Online Appendix for "Rare Disasters and Exchange Rates"

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This online appendix presents extensions of the model to business cycle shocks, and nominal bond risk premia. It also provides extra proofs, and technical complements to the calibration of the model.

VIII. A Setup with a Risk Factor and a Business Cycle Factor

VIII.A. Basic Theory with a Business Cycle

So far, we have only introduced one factor, so that, controlling for current productivity, exchange rates and risk premia are perfectly correlated. This is an undesirable feature. In this section, we extend our framework to a two-factor model with a risk factor and a business cycle factor (see Pavlova and Rigobon 2007, 2008 for a different framework with several factors).

We model country i's export sector productivity as follows: $\omega_{it} = \overline{\omega}_{it}(1 + y_{it})$ where $\overline{\omega}_{it}$ is the (stochastic) trend component of productivity and y_{it} is a deviation from the trend that we refer to as the business cycle factor. The trend $\overline{\omega}_{it}$ behaves according to:

$$\frac{\overline{\omega}_{it+1}}{\overline{\omega}_{it}} = \exp(g_\omega) \times \begin{cases} 1 & \text{if there is no disaster at } t+1, \\ F_{i,t+1} & \text{if there is a disaster at } t+1. \end{cases}$$

The business cycle factor y_{it} follows a linearity-generating process

$$\mathbb{E}_t[y_{i,t+1}] = \frac{1 + H_{i*}}{1 + H_{it}} \exp(-\phi_{y_i}) y_{it},$$

with the innovation uncorrelated with those of ω_{it} and M_t^*.

Proposition 10 (Business cycle factor). *The exchange rate is given by*

$$e_{it} = \frac{\overline{\omega}_{it}}{1 - \exp(-r_{ei})} \left(1 + \frac{\exp(-r_{ei} - h_*)}{1 - \exp(-r_{ei} - \phi_{H_i})} \widehat{H}_{it} + \frac{1 - \exp(-r_{ei})}{1 - \exp(-r_{ei} - \phi_{y_i})} y_{it}\right). \quad (52)$$

In the limit of small time intervals, the exchange rate is given by

$$e_{it} = \frac{\overline{\omega}_{it}}{r_{ei}} \left(1 + \frac{\widehat{H}_{it}}{r_{ei} + \phi_{H_i}} + \frac{r_{ei} y_{it}}{r_{ei} + \phi_{y_i}} \right). \tag{53}$$

In the limit of small time intervals, the interest rate is

$$r_{it} = r_{ei} - \lambda + \frac{-\frac{r_{ei}}{r_{ei}+\phi_{H_i}} \widehat{H}_{it} + \frac{r_{ei}\phi_{y_i}}{r_{ei}+\phi_{y_i}} y_{it}}{1 + \frac{\widehat{H}_{it}}{r_{ei}+\phi_{H_i}} + \frac{r_{ei} y_{it}}{r_{ei}+\phi_{y_i}}}. \tag{54}$$

The resilience $\widehat{H}_{it}$ affects the exchange rate and the interest rate in the same way as in the setup without the business cycle factor: a risky country with a low $\widehat{H}_{it}$ has a depreciated exchange rate e_{it} and a high interest rate r_{it}. As a result, the disaster factor captured by $\widehat{H}_{it}$ induces a negative correlation between e_{it} and r_{it}. In contrast, the business cycle factor induces a positive correlation between these two variables: a country with an above-trend export sector productivity y_{it} has an appreciated exchange rate e_{it} and a high interest rate r_{it}. The correlation between the exchange rate and the interest rate depends on the relative importance of the disaster factor and the business cycle factor.

Fama regressions with two factors. Denote by α' and β' respectively the constant term and the Fama coefficient of the Fama regression in the two-factor model:

$$\frac{e_{i,t+1} - e_{it}}{e_{it}} - \frac{e_{j,t+1} - e_{jt}}{e_{jt}} = \alpha' - \beta'(r_{it} - r_{jt}) + \varepsilon_{ij,t+1} \tag{55}$$

The next proposition relates the coefficient $\beta^{ND\prime}$ in a sample with no disasters and the coefficient $\beta^{\prime Full}$ in a full sample to their counterparts β^{ND} and β^{Full} in the one-factor model.

Proposition 11 (Fama regression with two factors). *Consider two countries i and j with $r_{ei} = r_{ej} = r_{ei}$, $\phi_{H_i} = \phi_{H_j} = \phi_{H_i}$, and $\phi_{y_i} = \phi_{y_j} = \phi_y$. Consider the limit of small time intervals as well as a small $\widehat{H}_{it}$ and $\widehat{H}_{jt}$. Let ν be the share of the interest rate differential variance due to $\widehat{H}_{it} - \widehat{H}_{jt}$:*

$$\nu = \frac{\left(\frac{r_{ei}}{r_{ei}+\phi_{H_i}}\right)^2 \operatorname{Var}\left(\widehat{H}_{it} - \widehat{H}_{jt}\right)}{\left(\frac{r_{ei}}{r_{ei}+\phi_{H_i}}\right)^2 \operatorname{Var}\left(\widehat{H}_{it} - \widehat{H}_{jt}\right) + \left(\frac{r_{ei}\phi_{y_i}}{r_{ei}+\phi_{y_i}}\right)^2 \operatorname{Var}\left(y_{it} - y_{jt}\right)}. \tag{56}$$

The coefficient $\beta^{ND\prime}$ (respectively $\beta^{\prime Full}$) in the Fama regression (55) for a sample with no dis-

asters (respectively for a full sample) is given by[26]

$$\beta^{ND\prime} = \nu \beta^{ND} + 1 - \nu \tag{57}$$
$$\beta^{\prime Full} = \nu \beta^{Full} + 1 - \nu.$$

In equation (57), $\beta^{ND\prime}$ is the weighted average of two Fama coefficients: the first coefficient, β^{ND}, corresponds to variations in exchange rates and interest rate differentials driven by the disaster factor; the second coefficient, 1, corresponds to variations in exchange rates and interest rate differentials driven by the business cycle factor. The weight ν is the share of the disaster factor in the variance of interest rate differentials.

VIII.B. Predicting the Exchange Rate with Forwards

Nominal yield curves contain a lot of information that is potentially useful for predicting exchange rates. We now explain how best to extract the relevant information to compute exchange rate risk premia. As above, the expected depreciation of the nominal exchange rate, up to second order terms and conditional on no disasters, is:

$$\mathbb{E}_t^{ND}\left[\frac{d\widetilde{e}_{it}}{\widetilde{e}_{it}}\right]/dt = g_{\omega_i} - \frac{\phi_{H_i}\widehat{H}_{it}}{r_{ei}+\phi_{H_i}} - \frac{r_{ei}\phi_{y_i}y_t}{r_{ei}+\phi_{y_i}} - \pi_{it} \tag{58}$$

It involves three factors that are also reflected in the nominal forward curve. Note, however, that it does not involve the inflation risk premium Π_t^i. So, an optimal combination of forward rates should predict expected currency returns with more accuracy than the simple Fama regression.

Boudoukh, Richardson and Whitelaw (BRW, 2014) propose to regress the exchange rate movement on the T-period forward rate from T periods ago. BRW regression:

$$\mathbb{E}_t\left[\frac{e_{i,t+1}-e_{it}}{e_{it}} - \frac{e_{j,t+1}-e_{jt}}{e_{jt}}\right] = \alpha^{Fwd}(T) - \beta^{Fwd}(T)\left(f_{t-T}^{i}(T+1) - f_{t-T}^{j}(T+1)\right) \tag{59}$$

Our model's prediction is summarized in the next Proposition.

Proposition 12 (Value of the β coefficient in the Fama regression with two factors and with forward rates). *Up to second order terms, in the BRW (59) regression with forward rates, the*

[26] The formula $\beta^{\prime Full} = \nu\beta^{Full} + 1 - \nu$ is valid even when B_t is not constant. The only difference in this case is that β^{Full} is no longer given by equation (27).

coefficients are:

$$\beta^{Fwd}(T) = \nu(T)\beta^{ND} + 1 - \nu(T) \tag{60}$$

and

$$\beta^{Fwd,Full}(T) = \nu(T)\beta^{Full} + 1 - \nu(T) \tag{61}$$

where β^{ND} *and* β^{Full} *are given by Eqs. (28) and (27), respectively, and*

$$\nu(T) = \frac{\left(\frac{r_{ei}}{r_{ei}+\phi_{H_i}}\right)^2 \mathrm{Var}\left(\widehat{H}_{it} - \widehat{H}_{jt}\right)\exp\left(-2\phi_{H_i}T\right)}{\left(\frac{r_{ei}}{r_{ei}+\phi_{H_i}}\right)^2 \mathrm{Var}\left(\widehat{H}_{it} - \widehat{H}_{jt}\right)\exp\left(-2\phi_{H_i}T\right) + \left(\frac{r_{ei}\phi_y}{r_{ei}+\phi_y}\right)^2 \mathrm{Var}\left(y_{it} - y_{jt}\right)\exp\left(-2\phi_y T\right)} \tag{62}$$

is the share of variance in the forward rate due to $\widehat{H}_{it} - \widehat{H}_{jt}$*. In particular, when* $\phi_{H_i} > \phi_y$*, the long horizon regression has a coefficient going to 1:* $\lim_{T\to\infty}\beta^{Fwd}(T) = \lim_{T\to\infty}\beta^{Fwd,Full}(T) = 1$.

Boudoukh, Richardson and Whitelaw (2014) find that $\beta^{Fwd}(T)$ increases toward 1 with the horizon. Our theory is consistent with this empirical finding. Indeed, to interpret Proposition 12, consider the case where risk premia are quickly mean-reverting, and the business cycle factor is slowly mean-reverting, $\phi_{H_i} > \phi_y$. [27] Then, for large T, $\nu(T)$ tends to 0, which means that, at long horizons, the forward rate is mostly determined by the level of the business cycle factor and not the risk premium. Hence, at a long horizon the model behaves like a model without risk premia and therefore generates a coefficient β close to 1.

IX. Discussion

The Backus-Smith puzzle. Recall that the (traditional) real exchange rate is (44):

$$\mathcal{E}_{it} = \left(\xi_{it}^{\sigma} + {e_{it}}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{63}$$

Call $C_{it} := \left[\left(C_{it}^{T}\right)^{\frac{\sigma-1}{\sigma}} + \frac{1}{\xi_{it}}\left(C_{it}^{NT}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ the consumption basket. We have:

$$\zeta_{it} C_{it}^{-\gamma} = \frac{M_t^* e^{\rho t}}{\mu_i}\mathcal{E}_{it}.$$

[27] The same reasoning would hold if we replaced the business cycle factor with inflation.

Hence, with two countries i, j:

$$\Delta \ln \frac{C_{it}}{C_{jt}} = \frac{1}{\gamma} \Delta \ln \frac{\zeta_{it}}{\zeta_{jt}} - \frac{1}{\gamma} \Delta \ln \frac{\mathcal{E}_{it}}{\mathcal{E}_{jt}}.$$

Hence, the correlation between relative consumption growth $\Delta \ln \frac{C_{it}}{C_{jt}}$ and movement $\Delta \ln \frac{\mathcal{E}_{it}}{\mathcal{E}_{jt}}$ in the relative exchange rate will not be perfect, because of the taste shock $\Delta \ln \frac{\zeta_{it}}{\zeta_{jt}}$. The model generates an imperfect correlation between total consumption and real exchange rates.

Predicting returns with the price-dividend ratio. Finally, Table VI verifies that our calibration of stocks also matches the pattern of predictability of the stock market with the price-dividend ratio, with the slope and R^2 rising with the horizon (at least for a while).

TABLE VI: Predicting Returns with the Price-Dividend Ratio

	Data			Model	
Horizon	Slope	s.e.	R^2	Slope	R^2
1 yr	0.11	(0.053)	0.04	0.15	0.08
4 yr	0.42	(0.18)	0.12	0.51	0.26
8 yr	0.85	(0.20)	0.29	0.86	0.39

Notes. Predictive regression for the expected stock return $r^{\text{Stock}}_{i,t\to t+T} = \alpha_T + \beta_T \ln (D_{it}/P_{it})$, at horizon T (annual frequency), up to an 8-year horizon. The data are from Campbell (2003, Table 10 and 11B)'s calculation for the US for the period 1891–1997.

X. Richer Nominal Model

X.A. *Basic Theory*

We now develop a richer model with an inflation-specific risk premium. This allows us to talk about a basic fact about the yield curve: on average, the (nominal) yield curve is upward sloping, i.e. long-term interest rates are higher than short term interest rates. We therefore extend the framework by incorporating inflation risk along the lines of Gabaix (2012): as inflation rises during disasters (on average), long-term bonds are riskier, which makes the yield curve slope upward. This will allow us to study the term premium across countries.

The variable part of inflation now follows the process:

$$\widehat{\pi}_{t+1} = \frac{1-\pi_*}{1-\pi_t} \cdot \left(\exp\left(-\phi_{\pi_i}\right) \widehat{\pi}_t + 1_{\{\text{Disaster at } t+1\}} \left(J_* + \widehat{J}_t \right) \right) + \varepsilon^{\pi}_{t+1} \tag{64}$$

In case of a disaster, inflation jumps by an amount $J_t = J_* + \widehat{J}_t$. This jump in inflation makes long-term bonds particularly risky. J_* is the baseline jump in inflation, $\widehat{J}_t$ is the mean-reverting deviation from the baseline. It follows a twisted auto-regressive process, and, for simplicity, does not jump during crises:

$$\widehat{J}_{t+1} = \frac{1-\pi_*}{1-\pi_t} \cdot \exp\left(\phi_\Pi\right) \widehat{J}_t + \varepsilon^J_{t+1}. \tag{65}$$

We define

$$\Pi_t := \frac{p_t \mathbb{E}_t \left[B^{-\gamma}_{t+1} F_{i,t+1} \right]}{1 + H_{i*}} \widehat{J}_t, \tag{66}$$

which is the mean-reverting part of the "risk adjusted" expected increase in inflation if there is a disaster. We parametrize the typical jump in inflation J_* in terms of a number $\kappa \leq (1-\rho_\pi)/2$:

$$\frac{p_t \mathbb{E}_t \left[B^{-\gamma}_{t+1} F_{i,t+1} J_* \right]}{1 + H_{i*}} = (1-\pi_*)\, \kappa \left(1 - \exp\left(-\phi_{\pi_i}\right) - \kappa\right).$$

κ represents a risk premium for the risk that inflation increases during disasters. Also, we define

$$\pi_{**} := \pi_* + \kappa \tag{67}$$

and $\psi_\Pi := \phi_{\pi_i} - \kappa$. They represent the "risk adjusted" trend and mean-reversion parameters in the inflation process. In the continuous-time limit,

$$\kappa \left(\phi_{\pi_i} - \kappa\right) = p_t \mathbb{E}_t \left[B^{-\gamma}_{t+1} F_{i,t+1} J_* \right] = J_* \left(H_{it} + p_t\right) \tag{68}$$

As before, we denote nominal variables with a tilde. The price of a long-term nominal bond yielding one unit of the currency at time $t+T$ is $\widetilde{Z}_t(T) = \mathbb{E}_t \left[\frac{M^*_{t+T} e_{t+T} Q_{t+T}}{M^*_t e_t Q_t} \right]$, where Q_t is the inverse of the price level.

The yield at maturity T, $\widetilde{Y}_t(T)$, and the forward rates $\widetilde{f}_t(T)$ are defined by $-\ln \widetilde{Z}_t(T) = \widetilde{Y}_t(T)\, T = \sum_{T'=1}^{T} \widetilde{f}_t(T')$. The forward rates can be derived in closed form. For completeness, we also import the part with business cycle risk from section VIII (the y_t term below).

PROPOSITION 13 (Forward rates with inflation risk premia). *In the continuous time limit, up to second order terms in* $\left(\widehat{H}_t, y_t, \pi_t, \Pi_t, \kappa\right)$*:*

$$\widetilde{f}_t(T) = r_{ei} - \lambda - \frac{r_{ei}}{r_{ei} + \phi_{H_i}} \exp\left(-\phi_{H_i} T\right) \widehat{H}_t + \frac{r_{ei}\phi_y}{r_{ei} + \phi_y} \exp\left(-\phi_y T\right) y_t + \\ + \pi_{**}\left(1 - \exp\left(-\phi_{\pi_i} T\right)\right) + \exp\left(-\phi_{\pi_i} T\right) \pi_t + \frac{\exp\left(-\phi_\pi T\right) - \exp\left(-\psi_\Pi T\right)}{\psi_\Pi - \phi_{\pi_i}} \Pi_t \tag{69}$$

where $\widehat{H}_t$ *is the transitory part of the country's resilience,* y_t *is the state of the business cycle,* π_t *is inflation,* Π_t *is the transitory part of the bond risk premium: they are all for a given country* i *(for simplicity, we omit here the index* i*).*

Proof. The proof is along the lines of Gabaix (2012, Theorem 2 and Lemma 2), and we only sketch it here as the mechanics are very similar. The first step is to calculate that $Y_t := M_t e_t Q_t \left(1, \widehat{H}_t, y_t, \widehat{\pi}_t, \Pi_t\right)$ is LG (to the leading order), with continuous time generator:

$$\omega = \left(r_{ei} - \lambda\right) I + \begin{pmatrix} 0 & -\frac{r_{ei}}{r_{ei}+\phi_{H_i}} & \frac{r_{ei}\phi_y}{r_{ei}+\phi_{H_i}} & 1 & 0 \\ 0 & \phi_{H_i} & 0 & 0 & 0 \\ 0 & 0 & \phi_y & 0 & 0 \\ -\kappa\left(\phi_{\pi_i} - \kappa\right) & 0 & 0 & \phi_{\pi_i} & -1 \\ 0 & 0 & 0 & 0 & \phi_J \end{pmatrix} \tag{70}$$

Then, we derive the bond price as $Z_t(T) = (1,0,0,0,0)' e^{-\omega T} Y_t$. The forward rate is then $f_t(T) = -\partial_T \ln Z_t(T)$. Here we report the limit for $\kappa \to 0$, which makes terms cleaner, and gives a sense in which the proposition is only up to second order terms. The nominal forward rate in (69) depends on real and nominal factors. The real factors are the resilience of the economy ($\widehat{H}_t$) and the expected growth rate of productivity ($-\phi_y y_t$). The nominal factors are inflation (π_t), and the variable component of the the risk premium for inflation jump risk (Π_t). $\square$

When a disaster occurs, inflation increases (on average). As very short term bills are essentially immune to inflation risk, while long-term bonds lose value when inflation is higher, long-term bonds are riskier, hence they get a higher risk premium. Hence, the yield curve slopes up on average – as implied by the term $\pi_{**}(1-\exp(-\phi_{\pi_i}T)) \sim \pi_{**}\phi_{\pi_i}T$.

Each of the three terms is multiplied by a term of the type $\exp(-\phi_{H_i}T)$. For small speeds of mean reversion ϕ_{H_i} the forward curve is fairly flat. The last term, however, is close to T for small maturities ($\frac{\exp(-\phi_{\pi_i}T)-\exp(-\psi_\Pi T)}{\psi_\Pi-\phi_{\pi_i}} \sim T$) and therefore creates a variable slope in the forward curve.

Hence, we obtain a rich forward curve. Gabaix (2012) shows that this type of yield curve generates a realistic term premium and volatility of the yield curve. Here, we have two extra terms: the country-specific resilience $\widehat{H}_t$, and the state of the business cycle y_t.

X.B. The Term Premium Across Countries

Proposition 13 implies that the short-term nominal interest rate is $\widetilde{r}_t = \widetilde{f}_t(0)$:

$$\widetilde{r}_t = r_{ei} - \lambda - \frac{r_{ei}}{r_{ei}+\phi_{H_i}}\widehat{H}_t + \frac{r_{ei}\phi_y}{r_{ei}+\phi_y}y_t + \pi_t,$$

so that on average, up to second order terms,

$$\mathbb{E}[\widetilde{r}_t] = r_{ei} - \lambda + \pi_*,$$

while the long-term nominal interest rate ($\widetilde{r}_t^{LT} = \lim_{T\to\infty}\widetilde{f}_t(T)$) is equal to:

$$\widetilde{r}^{LT} := r_{ei} - \lambda + \pi_* + \kappa.$$

It is independent of time, which is typical in these models.

The difference between the two rates is

$$\widetilde{r}^{LT} - \mathbb{E}[\widetilde{r}_t] = \kappa.$$

We will call this the "term premium".[28] It is also the expected excess return of long-term bonds conditional on no disasters. Below, we define p_* as the average value of the probability of disasters.

[28] Empirically, the term premium is often drawn from the properties of finite-maturity bonds, e.g. the 30-year bond, but for conceptual discussions very long-term bonds are clearer.

Proposition 14 (Slope of the yield curve / term premium) *On average (and up to second order terms), the term premium (the average value of long-term bond yields minus short term bond yields) in country i, κ_i, is the smaller positive root of*

$$\kappa_i \left(\phi_{\pi,i} - \kappa_i\right) = J_{i*} \left(H_{i*} + p_*\right). \tag{71}$$

The term premium κ_i is increasing with the expected increase of inflation in disasters (J_{i*}), and increasing *with resilience. In other words, controlling for the inflation process ($\phi_{\pi,i}$ and J_{i*}), countries that are relatively riskier (have a lower resilience H_{i*}) have a lower term premium κ_i than countries that are safer (have a higher resilience H_{i*}).*

Lustig, Stathopoulos and Verdelhan (2014) find evidence of this effect (see also Ang and Chen (2010) for related work). In their sample, more risky (high interest rate) countries have lower term premia than less risky (low interest rate) countries.

The intuition is the following. To make the benchmark starker, suppose that all countries have the same inflation processes (same $\phi_{\pi,i}$ and J_{i*}). Because a risky currency will depreciate in a disaster, a "term premium trade" portfolio long its long-term bond and short its short-term bond will have little value in a disaster: in the limit where the currency is due to entirely collapse in a disaster ($F_{i*} = 0$, so that $H_{i*} + p_* = 0$, and $\kappa_i = 0$) this "term premium trade" will have exactly zero payoff. Hence, perhaps surprisingly at first, it should have a zero risk premium.

However, the same "term premium trade" with a safe currency will do very poorly during disasters, as the value of long term bonds falls (on average) during disasters because of the increase in inflation: this trade is risky, hence commands a risk premium, which is the term premium, κ_i.[29] In some sense, disasters "democratize" risk premia, i.e. dull the differences in riskiness between bonds (here, between short-term and long-term bonds of risky countries), as extreme disasters just wipe out the value of all very risky bonds.[30]

[29] More precisely, consider small jumps and risk premia: inflation jumps by J_*, but with a speed of mean-reversion ϕ_{π_i}, so that the loss in value of long-term bonds is $\frac{J_*}{\phi_{\pi_i}}$. Hence, the risk premium κ satisfies $\kappa = p_t \mathbb{E}_t \left[B_{t+1}^{-\gamma} F_{i,t+1} \frac{J_*}{\phi_{\pi_i}}\right]$, i.e. $\kappa \phi_{\pi_i} = p_t \mathbb{E}_t \left[B_{t+1}^{-\gamma} F_{i,t+1} J_*\right]$, which is equation (68), up to a second order term in κ.

[30] A similar intuition holds in Gabaix (2012), Proposition 4, point (i).

XI. Proofs That Were Omitted in the Paper

Proof of Lemma 1. *Part (i).* The CES functional form gives:

$$u_{C_{it}^T} = \zeta_{it} \left(C_{it}^T\right)^{-\frac{1}{\sigma}} \left[\left(C_{it}^T\right)^{\frac{\sigma-1}{\sigma}} + \frac{1}{\xi_{it}} \left(C_{it}^{NT}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}(1-\gamma)-1} \tag{72}$$

$$u_{C_{it}^{NT}} = \frac{\zeta_{it}}{\xi_{it}} \left(C_{it}^{NT}\right)^{-\frac{1}{\sigma}} \left[\left(C_{it}^T\right)^{\frac{\sigma-1}{\sigma}} + \frac{1}{\xi_{it}} \left(C_{it}^{NT}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}(1-\gamma)-1} \tag{73}$$

hence

$$e_{it} = \frac{u_{C_{it}^{NT}}}{u_{C_{it}^T}} = \frac{1}{\xi_{it}} \left(\frac{C_{it}^{NT}}{C_{it}^T}\right)^{-\frac{1}{\sigma}}$$

Given

$$\frac{\partial L}{\partial C_{it}^T} = \mu_i \exp\left(-\rho t\right) u_{C_{it}^T} - M_t^* = 0,$$

we have

$$\left(C_{it}^T\right)^{-\gamma} \left[1 + \xi_{it}^{-\sigma} e_{it}^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}(1-\gamma)-1} = \frac{M_t^* e^{\rho t}}{\mu_i \zeta_{it}}$$

and

$$C_{it}^T \left[1 + \xi_{it}^{-\sigma} e_{it}^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}\left(1-\frac{1}{\gamma}\right)+\frac{1}{\gamma}} = \left(\frac{M_t^* e^{\rho t}}{\mu_i \zeta_{it}}\right)^{-\frac{1}{\gamma}}$$

This gives the values of C_{it}^T and C_{it}^{NT}. We must have the resource constraints:

$$\eta_{it}^{NT} = \iota_{it}^T + C_{it}^{NT}, \tag{74}$$

$$Y_{it}^T = \eta_{it}^T + \sum_{s=0}^{\infty} \exp(-\lambda s) \omega_{it} \iota_{i,t-s}^T, \tag{75}$$

$$nx_{it} = Y_{it}^T - C_{it}^T. \tag{76}$$

Consider an arbitrary sequence of ι_{it}^T (the simplest case being $\iota_{it}^T = 0$), and define the endowment to be $\eta_{it}^{NT} := \iota_{it}^T + C_{it}^{NT}$, $Y_{it}^T := nx_{it} + C_{it}^T$ and $\eta_{it}^T := Y_{it}^T - \sum_{s=0}^{\infty} \exp(-\lambda s) \omega_{it} \iota_{i,t-s}^T$. Then, these endowments generate the posited equilibrium value of the pricing kernel and net exports.

Part (ii). Processes M_t^* and ω_{it} pin down the process for e_{it} as per Proposition 2. This in turn pins down the value of $u_{C_{it}^T}$ and $u_{C_{it}^{NT}}$ as per equations (72)-(73), two values that can be matched with judicious values of ξ_{it} (given by calculating $\frac{u_{C_{it}^T}}{u_{C_{it}^{NT}}}$) and then ζ_{it}. The rest is as in part (i).

Proof of Lemma 2. The Lemma is proven in Farhi et al. (2015, Proposition 5), but for completeness we provide a proof. We call T the maturity of the option, and perform a Taylor expansion (in $T^{1/2}$) for small T. We consider puts with a given Garman-Kohlhagen delta equal to $-\Delta$ (with $\Delta > 0$), so that $1-K = O\left(T^{1/2}\right)$, and $V^P = O\left(T^{1/2}\right)$ (for a detailed explanation, see Farhi et al. (2015)). We define $d = \Phi^{-1}(\Delta)$. We call S the initial exchange rate, which we later normalize to 1.

Proposition 6 implies the following, where we drop all the terms of order $o(T)$:

$$
\begin{aligned}
V^P &= e^{(-R+\mu_i)T}(1-p_0T)\,V_{BS}^P\left(S, Ke^{(\mu_j-\mu_i)T}, \sigma_{i|j}\right) + e^{(-R+\mu_i)T}p_0T\mathbb{E}_0^D\left[B_1^{-\gamma}\left(Ke^{(\mu_j-\mu_i)T}F_{j,1} - SF_{i,1}\right)^+\right] \\
&= V_{BS}^P\left(S, Ke^{(\mu_j-\mu_i)T}, \sigma_{i|j}\right) + p_0T\mathbb{E}_0^D\left[B_1^{-\gamma}\left(KF_{j,1} - SF_{i,1}\right)^+\right] + O\left(T^{3/2}\right) \\
&= V_{BS}^P\left(S, Ke^{-r_jT}e^{(r_j+\mu_j-\mu_i)T}, \sigma_{i|j}\right) + p_0T\mathbb{E}_0^D\left[B_1^{-\gamma}\left(KF_{j,1} - SF_{i,1}\right)^+\right] + O\left(T^{3/2}\right) \\
&= e^{(r_j+\mu_j-\mu_i)T}V_{BS}^P\left(Se^{-r_iT-(r_j+\mu_j-\mu_i-r_i)T}, Ke^{-r_jT}, \sigma_{i|j}\right) + p_0T\mathbb{E}_0^D\left[B_1^{-\gamma}\left(KF_{j,1} - SF_{i,1}\right)^+\right] + O\left(T^{3/2}\right) \\
&= e^{(r_j+\mu_j-\mu_i)T}V_{BS}^P\left(Se^{-r_iT+(H_j-H_i)T}, Ke^{-r_jT}, \sigma_{i|j}\right) + p_0T\mathbb{E}_0^D\left[B_1^{-\gamma}\left(KF_{j,1} - SF_{i,1}\right)^+\right] + O\left(T^{3/2}\right) \\
&= V_{BS}^P\left(Se^{-r_iT}, Ke^{-r_jT}, \sigma_{i|j}\right) + \frac{\partial V_{BS}^P}{\partial S}\left(Se^{-r_iT}, Ke^{-r_jT}, \sigma_{i|j}\right) \times (H_j - H_i)\,T \\
&+ p_0T\mathbb{E}_0^D\left[B_1^{-\gamma}\left(KF_{j,1} - SF_{i,1}\right)^+\right] + O\left(T^{3/2}\right) \\
&= V_{BS}^P\left(Se^{-r_iT}, Ke^{-r_jT}, \sigma_{i|j}\right) - \Delta\,(H_j - H_i)\,T + p_0T\mathbb{E}_0^D\left[B_1^{-\gamma}\left(KF_{j,1} - SF_{i,1}\right)^+\right] \\
&+ O\left(T^{3/2}\right), \text{ using } \frac{\partial V_{BS}^P}{\partial S} = -\Delta \\
&= V_{BS}^P\left(Se^{-r_iT}, Ke^{-r_jT}, \sigma_{i|j}\right) + AT + O\left(T^{3/2}\right)
\end{aligned}
$$

where

$$
A = -\Delta\,(H_j - H_i) + p_0\mathbb{E}_0^D\left[B_1^{-\gamma}\left(KF_{j,1} - SF_{i,1}\right)^+\right].
$$

On the other hand, the option price can be expressed as a traditional Garman-Kohlhagen value, with an implied volatility $\sigma_{i|j} + \tilde{\sigma}$ (where $\tilde{\sigma}$ is the difference between implied volatility and realized volatility, and is of order $O\left(T^{1/2}\right)$) tuned to match the price:

$$
\begin{aligned}
V^P &= V_{BS}^P\left(Se^{-r_iT}, Ke^{-r_jT}, \sigma_{i|j} + \tilde{\sigma}\right) \\
&= V_{BS}^P\left(Se^{-r_iT}, Ke^{-r_jT}, \sigma_{i|j}\right) + \frac{\partial V_{BS}^P}{\partial \sigma}(Se^{-r_iT}, Ke^{-r_jT}, \sigma)\tilde{\sigma} + O\left(T^{3/2}\right) \\
&= V_{BS}^P\left(Se^{-r_iT}, Ke^{-r_jT}, \sigma_{i|j}\right) + \phi\left(d\right)\sqrt{T}\tilde{\sigma} + O\left(T^{3/2}\right),
\end{aligned}
$$

using $\frac{\partial V_{BS}^P}{\partial \sigma}(Se^{-r_iT}, Ke^{-r_jT}, \sigma_{i|j}) = \phi\left(d\right)\sqrt{T}$. Hence

$$
AT = \phi\left(d\right)\sqrt{T}\tilde{\sigma} + O\left(T^{3/2}\right)
$$

which gives:

$$
\tilde{\sigma}^{\text{Put}} = \frac{-\Delta\left(H_j - H_i\right) + p_0\mathbb{E}_0^D\left[B_1^{-\gamma}\left(KF_{j,1} - F_{i,1}\right)^+\right]}{\phi\left(d\right)}\sqrt{T} + O\left(T\right)
$$

Likewise, the implied volatility of a symmetric call is

$$
\tilde{\sigma}^{\text{Call}} = \frac{-\left(1-\Delta\right)\left(H_j - H_i\right) + p_0\mathbb{E}_0^D\left[B_1^{-\gamma}\left(F_{j,1} - KF_{i,1}\right)^+\right]}{\phi\left(-d\right)}\sqrt{T} + O\left(T\right)
$$

so that the risk reversal is:

$$
\begin{aligned}
RR = \tilde{\sigma}^{\text{Put}} - \tilde{\sigma}^{\text{Call}} &= \frac{\left(1-2\Delta\right)\left(H_j - H_i\right) + O(K-1)}{\phi\left(d\right)}\sqrt{T} + O\left(T\right) \\
&= \frac{\left(1-2\Delta\right)\left(H_j - H_i\right)}{\phi\left(d\right)}\sqrt{T} + O\left(T\right) \text{ as } K - 1 = O\left(T^{1/2}\right) \\
&= k_\Delta\left(H_j - H_i\right)\sqrt{T} + O\left(T\right)
\end{aligned}
$$

where $k_\Delta := \frac{1-2\Delta}{\phi(d)} = \frac{1-2\Delta}{\phi(\Phi^{-1}(\Delta))}$.

Proof of Lemma 3. The quick intuition is the following: the expected excess return of the stock claim in terms of the international numéraire is $p\mathbb{E}_t\left[B_{t+1}^{-\gamma}\left(1 - F_{Di,t+1}\right)\right]$, while the expected excess return on holding the currency is $p\mathbb{E}_t\left[B_{t+1}^{-\gamma}\left(1 - F_{i,t+1}\right)\right]$. Hence, the excess return on holding the domestic stock is the difference between the two, namely $p_t\mathbb{E}_t\left[B_{t+1}^{-\gamma}\left(F_{Di,t+1} - F_{i,t+1}\right)\right]$.

The derivation is as follows: conditional on no disasters, the return on a stock in the domestic currency is (we neglect the Jensen's inequality / Ito variance terms, which are second order):

$$
\begin{aligned}
\mathbb{E}r_{it}^{S} &= \frac{dP_{Dk,t}}{P_{Dk,t}} + \frac{D_{it}/e_{it}}{P_{Dk,t}}dt \\
&= \frac{dD_{it}}{D_{it}} - \frac{de_{it}}{e_{it}} + \frac{dh_{Dit}}{1+h_{Dit}} + \frac{D_{it}/e_{it}}{P_{Dk,t}}dt \\
&= g_{D_i} - g_{\omega_i} - \frac{dh_{it}}{1+h_{it}} + \frac{dh_{Dit}}{1+h_{Dit}} + \frac{r_{Di}}{1+h_{Dit}} \\
&= g_{D_i} - g_{\omega_i} + r_{Di} + \phi_{H_i} h_{it} - (\phi_{H_i} + r_{Di})\, h_{Dit},
\end{aligned}
$$

while $r_{f,it} = r_{ei} - \lambda - r_{ei} h_{it}$, so

$$
\begin{aligned}
\mathbb{E}r_{it}^{S} - r_{f,it} &= g_{D_i} - g_{\omega_i} + r_{Di} - r_{ei} + \lambda + (\phi_{H_i} + r_{ei})\, h_{it} - (\phi_{H_i} + r_{Di})\, h_{Dit} \\
&= A + \widehat{H}_{it} - \widehat{H}_{Dit},
\end{aligned}
$$

where the constant A is:

$$
\begin{aligned}
A &= g_{D_i} - g_{\omega_i} + r_{Di} - r_{ei} + \lambda \\
&= g_{D_i} - g_{\omega_i} + (R - g_{D_i} - h_{Di*}) - (R + \lambda - g_{\omega_i} - h_{i*}) + \lambda \\
&= h_{i*} - h_{Di*},
\end{aligned}
$$

so

$$
\mathbb{E}r_{it}^{S} - r_{f,it} = h_{i*} - h_{Di*} + \widehat{H}_{it} - \widehat{H}_{Dit} = H_{it} - H_{Dit} = -H_{D'it}.
$$

XII. Details of the Calibration

In the first calibration to generate Table III in the paper, we use the variables listed in the table below.

The value for ϕ_{H_i} is taken from Gabaix (2012), while the values for ϕ_{π_i} and σ_{π_i} are estimated. A value of π_* equal to 2% is consistent with average inflation in the US since 1990. We also set $g_{\omega,i} = g_{\omega,j} = 0$. This generates a real risk-free rate (in the absence of disasters) of 0.5%.

We use the following relationship to get estimates for σ_{H_i}. Recall that we are targeting a bilateral exchange rate volatility of 11% (annualized). Using the relationship between annual and monthly volatilities, this corresponds to a monthly targeted volatility of $11\%/\sqrt{12}$, or a

Symbol	Description	Value
r_{ei}	Exchange rate discount rate	6% (calibrated to this value)
ϕ_{H_i}	Speed of mean reversion (resilience)	18%
σ_{H_i}	Volatility of resilience	1.87% (calibrated to this value)
π_*	Mean inflation	2%
ϕ_{π_i}	Speed of mean reversion (inflation)	23%
σ_{π_i}	Volatility of inflation	0.6%
g_{ω_i}	Productivity growth rate	0%

monthly variance of $0.11^2/12$. Since these are bilateral variances, and using the relationship in equation (16) of the paper, we have that

$$\frac{0.11^2}{12 \times 2} = (\sigma_{H_i})^2/(r_{ei} + \phi_{H_i})^2.$$

Now, we can solve for σ_{H_i}.

Specifically,

$$\sigma_{H_i} = (r_{ei} + \phi_{H_i})\frac{0.11}{\sqrt{2}\sqrt{12}}$$

If r_{ei} and ϕ_{H_i} are in annualized units, then

$$\sigma_{H_i} = (r_{ei} + \phi_{H_i})\frac{0.11}{12\sqrt{2}\sqrt{12}}$$

This gives us the value of σ_{H_i} reported in Table I in the paper, and in the above table.[31]

The first step in the calibration is to simulate the series of resiliences H_i and H_j for countries i and j. We can simulate such a time series using equations (13) and (14) in the paper. However, we need to know σ_{ε^H}, the standard deviation of the residual in the equation

$$\hat{H}_{i,t+1} = \exp(-\phi_{H_i} + h_*) \times \hat{H}_{it}/(1 + H_{it}) + \varepsilon^H_{i,t+1}.$$

The methodology to compute this quantity follows.

Variance processes. Consider an LG process centered at 0,

$$dX_t = -\left(\phi + X_t\right) X_t dt + \sigma\left(X_t\right) dW_t,$$

[31] Note: in all the tables, only annualized values are reported. Annualization factors for different variables are reported in a table in the next section.

where W_t is a standard Brownian motion. Because of economic considerations, the support of X_t needs to be some $(X_{\min}, \infty)$ with $-\phi \leq X_{\min} < 0$. $X_{\min}$ cannot be less than $-\phi$ since the random variable X must always be mean reverting. For the simulation, we take $X_{\min} = -\phi$, maintaining full generality of the allowed domain. The following variance process makes this possible:

$$\sigma^2(X) = 2K(1 - X/X_{\min})^2, \tag{77}$$

with $K > 0$. K is in units of $[Time]^{-3}$. The average variance of X is: $\overline{\sigma}_X^2 = \mathbb{E}[\sigma^2(X_t)] = \int_{X_{\min}}^{X_{\max}} \sigma(X)^2 p(X)\, dX$ where $p(X)$ is the steady-state density of X_t. It can be calculated using the Forward Kolmogorov equation, which yields:

$$d \ln p(X)/dX = 2X(\phi + X)/\sigma^2(X) - d \ln \sigma^2(X)/dX.$$

Numerical simulations show that the process for volatility is fairly well approximated by: $\overline{\sigma}_X \simeq K^{1/2}\xi$ with $\xi = 1.3$.

Therefore, we use a value of K equal to $(\sigma_{H_i}/1.3)^2$.

This is the variance process that we use in the simulation as well, but in discrete instead of continuous time. In particular, we use

$$\hat{H}_{i,t+1} = \exp(-\phi_{H_i} + h_*) \times \hat{H}_{it}/(1 + H_{it}) + \sqrt{2K} \times (1 + \hat{H}_{it}/\phi_{H_i}) \times N_{i,t+1}^H;$$

where N_i is a set of standard Gaussian shocks with mean 0 and variance 1.[32] We similarly simulate the resilience for country j.

Methodology for the simulations. The methodology followed in the code is as follows:
1) Initialize variables in monthly units.
2) Simulate a sequence of 400 shocks (to remove possible dependencies on the starting value, we simulate 500 shocks, throwing away the first 100), obtain monthly time series of resilience and inflation values using the following equations (we use parameters and equations for country

[32] Throughout the calibration, we use Gaussian shocks to simulate innovations to the variable part of resilience. We always require this variable part to be greater than $-\phi_{H_i}$, to preserve the mean-reverting property of $\hat{H}$. Theoretically, with discrete Gaussian shocks, this bound may be violated, but in the continuous time limit, this violation happens with probability 0. As such we stick to this method of simulating the innovation shocks. If the shocks were uniform instead of Gaussian, this issue would not arise. For more details, see the online appendix of Gabaix (2012).

i below; needless to say, the same holds for country j as well):

$$H_{i,t+1} = H_{i*} + \hat{H}_{i,t+1}$$

$$\hat{H}_{i,t+1} = \exp(-\phi_{H_i} + h_*) \times \hat{H}_{it}/(1 + H_{it}) + \sqrt{2K_H} \times (1 + \hat{H}_{it}/\phi_{H_i}) \times N^H_{i,t+1};$$

$$\pi_{t+1} = \pi_* + \exp(-\phi_{\pi_i})\frac{1-\pi_*}{1-\pi_t}(\pi_t - \pi_*) + \sqrt{2K_\pi} \times (1 + (\pi_t - \pi_*)/\phi_{H_i}) \times N^\pi_{i,t+1};$$

where $K_H = (\sigma_{H_i}/1.3)^2$ and $K_\pi = (\sigma_{\pi_i}/1.3)^2$.

3) Use resilience, inflation values and other fundamentals to obtain a series of interest rates and spot rates from equations (16) and (22).

4) At every date t, using the two interest rates computed at t, and the bilateral volatility, we solve for a pair of strikes that would give the Put and Call deltas of 0.25 and -0.25. The formula used is the one in Garman-Kohlhagen (with maturity of 1 unit).

5) For the pair of strikes computed, we compute the values of Calls and Puts using equations (33) and (30) of the paper, again using monthly parameters and with maturity of 1 unit (which is a month in this case).

6) We use the computed prices of Puts and Calls to obtain implied volatilities using the Garman-Kohlhagen formula.

7) The risk reversal is computed as the difference in implied volatilities, and this is again in monthly units.

8) The desired moments are obtained and annualized.

Calibration for stocks. We will use additional parameters to perform the calibration for stocks, while holding fixed the parameters that generated Table III. The parameters are as follows: The value of H_{D*} is taken to imply a stock recovery rate of 66%, in line with Gabaix

Symbol	Description	Value
g_{D_i}	Dividend growth rate	2.5%
H_{D*}	Average dividend resilience	8.9%
$\phi_{H_{D_i}}$	Speed of mean reversion (dividend resilience)	13%
$\sigma_{H_{D_i}}$	Volatility of dividend resilience	3.11% (calibrated to this value)
σ_{D_i}	Volatility of dividends	8.43%

(2012). The value for $\phi_{H_{D_i}}$ is also according to Gabaix (2012). To obtain the value of σ_{D_i}, we first observe that the standard deviation of D_t/e_t is 11%, which implies a volatility of D_t, i.e. σ_{D_i} of 8.43%.

The value of $\sigma_{H_{D_i}}$ is set to generate stock return volatility of 18%. In practice, we fix a value of $\sigma_{H_{D_i}}$, run the simulations and obtain stock return volatility. If this volatility is below 18%, we raise $\sigma_{H_{D_i}}$ and continue by trial and error until we hit 18%. Due to the tractability of the model, this process does not take long at all. We obtain an annualized value of $\sigma_{H_{D_i}} = 3.11\%$ to obtain the desired stock return volatility. This annualized value is reported in the above table.

Once we fix a value for $\sigma_{H_{D_i}}$, we need to generate ε^{H_D}. On the one hand, since

$$\hat{H}_{D,t+1} = \frac{1 + H_{D*}}{1 + H_{D,t}} \exp(-\phi_{H_{D_i}})\hat{H}_{D,t} + \varepsilon_{t+1}^{H_D},$$

then, to respect the LG bound ($\hat{H}_{D,t+1} \geq -\phi_{H_{D_i}}$) and to ensure mean-reversion, we must have that $\varepsilon_{t+1}^{H_D} = 0$ whenever $\hat{H}_{D,t} = -\phi_{H_{D_i}}$. If not, there is a strictly positive probability that $\hat{H}_{D,t+1} < -\phi_{H_{D_i}}$, violating the LG bound. On the other hand, we must make sure that $\varepsilon_{t+1}^{H_D} - \varepsilon_{t+1}^{H}$ is uncorrelated with ε_{t+1}^{H}. Therefore, we use the following methodology.

Let $\varepsilon_{t+1}^{H_D} = \varepsilon_{t+1}^{H_D'} + \varepsilon_{t+1}^{H}$, where $\varepsilon_{t+1}^{H_D'}$ is uncorrelated with ε_{t+1}^{H}. Let σ_{H_i} and $\sigma_{H_{D_i}}$ be the standard deviations of ε_{t+1}^{H} and $\varepsilon_{t+1}^{H_D}$. Then $Cov(\varepsilon_{t+1}^{H_D}, \varepsilon_{t+1}^{H}) = \sigma_{H_i}^2$, and $corr(\varepsilon_{t+1}^{H_D}, \varepsilon_{t+1}^{H}) = \sigma_{H_i}/\sigma_{H_{D_i}} = \rho$. Thus, it is now sufficient to simulate $\varepsilon_{t+1}^{H_D}$ so that it has a correlation of ρ with ε_{t+1}^{H} and has the appropriate LG-implied vanishing properties.

The way to do this is by simulating

$$\varepsilon_{t+1}^{H_D} = \sqrt{2K_{H_D}} \left(1 + \frac{\hat{H}_{D,t}}{\phi_{H_D}}\right) \left[\rho Z_{t+1}^{H} + \sqrt{1-\rho^2} Z_{t+1}'\right],$$

where Z_{t+1}^{H} is the standard Gaussian random number drawn to simulate ε_{t+1}^{H}, (i.e. $\varepsilon_{t+1}^{H} = \sqrt{2K_H}(1 + \frac{\hat{H}_t}{\phi_{H_i}})Z_{t+1}^{H}$) and Z_{t+1}' is a set of standard Gaussian shocks such that $Cov(Z_{t+1}', Z_{t+1}^{H}) = 0$.

As before, K_H and K_{H_D} are set to generate appropriate values of the steady state volatility of ε^{H_D} and ε^{H}. In particular, $K_H = (\sigma_{H_i}/1.3)^2$ and $K_{H_D} = (\sigma_{H_{D_i}}/1.3)^2$.

We then use the equation

$$\hat{H}_{D,t+1} = \exp(-\phi_{H_{D_i}} + h_{D*}) \frac{\hat{H}_{D,t}}{1 + H_{D,t}} + \varepsilon_{t+1}^{H_D}$$

to generate the sequence of stock resiliences.

We also separately simulate a sequence of dividends to obtain prices and returns.

HML Calibration. Recall from the paper that we now alter the calibration done earlier by introducing a one-factor structure in currencies and stocks. We define the normalized resilience as:

$$h_{Dit} = \frac{1 + \frac{\hat{H}_{Dit}}{r_{Di} + \phi_{H_D i}}}{1 + \frac{\hat{H}_{it}}{r_{ei} + \phi_{H_i}}} \; ; \; h_{eit} = \frac{\hat{H}_{it}}{r_{ei} + \phi_{H_i}}.$$

We define $\varepsilon_{h_{eit}}$ as:

$$h_{ei,t+1} = \frac{1 + h_{ei*}}{1 + h_{eit}} \exp(-\phi_{H_i}) h_{eit} + \varepsilon_{ei,t+1}^{H}.$$

Now, we introduce aggregate shocks to the resilience innovations as follows (this automatically introduces a factor structure to normalized resiliences as well):

$$\varepsilon_{ei,t+1}^{H} = \beta_{eit} f_{e,t+1} + \eta_{ei,t+1},$$

where we set $\beta_{eit} = b(H_{at} - H_{it})(1 + \hat{H}_{it}/\phi_{H_i})$, and we set $f_{e,t+1} = Z_{t+1}$ where Z_{t+1} is a common innovation of i.i.d. standard Gaussian random variables and H_{at} is the average resilience across countries. We assume, as before, that the H_{i*} are the same across countries. The above formulation assures that $\beta_{eit} f_{e,t+1} = 0$ whenever $\hat{H}_{it} = -\phi_{H_i}$.

Now, since resiliences H are uncorrelated across countries, then, if we have a continuum of countries, H_{at} converges almost surely to H_*, and so $H_{at} - H_{it}$ converges almost surely to $\hat{H}_{it}$. The above reduces to

$$\beta_{eit} f_{e,t+1} = -b \hat{H}_{it} (1 + \hat{H}_{it}/\phi_{H_i}) Z_{t+1}.$$

Let $\eta_{ei,t+1} = c\sqrt{2K_H}(1 + \hat{H}_{it}/\phi_{H_i}) Z_{t+1}^{i}$, where Z_{t+1}^{i} is a shock to country i alone and c is a constant. Therefore,

$$\varepsilon_{ei,t+1}^{H} = -b\hat{H}_{it}(1 + \hat{H}_{it}/\phi_{H_i}) Z_{t+1} + c\sqrt{2K_H}(1 + \hat{H}_{it}/\phi_{H_i}) Z_{t+1}^{i},$$

where c is such that the volatility of $\varepsilon^H_{ei,t+1}$ matches the data. The above formulation ensures that $\varepsilon^H_{ei,t+1} = 0$ whenever $\hat{H}_{it} = -\phi_{H_i}$ and so the LG bounds are satisfied.

We use a value of b equal to 0.11 and c equal to 0.6. With these parameters, the share of idiosyncratic variance of resilience is 69%, in line with estimates.

To simulate the resilience innovations for stocks, we follow the below steps:

1) Recall that $\varepsilon^D_i = \varepsilon^{D'}_i + \varepsilon^H_{ei}$, where $\varepsilon^{D'}_i$ is orthogonal to ε^H_{ei}. We also have common innovations to $\varepsilon^{D'}_i$, so we simulate it as follows:

$$\varepsilon^{D'}_{i,t+1} = v\left[Z^D_{t+1} + Z^{\tilde{D}}_{i,t+1}\right],$$

where Z^D is a set of common innovations with unit variance and a correlation of $corr\left(f_{et}, f_{Dt}\right) = 0.65$ with Z. $Z^{\tilde{D}}$ is a set of idiosyncratic innovations with unit variance. v is such that $corr(\varepsilon^D_i, \varepsilon^H_{ei}) = \rho_1$, where $\rho_1 = \sigma_{He}/\sigma^D_{H_i}$. σ_{He} is the standard deviation of $\hat{H}_i$ and $\sigma^D_{H_i}$ is the standard deviation of $\hat{H}^D$. This ensures that $corr(\varepsilon^{D'}_i, \varepsilon^H_{ei}) = 0$.

2) From the simulated $\varepsilon^{D'}_i$, obtain $\varepsilon^D_i = \varepsilon^{D'}_i + \varepsilon^H_{ei}$, and obtain the time series of resilience for each country.

3) Separately simulate dividends to obtain prices and returns.

To obtain correlations, we perform the following steps:

1) Using the methodology outlined above, simulate the time series for resilience.

2) From the obtained time series, obtain exchange rates for both countries and the resulting (log of) bilateral exchange rate, and also obtain risk reversals.

3) Sort the real interest rate r_i into terciles, and obtain a corresponding series of "high shocks", "medium shocks" and "low shocks" of Z.

4) Look at the correlations of each tercile of Z with the corresponding bilateral exchange rate and the change in risk reversals at those indices.

Annualization Methodology. This section details the techniques used to convert numbers from both the calibration and the data to annualized figures. Note that all values reported are annualized. The "annualization factor" (Ann. Factor) is the number we use to multiply the value directly obtained (from data/calibration) to transform it into annual terms.

Moments	Ann. Factor (Data)	Ann. Factor(Calibration)
$\mathbb{E}(\|RR\|)$	1	$\sqrt{12}$
Std Dev(RR)	1	$\sqrt{12}$
Std Dev$(\Delta(RR))$	$\sqrt{12}$	$\sqrt{12} \times \sqrt{12}$
Std Dev(r_{it})(real)	1	12
Std Dev(r_{it})(nominal)	1	12
Std Dev$(\Delta(r_{it} - r_{jt}))$(real)	$\sqrt{12}$	$12 \times \sqrt{12}$
Std Dev$(\Delta(r_{it} - r_{jt}))$(nominal)	$\sqrt{12}$	$12 \times \sqrt{12}$
Std Dev$(\Delta \ln(e_{it}/e_{jt}))$	$\sqrt{12}$	$\sqrt{12}$
Carry Trade Return	1	12

Miscellaneous Notes. *Derivation of the carry trade return.* Recall that, for a process $X_{t+\Delta t} = (1 - \phi_X \Delta t) X_t + \sqrt{\Delta t} \sigma_\varepsilon \varepsilon_{t+1}$, the dispersion (in the limit of small time intervals) is $std(X_t) = \sigma_\varepsilon / \sqrt{2\phi_X}$. Hence, the standard deviation of the steady-state distribution of $\widehat{H}_t$ is:

$$std\left(\widehat{H}_t\right) = \frac{\sigma_{H_i}}{\sqrt{2\phi_{H_i}}}.$$

Note that these results hold exactly in the limit of small shocks, i.e. $std\left(\widehat{H}_t\right) / \left(\frac{\sigma_{H_i}}{\sqrt{2\phi_{H_i}}}\right) \to 1$ as $\sigma_{H_i} \to 0$.

Recall that, for a Gaussian variable $X \sim N(0, \sigma^2)$, $\mathbb{E}[X \mid X > 0] = -\mathbb{E}[X \mid X < 0] = \sqrt{\frac{2}{\pi}}\sigma$. As $\widehat{H}_t$ is approximately Gaussian (when the process goes to continuous time and $\widehat{H}_t$ is small, it is approximately an Ornstein-Uhlenbeck), we find that the carry trade return is

$$X^e = 2\sqrt{\frac{2}{\pi}} std\left(\widehat{H}_t\right) = \frac{2}{\sqrt{\pi \phi_{H_i}}} \sigma_{H_i}.$$

Details for the interpretation of the calibration in section V.A.

The calculations reported in the text come from:

$$\nu_{F_{it}} = \frac{\upsilon_{H_{it} - H_{jt}}}{\mathbb{E}[pB^{-\gamma}]} \frac{1}{\sqrt{2}},$$

where the $\sqrt{2}$ comes from the assumption of independent movements in H_{it} (it is easy to generalize) and

$$v_{p_t} = \frac{v_{H_{it}-H_{jt}}}{\mathbb{E}_t\left[B_{t+1}^{-\gamma}\left|F_{it+1}-F_{jt+1}\right|\right]}.$$

XIII. Allowing for Different Time Scales in Resilience

XIII.A. The Model with Two Time Scales for Resilience

There are different time scales in most measures of risk. For instance, the VIX index (of stock market volatility) features low-frequency epochs of low vs. high volatility (e.g. pre-2008 vs. post-2008), and high-frequency variations (e.g. temporary rises in volatility level, such as after bad macroeconomic news). To capture them, we propose an extension of the model with two time scales of resilience. We decompose resilience H_{it} as:

$$H_{it} = H_{i*} + \sum_{s=1}^{2} \widehat{H}_{ist}, \tag{78}$$

where $\widehat{H}_{ist}$ for $s = 1, 2$ are the two transitory components of resilience, one slow-moving, one fast-moving (s indicates the time scale). Their laws of motion are:

$$\widehat{H}_{is,t+1} = \frac{1 + H_{i*}}{1 + H_{it}} \exp(-\phi_{H_{is}}) \widehat{H}_{ist} + \varepsilon^{H}_{is,t+1}, \tag{79}$$

where $\mathbb{E}_t\left[\varepsilon^{H}_{is,t+1}\right] = \mathbb{E}^{D}_t\left[\varepsilon^{H}_{i,t+1}\right] = 0$.

We assume that $\phi_{H_{i1}} < \phi_{H_{i2}}$, so that $\widehat{H}_{i1t}$ is the slow component of resilience, and $\widehat{H}_{i2t}$ is its fast component. For instance, $\widehat{H}_{i1t}$ can capture the movements of resilience happening at business cycle frequency, and $\widehat{H}_{i2t}$ the movements happening at higher (e.g. monthly) frequency.

All our results are easily adapted to this multi-scale setup.

Proposition 15 (Modifications when there are two time scales for resilience). *The theory's basic formulas carry over to the setup with two time scales for resilience, with minor modifications as follows. In the limit of small time intervals, the exchange rate (16), interest rate (22),* *and the Fama coefficients become:*

$$e_{it} = \frac{\omega_{it}}{r_{ei}} \left(1 + \sum_{s=1}^{2} \frac{\widehat{H}_{ist}}{r_{ei} + \phi_{H_{is}}} \right), \tag{80}$$

$$r_{it} = r_{ei} - \lambda - r_{ei} \frac{\sum_{s=1}^{2} \frac{\widehat{H}_{ist}}{r_{ei}+\phi_{H_{is}}}}{1 + \sum_{s=1}^{2} \frac{\widehat{H}_{ist}}{r_{ei}+\phi_{H_{is}}}}, \tag{81}$$

$$\beta^{ND} = -\frac{\sum_{s=1}^{2} \frac{\phi_{H_{is}}}{\left(r_{ei}+\phi_{H_{is}}\right)^2} \operatorname{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst}\right)}{\sum_{s=1}^{2} \frac{r_{ei}}{\left(r_{ei}+\phi_{H_{is}}\right)^2} \operatorname{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst}\right)}, \tag{82}$$

while the expected return of the carry trade (25), and the risk-reversal (34) are unchanged.

Proof of Proposition 15. The value of e_{it} is derived as in Gabaix (2012, Proposition 12, Online Appendix). The proof for the interest rate is as above. For completeness, we state the value of the nominal Fama coefficient with two time scales.

$$\widetilde{\beta}^{ND} = \frac{-\sum_s \frac{\phi_{H_s} r_{ei}}{(r_{ei}+\phi_{H_s})^2} \operatorname{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst}\right) + \operatorname{Var}\left(\pi_{it} - \pi_{jt}\right)}{\sum_s \left(\frac{r_{ei}}{r_{ei}+\phi_{H_s}}\right)^2 \operatorname{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst}\right) + \operatorname{Var}\left(\pi_{it} - \pi_{jt}\right)}. \tag{83}$$

Proof sketch. We proceed as in the original proof of Proposition 5. Up to second order terms,

$$\begin{aligned} \mathbb{E}_t^{ND}\left[\frac{\widetilde{e}_{i,t+1-}\widetilde{e}_{it}}{\widetilde{e}_{it}}\right] &= -\sum_s \frac{\phi_{H_s}}{r_{ei}+\phi_{H_s}} \widehat{H}_{ist} - \pi_{it} + K_i \\ \widetilde{r}_{it} &= -\sum_s \frac{r_{ei}\widehat{H}_{ist}}{r_{ei}+\phi_{H_s}} + \pi_{it} + K_i', \end{aligned}$$

so

$$\begin{aligned} \widetilde{\beta}^{ND} &= \frac{-\operatorname{Cov}\left(\mathbb{E}_t^{ND}\left[\frac{\widetilde{e}_{i,t+1-}\widetilde{e}_{it}}{\widetilde{e}_{it}}\right] - \mathbb{E}_t^{ND}\left[\frac{\widetilde{e}_{j,t+1-}\widetilde{e}_{jt}}{\widetilde{e}_{jt}}\right], \widetilde{r}_{it} - \widetilde{r}_{jt}\right)}{\operatorname{Var}\left(\widetilde{r}_{it} - \widetilde{r}_{jt}\right)} \\ &= \frac{-\sum_s \frac{\phi_{H_s} r_{ei}}{(r_{ei}+\phi_{H_s})^2} \operatorname{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst}\right) + \operatorname{Var}\left(\pi_{it} - \pi_{jt}\right)}{\sum_s \left(\frac{r_{ei}}{r_{ei}+\phi_{H_s}}\right)^2 \operatorname{Var}\left(\widehat{H}_{ist} - \widehat{H}_{jst}\right) + \operatorname{Var}\left(\pi_{it} - \pi_{jt}\right)}. \end{aligned}$$

XIII.B. Calibration with Two Time Scales

XIII.B.1. Parameter Values

We present a calibration of the model. Our data is nominal; we therefore use the extension to a nominal setup. In order to match the observed autocorrelation structure of risk reversals, we use the two time scales model.[33] Up to second order terms, the differences in resiliences $H_{it} - H_{jt}$ are a sufficient statistic for the quantities of interest (which are bilateral, e.g. $\ln\left(\frac{e_{it}}{e_{jt}}\right)$, $r_{it} - r_{jt}$, etc.). Hence we specify parameters for the differences in resilience – rather than the absolute resilience H_{it} and H_{jt} and their correlation. These differences in resiliences could come from various combinations of shocks to the world disaster probability p_t, severity B_{t+1} and country-specific factors $F_{i,t+1}$. We discuss these later.

Table I summarizes the main inputs of the calibration. The justification is as follows.

Exchange rate and interest rate. We call Δ the time-difference operator, $\Delta x_t = x_t - x_{t-1}$, and $\sigma_x = stdev\left(\Delta x_t\right)$ the volatility of a variable x_t. For two countries, define the volatility of the bilateral exchange rate as $\sigma_e^{bil} = stdev\left(\Delta \ln \frac{e_{it}}{e_{jt}}\right)$ and the volatility of the difference in interest rates $\sigma_r^{bil} = stdev\left(\Delta\left(r_{it} - r_{jt}\right)\right)$. Equations (16) and (22) give $\sigma_r^{bil} = r_{ei}\sigma_e^{bil}$.[34] The above equation constrains our calibration.[35] We will match $\sigma_e^{bil} \simeq 11\%$. In the sample, the volatility of the nominal bilateral interest rate is $\sigma_{\widetilde{r}}^{bil} \simeq 0.7\%$. We therefore set $r_{ei} = 6\%$.[36]

The speeds of mean-reversion ϕ_{H_1}, ϕ_{H_2} and the variances of $H_{ist} - H_{jst}$ are chosen to roughly match the level and volatility of the risk reversals, their autocorrelations at different lags, as well as the volatility of the exchange rate. For the speed of mean-reversion of the slow component we take $\phi_{H_1} = 0.1$, which gives a half-life of $\ln 2/\phi_{H_i} = 7$ years, in line with estimates from

[33] We also performed a calibration with one time scale (available upon request). That calibration is essentially equally successful, except that it does not match the high-frequency movements of the risk reversals (volatility and autocorrelation), and it generates a close to perfect correlation between innovations to the exchange rate and innovations to resilience.

[34] To keep the model parsimonious, we assume no default risk on debt. This is the cleanest assumption for developed countries. Of course, in many cases (e.g. when pricing sovereign debt), default risk can be added without changing anything about the exchange rate.

[35] This expression also holds in the two time scales model (Proposition 15).

[36] The growth rate of productivity g_{ω_i} is irrelevant in practice, but for completeness we propose a specific value. We choose the growth rates so that in normal times consumption of non-tradables grows at a rate $g_c = 2.5\%$. We set $g_{\omega_i} = 0$, but results are not sensitive to the choice of this parameter. We make sure that the riskless domestic short-term rate is on average around 0.5%, which pins down the rate of time preference ρ. The parameter $r_{ei} = R + \lambda - g_{\omega_i} - h_{i*}$ is driven in the model by deeper combinations of underlying factors $p_t, B_{t+1}^{-\gamma}$, and F_{it} but mainly three parameters govern the key statistics that we present in Table III. We take $\lambda = 5.5\%$, which generates a real interest rate of $r_{ei} - \lambda = 0.5\%$. The underlying rate of time preference ρ is calibrated to match the value of r_{ei}. For simplicity, we take the recovery rate of productivity to be the average recovery rate of consumption, $F_{i*} = \mathbb{E}\left[B^{-\gamma}\right]^{1/\gamma}$. Hence we get the rate of time preference $\rho = 5.9\%$.

the exchange rate predictability literature (Rogoff 1996). For the speed of mean-reversion of the fast component, ϕ_{H_2}, we target the autocorrelations of the RR (Table VIII): the RR has a fast mean-reverting component with a half-life of about 4 months. We choose the volatilities of resilience to target the volatilities of RR and the exchange rate reported in Table VIII. For parsimony, we take the innovations to the fast and slow component ($H_{i1t} - H_{j1t}$ and $H_{i2t} - H_{j2t}$) to be uncorrelated.

TABLE VII: Key Parameter Inputs.

Exchange rate discount rate	$r_{ei} = 6\%$
Volatility of H	$\sigma_{H_{i1t}-H_{j1t}} = 1.74\%, \sigma_{H_{i2t}-H_{j2t}} = 3.97\%$
Mean reversion of resilience	$\phi_{H_1} = 10\%, \phi_{H_2} = 250\%$
Inflation: volatility and speed of mean-reversion	$\sigma_{\pi_i} = 0.6\%, \phi_{\pi_i} = 30\%$

Notes. This table reports the coefficients used in the model. σ_X is the average volatility, and ϕ_X is the speed of mean-reversion. The time unit is a year (the model is simulated at the monthly frequency, but for readability the numbers reported are all annualized).

Inflation. Data (e.g. on currency options) are nominal, and the essence of our model is real. We pick inflation parameters that are broadly in line with averages in our sample.

Carry trade returns. We proceed as is usual in the carry trade literature, see e.g. Farhi et al. (2015). However, to better capture disaster risk, we sort on risk reversals rather than interest rates. We divide countries into two equally-sized bins of resilience; the more risky countries are those in the bottom half according to resilience, whereas the less risky ones are in the top half. We define the carry trade as going long \$1 in the equally-weighted portfolio of risky countries and going short \$1 in the equally-weighted portfolio of safer countries (the ones with high $\widehat{H}_t$).

Interpreting resilience processes in terms of deeper disaster parameters. Resilience differentials are sufficient statistics for the calibration. We now discuss how their variations are related to deeper disaster parameters.

We take numbers from Barro and Ursua (2008).[37] The average probability of disasters is $\mathbb{E}[p] = 3.6\%$. An important parameter in the calibration is the risk-adjusted probability

[37] See also Gabaix (2012), Gourio (2012) and Wachter (2013) for related calibrations of disaster models in closed economies.

of disasters $\mathbb{E}\left[pB^{-\gamma}\right]$. Disasters are overweighted compared to their physical probability by a factor $\mathbb{E}\left[B^{-\gamma}\right]$. This factor is very sensitive to the severity of disasters and to the coefficient of relative risk aversion. We take $\gamma = 4$, which yields $\mathbb{E}\left[B^{-\gamma}\right]^{1/\gamma} = 0.66$. Hence, the "risk neutral" (i.e. risk-adjusted) probability of disasters equals $\mathbb{E}\left[pB^{-\gamma}\right] = 19.2\%$. Note that even though $\mathbb{E}\left[B^{-\gamma}\right]^{1/\gamma} = 0.66$, which corresponds to a risk-adjusted average size of disaster of 34%, the median disaster in Barro and Ursua (2008) is much smaller: because of risk aversion, the small possibility of a large disaster matters a lot.

This calibration, strictly speaking, relies on a stark idealization in which consumption is permanently affected after disasters. In practice, there is a partial recovery from disasters (Barro and Ursua 2008). For a given γ, that lowers the disaster risk premium (Gourio 2008). However, this can be remedied by increasing γ slightly. Indeed, Barro and Jin (2011) find an empirical power-law distribution of disaster sizes, so that a moderate γ can generate a very large (infinite for a large enough γ) risk premium. In addition, for our purposes, the idealization of a permanent disaster seems like a good compromise between parsimony and realism.

Our calibration only requires the law of motion of the resilience differential, $H_{it} - H_{jt} = p_t \mathbb{E}_t\left[B_{t+1}^{-\gamma}\left(F_{i,t+1} - F_{j,t+1}\right)\right]$. Results of the calibration do not depend on whether the shocks come from movements in p_t, $B_{t+1}^{-\gamma}$ or $F_{i,t+1} - F_{j,t+1}$.[38,39]

To interpret the volatility of $H_{it} - H_{jt} = p_t \mathbb{E}_t\left[B_{t+1}^{-\gamma}\left(F_{i,t+1} - F_{j,t+1}\right)\right]$, we present the standard deviation of changes in $H_{it} - H_{jt}$ over a horizon of one year. Generally, call this object v_{X_t} for the standard deviation of a variable X_t at a one-year horizon: $v_{X_t} = stdev\left(X_{t+1\text{year}} - X_t\right)$. We take some polar cases. If the innovations come entirely from idiosyncratic movements of F_{it} (keeping p_t and $B_{t+1}^{-\gamma}$ constant at $\mathbb{E}[p]$ and $\mathbb{E}\left[B^{-\gamma}\right]$), then $v_{F_{it}} = 12.9\%$. This is broadly in line with Gabaix (2012), who argues that a one-year horizon volatility $\nu_{F_{it}} \simeq 10\%$ for the resilience of the aggregate stock market is plausible and does not violate variance bounds from historical data: hence, that calibration seems acceptable too. Conversely, suppose that innovations to the resilience differential come entirely from movements in p_t (keeping $\mathbb{E}_t\left[B_{t+1}^{-\gamma}\left(F_{i,t+1} - F_{j,t+1}\right)\right]$ constant). With fixed values of F_{it}, e.g. $\left|F_{i,t+1} - F_{j,t+1}\right| = 0.4$ (similar to the numbers above) we get $v_{p_t} = 1.6\%$. This is of the same order of magnitude as the calibration in Wachter (2013), which uses $v_{p_t} \simeq 1.1\%$.

[38] For instance, movements in p_t generate a positive covariance between the innovations of H_{it} and H_{jt}, while idiosyncratic movements of $F_{i,t+1}$ and $F_{j,t+1}$ generate a 0 covariance. For the calibration, the covariance between the innovations to H_{it} and H_{jt} does not matter per se – only the variance of the innovations to $(H_{it} - H_{jt})$.

[39] This is true up to second order terms. We verify numerically that this is a good approximation.

XIII.B.2. Implications

Table VIII presents the main results from the calibration in Table VII.

TABLE VIII: Moments: Empirical and in the Model

Moments	Data	Calibration
Std Dev$(\Delta \ln \tilde{e}_{ijt})$	12.35%	11.4%
Carry Trade Return	3.44%	4.6%
Mean$(\|RR\|)$	1.31%	1.23%
Std Dev(RR)	1.24%	1.26%
Std Dev(ΔRR_{ijt})	2.60%	1.8%
Std Dev$(\tilde{r}_{it})$	1.38%	1.12%
Std Dev$(\Delta(\tilde{r}_{it} - \tilde{r}_{jt}))$	0.71%	1.17%
Corr$(\Delta \ln \tilde{e}_{ijt}, \Delta RR_{ijt})$	-0.58	-0.49
Corr$(\ln \tilde{e}_{ij,t+1}, \ln \tilde{e}_{ijt})$	0.88	0.98
Corr$(\Delta \ln \tilde{e}_{ij,t+1}, \Delta \ln \tilde{e}_{ijt})$	-0.13	-0.01
Corr$(\tilde{r}_{it} - \tilde{r}_{jt}, RR_{ijt})$	0.55	0.57
$A(1)$	0.77	0.90
$A(6)$	0.45	0.61
$A(12)$	0.31	0.45

Notes. The table reports the moments generated by the model, using the inputs from Table VII. The risk reversal RR_{ijt} is defined as the implied volatility of an out-of-the-money put minus the implied volatility of an out-of-the-money call, all at 25-delta. A high RR_{ijt} means that the price of protection from depreciation of currency i (against country j) is high. $\tilde{r}_{it}$ is the nominal interest rate. We define $\widetilde{e}_{ijt} = \frac{\widetilde{e}_{it}}{\widetilde{e}_{jt}}$, the nominal bilateral exchange rate between countries i and j: a high $\widetilde{e}_{ijt}$ means that currency i appreciates. Carry trade returns are the returns from a long-short portfolio going \$1 long (short) an equally-sized portfolio of high (low) RR countries. $A(k)$ is the autocorrelation of RR_{ijt} at lag k. $\Delta X_t = X_t - X_{t-1}$ is the time-difference, annualized. Time unit is a year (the model is estimated and simulated at a monthly frequency, but for readability the numbers reported above are all annualized).

The model hits the volatility of the bilateral exchange rate, i.e. generates the right amount of "excess volatility" in exchange rates. The model also roughly matches (and slightly undershoots) the size of disaster risk as measured by the average size of risk reversals.[40,41] At the same time, the model generates a moderate volatility of the interest rate, as in the data.

We showed earlier that in the model countries with high risk reversals have high interest rates and that increases in risk reversals are associated with depreciations of the exchange rate. The calibration shows that these predictions hold not just qualitatively, but also quantitatively: Table III reports the calibrated values of $\mathrm{Cov}\left(\widetilde{r}_{it} - \widetilde{r}_{jt}, RR_t\right)$ and $\mathrm{corr}(\Delta \ln \tilde{e}_{ijt}, \Delta RR_{ijt})$ and shows that they broadly match their empirical counterparts.

The carry trade generated by the model gives average returns in line with the empirical evidence (see Farhi et al. 2015 for more variants of the carry trade). Investing in countries with high risk reversals generates high expected returns. Indeed, the expected return of the carry trade (given positive RR) is about 3% per annum. Finally, the model generates the Fama coefficient of $\beta^{ND} = -0.40$, in line with estimates of the literature cited above.

We conclude that the disaster model can be made quantitatively broadly congruent with the empirical facts.

References for the online appendix

Ang, Andrew, and Joseph Chen, 2010. "Yield Curve Predictors of Foreign Exchange Returns." Working Paper, Columbia.

Lustig, Hanno, Andreas Stathopoulos and Adrien Verdelhan. 2014. "The Term Structure of Currency Carry Trade Risk Premia." Working Paper, UCLA.

[40] We take the mean of the absolute values of risk reversals because, by symmetry, the mean of risk reversals is 0.

[41] If we increased the value of $\mathbb{E}\left[pB^{-\gamma}\right]$, for example by slightly increasing γ, we could match better the average value of the RR and other moments, without requiring a larger volatility of the relative prospective recovery rate $F_{it} - F_{jt}$ or of the probability of disaster p_t. We thought it was more parsimonious to stick to the numbers from the previous literature for $\mathbb{E}\left[pB^{-\gamma}\right]$.