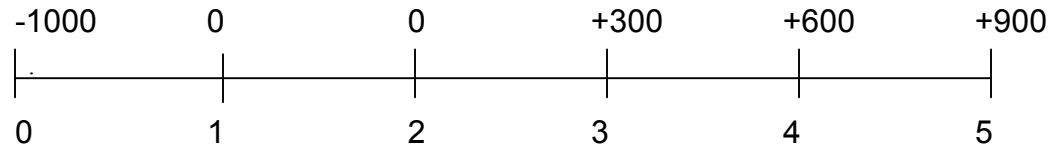


## NPV Versus IRR

W.L. Silber

I. Our favorite project A has the following cash flows:



We know that if the cost of capital is 18 percent we reject the project because the net present value is negative:

$$-1000 + \frac{300}{(1.18)^3} + \frac{600}{(1.18)^4} + \frac{900}{(1.18)^5} = NPV$$

$$-1000 + 182.59 + 309.47 + 393.40 = -114.54$$

We also know that at a cost of capital of 8% we accept the project because the net present value is positive:

$$-1000 + \frac{300}{(1.08)^3} + \frac{600}{(1.08)^4} + \frac{900}{(1.08)^5} = NPV$$

$$-1000 + 238.15 + 441.02 + 612.52 = 291.69$$

II. Thus, somewhere between 8% and 18% we change our evaluation of project A from rejecting it (when NPV is negative) to accepting it (when NPV is positive). We can calculate the point at which NPV shifts from negative to positive by searching for the value of  $r$ , called the internal rate of return (IRR) in the following equation, which makes the NPV=0.

$$-1000 + \frac{300}{(1+r)^3} + \frac{600}{(1+r)^4} + \frac{900}{(1+r)^5} = 0$$

More generally, if  $CF_i$  is the cash flow in period  $i$ , the IRR is that rate,  $r$ , such that:

$$CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_t}{(1+r)^t} = 0$$

In our case,  $CF_0 = -1000$ ,  $CF_3 = 300$ ,  $CF_4 = 600$  and  $CF_5 = 900$ . All the other  $CF_i = 0$ .

III. The IRR can, in general, only be derived by trial and error. Putting our values for the  $CF_i$  into a calculator (very carefully) we find the  $IRR = 14.668\%$ . We can check this result as follows:

$$-1000 + \frac{300}{(1.14668)^3} + \frac{600}{(1.14668)^4} + \frac{900}{(1.14668)^5} =$$

$$-1000 + 198.97 + 347.04 + 453.97 = -.02$$

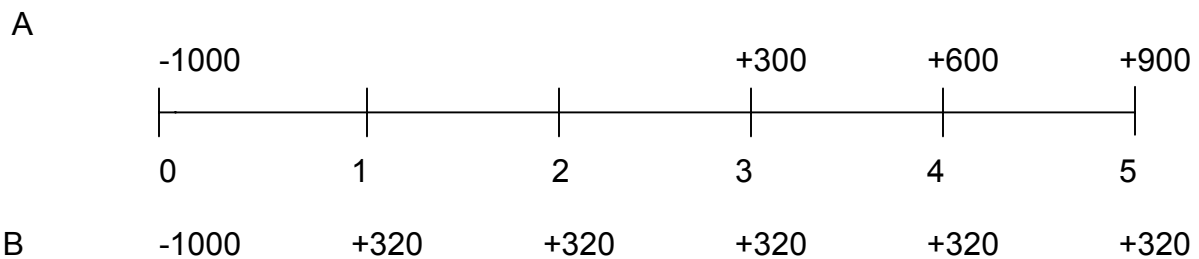
The sum is not exactly zero because of rounding.

IV. We can now formulate an alternative rule to accepting the project if  $NPV > 0$  and rejecting it if  $NPV < 0$ . In particular, we can recommend rejecting a project if the cost of capital is greater than the IRR (14.668% in this case) and we can recommend accepting a project if the cost of capital is less than the IRR. These two rules are equally acceptable in this case for determining whether project A will increase the value of the firm.

V. There are circumstances, however, where the IRR rule and the NPV rule provide conflicting advice. In particular, IRR and NPV may differ where there are two mutually

exclusive projects that must be ranked according to which one is best and where these two projects have very different timing of cash flows. Whenever there is a conflict between NPV and IRR the correct answer is provided by NPV. Let's see why.

VI. Suppose we want to compare project B with project A. The cash flows are described below, with B's cash flows equally distributed over time, while A's cash flow (as we saw) are delayed.



We have already solved for the IRR of project A, i.e.,  $IRR_A = 14.668\%$ . Solving for the IRR of project B produces,  $IRR_B = 18.03\%$ . Thus, the IRR rule ranks project B better than A. Let's see whether that is also true for the NPV rule, i.e., let's see if  $NPV_B$  is always greater than  $NPV_A$ . To implement the NPV rule we must calculate the NPV of A and B for alternative values of the cost of capital. This is done in table 1:

Table 1

Cost of Capital	$NPV_A$	$NPV_B$
20%	-175	-43
15%	-12	+73
10%	+194	+213
8%	+291	+277
5%	+458	+385

Notice that project B is better (has a higher NPV) than project A when the cost of capital is above 10% (above 20% both have negative NPVs, but B is less bad), while project A is better when the cost of capital is below 8%. In fact, you can calculate the exact cost of capital at which the recommendation switches by setting  $NPV_A = NPV_B$  and solving (after some algebraic manipulations) for the IRR under those circumstances. This IRR turns out to be 8.8169%.

VII. We see that the NPV rule says that project B is better than project A (the same ranking as the IRR rule) only for “high” values for the cost of capital, i.e., for a cost of capital greater than 8.8169%. For values of the cost of capital below 8.8169% project A is better. This makes considerable sense. When the cost of capital is “high” the delayed cash flows of project A are penalized considerably, while when the cost of capital is “low” the delayed cash flows are not penalized. Indeed, the low cost of capital makes those “large but delayed” cash flows quite valuable.

VIII. One way to understand the preference of NPV over IRR, more generally, is to recognize that NPV uses the “correct” rate, i.e., the cost of capital, to discount the cash flows, rather than an “arbitrary” rate, i.e., the IRR, that makes  $NPV = 0$ .

Another way to understand the superiority of the NPV rule is that the discounting process inherent in both the IRR and NPV techniques implicitly assumes the reinvestment of the cash flows at whatever discount rate is used, either IRR or the cost of capital. When the IRR is very high relative to the cost of capital it is unrealistic to assume reinvestment at that high rate. This is especially damaging when comparing

two investments with very different timing of cash flows. We will revisit this reinvestment assumption later, under our discussion of yield to maturity on coupon bonds, where its meaning will become clearer.