Coopetition in Platform-Based Retailing: On the Platform's Entry

Lian Qi
Rutgers Business School, Rutgers, The State University of New Jersey, New Brunswick, New Jersey 08854, USA,
lianqi@business.rutgers.edu

Haotian Song
School of Management, Zhejiang University, Hangzhou 310058, China, hasong@zju.edu.cn

Wenqiang Xiao
Stern School of Business, New York University, New York City, New York 10012, USA, wx2@stern.nyu.edu

We study the dynamic incentive interactions between a platform and a third-party seller over two stages, where the seller exerts product-value-enhancement effort in the first stage in anticipation of the platform's potential entry in the second stage. Upon entry, the platform can exert effort to boost the value of the product sold by the platform, with positive spillovers to the value of the product sold by the platform. We show the existence of both the protective and the receptive regimes, characterizing the necessary and sufficient conditions for each regime. These conditions are of thresholds type with respect to the parameters including the degree of spillover, the referral rate, and the competition intensity. In the protective (receptive) regime, the seller is worse (better) off with the platform's entry than without, thereby distorting effort downward (upward) from the first best to deter (induce) the platform's entry to the products of intermediate value. Notably, if the spillover effect is negative, only the protective regime will arise. We also provide the necessary and sufficient conditions under which the platform's non-entry commitment strictly improves the platform's profits by restoring the seller's effort to the first best, achieving the win-win outcome for both the platform and the seller.

Key words: platform-based incentive, platform entry, coopetition

1. Introduction

In the process of becoming "an everything store", Amazon enables third-party sellers to list and sell their products on Amazon Marketplace, a service launched in November 2000. The marketplace allows Amazon not only to earn a referral fee from every unit of third-party sales but also to expand the offerings on its site without having to invest in additional inventory. Meanwhile, third-party sellers gain access to Amazon's hundreds of millions of customers. These mutual benefits foster the collaborative relationship between Amazon and its third-party sellers, resulting in a steady increase in the third-party's sales revenue as a percentage of the total sales on Amazon.com in the last two decades that reached a record high of 58% in 2018 (Keyes, 2019).

Despite these symbiotic advantages, Amazon, as the owner of the marketplace with direct access to continuous flow-in product information such as prices and sales, sourcing channels and costs, etc., may find some products attractive to offer by itself after partnering with a third-party seller for a period of time, thereby competing directly with the seller. Zhu and Liu (2018) discover that in a 10-month time interval (from June 2013 to April 2014), Amazon had entered 4,852 (3%) of the 163,853 products that were initially offered only by third-party sellers. While such a strategic move allows Amazon to capture the entire profit margin of its own sales instead of just earning a fraction of the seller's margin via the referral fee, it is often perceived by the sellers as a free ride on their costly efforts invested earlier in discovering and improving the value of the products, the return of which is prone to Amazon's misappropriation by entering into the product space after seeing the success of the products. Consequently, those affected sellers may fall into victims of swimming with sharks. We call the effect of Amazon's entry to directly compete with a third-party seller the competition effect.

As much as Amazon's entry to directly compete with the seller takes a bite from the seller's sales, it can lift up the total sales with Amazon's subsequent marketing efforts in enhancing the product value, e.g., by prominent display on its web pages, advertising, and superior delivery service. As evidence, Zhu and Liu (2018) find that Amazon's entry lowers the sales rankings of the affected products by 57% on average, implying the increase of total sales because sales rankings are (log-linear) negatively correlated with sales volumes. Further, Amazon's Buy Box, a simplified check-out process taking 80% of the entire purchases on Amazon Marketplace, is won by Amazon with 65% chance, leaving 35% chance to the seller due to the winner rotation when both Amazon and the third-party seller sell the same product (Bettadapura, 2018). This suggests a nontrivial fraction of the increase in the total sales due to Amazon's efforts in improving the product value may spill over to the third-party seller. We call the effect of Amazon's entry to improve product value with some positive spillovers to the third-party seller the spillover effect.

The preceding evidence suggests that to fully characterize the dynamic incentives of Amazon's entry problem, it is important to consider both the negative competition effect and the positive spillover effect, both of which would impact the seller's product-value-enhancement effort decision in the earlier stage and thus Amazon's entry decision and its subsequent product-value-enhancement effort decision in the later stage. The consideration of both effects that result in the coopetition between Amazon and the seller sets our paper apart from the existing literature on the strategic incentives of Amazon's entry (e.g., Jiang et al. 2011).

Specifically, we model the entry problem of the platform such as Amazon over two stages. In the first stage, a third-party seller is the only one selling a product on the platform. The product value, defined to be the intercept of the inverse demand, is jointly determined by an exogenous base value and an endogenous product-value-enhancement effort level, exerted by the seller only in the first stage. The seller makes the quantity decision, which together with the product value determines the sales price. A fixed fraction of the sales price is paid by the seller to the platform as the referral fee. In the second stage, after inferring the product value carried over from the first stage, the platform decides whether or not to enter the product space. Upon non-entry, the second stage's problem is identical to that of the first stage. Upon entry, the platform decides the product-value-enhancement effort with positive spillovers to the value of the product sold by the seller as well; then the two parties are engaged in Cournot quantity competition.

We show the existence of both the protective and the receptive regimes, characterizing the necessary and sufficient conditions for each regime. These conditions are of thresholds type with respect to the parameters including the degree of spillover, the referral rate, and the competition intensity. Specifically, we show that the seller should adopt the receptive standpoint to induce the platform's entry under any one of the following circumstances: 1) sufficiently large degree of spillover; 2) intermediate referral rate; 3) sufficiently small competition intensity; and 4) moderately large competition intensity. In the protective (receptive) regime, the seller is worse (better) off with the platform's entry than without, thereby distorting effort downward (upward) from the first best to deter (induce) the platform's entry to the products with intermediate value. We show that under the protective regime with intermediate product value, the platform's non-entry commitment strictly improves the platform's profits by restoring the seller's effort to the first best, achieving the win-win outcome for both the platform and the seller.

The remainder of the paper is organized as follows. We review the related literature in Section 2 and describe the model in Section 3. Section 4 presents the equilibrium analysis and the results. Section 5 characterizes the win-win regime of the platform's commitment of non-entry. Section 6 checks the robustness and Section 7 concludes.

2. Literature Review

The most closely related papers are Jiang et al. (2011) and Zhu and Liu (2018), both of which study the platform's entry into its third-party sellers' product spaces. Jiang et al. (2011) build a signaling model to examine the platform's entry into the product space of a third-party seller with private base demand information. They show that the seller may ex ante underinvest in her demand-enhancement efforts for demand masking purpose as a strategic response to discourage the platform's entry. They assume that the platform will become the monopoly after its entry and the seller will be forced out of the product space. We relax this assumption by allowing both the platform and the seller to coexist in selling the same product after the platform's entry. This is partly motivated by the empirical findings that the majority of sellers remain selling in Marketplace even after Amazon selling the same product and that Amazon's entry increases a seller's likelihood of abandoning the product on Amazon only by 6% (Zhu and Liu 2018). More importantly, by considering the platform's product-value-enhancement efforts (absent in Jiang et al. 2011), our model captures the spillover effect, a phenomenon with empirical evidence from the literature on the more general platform-owners' entry into their complementors' product spaces (to be reviewed later). The interaction between the competition effect and the spillover effect leads to the drastically different result from those of others: The seller may ex ante overinvest in her efforts to induce the platform's entry. Such a finding, together with the empirical evidence from the literature on platform-owners' entry that some complementors indeed increased their innovation efforts in response to platforms' future entry (to be reviewed later), suggests that our work complements Jiang et al. (2011) by providing a more multifaceted characterization of the seller's strategic response.

The empirical work by Zhu and Liu (2018) reports that over a 10-month period, Amazon entered 4,852 (3%) of the 163,853 products that were initially offered only by third-party sellers. Interestingly, they empirically verify that Amazon is more likely to enter those successful product spaces (measured by sales revenue and product ratings). Our theoretical result on the platform's optimal entry strategy is consistent with this empirical finding and thus provides theoretical justification for such an entry strategy. They also find that Amazon's entry lowers the sales rankings of the affected products by 57% (and hence improves the total sales), suggesting the importance of considering the spillover effect for a more complete characterization of the multifaceted nature of the platform-owner's entry problem. This is the key new feature that sets our work apart from the seminal work by Jiang et al. (2011).

Our paper falls into the growing body of literature on the more general problem of platform-owner's entry into its complementors' product spaces. The vast majority of papers in this stream of literature are empirical and cross-disciplinary. Gawer and Cusumano (2002) find that Intel tries to avoid competing directly with complementors and enters only to product spaces in which it is not satisfied, with the intention that competition can foster innovation. Intel's entry strategy of picking low-demand products is in sharp contrast with that of Amazon reported by Zhu and Liu (2018). Zhu and Liu (2018) reconcile this difference by identifying the level of platform-specific investment as the key driver for the platform's strategic entry decision.

Within this stream, more related to ours are the papers focusing on distinct platforms and providing the empirical evidences of the spillover effect caused by the platform-owner's entry and/or of the complementors' strategic response in increasing ex ante product-related efforts (before the platform's entry). In particular, Li and Agarwal (2017) find that Facebook's entry into photo-sharing (via its integration of Instagram) increases consumer awareness, boosts the overall market for photo-sharing apps, and causes a positive spillover effect on big third-party apps. Kang (2017) finds that Google's introduction of Health apps on their mobile systems has a positive effect on its complementors developing health apps. Cennamo et al. (2018) show a positive entry effect of console manufacturers on third-party game publishers in the video-game industry. They explain that the underlying rationale is that game popularity declines rapidly and hence the overall market expansion effect dominates the competitive effect in the video-game industry. Wen and Zhu (2018) find that Google's entry threat boosts the innovation (prior to entry) by 7.8% from those affected but popular third-party mobile apps on Google's Android system. See Zhu (2019) for a comprehensive review of this stream of literature. Motivated by these empirical findings, our theoretical work, by considering both the competition effect and the spillover effect, identifies simple conditions (in terms of the degree of spillover, the referral rate, and the competition intensity) under which one effect is stronger than the other, resulting in either protective or inducive strategy from the complementor.

The platform's entry to form a coopetition relationship with its third-party seller also bears some similarities with the supplier encroachment problem, in which a supplier opens an online channel to compete directly with its retailer. The platform's entry decision, like the supplier's encroachment decision, turns the initially collaborative relationship into the more complex coopetitive relationship later on. Aryal et al. (2007) show that although the supplier encroachment introduces competition between the supplier and the retailer, both of them may be better off due to the insight that the supplier encroachment may induce the supplier to lower the wholesale price, thereby mitigating the classical double marginalization problem. Li et al. (2014) further add a new effect to the interplay between the competition effect and the double marginalization mitigation effect, i.e., the signaling cost effect such that the retailer with private demand information intends to use under-order as a signal to mitigate the supplier's competitive reaction. Huang et al. (2018) examine the strategic interaction between the retailer's demand information sharing decision in the early stage and the supplier's encroachment decision in the late stage, whereas Guan et al. (2019) examine the strategic interaction between the retailer's inventory withholding decision in the early stage and the supplier's encroachment decision in the late stage. Zhang et al. (2020) study the practice of manufacturer encroachment in a supply chain, where the manufacturer competes with the retailer by introducing a direct channel, and analyze three advertising schemes and their influences. Ha et al. (2022) explore the channel choice problem of online platforms that enhance demand through service efforts and derive conditions for the agency channel, the reselling channel, or the dual channel to emerge at equilibrium. Zhang et al. (2021) investigate a platform service supply chain involving a supplier and a service platform, where the platform invests in retail services and the supplier decides whether to encroach on the retail market by opening a direct channel on the platform. They find that the platform can benefit from supplier encroachment due to reduced double marginal effects and profit sharing. Despite our focus on a distinct business problem, our model bears a similar two-stage dynamic incentive interaction. However, in our model, the seller exerts product-value-enhancement effort in the early stage in anticipation of the platform's both entry decision and product-value-enhancement effort in the later stage, resulting in the interplay between the competition effect and the spillover effect, the latter of which is absent in the literature on supplier encroachment.

The spillover effect has been studied in the literature on private label introduction. Chen et al. (2011) show that a retailer's private label introduction can help mitigate the double marginalization problem of the national brand, and characterizes conditions under which this is beneficial for the supply chain. Zhou et al. (2019) introduce the spillover either from the private label to the national brand or vice versa, and show that the manufacturer may benefit from the retailer's private label introduction. Our study is distinct from theirs in at least two important aspects. First, they study a manufacturer selling a national brand to a retailer selling a private label, whereas we consider the contractual relationship between a platform and its third-party seller. Second, we have a distinct focus on how both the competition effect and the spillover effect distort the seller's effort in the first-stage in anticipation of the platform's entry and effort decisions in the second-stage.

3. Model

Consider a third-party seller (she, indexed by S) selling a single product on an online platform (it, indexed by P) over two stages. The seller can purchase the product from a supplier at per unit cost that is normalized to zero without loss of generality. In the first stage, with the product's base value a perceived by the seller, the seller invests in sales service effort. This effort elevates the product's value from a to $a + e_S$, incurring a cost $C_S(e_S)$, borne by the seller. This effort reflects the seller's initial focus on quality enhancement during the early stages of product introduction. Such effort includes providing excellent customer service, enriching product descriptions, promptly responding to customer inquiries, ensuring robust after-sales support, and improving the product's perceived quality through consumer ratings. This period is pivotal for laying a solid groundwork for the product's reputation, as consumer ratings accumulated early on significantly shape future customers' perceptions. The early positive impressions and high-quality interactions often have a lasting effect on a product's market position (see, e.g., Cho and Janda 2022). Consequently, we assume that the enhanced perceived product value $(a + e_S)$ is maintained in the subsequent stage. In Section 6.2, we expand the model to consider the possibility of only a partial carryover of this enhanced value.

Similar to Jiang et al. (2011), we assume that the enhanced product value $a + e_S$, together with the seller's order quantity q_1, determines the average selling price per unit in the first stage, denoted by p_1, according to the following linear inverse demand function:

$$p_1 = a + e_S - q_1.$$

That is, all q_1 units are sold at the average price p_1 per unit. At the end of the first stage, the seller collects the total sales revenue $q_1 p_1$ and pays the platform the referral fee of

fq_1p_1. We assume that the referral rate f is exogenously given in $[0,1)$, which is consistent with the common practice of charging merchants a constant referral rate for each product category. Consequently, the seller's profit in the first stage is

$$\Pi_1^S = q_1p_1(1-f) - C_S(e_S)$$

and that of the platform is

$$\Pi_1^P = fq_1p_1.$$

Similar to Jiang et al. (2011), we assume that the product base value a is observable by the seller but not by the platform. This assumption necessitates our two-stage model to reflect the practice that the platform delays its entry decision to the second stage in order to cherry-pick products after more information is available to the platform. At the beginning of the second stage, the platform observes the average per-unit selling price p_1 and the sales quantity q_1, based on which it can infer the enhanced product value $a+e_S$ in the first stage. Because of the lasting effect of the seller's effort, the product base value perceived by customers at the beginning of the second period remains the same as $a+e_S$, based on which the platform then decides whether or not to enter the product space.

Under the platform's non-entry decision, the seller is the only party selling the product, and the second stage's problem is identical to the first stage's problem except that the seller no longer makes the effort decision in the second stage. Note that the assumption of muting the seller's effort decision in the second stage is made only for expositional simplicity and can be relaxed by adding the seller's effort decision in the second stage as well without qualitatively changing the results. The seller decides the order quantity q_S, which together with the carryover product value $a+e_S$, determines the average per-unit selling price $p_S = a+e_S-q_S$. The seller then collects the sales revenue q_Sp_S and pays the platform the referral fee of fq_Sp_S. Consequently, under non-entry, the seller's profit in the second stage is

$$\Pi_{2,N}^S = q_Sp_S(1-f) \tag{1}$$

and that of the platform is

$$\Pi_{2,N}^P = fq_Sp_S, \tag{2}$$

where the subscript N refers to non-entry.

Under the platform's entry decision, the platform incurs a fixed entry cost K by establishing the sourcing channel at the same per unit purchase cost, which has been normalized to zero. The assumption of identical purchase costs can be relaxed without qualitatively altering the results. The platform then exerts product-value-enhancement effort e_P (e.g., devoting resources in advertisement and prominent display), which increases the value of the product sold by the platform from $a+e_S$ to $a+e_S+e_P$ but at the cost $C_P(e_P)$, borne by the platform. This effort effectively epitomizes the platform's marketing endeavors in reality, such as online advertising and the use of sponsored product placements (e.g., Amazon sponsored products), playing a complementary role to the seller's quality effort. Meanwhile, we recognize the platform's unique capability to amplify the product's reach and reinforce its market presence. Leveraging its marketing prowess and reach, the platform can substantially boost product awareness and visibility, benefiting not just its own sales but also providing nonnegative spillover effects to the seller by increasing the product's overall market presence. We model this spillover effect as increasing the value of the product sold by the seller from $a+e_S$ to $a+e_S+\delta e_P$, where $\delta \in [0,1]$ represents the degree of the spillover effect. We will also examine the negative spillover effect, i.e., $\delta < 0$, in Section 6.4.

After the platform's effort investment, the seller and the platform are engaged in Cournot competition with product value being $a+e_S+\delta e_P$ and $a+e_S+e_P$, respectively. Let q_S and q_P be the order quantities of the seller and of the platform, respectively. Then the average per unit selling prices of the seller and the platform in the second stage are respectively given as follows:

$$p_S = a + e_S + \delta e_P - q_S - \beta q_P$$
$$p_P = a + e_S + e_P - q_P - \beta q_S,$$

where $\beta \in [0,1]$ expresses the degree of differentiation (perceived by the customers) between the seller and the platform and reflects the competition intensity, with $\beta = 1$ representing the case of perfect substitutes. The inverse demand functions are derived from maximizing a quadratic customer utility function, following the procedure established in the celebrated work by Singh and Vives (1984). The utility function therein confirms our above explanations of the intercepts (i.e., $a+e_S+\delta e_P$ and $a+e_S+e_P$) and the coefficient β. Further elaboration can be found in Appendix E.

Under the above linear competition model, the seller's profit in the second stage is

$$\Pi_{2,E}^{S} = q_S p_S (1 - f) \tag{3}$$

and the platform's profit in the second stage is

$$\Pi_{2,E}^{P} = q_S p_S f + q_P p_P - C_P(e_P) - K, \tag{4}$$

where the subscript E refers to entry. Figure 1 summarizes the sequence of events in the two-stage game between the platform and the third-party seller.

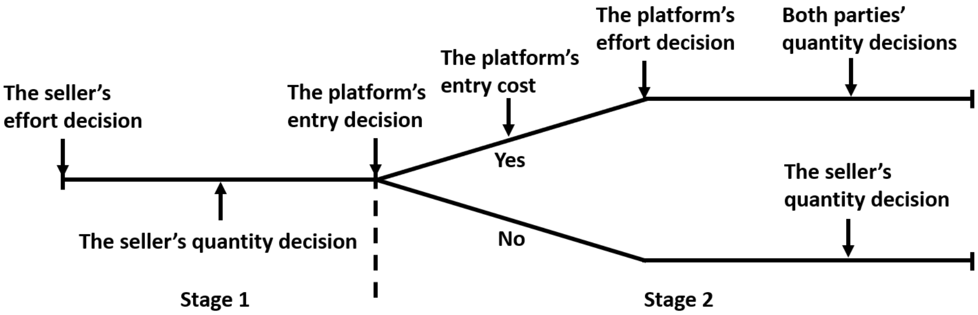


Figure 1 **The sequence of events over the two stages.**

For expositional simplicity, we assume the effort cost functions are quadratic, i.e., $C_i(e_i) = c_i e_i^2$, for $i \in \{P, S\}$, noting that our analysis can be extended to any convex effort cost functions. We normalize $c_P = 1/2$ and assume $c_S > 1/2$ implying that the platform is more cost-efficient than the seller in enhancing product value perceived by customers. We summarize the mathematical notation in Table 1.

4. Equilibrium Analysis and Results

We use the backward analysis to characterize the subgame perfect Nash equilibrium of the two-stage game. We start by deriving the two parties' equilibrium quantity decisions in the second stage given that the platform has entered and exerted effort. We then analyze the platform's optimal effort decision and its optimal entry decision given the enhanced product value by comparing the platform's profits with and without entry in the second stage. We finally characterize the seller's optimal effort and quantity decisions in the first stage.

The Optimal Quantity and Effort Decisions in the Second Stage

Table 1 Notation

	Symbol	Description
Constant	f	The referral rate
	δ	The degree of spillover effect of the platform's effort
	β	The competition intensity
	c_S	The seller's effort cost coefficient
	c_P	The platform's effort cost coefficient
	K	The platform's entry cost
	a	The product base value
Stage 1	e_S	The seller's effort decision
	q_1	The seller's quantity decision
	p_1	The product price
	Π_1^S	The seller's profit
	Π_1^P	The platform's profit
Stage 2	e_P	The platform's effort decision
	q_S	The seller's quantity decision
	q_P	The platform's quantity decision
	p_S	The product price on the seller's end
	p_P	The product price on the platform's end.
	$\Pi_{2,N}^S$	The seller's profit without the platform's entry
	$\Pi_{2,N}^P$	The platform's profit without entry
	$\Pi_{2,E}^S$	The seller's profit with the platform's entry
	$\Pi_{2,E}^P$	The platform's profit with its entry

Suppose the platform has entered the seller's product space and exerted effort so that the respective enhanced product values of the seller and the platform are $a + e_S + \delta e_P$ and $a + e_S + e_P$. The following lemma characterizes the two parties' equilibrium quantity decisions. All the proofs are relegated to Appendix A.

LEMMA 1. *Given that the platform has entered the seller's product space and that the enhanced product values are* $a+e_S+\delta e_P$ *and* $a+e_S+e_P$ *for the products sold by the seller and by the platform, respectively, the two parties' equilibrium quantity decisions in the second stage are* $q_S^* = \left[\frac{2-\beta}{4-\beta^2(1+f)}(a+e_S+e_P) - \frac{2(1-\delta)}{4-\beta^2(1+f)}e_P\right]^+$ *and* $q_P^* = \frac{a+e_S+e_P-\beta(1+f)q_S^*}{2}$.

Lemma 1 implies that the seller's order quantity in the second stage upon the platform's entry may drop to zero, forcing the seller to opt out of the market. This may occur because despite the same competition intensity β for both the two parties, the seller's enhanced product value $a+e_S+\delta e_P$ is inferior to that of the platform's product. Further, the referral fees paid by the seller to the platform weaken the seller's competitiveness relative to the platform, resulting in the possible exit of the seller at the equilibrium. The following lemma characterizes the platform's optimal effort decision given that it has entered the product space with the enhanced product value $a+e_S$. It also provides the analytical condition for the case of the seller's presence in the second stage.

LEMMA 2. *Given that the platform has entered the seller's product space and that the enhanced product value from the first stage is* $a+e_S$*, if*

$$4(1+\delta)-2\beta(2-\delta+\delta^2)+\beta^3(1+f)-\beta^2(2+f+\delta f)>0, \tag{5}$$

then the platform's optimal product-value-enhancement effort in the second stage is

$$e_P^*=\frac{(a+e_S)(8+8\delta f-4\beta(1+\delta)(1+f)+\beta^3(1+\delta)f(1+f)-2\beta^2(f+\delta(-1+f^2)))}{8-8\delta^2 f+8\beta\delta(1+f)-2\beta^3\delta f(1+f)+\beta^4(1+f)^2+2\beta^2(-4-3f+\delta^2(-1+f^2))},$$

and the seller remains in the second stage, i.e., $q_S^*>0$*; otherwise,* $e_P^*=\frac{(2-\beta)(a+e_S)}{\beta-2\delta}$ *and the seller opts out in the second stage, i.e.,* $q_S^*=0$.

It is verifiable that the left-hand side of (5) increases in δ and decreases in both β and f. This implies that everything else being equal, the seller will quit the market in the second stage upon the platform's entry for sufficiently small value of δ, for sufficiently large value of β, or for sufficiently large value of f. This result is intuitive because a decrease of δ, an increase of β, and an increase of f all weaken the seller's competitiveness relative to the platform in the quantity competition game of the second stage. We assume that Condition (5) holds in the remaining discussions in the paper, since the problem does not involve seller-platform interactions and turns to be trivial if the seller is doomed to leave the product space after the entry of the platform.

The Platform's Optimal Entry Decision in the Second Stage

After characterizing the platform's optimal effort decision and the two parties' optimal quantity decisions upon the platform's entry, we now proceed to characterize the platform's optimal entry decision given that the enhanced product value from the first stage is $a+e_S$, which has been inferred by the platform after observing the average per-unit selling price p_1 and the sales quantity q_1 in the first stage, i.e., $a+e_S=p_1+q_1$. We denote the platform's optimal profit in the second stage by $\Pi_{2,E}^P$ if it enters the seller's product space, and by $\Pi_{2,N}^P$ if it does not enter. The following lemma shows that the platform's optimal entry strategy is of the simple threshold type based on the platform's inferred value of $a+e_S$.

PROPOSITION 1. *Given that the platform has inferred that the enhanced product value from the first stage is* $a+e_S$*, the platform is better off by entering the seller's product space* *than not entering, i.e.,* $\Pi^P_{2,E} > \Pi^P_{2,N}$ *if and only if* $a + e_S > \Delta$*, where* Δ *is formulated in the proof. Furthermore,* Δ *has the following properties:*

- *as* δ *increases,* Δ *first increases and then decreases;*
- *as* f *increases,* Δ *first increases and then decreases;*
- *as* β *increases,* Δ *first increases, then decreases and finally increases.*

In deciding whether or not to enter the seller's product space, the platform needs to compare the potential revenue including both its own product sales revenue and the referral fees from the seller against the fixed entry cost K. Because the total revenue increases in the product value $a + e_S$ (inferred from the sales quantity and price in the first stage), the total revenue dominates the fixed entry cost for sufficiently high value of $a + e_S$. This explains the intuition behind the optimality of the threshold-type entry strategy given in Proposition 1. While intuitive, Proposition 1 provides theoretical justification for the empirical finding by Zhu and Liu (2018) that the platform is more likely to enter into product spaces with high product sales and consumer ratings, and also for the anecdotal observations by Jiang et al. (2011) that the platform has a prominent presence in top-ranked brackets (in sales) across many product categories with diminishing presence in lower ranked brackets. It is in contrast with the practice of those platforms (e.g., Intel) focusing on motivating innovation and thereby entering only those underperforming products (Gawer and Cusumano 2002, Gawer and Henderson 2007).

The monotonicity properties of Δ outlined in Proposition 1 imply how the platform's entry decision depends on the parameters (δ, f, β) after observing the product's enhanced value $a + e_S$ in the first stage. First, Proposition 1 implies that everything else being equal, the platform should enter only when the degree of spillover δ is either sufficiently small or sufficiently large but not intermediate. A sufficiently small value of δ gives the platform a significant competitive edge over the seller in terms of enhanced product value and thus a stronger incentive to enter. Interestingly, the platform should also enter when δ is sufficiently large but for a distinct reason. A sufficiently large value of δ means that the seller's product value benefits substantially from the spillover effect, enabling the platform to derive large profits through the referral fee and therefore strengthening its incentive to enter.

Second, the impact of the referral fee f on the platform's entry decision is similar to that of δ, i.e., the platform should enter only when the referral fee f is either sufficiently small or sufficiently large but not intermediate. Under a sufficiently small value of f, the platform has little concern about whether or not its entry will hurt its referral fee. Under a sufficiently large value of f, the platform's post-entry marketing effort that spills over to the seller may result in significant increase in the referral fees, well offsetting the damage caused by the competition.

Third, everything else being equal, the platform should enter only when the competition intensity β is either sufficiently small or moderately large, but not moderately small and sufficiently large. When β is sufficiently small, the competition caused by the platform's entry has very little damage to the seller's profits as well as to the platform's referral fees. As β increases, both the platform's direct sales profits and the referral fees decrease, weakening the platform's entry incentive. Interestingly, as β further increases in the moderate range, the platform's post-entry quantity decision q_P^* drops to the extent that the net competition effect βq_P^* on the seller starts decreasing, resulting in an increase in the seller's profits as well as in the referral fees. Therefore, the platform's entry incentive strengthens. This counterintuitive outcome, diverging from traditional competition frameworks, results from the unique dynamics of the e-commerce environment where the platform must weigh both direct sales and referral fees simultaneously. This strategic entry regime, i.e., when β is moderately large, exists only for sufficiently large values of f and δ, where the combined positive effects of the referral fee and entry spillover are significant. However, as β continues to rise to a sufficiently high level, the platform becomes even more conservative in sales, leading to a more aggressive selling stance from the seller. Despite accruing considerable referral fees, the platform's direct sales revenue suffers significantly, rendering the total profits less favorable compared to the non-entry scenario. Thus, with sufficiently large β, the platform withdraws from direct competition, solely focusing on earning through referral fees.

Overall, the monotonicity results in Proposition 1 reveal how the interplay among the referral-fee effect, the spillover effect, and the competition effect influences the platform's strategic entry decisions, underpinning the complex dynamics between market forces and strategic behavior in e-commerce platform-seller ecosystems.

The Seller's Optimal Quantity and Effort Decisions in the First Stage

Finally, we are ready to characterize the seller's optimal decision on the sales service effort level e_S in the first stage in anticipation of the two parties' optimal responses in the second stage, which have been characterized in Lemmas 1 and 2.

The difficulty lies in the fact that the seller's total profits as a function of e_S, denoted by $\Pi^S(e_S)$, take distinct functional forms depending on whether or not the platform will enter the seller's product space. We denote the seller's total profits by $\Pi^S_N(e_S)$ and by $\Pi^S_E(e_S)$ under the platform's entry and non-entry, respectively. Although the simple threshold-type entry strategy given in Proposition 1 allows us to write the seller's total profits in the two mutually exclusive and exhaustive cases as follows:

$$\Pi^S(e_S) = \begin{cases} \Pi^S_N(e_S) & \text{if } e_S + a \leq \Delta \\ \Pi^S_E(e_S) & \text{if } e_S + a > \Delta \end{cases},$$

the characterization of the seller's optimal effort decision requires analyzing the seller's preference over the platform's entry and the platform's non-entry, i.e., the analytical comparison between $\Pi^S_N(e_S)$ and $\Pi^S_E(e_S)$. To this end, by Lemmas 1 and 2, we have the following closed-form expressions for $\Pi^S_N(e_S)$ and $\Pi^S_E(e_S)$:

$$\Pi^S_N(e_S) = \frac{(1-f)(a+e_S)^2}{2} - c_S e_S^2 \tag{6}$$

and

$$\Pi^S_E(e_S) = \left(\frac{1}{4} + H\right)(1-f)(a+e_S)^2 - c_S e_S^2 \tag{7}$$

where

$$H = \begin{cases} \frac{(4(1+\delta)-2\beta(2-\delta+\delta^2)+\beta^3(1+f)-\beta^2(2+f+\delta f))^2}{(8-8\delta^2 f+8\beta\delta(1+f)-2\beta^3\delta f(1+f)+\beta^4(1+f)^2+2\beta^2(-4-3f+\delta^2(-1+f^2)))^2} & \text{if (5) holds} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

Interestingly, the above closed-form expressions imply that for any given e_S, the seller prefers the platform's entry to non-entry, i.e., $\Pi^S_E(e_S) > \Pi^S_N(e_S)$ if and only if $H > 1/4$, which depends only on the three parameters, i.e., δ, f, and β. We define Ω to be the set of values (δ, f, β) so that the condition $H > 1/4$ holds, i.e., $\Omega = \{(\delta, f, \beta) | H > 1/4\}$. Let $\overline{\Omega}$ be the complementary set of Ω, i.e., $\overline{\Omega} = \{(\delta, f, \beta) | H \leq 1/4\}$. The following proposition provides the necessary and sufficient conditions to fully characterize the seller's preference over the platform's entry and non-entry, with respect to the three parameters δ, f, and β in the sequel.

Proposition 2. *For any e_S, the seller prefers the platform's entry to non-entry, i.e., $\Pi_E^S(e_S) > \Pi_N^S(e_S)$, if and only if $(\delta, f, \beta) \in \Omega$. Specifically, for any given f and β, $(\delta, f, \beta) \in \Omega$ if and only if $\delta > \tilde{\delta}(f, \beta)$; for any given δ and β, $(\delta, f, \beta) \in \Omega$ if and only if $f > \tilde{f}(\delta, \beta)$; for any given δ and f, $(\delta, f, \beta) \in \Omega$ if and only if $\beta < \beta_1(\delta, f)$ or $\beta > \beta_2(\delta, f)$ where $\beta_1(\delta, f) \leq \beta_2(\delta, f)$. The thresholds $\tilde{\delta}$, $\tilde{f}$, β_1 and β_2 are defined in the proof.*

Because the platform's entry intensifies the product market competition and places the seller at a disadvantageous competitive position due to the referral fees, the platform's entry may paint a gloomy picture for the seller. However, by considering a salient feature in practice that the platform can improve the product value perceived by customers via investing resources in advertisement and prominent display in search results with positive spillovers to the seller, we show in Proposition 2 that the seller may strictly prefer the platform's entry to non-entry in the regime Ω. Although the existence of such a regime is not surprising due to the positive spillover effect, Proposition 2 provides non-trivial necessary and sufficient conditions with respect to (δ, f, β) under which the spillover effect dominates the competition effect implying that the seller should be receptive for inviting the platform's entry instead of following the conventional wisdom to deter the platform's entry.

For any given f and β, as the degree of spillover δ increases, the spillover effect strengthens and so does the seller's competitiveness in the second stage, implying that the seller's standpoint of the platform's entry reverses from being protective to being receptive when δ exceeds the threshold $\tilde{\delta}(f, \beta)$. For any given δ and β, as the referral rate f begins to increase from zero, the platform finds greater motivation to invest in enhancing the product's value through marketing effort e_P^*. This is because such enhancement efforts not only boost the platform's own sales but also amplify the seller's sales, the latter of which translates into increased referral fees for the platform. As e_P^* escalates, so does the spillover effect, indicating that the seller's perspective on the platform's market entry shifts from a protective stance to a receptive one once f surpasses a certain threshold $\tilde{f}(\delta, \beta)$.

Conventional wisdom suggests that as the competition intensity β increases, the seller's preference should switch from being receptive to being protective, because the platform's entry poses a greater threat with heightened competition intensity. Proposition 2 shows that such a wisdom is partially correct. Indeed, as β increases, the seller's preference switches from being receptive to being protective. However, as β further increases, the seller's preference is reversed again from being protective to being receptive. Such a counterintuitive result is driven by the strategic reaction from the platform in response to the increase of β, as explained earlier in the discussion of Proposition 1 with respect to β. That is, the platform, caring both the referral fees and its direct sales profits, reduces its quantity decision q_P^* to the extent that the net competition effect βq_P^* starts decreasing in β, thereby reversing the seller's preference to being receptive for sufficiently high β.

Proposition 2 provides the following managerial implications for the seller and the platform. To the extent that the spillover effect is present, i.e., the platform's entry and its subsequent investment in enhancing the product value benefits not only its own product but also those sold by the seller, the platform's entry may not always paint a gloomy picture for the seller. Instead, the seller should adopt the receptive standpoint to induce the platform's entry under any one of the following (overlapping) circumstances: (i) sufficiently large value of δ; (ii) sufficiently large value of f; (iii) sufficiently small value of β; and (iv) sufficiently large value of β. Under these circumstances, it is useful for the platform to be transparent about the potential benefits of the platform's investment in product space that will be passed to the seller, e.g., the nature and scale of investments in enhancing product value, data evidence on how much the platform's entry influences the total sales and the seller's sales, etc. In so doing, the platform can mitigate the seller's concern of uneven competition caused by the platform's entry thereby strengthening the seller's tie to the platform.

In the remainder of the section, we examine how the seller should make her effort and quantity decision in the first stage. We carry out our analysis in the two mutually exclusive and exhaustive regimes $\overline{\Omega}$ and Ω in the sequel. In the protective regime $\overline{\Omega}$, we analyze how the seller should make her effort decision to deter the platform's entry as much as possible. In the receptive regime Ω, we characterize how the seller should make her effort decision to induce the platform's entry as much as possible.

To this end, we introduce two benchmark effort levels. Let $e_N^B(a)$ ($e_E^B(a)$) be the seller's first best effort level assuming the platform will never (always) enter the seller's product space in the second stage. By definition, $e_N^B(a)$ and $e_E^B(a)$ are the unconstrained maximizers of $\Pi_N^S(e_S)$ and $\Pi_E^S(e_S)$, respectively. It then follows from the first-order necessary condition that

$$e_N^B(a) = \frac{(1-f)a}{f+2c_S-1}, \tag{9}$$

$$e_E^B(a) = \frac{a(1-f)(1+4H)}{4c_S - (1-f)(1+4H)}. \tag{10}$$

The following two propositions characterize the seller's optimal effort and order quantity decision in the first stage in the protective regime $\overline{\Omega}$ and in the receptive regime Ω, respectively.

PROPOSITION 3. *In the protective regime, i.e.,* $(\delta, f, \beta) \in \overline{\Omega}$, *the seller's optimal effort decision in the first stage, denoted by* $e_S^*(a)$ *is:* $e_S^*(a) = e_N^B(a)$ *for* $a \leq \underline{a}_1$, $e_S^*(a) = \Delta - a$ *for* $a \in (\underline{a}_1, \bar{a}_1)$, *and* $e_S^*(a) = e_E^B(a)$ *for* $a \geq \bar{a}_1$; *the seller's optimal order quantity decision in the first stage, denoted by* q_1^*, *is* $q_1^* = (a + e_S^*(a))/2$. *The thresholds* $\underline{a}_1$ *and* $\bar{a}_1$ *are defined in the proof.*

In the first stage, the seller anticipates that the platform will adopt the threshold-type entry strategy by cherry-picking only those products with sufficiently high value. Therefore, the seller is incentivized to deviate from these benchmark effort levels if doing so can alter the platform's entry decision in the seller's favor. In particular, in the protective regime $\overline{\Omega}$, the seller may underinvest (relative to the benchmark $e_N^B(a)$) in her effort so that the platform, who would have found the product attractive had she followed the first best effort decision, now finds the product space no longer attractive for entry because of the seller's strategic underinvestment in effort relative to the first best. Proposition 3 reveals that such an underinvestment in effort relative to the first best arises only when the product base value is intermediate, i.e., $a \in (\underline{a}_1, \bar{a}_1)$. When a is sufficiently small, i.e., $a \leq \underline{a}_1$, or a is sufficiently large, $a \geq \bar{a}_1$, the seller's best response is to follow the first-best decision because doing so would not result in the platform's entry anyway under the former and because the required effort distortion is so large that the seller gives up effort distortion under the latter. Figure 2 illustrates how the seller should vary her optimal effort level based on the product base value a, with the downward distortion in effort arising for intermediate values of a to deter the platform's entry. Moreover, the amount of downward distortion in effort required to deter the platform's entry increases when the product base value a increases over the intermediate range.

PROPOSITION 4. *In the receptive regime, i.e.,* $(\delta, f, \beta) \in \Omega$, *the seller's optimal effort decision in the first stage, denoted by* $e_S^*(a)$ *is:* $e_S^*(a) = e_N^B(a)$ *for* $a < \underline{a}_2$, $e_S^*(a) = \Delta + \epsilon - a$ *for* $a \in [\underline{a}_2, \bar{a}_2]$ *where* ϵ *is infinitesimally small, and* $e_S^*(a) = e_E^B(a)$ *for* $a > \bar{a}_2$; *the seller's*

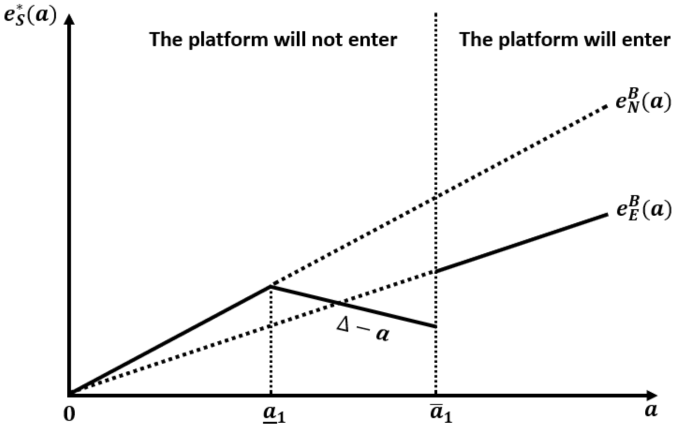


Figure 2 **The protective regime** $(\delta, f, \beta) \in \overline{\Omega}$.

optimal order quantity decision in the first stage, denoted by q_1^**, is* $q_1^* = (a + e_S^*(a))/2$*. The thresholds* $\underline{a}_2$ *and* $\bar{a}_2$ *are defined in the proof.*

Compared with the protective regime, contrast arises under the receptive regime. Instead of distorting her effort downward from the first best in the protective regime, Proposition 4 reveals that the seller in the receptive regime should distort her effort upward from the first best to induce the platform's entry. The upward distortion in effort from the first best arises only when the product base value is intermediate, i.e., $a \in [\underline{a}_2, \bar{a}_2]$. This is because when a is sufficiently small, inducing the platform's entry requires too much of effort distortion which is too costly for the seller and when a is sufficiently large, there is no need for the seller to distort effort away from the first best as the platform will enter even under the seller's the first best effort. Moreover, the amount of upward distortion in effort required to induce the platform's entry decreases when the product base value a increases over the intermediate range. This is illustrated in Figure 3.

The implications of Propositions 3 and 4 are as follows. In practice, the platform's entry into the product spaces initially owned by its third-party sellers has often triggered complaints and protective actions from some sellers such as deliberately reducing product-value enhancement services, e.g., McFarland (2009, 2010). This is consistent with our analytical result under the protective regime where the seller indeed distorts her effort downward from the first best to deter the platform's entry for intermediate values of a. However, Proposition 4 discovers that the opposite result may arise at the equilibrium, i.e., the seller distorts her effort upward from the first best under the receptive regime with intermediate values of a. We not only show the existence of overinvestment but also

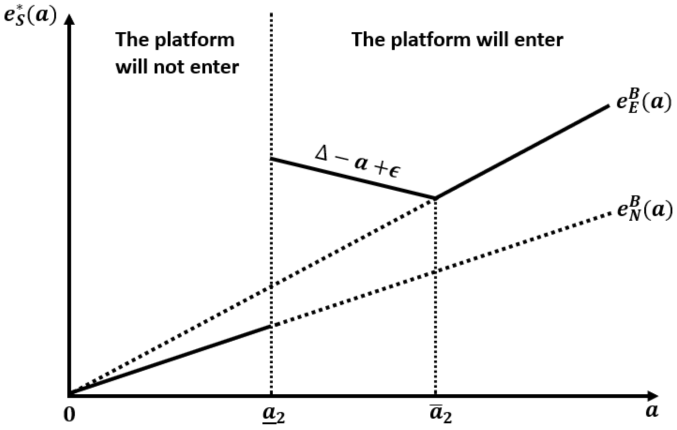
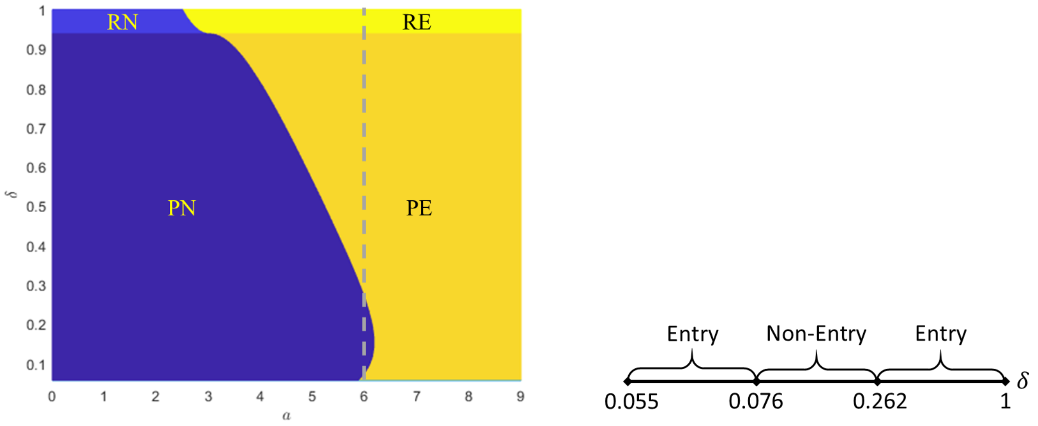


Figure 3 **The receptive regime** $(\delta, f, \beta) \in \Omega$.

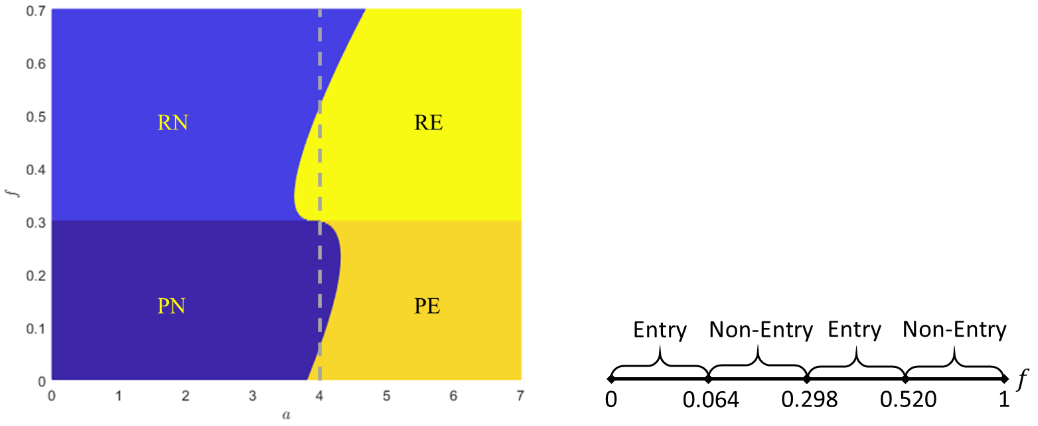
provide clean necessary and sufficient conditions as to when it happens, i.e., $(\delta, f, \beta) \in \Omega$ (see Proposition 2 for equivalent and more intuitive representations in terms of δ, f, and β respectively) and $a \in [\underline{a}_2, \bar{a}_2]$. Our finding of the seller's overinvestment in effort to induce the platform's entry is in contrast with that by Jiang et al. (2011) where only underinvestment in the seller's product value enhancement effort arises.

Finally, we close this section by presenting numerical results as to how the model parameters a, δ, f, and β influence the platform's entry decision and the seller's preference in the two-stage equilibrium outcome. The regime RN (RE) stands for the scenario where the seller is receptive but the platform chooses non-entry (entry). The regime PN (PE) represents the scenario where the seller is protective and the platform chooses non-entry (entry). As representatives, Figures 4-6 depict the various regimes in the equilibrium with respect to (a, δ), (a, f) and (a, β), respectively, with other parameters held constant. Several observations are noteworthy. First, the platform's entry decisions in the equilibrium with respect to δ and β are consistent with the analytical results in Proposition 1 (see Figures 4b and 6b), although the latter characterizes the platform's optimal entry decision after inferring the value of $a + e_S$ instead of the final equilibrium outcome. Second, some of the regimes characterized in Proposition 1 may degenerate especially for sufficiently large or small value of a. Third, how f impacts the platform's entry or non-entry in the equilibrium outcome differs from that in Proposition 1 (see Figure 5b). The non-entry arises for sufficiently large f in the equilibrium because with large f, the seller has little incentive to actually exert effort to induce the platform's entry despite the fact that the seller is receptive.



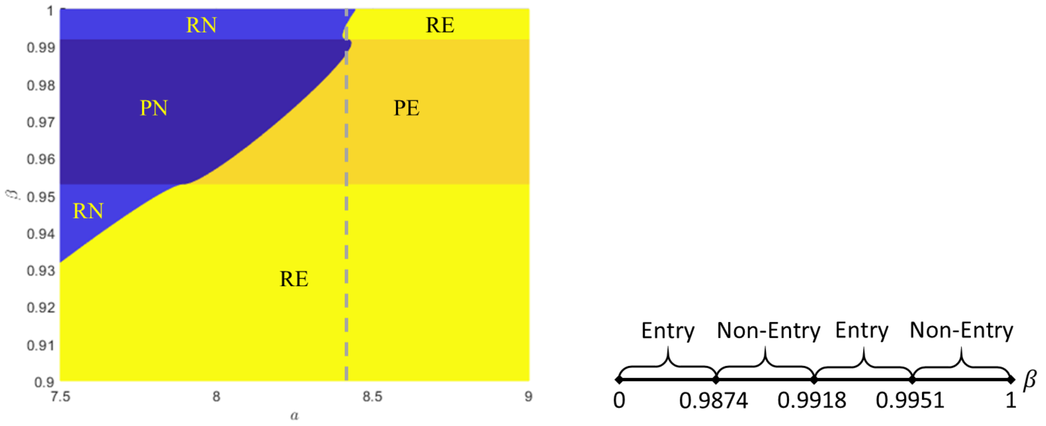
(a) Regimes with respect to (a, δ) (b) How regimes switch when $a = 6$

Figure 4 **Regimes of entry and non-entry (parameters:** $c_S = 0.65$, $K = 10$, $f = 0.19$ **and** $\beta = 0.86$**).**



(a) Regimes with respect to (a, f) (b) How regimes switch when $a = 4$

Figure 5 **Regimes of entry and non-entry (parameters:** $c_S = 0.65$, $K = 10$, $\delta = 0.88$ **and** $\beta = 0.92$**).**



(a) Regimes with respect to (a, β) (b) How regimes switch when $a = 8.41$

Figure 6 **Regimes of entry and non-entry (parameters:** $c_S = 0.65$, $K = 10$, $\delta = 0.62$ **and** $f = 0.751$**).**

5. Win-Win Regime of the Platform's Commitment of Non-Entry

Recall from Proposition 3 that the seller under the protective regime $\bar{\Omega}$ with intermediate product base value $a \in (\underline{a}_1, \bar{a}_1)$ should strategically distort her effort downward from the first best effort to successfully deter the platform's entry. In this scenario, the platform can benefit from the non-entry commitment because in so doing the seller would restore her effort to the first best thereby improving the platform's profits without altering the platform's actual entry decision. In other scenarios, e.g., the receptive regime with intermediate product base value $a \in [\underline{a}_2, \bar{a}_2]$, such a non-entry commitment may hurt the platform because it eliminates the seller's effort upward distortion thereby lowering the platform's profits without altering the platform's entry decision. An important question, which is also the core question addressed by Jiang et al. (2011) albeit in a different setting, is under what conditions such a non-entry commitment achieves win-win for both the platform and the seller. The following proposition identifies the necessary and sufficient condition with respect to the model parameters (δ, f, β) and a under which the platform is strictly better off under the non-entry commitment. Further, it reveals that when these conditions are met, the platform's non-entry commitment also strictly benefits the seller, thereby achieving the win-win outcome.

Proposition 5. *If the platform commits non-entry in advance, it is strictly better off when the product's value* $a \in (\underline{a}_1, \bar{\bar{a}}_1)$ *and* $(\delta, f, \beta) \in \overline{\Omega}$*, where* $\bar{\bar{a}}_1$*, defined in the proof, satisfies* $\bar{\bar{a}}_1 > \bar{a}_1$*. Under the same conditions, the platform's non-entry commitment also strictly improves the seller's profits.*

The driver for the platform to potentially benefit from the non-entry commitment is that such a commitment may boost the seller's effort in the first stage thereby increasing the platform's referral fees. The increase in the platform's referral fees may outweigh the platform's gain from direct sales after entry. Clearly, the platform's non-entry commitment would not result in the seller's effort boost under the receptive regime Ω, because the platform's entry induces the seller to exert more effort under the receptive regime. It remains to examine when the platform benefits from the non-entry commitment under the protective regime $\overline{\Omega}$. Recall from Figure 2 that there are three cases with respect to the product base value a. When a is sufficiently small, i.e., $a \leq \underline{a}_1$, the seller's optimal effort level $e_S^*(a)$ without commitment is already the first best effort for the seller under the platform's non-entry, implying that the platform's non-entry commitment will not influence the seller's effort. When a is intermediate, i.e., $a \in (\underline{a}_1, \bar{a}_1)$, the seller's optimal effort level $e_S^*(a)$ without commitment is downward distorted from her first best effort to successfully deter the platform's entry, implying that the platform's non-entry commitment can restore the seller's effort up to the first best without altering the entry decision thereby making the platform strictly better off. This explains that both parties benefit from the platform's non-entry commitment for those products with product base value $a \in (\underline{a}_1, \bar{a}_1)$. Interestingly, Proposition 5 shows that the win-win regime goes beyond the case of intermediate a and includes those $a \in [\bar{a}_1, \bar{\bar{a}}_1)$. Now, the platform's commitment to non-entry alters the equilibrium outcome from entry to non-entry, resulting in greater effort incentives for the seller by ensuring that she completely owns the product space in the second stage. Such an effort boost is beneficial for the platform via the increased referral fees, outweighing the gains from direct sales. However, when a is overly large such that $a \geq \bar{\bar{a}}_1$, the product market is so attractive that the platform's gains from direct sales via entry become the dominant factor, outweighing the gains in referral fees via the commitment of non-entry. Therefore, the commitment of non-entry hurts the platform's overall profits. This explains the necessary and sufficient conditions, i.e., $a \in (\underline{a}_1, \bar{\bar{a}}_1)$ and $(\delta, f, \beta) \in \overline{\Omega}$, in Proposition 5.

Despite that a is unknown to the platform at the beginning of the first stage, Proposition 5 implies that the platform should definitely not make the non-entry commitment for the products falling into the receptive regime. For those products falling into the protective regime, such a non-entry commitment should also be avoided if there is clear evidence that the product value is either sufficiently low or sufficiently high.

6. Robustness

We examine the robustness of our core results by exploring four extensions. The first extension involves contemplating an alternate sequence of events during the second stage. The second extension investigates a scenario where the carryover effect of the seller's product value enhancement from Stage 1 to Stage 2 is partial, rather than complete. The third extension considers the effort cost functions in a cumulative manner. The fourth extension assesses the consequences of a negative spillover effect, i.e., $\delta < 0$.

6.1. Simultaneous Effort and Quantity Decisions in the Second Stage

Instead of assuming that the platform makes the effort decision prior to the two parties' quantity decisions in the base model, we now consider the sequence where the platform makes its effort and quantity decision in parallel with the seller's quantity decision. We provide the equilibrium analysis for this modified game in Appendix B, based on which we discuss the robustness of our core results with respect to such an alternative sequence in the following.

First, similar to Proposition 1, we show that the platform's optimal entry strategy remains to be the threshold-type, but with the entry threshold (denoted by Δ_{sim}) no less than that of the base model, i.e., $\Delta_{sim} \geq \Delta$. This implies that with everything else being equal, the platform's entry incentive is weakened by altering the sequential effort/quantity decisions to the simultaneous decisions. The intuition is similar to that of the first mover advantage, where the platform is better off by moving first in making the effort decision to influence the subsequent quantity competition.

Second, we establish the necessary and sufficient conditions for both the receptive and the protective regimes, denoted by Ω_{sim} and $\bar{\Omega}_{sim}$. Similar to Proposition 2, we show that $\Omega_{sim} = \{(\delta, f, \beta) | H_{sim} > 1/4\}$, but with $H_{sim} > H$ for any given (δ, f, β), implying that with everything else being equal, the seller is more likely to be receptive to the platform's entry under the simultaneous sequence than that under the sequential sequence. The intuition is that under the simultaneous sequence, the platform can no longer use its effort decision to influence the quantity competition. Interestingly, we prove that unlike the base model where the seller may be forced out of the competition in the second stage, the seller will always remain in the second stage under the simultaneous sequence setting.

Third, we are able to show that Propositions 3 to 5 continue to hold with proper modifications to the thresholds, implying that the qualitative insights with respect to the effort distortions and the win-win regime of the platform's non-entry commitment remain unchanged.

6.2. Partial Carryover Effect in the Second Stage

The discussion thus far has been premised on the assumption that there is a complete carryover of the seller's enhanced product value from Stage 1 to Stage 2. In this extended scenario, we propose a modification where the enhancement in product value by the seller in Stage 1 is only partially retained in Stage 2. To quantify this, we introduce a carryover parameter $\gamma \in [0, 1]$ such that the enhanced product value carried over to Stage 2 is $\gamma(a + e_S)$. Clearly, when $\gamma = 1$, the model boils down to the original one. Consequently, when the platform opts out of entry at the beginning of Stage 2, the seller's per unit selling price is $p_S = \gamma(a+e_S) - q_S$; and when the platform decides to enter the product space, the two parties' per unit selling prices become:

$$p_S = \gamma(a+e_S) + \delta e_P - q_S - \beta q_P$$
$$p_P = \gamma(a+e_S) + e_P - q_P - \beta q_S.$$

The detailed analysis, provided in Appendix C, follows the standard backward induction. Moreover, the results can be derived by simply adjusting $a+e_S$ to $\gamma(a+e_S)$ within the analysis of the original full-carryover model. Discussions pertaining to the robustness of our core findings are outlined as follows.

First, because the only modification is the substitution of $a+e_S$ with $\gamma(a+e_S)$, it suffices to replace $a+e_S$ with $\gamma(a+e_S)$ in the expressions of equilibrium quantities and the platform's equilibrium effort. As a result, the platform's optimal entry strategy continues to follow a threshold policy, such that the platform will enter the seller's product space if and only if $\gamma(a+e_S) > \Delta$, with Δ being the same as defined in Proposition 1. In other words, the platform's entry incentive is moderated by the partial-carryover parameter γ, all other factors remaining equal.

Second, and somewhat intriguingly, we find that the receptive and protective regimes remain to be $\Omega = \{(\delta, f, \beta)|H > 1/4\}$ and $\bar{\Omega} = \{(\delta, f, \beta)|H \leq 1/4\}$ with H being unchanged. This is because, irrespective of the platform's decision to enter the seller's product space in the second stage, the seller's enhanced product value, $\gamma(a+e_S)$, is uniform. Combining with the same Stage-1 profit of the seller, the comparison between Π_N^S and Π_E^S essentially reduces to comparing $\gamma/4$ and γH and further to comparing $1/4$ and H. This immediately ensures the same equilibrium structures of the seller's Stage-1 effort decision as delineated in Propositions 3 and 4. In scenarios where the product base value is intermediate, the seller's effort distortions, both upward and downward, continue to manifest in the protective and receptive regimes, respectively.

Note that we have initially posited that the partial carryover effect is uniformly applied to both the seller and the platform. However, our model is adaptable to include asymmetric carryover effects by differentiating the carryover coefficients: γ_S for the seller and γ_P for the platform. This adjustment leads to modified unit selling prices expressed as:

$$p_S = \gamma_S(a+e_S) + \delta e_P - q_S - \beta q_P,$$
$$p_P = \gamma_P(a+e_S) + e_P - q_P - \beta q_S.$$

Incorporating these distinct parameters introduces additional complexity to the model's analysis. The relative magnitudes of γ_S and γ_P play a critical role in determining potential equilibrium outcomes, including scenarios where corner cases such as $q_S^* = 0$ or $q_P^* = 0$ may arise. We can establish a condition akin to Condition (5) that ensures competitive engagement by both parties in Stage 2. Consequently, the primary insights from our analysis are preserved. This general model includes the independent effects of both parties' efforts as a special case, with $\gamma_P = 0$ and $\delta = 0$.

6.3. Cumulative Effort Cost

In this section, we consider an alternative form of effort cost, which is in a cumulative sense. In particular, the platform's effort cost of enhancing the product value from $a + e_S$ to $a + e_S + e_P$ in Stage 2 is $c_P((a + e_S + e_P)^2 - (a + e_S)^2)$. Introducing this extra parameter c_P indeed adds a layer of complexity to the game. However, the choice to use a general cost coefficient c_P is deliberate for two primary reasons. First, the cumulative nature makes it inherently more costly for the platform to justify exerting additional effort in Stage 2 due to the escalating cost implications. Maintaining the platform's cost coefficient at $c_P = 1/2$ could potentially deter the platform from making any effort in Stage 2, thus simplifying the analysis to a trivial case where the platform's actions do not contribute to the dynamics of competition and cooperation. Second, by allowing c_P to be a general coefficient, we can further demonstrate the flexibility and robustness of our model. This approach not only accommodates a broader range of scenarios but also underscores the adaptability of our core results to variations in the cost structure.

The analysis is detailed in Appendix D. Below, we provide an outline of the important steps towards the equilibrium outcomes. First, altering the cost functions does not affect the two parties' quantity decisions in Stage 2. Consequently, q_S^* and q_P^* remain unchanged from those in Lemma 1. In focusing on the nontrivial regime, where both the seller and the platform are actively competing and the platform opts to exert positive effort, we establish a condition analogous to Condition (5) from the original model, here denoted as (5_{cum}).

Second, given the same pair of optimal quantities, we derive the platform's optimal effort e_P^* in Stage 2. Despite the inclusion of the cost coefficient c_P in the model, the platform's optimal effort e_P^* in Stage 2 continues to be a function of $(a + e_S)$. It follows that the platform's profit difference between the one under its entry and the one without entry is a function of $(a+e_S)^2$. This subsequently leads to the threshold-type entry policy, i.e., the platform will enter the product space if and only if $a+e_S>\Delta_{cum}$.

Third, by comparing the seller's profit under no entry, $\Pi_N^S(e_S)$ with that under the entry, $\Pi_E^S(e_S)$, we delineate the necessary and sufficient conditions for identifying the receptive and protective regimes. Remarkably, the transition to a cumulative form of effort cost does not skew this comparative analysis, leading to the receptive regime $\Omega_{cum}=\{(\delta,f,\beta)|H_{cum}>1/4\}$ and the protective regime $\bar{\Omega}_{cum}=\{(\delta,f,\beta)|H_{cum}\leq 1/4\}$. The subsequent analysis is the same as that in the main model. As a result, the propositions laid out in the original framework (specifically Propositions 3 to 5) continue to be applicable, albeit with necessary adjustments to the thresholds to accommodate the new cost considerations.

The robustness of the original model's propositions within this revised model illustrates that the foundational strategic insights remain valid. The necessary adjustments to the thresholds and regimes to reflect the new cost dynamics do not detract from the applicability of these propositions but rather enhance the model's relevance by accommodating a broader range of cost considerations.

6.4. Negative Spillover Effect Parameter

By allowing δ to potentially be negative, the model accommodates a broader range of competitive dynamics. This captures scenarios where the platform's entry erodes the seller's market share or diminishes the perceived value of the seller's offering, diverging from the initial assumption that the entry would generally yield positive or at least neutral spillover effects.

The analysis and all equilibrium expressions remain unchanged. Notably, we find that, when $\delta<0$, the set corresponding to the receptive regime, $\Omega=\{(\delta,f,\beta)|H>1/4\}$, is empty. This means that, with the adverse effect on the seller due to the platform's entry, there ceases to be a scenario in which the seller is receptive to the platform's presence. This finding is intuitive: the negative spillover effect implies that any effort by the platform in Stage 2 invariably disadvantages the seller while benefiting the platform, leading to the seller consistently preferring the platform's absence to its presence.

Given the non-existence of the receptive regime with negative δ, Proposition 4, which assumes such a regime, is rendered inapplicable. Thus, the game's dynamics invariably fall within the protective regime. Consequently, Proposition 3 and Proposition 5 remain valid.

In summary, extending the model to include negative δ values not only enriches the theoretical framework by introducing a more comprehensive range of competitive dynamics but also enhances the model's practical relevance. It reflects the complex reality of platform-seller interactions, where the consequences of platform entry can vary widely, from highly beneficial to significantly detrimental.

7. Conclusions

Our investigation delves into the nuanced dynamics of incentive interactions between an online platform and a third-party seller, structured across two pivotal stages. This exploration is particularly relevant in today's digital marketplace, where the balance of competition and cooperation shapes the landscape of e-commerce. In the initial stage, the seller is motivated to enhance the value of their product, anticipating the potential for the platform to enter their market space in the subsequent stage. Such entry by the platform not only escalates market competition for the seller but also introduces a complex interplay of competitive and cooperative dynamics.

Drawing inspiration from Amazon's exemplary role in elevating product value—through strategies such as prominent product displays and optimizing logistics/delivery services—and its practice of sharing the benefits of enhanced product marketability with sellers (for instance, by rotating the winner of the Buy Box), our model incorporates the platform's capacity to exert effort in product-value enhancement that benefits the seller as well. This dual focus allows us to capture both the competitive and the spillover effects of the platform's market entry.

Our findings unveil a strategic paradigm, the "receptive regime," where the seller stands to gain more from the platform's market entry than from abstaining. We pinpoint several conditions under which the seller should welcome the platform's entry: significant values of the degree of spillover and the referral rate, alongside the strategic calibration of the competition intensity. Interestingly, while the benefits of high values of the degree of spillover and the referral rate, and low values of the competition intensity are intuitively understood, the advantageous scenario under moderately high values of the competition intensity unveils the intricate influence of these parameters on the platform's strategic decisions post-entry, thereby affecting the seller's preferences.

In this intricate dance of competition and collaboration, transparency emerges as a key strategy for the platform. By openly communicating the potential benefits of its investments in the product space-such as the nature and scale of value-enhancing efforts and tangible data on the impact of platform entry on sales figures—the platform can alleviate the seller's concerns about entering an uneven playing field. Strategies such as rotating the Buy Box winner ensure that the gains from increased market size are equitably shared, reinforcing the seller's commitment to the platform.

Moreover, our findings contribute to a deeper understanding of the investment dynamics in product value enhancement. We uncover scenarios of both underinvestment and overinvestment in seller efforts, aligning with prior research by Jiang et al. (2011), while also adding a layer of complexity to the strategic decision-making process. This overinvestment is particularly pronounced in the 'receptive regime,' when the product's base value occupies a middle ground. This insight underscores the importance of aligning incentives between platforms and sellers to optimize outcomes.

In a strategic advisory to platforms, we highlight the importance of identifying products within a 'protective regime' and making clear commitments to sellers regarding market entry strategies. Such upfront commitments can recalibrate seller efforts towards optimal levels, ensuring a harmonious and productive platform-seller relationship. This approach not only enhances the platform's ecosystem but also ensures a thriving, competitive marketplace that ultimately benefits consumers.

Our study contributes to the strategic dialogue on platform economics, offering actionable insights for both platforms and third-party sellers. By understanding the delicate balance between competition and cooperation, and by strategically managing the dynamics of platform entry, stakeholders can forge more resilient and prosperous e-commerce ecosystems.

References

Arya, A., Mittendorf, B., Sappington. D. E. M. (2007). The bright side of supplier encroachment. *Marketing Science* 26(5):651–659.

Bettadapura, K. (2018). How online merchants can win Amazon's Buy Box. *Forbes* (Mar. 23).

Cennamo, C., Gu, Y., Zhu, F. (2018). *Value co-creation and capture in platform markets: Evidence from a creative industry* (Working Paper). Harvard Business School.

Chen, L., Gilbert, S.M., Xia, Y. (2011). Private labels: Facilitators or impediments to supply chain coordination. *Decision Science*. 42(3):689–720.

Cho, J. Janda, S. (2022). Perception carryover in cross-buying: The role of interpurchase time and product locus. *Journal of Marketing Analytics*. 4(11):809-819.

Gawer, A., Cusumano, M. A. (2002). *Platform leadership: How Intel, Microsoft, and Cisco drive industry innovation*. Cambridge, MA: Harvard Business School Press.

Gawer, A., Henderson, R. (2007). Platform owner entry and innovation in complementary markets: Evidence from Intel. *Journal of Economics and Management Strategy*, 16(1):1–34.

Guan, H., Gurnani, H., Geng, X., & Luo, Y. (2019) Strategic Inventory and Supplier Encroachment. *Manufacturing & Service Operations Management* 21(3):536-555.

Ha, AY., Tong, S., & Wang, Y. (2022) Channel structures of online retail platforms. *Manufacturing & Service Operations Management* 24(3):1547-1561.

Huang, S., Guan, X., Chen, Y. (2018). Retailer information sharing with supplier encroachment. *Production and Operations Management*. 27(6):1133-1147.

Jiang, B., Jerath, K. Srinivasan, K. (2011). Firm strategies in the "mid tail" of platform-based retailing. *Marketing Science*, 30(5):757-775.

Kang, H.Y. (2017). Intra-platform envelopment: The coopetitive dynamics between the platform owner and complementors. *Academy of Management Proceedings*. 2017(1), 11205.

Keyes, D. (2019). Jeff Bezos says third parties are besting Amazon — here's why that's good for the firm. *Business Insider* (Apr. 12).

Li, Z., Agarwal, A. (2017). Platform integration and demand spillovers in complementary markets: Evidence from Facebook's integration of Instagram. *Management Science*, 63(10):3438–3458.

Li, Z., Gilbert, S. M., Lai, G. (2014) Supplier Encroachment Under Asymmetric Information. *Management Science* 60(2):449-462.

McFarland, A. (2009). Amazon lets us pay them to grow. *Adam McFarland* (blog), October 18, http://www.adam-mcfarland.net/2009/10/18/amazon-lets-us-pay-them-to-grow/.

McFarland, A. (2010). How Amazon exploits the "mid-tail." *Adam McFarland* (blog), July 6, http://www.adam-mcfarland.net/2010/07/06/how-amazon-exploits-the-mid-tai/.

Sing, N. Vives, X. (1984). Price and Quantity Competition in a Differentiated Duopoly. *The RAND Journal of Economics*. 15(4):546–554.

Wen, W., Zhu, F. (2019). Threat of platform-owner entry and complementor responses: Evidence from the mobile app market. *Strategic Management Journal*, 40(9):1336-1367.

Zhang, J., Cao, Q., He, X. (2020). Manufacturer encroachment with advertising. *Omega*, 91, 102013.

Zhang, X., Li, G., Liu, M., Sethi, SP. (2021). Online platform service investment: A bane or a boon for supplier encroachmen. *International Journal of Production Economics*, 235, 108079.

Zhou, Y., Liu, T., Cai, G. (2019). Impact of In-Store Promotion and Spillover Effect on Private Label Introduction. *Service Science*. 11(2):96-112.

Zhu, F. (2019). Friends or foes? Examining platform owners' entry into complementors' spaces. *Journal of Economics & Management Strategy*, 28(1):23–28.

Zhu, F. Liu, Q. (2018). Competing with Complementors: An Empirical Look at Amazon.com. *Strategic Management Journal*, 39(10):2618-2642.

Online Supplement to “Coopetition in Platform-Based Retailing: On the Platform's Entry”

Lian Qi, Haotian Song, Wenqiang Xiao

Appendix A: Proofs

Proof of Lemma 1. It follows from (3) and (4) that we can write the first-order derivatives in the following:

$$\frac{\partial \Pi_{2,E}^{S}}{\partial q_S} = (a + e_S + \delta e_P - 2q_S - \beta q_P)(1 - f),$$

and

$$\frac{\partial \Pi_{2,E}^{P}}{\partial q_P} = a + e_S + e_P - 2q_P - \beta(1 + f)q_S,$$

which brings us the closed-form expressions for the equilibrium quantity decisions q_S^* and q_P^* as stated in Lemma 1.

Proof of Lemma 2. Given a and e_S, for e_P that makes $q_S^* > 0$, substituting the quantity decisions in the second stage with the equilibrium quantities given in Lemma 1, we can rewrite the platform's profit in the second stage with entry and derive the optimal effort level

$$e_P^* = \frac{(a + e_S)(8 + 8\delta f - 4\beta(1 + \delta)(1 + f) + \beta^3(1 + \delta)f(1 + f) - 2\beta^2(f + \delta(-1 + f^2)))}{8 - 8\delta^2 f + 8\beta\delta(1 + f) - 2\beta^3\delta f(1 + f) + \beta^4(1 + f)^2 + 2\beta^2(-4 - 3f + \delta^2(-1 + f^2))}.$$

Substituting this optimal effort level into condition $q_S^* > 0$, we drive condition (5).

Therefore, under condition (5), e_P^* derived above is positive and optimal and its denominator is positive as well. Otherwise, substituting $q_S^* = 0$ and q_P^* given in Lemma 1, we can rewrite the platform's profit in the second stage with entry and derive the optimal effort level $e_P^* = a + e_S$, which, however, results in positive q_S^* based on the formula given in Lemma 1. Consequently, the optimal e_P^* is that satisfying $q_S^* = 0$ and is $\frac{(2-\beta)(a+e_S)}{\beta - 2\delta}$, whose denominator can be verified to be positive when (5) is violated.

To support the proof of Propositions 1 and 2, we first establish the following lemma that segments the (δ, f, β) space into two distinct subspaces.

Lemma 3. *For any given f and β, the following function (11) is positive if and only if $\delta > \tilde{\delta}(f, \beta)$; for any given δ and β, it is positive if and only if $f > \tilde{f}(\delta, \beta)$; for any given δ and f, it is positive if and only if $\beta < \beta_1(\delta, f)$ or $\beta > \beta_2(\delta, f)$ where $\beta_1(\delta, f) \le \beta_2(\delta, f)$.*

$$-\beta^4(1 + f)^2 + 8\delta(1 + \delta f) + 2\beta^3(1 + f)(1 + \delta f) - 4\beta(2 + \delta + \delta^2 + 2\delta f) + \beta^2(-2\delta f + 4(1 + f) - 2\delta^2(-1 + f^2)). \tag{11}$$

Proof of Lemma 3. a) We first assess (11) as a quadratic function of δ, whose first order derivative with respect to δ can be verified to be nonnegative at $\delta = 1$ when $0 \le \beta \le 1$ and $0 \le f < 1$. Additionally, (11) reduces to $-\beta(-2 + \beta + \beta f)(-4 + \beta^2 + \beta^2 f)$ when $\delta = 0$, which is always non-positive under conditions $0 \le \beta \le 1$ and $0 \le f < 1$.

When (11) is a downward parabola of δ, since (11) increases in δ at $\delta = 1$, we conclude part (a) of this conclusion, where $\tilde{\delta}(\beta, f)$ is the smaller root of the following equation (when no real root exists, (11) is negative and we assign a value bigger than 1 to $\tilde{\delta}(\beta, f)$).

$$-\beta^4(1+f)^2+8\delta(1+\delta f)+2\beta^3(1+f)(1+\delta f)-4\beta(2+\delta+\delta^2+2\delta f)+\beta^2(-2\delta f+4(1+f)-2\delta^2(-1+f^2))=0. \tag{12}$$

When (11) is an upward parabola of δ, since (11) increases in δ at $\delta=1$ and is non-positive at $\delta=0$, the unique $\tilde{\delta}(\beta,f)$ that satisfies part (a) of this conclusion exists and is the bigger root of (12).

When (11) is a degenerate parabola, again, since (11) increases in δ at $\delta=1$ (in this case, it is strictly increasing), there is a unique $\tilde{\delta}(\beta,f)$, which is the only root of (12).

b) Looking closely at (11), which is a quadratic function of f, we can show that, given $\beta\in[0,1]$ and $\delta\in[0,1]$, it is a concave parabola of f that is maximized at $-\frac{\beta^4+\beta^2(-2+\delta)+4\beta\delta-4\delta^2-\beta^3(1+\delta)}{\beta^2(\beta^2-2\beta\delta+2\delta^2)}>1$. Therefore, part (b) of this conclusion stands with $\tilde{f}(\beta,\delta)$ equal to the smaller root of (12).

c) We further analyze (11) as a continuous quartic function of β, which has three critical points (we denote them by $\beta_{01}\le\beta_{02}\le\beta_{03}$) and approaches $-\infty$ when β approaches $+\infty$ or $-\infty$.

Its first-order derivative with respect to β is a continuous cubic function, which has two critical points (we denote them by $\beta_{11}\le\beta_{12}$) and approaches $-\infty$ $(+\infty)$ when β approaches $+\infty$ $(-\infty)$. Therefore, $\beta_{01}\le\beta_{11}\le\beta_{02}\le\beta_{12}\le\beta_{03}$.

Its second-order derivative with respect to β is a downward parabola, which has one critical point (we denote it by β_{21}). Following the definition, $\beta_{11}\le\beta_{21}\le\beta_{12}$.

Its third-order derivative with respect to β is a decreasing line, which is positive when $\beta=0$ and is negative when $\beta=1$. Thus $0<\beta_{21}$.

Therefore, $0<\beta_{21}\le\beta_{12}\le\beta_{03}$. Additionally, this first order derivative equals $-8-4\delta-4\delta^2-8\delta f\le 0$ when $\beta=0$, so $\beta_{01}\le 0\le\beta_{02}$.

Finally, if $1\ge\beta_{03}$, then $1\ge\beta_{03}\ge\beta_{12}$, which implies that both first and second order derivatives are non-positive at $\beta=1$. This, however, is impossible when $0\le\delta\le1$ and $0\le f<1$, so $\beta_{01}\le0<1<\beta_{03}$, and hence there exist $\beta_1(\delta,f)\le\beta_2(\delta,f)$ satisfying part (c) of this conclusion, where $\beta_1(\delta,f)$ $(\beta_2(\delta,f))$ is the only possible root of (12) in $\beta\in[\beta_{01},\beta_{02}]$ $(\beta\in[\beta_{02},\beta_{03}])$. (When no real root of (12) exists in $[\beta_{01},\beta_{02}]$, since one can verify that (11) is nonnegative at $\beta=0$ when $0\le\delta\le1$ and $0\le f<1$, (11) is positive and we set $\beta_1(\delta,f)=\beta_2(\delta,f)$ equal a value bigger than 1. When a real root exists in $[\beta_{01},\beta_{02}]$ but no real root exists in $[\beta_{02},\beta_{03}]$, (11) is negative in $[\beta_{02},\beta_{03}]$, and we set $\beta_1(\delta,f)$ the only possible root in $\beta\in[\beta_{01},\beta_{02}]$ and set $\beta_2(\delta,f)$ a value bigger than 1.)

Proof of Proposition 1. Take any e_S. The seller's profit in the second stage under the platform's non-entry is $\Pi^S_{2,N}=q_2[(a+e_S-q_2)(1-f)]$, which is maximized at $q_2^*=\arg\max_{q_2}\Pi^S_{2,N}=\frac{a+e_S}{2}$, implying that the platform's optimal profit in the second stage under non-entry is

$$\Pi^P_{2,N}=\frac{f(a+e_S)^2}{4}.$$

This, together with the result in Lemma 2, implies that the difference between the platform's optimal profits in the second stage under entry and non-entry is

$$\begin{aligned}&\Pi^P_{E-N}(e_S)=\Pi^P_{2,E}-\Pi^P_{2,N}\\&=-\frac{(a+e_S)^2(\beta^4f(1+f)^2+8\beta(1+f)(2+\delta f)-2\beta^3f(1+f)(2+\delta f)-8(2+2\delta f+\delta^2(-1+f)f)+2\beta^2(-2-(2+\delta^2)f-(2-2\delta+\delta^2)f^2+\delta^2f^3))}{(4(8-8\delta^2f+8\beta\delta(1+f)-2\beta^3\delta f(1+f)+\beta^4(1+f)^2+2\beta^2(-4-3f+\delta^2(-1+f^2))))}-K.\end{aligned}$$

when (5) holds, and is the following otherwise.

$$-\frac{(-2+\beta)^2(a+e_S)^2}{2(\beta-2\delta)^2}+\frac{(-1+\delta)^2(a+e_S)^2}{(\beta-2\delta)^2}-(a+e_S)^2f/4-K.$$

The above closed-form expression for $\Pi^P_{E-N}(e_S)$ implies that the platform is strictly better off under entry relative to non-entry if and only if $\Pi^P_{E-N}(e_S)>0$, or equivalently, $a+e_S>\Delta$ where

$$\Delta=\sqrt{-\frac{K(4(8-8\delta^2f+8\beta\delta(1+f)-2\beta^3\delta f(1+f)+\beta^4(1+f)^2+2\beta^2(-4-3f+\delta^2(-1+f^2))))}{(\beta^4f(1+f)^2+8\beta(1+f)(2+\delta f)-2\beta^3f(1+f)(2+\delta f)-8(2+2\delta f+\delta^2(-1+f)f)+2\beta^2(-2-(2+\delta^2)f-(2-2\delta+\delta^2)f^2+\delta^2f^3))}}.$$

when (5) holds; otherwise,

$$\Delta=2(\beta-2\delta)\sqrt{-\frac{K}{4+8\delta-4\delta^2(1-f)+\beta^2(2+f)-4\beta(2+\delta f)}}.$$

Note that the functions in the square roots of Δ formulated above are always nonnegative given $f\in[0,1)$ and $0\le\delta,\beta\le 1$.

To show how δ, f and β affect Δ, it is equivalent to investigating how they affect Δ^2/K. As noted in the text, only the case when (5) holds is our focus in this paper. Since the denominator of Δ is never equal to zero when $f\in[0,1)$ and $0\le\delta,\beta\le 1$, Δ is a continuous function when δ, f and β vary in their corresponding ranges.

We first show the relationship between δ and Δ^2/K. We have $\frac{\partial\Delta^2/K}{\partial\delta}=\frac{Num_\delta}{Denom_\delta}$, where the denominator and the numerator are

$$\begin{aligned}Denom_\delta&=(\beta^4f(1+f)^2+8\beta(1+f)(2+\delta f)-2\beta^3f(1+f)(2+\delta f)\\&\quad-8(2+2\delta f+\delta^2(-1+f)f)+2\beta^2(-2-(2+\delta^2)f-(2-2\delta+\delta^2)f^2+\delta^2f^3))^2\\Num_\delta&=16((-4\beta+2\beta^2+8f-4\beta f+\beta^3f-2\beta^2f^2+\beta^3f^2)\delta-(-8+4\beta+4\beta f+2\beta^2f-\beta^3f-\beta^3f^2))\\&\quad\times(f(-4+\beta^2f)\delta-(-4\beta+2\beta^2+4f-4\beta f+\beta^3f-\beta^2f^2+\beta^3f^2))\end{aligned}$$

The denominator in the above equation is positive, so we only need to check the numerator, which is the product of two terms. One can verify that $-8+4\beta+4\beta f+2\beta^2f-\beta^3f-\beta^3f^2$ in the first term of the numerator and $f(-4+\beta^2f)$ in its second term are both negative when $0\le\beta\le 1$ and $0\le f<1$. Since $-4\beta+2\beta^2+8f-4\beta f+\beta^3f-2\beta^2f^2+\beta^3f^2$ in the first term of the numerator is no less than $-4\beta+2\beta^2+4f-4\beta f+\beta^3f-\beta^2f^2+\beta^3f^2$ in the second term in $\beta\in[0,1]$ and $f\in[0,1)$, if the latter is non-negative, the former is non-negative too, when the above derivative is non-positive in $\delta\in[0,1]$, so Δ is decreasing in δ. If the latter is negative and the former is non-negtive, the numerator is a concave parabola that is non-negative at $\delta=0$ and negative $\delta=1$, so Δ is increasing and then decreasing in δ; now if both are negative, the numerator is a convex parabola whose two intersections with the δ axis are both bigger than $\delta=1$, so Δ is increasing in δ.

As for the relationship between f and Δ^2/K, we check the first order derivative of Δ^2/K with regard to f. Its denominator is again positive. Its numerator, which we use $Num_{(\Delta^2/K,f)}$ to represent, is quite complicated. Following Lemma 3, we discuss how $Num_{(\Delta^2/K,f)}$ changes with f using two cases. When (11) is not positive, $Num_{(\Delta^2/K,f)}$ can be verified to be non-negative in $\beta,\delta\in[0,1]$ and $f\in[0,1)$, so Δ increases in f. When (11) is positive, one can verify that the first order derivative of $Num_{(\Delta^2/K,f)}$ with respect to f is always non-positive, its second order derivative is a convex parabola of f that is always positive and its third order derivative is an increasing linear function of f that is always negative in $\beta,\delta\in[0,1]$ and $f\in[0,1)$. Following the similar analysis approach in the proof of part (c) of Lemma 3, we conclude that $Num_{(\Delta^2/K,f)}$ (and hence Δ) is positive (increasing) and then negative (decreasing) as f increases in $[0,1)$ when (5) holds and (11) is positive. The relationship between f and Δ in this proposition therefore follows from Lemma 3 such that, given (δ,β) and with the increase of f, (11) is first no bigger than zero and then positive.

Finally, we check the relationship between β and Δ^2/K. We have $\frac{\partial\Delta^2/K}{\partial\beta}=\frac{Num_\beta}{Denom_\beta}$, where the denominator and the numerator are

$$\begin{aligned}Denom_\beta=&(\beta^4f(1+f)^2+8\beta(1+f)(2+\delta f)-2\beta^3f(1+f)(2+\delta f)\\&-8(2+2\delta f+\delta^2(-1+f)f)+2\beta^2(-2-(2+\delta^2)f-(2-2\delta+\delta^2)f^2+\delta^2f^3))^2\\Num_\beta=&16((1+f)\beta-(2+\delta f-\delta^2f))\times(-16(1+\delta)(1+f)+8\beta^2(1+\delta)f(1+f)\\&-8\beta^3(1+f)^2+\beta^5f(1+f)^2+8\beta(4-\delta+\delta^2+2f)-\beta^4(1+f)(-2+(1+\delta)f^2)).\end{aligned}$$

The denominator in the above equation is positive, so we only need to check the numerator, which is the product of two terms. One can easily verify that the first term $(1+f)\beta-(2+\delta f-\delta^2f)$ is negative in $\beta,\delta\in[0,1]$ and $f\in[0,1)$, so we only need to check the second term

$$Num_{(\Delta^2/K,\beta)}=-16(1+\delta)(1+f)+8\beta^2(1+\delta)f(1+f)-8\beta^3(1+f)^2+\beta^5f(1+f)^2+8\beta(4-\delta+\delta^2+2f)-\beta^4(1+f)(-2+(1+\delta)f^2).$$

Similar to the proof above showing the relationship between f and Δ^2/K, two cases based on the value of (11) are considered. When (11) is positive, $Num_{(\Delta^2/K,\beta)}$ can be verified to be negative in $\beta,\delta\in[0,1]$ and $f\in[0,1)$, so Δ increases in β. When (11) is not positive, we can readily verify that the first order derivative of $Num_{(\Delta^2/K,\beta)}$ with respect to β is always positive in $\beta\in[0,1]$, its second order derivative is non-negative at $\beta=0$ and negative at $\beta=1$, its third order derivative is a convex parabola of β that is negative at $\beta=0$ and non-negative at $\beta=1$, where $\delta\in[0,1]$ and $f\in[0,1)$. Following the similar analysis approach in the proof of part (c) of Lemma 3, we conclude that $Num_{(\Delta^2/K,\beta)}$ (and hence Δ) is in general negative (increasing) and then positive (decreasing) as β increases in $[0,1]$ when (5) holds and (11) is not positive. The relationship between β and Δ in this proposition therefore follows from the last portion of Lemma 3 such that, given (δ,f) and with the increase of β, (11) is first positive, and then turns to be non-positive and finally is positive again.

Proof of Proposition 2. $\Pi_E^S(e_S)>\Pi_N^S(e_S)$ will not hold if (5) is violated, when the seller is forced out in the second stage with zero stage-2 profit. In the following, we compare $\Pi_E^S(e_S)$ and $\Pi_N^S(e_S)$ when (5) holds. It follows from (6) and (7) that

$$\begin{aligned}\Pi_E^S(e_S)-\Pi_N^S(e_S)&=(\tfrac{(4(1+\delta)-2\beta(2-\delta+\delta^2)+\beta^3(1+f)-\beta^2(2+f+\delta f))^2}{(8-8\delta^2f+8\beta\delta(1+f)-2\beta^3\delta f(1+f)+\beta^4(1+f)^2+2\beta^2(-4-3f+\delta^2(-1+f^2)))^2}-\tfrac{1}{4})(a+e_S)^2(1-f)\\&=(\tfrac{Num(f,\beta,\delta)}{4(8-8\delta^2f+8\beta\delta(1+f)-2\beta^3\delta f(1+f)+\beta^4(1+f)^2+2\beta^2(-4-3f+\delta^2(-1+f^2)))^2})(a+e_S)^2(1-f),\end{aligned}$$

where

$$\begin{aligned}&Num(f,\beta,\delta)\\=&\ 4(4(1+\delta)-2\beta(2-\delta+\delta^2)+\beta^3(1+f)-\beta^2(2+f+\delta f))^2\\&-(8-8\delta^2f+8\beta\delta(1+f)-2\beta^3\delta f(1+f)+\beta^4(1+f)^2+2\beta^2(-4-3f+\delta^2(-1+f^2)))^2\\=&\ (-\beta^4(1+f)^2+8\delta(1+\delta f)+2\beta^3(1+f)(1+\delta f)-4\beta(2+\delta+\delta^2+2\delta f)+\beta^2(-2\delta f+4(1+f)-2\delta^2(-1+f^2)))\\&\times(\beta^4(1+f)^2-2\beta^3(1+f)(-1+\delta f)+8(2+\delta-\delta^2f)-4\beta(2+\delta^2-\delta(3+2f))+2\beta^2(-6-4f-\delta f+\delta^2(-1+f^2)))\end{aligned}$$

One can verify that the second term above is always positive when $\beta \in [0,1]$, $\delta \in [0,1]$ and $f \in [0,1)$, along with condition (5), so whether $\Pi_E^S(e_S)$ is bigger than $\Pi_N^S(e_S)$ or not depends on the first term above, which is (11). This proposition therefore follows from Lemma 3.

Proof of Proposition 3. It follows from the platform's optimal threshold entry strategy characterized in Lemma 1 that the seller's two-stage profit as a function of her effort e_S is

$$\Pi^S(e_S) = \begin{cases} \Pi_N^S(e_S) & \text{if } e_S \leq \Delta - a \\ \Pi_E^S(e_S) & \text{if } e_S > \Delta - a \end{cases}$$

where $\Pi_N^S(e_S)$ and $\Pi_E^S(e_S)$ are defined in Equations (6) and (7), respectively. Hence, the seller's optimal effort $e_S^*(a)$ is the maximizer of function $\Pi^S(e_S)$. Because $\Pi_N^S(e_S)$ and $\Pi_E^S(e_S)$ are concave functions with unique unconstrained maximizers $e_N^B(a)$ and $e_E^B(a)$ given in Equations (9) and (10), respectively, $e_S^*(a)$ must be one of the following three values in the regime $(\delta, f, \beta) \in \overline{\Omega}$: the two unconstrained maximizers $e_N^B(a)$ and $e_E^B(a)$, and the cutoff value $\Delta - a$. In the remainder of the proof, for the regime $(\delta, f, \beta) \in \overline{\Omega}$, we identify three distinct subregimes, in each of which we show precisely which one of the three values the seller's optimal effort $e_S^*(a)$ should take. Note that this includes not only the case that (5) holds and $H \leq 1/4$, but also the case that (5) does not hold so that $H = 0$.

Because $e_N^B(a)$ strictly increases in a, we can define $\underline{a}_1 = \{a | a + e_N^B(a) = \Delta\}$, which uniquely exists. Define $M(a) = \Pi_N^S(\Delta - a) - \Pi_E^S(e_E^B(a))$. By the definition of $\underline{a}_1$, we have $M(\underline{a}_1) = \Pi_N^S(e_N^B(\underline{a}_1)) - \Pi_E^S(e_E^B(\underline{a}_1)) \geq 0$, where the inequality follows from Proposition 2. It follows from Equations (6) and (7) that $dM(a)/da = 2c_S(\Delta - a - e_E^B(a))$, implying that $dM(a)/da|_{a=\underline{a}_1} = 2c_S[e_N^B(\underline{a}_1) - e_E^B(\underline{a}_1)] \geq 0$, where the inequality is due to $e_N^B(a) - e_E^B(a) \geq 0$ for any $a > 0$ when $H \leq 1/4$, a result that is verifiable from Equations (9) and (10). This, together with the fact that $a + e_E^B(a)$ increases in a, implies that the sign of $dM(a)/da$ changes from non-negative to negative only once when a increases over $a \geq \underline{a}_1$. Recall that $M(\underline{a}_1) \geq 0$. Therefore, there exists a largest value that is no less than $\underline{a}_1$ and satisfies $M(a) = 0$. Formally, we define $\bar{a}_1 = \max\{a | a \geq \underline{a}_1 \text{ and } \Pi_N^S(\Delta - a) = \Pi_E^S(e_E^B(a))\}$. Note that $\bar{a}_1$ uniquely exists. It is worth mentioning that for $a \geq \bar{a}_1$, $dM(a)/da < 0$ implying that $a + e_E^B(a) > \Delta$.

Case 1.1. $a \leq \underline{a}_1$. By the definition of $\underline{a}_1$, we have $a + e_N^B(a) \leq \Delta$, implying that the unconstrained maximizer $e_N^B(a)$ satisfies the constraint $e_N^B(a) \leq \Delta - a$. This, together with the result from Proposition 2 that $\Pi_N^S(e_S) \geq \Pi_E^S(e_S)$ for any e_S when $H \leq 1/4$, implies that $e_S^*(a) = e_N^B(a)$ and the platform will not enter.

Case 1.2. $a \in (\underline{a}_1, \bar{a}_1)$. By the definition of $\underline{a}_1$, we have $a + e_N^B(a) > \Delta$, implying that it suffices to compare $\Pi_N^S(\Delta - a)$ with $\sup_{e_S > \Delta - a} \Pi_E^S(e_S)$ to determine $e_S^*(a)$. Because $a < \bar{a}_1$, we have $\Pi_N^S(\Delta - a) \geq \Pi_E^S(e_E^B(a))$, implying that $\Pi_N^S(\Delta - a) \geq \sup_{e_S > \Delta - a} \Pi_E^S(e_S)$ because $e_E^B(a)$ is the unconstrained maximizer of $\Pi_E^S(e_S)$. Therefore, $e_S^*(a) = \Delta - a$ and the platform will not enter.

Case 1.3. $a \geq \bar{a}_1$. Similar to Case 1.2, it suffices to compare $\Pi_N^S(\Delta - a)$ with $\sup_{e_S > \Delta - a} \Pi_E^S(e_S)$ to determine $e_S^*(a)$. By definition of $\bar{a}_1$, we have $\Pi_N^S(\Delta - a) \leq \Pi_E^S(e_E^B(a))$. This, together with the earlier mentioned result that $a + e_E^B(a) > \Delta$ for $a \geq \bar{a}_1$, implies that $e_S^*(a) = e_E^B(a)$ and the platform will enter.

The seller's best quantity decision in stage 1 naturally follows from the first order condition based on her stage-1 profit function.

Proof of Proposition 4. The proof of this proposition when $(\delta, f, \beta) \in \Omega$ is very similar to that of Proposition 3 for $(\delta, f, \beta) \in \overline{\Omega}$, except that, when $(\delta, f, \beta) \in \Omega$, $e_S^*(a)$ may take a value that infinitesimally approaches the cutoff $\Delta - a$ from the right, which with some loss of rigor we denote by $\Delta - a + \epsilon$ with ϵ being an infinitesimally small positive value. Note that this can only be the case that (5) holds and $H > 1/4$, since when (5) does not hold, $H = 0$.

We define two thresholds: $\underline{a}_2 = \inf\{a | \Pi_N^S(e_N^B(a)) \leq \Pi_E^S(\Delta - a)\}$, and $\bar{a}_2 = \{a | a + e_E^B(a) = \Delta\}$. Note that $\bar{a}_2$ is well defined because $e_E^B(a)$ increases in a. Further, when $a = \bar{a}_2$, the inequality in the definition of $\underline{a}_2$ holds because $\Pi_N^S(e_N^B(\bar{a}_2)) < \Pi_E^S(e_N^B(\bar{a}_2)) \leq \Pi_E^S(e_E^B(\bar{a}_2)) = \Pi_E^S(\Delta - \bar{a}_2)$, where the first inequality is due to Proposition 2, the second inequality is due to the fact $e_E^B(a)$ is the maximizer of $\Pi_E^S(e)$, and the last equality is by the definition of $\bar{a}_2$. Hence, $\underline{a}_2 \leq \bar{a}_2$. Moreover, $\underline{a}_2$ is well defined because $d\Pi_N^S(e_N^B(a))/da = 2c_S e_N^B(a)$ is less than $d\Pi_E^S(\Delta - a)/da = 2c_S(\Delta - a)$ under the condition that $a + e_N^B(a) < \Delta$, which is satisfied for all $a \leq \bar{a}_2$ (because $e_E^B(a) > e_N^B(a)$ for all a, a result that is verifiable from Equations (9) and (10)).

Case 2.1. $a > \bar{a}_2$. By the definition of $\bar{a}_2$, we have $a + e_E^B(a) > \Delta$, implying that the unconstrained maximizer $e_E^B(a)$ satisfies the constraint $e_E^B(a) > \Delta - a$. This, together with the result from Proposition 2 that $\Pi_N^S(e_S) < \Pi_E^S(e_S)$ for any e_S, implies that $e_S^*(a) = e_E^B(a)$ and the platform will enter.

Case 2.2. $a \in [\underline{a}_2, \bar{a}_2]$. By the definition of $\underline{a}_1$, we have $\Pi_N^S(e_N^B(a)) \leq \Pi_E^S(\Delta - a)$. This, together with the definition of $\bar{a}_2$ that $e_E^B(a) \leq \Delta - a$, implies that $e_S^*(a) = \Delta - a + \epsilon$ with ϵ being an infinitesimally small positive value and the platform will enter.

Case 2.3. $a < \underline{a}_2$. By the definition of $\underline{a}_2$, we have $\Pi_N^S(e_N^B(a)) > \Pi_E^S(\Delta - a)$. This, together with the fact that $e_N^B(a) \leq \Delta - a$ and $e_E^B(a) \leq \Delta - a$, implies that $e_S^*(a) = e_N^B(a)$ and the platform will not enter.

The seller's best quantity decision in stage 1 naturally follows from the first order condition based on her stage-1 profit function.

Proof of Proposition 5. If $(\delta, f, \beta) \in \Omega$, then it follows from Proposition 2 that the seller prefers the platform's entry to non-entry, implying that the platform's commitment of non-entry cannot increase the seller's effort in the first stage relative to not making such a commitment. Therefore, it suffices to restrict our attention to the regime $(\delta, f, \beta) \in \overline{\Omega}$. Note that this includes not only the case that (5) holds and $H \leq 1/4$, but also the case that (5) does not hold so that $H = 0$.

In the subregime where $a \geq \bar{a}_1$, the platform's profit without non-entry commitment is

$$f p_1 q_1 + f p_S q_S + p_P q_P - \frac{e_P^2}{2} - K,$$

where $p_1 = q_1 = \frac{a + e_E^B(a)}{2}$, and p_S, q_S, p_P, q_P and e_P are determined based on Lemmas 1 and 2. If the platform commits non-entry, the platform's total profit is $2fp_1q_1$, where q_1 and p_1 respectively equal $\frac{a + e_N^B(a)}{2}$ and $\frac{a + e_N^B(a)}{2}$. Let $\Delta\Pi^P(a)$ be the difference between the platform's profit without non-entry commitment and the platform's profit with non-entry commitment. With some straightforward algebra, it is verifiable that $\Delta\Pi^P(a)$ is a parabola that is either concave or convex.

Case 1. The parabola $\Delta\Pi^P(a)$ is concave. Because $\Delta\Pi^P(a)|_{a=0} \leq 0$, along with the facts that $\Delta\Pi^P(a)$ is a concave parabola and $\frac{\partial \Delta\Pi^P(a)}{\partial a}|_{a=0} = 0$, we have that for all $a \geq \bar{a}_1$, $\Delta\Pi^P(a) < 0$ implying that the platform is strictly better off under the non-entry commitment. In this case, we define $\bar{\bar{a}}_1 = +\infty$.

Case 2. The parabola $\Delta\Pi^P(a)$ is convex. Because $\Delta\Pi^P(a)|_{a=0} \leq 0$, we define $\tilde{a}$ to be the smallest solution satisfying $\Delta\Pi^P(a) = 0$ in $a \geq 0$. We define $\bar{\bar{a}}_1 = \max\{\tilde{a}, \bar{a}_1\}$. Therefore, the platform is strictly better off under the non-entry commitment when $a \in (\bar{a}_1, \bar{\bar{a}}_1)$ and is (weakly) worse off under the non-entry commitment when $a \geq \bar{\bar{a}}_1$.

In the subregime where $a \in (\underline{a}_1, \bar{a}_1)$, it follows from Proposition 3 that the platform's non-entry commitment can restore the seller's effort back to the higher level $e_N^B(a)$ without changing the equilibrium entry outcome, implying that the platform is strictly better off under the non-entry commitment. In the subregime where $a \leq \underline{a}_1$, it follows from Proposition 3 that the platform's non-entry commitment has no impact on the equilibrium outcome. To combine the results in the three subregimes, we have that in the regime $(\delta, f, \beta) \in \overline{\Omega}$, the platform is strictly better off under the non-entry commitment if and only if $a \in (\underline{a}_1, \bar{\bar{a}}_1)$.

As for the seller, by definition of $e_N^B(a)$, under the platform's non-entry commitment, the seller's optimal effort in the first stage is equal to the unconstrained optimum, i.e., $e_N^B(a)$, and the seller's total profits are $\Pi_N^S(e_N^B(a))$. Without the platform's non-entry commitment, it follows from Proposition 3 that the seller's total profits are $\Pi_N^S(e_S^*(a))$ for $a \in (\underline{a}_1, \bar{a}_1)$ and $\Pi_E^S(e_E^B(a))$ for $a \in [\bar{a}_1, \bar{\bar{a}}_1)$. The former is strictly less than $\Pi_N^S(e_N^B(a))$ because $e_N^B(a)$ is the unconstrained maximizer, and the latter is strictly less than $\Pi_N^S(e_N^B(a))$ as well because $\Pi_E^S(e_E^B(a)) \leq \Pi_N^S(e_E^B(a)) < \Pi_N^S(e_N^B(a))$ where the first inequality follows from Proposition 2 (in regime $\overline{\Omega}$) and the second inequality follows from the definition of $e_N^B(a)$. Therefore, we have proved that the seller earns strictly higher profits under the platform's commitment of non-entry than that without such a commitment, when $(\delta, f, \beta) \in \overline{\Omega}$ and $a \in (\underline{a}_1, \bar{\bar{a}}_1)$.

Appendix B: Simultaneous Effort and Quantity Decisions in the Second Stage

1) Second stage decisions under the platform's entry: Given that the platform has entered the seller's product space and two parties make quantity decisions at the same time when the platform makes the effort decision, it follows from (3) and (4) that we can write down the following first-order derivatives:

$$\frac{\partial \Pi_{2,E}^S}{\partial q_S} = (a + e_S + \delta e_P - 2q_S - \beta q_P)(1 - f)$$
$$\frac{\partial \Pi_{2,E}^P}{\partial q_P} = a + e_S + e_P - 2q_P - \beta(1 + f)q_S$$
$$\frac{\partial \Pi_{2,E}^P}{\partial e_P} = -e_P + q_P + \delta f q_S.$$

Solving the corresponding first-order conditions while ensuring that all the solutions are nonnegative brings us the following closed-form expressions for the equilibrium decisions q_S^*, q_P^* and e_P^*:

The seller's equilibrium quantity decision in the second stage

$$q_S^* = \frac{(1 - \beta + \delta)(a + e_S)}{2 - 2\delta^2 f - \beta^2(1 + f) + \beta(\delta + 2\delta f)},$$

and the platform's equilibrium quantity decision and product-value-enhancement effort in the second stage

$$q_P^* = \frac{(2 - \beta - \beta f + \delta f - \delta^2 f)(a + e_S)}{2 - 2\delta^2 f - \beta^2(1 + f) + \beta(\delta + 2\delta f)}$$
$$e_P^* = \frac{(2 - \beta - \beta f + 2\delta f - \beta\delta f)(a + e_S)}{2 - 2\delta^2 f - \beta^2(1 + f) + \beta(\delta + 2\delta f)}.$$

Note that under the simultaneous decision setting, the seller will no longer be forced out of the product space after the entry of the platform.

2) The platform's entry decision. Since the seller's profit in the second stage under the platform's non-entry is $\Pi^S_{2,N} = q_2[(a+e_S-q_2)(1-f)]$, which is maximized at $q_2^* = \arg\max_{q_2} \Pi^S_{2,N} = \frac{a+e_S}{2}$, the platform's optimal profit in the second stage under non-entry is

$$\Pi^P_{2,N} = \frac{f(a+e_S)^2}{4}.$$

This, together with the two parties' second stage decisions derived above for the case of the platform's entry, implies that the difference between the platform's optimal profits in the second stage under entry and non-entry is

$$\Pi^P_{E-N}(e_S) = \Pi^P_{2,E} - \Pi^P_{2,N} = -\frac{(a+e_S)^2 Denom(f,\beta,\delta)}{4(2-2\delta^2 f-\beta^2(1+f)+\beta(\delta+2\delta f))^2} - K,$$

where the coefficient of $(a+e_S)^2$ is guaranteed to be nonnegative and

$$\begin{aligned} Denom(f,\beta,\delta) = -[&-8-8\delta f-4\delta^2(-3+f)f+8\delta^3 f^2 \\ &+4\delta^4(-1+f)f^2+\beta^4 f(1+f)^2-2\beta^3 f(1+f)(2+\delta+2\delta f) \\ &-4\beta(-2+(-2+\delta+2\delta^2)f+\delta(-1+4\delta)f^2+2\delta^3 f^3) \\ &+\beta^2(4\delta f(2+3f)-2(1+f^2)+\delta^2 f(1+6f+8f^2))]. \end{aligned}$$

The above closed-form expression for $\Pi^P_{E-N}(e_S)$ implies that the platform is strictly better off under entry relative to non-entry if and only if $\Pi^P_{E-N}(e_S) > 0$, or equivalently, $a+e_S > \Delta_{sim}$ where

$$\Delta_{sim} = \sqrt{\frac{4(2-2\delta^2 f-\beta^2(1+f)+\beta(\delta+2\delta f))^2 K}{Denom(f,\beta,\delta)}}.$$

One can easily verify that $\Delta_{sim} \geq \Delta$ provided in Proposition 1.

We can also define

$$H_{sim} = \frac{(1-\beta+\delta)^2}{(2-2\delta^2 f-\beta^2(1+f)+\beta(\delta+2\delta f))^2}$$

and then the expressions of $\Pi^S_N(e_S)$, $\Pi^S_E(e_S)$, $e^N_B(a)$, $e^E_B(a)$ and $\Omega/\bar{\Omega}$ under the simultaneous decision setting will be symbolically same to those under the sequential setting except that we replace H in the formulas using H_{sim}, which can be readily verified to be strictly bigger than H.

3) Receptive and protective regimes: Similar to the proof of Proposition 2, we have

$$\Pi^S_E(e_S) - \Pi^S_N(e_S) = \frac{Num(f,\beta,\delta)}{4(2-2\delta^2 f-\beta^2(1+f)+\beta(\delta+2\delta f))^2}(a+e_S)^2(1-f)$$

where

$$\begin{aligned} Num(f,\beta,\delta) &= -(-4+2\beta+\beta^2-2\delta-\beta\delta+\beta^2 f-2\beta\delta f+2\delta^2 f) \\ &\quad \times(-2\beta+\beta^2+2\delta-\beta\delta+\beta^2 f-2\beta\delta f+2\delta^2 f) \\ &= -[(\beta^2-2\beta\delta+2\delta^2)f-4+2\beta+\beta^2-2\delta-\beta\delta] \\ &\quad \times[(\beta^2-2\beta\delta+2\delta^2)f-(2-\beta)(\beta-\delta)] \end{aligned}$$

Note that $(\beta^2-2\beta\delta+2\delta^2)$ is always nonnegative. The first term in $Num(f,\beta,\delta)$ is thus no larger than

$$(\beta^2-2\beta\delta+2\delta^2)\times 1-4+2\beta+\beta^2-2\delta-\beta\delta=2\beta^2+\beta(2-3\delta)+2(-2+\delta)(1+\delta)$$

which is negative given $0\le\delta,\beta\le 1$. Therefore, the sign of $Num(f,\beta,\delta)$ is the same as that of its second term $(\beta^2-2\beta\delta+2\delta^2)f-(2-\beta)(\beta-\delta)$. This term is linear increasing in f and convex in β and δ. Setting $(\beta^2-2\beta\delta+2\delta^2)f-(2-\beta)(\beta-\delta)=0$ yields:

- Given (β,δ) fixed, we have

$$f=\frac{(2-\beta)(\beta-\delta)}{\beta^2-2\beta\delta+2\delta^2}$$

- Given (f,β) fixed, we have

$$\delta_1=\frac{-2+\beta+2\beta f-\sqrt{4-4\beta+\beta^2+8\beta f-4\beta^2 f-4\beta^2 f^2}}{4f}$$
$$\delta_2=\frac{-2+\beta+2\beta f+\sqrt{4-4\beta+\beta^2+8\beta f-4\beta^2 f-4\beta^2 f^2}}{4f}$$

It can be shown that given $0\le f<1$ and $0\le\beta\le 1$, the discriminant is always positive and $\delta_1\le 0\le\delta_2<1$.

- Given (f,δ) fixed, we have

$$\beta_1=\frac{2+\delta+2\delta f-\sqrt{4-4\delta+\delta^2-4\delta^2 f-4\delta^2 f^2}}{2(1+f)}$$
$$\beta_2=\frac{2+\delta+2\delta f+\sqrt{4-4\delta+\delta^2-4\delta^2 f-4\delta^2 f^2}}{2(1+f)}$$

The discriminant can be negative. When it is nonnegative, one can verify that $0\le\beta_1$ given $0\le f<1$ and $0\le\beta\le 1$.

Hence, $\Pi_E^S(e_S)-\Pi_N^S(e_S)>0$ is equivalent to the following three conditions:

- Given (β,δ) fixed, $f>\tilde{f}(\beta,\delta)$ with

$$\tilde{f}(\beta,\delta)=\min\left\{\left[\frac{(2-\beta)(\beta-\delta)}{\beta^2-2\beta\delta+2\delta^2}\right]^+,1\right\}.$$

- Given (f,β) fixed, $\delta>\tilde{\delta}(f,\beta)$ with

$$\tilde{\delta}(f,\beta)=\frac{-2+\beta+2\beta f+\sqrt{4-4\beta+\beta^2+8\beta f-4\beta^2 f-4\beta^2 f^2}}{4f}.$$

- Given (f,δ) fixed, $\beta\in[0,\tilde{\beta}_1(f,\delta))\cup(\tilde{\beta}_2(f,\delta),1]$ where $\tilde{\beta}_1(f,\delta)=\tilde{\beta}_2(f,\delta)$ take any value bigger than 1 when $4-4\delta+\delta^2-4\delta^2 f-4\delta^2 f^2<0$ or the following otherwise.

$$\tilde{\beta}_1(f,\delta)=\max\left\{\frac{2+\delta+2\delta f-\sqrt{4-4\delta+\delta^2-4\delta^2 f-4\delta^2 f^2}}{2(1+f)},0\right\}$$
$$\tilde{\beta}_2(f,\delta)=\min\left\{\frac{2+\delta+2\delta f+\sqrt{4-4\delta+\delta^2-4\delta^2 f-4\delta^2 f^2}}{2(1+f)},1\right\}$$

Appendix C: Partial Carryover Effect in the Second Stage

The analysis adheres to the backward induction approach utilized previously. Moreover, the results can be derived by simply adjusting $a+e_S$ to $\gamma(a+e_S)$ within the analysis of the original full-carryover model.

1) Second-stage decision under the platform's entry. Because the only modification is the substitution of $a+e_S$ with $\gamma(a+e_S)$, it suffices to replace $a+e_S$ with $\gamma(a+e_S)$ in the expressions of equilibrium quantities and the platform's equilibrium effort.

LEMMA 4. *Given that the platform has entered the seller's product space and that the enhanced product values are* $\gamma(a+e_S)+\delta e_P$ *and* $\gamma(a+e_S)+e_P$ *for the products sold by the seller and by the platform, respectively, the two parties' equilibrium quantity decisions in the second stage are* $q_S^* = \left[\frac{2-\beta}{4-\beta^2(1+f)}(\gamma(a+e_S)+e_P) - \frac{2(1-\delta)}{4-\beta^2(1+f)}e_P\right]^+$ *and* $q_P^* = \frac{\gamma(a+e_S)+e_P-\beta(1+f)q_S^*}{2}$.

2) The platform's entry decision.

LEMMA 5. *Given that the platform has entered the seller's product space and that the enhanced product value from the first stage is* $\gamma(a+e_S)$*, if* (5) *holds, then the platform's optimal product-value-enhancement effort in the second stage is*

$$e_P^* = \frac{\gamma(a+e_S)(8+8\delta f-4\beta(1+\delta)(1+f)+\beta^3(1+\delta)f(1+f)-2\beta^2(f+\delta(-1+f^2)))}{8-8\delta^2 f+8\beta\delta(1+f)-2\beta^3\delta f(1+f)+\beta^4(1+f)^2+2\beta^2(-4-3f+\delta^2(-1+f^2))},$$

and the seller remains in the second stage, i.e., $q_S^*>0$*; otherwise,* $e_P^* = \frac{(2-\beta)\gamma(a+e_S)}{\beta-2\delta}$ *and the seller opts out in the second stage, i.e.,* $q_S^*=0$.

In this partial-carryover model, the expression of $\Pi^P_{E-N}(e_S)=\Pi^P_{2,E}-\Pi^P_{2,N}$ differs from the original one only by the inclusion of an additional multiplier γ^2. Consequently, the entry policy follows $\gamma(a+e_S)>\Delta$, and the threshold Δ stays unchanged.

3) Receptive and protective regimes. Based on the equilibrium quantities and the platform's equilibrium effort, we have

$$\Pi^S_N = \frac{1+\gamma^2}{4}(1-f)(a+e_S)^2 - c_S e_S^2$$
$$\Pi^S_E = \left(\frac{1}{4}+\gamma^2 H\right)(1-f)(a+e_S)^2 - c_S e_S^2$$

where

$$H = \begin{cases} \frac{(4(1+\delta)-2\beta(2-\delta+\delta^2)+\beta^3(1+f)-\beta^2(2+f+\delta f))^2}{(8-8\delta^2 f+8\beta\delta(1+f)-2\beta^3\delta f(1+f)+\beta^4(1+f)^2+2\beta^2(-4-3f+\delta^2(-1+f^2)))^2} & \text{if (5) holds} \\ 0 & \text{otherwise.} \end{cases}$$

Notably, H stays unchanged. Moreover,

$$\Pi^S_E(e)-\Pi^S_N(e) = \left(H-\frac{1}{4}\right)\gamma^2(a+e_S)^2(1-f)$$

which indicates that Proposition 2 does not require any modifications. This includes the regime $\Omega = \{(\delta,f,\beta)|H>1/4\}$ and all associated thresholds. Furthermore, in scenarios where the product base value is intermediate, the seller's effort distortions, both upward and downward, continue to manifest in the protective and receptive regimes, respectively. In the protective regime, the platform can still achieve a better outcome by opting for a non-entry commitment when the inferred base value of the product, a, is either intermediate or exceeds the intermediate range without being excessively large.

Appendix D: Cumulative Effort Cost

We assume the platform's effort cost of enhancing the product value from $a+e_S$ to $a+e_S+e_P$ in Stage 2 to be $c_P((a+e_S+e_P)^2-(a+e_S)^2)$.

The only modifications on the profit functions are the cost functions. The two parties' optimal quantities are the same as those in Lemma 1.

1) The platform's entry decision. Given $a+e_S$, for e_P that makes $q_S^* > 0$, substituting the quantity decisions in Stage 2 with the equilibrium quantities given in Lemma 1, we have $\Pi_{2,E}^P$ as a function of e_P and derive the optimal effort by the first-order condition

$$e_P^* = \left[-\frac{(a+e_S)(-8+32c_P-8\delta f+4\beta(1+\delta)(1+f)-\beta^3(1+\delta)f(1+f)+2\beta^4c_P(1+f)^2+2\beta^2(f-8c_P(1+f)+\delta(-1+f^2)))}{(2(4\beta\delta(1+f)-\beta^3\delta f(1+f)-4(1+\delta^2 f)+c_P(-4+\beta^2(1+f))^2+\beta^2(f+\delta^2(-1+f^2))))}\right]^+$$

It is important to note that e_P^* is still a function of $(a+e_S)$. In the following, we focus on the nontrivial regime where both parties are actively engaged in competition and the platform chooses to exert positive effort. Akin to Condition (5), this nontrivial regime can be parameterized by a condition regarding (δ, f, β), which we refer to as Condition (5_{cum}). The platform's Stage-2 profit difference is still a function of $(a+e_S)^2$:

$$\begin{aligned}\Pi_{E-N}^P(e_S) &= \Pi_{2,E}^P - \Pi_{2,N}^P \\ &= \frac{(a+e_S)^2 Denom_{cum}(f,\beta,\delta)}{(4(4\beta\delta(1+f)-\beta^3\delta f(1+f)-4(1+\delta^2 f)+c_P(-4+\beta^2(1+f))^2+\beta^2(f+\delta^2(-1+f^2))))} - K\end{aligned}$$

where

$$\begin{aligned}Denom_{cum}(f,\beta,\delta) = (&4c_P^2(-4+\beta^2(1+f))^2+\delta f(8+4\delta(-1+f)-4\beta(1+f)+\beta^3 f(1+f)+\beta^2(-2f+\delta(1+f-f^2))) \\ &- c_P(16+32\delta f-16\beta\delta(1+f)+4\beta^3\delta f(1+f)+\beta^4 f(1+f)^2-4\beta^2(1+3f+f^2+2\delta(-1+f^2))))\end{aligned}$$

The platform's entry decision will follow a threshold policy: the platform will enter the seller's product space iff $a+e_S > \Delta_{cum}$, where

$$\Delta_{cum} = \sqrt{\frac{K(4(4\beta\delta(1+f)-\beta^3\delta f(1+f)-4(1+\delta^2 f)+c_P(-4+\beta^2(1+f))^2+\beta^2(f+\delta^2(-1+f^2))))}{Denom_{cum}(f,\beta,\delta)}}$$

3) Receptive and protective regimes. Define

$$H_{cum} = \begin{cases} \frac{(-1+\delta)^2(-4+2\beta\delta+\beta^2 f-4c_P(-4+\beta^2(1+f)))^2}{4(4\beta\delta(1+f)-\beta^3\delta f(1+f)-4(1+\delta^2 f)+c_P(-4+\beta^2(1+f))^2+\beta^2(f+\delta^2(-1+f^2)))^2} & \text{if } (5_{cum}) \text{ holds} \\ 0 & \text{otherwise.} \end{cases}$$

The seller's profit without or with the platform's entry can be expressed as

$$\Pi_N^S(e_S) = \frac{(1-f)(a+e_S)^2}{2} - c_S e_S^2$$
$$\Pi_E^S(e_S) = \left(\frac{1}{4}+H_{cum}\right)(1-f)(a+e_S)^2 - c_S e_S^2$$

The cumulative form of effort cost does not affect the comparison between the two profits, leading to the receptive regime $\Omega_{cum} = \{(\delta, f, \beta) | H_{cum} > 1/4\}$ and the protective regime $\bar{\Omega}_{cum} = \{(\delta, f, \beta) | H_{cum} \leq 1/4\}$.

The seller's optimal effort $e_S^*(a)$ is the maximizer of the function

$$\Pi^S(e_S) = \begin{cases} \Pi_N^S(e_S) & \text{if } e_S \leq \Delta_{cum} - a \\ \Pi_E^S(e_S) & \text{if } e_S > \Delta_{cum} - a \end{cases}$$

The unconstrained maximizers of $\Pi_N^S(e_S)$ and $\Pi_E^S(e_S)$ take the same forms as those in Equations (9) and (10), except that H is modified to H_{cum}.

The subsequent analysis is the same as that in the main model. As a result, the propositions laid out in the original framework (specifically Propositions 3 to 5) continue to be applicable, albeit with necessary adjustments to the thresholds to accommodate the new cost considerations. Specifically, as a increases, $e_S^*(a)$ transitions through three distinct phases: it equates to $e_N^B(a)$, to $\Delta - a$, and then to $e_E^B(a)$. The threshold values are again defined as: $\underline{a}_1 = \{a | a + e_N^B(a) = \Delta_{cum}\}$, $\bar{a}_1 = \max\{a | a \geq \underline{a}_1$ and $\Pi_N^S(\Delta_{cum} - a) = \Pi_E^S(e_E^B(a))\}$, $\underline{a}_2 = \inf\{a | \Pi_N^S(e_N^B(a)) \leq \Pi_E^S(\Delta_{cum} - a)\}$ and $\bar{a}_2 = \{a | a + e_E^B(a) = \Delta_{cum}\}$. The phase where $e_S^*(a) = \Delta - a$ is particularly noteworthy as it represents a strategic adjustment by the seller: downward distortion of effort in the protective regime to deter the platform from entering the market, and upward distortion of effort in the receptive regime to encourage platform entry. Moving forward, in the subregime where $a \geq \bar{a}_1$, we can still define $\Delta\Pi^P(a)$ as the difference between the platform's profit without non-entry commitment and the platform's profit with non-entry commitment and consequently define the threshold value $\bar{\bar{a}}_1$ such that both parties are strictly better off under the platform's non-entry commitment when $a \in (\underline{a}_1, \bar{\bar{a}}_1)$ and $(\delta, f, \beta) \in \bar{\Omega}_{cum}$.

Appendix E: Inverse Demand Function Derivation

The inverse demand functions are derived through maximizing a quadratic customer utility function as in the celebrated work by Singh and Vives (1984). A representative customer's utility function is given by

$$[a_1 q_1 + a_2 q_2 - (b_{11} q_1^2 + 2b_{12} q_1 q_2 + b_{22} q_2^2)/2] - (p_1 q_1 + p_2 q_2)$$

where q_i is the amount of goods i and p_i is its price, a_i and b_{ij} are positive, with $b_{11} b_{22} - b_{12}^2 > 0$, $a_1 b_{22} - a_2 b_{12} > 0$ and $a_2 b_{11} - a_1 b_{12} > 0$. Maximizing the utility with respect to the quantities yields the following linear demand structure:

$$p_1 = a_1 - b_{11} q_1 - b_{12} q_2$$
$$p_2 = a_2 - b_{12} q_1 - b_{22} q_2$$

Apply this framework to our specific context. In Stage 1, where the market only has a third-party seller, the customer's utility is $(a + e_S) q_1 - q_1^2/2 - p_1 q_1$. Maximizing this utility with respect to q_1 yields the first-order condition

$$(a + e_S) - q_1 - p_1 = 0 \implies p_1 = a + e_S - q_1.$$

In Stage 2 without the platform's entry, the same procedure repeats and we continue to have the above inverse demand function. In Stage 2 with the platform's entry, the third-party seller competes with the platform. The customer's utility becomes

$$[(a + e_S + \delta e_P) q_S + (a + e_S + e_P) q_P - (q_S^2 + 2\beta q_S q_P + q_P^2)/2] - (p_S q_S + p_P q_P).$$

Maximizing the utility with respect to the quantities yields

$$p_S = a + e_S + \delta e_P - q_S - \beta q_P$$
$$p_P = a + e_S + e_P - \beta q_S - q_P.$$