

calc ; flo = 1/(lo\*sp0\*sqr(2\*pi)) \* exp(-.5\*((log(lo)-mu0)/sp0)^2) $

calc ; plo = phi((log(lo)-mu0)/sp0)

; uo = exp(sp0\*inp(plo+.95)+mu0) $

calc ; fuo = 1/(uo\*sp0\*sqr(2\*pi)) \* exp(-.5\*((log(uo)-mu0)/sp0)^2) $

calc ; puo = 1-phi((log(uo)-mu0)/sp0) $

calc ; list ; lo;uo; flo-fuo ; plo ; puo $

endproc $

exec;n=20$

First 3...

[CALC] LO = .3286204

[CALC] UO = 25.0149757

[CALC] \*Result\*= .1544861

[CALC] PLO = .0242100

[CALC] PUO = .0257900

Calculator: Computed 5 scalar results

[CALC] LO = .3236204

[CALC] UO = 24.6631367

[CALC] \*Result\*= .1525084

[CALC] PLO = .0234314

[CALC] PUO = .0265686

Calculator: Computed 5 scalar results

[CALC] LO = .3186204

[CALC] UO = 24.3286542

[CALC] \*Result\*= .1504947

[CALC] PLO = .0226624

[CALC] PUO = .0273376

Iterations 58 and 59

Calculator: Computed 5 scalar results

[CALC] LO = .0436204

[CALC] UO = 17.9351365

[CALC] \*Result\*= .0008362

[CALC] PLO = .0000721

[CALC] PUO = .0499279

Calculator: Computed 5 scalar results

This is the solution. The 95% interval is much narrower than (.334,25.338)

**[CALC] LO = .0386204**

**[CALC] UO = 17.9301104**

[CALC] \*Result\*= -.0007449

[CALC] PLO = .0000459

[CALC] PUO = .0499541

**Applications**

?=======================================================================

? Chapter 4 Application 1

?=======================================================================

Read $

Year GasExp Pop Gasp Income PNC PUC PPT PD PN PS

1953 7.4 159565 16.668 8883 47.2 26.7 16.8 37.7 29.7 19.4

...

2004 224.5 293951 123.901 27113 133.9 133.3 209.1 114.8 172.2 222.8

Sample ; 1 - 52 $

Create ; G = 1000000\*gasexp/(gasp\*pop)$

Create ; t = year - 1952 $

Namelist ; X = one,income, gasp,pnc,puc,ppt,pd,pn,ps,t$

?=======================================================================

? a. Basic regression

?=======================================================================

Regress ; Lhs = g ; Rhs = X $

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=G Mean = 4.935619 |

| Standard deviation = 1.059105 |

| WTS=none Number of observs. = 52 |

| Model size Parameters = 10 |

| Degrees of freedom = 42 |

| Residuals Sum of squares = .4985489 |

| Standard error of e = .1089505 |

| Fit R-squared = .9912852 |

| Adjusted R-squared = .9894177 |

| Model test F[ 9, 42] (prob) = 530.82 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| 1.10587817 .56937860 1.942 .0588

INCOME | .00021575 .517619D-04 4.168 .0001 16805.0577

GASP | -.01108386 .00397812 -2.786 .0080 51.3429615

PNC | .00057735 .01284414 .045 .9644 87.5673077

PUC | -.00587463 .00487032 -1.206 .2345 77.8000000

PPT | .00690726 .00483613 1.428 .1606 89.3903846

PD | .00122888 .01188175 .103 .9181 78.2692308

PN | .01269051 .01259799 1.007 .3195 83.5980769

PS | -.02802781 .00799625 -3.505 .0011 89.7769231

T | .07250369 .01418280 5.112 .0000 26.5000000

?=======================================================================

? b. Hypothesis that b(NC) = b(UC) $

?=======================================================================

Calc ; list ; (b(4)-b(5))/sqr(varb(4,4)+varb(5,5)-2\*varb(4,5)) $

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

Result = .494883

?=======================================================================

? c. Elasticities. In each case, elasticity = b\*xbar/ybar

?=======================================================================

Calc ; g2004 = g(52)$

Calc ; i2004 = income(52)$

Calc ; pg2004 = gasp(52)$

Calc ; ppt2004 = ppt(52)$

Calc ; list ; ei = b(2)\*i2004/g2004

; ep = b(3)\*pg2004/g2004

; eppt = b(6)\*ppt2004/g2004$

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

EI = .948988

EP = -.222792

EPPT = .234311

?=======================================================================

? d. Log regression

?=======================================================================

Create ; logg = log(g) ; logpg = log(gasp) ; logi = log(income)

; logpnc=log(pnc) ; logpuc = log(puc) ; logppt = log(ppt)

; logpd = log(pd) ; logpn = log(pn) ; logps = log(ps) $

Namelist ; LogX = one,logi,logpg,logpnc,logpuc,logppt,logpd,logpn,logps,t$

Regress ; lhs = logg ; rhs = logx $

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LOGG Mean = 1.570475 |

| Standard deviation = .2388115 |

| WTS=none Number of observs. = 52 |

| Model size Parameters = 10 |

| Degrees of freedom = 42 |

| Residuals Sum of squares = .3812817E-01 |

| Standard error of e = .3012994E-01 |

| Fit R-squared = .9868911 |

| Adjusted R-squared = .9840821 |

| Model test F[ 9, 42] (prob) = 351.33 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -7.28719016 2.52056245 -2.891 .0061

LOGI | .99299135 .25037574 3.966 .0003 9.67214751

LOGPG | .06051812 .05401018 1.120 .2689 3.72930296

LOGPNC | -.15471632 .26696298 -.580 .5653 4.38036654

LOGPUC | -.48909058 .08519952 -5.741 .0000 4.10544881

LOGPPT | .01926966 .13644891 .141 .8884 4.14194132

LOGPD | 1.73205775 .25988611 6.665 .0000 4.23906603

LOGPN | -.72953933 .26506853 -2.752 .0087 4.23689080

LOGPS | -.86798166 .35291106 -2.459 .0181 4.17535768

T | .03797198 .00751371 5.054 .0000 26.5000000

?=======================================================================

? e. Correlations of Price Variables

?=======================================================================

Namelist ; Prices = pnc,puc,ppt,pd,pn,ps$

Matrix ; list ; xcor(prices) $

Correlation Matrix for Listed Variables

PNC PUC PPT PD PN PS

PNC 1.00000 .99387 .98074 .99327 .98853 .97849

PUC .99387 1.00000 .98242 .98783 .98220 .97685

PPT .98074 .98242 1.00000 .95847 .98986 .99751

PD .99327 .98783 .95847 1.00000 .97734 .95633

PN .98853 .98220 .98986 .97734 1.00000 .99358

PS .97849 .97685 .99751 .95633 .99358 1.00000

?=======================================================================

? f. Renormalizations of price variables

?=======================================================================

/\*

In the linear case, the coefficients would be divided by the same

scale factor, so that x\*b would be unchanged, where x is a variable

and b is the coefficient. In the loglinear case, since log(k\*x)=

log(k)+log(x), the renomalization would simply affect the constant

term. The price coefficients woulde be unchanged.

\*/

?=======================================================================

? g. Oaxaca decomposition

?=======================================================================

Dates ; 1953 $

Period ; 1953-1973 $

Matrix ; xb0 = Mean(logx)$

Regress ; lhs = logg ; rhs = logx $

Matrix ; b0 = b ; v0 = varb $

Calc ; yb0 = ybar $

Period ; 1974-2004 $

Matrix ; xb1 = mean(logx) $

Regress ; lhs = logg ; rhs = logx $

Matrix ; b1 = b ; v1 = varb $

Calc ; yb1 = ybar $

? Now the decomposition

Calc ; list ; dybar = yb1 - yb0 $ Total

Calc ; list ; dy\_dx = b1'xb1 - b1'xb0 $ Change due to change in x

Calc ; list ; dy\_db = b1'xb0 - b0'xb0 $

Matrix ; vdb = v1+v0 ; vdb = xb0'[vdb]xb0 $

Calc ; sdb = sqr(vdb)

; list ; lower = dy\_db - 1.96\*sqr(vdb)

; upper = dy\_db + 1.96\*sqr(vdb) $

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

DYBAR = .395377

DY\_DX = .122745

DY\_DB = .272631

LOWER = .184844

UPPER = .360419

?=======================================================================

? Chapter 4 Application 2

?=======================================================================

Create ; lc = log(cost/pf) ; lpl=log(pl/pf) ; lpk=log(pk/pf)$

Create ; lq = log(q) ; lqq = .5\*lq\*lq $

Namelist ; x = one,lq,lqq,lpk,lpl $

? a. Cost function

Regress; lhs = lc ; rhs = x ; printvc $

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LC Mean = -.3195570 |

| Standard deviation = 1.542364 |

| WTS=none Number of observs. = 158 |

| Model size Parameters = 5 |

| Degrees of freedom = 153 |

| Residuals Sum of squares = 2.904896 |

| Standard error of e = .1377906 |

| Fit R-squared = .9922222 |

| Adjusted R-squared = .9920189 |

| Model test F[ 4, 153] (prob) =4879.59 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -6.81816332 .25243920 -27.009 .0000

LQ | .40274543 .03148312 12.792 .0000 8.26548908

LQQ | .06089514 .00432530 14.079 .0000 35.7912728

LPK | .16203385 .04040556 4.010 .0001 .85978893

LPL | .15244470 .04659735 3.272 .0013 5.58162250

1 2 3 4 5

+----------------------------------------------------------------------

1| .06373 -.00238 .00031 .00399 -.01047

2| -.00238 .00099 -.00013 .00010 -.00020

3| .00031 -.00013 .1870819D-04 -.1493338D-04 .2453652D-04

4| .00399 .00010 -.1493338D-04 .00163 -.00102

5| -.01047 -.00020 .2453652D-04 -.00102 .00217

?=======================================================================

? b. capital price coefficient

?=======================================================================

Wald ; fn1 = 1 - b\_lpk - b\_lpl $

+-----------------------------------------------+

| WALD procedure. Estimates and standard errors |

| for nonlinear functions and joint test of |

| nonlinear restrictions. |

| Wald Statistic = 266.36109 |

| Prob. from Chi-squared[ 1] = .00000 |

+-----------------------------------------------+

+--------+--------------+----------------+--------+--------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]|

+--------+--------------+----------------+--------+--------+

Fncn(1) | .68552145 .04200352 16.321 .0000

?=======================================================================

? c. efficient scale

?=======================================================================

Wald ; fn1 = exp((1-b\_lq)/b\_lqq) $

+-----------------------------------------------+

| WALD procedure. Estimates and standard errors |

| for nonlinear functions and joint test of |

| nonlinear restrictions. |

| Wald Statistic = 21.74979 |

| Prob. from Chi-squared[ 1] = .00000 |

+-----------------------------------------------+

+--------+--------------+----------------+--------+--------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]|

+--------+--------------+----------------+--------+--------+

Fncn(1) | 18177.1045 3897.59890 4.664 .0000

Calc ; qstar = waldfns(1) ; vqstar = varwald(1,1)

; list ; lower = qstar - 1.96\*sqr(vqstar)

; upper = qstar + 1.96\*sqr(vqstar) $

?=======================================================================

? d. Raw data

?=======================================================================

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

LOWER = 10537.810653

UPPER = 25816.398344

Create ; output = q $

Sort ; lhs = output $

/\*

The estimated efficient scale is 18177. There are 25 firms in the sample that have output larger than this. As noted in the problem, many of the largest firms in the sample are aggregates of smaller ones, so it is difficult to draw a conclusion here. However, some of the largest firms (Southern, American Electric power) are singly counted, and are much larger than this scale. The important point is that much of the output in the sample is produced by firms that are smaller than this efficient scale. There are unexploited

economies of scale in this industry.

\*/

Chapter 5

Hypothesis Tests and Model Selection

**Exercises**

1. The estimated covariance matrix for the least squares estimator is

*s*2(**X′X**)-1 = 

where *s*2 = 520/(29‑3) = 20. Then, the test may be based on *t* = (.4 + .9 - 1)/[.410 + .256 - 2(.051)]1/2 = .399. This is smaller than the critical value of 2.056, so we would not reject the hypothesis.

2. In order to compute the regression, we must recover the original sums of squares and cross products for **y**. These are **X′y** = **X′Xb** = [116, 29, 76]**′.** The total sum of squares is found using *R*2 = 1 ‑ **e′e**/**y′M**0**y**, so **y′M**0**y** = 520 / (52/60) = 600. The means are = 0, = 0, = 4, so, **y′y** = 600 + 29(42) = 1064. The slope in the regression of **y** on **x**2 alone is *b*2 = 76/80, so the regression sum of squares is *b22*(80) = 72.2, and the residual sum of squares is 600 ‑ 72.2 = 527.8. The test based on the residual sum of squares is *F* = [(527.8 ‑ 520)/1]/[520/26] = .390. In the regression of the previous problem, the *t*‑ratio for testing the same hypothesis would be *t* = .4/(.410)1/2 = .624 which is the square root of .39.

3. For the current problem, **R** = [**0**,**I**] where **I** is the last *K*2 columns. Therefore, **R**(**X′X**)-1**R**′ is the lower right *K*2×*K*2 block of (**X′X**)-1. As we have seen before, this is (**X**2**′M**1**X**2)-1. Also, (**X′X**)-1**R′** is the last *K*2 columns of (**X′X**)-1. These are (**X′X**)-1**R′** =  Finally, since **q** = **0**, **Rb** ‑ **q** = (**0b**1 + **Ib**2) ‑ **0** = **b**2. Therefore, the constrained estimator is

**b\* =** (**X2′M1X2**)**b2,** where **b**1 and **b**2 are the multiple regression coefficients in the regression of **y** on both **X**1 and **X**2. Collecting terms, this produces **b\* =** . But, we have from (3-18) that **b**1 = (**X**1**′X**1)-1**X**1**′y** - (**X**1**′X**1)-1**X**1**′X**2**b**2 so the preceding reduces to **b\* =**  which was to be shown.

If, instead, the restriction is **β**2 = **β**20then the preceding is changed by replacing **Rβ** ‑ **q** = **0** with

**Rβ** ‑ **β**20 = **0**. Thus, **Rb** ‑ **q** = **b**2 ‑ **β**20**.** Then, the constrained estimator is

**b\* =** (**X2′M1X2**)(**b2 - β**20)

or

**b\* =** 

Using the result of the previous paragraph, we can rewrite the first part as

**b1\*** = (**X**1**′X**1)-1**X**1**′y** - (**X**1**′X**1)-1**X**1**′X**2**β20** = (**X**1**′X**1)-1**X**1**′**(**y** - **X**2**β**20)

which was to be shown.

4. By factoring the result in (5‑23), we obtain **b**\* = [**I** ‑ **CR**]**b** + **w** where **C** = (**X′X**)-1**R′**[**R**(**X′X**)-1**R′**]-1 and **w** = **Cq**. The covariance matrix of the least squares estimator is

Var[**b**\*] = [**I** ‑ **CR**]σ2(**X′X**)-1[**I** ‑ **CR**]**′**

= σ2(**X′X**)-1 + σ2**CR**(**X′X**)-1**R′C′** ‑ σ2**CR**(**X′X**)-1 ‑ σ2(**X′X**)-1**R′C′**.

By multiplying it out, we find that **CR**(**X′X**)-1 = (**X′X**)-1**R′**(**R**(**X′X**)-1**R′**)-1**R**(**X′X**)-1 = **CR**(**X′X**)-1**R′C′**

so Var[**b**\*] = σ2(**X′X**)-1 ‑ σ2**CR**(**X′X**)-1**R′C′** = σ2(**X′X**)-1 ‑ σ2(**X′X**)-1**R′**[**R**(**X′X**)-1**R′]**-1**R**(**X′X**)-1

This may also be written as Var[**b**\*] = σ2(**X′X**)-1{**I** ‑ **R′**(**R**(**X′X**)-1**R′**)-1**R**(**X′X**)-1}

= σ2(**X′X**)-1{[σ2(**X′X**)-1]-1 ‑ **R′**[**R**σ2(**X′X**)-1**R′**]-1**R**}σ2(**X′X**)-1

Since Var[**Rb**] = **R**σ2(**X′X**)-1**R′** this is the answer we seek.

5. The variance of the restricted least squares estimator is given in the second equation in the previous exercise. We know that this matrix is positive definite, since it is derived in the form **B′**σ2(**X′X**)-1**B′**, and σ2(**X′X**)-1 is positive definite. Therefore, it remains to show only that the matrix subtracted from Var[**b**] to obtain Var[**b**\*] is positive definite. Consider, then, a quadratic form in Var[**b**\*]

**z′**Var[**b**\*]**z** = **z′**Var[**b**]**z** ‑ σ2**z′**(**X′X**)-1(**R′**[**R**(**X′X**)-1**R′**]-1**R**)(**X′X**)-1**z**

= **z′**Var[**b**]**z** ‑ **w′**[**R**(**X′X**)-1**R′**]-1**w** where **w** = σ**R**(**X′X**)-1**z**.

It remains to show, therefore, that the inverse matrix in brackets is positive definite. This is obvious since its inverse is positive definite. This shows that every quadratic form in Var[**b**\*] is less than a quadratic form in Var[**b**] in the same vector.

6. The result follows immediately from the result which precedes (5‑28). Since the sum of squared residuals must be at least as large, the coefficient of determination, *COD* = 1 ‑ sum of squares / Σ*i* (*yi* -)2,

must be no larger.

7. For convenience, let **F** = [**R**(**X′X**)-1**R′**]-1. Then, **λ** = **F**(**Rb** ‑ **q**) and the variance of the vector of Lagrange multipliers is Var[**λ**] = **FR**σ2(**X′X**)-1**R′F** = σ2**F**. The estimated variance is obtained by replacing σ2 with *s*2. Therefore, the chi‑squared statistic is

χ2 = (**Rb** ‑ **q**) **′F′**(*s*2**F**)-1**F**(**Rb** ‑ **q**) = (**Rb** ‑ **q**) **′**[(1/*s*2)**F**](**Rb** ‑ **q**)

= (**Rb** ‑ **q**) **′**[**R**(**X′X**)-1**R′**]-1(**Rb** ‑ **q**)/[**e′e**/(*n* ‑ *K*)]

This is exactly *J* times the *F* statistic defined in (5‑19) and (5‑20). Finally, *J* times the *F* statistic in (5‑20) equals the expression given above.

8. We use (5‑28) to find the new sum of squares. The change in the sum of squares is

**e**\***′e**\* ‑ **e′e** = (**Rb** ‑ **q**) **′**[**R**(**X′X**)-1**R′**]-1(**Rb** ‑ **q**)

For this problem, (**Rb** ‑ **q**) = *b*2 + *b*3 ‑ 1 = .3. The matrix inside the brackets is the sum of the 4 elements in the lower right block of (**X′X**)-1. These are given in Exercise 1, multiplied by *s*2 = 20. Therefore, the required sum is [**R**(**X′X**)-1**R′**] = (1/20)(.410 + .256 ‑ 2(.051)) = .028. Then, the change in the sum of squares is

.32 / .028 = 3.215. Thus, **e′e** = 520, **e**\***′e**\* = 523.215, and the chi‑squared statistic is 26[523.215/520 ‑ 1] = .16. This is quite small, and would not lead to rejection of the hypothesis. Note that for a single restriction, the Lagrange multiplier statistic is equal to the *F* statistic which equals, in turn, the square of the *t* statistic used to test the restriction. Thus, we could have obtained this quantity by squaring the .399 found in the first problem (apart from some rounding error).

9. First, use (5‑28) to write **e\*′e\*** = **e′e** + (**Rb** ‑ **q**)**′**[**R**(**X′X**)-1**R′**]-1(**Rb** ‑ **q**). Now, the result that *E*[**e′e**] = (*n* ‑ *K*)σ2 obtained in (4-17) must hold here, so *E*[**e\*′e\***] = (*n* ‑ *K*)σ2 + *E*[(**Rb** ‑ **q**)**′**[**R**(**X′X**)-1**R′**]-1(**Rb** ‑ **q**)].

Now, **b** = **β** + (**X′X**)-1**X′ε**, so **Rb** ‑ **q** = **Rβ** ‑ **q** + **R**(**X′X**)-1**X′ε**. But, **Rβ** ‑ **q** = **0**, so under the hypothesis, **Rb** ‑ **q** = **R**(**X′X**)-1**X′ε**. Insert this in the result above to obtain

*E*[**e\*′e\***] = (*n*‑*K*)σ2 + *E*[**ε′X**(**X′X**)-1**R′**[**R**(**X′X**)-1**R′**]-1**R**(**X′X**)-1**X′ε**]. The quantity in square brackets is a scalar, so it is equal to its trace. Permute **ε′X**(**X′X**)-1**R′** in the trace to obtain

*E*[**e\*′e\***] = (*n* ‑ *K*)σ2 + *E*[*tr*{[**R**(**X′X**)-1**R′**]-1**R**(**X′X**)-1**X′εε′X**(**X′X**)-1**R′**]}

We may now carry the expectation inside the trace and use E[**εε′**] = σ2**I** to obtain

*E*[**e\*′e\***] = (*n ‑ K*)σ2 + tr{[**R**(**X′X**)-1**R′**]-1**R**(**X′X**)-1**X′**σ2**IX**(**X′X**)-1**R′**]}

Carry the σ2 outside the trace operator, and after cancellation of the products of matrices times their inverses, we obtain *E*[**e\*′e\***] = (*n ‑ K*)σ2 + σ2tr[**I***J*] = (*n* ‑ *K* + *J*)σ2.

10. Show that in the multiple regression of **y** on a constant, **x**1, and **x**2, while imposing the restriction

β1 + β2 = 1 leads to the regression of **y** ‑ **x**1 on a constant and **x**2 ‑ **x**1.

For convenience, we put the constant term last instead of first in the parameter vector. The constraint is **Rb ‑ q** = **0** where **R** = [1 1 0] so **R**1 = [1] and **R**2 = [1,0]. Then, β1 = [1]-1[1 ‑ β2] = 1 ‑ β2. Thus, **y** = (1 ‑ β2)**x**1 + β2**x**2 + α**i** + **ε** or **y** ‑ **x**1 = β2(**x**2 ‑ **x**1) + α**i** + **ε**.

11. If the regression model is **y** =**X**1****1 + **X**2****2 + **ε**, and **y** is regressed on **X**1 alone, then E[**b**1] = ****1 + **P**1.2****2. The prediction will be **y** = **X**1**b**1. The prediction error will be **e**1 = (**y** – **X**1**b**1) = (**X**1****1 + **X**2****2 + **ε** - **X**1**b**1). The expected value of the prediction error is E[**e**1|*X*1] = E[{**X**1(****1 – **b**1) + **X**2****2 + **ε**}|**X**1]. Taking the three parts separately, E[{**X**1(****1 – **b**1)|**X**1] = **X**1****1 – **X**1****1 - **X**1**P**1.2****2; E[**X**2****2|**X**1] = E[X2|**X**1]****2; E[**ε**|**X**1] = **0**. Thus,

E[**e**1] = E[**X**2|**X**1]****2 – **X**1**P**1.2****2. There is more that can be said at this point. The problem assumes that E[**X**2|**X**1] is linear. It follows that the matrix **P**1.2 actually estimates the slopes in these linear functions. Thus, **P**1.2 estimates this linear function, **Π**1.2 and the forecast error is generally **X**1(**Π**1.2 – **P**1.2). This has expectation zero if the linear regression of columns of **X**2 on X1 produces unbiased estimators of the columns of **Π**1.2. The end result is that omission of variables **X**2 from the regression does not necessarily produce biased forecasts; the forecast error does not necessarily have a nonzero mean. It depends on how **X**1 predicts **X**2.

The result cited is E[b1] = β1 + P1.2β2 where P1.2 = (X1′X1)-1X1′X2, so the coefficient estimator is biased. If the conditional mean function *E*[X2|X1] is a linear function of X1, then the sample estimator P1.2 actually is an unbiased estimator of the slopes of that function. (That result is Theorem B.3, equation (B-68), in another form). Now, write the model in the form

y = X1β1 + E[X2|X1]β2 + ε + (X2 - E[X2|X1])β2

So, when we regress y on X1 alone and compute the predictions, we are computing an estimator of

X1(β1 + P1.2β2) = X1β1 + E[X2|X1]β2. Both parts of the compound disturbance in this regression ε and

**(X2 - E[X2|X1])β2** have mean zero and are uncorrelated with **X**1 and E[**X**2|**X**1], so the prediction error has mean zero. The implication is that the forecast is unbiased. Note that this is not true if E[**X**2|X1] is nonlinear, since **P**1.2 does not estimate the slopes of the conditional mean in that instance. The generality is that leaving out variables wil bias the coefficients, but need not bias the forecasts. It depends on the relationship between the conditional mean function E[**X**2|**X**1] and **X**1**P**1.2.

12. The “long” estimator, b1.2 is unbiased, so its mean squared error equals its variance, σ2(X1′M2X1)-1

The short estimator, b1 is biased; E[b1] = β1 + P1.2β2. It’s variance is σ2(X1′X1)-1. It’s easy to show that this latter variance is smaller. You can do that by comparing the inverses of the two matrices. The inverse of the first matrix equals the inverse of the second one minus a positive definite matrix, which makes the inverse smaller hence the original matrix is larger - Var[b1.2] > Var[b1]. But, since b1 is biased, the variance is not its mean squared error. The mean squared error of b1 is Var[b1] + bias×bias′. The second term is P1.2β2β2′P1.2′. When this is added to the variance, the sum may be larger or smaller than Var[b1.2]; it depends on the data and on the parameters, β2. The important point is that the mean squared error of the biased estimator *may* be smaller than that of the unbiased estimator.

13. The log likelihood function at the maximum is

ln*L* = -*n*/2[1 + ln2π + ln(e′e/*n*)]

= -*n*/2{1 + ln2π + ln[*nSyy*(1 – *R*2)]}

= -*n*/2{1 + ln2π + ln(*nSyy*) + ln(1-*R*2)} where *Syy* = 

since *R*2 = 1 - e′e/Syy . The derivative of this expression is ∂ln*L*/∂*R*2 = (-*n*/2){1/(1-*R*2)}(-1) which is always positive. Therefore, the log likelihood increases when *R*2 increases.

14. An inconvenient way to obtain the result is by repeated substitution of *Ct*-1, then *Ct*-2 and so on. It is much easier and faster to introduce the lag operator used in Chapter 20. Thus, the alternative model is

*Ct* = γ1 + γ2*Yt* + γ3*LCt* + ε1*t* where LCt = Ct-1.

Then, (1 – γ3*L*)*Ct* = γ1 + γ2*Yt* + ε1*t*.  
Now, multiply both sides of the equation by 1/(1-γ3*L*) = 1 + γ3*L* + γ32*L*2 + … to obtain

***Ct =* γ1/(1 - γ3) + γ2*Yt* + γ2γ3*Yt*-1 + γ2γ3*sYt-s* + γ3*s*ε*t-s*.**

**Applications**

?=======================================================================

? Application 5.1 Wage Equation

?=======================================================================

Read;File=...; nvar=5;nobs=17919$

? This creates the group count variable.

Regress ; Lhs = one ; Rhs = one ; Str = ID ; Panel $

? This READ merges the smaller file into the larger one.

Read;File=... ; names=ability,med,fed,bh,sibs? ; group=\_groupti

;nvar=5;nobs=2178$

Names=id,educ,lwage,pexp,t;

namelist ; x1=one,educ,pexp,ability$

namelist ; x2=med,fed,bh,sibs$

?=======================================================================

? a. Long regression

?=======================================================================

regress ; lhs= lwage ; rhs = x1,x2 $

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LWAGE Mean = 2.296821 |

| Standard deviation = .5282364 |

| WTS=none Number of observs. = 17919 |

| Model size Parameters = 8 |

| Degrees of freedom = 17911 |

| Residuals Sum of squares = 4119.734 |

| Standard error of e = .4795950 |

| Fit R-squared = .1760081 |

| Adjusted R-squared = .1756861 |

| Model test F[ 7, 17911] (prob) = 546.55 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| .98965433 .03389449 29.198 .0000

EDUC | .07118866 .00225722 31.538 .0000 12.6760422

PEXP | .03951038 .00089858 43.970 .0000 8.36268765

ABILITY | .07736880 .00493359 15.682 .0000 .05237402

MED | .709887D-04 .00169543 .042 .9666 11.4719013

FED | .00531681 .00133795 3.974 .0001 11.7092472

BH | -.05286954 .00999042 -5.292 .0000 .15385903

SIBS | .00487138 .00179116 2.720 .0065 3.15620291

?=======================================================================

? b. F test

?=======================================================================

Calc ; list ; fstat = Rsqrd/(kreg-1)/((1-rsqrd)/(n-kreg)) $

+------------------------------------+

FSTAT = 14.025040

Calc ; r1 = rsqrd ; df1=n-kreg$

Matrix ; b1 = b ; v1 = varb $

Matrix ; b1 =b1(5:8) ; v1=varb(5:8,5:8)$

Regress ; lhs = lwage ; rhs = x1 $

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LWAGE Mean = 2.296821 |

| Standard deviation = .5282364 |

| WTS=none Number of observs. = 17919 |

| Model size Parameters = 4 |

| Degrees of freedom = 17915 |

| Residuals Sum of squares = 4132.637 |

| Standard error of e = .4802919 |

| Fit R-squared = .1734272 |

| Adjusted R-squared = .1732888 |

| Model test F[ 3, 17915] (prob) =1252.94 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| 1.02722913 .03004146 34.194 .0000

EDUC | .07376210 .00221425 33.312 .0000 12.6760422

PEXP | .03948955 .00089835 43.958 .0000 8.36268765

ABILITY | .08289072 .00459996 18.020 .0000 .05237402

?=======================================================================

? c. F test for hypothesis that coefficients on X2 are zero

?=======================================================================

Calc ; list ; fstat = (r1-rsqrd)/(col(x2))/((1-r1)/(df1)) $

+------------------------------------+

FSTAT = 14.025040

?=======================================================================

? c. Wald test for hypothesis that coefficients on X2 are zero

?=======================================================================

Matrix ; List ; Wald = b1'<v1>b1 $

Matrix WALD has 1 rows and 1 columns.

1

+--------------

1| 56.10016

Note Wald = 4\*F, as expected.

?=======================================================================

? Application 5.2 Translog Cost Function

?=======================================================================

? First prepare the data

?

Create ; lpk=log(pk);lpl=log(pl);lpf=log(pf)$

create ; lpk2=.5\*lpk^2 ; lpl2=.5\*lpl^2 ; lpf2=.5\*lpf^2$

Create ; lpkf=lpk\*lpf ; lplf=lpl\*lpf ; lpkl=lpk\*lpl $

Create ; lq = log(q) ; lq2 = .5\*lq^2 $

Create ; lqk=lq\*lpk ; lql=lq\*lpl ; lqf=lq\*lpf $

Create ; lc = log(cost) $

Create ; lcpf = log(cost/pf) $

Create ; lpkpf=log(pk/pf) ; lplpf=log(pl/pf) $

Create ; lpkpf2=.5\*lpkpf^2 ; lplpf2=.5\*lplpf^2 ; lplfpkf=lplpf\*lpkpf $

Create ; lqlpkf=lq\*lpkpf ; lqlplf=lq\*lplf $

?=======================================================================

? a. Beta is a,b,dk,dl,df,pkk,pll,pff,pkl,pkf,plf,c,tqk,tql,tqf

?=======================================================================

Restrictions are

0,0,1,1,1,0,0,0,0,0,0,0,0,0,0 1

0,0,0,0,0,1,0,0,1,1,0,0,0,0,0 0

R = 0,0,0,0,0,0,1,0,1,0,1,0,0,0,0 q = 0

0,0,0,0,0,0,0,1,0,1,1,0,0,0,0 0

0,0,0,0,0,0,0,0,0,0,0,0,1,1,1 0

?=======================================================================

? b. Testing the theory

?=======================================================================

Namelist ; X1=one,lq,lpk,lpl,lpf,lpk2,lpl2,lpf2,lpkl,lpkf,lplf,lq2,lqk,lq...

Namelist ; X0=one,lq,lpkf,lplf,lpkpf2,lplpf2,lplfpkf,lq2,lqlpkf,lqlplf$

Regress ; lhs = lc ; rhs=x0 $

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LC Mean = 3.071619 |

| Standard deviation = 1.542734 |

| WTS=none Number of observs. = 158 |

| Model size Parameters = 10 |

| Degrees of freedom = 148 |

| Residuals Sum of squares = 2.634416 |

| Standard error of e = .1334170 |

| Fit R-squared = .9929498 |

| Adjusted R-squared = .9925211 |

| Model test F[ 9, 148] (prob) =2316.03 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -1.13340208 1.04296294 -1.087 .2789

LQ | .02244828 .12717485 .177 .8601 8.26548908

LPKF | -.02309567 .14153592 -.163 .8706 14.4192992

LPLF | -.01690697 .09185395 -.184 .8542 30.4387314

LPKPF2 | -.04730093 .21017152 -.225 .8222 .42211776

LPLPF2 | -.03419034 .06850142 -.499 .6184 15.6173009

LPLFPKF | -.00741233 .11649585 -.064 .9494 4.84868706

LQ2 | .05544306 .00446607 12.414 .0000 35.7912728

LQLPKF | .03562155 .02862683 1.244 .2153 7.15696461

LQLPLF | .01279036 .00375187 3.409 .0008 251.570118

Calc ; ee0 = sumsqdev $

Regress ; lhs = lcpf ; rhs = x1 $

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LCPF Mean = -.3195570 |

| Standard deviation = 1.542364 |

| WTS=none Number of observs. = 158 |

| Model size Parameters = 15 |

| Degrees of freedom = 143 |

| Residuals Sum of squares = 2.464348 |

| Standard error of e = .1312753 |

| Fit R-squared = .9934018 |

| Adjusted R-squared = .9927558 |

| Model test F[ 14, 143] (prob) =1537.82 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -76.2592615 38.2800363 -1.992 .0483

LQ | -1.08042535 .37554512 -2.877 .0046 8.26548908

LPK | 6.38079702 4.52920686 1.409 .1611 4.25096457

LPL | 14.7182926 7.08482345 2.077 .0395 8.97279814

LPF | -1.89473291 2.84231282 -.667 .5061 3.39117564

LPK2 | -.32741427 .44070869 -.743 .4587 9.05539681

LPL2 | -1.53852735 .69240298 -2.222 .0279 40.2700121

LPF2 | -.07350556 .18203881 -.404 .6870 5.78602018

LPKL | -.57205049 .37189026 -1.538 .1262 38.1346773

LPKF | -.02402470 .24632928 -.098 .9224 14.4192992

LPLF | .16228289 .27007181 .601 .5489 30.4387314

LQ2 | .05297849 .00471336 11.240 .0000 35.7912728

LQK | .04014440 .02979137 1.348 .1799 35.1677247

LQL | .13104059 .03828401 3.423 .0008 74.2063474

LQF | .05865220 .02554928 2.296 .0232 28.0107601

Calc ; ee1 = sumsqdev $

Calc ; list ; Fstat = ((ee0 - ee1)/5)/(ee1/(158-15))$

+------------------------------------+

FSTAT = 1.973714

--> Calc ; list ; ftb(.95,5,143)$

+------------------------------------+

Result = 2.277490

The F statistic is small; the theory is not rejected.

?=======================================================================

? c. Testing homotheticity

?=======================================================================

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LCPF Mean = -.3195570 |

| Standard deviation = 1.542364 |

| WTS=none Number of observs. = 158 |

| Model size Parameters = 10 |

| Degrees of freedom = 148 |

| Residuals Sum of squares = 2.634223 |

| Standard error of e = .1334121 |

| Fit R-squared = .9929469 |

| Adjusted R-squared = .9925180 |

| Model test F[ 9, 148] (prob) =2315.08 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -2.78239562 1.04292476 -2.668 .0085

LQ | .01362521 .12717020 .107 .9148 8.26548908

LPKF | -.06044098 .14153074 -.427 .6700 14.4192992

LPLF | -.07639000 .09185059 -.832 .4069 30.4387314

LPKPF2 | -.10507269 .21016383 -.500 .6178 .42211776

LPLPF2 | -.00146323 .06849891 -.021 .9830 15.6173009

LPLFPKF | .01806822 .11649158 .155 .8770 4.84868706

LQ2 | .05565578 .00446590 12.462 .0000 35.7912728

LQLPKF | .03824257 .02862578 1.336 .1836 7.15696461

LQLPLF | .01296202 .00375173 3.455 .0007 251.570118

Regress ; lhs = lcpf ; Rhs = x0 ; cls:b(9)=0,b(10)=0$

+----------------------------------------------------+

| Linearly restricted regression |

| Ordinary least squares regression |

| LHS=LCPF Mean = -.3195570 |

| Standard deviation = 1.542364 |

| WTS=none Number of observs. = 158 |

| Model size Parameters = 8 |

| Degrees of freedom = 150 |

| Residuals Sum of squares = 2.896172 |

| Standard error of e = .1389526 |

| Fit R-squared = .9922456 |

| Adjusted R-squared = .9918837 |

| Model test F[ 7, 150] (prob) =2741.96 (.0000) |

| Restrictns. F[ 2, 148] (prob) = 7.36 (.0009) |

| Not using OLS or no constant. Rsqd & F may be < 0. |

| Note, with restrictions imposed, Rsqd may be < 0. |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -6.20547247 .37175165 -16.693 .0000

LQ | .40111764 .03208201 12.503 .0000 8.26548908

LPKF | -.05918207 .14502101 -.408 .6838 14.4192992

LPLF | .03234530 .08668866 .373 .7096 30.4387314

LPKPF2 | -.20340518 .21249945 -.957 .3400 .42211776

LPLPF2 | -.00516132 .06888408 -.075 .9404 15.6173009

LPLFPKF | .08684971 .10534811 .824 .4110 4.84868706

LQ2 | .06103878 .00440807 13.847 .0000 35.7912728

LQLPKF | -.138778D-16 .517639D-09 .000 1.0000 7.15696461

LQLPLF | .000000 .915064D-10 .000 1.0000 251.570118

Calc ; list ; ftb(.95,2,148)$

+------------------------------------+

Result = 3.057197

The F statistic of 7.36 is larger than the critical value of 3.057. The hypothesis is rejected.

?=======================================================================

? d. Testing generalized Cobb-Douglas against full translog.

?=======================================================================

Regress ; lhs = lcpf ; rhs = x0 ;cls:b(5)=0,b(6)=0,b(7)=0,b(9)=0,b(10)=0$

+----------------------------------------------------+

| Linearly restricted regression |

| Ordinary least squares regression |

| LHS=LCPF Mean = -.3195570 |

| Standard deviation = 1.542364 |

| WTS=none Number of observs. = 158 |

| Model size Parameters = 5 |

| Degrees of freedom = 153 |

| Residuals Sum of squares = 3.191949 |

| Standard error of e = .1444383 |

| Fit R-squared = .9914536 |

| Adjusted R-squared = .9912302 |

| Model test F[ 4, 153] (prob) =4437.33 (.0000) |

| Restrictns. F[ 5, 148] (prob) = 6.27 (.0000) |

| Not using OLS or no constant. Rsqd & F may be < 0. |

| Note, with restrictions imposed, Rsqd may be < 0. |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -5.07718678 .18072495 -28.093 .0000

LQ | .41724916 .03285950 12.698 .0000 8.26548908

LPKF | .00903097 .01466874 .616 .5391 14.4192992

LPLF | -.03131901 .00770196 -4.066 .0001 30.4387314

LPKPF2 | -.582867D-15 .127559D-07 .000 1.0000 .42211776

LPLPF2 | -.328730D-15 .986857D-08 .000 1.0000 15.6173009

LPLFPKF | .461436D-15 .201473D-07 .000 1.0000 4.84868706

LQ2 | .05956626 .00452575 13.162 .0000 35.7912728

LQLPKF | -.555112D-16 .538074D-09 .000 1.0000 7.15696461

LQLPLF | -.693889D-17 .223074D-09 .000 1.0000 251.570118

Calc ; list ; ftb(.95,5,148)$

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

Result = 2.275319

The F statistic of 6.27 is larger than the critical value of 2.275. The hypothesis is rejected.

?=======================================================================

? e. Testing Cobb-Douglas against full translog.

?=======================================================================

Matrix ; b2=b(5:10) ; v2=varb(5:10,5:10) $

Matrix ; list ; Fcd = 1/6 \* b2'<v2>b2 $

Matrix FCD has 1 rows and 1 columns.

1

+--------------

1| 28.87144

Calc ; list ; ftb(.95,6,148)$

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

Result = 2.160352

The F statistic of 28.871 is larger than the critical value of 2.16. The hypothesis is rejected.

?=======================================================================

? f. Testing generalized Cobb-Douglas against homothetic translog.

?=======================================================================

Regress ; Lhs = lcpf ; rhs = one,lq,lpkf,lplf,lpkpf2,lplpf2,lplfpkf,lq2

; cls:b(5)=0,b(6)=0,b(7)=0$

+----------------------------------------------------+

| Linearly restricted regression |

| Ordinary least squares regression |

| LHS=LCPF Mean = -.3195570 |

| Standard deviation = 1.542364 |

| WTS=none Number of observs. = 158 |

| Model size Parameters = 5 |

| Degrees of freedom = 153 |

| Residuals Sum of squares = 3.191949 |

| Standard error of e = .1444383 |

| Fit R-squared = .9914536 |

| Adjusted R-squared = .9912302 |

| Model test F[ 4, 153] (prob) =4437.33 (.0000) |

| Restrictns. F[ 3, 150] (prob) = 5.11 (.0022) |

| Not using OLS or no constant. Rsqd & F may be < 0. |

| Note, with restrictions imposed, Rsqd may be < 0. |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -5.07718678 .18072495 -28.093 .0000

LQ | .41724916 .03285950 12.698 .0000 8.26548908

LPKF | .00903097 .01466874 .616 .5391 14.4192992

LPLF | -.03131901 .00770196 -4.066 .0001 30.4387314

LPKPF2 | -.199840D-14 .243505D-07 .000 1.0000 .42211776

LPLPF2 | -.746798D-15 .608762D-08 .000 1.0000 15.6173009

LPLFPKF | .140166D-14 .121752D-07 .000 1.0000 4.84868706

LQ2 | .05956626 .00452575 13.162 .0000 35.7912728

Calc ; list ; ftb(.95,3,150) $

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

Result = 2.664907

?

?=======================================================================

? g. We have not rejected the theory, but we have rejected all the

? functional forms

? except the nonhomothetic translog. Just like Christensen and Greene.

?=======================================================================

?=======================================================================

? Application 5.3 Nonlinear restrictions

?=======================================================================

sample;1-52$

name;x=one,logpg,logi,logpnc,logpuc,logppt,t,logpd,logpn,logps$

?=======================================================================

? a. Simple hypothesis test

?=======================================================================

Regr;lhs=logg;rhs=x$

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LOGG Mean = 1.570475 |

| Standard deviation = .2388115 |

| WTS=none Number of observs. = 52 |

| Model size Parameters = 10 |

| Degrees of freedom = 42 |

| Residuals Sum of squares = .3812817E-01 |

| Standard error of e = .3012994E-01 |

| Fit R-squared = .9868911 |

| Adjusted R-squared = .9840821 |

| Model test F[ 9, 42] (prob) = 351.33 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -7.28719016 2.52056245 -2.891 .0061

LOGPG | .06051812 .05401018 1.120 .2689 3.72930296

LOGI | .99299135 .25037574 3.966 .0003 9.67214751

LOGPNC | -.15471632 .26696298 -.580 .5653 4.38036654

LOGPUC | -.48909058 .08519952 -5.741 .0000 4.10544881

LOGPPT | .01926966 .13644891 .141 .8884 4.14194132

T | .03797198 .00751371 5.054 .0000 26.5000000

LOGPD | 1.73205775 .25988611 6.665 .0000 4.23906603

LOGPN | -.72953933 .26506853 -2.752 .0087 4.23689080

LOGPS | -.86798166 .35291106 -2.459 .0181 4.17535768

Calc;r1=rsqrd$

Regr;lhs=logg;rhs=one,logpg,logi,logpnc,logpuc,logppt,t$

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LOGG Mean = 1.570475 |

| Standard deviation = .2388115 |

| WTS=none Number of observs. = 52 |

| Model size Parameters = 7 |

| Degrees of freedom = 45 |

| Residuals Sum of squares = .1014368 |

| Standard error of e = .4747790E-01 |

| Fit R-squared = .9651249 |

| Adjusted R-squared = .9604749 |

| Model test F[ 6, 45] (prob) = 207.55 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -13.1396625 2.09171186 -6.282 .0000

LOGPG | -.05373342 .04251099 -1.264 .2127 3.72930296

LOGI | 1.64909204 .20265477 8.137 .0000 9.67214751

LOGPNC | -.03199098 .20574296 -.155 .8771 4.38036654

LOGPUC | -.07393002 .10548982 -.701 .4870 4.10544881

LOGPPT | -.06153395 .12343734 -.499 .6206 4.14194132

T | -.01287615 .00525340 -2.451 .0182 26.5000000

Calc;r0=rsqrd$

Calc;list;f=((r1-r0)/2)/((1-r1)/(n-10))$

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

F = 34.868735

The critical value from the *F* table is 2.827, so we would reject the hypothesis.

?=======================================================================

? b. Nonlinear restriction

?=======================================================================

Since the restricted model is quite nonlinear, it would be quite cumbersome to estimate and examine the loss in fit. We can test the restriction using the unrestricted model. For this problem,

**f** = [*nc* - *uc*, *nc**s* - *pt**d*] ****

The matrix of derivatives, using the order given above and **"** to represent the entire parameter vector, is

**G** = = . The parameter estimates are

Thus, **f** = [-.17399, .10091]****. The covariance matrix to use for the tests is **G***s*2(**XX**)-1**G**

The statistic for the joint test is 2 = **f**[**G**s2(**XX**)-1**G**]-1**f** = .4772. This is less than the critical value for a

chi-squared with two degrees of freedom, so we would not reject the joint hypothesis. For the individual hypotheses,

we need only compute the equivalent of a *t* ratio for each element of **f**. Thus,

*z*1 = -.6053

and *z*2 = .2898

Neither is large, so neither hypothesis would be rejected. (Given the earlier result, this was to be expected.)

?=======================================================================

? c. Computations for nonlinear restriction

?=======================================================================

sample;1-52$

name;x=one,logpg,logi,logpnc,logpuc,logppt,t,logpd,logpn,logps$

Regr;lhs=logg;rhs=x$

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LOGG Mean = 1.570475 |

| Standard deviation = .2388115 |

| WTS=none Number of observs. = 52 |

| Model size Parameters = 7 |

| Degrees of freedom = 45 |

| Residuals Sum of squares = .1014368 |

| Standard error of e = .4747790E-01 |

| Fit R-squared = .9651249 |

| Adjusted R-squared = .9604749 |

| Model test F[ 6, 45] (prob) = 207.55 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -13.1396625 2.09171186 -6.282 .0000

LOGPG | -.05373342 .04251099 -1.264 .2127 3.72930296

LOGI | 1.64909204 .20265477 8.137 .0000 9.67214751

LOGPNC | -.03199098 .20574296 -.155 .8771 4.38036654

LOGPUC | -.07393002 .10548982 -.701 .4870 4.10544881

LOGPPT | -.06153395 .12343734 -.499 .6206 4.14194132

T | -.01287615 .00525340 -2.451 .0182 26.5000000

Calc;r1=rsqrd$

Regr;lhs=logg;rhs=one,logpg,logi,logpnc,logpuc,logppt,t$

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LOGG Mean = 1.570475 |

| Standard deviation = .2388115 |

| WTS=none Number of observs. = 52 |

| Model size Parameters = 7 |

| Degrees of freedom = 45 |

| Residuals Sum of squares = .1014368 |

| Standard error of e = .4747790E-01 |

| Fit R-squared = .9651249 |

| Adjusted R-squared = .9604749 |

| Model test F[ 6, 45] (prob) = 207.55 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -13.1396625 2.09171186 -6.282 .0000

LOGPG | -.05373342 .04251099 -1.264 .2127 3.72930296

LOGI | 1.64909204 .20265477 8.137 .0000 9.67214751

LOGPNC | -.03199098 .20574296 -.155 .8771 4.38036654

LOGPUC | -.07393002 .10548982 -.701 .4870 4.10544881

LOGPPT | -.06153395 .12343734 -.499 .6206 4.14194132

T | -.01287615 .00525340 -2.451 .0182 26.5000000

Calc;r0=rsqrd$

Calc;list;fstat=((r1-r0)/2)/((1-r1)/(n-10))$

+------------------------------------+

FSTAT = 34.868735

Calc;list;ftb(.95,3,42)$

+------------------------------------+

Result = 2.827049

REGR;Lhs=logg;rhs=x$

Calc ; ds=b(10);dd=-b(8);gpt=-b(6);gnc=b(4)$

Matr;gm=[0,0,0,1,-1,0,0,0,0,0 / 0,0,0,ds,0,dd,0,gpt,0,gnc]$

Calc;f1=b(4)-b(6) ; f2=b(4)\*b(10)-b(6)\*b(8)$

Matrix;list;f=[f1/f2]$

Matrix F has 2 rows and 1 columns.

1

+--------------

1| -.17399

2| .10091

Matrix;list;vf=gm\*varb\*gm'$

Matrix VF has 2 rows and 2 columns.

1 2

+----------------------------

1| .08263 -.08059

2| -.08059 .12129

Matrix;list;Wald=f'<vf>f$

Matrix WALD has 1 rows and 1 columns.

1

+--------------

1| .47716

Calc;list;z1=f(1)/sqr(vf(1,1))$

+------------------------------------+

Z1 = -.605278

Calc;list;z2=f(2)/sqr(vf(2,2))$

+------------------------------------+

Z2 = .289760

The J test in Example 5.7 is carried out using over 50 years of data. It is optimistic to hope that the underlying structure of the economy did not change in 50 years. Does the result of the test carried out in Example 8.2 persist if it is based on data only from 1980 to 2000? Repeat the computation with this subset of the data.

?====================================

? Example Application 5.4

?====================================

Dates ; 1950.1 $

Period ; 1950.1 - 2000.4 $

Create ; Ct = Realcons ; Yt = RealDPI $

Create ; Ct1 = Ct[-1] ; Yt1 = Yt[-1] $

? Example 7.2

Period ; 1950.2 - 2000.4 $

Regress; Lhs = Ct ; Rhs = one,Yt,Yt1 ; Keep = CY $

Regress; Lhs = Ct ; Rhs = one,Yt,Ct1 ; Keep = CC $

Regress; Lhs = Ct ; Rhs = one,Yt,Yt1,CC $

+----------------------------------------------------+

| Ordinary least squares regression |

| Model was estimated May 12, 2007 at 08:56:19AM |

| LHS=CT Mean = 3008.995 |

| Standard deviation = 1456.900 |

| WTS=none Number of observs. = 203 |

| Model size Parameters = 4 |

| Degrees of freedom = 199 |

| Residuals Sum of squares = 73550.21 |

| Standard error of e = 19.22496 |

| Fit R-squared = .9998285 |

| Adjusted R-squared = .9998259 |

| Model test F[ 3, 199] (prob) =\*\*\*\*\*\*\* (.0000) |

| Diagnostic Log likelihood = -886.1351 |

| Restricted(b=0) = -1766.209 |

| Chi-sq [ 3] (prob) =1760.15 (.0000) |

| Info criter. LogAmemiya Prd. Crt. = 5.931932 |

| Akaike Info. Criter. = 5.931926 |

| Autocorrel Durbin-Watson Stat. = 2.0256102 |

| Rho = cor[e,e(-1)] = -.0128051 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -.60444607 3.43245774 -.176 .8604

YT | .31456542 .04619552 6.809 .0000 3352.09360

YT1 | -.33004915 .04591940 -7.188 .0000 3325.25222

CC | 1.01450597 .01613899 62.861 .0000 3008.99507

Regress; Lhs = Ct ; Rhs = one,Yt,Ct1,CY $

+----------------------------------------------------+

| Ordinary least squares regression |

| Model was estimated May 12, 2007 at 08:56:19AM |

| LHS=CT Mean = 3008.995 |

| Standard deviation = 1456.900 |

| WTS=none Number of observs. = 203 |

| Model size Parameters = 4 |

| Degrees of freedom = 199 |

| Residuals Sum of squares = 73550.21 |

| Standard error of e = 19.22496 |

| Fit R-squared = .9998285 |

| Adjusted R-squared = .9998259 |

| Model test F[ 3, 199] (prob) =\*\*\*\*\*\*\* (.0000) |

| Diagnostic Log likelihood = -886.1351 |

| Restricted(b=0) = -1766.209 |

| Chi-sq [ 3] (prob) =1760.15 (.0000) |

| Info criter. LogAmemiya Prd. Crt. = 5.931932 |

| Akaike Info. Criter. = 5.931926 |

| Autocorrel Durbin-Watson Stat. = 2.0256102 |

| Rho = cor[e,e(-1)] = -.0128051 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -865.712368 120.569071 -7.180 .0000

YT | 9.82505250 1.36759557 7.184 .0000 3352.09360

CT1 | 1.02780685 .01635059 62.861 .0000 2982.97438

CY | -10.6765577 1.48541853 -7.188 .0000 3008.99507

? Application 7.1. We use only the 1980 data, so we

? start in quarter 2 of 1980 even though data are

? available for the last quarter of 1979.

Period ; 1980.2 - 2000.4 $

Regress; Lhs = Ct ; Rhs = one,Yt,Yt1 ; Keep = CY $

Regress; Lhs = Ct ; Rhs = one,Yt,Ct1 ; Keep = CC $

Regress; Lhs = Ct ; Rhs = one,Yt,Yt1,CC $

+----------------------------------------------------+

| Ordinary least squares regression |

| Model was estimated May 12, 2007 at 08:58:19AM |

| LHS=CT Mean = 4503.230 |

| Standard deviation = 879.3593 |

| WTS=none Number of observs. = 83 |

| Model size Parameters = 4 |

| Degrees of freedom = 79 |

| Residuals Sum of squares = 43603.43 |

| Standard error of e = 23.49345 |

| Fit R-squared = .9993123 |

| Adjusted R-squared = .9992862 |

| Model test F[ 3, 79] (prob) =\*\*\*\*\*\*\* (.0000) |

| Diagnostic Log likelihood = -377.7300 |

| Restricted(b=0) = -679.9419 |

| Chi-sq [ 3] (prob) = 604.42 (.0000) |

| Info criter. LogAmemiya Prd. Crt. = 6.360511 |

| Akaike Info. Criter. = 6.360436 |

| Autocorrel Durbin-Watson Stat. = 1.8153241 |

| Rho = cor[e,e(-1)] = .0923379 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| 39.6958824 37.1402619 1.069 .2884

YT | .20222923 .07364203 2.746 .0075 4987.32410

YT1 | -.25661196 .07221392 -3.553 .0006 4951.70482

CC | 1.04938412 .04670690 22.467 .0000 4503.23012

Regress; Lhs = Ct ; Rhs = one,Yt,Ct1,CY $

+----------------------------------------------------+

| Ordinary least squares regression |

| Model was estimated May 12, 2007 at 08:58:19AM |

| LHS=CT Mean = 4503.230 |

| Standard deviation = 879.3593 |

| WTS=none Number of observs. = 83 |

| Model size Parameters = 4 |

| Degrees of freedom = 79 |

| Residuals Sum of squares = 43603.43 |

| Standard error of e = 23.49345 |

| Fit R-squared = .9993123 |

| Adjusted R-squared = .9992862 |

| Model test F[ 3, 79] (prob) =\*\*\*\*\*\*\* (.0000) |

| Diagnostic Log likelihood = -377.7300 |

| Restricted(b=0) = -679.9419 |

| Chi-sq [ 3] (prob) = 604.42 (.0000) |

| Info criter. LogAmemiya Prd. Crt. = 6.360511 |

| Akaike Info. Criter. = 6.360436 |

| Autocorrel Durbin-Watson Stat. = 1.8153241 |

| Rho = cor[e,e(-1)] = .0923379 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -856.107861 221.141722 -3.871 .0002

YT | 1.21490273 .32340906 3.757 .0003 4987.32410

CT1 | .98759074 .04395654 22.467 .0000 4465.65542

CY | -1.13474451 .31933175 -3.553 .0006 4503.23012

?

? The results are essentially the same. This suggests

? that neither model is right.

The regressions are based on real consumption and real disposable income. Results for 1950 to 2000 are given in the text. Repeating the exercise for 1980 to 2000 produces: for the first regression, the estimate of α is 1.03 with a t ratio of 23.27 and for the second, the estimate is -1.24 with a t ratio of -3.062. Thus, as before, both models are rejected. This is qualitatively the same results obtained with the full 51 year data set.