Calculating the Annual Return (Realized Compound Yield) on a Coupon Bond

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Objective:

To show that the annual return actually earned on a coupon-bearing bond will equal its yield to maturity only if you can and do reinvest the coupons at the yield to maturity.

PROOF FOR ANNUAL PAY BONDS

1. Assume: F = 1000 C = $80 t=4 years

2. If P=100 we know that YTM = 8%

3. Definition of Annual Return

$$r_{ann} = \left(\frac{V_t}{V_0} \right)^{1/t} - 1,$$

where V_t = $ amount at the end and V_0 is the $ amount at the beginning. In our case

$$V_0 = \$1000 \quad \text{and} \quad t = 4$$

therefore

$$r_{ann} = \left(\frac{V_t}{\$1000} \right)^{1/4} - 1$$

4. To calculate r_{ann} we must calculate V_t. To calculate V_t we must account for the reinvestment of the annual 8% coupon (=$80 per annum). Assuming we reinvest these coupons at 8%, we have the following cash flows on the bond:

Cash Flows

	Yr 1	Yr 2	Yr 3	Reinvest	Yr 4
1st coupon	$80	⇒		*$(1.08)^3$	= $100.78
2nd coupon		$80	⇒	*$(1.08)^2$	= $93.31
3rd coupon			$80	*(1.08)	= $86.40
4th coupon + principal					$1080.00
FINAL TOTAL (V_t) =					$1360.49

5. In this case,

$$r_{ann} = \left(\frac{\$1360.49}{\$1000} \right)^{1/4} - 1 = .08$$

Thus

$r_{ann} = YTM$ if you reinvest the coupons at the YTM

6. If you reinvest the coupons at more than 8% you accumulate more than $1360.49 and earn an annual return > .08 and if you reinvest the coupon at less than 8% you accumulate less than $1360.49 and earn an annual return < .08.

FOR SEMI-ANNUAL PAY BONDS: AN EXERCISE

1. Assume: F = $1000 C/2 = $40 t = 4 years

2. If P = 100 we know YTM = 8%

3. Calculate the annual return assuming you reinvest the coupons at the YTM/2 or at .08/2 = .04

4. What is the relationship between YTM and r_{ann} in this case?